

# The Role of Labelled Deductive Systems in a Formal System for Defeasible Argumentation

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## Abstract

There has been an increasing demand of a variety of logical systems, prompted by applications of logic in AI, logic programming and other related areas. Labelled Deductive Systems (LDS) [Gab96] were developed as a flexible methodology to formalize such a kind of complex logical systems.

In the last decade, defeasible argumentation [SL92, PV99, Ver96, BDKT97] has proven to be a confluence point for many approaches to formalizing commonsense reasoning. Different formalisms have been developed, many of them sharing common features.

This paper outlines an *argumentative* LDS, in which the main issues concerning defeasible argumentation are captured within a unified logical framework. The proposed framework is defined in two stages. First, defeasible inference will be formalized by characterizing a *defeasible* LDS. That system will be then extended in order to obtain an *argumentative* LDS.

## 1 Introduction and motivations

From the early '90s there have been several attempts to find an unified framework for non monotonic reasoning (NMR). Recent work [PV99, BDKT97] has shown that *defeasible argumentation* constitutes a confluence point for characterizing many different approaches to NMR. Nevertheless, the evolution of different, alternative formalisms for defeasible argumentation has resulted in a number of models which share some common issues (the notion of argument, attack between arguments, defeat, justifying arguments, etc.).

We are concerned in studying these aspects within a logical system, using the MTDR framework as a basis [SL92, SCG94].<sup>1</sup> In our logical framework we want to capture the main issues involved in defeasible argumentation by specifying a suitable underlying logical language and its associated inference rules. In order to accomplish this goal we will make use of the so-called *labelled deductive systems* [Gab96]. Labelled Deductive Systems (LDS for short) offer an attractive approach to formalizing complex systems, since they allow to characterize the different components involved in a logical system by using *labels*. As an ultimate result we want to be able to define an *argumentative LDS*.

This paper is structured as follows. In section 2 we will briefly outline our approach. Then in section 3 we will define a knowledge representation language  $\mathcal{L}_{KR}$  as well as a labelling language  $\mathcal{L}_{Labels}$ , both of which will be used as the object language in our formalization. Section 4 presents

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<sup>1</sup>In what follows, we assume that the reader is familiar with defeasible argumentation under the MTDR framework (see [SL92, SCG94, Che96] for details).

a characterization of the inference rules needed for capturing *defeasible inference*. Next, in section 5 we will discuss some of the features needed for extending defeasible inference to define an argumentative LDS. Finally, in section 6 we will discuss the most important conclusions that have been obtained.

## 2 An outline of our approach

When modelling the behavior of an intelligent agent in an argumentative setting, it is common to provide that agent with the usual features in knowledge-base systems, namely a *knowledge base* and an *inference engine*. In the case of MTDR, the knowledge base involves incomplete and potentially inconsistent information. In order to infer a *justified conclusion*  $h$ , the MTDR inference procedure involves two levels. First, the agent must be able to find a tentative piece or reasoning, viz. an *argument*  $\mathcal{A}$ , supporting  $h$ . Second, the agent should determine that the argument  $\langle \mathcal{A}, h \rangle$  is a *justification*, by being ultimately preferred over any other argument which *defeats*  $\langle \mathcal{A}, h \rangle$ . Determining whether an argument is a justification involves a recursive procedure, in which arguments, defeaters, defeaters of defeaters, and so on, must be taken into account.

We want to capture both defeasible knowledge representation and argumentative inference within a *logical system*  $(\Gamma, \vdash)$ , in which  $\Gamma$  represents the agent's knowledge base and  $\vdash$  stands for a consequence relation. Traditionally, a logical system  $(\Gamma, \vdash)$  allows the inference of new wffs from those available in  $\Gamma$  using the rules of inference that characterize the notion of logical consequence  $\vdash$ . In order to formalize defeasible argumentation within a logical system (which involves the well-known problems associated with non monotonic reasoning), we will make use of a powerful methodology, called *labelled deductive systems* [Gab96]. In LDS, the usual notion of formula is replaced by the notion of *labelled formula* expressed as *Label*: $\alpha$ , where *Label* represents a label associated with the wff  $\alpha$ . Inference rules that characterize the notion of consequence in an LDS will be augmented in order to include labels.

