Embe ... ries

luction plays
easoning. In a
hat goes from observations
. That is to say, abductive
e theory, enable deductions
there ... severa ... uch expla ... ons for a given observation.
oduction is regarded as an inference to the best explanation.
uction is the explanation of *anomalous* observations, *i. e.*,
with the current theory. It is perhaps impossible to do such
For this reason, in this work we will consider the problem of
onotonic theories. Our inference system is based on a natural
licational segment of a relevant logic, much similar to the $R_{\rightarrow}$
[1]. Then we will discuss some issues arising the pragmatic
ces in nonmonotonic theories.

nomalous Observations

ies of abduction in nonmonotonic theories, first we will give a formal
onotonic reasoning system, and then include an explicit rule for abduc-
stem regards defeasible rules $a(X) \succ\!\!- b(X)$[1] as material implications
ns inference rule (that is, contraposition, left strengthening, right weaken-
e explicitly left out). Defeasible rules can be activated in MP only when
y instantiated, *i. e.*, there is a ground substitution for X such that all the
been inferred. The reasoning system, then, will chain inferences in a way
sical) deductions, with the addition of inferences in which a fully activated
used. This chains of inferences are named *(sub)-theories* [2, 6] or*arguments*
le rule can be regarded as a *prima facie* material implication, then an argument
acie proof or a *prediction* for A. We can then extend the (classical) consequence
new operator $\mid\!\sim$, where $\mathcal{T} \mid\!\sim A$ means that there is an argument for A in theory

nt and consequent of defeasible rules are restricted to be *sets of literals* (interpreted as a conjunction),
of free variables.

Since we may reasonably expect that these inferences will eventually generate a pair of contradictory literals, and since we want to avoid (classical) trivialization, then our reasoning system must incorporate some kind of strengthening or restriction among the structural rules. For this reasoning, we adapted a presentation of the implicational segment of a relevant logic, much similar to the $R_{\rightarrow}$ system of Anderson and Belnap [1]. Then, the reiteration rule is restricted to sentences that were inferred within the same subproof. If we need reiteration of a sentence S of a previous step outside the subproof, then we must either introduce S as a new assumption, or reproduce the inference steps that leads to the inference of S. To take due care of this, we establish an index schema that labels premise introduction and its ulterior discharge by means of $\rightarrow_I$. The use of a defeasible rule is regarded as a restricted *modus ponens* that also introduces a new hypothesis. The labeling schema obeys a simple set of cases (the subindices I and J denote sets of indices, and i and j denote individual indices).

- (*Premise*) An hypothetical premise a is introduced with an index i never used before (we will use the sequence of natural numbers).
- ($\rightarrow_I$) From a (sub-)demonstration for b_I from premise a_j (with $j \in I$) to infer $(a \rightarrow b)_{I-\{j\}}$.
- (*Reit.*) Reiteration of a sentence retains the indices.
- ($\rightarrow_E$) In the *modus ponens* rule, the consequent retains the indices of the major and the minor premises: from a_I and $(a \rightarrow b)_J$ to infer $b_{I \cup J}$.

We now add the case for defeasible rules.

- ($\succ\!\!-_E$) From a (sub-) demonstration of $a(t)_I$ and $a(X) \succ\!\!- b(X)$ to infer $a(t) \rightarrow b(t)_{I \cup \{k\}}$, where k is an index never used before.[2].

Example **2.1** *Suppose that in our knowledge base we have*
$a, a \succ\!\!- b, b \succ\!\!- c, a \succ\!\!- \neg c$

In this situation, we may establish the following reasoning lines:

1	[$a_{\{1\}}$	*Premise*
2	$(a \succ\!\!- b)_{\{2\}}$	*Defeasible rule*
3	$b_{\{1,2\}}$	$1, 2, \succ\!\!-_E$
4	$(b \succ\!\!- c)_{\{3\}}$]	*Defeasible rule*
5	$c_{\{1,2,3\}}$	$3, 4, \succ\!\!-_E$

1	[$a_{\{1\}}$	*Premise*
2	$(a \succ\!\!- \neg c)_{\{4\}}$]	*Defeasible rule*
3	$\neg c_{\{1,4\}}$	$1, 2, \succ\!\!-_E$

Given a theory $\mathcal{T}$ and an observed evidence e, then e is surprising if neither $\mathcal{T} \vdash e$, nor $\mathcal{T} \vdash \neg e$, and e is anomalous if $\mathcal{T} \vdash \neg e$. The first situation has received considerable interest since Peirce, who coined the word *abduction*, and characterized it as the third member of the triad of

[2] Both $a(X)$ and $a(t)$ denote sets of literals. X is a tuple of free variables, t is a tuple of ground terms such that X may be substituted for t in $a(X)$.

