Universidad Carlos III de Madrid
www.uc3m.es

TESIS DOCTORAL

Essays on Heterogeneous Agent Models

Autor:

Omar Rachedi

Director/es:

Matthias Kredler

DEPARTAMENTO DE ECONOMÍA

Getafe, mayo de 2015

Universidad Carlos III de Madrid
www.uc3m.es

TESIS DOCTORAL

ESSAYS ON HETEROGENEOUS AGENT MODELS

Autor: ***Omar Rachedi***

Director: **Matthias Kredler**

Firma del Tribunal Calificador:

Firma

Presidente:

Vocal:

Secretario:

Calificación:

Getafe, de de

A Nonna

Acknowledgements

I would like to express my sincere gratitude to my adviser, Matthias Kredler, and also to Juanjo Dolado, Andres Erosa, Salvador Ortigueira, Manuel Santos and Hernan Seoane. Their guidance and support throughout my time as a graduate student has been crucial in my (not yet completed) development as an economist.

This dissertation also benefited from comments, suggestions, and support from many other people, including Antonia Diaz, Pedro Gomes, Claudio Michelacci and Pedro Teles. I extend my sincere gratitude to them for their help.

I wanna thank my fantastic colleagues Pedro Sant'Anna, Mehdi Hamidisahneh, Iacopo Morchio, Arancha Crespo, Nikolas Tsakas, Xiajoun Song, Lovleen Kushwah, Francesco Risi, Olga Croitorov, Joseph Gomes, Robert Kirby, Lian Allub and Anett Erdmann. Our daily discussions about research and crazy anecdotes have been a very important part of my life as a PhD student. I do hope our paths cross again someday!

My life in Madrid wouldn't have been the same without Anahi Francis Osorio, Jan Bietenbeck, Alessandro Galesi, Alessandro Peri and Marco Serena. I thank Anahi for sharing her time (and patience) with me. I thank Jan for five amazing years of "flatemateship". I owe Alessandro & Alessandro a burgeoning of things: they taught me nearly everything I know on macroeconomics and Call of Duty. It's a pity that they couldn't be very effective in teaching Marco how to play. Anyway, I'm still very grateful to him for always helping me when I needed it most.

Finally, I would like to thank my family. My parents, my brothers, my sister, my brother in law and my nephews. Thanking them just for this thesis would be extremely reducing since I owe them three decades of unconditional support and patience.

Abstract

There is a continued interest among economists on the interconnections between financial markets, credit markets and the real economy. The three main chapters of this dissertation contribute to the understanding of how financial and credit frictions - either at the firm or household level - can affect the real economy, and even trigger a financial crisis.

Chapter 1 studies the causes of financial crises. I show that shocks to the volatility of total factor productivity (TFP) can generate endogenous variations in loan-to-value (LTV) ratios and trigger credit crunches, without appealing to financial shocks. Using a panel of countries, I find that financial crises coincide with the reversal of a long period of low volatility of TFP. To explain this new fact, I develop a general equilibrium model in which volatility shocks to TFP interact with an occasionally binding borrowing constraint and housing serves as collateral. I introduce search frictions in the housing market to capture the liquidity of housing and endogenize the LTV ratio: households borrow at higher LTV ratios when the collateral is more liquid. In this environment, volatility shocks cause financial crises by changing the liquidity of the collateral. In a quantitative exercise, I feed the model with the stochastic volatility of the U.S. Solow residual. I find that the interaction of volatility shocks and search frictions in the housing market increases the frequency of financial crises by 47% and the associated output drop by 30%. In addition, volatility shocks generate volatile LTV ratios, thus providing a foundation for financial shocks.

Chapter 2 studies whether households' limited attention to the stock market can quantitatively account for the bulk of asset prices. I address this question introducing an observation cost in a production economy with heterogeneous agents, incomplete markets and idiosyncratic risk. In this environment inattention changes endogenously over time and across agents. I calibrate the observation cost to match the observed duration of inattention of the median agent in the data. The model generates limited participation in the stock market, a weak correlation between consumption growth and stock returns, and countercyclical dynamics for both the stock returns volatility and the excess return. It also generates forms of predictability in stock returns and consumption growth. Nonetheless, the level of the equity premium is still low, around 1%. Finally, I find that inattention affects asset prices if borrowing constraints are tight enough.

Chapter 3 - which is joint work with Alessandro Peri - studies the series of US annual corporate default rates from 1950 until 2012. We document the presence of one structural break in the unconditional mean, which is dated in 1986. Meanwhile credit spreads hardly moved. We present a dynamic equilibrium model where the development of credit markets accounts for this empirical evidence. Financial development increases both the default rate and firms' expected recovery rates. These two effects offset each other and translate into constant credit spreads. In the model financial development explains 64% of the rise in

default rates and predicts just a 2 basis point increase in the credit spreads. Furthermore, the model accounts for a number of trends that characterized public firms over the last decades: the fall in the number of firms distributing dividends, the rise in the degree of dividend smoothing, and the increase in the volatility of public firms.

Contents

List of Tables

List of Figures

CHAPTER 1

The Calm Before the Storm: Time Varying Volatility and the Origins of Financial Crises

1.1 Introduction

What causes financial crises? Major credit crunches are usually considered events which originate in the financial sector. In this paper, I show that shocks to the volatility of total factor productivity (TFP) can generate endogenous variations in loan-to-value (LTV) ratios and trigger financial crises, without appealing to financial shocks. My focus on volatility links to the financial instability hypothesis of Minsky (1992), which conjectures that long periods of low fluctuations can lead to a crisis. This is a phenomenon that Brunnermeier and Sannikov (2013) refer to as the volatility paradox: it is the calm that generates the storm.

I first establish in the data an association between the volatility of TFP and financial crises which is consistent with the volatility paradox. I build a panel of crises across countries and find that the volatility of TFP is around 12% below trend over the two years preceding a financial crisis, before jumping up. This rise in volatility leads the burst of the crisis.

To explain this new fact, I develop a general equilibrium model in which volatility shocks interact with an occasionally binding borrowing constraint and housing serves as collateral. I introduce search frictions in the housing market to capture the liquidity of housing and endogenize the maximum LTV ratio (i.e., the maximum amount households can borrow given the value of their assets). Households borrow at higher LTV ratios when the housing market is more liquid. In the model, financial crises happen when the LTV ratio drops and the borrowing constraint becomes binding, which forces households to deleverage. The constraint binds with a probability that depends on the optimal choices of the households.

In this environment, volatility shocks affect the frequency of credit crunches by changing the liquidity of the collateral. A long period of low volatility sows the seeds of the crisis by boosting housing liquidity, which raises both the LTV ratio and households' leverage;

then a sudden volatility spike freezes out the liquidity of the housing market and reduces the LTV ratio, forcing households to sharply deleverage.

In a quantitative exercise, I feed the model with the stochastic volatility of the Solow residual of the U.S. economy, which I estimate by Bayesian techniques. I use global numerical methods to solve the model and preserve its non-linear dynamics. I find that the interaction of volatility shocks and search frictions in the housing market increases the frequency of financial crises by 55% and the associated output drop by 58%. I show that financial crises are characterized by deflationary spirals à la Fisher (1933) in both the house price *and* the LTV ratio, a novel mechanism which amplifies the severity of a downturn. The initial drop in the LTV ratio forces households to deleverage, generating a decline in both house prices and housing liquidity, which eventually decreases even further the LTV ratio in a deflationary loop. Furthermore, the model accounts for around half of the observed time variation in LTV ratios. Hence, the interaction of volatility shocks and search frictions in the housing market provides a rationale for financial shocks.

The mechanism of the paper works through changes in the liquidity of housing, which eventually modify the maximum LTV ratio at which households borrow. In the model, periods of low volatility boost housing investment. As more households look for a house, sellers are more likely to meet with a buyer. The higher liquidity of housing relaxes the LTV ratio and generates a credit and an investment boom which reinforce each other. This spiral builds up systemic risk because the economy becomes fragile to the realizations of adverse shocks at high levels of households' leverage. Indeed, the dynamics of the model are non-linear. As in Mendoza (2010), Bianchi (2012) and Bianchi and Mendoza (2013), negative shocks generate only mild recessions at low levels of leverage. Instead, when households are highly indebted, a sudden peak in volatility can dry up the liquidity of housing and lower the LTV ratio down to the point that the borrowing constraint becomes binding. Agents are then forced to deleverage and fire sell their houses, triggering a debt deflationary spiral in both the house price and LTV ratio, which turns the credit boom into a bust.

The search frictions in the housing market creates a direct link between the liquidity of the collateral and households' borrowing capacity, a novel mechanism in the literature of general equilibrium models with financial frictions. While standard models à la Kiyotaki and Moore (1997) usually assume that the LTV ratio is exogenous, in this paper the ratio is endogenous. The link between housing liquidity, collateral values and the LTV ratio follows Brunnermeier and Pedersen (2009), in which market liquidity directly determines households' funding liquidity, that is, the ease at which households can access new loans. More precisely, a house has a high collateral value if lenders can sell it both quickly and at a high price in case they seize it.[1] In equilibrium the LTV ratio is the ratio between

[1] The role of the collateral liquidity is already pointed out in Del Negro et *al.* (2011) and Kiyotaki and Moore (2012). These papers exogenously impose the degree of collateral illiquidity, ruling out any feedback effect between market liquidity and households' funding liquidity.

the option value of a vacant house - the value of a house on sale that is expected to be sold in the future and does not yield any dividend or utility to its owner - and the market value of housing. The wedge between these two prices widens in illiquid markets because houses are expected to be on sale for a longer time. Through this channel, changes in the liquidity of the housing market alter the value of the collateral asset and affect households' borrowing capacity. From this perspective, this paper follows the contributions of Fostel and Geanakoplos (2008) and Geanakoplos (2010) on the importance of endogenizing the LTV ratio to match of the dynamics of macroeconomic and financial variables.

How can volatility affect housing investment? The volatility shocks propagate into the real economy through the frictional housing market. Indeed, search frictions generate adjustment costs and partial irreversibilities in housing investment. On the one hand, households incur search costs whenever they look for a house. On the other hand, agents sell their properties at a discounted price when the housing market is illiquid. Hence housing investment is expensive to reverse. As shown in Bloom (2009), in this environment agents become more cautious in uncertain times: agents reduce ex-ante their investment propensity to avoid incurring ex-post the costs of frequently adjusting the housing stock. In the quantitative analysis, I show that volatility shocks barely change the frequency of financial crises if there are no search frictions in housing market. Even in the case I consider a frictional housing market, volatility shocks matter only if households' LTV ratio is not constant and does depend on housing liquidity. From this point of view, this paper contributes to the literature on the real effects of volatility shocks (e.g., Justiniano and Primiceri, 2008; Bloom, 2009; Fernandez-Villaverde et *al.*, 2011) on two dimensions. First, I find a new propagation mechanism for the volatility shocks: the interaction of search frictions in the market of the collateral asset and an endogenous LTV ratio. Second, I show that changes in the *exogenous volatility* of productivity may generate sharp movements in the *endogenous volatility* of output and credit when households' leverage is excessively high.[2]

The presence of the volatility shocks and its effect of house prices relates to Bansal and Yaron (2004), where the stochastic volatility of consumption growth accounts for the time variation of risk premia. The importance of aggregate volatility is also stressed in Bansal et *al.* (2014), who point out that macroeconomic volatility is a primary source of risk affecting asset prices. I complement this literature by showing that the introduction of a stochastic volatility in a production economy with CRRA preferences can generate sharp fluctuations in house prices around a financial crisis.