As pointed out before, the agent's knowledge base  $\Gamma$  will contain incomplete, potentially inconsistent information. Hence we will provide our intelligent agent with a *defeasible LDS*  $(\Gamma, \vdash_{Arg})$  which will allow him to arrive to tentative conclusions. Those conclusions will correspond to labelled formulas *label*:*wff*, where *label* will be associated with the notion of *argument* (as defined originally in [SL92]).

In other words, the consequence relation  $\vdash_{Arg}$  will allow our agent to derive labelled wffs having the form *argument*:*conclusion*, where *argument* will be a wff in a *labelling language*  $\mathcal{L}_{Labels}$ , and *conclusion* will be a ground literal in a *knowledge representation language*  $\mathcal{L}_{KR}$ . In this setting, an argument will represent a tentative proof our intelligent agent can build in order to support  $p$ . However, our agent could also be able to build an argument supporting  $\neg p$  from the knowledge available in  $\Gamma$ . This leads to a comparative, recursive analysis of arguments in which a given argument should be compared with all those *counter-arguments* which may defeat it. To model this process, our approach will consist in extending the consequence relationship  $\vdash_{Arg}$ , in order to obtain a new consequence relationship  $\vdash_{\mathcal{T}}$ . Those wffs derivable from  $\Gamma$  via  $\vdash_{\mathcal{T}}$  will correspond to *acceptable dialectical trees* [SCG94] for a given argument. These new labelled wffs will have the form *dialectical tree*:*conclusion*.

The elements in our ontology are summarized in figure 1. The lower level represents the knowledge base  $\Gamma$ , from which our agent will be able to build arguments using a defeasible LDS. In

Argumentative LDS $(\Gamma, \vdash_{\mathcal{A}})$
Defeasible LDS $(\Gamma, \vdash_{Arg})$
Knowledge Base $\Gamma$

Figure 1: Formalizing argumentation using LDS

order to decide whether an *argument* is *justified* or not, a comparison among arguments is needed, which results in computing an acceptable dialectical tree. This will be captured by the consequence relation  $\vdash_{\mathcal{A}}$  within an *argumentative* LDS.<sup>2</sup>

### 3 Knowledge Representation

In this section we will introduce a *knowledge representation language*  $\mathcal{L}_{KR}$  for performing defeasible inference, together with a *labelling language*  $\mathcal{L}_{Labels}$ . These languages will be used to define the object language  $\mathcal{L}_{Arg}$  to be used in our defeasible LDS. Our approach is based on the one introduced by Modgil in [Mod98], adapted for our purposes.

Following Gabbay's terminology [Gab96], the basic information units in  $\mathcal{L}_{Arg}$  will be called *declarative units*, having the form *Label:wff*. In our approach we will restrict wffs in labelled formulas to ground literals. As we will see along this section, a ground literal can be understood as *conclusion* of an *argument*, which will be defined by the label.

A label in a formula  $L:\alpha$  will provide three elements which are convenient to take into account when formalizing defeasible argumentation, namely:

1. For every declarative unit  $L:\alpha$  the label  $L$  will distinguish whether that declarative unit corresponds to *defeasible* or *non-defeasible* information.
2. The label  $L$  will also provide an *unique name* to identify a wff in the knowledge base  $\Gamma$ .
3. When performing the inference of a declarative unit  $L:\alpha$  from a set  $\Gamma$  of declarative units, the label  $L$  will provide a *trace* of the wffs needed in the derivation of  $L:\alpha$  from  $\Gamma$ .