syllogistic reasoning (together with deduction and induction)[3]. The second situation (abduction of anomalous observations) has received only an occasional attention. It is a well established fact that monotonic theories cannot accommodate anomalous observations. For this reason, research in this direction must focus in abduction in nonmonotonic theories. For this reason, our final step is to propose a rule for abduction. This rule is based on several considerations (which cannot be discussed at length here because of space limitations). For this reason, we will give here only a short motivation. Given a nonmonotonic theory $\mathcal{T}$ (*i. e.*, a theory that may have defeasible rules), an abduction for an observation O should be a hypothetical explanation H that is compatible with $\mathcal{T}$, neither $\mathcal{T}$ nor H should jointly (but not separately) explain O, and any other explanation H' should also explain H itself. Formally:

- (*Abd.*) From O_k to infer $H_{S\cup\{k\}}$ iff
  1. $H_{S\cup\{k\}} \cup \mathcal{T} \not\vdash \bot$, *(H is consistent with $\mathcal{T}$)*
  2. $\mathcal{T} \not\mid\!\sim O_k$, *(there is no argument for O in $\mathcal{T}$)*
  3. $H_{S\cup\{k\}} \not\mid\!\sim O_k$, *(there is no argument for O in H)*
  4. $\mathcal{T} \cup H_{S\cup\{k\}} \mid\!\sim O_k$, *(there an argument for O in $\mathcal{T} \cup H$)*
  5. Any other set H' that satisfies the four conditions above is such that $H' \cup \mathcal{T} \mid\!\sim H_{S\cup\{k\}}$ (*i. e.*, H is the most "shallow" explanation for O).

Example **2.2** *Suppose that in a knowledge-based system we find the rules*

$w(X) \succ\!\!- i(X)$	*Normally if X has a work, then X receives an income.*
$w(X) \succ\!\!- t(X)$	*Normally if X has a work, then X pays taxes.*
$w(X) \succ\!\!- \neg s(X)$	*Normally if X has a work, then X does not study.*
$s(X) \succ\!\!- w(X)$	*Normally if X studies, then X has a work.*
$c(X) \succ\!\!- s(X)$	*Normally if X has a scholarship, then X studies.*
$c(X) \succ\!\!- i(X)$	*Normally if X has a scholarship, then X receives an income.*
$c(X) \succ\!\!- \neg t(X)$	*Normally if X has a scholarship, then X does not pay taxes.*

Given this, what can we expect about Scott, of whom we only know he pays taxes?

1	$t(Scott)_{\{1\}}$	*Premise*
2	$(w(X) \succ\!\!- t(X))_{\{2\}}$	*Defeasible rule*
3	$w(Scott)_{\{1,2\}}$	$1, 2,$ *Abduction (Explanation)*
4	$w(Scott)_{\{1,2\}}$	$3,$ *Reit.*
5	$(w(X) \succ\!\!- i(X))_{\{3\}}$	*Defeasible rule*
6	$i(Scott)_{\{1,2,3\}}$	$4, 5, \rightarrow_E$ *(Prediction)*
7	$w(Scott)_{\{1,2\}}$	$3,$ *Reit.*
8	$(w(X) \succ\!\!- \neg s(X))_{\{4\}}$	*Defeasible rule*
9	$\neg s(Scott)_{\{1,2,4\}}$	$7, 8, \rightarrow_E$ *(Prediction)*

[3]This shortest account of Peirce is surely unfair, since his purpose was much wider, for in his semiotic analysis of inference, abduction was central as the source of creativity and new knowledge [3].

By abduction, we can show that $t(Scott)$ *because* $w(Scott)$ *(he pays taxes because he works), and from this inference, we can predict that he has an income, and that he does not study. It is a desirable feature here that further (iterated) abductions (for example,* $c(Scott)$ *because* $i(Scott)$*) are blocked for being inconsistent (see the next Section).*

If we knew about another person, say Kim*, of whom we know only that she receives an income, then we can generate two abductive explanations for her income. The first one,* $i(Kim)$ *because* $w(Kim)$*, allow further predictions (*$t(Kim)$ *and* $\neg s(Kim)$*). The second one,* $i(Kim)$ *because* $c(Kim)$*, allow other predictions (*$s(Kim)$ *and* $\neg t(Kim)$*). In this situation we have two unrelated explanations, of which we can not make any preference (again, see next Section). However, further knowing that, for instance,* $s(Kim)$*, will block the first explanation in favor of the second.*

3 Combining Defeaters

A final issue we wish to discuss is the defeat among various arguments in a nonmonotonic theory. This situation arises if we admit the possibility of *iterating* abductive explanations. In the Sect. 3, we introduced a "shallow" abductive operator, but it can be iterated to produce "deeper" explanations.