Finally, this paper provides a quantitative theory of financial crises which sheds lights

[2]What is the interpretation of the volatility shocks? Carvalho and Gabaix (2013) show that the changes from the manufacturing sector towards the service and financial sector can account for the movements in the volatility of U.S. macroeconomic variables over the last decades. Alternatively, Bloom (2009) finds that time variations in aggregate volatility are correlated with changes in the cross-sectional dispersion of firms' growth rates. Christiano et *al.* (2014) shows that shocks to the dispersion of firms' productivity are a key source of business cycle fluctuations.

on the debate on the causes of the last recession. For instance, Gilchrist and Zakrajsek (2012) and Jermann and Quadrini (2012) show that the recent crisis has been driven by a large negative financial shock that generated a credit crunch and consequently a sharp drop in investment and employment. These results have consolidated the view that the cause of the recent crisis is the disruption of *credit supply* due to the breakdown of banks.

Yet, this explanation is at odds with the empirical evidence provided by Mian and Sufi (2009, 2011), who find that the housing market in the U.S. started to slump around 2006, much earlier than the collapse of Bear Stearns and Lehman Brothers. In this vein, the deterioration of the balance sheet of the households - rather than the one of the financial intermediaries - has triggered the Great Recession.

To reconcile these different views, I propose a mechanism which is only based on real shocks - especially on innovations to the volatility of TFP - and works entirely through variations in *credit demand*. In the model there is no bank. However, volatility shocks drive changes in housing liquidity, which affect households' collateral values and eventually modify the LTV ratio. This mechanism makes households' leverage to move over time even when the house price does not change. Hence, the interaction of volatility shocks and a frictional housing market generates dynamics in the LTV ratio that are observationally equivalent to a financial shock, although they entirely hinge on credit demand. This result suggests that financial shocks should not necessarily be interpreted as if they were originated in the financial sector, and could rather be caused by shifts in credit demand.

1.1.1 Related Literature

This paper is connected to three strands of literature. First, I complement the empirical evidence provided by Reinhart and Rogoff (2009), Mendoza and Terrones (2012), Schularick and Taylor (2012), and Jorda et *al.* (2013, 2015) on the dynamics of macroeconomic variables around financial crises. These authors show that financial crises are actually credit booms gone bust. I document that although aggregate volatility does not display strong comovements with recessions, it is characterized by large swings around financial crises. Second, this paper contributes to the debate on the recent house price boom and bust. Global imbalances are often referred to as the main cause of the house price boom. For instance, Justiniano et *al.* (2014) show that the global savings glut accounts for around one fourth of the increase in U.S. house prices in the early 2000's. Yet, Favilukis et *al.* (2013) argue that the boom and bust in the housing market is explained by financial development in the mortgage market. While there is a burgeoning evidence on the improvements in financial markets in recent years, it is harder to understand the reversal of the process of financial development amidst the financial crisis. In my model movements in the LTV ratio - due to changes in housing liquidity - provide a rationale to both the process of financial development and its reversal. The relaxation of credit conditions in the mortgage market can be explained by the high liquidity of the housing market in the

2000's. Analogously, the liquidity freeze around the crisis can account for the reversal of the process of financial development. Third, this paper relates to the literature on search frictions in the housing market, which follows the contribution of Wheaton (1990). For example, Diaz and Jerez (2013) show that a model with a frictional housing market can reproduce the house price volatility. I add to this literature by showing that changes in housing liquidity affect the frequency of financial crises.[3]

1.2 Evidence on Volatility and Financial Crises

In this Section I document a new stylized fact on the dynamics of the volatility of TFP around financial crises. I find that crises coincide with the reversal of a long period of low fluctuations.

1.2.1 Data on Volatility and Financial Crises

I build a panel of 20 developed countries from 1980 until 2013 to understand how volatility is related to financial crises. The countries covered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. For any of these countries, I consider an indicator of aggregate volatility at an annual frequency: the stochastic volatility of total factor productivity (TFP). I compute the series of TFP z_t for each country using data from the Penn World Tables. Then, I posit that in each country TFP follows a first-order autoregressive process with stochastic volatility

$$\begin{aligned} z_t &= \rho_z z_{t+1} + e^{\sigma_t}\epsilon_{z,t}, && \epsilon_{z,t} \sim N(0,1) \\ \sigma_t &= (1-\rho_\sigma)\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta\epsilon_{\sigma,t}, && \epsilon_{\sigma,t} \sim N(0,1) \end{aligned} \tag{1.1}$$

where ρ_z denotes the persistency of the level equation of TFP, ρ_σ is the persistency of the volatility equation, $\bar{\sigma}$ is the long-run mean of the volatility of TFP and η captures the degree of stochastic volatility in the process. $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ denote the innovations to the level and volatility of TFP, respectively. I assume that both $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are independent to each other.

Since the innovations $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are unknown to the econometrician, I need to apply a filter to the data to estimate the parameters of the process (1.1). In this framework the Kalman filter is unsuitable because it applies only to linear series, while here the shocks to the volatility enter non-linearly in the level equation of TFP. I evaluate the likelihood of

[3]The setting of the housing market in my paper follows Ungerer (2013), which shows that monetary policy affects aggregate leverage through a borrowing margin that depends on housing liquidity. Instead, I focus on the link between volatility, housing liquidity and financial crises, and emphasize the Fisherian deflation in LTV ratios amidst a crisis.

this process by appealing to the Sequential Importance Sampling particle filter introduced in Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et *al.* (2011). The estimation of the stochastic volatility of TFP closely follows Born and Pfeifer (2013). I use Bayesian techniques to estimate the likelihood of the process of productivity. I elicit some unrestrictive priors, and after deriving the likelihood of the process for some given parameters with the SIS particle filter, I maximize the posterior likelihood using a random walk Metropolis-Hastings algorithm with 20000 replications, out of which the first 5000 represent burn-in draws. Finally, I recover the historical distribution of the volatility of TFP using the backward-smoothing routine of Godsill et *al.* (2004).

I also consider different measures of volatility as robustness checks. First, following Bloom (2009), I proxy aggregate volatility with the logarithm of the variance of daily stock returns within a year. Second, I compute the volatility of quarterly GDP growth over a moving window of 20 quarters.

Finally, I take the dates of financial crises from multiple sources, that is, Bordo et *al.* (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et *al.* (2015). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. The dates of recessions are instead given by the OECD recession indicators. Overall, the panel covers 29 events of financial crises and 118 events of recessions. I report the dates of crises and recessions by country and all the sources of the data in Appendix 1.A.

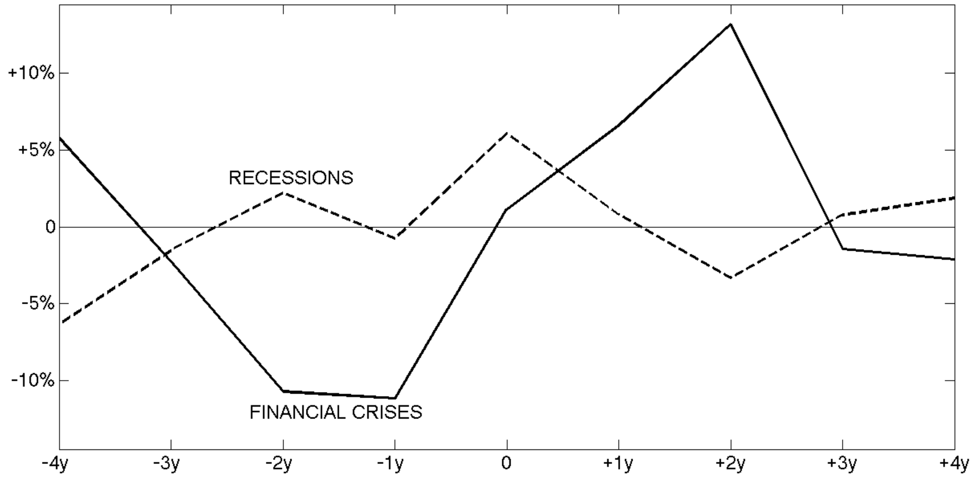
1.2.2 The Dynamics Around Crises and Recessions

This Section studies the dynamics of volatility around financial crises and recessions. For any country and for any financial crisis and recession, I take the series of aggregate volatility in a time window of nine years around the event of interest, that is, from four years before either the financial crisis or the recession up to four years afterwards. Then, I consider the series defined by the average observations across events for any year of the window as the typical pattern around financial crises and recessions. For example, to define the typical level of volatility the year preceding a financial crisis, I take the volatility of the Solow residual one year before each of the 29 financial crises of my sample and then compute the mean.

Figure 1.1 displays the typical dynamics of aggregate volatility around financial crises and recessions. The figure documents that aggregate volatility asymmetrically varies around crises and recessions. While there are negligible deviations from trend during recessions, the behaviour of volatility around crises is characterized by large swings. Crises tend to be preceded by years in which volatility is around 10% below trend and the burst of the crisis pushes volatility up to around 13% above trend.

FIGURE 1.1

AGGREGATE VOLATILITY AROUND CRISES AND RECESSIONS.



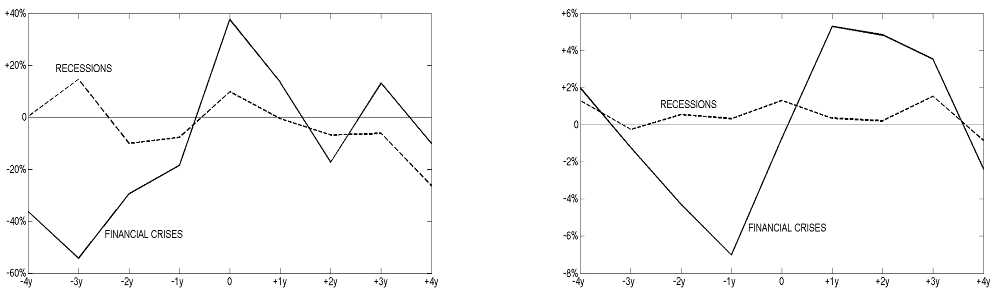
The figure plots the average values of the deviations from the trend of the stochastic volatility of countries' total factor productivity around recessions and financial crises (9 year window). The continuous line indicates the dynamics around financial crises, while the dashed line refers to recession. The dates of financial crises are taken from Reinhart and Rogoff (2009). Recessions are derived from the OECD recession indicators.