Wffs in our knowledge representation language  $\mathcal{L}_{KR}$  will be a subset of a classic propositional language  $\mathcal{L}$ , restricted to *implications* and *facts*. A modality (label) will be attached to both kinds of wffs: *defeasible* and *non-defeasible*. Formally:

**DEFINITION 3.1** (*Language  $\mathcal{L}_{KR}$* ). The knowledge representation language  $\mathcal{L}_{KR}$  will be composed of

1. A countable set of *propositional atoms*, possibly subindicated. We will denote propositional atoms with lowercase letters. Example:  $a, b, c, d, e, \dots, a_1, a_2, a_3$  are propositional atoms.
2. Logical connectives  $\wedge, \neg$  and  $\leftarrow$ .

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<sup>2</sup>For reasons of space, this paper focuses on the defeasible LDS, discussing only some of the aspects involved in defining the argumentative LDS.

The set of all atoms in  $\mathcal{L}_{KR}$  will be denoted as  $Atoms(\mathcal{L}_{KR})$ .  $\square$

DEFINITION 3.2 (*Wffs in  $\mathcal{L}_{KR}$* ). Wffs in  $\mathcal{L}_{KR}$  will be defined as follows:

1. If  $\alpha$  is an atom in  $\mathcal{L}_{KR}$ , then  $\alpha$  and  $\neg\alpha$  are wffs called *literals* in  $\mathcal{L}_{KR}$ . We will denote as  $Lit(\mathcal{L}_{KR})$  the set of all literals in  $\mathcal{L}_{KR}$ .
2. If  $\alpha_1, \dots, \alpha_k, \beta$  are literals in  $\mathcal{L}_{KR}$ , then  $\beta \leftarrow \alpha_1, \dots, \alpha_k$  is a wff in  $\mathcal{L}_{KR}$ , called *implication*.<sup>3</sup>

Note that implications are written as logic programming clauses. More precisely, our formalization will follow the approach used in *defeasible* logic programming [Gar97] for representing knowledge. We will denote as  $Wffs(\mathcal{L}_{KR})$  the set of all wffs in  $\mathcal{L}_{KR}$ .  $\square$

For the sake of simplicity, when referring to the language  $\mathcal{L}_{KR}$  the following conventions will be used: Greek lowercase letters  $\alpha, \beta, \gamma$  will refer to any wff in  $\mathcal{L}_{KR}$ . Greek uppercase letters  $\Upsilon, \Phi, \Gamma$  will refer to a *set* of wffs in  $\mathcal{L}_{KR}$ . The conjunction  $\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_k$  will be simply written as  $\alpha_1, \alpha_2, \dots, \alpha_k$ .

Separately we will define a labelling language  $\mathcal{L}_{Labels}$ , which will be associated with wffs in  $\mathcal{L}_{KR}$ .

DEFINITION 3.3 (*Labelling constants*). A set *Labels* of *labelling constants* will include constant names having the form  $n_i$  or  $d_i$ . Thus *Labels* constitutes the enumerable set:

$$Labels = \{ n_1, n_2, \dots, d_1, d_2, \dots \}$$

A *set* of labelling constants will be denoted as  $L_1, L_2, \dots, L_k$ .  $\square$

DEFINITION 3.4 (*Labelling language  $\mathcal{L}_{Labels}$* ). A label  $L$  in our labelling language  $\mathcal{L}_{Labels}$  can be either an *argument label* or a *dialectical label*, defined as follows:

1. An *argument label* will be a tuple  $\langle L_i, \Phi \rangle$  where  $L_i \subseteq Labels$ , and  $\Phi \subseteq \wp(Wffs(\mathcal{L}_{KR}))$ . The set of all argument labels that can be defined from *Labels* and  $\mathcal{L}_{KR}$  will be denoted as  $ArgumL(Labels, \mathcal{L}_{KR})$ .
2. If  $\langle L_i, \Phi \rangle$  is an argument label, then

$$\begin{aligned} & \mathcal{T}_j^U(\langle L_i, \Phi \rangle), \text{ with } j \in \text{Nat} \\ & \mathcal{T}_k^D(\langle L_i, \Phi \rangle), \text{ with } k \in \text{Nat} \end{aligned}$$

are *dialectical labels* in  $\mathcal{L}_{Labels}$ . For the sake of simplicity, we will write  $\mathcal{T}_k^D$  to denote a generic dialectical label  $\mathcal{T}_k^D(\langle L_i, \Phi \rangle)$ , for a given argument label  $\langle L_i, \Phi \rangle$ . We will also write  $\mathcal{T}_k$  to denote either the functor  $\mathcal{T}_k^D$  or the functor  $\mathcal{T}_k^U$

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<sup>3</sup>We introduce this term in order to denote a relationship between a consequent (literal) and an antecedent (set of literals). This relationship can be “weak” (defeasible) or “strong” (non-defeasible). The corresponding semantics will be characterized in terms of natural deduction rules to be introduced later.

3. If  $T_1, \dots, T_k$  are dialectical labels, then  $T_n^U(T_1, \dots, T_k)$ , with  $j \in \text{Nat}$ ,  $n \notin \{1 \dots k\}$ , and  $T_m^D(T_1, \dots, T_k)$ , with  $k \in \text{Nat}$ ,  $m \notin \{1 \dots k\}$  will also be dialectical labels in  $\mathcal{L}_{Labels}$ . The set of all dialectical labels that can be defined from  $Labels$  and  $\mathcal{L}_{KR}$  will be denoted as  $DialectL(Labels, \mathcal{L}_{KR})$

The set of all wffs in  $\mathcal{L}_{Labels}$  will be the set  $ArgumL(Labels, \mathcal{L}_{KR}) \cup DialectL(Labels, \mathcal{L}_{KR})$ .

□

It should be noted that in order to characterize a *defeasible* LDS argument labels will suffice; dialectical labels will be used in characterizing an *argumentative* LDS, as discussed in section 5.

**DEFINITION 3.5** (*Defeasible Labelled Language  $\mathcal{L}_{Arg}$* ). If  $\mathcal{L}_{Labels}$  is a labelling language, and  $\mathcal{L}_{KR}$  is a knowledge representation language, then the *defeasible labelled language*, denoted  $\mathcal{L}_{Arg}$ , is defined as  $\mathcal{L}_{Arg} = (\mathcal{L}_{Labels}, \mathcal{L}_{KR})$  □

**DEFINITION 3.6** (*Declarative Unit*). Given a language  $\mathcal{L}_{Arg}$ , a *declarative unit* will be a pair  $Label:\alpha$ , where  $Label$  is a label written in the language  $\mathcal{L}_{Labels}$ , and  $\alpha$  is a wff in  $\mathcal{L}_{KR}$ . We will distinguish two special kinds of declarative units:

- If  $Label$  is an argument label, then  $Label:\alpha$  will be called an *argumentative declarative unit*.
- If  $Label:\alpha$  is an argumentative declarative unit, where  $Label = \langle L_i, \Phi \rangle$  is such that  $L_i$  is a singleton, then  $Label:\alpha$  will be called an *atomic declarative unit*.

□

**EXAMPLE 3.1** Let  $Labels = \{ n_1, n_2, n_3, d_1 \}$ , and let  $\mathcal{L}_{KR} = \{ a, b, b \leftarrow a, c \leftarrow b \}$ . Then  $\langle \{n_1\} \{a\} \rangle : a$ ,  $\langle \{n_2\} \{b\} \rangle : b$ ,  $\langle \{n_3\} \{b \leftarrow a\} \rangle : b \leftarrow a$  and  $\langle \{n_1\}, \{c \leftarrow b\} \rangle : c \leftarrow b$   $\langle \{n_1, n_2\} \{a, b\} \rangle : a \wedge b$   $\langle \{n_1, d_2, d_3\} \{a, b, a, c \leftarrow b\} \rangle : c$  are argumentative declarative units in  $\mathcal{L}_{Arg}$ . Note that the first four declarative units are atomic. □

From now on we will refer to a declarative unit  $Label:\alpha$  by the abbreviated form *du*. We will refer to  $Label$  as the “label” associated with  $Label:\alpha$ . Greek uppercase letters  $\Gamma, \Pi, \Upsilon$  will be used to refer to sets of *dus*, when no ambiguity arises.