Example **3.1** *(After [4] and [5]). Suppose we have the following theory.*

$$\mathcal{T}=\{ \quad r(T) \succ\!\!- wr(T), \qquad \text{(if it rains, the road is wet)},$$
$$r(T) \succ\!\!- wl(T), \qquad \text{(if it rains, the lawn is wet)},$$
$$r(T) \succ\!\!- \neg s(T), \qquad \text{(if it rains, it's not sunny)},$$
$$s(T) \succ\!\!- \neg r(T), \qquad \text{(if it's sunny, it does not rain)},$$
$$so(T) \succ\!\!- wl(T), \qquad \text{(if the sprinklers are on, the lawn is wet)},$$
$$s(T) \wedge h(T) \succ\!\!- so(T), \qquad \text{(if it's sunny and hot, the sprinklers are on)},$$
$$wl(T) \succ\!\!- ws(T), \qquad \text{(if the lawn is wet, the shoes are wet)},$$
$$wr(T) \succ\!\!- ws(T) \ \}. \qquad \text{(if the road is wet, the shoes are wet)}.$$

In this situation, suppose we observe that our shoes are wet ($E = ws(today)$*). The possible (shallow) explanations for this are that either the road is wet, or that the lawn is wet, or both. However, none of these suffices to generate a "most specific" explanation.*

To generate a more specific explanation we can iterate the abductive inference, that is, to generate a new "evidence" set E' that contains E plus any of the independently generated explanations, and then use this new context to try to generate a new abductive explanation. This procedure may be easy to formalize, but, as we will see, it may be that an argument is conflicting with some of these abductive hypotheses, and a criterion for combining defeat should be taken into account.

Definition **3.1** *Given a context* $\mathcal{T} \cup E$ *with an underlying knowledge* $\mathcal{K}$*. Then*

1. *The set of argumentative supported conclusions* A_c *are generated from* $E \cup \mathcal{T}$*.*
2. *The set of abductive explanations* B_c *are generated from* $E \cup \mathcal{T}$*.*
3. *If there is a pair of contradictory literals* $a \in A_c$ *and* $b \in B_c$*, then either*

   (I) Any argument A *for* a *defeats any argument generated with* b*.*

(II) Any argument generated with b defeats any argument generated with a.

(III) Any of the defeasible rules used in the arguments for a or in the explanations for b is defeated (syntactically blocked).

4. Firm conclusions C are the members of A_c and B_c that were not defeated in 3.

If we need to iterate abduction, then the firm conclusions C are added to the "evidence" E, and the process is repeated.

Example **3.2** *Suppose we are in the situation of Example 3.1, and we observe our shoes wet ($ws(today)$), and we remember that today it was sunny ($s(today)$). Then, what can we conclude? The most general abductive explanations are $wr(today)$ and $wl(today)$. By the moment, any of these explanations is compatible with the observations and there is no defeat. If we iterate the abductive process, we find that $r(today)$ is explanation for $wr(today)$, and $r(today)$ or $so(today)$ are explanations for $wl(today)$.*

Following strategy I, we assimilate $so(today)$ as the only tenable explanation for $ws(today)$, that is, we conjecture that the sprinkler was on, it got the lawn wet, and then our shoes got wet. If we push this further, we can also conjecture that today it was hot in addition of being sunny.

Instead, if we follow strategy II, then the explanation $r(today)$ blocks our remembrance of being sunny. Then, our explanation now is that it rained, the rain got the road and the lawn wet, and then our shoes got also wet.

If we use strategy III, then both previous explanations are valid and compatible, and we reject the rules that mutually exclude $r(today)$ and $s(today)$, that is, we suppose that today it may be hot and sunny at one time, and rainy at another.

References

[1] Alan R. Anderson and Nuel Belnap. *Entailment: The Logic of Necesity and Relevance*. Princeton University Press, 1975.

[2] Gehrard Brewka. Cumulative Default Logic: in Defense of Nonmonotonic Default Rules. *Artificial Intelligence*, 50(2):183–205, 1991.

[3] C. Hartshorne *et. al.* *Collected Papers of Charles S. Peirce*. Harvard University Press, Cambridge, MA, 1958.

[4] Kurt Konolige. Abduction versus Closure in Causal Theories. *Artificial Intelligence*, 53(2-3):255–272, 1992.

[5] Judea Pearl. *Probabilistic Reasoning in Intelligent Systems*. Morgan Kaufmann, San Mateo, California, 1988.

[6] D. Poole, R. Goebel, and R. Aleliunas. Theorist: a Logical Reasoning System for Defaults and Diagnosis. In *The Knowledge Frontier*, pages 331–352, New York, 1987. Springer.

[7] Guillermo R. Simari and Ronald P. Loui. A Mathematical Treatment of Defeasible Reasoning and its Implementation. *Artificial Intelligence*, 53(2-3):125–158, 1992.

[8] G. A. W. Vreeswijk. Abstract Argumentation Systems. *Artificial Intelligence*, 90(2):225–279, 1997.