Figure 1.2 shows that aggregate volatility maintains the same dynamics around crises and recessions even when it is measured as the variance within a year of daily stock returns or as the variance of quarterly GDP growth rates over a moving window of 20 quarters. I also find a similar dynamics when using yet other measures of volatility, see Appendix 1.B.

FIGURE 1.2

DIFFERENT MEASURES OF AGGREGATE VOLATILITY.

(a) Volatility of Stock Returns (b) Volatility of GDP Growth



Note: The figure plots the dynamics of different measures of aggregate volatility around recessions and financial crises (9 year window). In Panel (a) the volatility is measured as the variance of daily stock market returns within a year. In Panel (b) the volatility refers to the variance of quarterly GDP growth rates computed over a moving window of 20 quarters. The continuous line indicates the dynamics around financial crises, while the dashed line refers to recession. The dates of financial crises are taken from Reinhart and Rogoff (2009). Recessions are derived from the OECD recession indicators.

I argue that this evidence points out a new stylized fact on the dynamics of volatility around financial crises which is consistent with the volatility paradox of Brunnermeier and Sannikov (2013). [4]

1.2.3 Volatility Shocks and the Housing Market

The previous analysis points out that changes in the level of aggregate volatility tend to coincide with the build-up of risk and the burst of a financial crisis. What is the mechanism behind this result? In this Section I show that volatility shocks are propagated into the real economy through the housing market. I run a structural VAR model, in which I compute the response of house price, the quantity of house sold and a measure of liquidity of housing to an unexpected increase in volatility.

The VAR is estimated using with monthly data from January 1963 until December 2013 on the level of S&P 500 returns, an indicator of volatility, the Federal Funds Rate, the consumer price index, industrial production and three variables on the housing markets related to price, quantity and liquidity.

I borrow the volatility indicator from Bloom (2009). This variable identifies a number of large and arguably exogenous peaks of stock market volatility, and is defined such that it equals 1 in each of these dates and zero otherwise. These dates coincide with events like the assassination of Kennedy, the Arab-Israeli War, the Gulf War and the 9/11 attack. The identification restriction posits that within a month the volatility indicator reacts only to the level of the S&P stock returns, but not to any of the aforementioned macroeconomic variable. The presence of the stock returns allows me to disentangle volatility shocks from any change in the level of stock market data. As housing market variables, I consider the median sales price of new one family homes sold[5], the number of new one family homes sold and the months supply provided by the Census Bureau. The latter is the ratio of houses for sale to houses sold and measures the number of months a house for sale is expected to last on the market. Hereafter I refer to this variable as the time on the market.

The benchmark ordering of the VAR considers the level of S&P 500 returns and the indicator of volatility first, then the interest rate, the consumer price index and the house price index, and finally the quantities with the industrial production, the level of sold houses and the time on the market. Figure 1.3 reports the response of house prices, the number of houses sold and the time on the market to a positive one standard deviation shock to volatility. Panel (b) shows that house prices respond very sluggishly to an increase in volatility, and start declining only around 10 months after the realization of the shock. At the peak, the response is around −0.001% below the baseline, which gives an

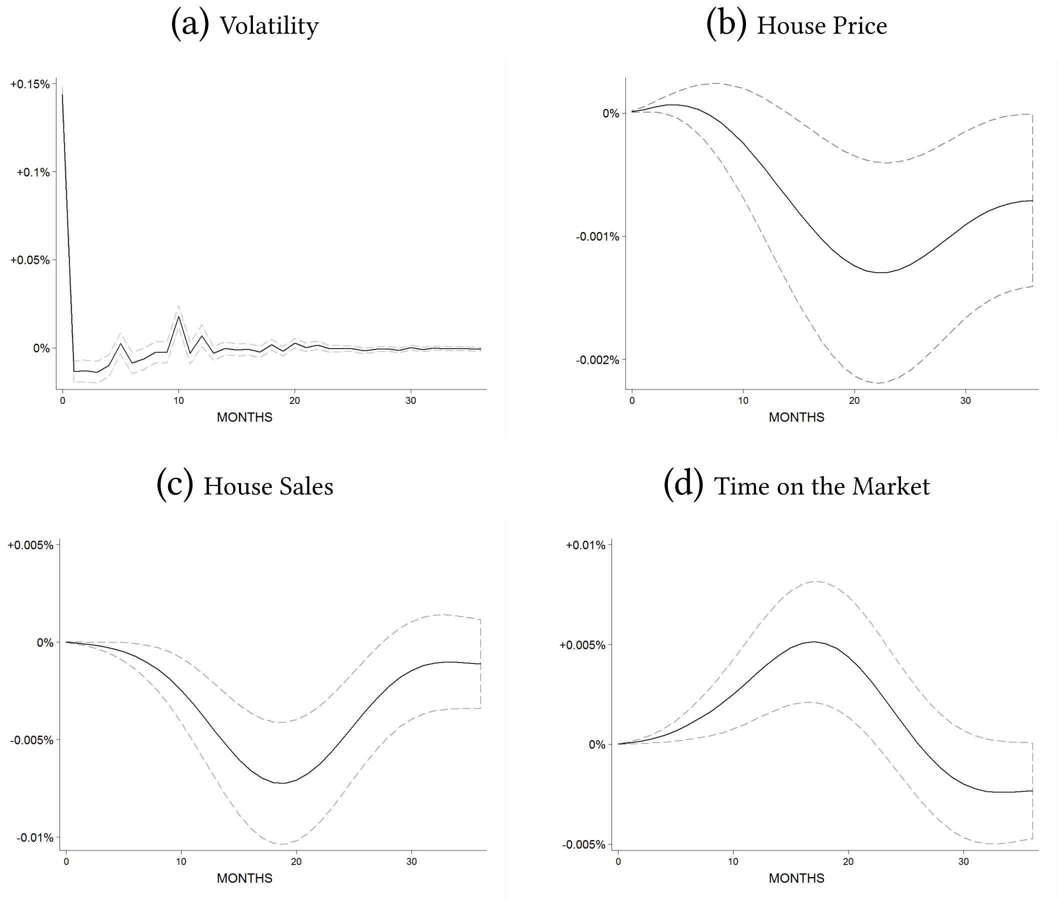
[4] Do volatility shocks cause financial crises? Figure 1.1 cannot identify whether the rise in the volatility of TFP causes the financial crisis or vice versa. Figure 1.2 shows that rises in aggregate volatility tends to lead the occurrence of financial crises. In Appendix 1.B, I plot the VIX index amidst the Great Recession, and I show that the VIX rose by around 60% at the beginning of 2007, well before the burst of the crisis.

[5] Results do not change when using the Conventional Mortgage Home Price Index, see Appendix 1.B.1.

annualized rate of −1.21%. Instead, an increase in volatility reduces the number of houses sold at peak by around −0.0065% on a monthly basis, which gives an annualized rate of −8.08%. Finally, a volatility shock raises the expected time on the market of a house in sale by 0.005% on a monthly basis, which corresponds to an annualized rate of 6.17%. This evidence suggests that volatility shocks do affect the housing market, mostly through changes in the number of houses sold and the time on the market of a house on sale.

FIGURE 1.3

VOLATILITY SHOCKS AND THE HOUSING MARKET.

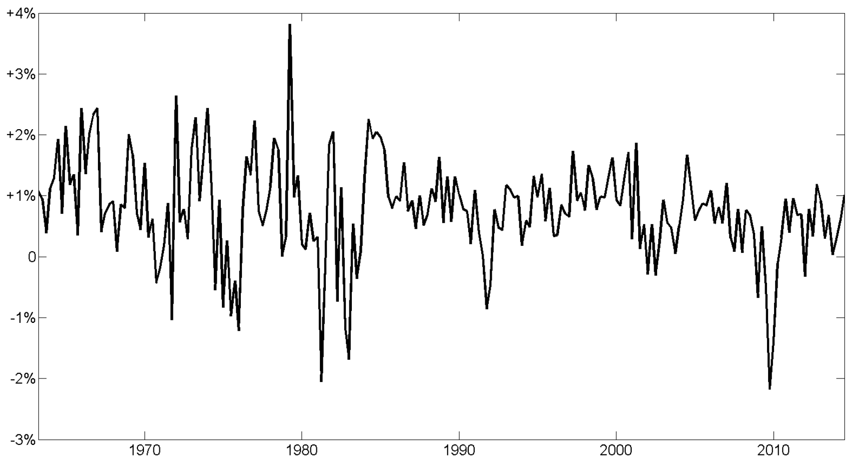


Note: VAR estimated from January 1963 to December 2013. The dashed lines are 1 standard-error bands around the response to a volatility shock. The coordinates indicate percent deviations from the baseline. The time on the market is measured by the monthly supply of homes, that is the ratio of houses for sale to houses sold.

This evidence is consistent with the dynamics of GDP growth volatility and housing market liquidity over the last decades. Figure 1.4 shows that the volatility of GDP growth rates has decreased starting from the 1980's, a phenomenon which is known as the Great Moderation. Stock and Watson (2002) document that in those years the standard deviations of GDP, consumption and investment have decreased by 41%, 38% and 22%, respectively. This trend has been partially reversed during the last recession. Figure 1.5 displays that the behavior of the housing market liquidity comoves with the volatility of the macroeconomic environment. Periods of low fluctuations experience a low time on the market, while turbulent periods - such as the oil crises in the 1970's and the Great Recession - have a much lower liquidity. Interestingly, the last period of the Great Moderation coincides with a historical low time on the market of a house of sale, at around 3.5 months. In the Great Recession, the time on the market peaked up to an all-time maximum of around 12 months.

FIGURE 1.4

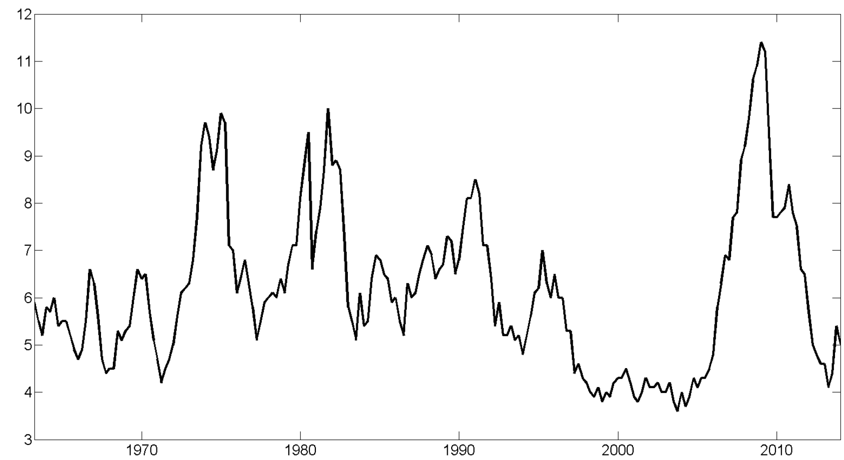
U.S. GDP GROWTH RATE



Note: The figure plots the quarterly series of US GDP growth rate from 1970Q1 until 2013Q4. The series is computed as the first difference of the log real GDP.