### 3.1 Argumentative Theories

Intuitively, a *theory*  $\Gamma \subset Wffs(\mathcal{L}_{Arg})$  will constitute the knowledge base from which an intelligent agent will perform its inference process. Such a theory  $\Gamma$  will be defined in terms of argumentative *dus*, distinguishing:

- *Non-defeasible* information, characterized by argumentative *dus* of the form  $\langle \{n_i\}, \emptyset \rangle : \alpha$
- *Defeasible* information, characterized by argumentative *dus* of the form  $\langle \{d_i\}, \Phi \rangle : \alpha$  such that  $\Phi \vdash \alpha$ .<sup>4</sup>

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<sup>4</sup>Usually the *du*  $\langle \{d_i\}, \{ \alpha \} \rangle : \alpha \in \Gamma$  will be associated with *defeasible facts*, and  $\langle \{d_i\}, \{ \beta \leftarrow \alpha \} \rangle : \beta \leftarrow \alpha \in \Gamma$  will be associated with *defeasible rules*.

Labelling constants  $n_i$  and  $d_i$  will denote unique names for declarative units. In this ontology, a declarative unit  $\langle \{d_i\}, \{\beta \leftarrow \alpha\} : \beta \leftarrow \alpha \rangle$  will stand for the defeasible rule  $\beta \leftarrow \alpha$  in the MTDR framework [SL92, SCG94].

Next we will focus on sets of declarative units (theories) that respect certain requirements to constitute an acceptable knowledge base for an intelligent agent. This situation is formalized through the following definition:

**DEFINITION 3.7** (*Argumentative theory*  $\Gamma$ ). Let  $\Gamma = \{ \gamma_1, \gamma_2, \dots, \gamma_k \}$  be a finite set of declarative units in  $\mathcal{L}_{Arg}$ .

Let  $Wffs-S(\Gamma)$  be the set of all wffs in  $\mathcal{L}_{KR}$  associated with non-defeasible declarative units in  $\Gamma$ , *i.e.*,  $Wffs-S(\Gamma) = \{ \alpha \mid \langle \{n_i\}, \emptyset \rangle : \alpha \in \Gamma \}$ . We will say that  $\Gamma$  is an *argumentative theory* if

- $\gamma_i$  is an atomic declarative unit, for  $i = 1 \dots k$
- $Wffs-S(\Gamma) \not\vdash p, \bar{p}$ , where  $p$  is a literal in  $\mathcal{L}_{KR}$  and  $\bar{p}$  denotes its complement.

□

The labelling system makes it easier to formalize a proof theory based on natural deduction. For every inference rule, labels propagate information from the premises to the conclusion. In this way, given a *du*  $\langle Labels, \Phi \rangle \alpha$  that has been inferred from a given theory  $\Gamma$ , the set *Labels* will provide a ‘history’ of the proof carried out to conclude  $\alpha$ . If  $\Phi = \emptyset$ , then no defeasible information was needed in order to conclude  $\alpha$ . Therefore wffs of the form  $\langle Labels, \emptyset \rangle \alpha$  will correspond to non-defeasible inferences. On the contrary,  $\langle Labels, \Phi \rangle \alpha$ ,  $\Phi \neq \emptyset$ , denotes a defeasible inference, and  $\Phi$  is the set of facts, presumptions, defeasible rules and non-defeasible rules needed to conclude  $\alpha$ . Intuitively,  $\alpha$  will be ‘supported’ by the argument  $\Phi$ .<sup>5</sup> Some inference rules for labelling will incorporate additional *preconditions* which should be satisfied for an inference rule to be applied. This preconditions are mainly intended for ensuring that the defeasible wff  $\langle Labels, \Phi \rangle \alpha$  can be inferred only if certain consistency checks are satisfied.