FIGURE 1.5

TIME ON THE MARKET OF HOUSES ON SALES IN THE U.S.



Note: The figure plots the quarterly series of the time on the market of houses on sale from 1963Q1 until 2013Q4. The time on the market series is given by the month supply of new one family houses from the Census Bureau.

1.3 The Model

1.3.1 Environment

In the economy there is a continuum of identical families that consist of a continuum of members. Although members live in different dwellings, there is perfect risk-sharing within the family. Families access a production function which assembles labor and housing to produce a consumption good. The technology is subject to aggregate productivity shocks with stochastic volatility.

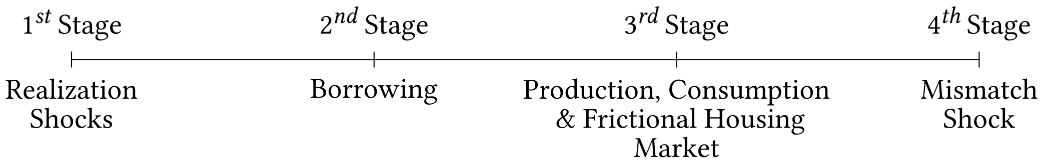
Family members trade real estate properties on a frictional market, such that there is a probability that a house on sale will not be matched with a buyer.

Families borrow from foreign lenders, and lack of commitment to repay debt. If families renege on debt, lenders seize their housing stock. To avoid the repudiation of debt, lenders impose a constraint on families' borrowing capacity. In equilibrium, families cannot borrow more than the collateral value of housing.

The role of housing is threefold: it provides utility services, it is a production input and it acts as the collateral asset.

Timing

Every period is split into four different stages. In the first one families observe the current realizations of the shocks. In the second one families borrow from the foreign lenders. This stage serves as a rationale for having in equilibrium a borrowing constraint that depends on current values of families' collateral. In the third stage production takes place and family members trade real estate properties on a frictional housing market. Finally, in the fourth stage a fraction of homeowners is hit by a mismatch shock and forced to leave the houses, which become vacant.



1.3.2 Families

The economy is populated by a continuum of families $i \in [0, 1]$. Each family consists of a continuum of ex-ante identical infinitely-lived members of measure one. Each member lives in a different dwelling and can own at most one house. Although family members individually trade houses, they pool their revenues within the family.

Each family maximizes the sum of their members' life-time utilities

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{i,t}, l_{i,t}, h_{i,t+1}\right) \tag{1.2}$$

where β is the time discount factor of family members, $c_{i,t}$ denotes the consumption of the family, $l_{i,t}$ is the level of leisure and $h_{i,t+1}$ is the end-of-period level of housing services which is assumed to be proportional to the end-of-period stock of occupied housing.

Families access a decreasing return to scale technology that uses labor force $n_{i,t}$, rented at the equilibrium wage w_t, and the stock of occupied housing $h_{i,t}$ to produce a homogeneous consumption good, as follows

$$y_{i,t} = e^{z_t} F\left(n_{i,t}, h_{i,t}\right). \tag{1.3}$$

The consumption good $y_{i,t}$ is sold on a frictionless market, and is the numeraire of the economy. The production function is subject to an aggregate productivity shock z_t, which follows an autoregressive motion with stochastic volatility

$$z_t = \rho_z z_{t-1} + e^{\sigma_t} \epsilon_{z,t} \tag{1.4}$$

$$\sigma_t = (1 - \rho_\sigma)\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta \epsilon_{\sigma,t} \tag{1.5}$$

where ρ_z denotes the persistence of the level of productivity, ρ_σ is the persistence of the volatility of productivity, $\bar{\sigma}$ is the long-run mean of volatility and η captures the degree of stochastic volatility of the process. When $\eta = 0$, the process reduces to a standard autoregression motion. Finally, $\epsilon_{z,t}$ and ϵ_{σ_t} denote the innovations to the level and volatility of productivity. I assume that they are i.i.d. following normal distributions $N\left(0, \sigma_{\epsilon_z}\right)$ and $N\left(0, \sigma_{\epsilon_\sigma}\right)$, respectively.

I appeal to this specification for aggregate productivity because the dynamics over time of the level and the volatility of aggregate productivity are pinned down by two different shocks, $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$, respectively. The two different sources of uncertainty, one related to the level and the other one linked to volatility, allows me to disentangle the role of volatility shocks and their contribution to the quantitative results of the model.[6]

1.3.3 The Housing Market

In the model houses are either occupied or vacant. Each family $i \in [0, 1]$ has a fraction of $h_{i,t}$ members which occupy a house and a fraction of $v_{i,t}$ members which own a house that does not fit their needs. I refer to the latter as vacant housing. I assume that vacant houses cannot be used as a production input, do not provide utility services and cannot

[6]E.g., in a GARCH model a unique shock drives the dynamics over time of both the level and the volatility of the process. I refer to Fernández-Villaverde et *al.* (2011) for further discussion on the topic.

be pledged as collateral. I further consider a fixed unit supply of houses.[7]

Real estate properties are traded on a frictional housing market. The search frictions capture in a reduced form the fact that matching in the housing market is time consuming. On one side of the market, each family has $v_{i,t}$ members who own vacant housing, which are put up on sale. On the other side of the market, there are $1 - h_{i,t}$ members which do not occupy a house and seek to buy one on the frictional market. Family members exercise a search effort $s_{i,t}$ - in units of time - in order to match with a seller. I assume that every unit of search effort comes at a monetary cost $\kappa s_{i,t}^2$. The ratio between the total amount of buyers (measured in efficiency units) to the total supply of houses on sale defines the tightness of the housing market

$$\theta_t = \frac{\int_0^1 \left(1 - h_{i,t}\right) s_{i,t} \, \mathrm{d}i}{\int_0^1 v_{i,t} \mathrm{d}i}. \tag{1.6}$$

A high market tightness θ_t indicates that the housing market is hot, that is, there are more buyers than sellers.

Following Wheaton (1990), the aggregate number of successful matches m_t is defined by a constant return to scale Cobb-Douglas matching function

$$m_t = \left(\int_0^1 \left(1 - h_{i,t}\right) s_{i,t} \, \mathrm{d}i\right)^{1-\gamma} \left(\int_0^1 v_{i,t} \mathrm{d}i\right)^{\gamma} \tag{1.7}$$

where $\gamma \in (0, 1)$. Upon a match, the transaction price of the house q_t^{mkt} is defined by a Nash bargaining problem, which I describe in Section 1.3.6. The matching function (1.7) stipulates that not all the houses supplied to the market are matched to a buyer. Indeed, the probability at which family members sell houses is

$$P_t^{\mathrm{sell}} = \frac{m_t}{\int_0^1 v_{i,t} \mathrm{d}i} = \theta_t^{1-\gamma}$$

which is increasing in the market tightness θ_t. The probability at which family members meet with buyers raises in hot housing market because there is a disproportionately larger amount of buyers exerting a high effort. Instead, the probability that a family member meets with a seller equals

$$P_t^{\mathrm{buy}} = \frac{m_t}{\int_0^1 \left(1 - h_{i,t}\right) s_{i,t} \, \mathrm{d}i} = \theta_t^{-\gamma}.$$

The probability of buying a house negatively depends on the tightness of the market. In a hot market, there are much more buyers than sellers, and any given family member is

[7] Davis and Heathcote (2007) find that the trend and volatility of US house prices are mostly driven by fluctuations in the price of land. Liu et *al.* (2013) show that fluctuations in land prices are a driving force of business cycle. In this vein, the housing stock in fixed supply of my model can be interpreted as land.

less likely to meet with a seller.

In this environment a family member manages to sell its house only with a probability P_t^{sell}. With the remaining probability $1 - P_t^{\text{sell}}$, the house keeps being on sale on the future period. Since vacant houses cannot be used either as production input and as collateral asset, I can define the option value of a house q_t^{opt} - the value of a house on sale that does not yield any utility service or dividend to the owner - when the frictional market opens as

$$q_t^{\text{opt}} = P_t^{\text{sell}} q_t^{\text{mkt}} + \left(1 - P_t^{\text{sell}}\right) \mathbb{E}_t \left[\Lambda_{t+1} q_{t+1}^{\text{opt}}\right]. \tag{1.8}$$

Equation (1.8) stipulates that the option value of housing depends on the liquidity of the housing market, the housing price and the continuation value of a vacant house. On the one hand, vacant houses have no option value when their selling probability in any future period goes to zero. On the other hand, the option value of vacant houses equals the market value of houses - as priced by the frictional market - when the current frictional market is perfectly liquid, that is, $P_t^{\text{sell}} = 1$. Notice that the option value q_t^{opt} is the actual value of houses put up on sale by the family members which sell their shelters. As long as the frictional market is partially illiquid, then $q_t^{\text{opt}} \leq q_t^{\text{mkt}}$, and the relevant house price for a seller is lower than the relevant house price for a buyer. Hence, the structure of the housing market endogenously generates a bid-ask spread $q_t^{\text{mkt}} - q_t^{\text{opt}}$ which depends on the liquidity of the frictional market.

Finally, I assume that a fraction ψ of homeowners is hit by a mismatch shock after that trading in the housing market has taken place.[8] Sellers cease to occupy their own dwelling, which adds to the stock of vacant housing that is carried over the next period. The laws of motion of occupied housing and vacant housing are

$$h_{i,t+1} = (1 - \psi) \left(h_{i,t} + P_t^{\text{buy}} s_{i,t} \left(1 - h_{i,t}\right)\right) \tag{1.9}$$

$$v_{i,t+1} = \left(1 - P_t^{\text{sell}}\right) v_{i,t} + \psi \left(h_{i,t} + P_t^{\text{buy}} s_{i,t} \left(1 - h_{i,t}\right)\right) \tag{1.10}$$

1.3.4 Borrowing Constraint

At the beginning of each period families observe the realizations of the aggregate shocks and then decide how much to borrow $d_{i,t+1}$. Families borrow from risk-neutral foreign investors, which inelastically supply funds at the gross interest rate R.[9] Families need

[8] The presence of the mismatch shock is often assumed in the literature of search frictions in the housing market, and dates back to Wheaton (1990). The shock allows to have always some vacant house in equilibrium. The mismatch shock can be interpreted by job mobility across locations, which forces homeowners to sell their real estate before relocating to a new city. Also a change in the number of people within a family could force homeowners to sell their house and buy a different one. The mismatch shock is analogous to the exogenous separation shock used in the search models of the labor market, see Pissarides (2000).

[9] This assumption is consistent with the analysis of Mendoza and Quadrini (2009) and Warnock and Warnock (2009) on the effects of US foreign capital inflows on the Treasury bill interest rates since mid 1980's.

also to purchase a fraction ν of the labor cost $w_t n_{i,t}$ in advance of production. Hence, they receive a working capital loan from the foreign investors. Working capital loans are repaid within the period and do not carry interest payments.

Families lack full commitment and can immediately decide to renege on their debt. In such a case, the lenders seize the housing stock $h_{i,t}$. Under the further assumptions that financial contracts are not exclusive, families can renege on their debt only at the beginning of each period and there is no additional penalty in repudiating the debt, Appendix 1.D.1 shows that in equilibrium the collateral constraint equals

$$\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq \underbrace{q_t^{\text{opt}} h_{i,t}}_{\text{Collateral Value of Housing}} . \tag{1.11}$$

As in Iacoviello (2005), families' borrowing capacity is determined by the collateral value of the housing. In Iacoviello (2005) the collateral value of housing equals to an exogenous fraction of its market value. In my model the collateral value of families' housing stock is always lower than its market value, as long as the housing market is not perfectly liquid, and there is a spread between the house price q_t^{mkt} and the option value of a house q_t^{opt}.

Multiplying and dividing the right-hand side of the constraint by the price of occupied housing q_t^{mkt}, the constraint becomes

$$\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq \underbrace{\frac{q_t^{\text{opt}}}{q_t^{\text{mkt}}}}_{\text{Maximum Loan-to-Value Ratio}} \times \underbrace{q_t^{\text{mkt}} h_{i,t}}_{\text{Market Value of Housing}} . \tag{1.12}$$

Equation (1.12) shows that the collateral value of agents depends on the market value of their housing stock, multiplied by a factor which defines the maximum LTV ratio. Standard models usually assume that the degree of pledgeability of the collateral is an exogenous parameter. Instead, in this framework the LTV ratio is endogenous and depends on the liquidity of the housing market. When the housing market liquidity freezes out, the low probability of selling vacant houses raises the wedge between the prices of occupied housing q_t^{mkt} and vacant housing q_t^{opt}. As a result, the LTV ratio $\frac{q_t^{\text{opt}}}{q_t^{\text{mkt}}}$ decreases. Therefore, Equation (1.12) defines the direct link through which the liquidity of the housing market endogenously determines agents' borrowing capacity. In this environment the LTV ratio moves over time because of the changes in the liquidity of the housing market.

1.3.5 Decentralized Equilibrium

The families use output net of the labor cost $z_t F\left(n_{i,t}, h_{i,t}\right) - n_{i,t} w_t$, the revenues from supplying labor $(1 - l_{i,t}) w_t$, the new amount of borrowing $\frac{d_{i,t+1}}{R}$ and the revenues from selling houses $q_t^{\text{mkt}} p_t^{\text{sell}} v_t$, to finance consumption $c_{i,t}$, the searching cost $\kappa s_{i,t}^2$, the repayment of debt $d_{i,t}$, and the purchases of new occupied houses $q_t^{\text{mkt}} P_t^{\text{buy}} s_{i,t} \left(1 - h_{i,t}\right)$. Therefore, families' budget constraint reads

$$c_{i,t} + \kappa s_{i,t}^2 + q_t^{\text{mkt}} P_t^{\text{buy}} s_{i,t} \left(1 - h_{i,t}\right) + d_{i,t} = \left[e^{z_t} F\left(n_{i,t}, h_{i,t}\right) - n_{i,t} w_t\right] + \dots$$
$$\dots + (1 - l_{i,t}) w_t + q_t^{\text{mkt}} P_t^{\text{sell}} v_t + \frac{d_{i,t+1}}{R}. \quad (1.13)$$

Hereafter, I focus on a symmetric competitive equilibrium. Since families are all ex-ante identical and there is no source of idiosyncratic uncertainty, families face the same budget and borrowing constraint, and take identical optimal choices. Therefore, I drop the subscripts from all the variables of the model.

The states of the families' problem are given by its stock of occupied houses h_t, the level of debt d_t, aggregate bond holdings D_t, the aggregate stock of occupied houses H_t and finally the level and volatility of productivity, z_t and σ_t. Since the stock of housing is in fixed supply, families do not need to take in account the stock of vacant houses v_t.

As long as prices depends on the aggregate level of bond holdings, and optimal decisions depend on current and future prices, families have to forecast also future aggregate bond holdings. I denote by $\Gamma_D(H, D, z, \sigma)$ the law of motion of aggregate bond holding D perceived by any family, and $\Gamma_H(H, D, z, \sigma)$ is the law of motion of the aggregate stock of housing occupied by the families H. Then, the individual maximization problem is

$$V(h, d; H, D, z, \sigma) = \max_{c,l,n,s,d'} \left\{ U\left(c, l, h'\right) + \beta \mathbb{E}_{z',\sigma'|z,\sigma}\left[V\left(h', d'; H', D', z', \sigma'\right)\right]\right\}$$

$$s.t. \quad c + d + C_h = e^z F(n, h) + \frac{d'}{R} + G_h \quad (1.14)$$

$$C_h = \kappa s^2 + q^{\text{mkt}}(H, D, z, \sigma) P^{\text{buy}}(H, D, z, \sigma) s (1 - h) \quad (1.15)$$

$$G_h = q^{\text{mkt}}(H, D, z, \sigma) P^{\text{sell}}(H, D, z, \sigma) v \quad (1.16)$$

$$h' = (1 - \psi)\left(h + P^{\text{buy}}(H, D, z, \sigma) s (1 - h)\right) \quad (1.17)$$

$$\frac{d'}{R} + \nu w(H, D, z, \sigma) n \leq q^{\text{opt}}(H, D, z, \sigma) h \quad (1.18)$$

$$D' = \Gamma_D(H, D, z, \sigma) \quad (1.19)$$

$$H' = \Gamma_H(H, D, z, \sigma) \quad (1.20)$$

subject to the law of motion for the TFP shocks as described by Equation (1.4). Equation (1.14) denotes the budget constraint, Equation (1.15) defines the total cost of trading housing C_h, Equation (1.16) is instead the total gain from trading housing G_h, Equation (1.17) denotes the law of motion of occupied houses, Equation (1.18) is the borrowing constraint and Equations (1.19) - (1.20) stipulate the perceived laws of motion for total bond holdings and occupied housing. Note that in the symmetric equilibrium $n_t = 1 - l_t$.

Upon observing the states of the economy, agents decide the optimal policy on consumption $\hat{c}(h, d; H, D, z, \sigma)$, working hours $\hat{n}(h, d; H, D, z, \sigma)$, the search effort in the housing market $\hat{s}(h, d; H, D, z, \sigma)$, and the amount of resources to borrow from the foreign investors $\hat{d}'(h, d; H, D, z, \sigma)$. In equilibrium, the perceived level of aggregate bond holdings $\Gamma_D(H, D, z, \sigma)$ has to coincide with the individual policy $\hat{d}'(h, d; H, D, z, \sigma)$, and the same applies for the law of motion of the aggregate stock of occupied housing $\Gamma_H(H, D, z, \sigma)$. Appendix 1.C reports the definition of equilibrium and the first-order conditions of the problem.

1.3.6 Nash Bargaining

The price q_t^{mkt} of an occupied house $h_{i,t}$ which is sold on the frictional market is determined through the following Nash bargaining problem

$$q_t^{\text{mkt}} \equiv \arg\max_{q_t^{\text{mkt}}} \left\{ S\left(q_t^{\text{mkt}}\right)^{\zeta} B\left(q_t^{\text{mkt}}\right)^{1-\zeta} \right\} \tag{1.21}$$

$$s.t. \quad S\left(q_t^{\text{mkt}}\right) = q_t^{\text{mkt}} - \mathbb{E}_t\left[\Lambda_{t+1} q_{t+1}^{\text{opt}}\right] \geq 0 \tag{1.22}$$

$$B\left(q_t^{\text{mkt}}\right) = V_t^H - q_t^{\text{mkt}} \geq 0 \tag{1.23}$$

where $S\left(q_t^{\text{mkt}}\right)$ is sellers' surplus in case of a transaction, $B\left(q_t^{\text{mkt}}\right)$ denotes buyers' surplus, ζ is sellers' bargaining power, and V_t^H is the fundamental value that families attribute to a new unit of occupied housing. The expected future price of vacant houses is the outside opportunity for a family member that does not manage to sell its house.

In the symmetric competitive equilibrium each family has the same fundamental value of occupying a house and therefore the identity of the buyer does not matter on the specification of the house price. Indeed, in equilibrium the price of a occupied house is

$$q_t^{\text{mkt}} = \zeta V_t^H + (1-\zeta)\mathbb{E}_t\left[\Lambda_{t+1} q_{t+1}^{\text{opt}}\right]. \tag{1.24}$$

Families' fundamental value of housing can be derived using the envelope condition on the optimal stock of occupied housing, which yields

$$\begin{aligned} V_t^H = &\psi \mathbb{E}_t\left[\Lambda_{t+1}\left(P_{t+1}^{\text{sell}} q_{t+1}^{\text{mkt}} + \left(1 - P_{t+1}^{\text{sell}}\right) q_{t+1}^{\text{opt}}\right)\right] + \dots \\ &\dots + (1-\psi)\,\mathbb{E}_t\left[\Lambda_{t+1}\left(V_{t+1}^H + \frac{U_{h_{t+1}}}{U_{c_{t+1}}} + e^{z_{t+1}} F_{h_{t+1}} + \frac{\phi_{t+1}}{U_{c_{t+1}}} q_{t+1}^{\text{opt}}\right)\right] \end{aligned} \tag{1.25}$$

where Y_{x_t} denote the derivatives of the function $Y(\cdot)$ with respect the term x_t and ϕ_t is the Lagrange multiplier associated to the borrowing constraint. The fundamental value of a marginal house bought by a family member can be interpreted as follows. First, with probability ψ the new homeowner is hit by the mismatch shock and forced to sell the house. The house is successfully matched with a buyer with a probability P_{t+1}^{sell} and keeps being on sale with the remaining probability. If the family member is not hit by the mismatch shock, then she will effectively occupy the house over the following period. In this case, the family receives the utility service from occupying the house, uses the house as an input in the production function and gains the marginal productivity $e^{z_{t+1}}F_{h_{t+1}}$. Moreover, the family enjoys the continuation value of owning the house V_{t+1}^{H}. Finally, the ownership of an additional house increases the collateral value of families' housing stock, relaxing its borrowing constraint. Thereby, families can access a larger loan, increase consumption and raise its utility level by $\frac{\phi_{t+1}}{U_{c_{t+1}}} q_{t+1}^{\text{opt}}$.

1.3.7 Characterization of the Decentralized Equilibrium

Proposition 1. *In a steady-state equilibrium the LTV ratio $\frac{q_t^{opt}}{q_t^{mkt}}$ positively depends on the liquidity of the housing market. Proof. See Appendix 1.D.2.*

In this model the LTV ratio is endogenous and depends on the liquidity of the housing market. When the market heats up, the ratio increases and therefore families' borrowing capacity increases. Analogously, a liquidity freeze tightens the LTV ratio, decreasing families' borrowing capacity. This result implies that the observed movements in maximum LTV ratios could be partially accounted for by changes in the liquidity of the housing market.