**EXAMPLE 3.2** Consider an intelligent agent whose theory  $\Gamma$  includes the following wffs:

$$\begin{aligned} &\langle \{n_1\}, \emptyset \rangle : \neg f \leftarrow p, \\ &\langle \{n_2\}, \emptyset \rangle : p \\ &\langle \{n_3\}, \emptyset \rangle : b, \\ &\langle \{d_1\}, \{ f \leftarrow b \} \rangle : f \leftarrow b \end{aligned}$$

Then it should not be valid to infer  $\langle \{n_1, n_2, n_3, d_1\}, \{ p, b, \neg f \leftarrow b, f \leftarrow b \} \rangle : f$ , since the set  $\{p, b, \neg f \leftarrow b, f \leftarrow b\}$  would entail an inconsistency ( $f$  and  $\neg f$ ). □

In the next section we will introduce an inference relation which will allow us to capture the notion of *consistent proof* involving defeasible information. The inference relation will ensure that only consistent proofs can be derived. These consistent proofs will be called *generalized arguments*.

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<sup>5</sup>In [Mod98] this set is called *justificational context*.

## 4 Argument Construction as a Consequence Relation $\vdash_{Arg}$

Our goal will be to define a logical system  $(\Gamma, \vdash_{Arg})$ , where  $\Gamma$  is a knowledge base as previously described, and  $\vdash_{Arg}$  is a consequence relation. The object language will be  $\mathcal{L}_{Arg}$ , and inference rules will be formulated in a natural deduction style.

### 4.1 Natural deduction rules characterizing $\vdash_{Arg}$

- **Introducing non-defeasible information (I-N):** Any wff in  $\Gamma$  corresponding to non-defeasible information can be introduced in a proof.

$$\frac{}{\Gamma, \langle n_i, \emptyset \rangle \alpha}$$

for any  $\langle Labels_1, \Phi_1 \rangle \alpha \in \Gamma$ .

- **Introducing defeasible information (I-D):** Any wff in  $\Gamma$  corresponding to defeasible information can be introduced in a proof if it is consistent wrt  $Wffs-S(\Gamma)$ .

$$\frac{Wffs-S(\Gamma) \cup \Phi_1 \not\vdash \perp}{\Gamma, \langle d_i, \Phi_1 \rangle \alpha}$$

for any  $\langle d_i, \Phi_1 \rangle \alpha \in \Gamma$ .

- **Introducing conjunction (I- $\wedge$ ):** If  $\langle Labels_1, \Phi_1 \rangle \alpha$  and  $\langle Labels_2, \Phi_2 \rangle \beta$  are *dus* such that  $Wffs-S(\Gamma) \cup \Phi_1 \cup \Phi_2 \not\vdash \perp$ , then the conjunction  $\alpha \wedge \beta$  can be derived. Formally:

$$\frac{\Gamma, \langle Labels_1, \Phi_1 \rangle : \alpha \quad \Gamma, \langle Labels_2, \Phi_2 \rangle : \beta \quad Wffs-S(\Gamma) \cup \Phi_1 \cup \Phi_2 \not\vdash \perp}{\Gamma, \langle Labels_1 \cup Labels_2, \Phi_1 \cup \Phi_2 \rangle \alpha \wedge \beta}$$

- **Eliminating implication (E- $\rightarrow$ ):** As a precondition for applying *modus ponens*, a similar criterion as the one used in the previous rule will be applied.

$$\frac{\Gamma, \langle Labels_1, \Phi_1 \rangle : \beta \leftarrow \alpha \quad \Gamma, \langle Labels_2, \Phi_2 \rangle : \alpha \quad Wffs-S(\Gamma) \cup \Phi_1 \cup \Phi_2 \not\vdash \perp}{\Gamma, \langle Labels_1 \cup Labels_2, \Phi_1 \cup \Phi_2 \rangle \beta}$$