Proposition 2. *The house price q_t^{mkt} negatively depends on the current shadow value of families' borrowing constraint. Proof. See Appendix 1.D.3.*

When families become borrowing constrained, they decrease the level of search effort on the housing market generating a fire sale spiral which is detrimental for house prices q_t^{mkt}. In this environment fire sales negatively affect both families' collateral value and their LTV ratio, triggering a deflationary spiral in both the house price and the LTV ratio.

Proposition 3. *The tightness of the housing market equals family members' search effort.*

The behaviour of the frictional housing market is starkly simplified in a symmetric competitive equilibrium. Indeed, in this equilibrium every member opts for the same level of search effort, implying that $\int_0^1 \left(1 - h_{i,t}\right) s_{i,t} \mathrm{d}i = (1 - h_t)\, s_t$. As a result, the equilibrium market tightness becomes

$$\theta_t = \frac{(1 - h_t)\, s_t}{v_t} = s_t$$

since the total housing stock is an unitary fixed supply. The tightness of the housing market entirely depends on the search effort exerted by buyers. Hence, the housing market is hot as long as the level of effort is high. This result further implies that in each period the probability of selling a house is $P_t^{\text{sell}} = \theta_t^{1-\gamma} = s_t^{1-\gamma}$. Hot housing market are characterized by a high level of effort from buyers and a high probability of selling a house. The opposite applies in cold markets.

Instead, the probability of buying a house equals $P_t^{\text{buy}} = \theta_t^{-\gamma} = s_t^{-\gamma}$. Given this probability, the total amount of houses bought by a family member becomes $P_t^{\text{buy}} s_t (1 - h_t) = s_t^{1-\gamma} (1 - h_t)$, which is increasing in the level of search effort exerted by family members.

Implication 1. *The frictional housing market generates partial irreversibilities in housing investment.*
The price at which families purchase housing q_t^{mkt} is higher than the expected price at which they sell houses q_t^{opt}. As in Duffie et *al.* (2005), the bid-ask spread depends on the presence of the search frictions. This spread implies that the housing investment is partial irreversible (i.e., the marginal gain of disinvestment is lower than the marginal cost of investment) and the degree of irreversibility fluctuates over time as a function of housing market liquidity. Investment is more irreversible in cold housing market.

Implication 2. *Partial irreversibilities in investment together with the presence of a decreasing to scale production function allow volatility shocks to have real effects: an increase in volatility freezes housing investment.*
Partial irreversibilities in investment coupled together with a decreasing return to scale production function make changes in volatility to have real effects. When it is expensive to reverse housing investment, family members become cautious and lower their search effort in uncertain times. Thus, a high volatility reduces the liquidity of the housing market. Instead, in a stable macroeconomic environment, agents increase their search effort and the housing market heats up. Decreasing returns to scale are key for this result. Caballero (1991) shows that a higher uncertainty decreases investment only in environment in which asymmetric adjustment costs interact with either imperfect competition or decreasing returns to scale technologies. If profits are convex in demand or costs, then a higher uncertainty actually rises expected profits leading to an investment boom.

1.4 Quantitative Analysis

I calibrate one period of the model to correspond to one quarter. To understand the quantitative relevance of the link between volatility, liquidity and financial crises, I estimate the shocks to both the level and the volatility of the aggregate total factor productivity of the U.S. economy using quarterly data from 1947Q2 until 2013Q4. The level and volatility shocks are estimated using Bayesian Sequential Monte Carlo methods.

I calibrate most of the parameters of the model to the values either estimated or used in previous papers. The main parameters which I calibrate to an empirical targets is the cost of searching effort in the housing market. Indeed, in Section 1.3.7 shows that the probabilities of buying and selling a house in the frictional market depend on the search effort exerted by the families. If the effort was costless, then the search frictions would be offset by an infinitely amount of search effort exerted by family members, and the liquidity of the housing market would be perfect. I calibrate the cost of search effort to match the long-run mean of the time on the market of a house on sale.

The model is solved numerically using global methods, which do not rely on approximations based on Taylor expansions around the steady state. Although the algorithm is much more time intensive, it preserves the non-linear dynamics of the model.

1.4.1 Estimating the Volatility of Total Factor Productivity

In the model the ultimate source of the build-up of risk and burst of financial crises is given by shocks to the level and the volatility of TFP. To understand the quantitative relevance of this mechanism, I take the actual series of level shocks and volatility shocks to TFP from the data. Namely, I derive the Solow residual of the economy using quarterly data on output, capital and labor from 1947Q2 until 2013Q4 and apply a one sided HP filter with parameter $\lambda = 1600$.As in Section 1.2, I estimate the process using Bayesian methods. I elicit some unrestrictive priors, and after deriving the likelihood of the process for some given parameters with the Sequential Importance Sampling (SIS) particle filter introduced in Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et *al.* (2011), I maximize the posterior likelihood using a random walk Metropolis-Hastings algorithm with 25000 replications, out of which the first 5000 represent burn-in draws. Finally, I recover the historical distribution of the time varying volatility using the backward-smoothing routine of Godsill et *al.* (2004).

1.4.2 Estimation Results

I elicit Beta priors centred around 0.90 for both the autocorrelation coefficients of the level equation ρ_z and the volatility equation ρ_σ. The implicit assumption is that both the level and the volatility are known to be highly persistent over time. For the degree of stochastic volatility η, I elicit a a Gamma prior with mean 0.315 and standard deviation 0.03 following the posterior estimate of Born and Pfeifer (2013), who derive the stochastic volatility of the U.S. Solow residual using data from 1970 on. Finally, I define a uniform distribution between -11 and -3 for the long-run log volatility $\bar{\sigma}$.

Table 1.1 reports the median, the 5-th and 95-th quantiles of the posterior distribution of each parameter. I find a strong persistence in both the level and volatility of TFP. The latter is very important because the mechanism of the model relies on the existence of a long period of low volatility which fosters a credit boom, and boosting households' leverage. The process is also characterized by a high degree of stochastic volatility. A one standard deviation increase in the volatility shocks raises the volatility of the innovation to the level of TFP by $(e^{\eta} - 1) \times 100 = 32.4\%$.

Table 1.1
Estimation of the Stochastic Volatility of TFP

Parameter	**Prior**			**Posterior**		
	Distribution	Mean	Std. Dev.	Median	5 Percent	95 Percent
ρ_z	Beta	0.90	0.10	0.8137	0.7500	0.8734
ρ_σ	Beta	0.90	0.10	0.7949	0.6071	0.9065
η	Gamma	0.315	0.03	0.2805	0.2362	0.3267
$\bar{\sigma}$	Uniform	-7.00	2.30	-5.3869	-5.5792	-5.2130

Note: ρ_z denotes the autocorrelation parameter of the level equation, while ρ_σ is the autocorrelation of the volatility equation. η captures the degree of stochastic volatility in the process, and $\bar{\sigma}$ denotes the long-run log volatility.

1.4.3 Calibration Exercise

Most of the parameters of the model are targeted to values estimated or used in previous papers. In particular, the calibration closely follows Bianchi and Mendoza (2013). The crucial parameter which is calibrated is the cost of exerting search effort on the housing market, which determines the behavior of the tightness of the market and eventually the level of housing liquidity.

I consider the following utility function for the families

$$U\left(c_t, l_t, h_{t+1}\right) = \frac{\left(c_t^{\xi} h_{t+1}^{1-\xi} - \mu \frac{(1-l_t)^{1+\omega}}{1+\omega}\right)^{1-\delta} - 1}{1-\delta}$$

The parameters δ, μ and ω denote the risk aversion, the degree of disutility from working and the inverse of the Frisch elasticity of family members. The parameter ξ governs the substitutability between consumption and housing.[10] This utility function belongs to the class of preferences introduced in Greenwood et *al.* (1988), and rules out any wealth effect on the labor supply, which would counter-factually lead to an increase in labor supply during a crisis. I set the disutility of work as $\mu = \alpha_n$ to have mean hours that equal 1.

[10] The Cobb-Douglas function in consumption expenditures reflects the fact that expenditure shares on housing are constant over time and across metropolitan areas, see Davis and Ortalo-Magné (2011).

Then, the Frisch elasticity is defined as $1/\omega = 1$ and I set the risk aversion $\delta = 2$ as in Bianchi and Mendoza (2013), whereas $\xi = 0.76$ as in Davis and Ortalo-Magné (2011), who find that the share of households' expenditure in housing is constant over time around a value of 24%. The subjective time discount factor is set to the standard value at the quarterly frequency of $\beta = 0.99$.

I stipulate a decreasing return to scale production function

$$y_t = e^{z_t} n_t^{\alpha_n} h_t^{\alpha_h}$$

where $\alpha_n + \alpha_h < 1$. The parameter α_h is calibrated to match the ratio of housing stock value over the GDP. Using data from the Flow of Funds from 1952Q1 until 2013Q4, the ratio of the market value of the real estate of the private nonfinancial sector over GDP is 2.24. In the model, this average is matched by a value of $\alpha_h = 0.11$. Instead, the labor share is set to the standard value of $\alpha_n = 0.64$. Overall, the returns to scale of the technology sum up to 0.75. Finally, the productivity process z_t inherits the data generator process estimated in the previous Section.

I calibrate the gross real interest rate to $R = 1.0065$, that is the value that Bianchi and Mendoza (2013) estimate for the average of the ex-post real interest rate on three months Treasury Bills over the last three decades. Instead, the working capital coefficient is set to $\nu = 0.17$. To compute this value, I use firms' M1 money holdings to proxy for their working capital. Since two-thirds of the total M1 stock are held by firms, M1 accounts on average for 16% of annual GDP over the period 1959Q1-2013Q4, and the 0.64 labor share defined above, I set $\nu = 4 * (2/3) * 0.16/0.64 = 0.68$. Hence, firms maintain 68 percent of their quarterly wage bill in cash. This number is very close to the value of 0.63 used in Schmitt-Grohe and Uribe (2007).

Finally, I calibrate the parameters characterizing the dynamics of the housing market as follows. I define the mismatch shock to be equal to $\psi = 0.0278$ to match the average stay in a house of 9 years reported by Ngai and Tenreyro (2014). The parameter of the matching function which refers to the houses supplied to the market by the real estate sector is set to $\gamma = 0.21$ following the value estimated in Genesove and Han (2012). The bargaining power of the seller is set such as $\zeta = \gamma$ so that the Hosios (1990) condition holds. Finally, the monetary cost of exerting searching effort in the frictional market is calibrated to match the average time on the market of a house on sale using data from 1963Q1 until 2013, which is 6.21 months. In this way, I find a value of $\kappa = 0.67$.

Table 1.2
Calibrated Parameters

Parameter	Value	Target/Source
Disutility from work	$\mu = \alpha_n$	Normalization
Inverse Frisch elasticity	$\omega = 1$	Bianchi and Mendoza (2013)
Substitutability consumption/housing	$\xi = 0.76$	Davis and Ortalo-Magné (2011)
Risk aversion	$\delta = 2$	Standard value
Time discount	$\beta = 0.99$	Standard value
Share labor	$\alpha_n = 0.64$	Standard value
Share housing	$\alpha_h = 0.11$	Ratio real estate value over GDP=2.24
Gross real interest rate	$R = 1.0065$	Average return Treasury Bills
Working capital parameter	$\nu = 0.68$	Ratio M1 over GDP held by firms
Mismatch shock	$\psi = 0.0278$	Ngai and Tenreyro (2014)
Sellers' matching function parameter	$\gamma = 0.21$	Genesove and Han (2012)
Sellers' bargaining power	$\zeta = \gamma$	Hosios' condition
Cost searching effort	$\kappa = 0.67$	Average TOM house on sale

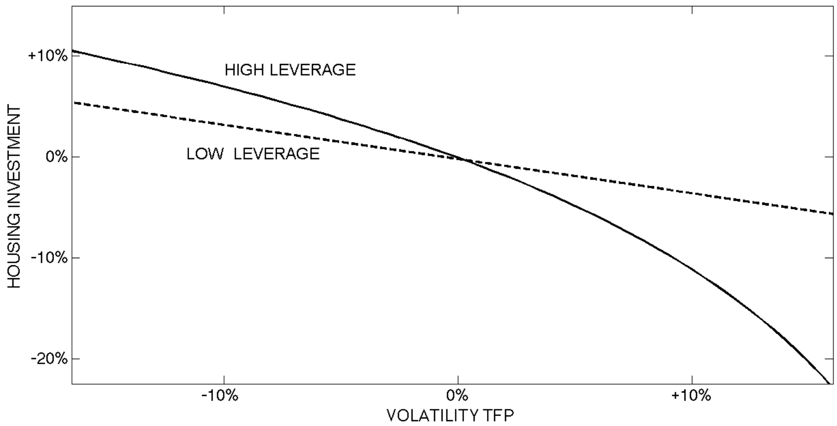
Note: The table report the calibrated value of all the parameters of the model, except for the DGP of the technology shock. TOM refers to the expected time on the market.

1.4.4 Quantitative Results

Real Effects of Volatility Shocks

How do volatility shocks affect the real economy? Figure 1.6 plots the policy function of housing investment (i.e., searching effort in the housing market) as a function of the volatility of TFP - at different levels of households' leverage. I report the values of both housing investment and TFP volatility as percentage deviations from their ergodic steady-state.

FIGURE 1.6
POLICY FUNCTION OF HOUSING INVESTMENT



The figure plots the policy function of housing investment (i.e., searching effort in the housing market) as a function of the volatility of TFP - for two different levels of households leverage, low and high. Housing investment and TFP volatility are defined as percentage deviations from the ergodic steady state.

Figure 1.6 shows that housing investment is decreasing in the level of TFP volatility. The higher the volatility, the more household members are discouraged to search for a house, the lower overall housing investment. Interestingly, the relationship between housing investment and TFP volatility does depend on households' leverage. When leverage is low, a 10% increase in the volatility of TFP reduces housing investment by around -3.5%. Instead, when households' leverage is high, the same change in volatility implies a fall in housing investment by around -7.5%. These differences are due to the fact that households' borrowing constraint is more likely to bind at high levels of leverage. When the constraint becomes binding, a small shock to productivity propagate much more into the real economy, because households have no additional borrowing capacity to smooth out the effects of shocks.

In addition, the relationship between housing investment and volatility becomes highly non-linear at high levels of leverage. Indeed, when leverage is high and volatility peaks, then the borrowing constraint becomes binding, which forces households to sharply reduce their debt and fire sell their housing stock.

Frequency and Severity of Financial Crises.

I compare the quantitative performance of five different economies with the data. In the first economy, which I refer to as the "RBC" case, I consider a standard model in which there are only level shocks to TFP and the housing market is perfectly liquid. In the second alternative, which I refer to as the "Search Frictions" economy, I add search frictions in the housing market in the "RBC" economy. This second case disentangles the role of search frictions alone in capturing the dynamics of financial crises. In the third case, which I refer to as the "Stochastic Volatility" economy, I add the volatility shocks to TFP to the "RBC" economy. Hence, this case disentangles the role of stochastic volatility once it is introduced in a standard framework with a perfectly liquid housing market. In the fourth and fifth economies I consider a "RBC" economy with *both* search frictions in the housing market and volatility shocks to TFP. The only difference between these two economies is that in the "Fixed LTV" economy I consider a constant LTV ratio at 100%, while in the "Stochastic LTV" economy I consider a LTV ratio which is endogenous and moves over time as a function of housing liquidity. These two economies disentangle the role of changes in the collateral liquidity as a propagation mechanism for the volatility shocks.

The addition of stochastic volatility - throughout the five economies - does not alter the unconditional mean of volatility. The presence of a stochastic volatility implies only a time varying pattern around the unconditional mean. The quantity of aggregate risk is the same in all the scenarios I compare.

Table 1.3 reports the results of the model on the frequency and the severity of financial crises, on a sample of simulated data over 10,000 periods. I compare the frequency and severity of financial crises implied by the five economies with the actual moments recovered from U.S. data. I define a financial crisis in the model as the state in which aggregate credit growth falls down by more than one standard deviation. According to this definition, over the last century there have been two financial crises: in 1929 and in 2007. As measures of severity, I consider the cumulative drop of output growth, employment drop and credit growth during the two years following a financial crisis (i.e, on the year upon the occurrence of the crisis and the following one).

The second column of Table 1.3 shows that a standard RBC model generates too few and mild crises. This economy accounts for around 39% of the frequency of crises and 52% of the output drop. Introducing search frictions raises the probability of experiencing a financial crisis by 15% and the associated drop in output, employment and credit by 12%, 18% and 11%, respectively. Hence, changes in the liquidity of the collateral improve the performance of the model to a limited extent, when the ultimate source of exogenous variation is given by shocks to the level of TFP. Instead, if I consider the role of stochastic volatility of TFP into a standard RBC economy, the "Stochastic Volatility" case, I find that volatility shocks barely improves the predictions of the model. A time varying volatility amplifies the severity of crises just by 8%, while the drops in output, employment and credit hardly change. These results point out that volatility shocks need a propagation mechanism in order to have a relevant role in capturing the dynamics of credit crunches.

TABLE 1.3
RESULTS

	Data	**RBC**	**Search Frictions**	**Stochastic Volatility**	**Search Frictions & Stochastic Volatility**	
					Fixed LTV	**Stochastic LTV**
Frequency Crises	2.00%	0.78%	0.90%	0.84%	0.95%	1.21%
Output Drop	−9.78%	−5.07%	−5.68%	−5.11%	−6.03%	−7.99%
Employment Drop	−10.29%	−4.20%	−4.96%	−4.37%	−5.14%	−6.03%
Credit Drop	−12.07%	−7.06%	−7.81%	−7.52%	−8.40%	−9.25%

Note: The output drop, employment drop and credit drop refer to the fall in output growth, employment growth and credit growth over the two years following a financial crisis (i.e., upon the year of occurence and the following one). In the model, a financial crisis correspond to the state in which aggregate credit falls down by more than one standard deviation. The "RBC" refers to an economy with only level shocks to TFP and a perfectly liquid housing market. The "Search Frictions" refers to a RBC economy with a frictional housing market. The "Stochastic Volatility" refers to a RBC economy with stochastic volatility. The "Search Frictions & Stochastic Volatility" refers to a RBC economy with stochastic volatility and a frictional housing market. This economy is studied in two different cases. In the first one, "Fixed LTV", the LTV ratio is fixed at 100%. In the second one, "Stochastic LTV", the LTV ratio is endogenous and moves over time as a function of housing liquidity.

When I consider the benchmark economy with both search frictions and stochastic volatility of TFP, in which the LTV ratio is endogenous and moves over time as a function of liquidity (the "Search Frictions & Stochastic Volatility - Stochastic LTV" case), then the frequency of crises implied by the model raises to 1.21%, with an associated drop in output, employment and credit of -7.99%, -6.03% and -9.25%. Thus, the interaction of volatility shocks and search frictions in the housing market raises the probability of experiencing a crisis by around 55% with respect the basic RBC economy and accounts for around 60% of the observed frequency of crises. As far as the severity of the crisis is concerned, volatility shocks and search frictions boost the drops of output in employment by around 58% and 44%, and the fall in credit by 31%.

The results of the "Search Frictions & Stochastic Volatility - Fixed LTV" disentangle the contribution of the changes in the collateral liquidity on the performance of the model. When the LTV ratio is kept constant at 100%, then the predictions of the model worsens, both in terms of frequency and severity of crises. The comparison between the "Fixed

LTV" and "Stochastic LTV" economies shows that changes in the liquidity of the collateral accounts for most of the increase in the frequency of financial crises of the benchmark economy.

These results point out that either search frictions or volatility shocks can improve the performance of the model, although falling short in capturing the characteristics of crises as in the data. The interaction of a frictional housing market and volatility shocks accounts for half of the probability of experiencing a crisis, and its corresponding drop in output and credit. The changes in the liquidity of the collateral, that eventually modifies the LTV ratio, represent the key mechanism through which volatility shocks propagate into the real economy. This finding adds to the literature on the real effects of volatility shocks, such as Justiniano and Primiceri (2008), Bloom (2009) and Fernandez-Villaverde et *al.* (2011), by pointing out that search frictions in the housing market amplifies the effects of the changes in volatility.

Dynamics of Aggregate Productivity around Financial Crises.

Table 1.3 shows that the volatility shocks to aggregate productivity raises the occurrence of financial crises by around 55%. What is then the dynamics of these shocks around financial crises? Figure 1.7 plots the behavior of the level and volatility of TFP around a crisis, as implied by the model.

The graphs show that the period preceding a crisis is characterized by a high level and a low volatility of productivity. In the model, a financial crisis bursts after a long period of expansion in the economic activity. For example, the level of productivity is 3% above its long-run mean three years before a crisis, while the volatility of productivity is around 15% below mean. This long period of high level of productivity with low volatility generates a credit and investment boom which reinforce each other, raising the equilibrium LTV ratio and ultimately boosting households' leverage. In this way, these realizations of high productivity and low volatility builds up systemic risk. Indeed, a joint 2.5% drop in the level of productivity and a rise in volatility of around 27% turn the borrowing constraint into binding and trigger a crisis. Hence, a crisis coincides with both a slump in the level of productivity and a sudden spike in volatility which follow a long period of high productivity with low volatility.