**DEFINITION 4.1 (Generalized argument).** Let  $\Gamma$  be a theory, such that  $\Gamma \vdash_{Arg} \langle Labels, \Phi \rangle \alpha$ . Then  $\langle Labels, \Phi \rangle$  will be called a *generalized argument* for  $\alpha$ .  $\square$

**DEFINITION 4.2 (Defeasible LDS).** Let

$$(\Gamma, \vdash_{Arg})$$

be a logical system, where  $\Gamma \subseteq Wffs(\mathcal{L}_{Arg})$  is a set of argument declarative units such that  $Wffs-S(\Gamma) \not\vdash \perp$ , and  $\vdash_{Arg}$  is the consequence relation characterized by the inference rules I-D, I-N, (I- $\wedge$ ) and (E- $\rightarrow$ ). This logical system will be called *defeasible labelled deductive system*.  $\square$

EXAMPLE 4.1 Let  $\Gamma$  be the argumentative theory shown below on the left. On the right side some possible inferences (and non-inferences) resulting from  $\vdash_{Arg}$  are shown.

$$\begin{array}{l}
\Gamma = \{ \langle n_1, \emptyset \rangle : d \leftarrow c, \\
\langle n_2, \emptyset \rangle : \neg d, \\
\langle n_3, \emptyset \rangle : e_1, \\
\langle n_4, \emptyset \rangle : e_2, \\
\langle d_1, \{c \leftarrow a, b\} \rangle : c \leftarrow a \wedge b, \\
\langle d_2, \{a \leftarrow e_1\} \rangle : a \leftarrow e_1, \\
\langle d_3, \{b \leftarrow e_2\} \rangle : b \leftarrow e_2 \} \\
\Gamma \vdash_{Arg} \langle d_2, n_3, \{a \leftarrow e_1\} \rangle : a \\
\Gamma \vdash_{Arg} \langle d_3, n_4, \{b \leftarrow e_2\} \rangle : b \\
\Gamma \vdash_{Arg} \langle d_2, n_3, d_3, n_4, \{a \leftarrow e_1, b \leftarrow e_2\} \rangle : a \wedge b \\
\Gamma \not\vdash_{Arg} \langle d_1, d_2, n_3, d_3, n_4, \{c \leftarrow a, b, a \leftarrow e_1, b \leftarrow e_2\} \rangle : c
\end{array}$$

□

For the sake of simplicity, the argumentative  $du \langle L_1, \Phi \rangle : \alpha$  will be usually written as  $\mathcal{A}_1 : \alpha$ .

## 5 Towards the Definition of an Argumentative LDS

So far a formalization of defeasible inference has been introduced, in which the notion of *generalized argument* has been formalized. However, given an argumentative theory  $\Gamma$  an intelligent agent can obtain different, conflicting arguments. Thus our agent could be able to find that  $\Gamma \vdash_{Arg} \mathcal{A}_1 : \alpha$  and  $\Gamma \vdash_{Arg} \mathcal{A}_2 : \neg \alpha$ . In order to decide among conflicting arguments a preference criterion is needed, as well as a global analysis in which the attack relationships between those conflicting arguments can be captured. This analysis will be formalized in terms of an *acceptable dialectical tree* [SCG94].

It should be remarked that arguments in the Simari-Loui sense [SL92] should be *minimal*. Therefore, in order to analyze generalized arguments in a dialectical setting it will be required that they are minimal. This will lead to defining a new logical system, an *argumentative LDS*  $(\Gamma, \vdash_{\mathcal{T}})$ , extending the consequence relation  $\vdash_{Arg}$  with new inference rules.

When defining  $\vdash_{\mathcal{T}}$ , inference rules for constructing acceptable dialectical trees are needed. In order to conceptualize these trees, we will use an approach similar to the one used when formalizing classical logic under natural deduction. We will start by stating the preconditions for introducing *axioms*, which in our case correspond to the most simple acceptable dialectical trees. Next we will introduce some of the inference rules needed for characterizing  $\vdash_{\mathcal{T}}$ .

- **Introducing an ‘undefeated’ acceptable dialectical tree:** For any generalized argument  $\mathcal{A}_1 : \alpha$  such that a) it is minimal and b) it has no defeaters, the labelled wff  $T_1^U(\mathcal{A}_1) : \alpha$  can be inferred (denoting that there exists a dialectical tree whose root is an undefeated node  $\mathcal{A}_1 : \alpha$ )

$$\frac{\Gamma, \mathcal{A}_1 : \alpha \quad \nexists \mathcal{A}' \subseteq \mathcal{A}_1 : \mathcal{A}' : \alpha \quad \nexists \mathcal{A}_2 : \beta \gg_{\text{def}} \mathcal{A}_1 : \alpha}{\Gamma, T_1^U(\mathcal{A}_1) : \alpha}$$

- **Introducing a ‘defeated’ acceptable dialectical tree:** For any generalized argument  $\mathcal{A}_1 : \alpha$  such that a) it is minimal and b) it has (at least) a defeater  $\mathcal{A}_2 : \beta$ , the labelled wff  $T_1^D(\mathcal{A}_1, T_2^U(\mathcal{A}_2)) : \alpha$  can be inferred (denoting that there exists a dialectical tree whose root is a defeated node  $\mathcal{A}_1 : \alpha$ )



$$\frac{\Gamma, \mathcal{A}_1:\alpha \quad \exists \mathcal{A}' \subseteq \mathcal{A}_1: \mathcal{A}':\alpha \quad \exists \mathcal{A}_2:\beta \gg_{\text{def}} \mathcal{A}_1:\alpha}{\Gamma, \mathcal{T}_1^D(\mathcal{A}_1, \mathcal{T}_2^U(\mathcal{A}_2)):\alpha}$$

Of course additional inference rules would be needed (not presented in this paper for space limitations) to fully characterize the notion of acceptable dialectical tree. Our final goal will be to capture the notion of justification in terms of *derivability* within such an argumentative LDS. Justified literals would be those supported by ultimately ‘undefeated’ acceptable dialectical trees. This situation could be formalized by the following definition:

**DEFINITION 5.1** (*Justification – preliminary version*). Let  $\Gamma$  be an argumentative theory, such that

$$\Gamma \vdash_{\mathcal{T}} \mathcal{T}^U:\alpha$$

Then the (ground) literal  $\alpha$  will be *justified*, and the declarative unit  $\mathcal{T}^U:\alpha$  will be called its *justification*.  $\square$

## 6 Conclusions

Labelled Deductive Systems offer a powerful tool for formalizing different logical frameworks. In this paper we have outlined a formalization of an argumentative system in terms of LDS. On the one hand, the notion of label allows to capture the concept of *argument* as a set of wffs supporting a given proposition. On the other hand, the concept of *dialectical tree* can be also captured by a complex label, defined in terms of more simple ones. We are currently working on a complete formalization of an argumentative framework using LDS.

Having a formal system that models the process of defeasible argumentation will allow to analyze different aspects associated with characterizing argumentative frameworks (such as argumentation protocols, resource-bounded reasoning, etc.). Since labelled formulas can on its turn be labelled, meta-level features can be captured in a natural way in LDS. Thus, as suggested originally by Gabbay [Gab96], we could think about wffs of the form

$$\textit{Agent}:(\textit{dialectical tree: conclusion})$$

to model argumentative reasoning in a multiagent system. Appropriate inference rules would characterize which interactions among agents are valid, specifying an *argumentative protocol* [Lou98, CS96] in terms of preconditions in inference rules. Research in this direction is currently being pursued.

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