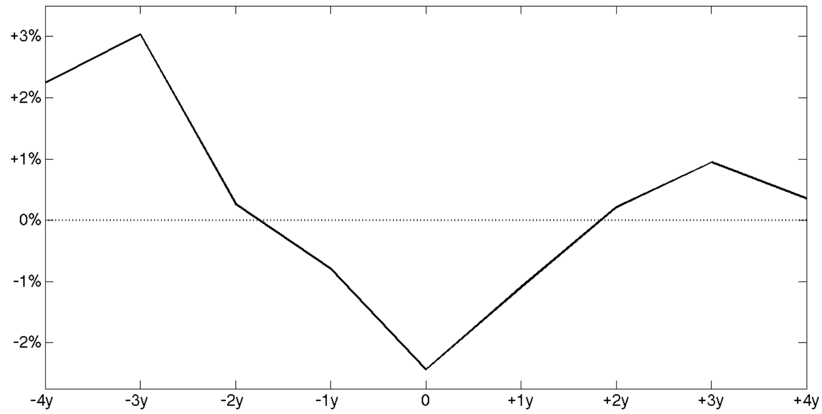
To understand the role of the changes in households' collateral values, I plot the dynamics of house price and the LTV ratio around financial crises in Figure 1.8. It shows that financial crises are preceded by an inflationary spiral in both the house prices and the LTV ratio which relaxes households' credit limit and raises the level of leverage and therefore systemic risk in the economy. Afterwards, the inflationary spirals are abruptly reversed into deflationary spirals amidst the burst of the financial crisis.

In the model a financial crisis is preceded by a boom in house price of around 8% above mean, which is then turned into a house price bust at 8% below trend. A similar dynamics

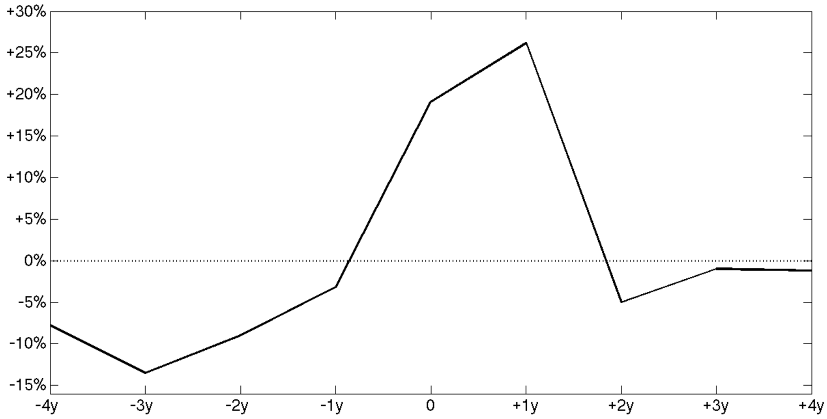
Figure 1.7

Dynamics of Aggregate Volatility around Financial Crises.

(a) Level of Aggregate Productivity



(b) Volatility of Aggregate Productivity

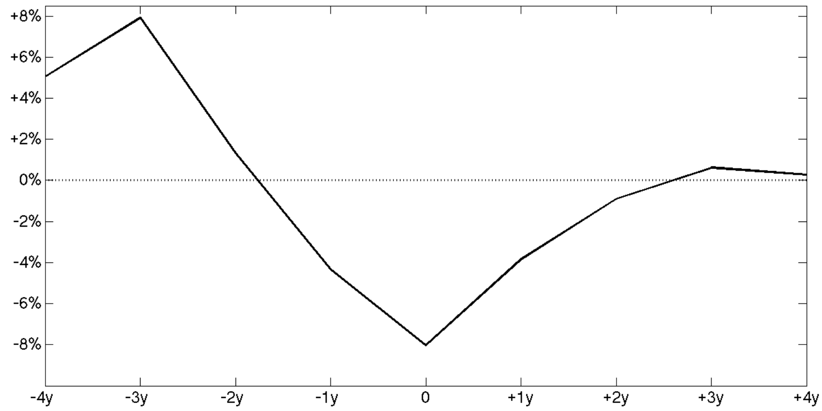


Note: The figure plots the average values of the deviations from the long-run mean of the level z_t (Panel a) and the volatility σ_t (Panel b) of total factor productivity in a 9 year window around financial crises. The solid line denotes the dynamics implied by the benchmark model, whereas the dashed line denotes the dynamics in the data. A financial crisis is defined as the state in which aggregate credit growth drops down by more than one standard deviation.

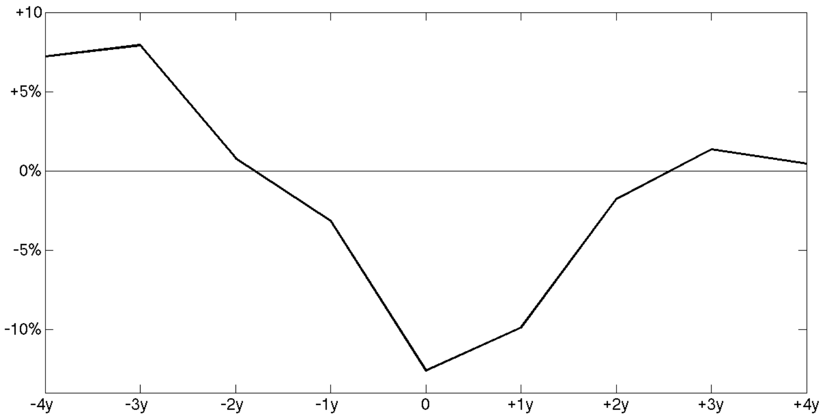
characterizes the LTV ratio. The ratio ranges around is 8% above mean before a financial crisis and it then turned into a very large drop, down to 13% below mean. So, amidst the financial crisis house price collapses by around 16% while the LTV ratio drops down by a larger extent, around 21%. This result underlies the key role of the novel mechanism of this paper - the endogenous boom and bust in the LTV ratio - in accounting for the frequency and the severity of financial crises.

In addition, the behavior of aggregate credit around crises can be used to compare the prediction of my theory with competitive explanations. Indeed, a rare disaster shock as in

FIGURE 1.8
HOUSE PRICE AND LOAN-TO-VALUE RATIO AROUND FINANCIAL CRISES.
(a) House Price



(b) Loan-to-Value Ratio



Note: The figure plots the average values of the deviations from the long-run mean of the house price q_t^{mkt} (Panel a) and the loan-to-value ratio $\frac{q_t^{\text{mkt}}}{q_t^{\text{opt}}}$ (Panel b) in a 9 year window around financial crises. The solid line denotes the dynamics implied by the benchmark model, whereas the dashed line denotes the dynamics in the data. A financial crisis is defined as the state in which aggregate credit growth drops down by more than one standard deviation.

Barro (2006) and Gourio (2012) is able to generate a financial crises in which both output and credit significantly drop. Yet, such a theory could not explain the period of credit and output boom which precedes a financial crisis. From this perspective, a model with volatility shocks can generate both the upside risk and the downside risk that is necessary to account for the very nature of financial crises: credit booms which turn into bust.

Time Variation in the Loan-to-Value Ratio.

In this Section I study the time variation in the LTV ratio $\frac{q_t^{\mathrm{opt}}}{q_t^{\mathrm{mkt}}}$ implied by the model. Table 1.4 reports the standard deviation of the LTV ratio in the data and in the four economies I consider. I report the average standard deviation of the LTV ratio together with the standard deviations conditional on whether the economies is in normal times or in crisis times. To derive the data counterpart of the LTV ratio of my model, I follow Jermann and Quadrini (2012). First, I log-linearize the collateral constraint defined in Equation (1.12). I assume that the collateral constraint is always binding and derive the series of the LTV ratio as the residual once I substitute each variable with its observable counterpart in the data. I take data on employment, wage, total liabilities and the market value of real estate. In this way, I obtain the value of the LTV ratio over time, a series which Jermann and Quadrini (2012) refer to as an exogenous financial shock.[11]

Table 1.4

Standard Deviation LTV Ratio

	Data	**RBC**	**Search Frictions**	**Stochastic Volatility**	**Search Frictions & Stochastic Volatility**	
					Fixed LTV	**Stochastic LTV**
Average	3.48%	0%	1.22%	0%	0%	1.95%
Normal Times	3.41%	0%	1.01%	0%	0%	1.50%
Crisis Times	5.52%	0%	3.23%	0%	0%	4.25%

Note: The LTV ratio is the ratio between the option value and the actual price of housing, $\frac{q_t^{\mathrm{opt}}}{q_t^{\mathrm{mkt}}}$. In the model, “Crisis Times” correspond to the states in which aggregate credit falls growth drops by more than one standard deviation. The “RBC” refers to an economy with only level shocks to TFP and a perfectly liquid housing market. The “Search Frictions” refers to a RBC economy with a frictional housing market. The “Stochastic Volatility” refers to a RBC economy with stochastic volatility. The “Search Frictions & Stochastic Volatility” refers to a RBC economy with stochastic volatility and a frictional housing market. This economy is studied in two different cases. In the first one, “Fixed LTV”, the LTV ratio is fixed at 100%. In the second one, “Stochastic LTV”, the LTV ratio is endogenous and moves over time as a function of housing liquidity.

Table 1.4 shows that as long as in the model the housing market is perfectly liquid and q_t^{opt} equals q_t^{mkt}, the LTV ratio is constant and equals 1. This is case in all the economies

[11] The standard deviation of the series does not change if either I consider only the data on real estate and liabilities of the household sector or I consolidate it with with the non-financial business sector.

without search frictions. Also in the economy with a fixed LTV ratio the standard deviation is zero by construction. Instead, the LTV ratio changes over time once I allow for a frictional housing market. In the "Search Frictions" economy, the standard deviation of the ratio is 1.22%. It equals 1.01% in normal times and it peaks up to 3.23% in crisis times. When I add volatility shocks, the standard deviation becomes 1.91%, with a value of 1.50% in normal times and 4.25% in crisis times. These results show that volatility shocks amplify the variation in the borrowing margin by around 60% on average, and account for 56% of the observed standard deviation of the borrowing margin. Moreover, Table 1.4 shows that although the model falls short in accounting for the volatility of LTV ratios in normal times, it provides a much better approximations in crisis times. The benchmark model accounts for around 77% of the standard deviation of LTV ratios amidst a financial crisis. Indeed, volatility shocks do not generate much variation in LTV ratios in good times. Instead, when the households' borrowing constraint becomes binding, changes in the level and volatility of TFP trigger a Fisherian deflation spiral in the house price and housing liquidity which amplifies the fluctuations in the LTV ratio.

Overall Table 1.4 shows that volatility shocks can be accounted for as a possible foundation of the financial shocks à la Jermann and Quadrini (2012), especially in crisis times. Hence, this model provides a quantitative theory of time varying LTV ratios which can be tested using data on housing market liquidity.

Moreover, in the model the changes in LTV ratios are driven by credit demand motives, because is no role for credit supply in the form of banks. The implications of this result are twofold. First, this evidence suggests that financial shocks should not necessarily be interpreted as if they were originated in the financial sector. Second, the findings of this paper can help reconciling different views on the cause of the last recession. Through the lenses of this paper, the fact the drop in investment, credit and employment amidst the Great Recession can be accounted for by a large negative financial shock - as shown in Jermann and Quadrini (2012) and Gilchrist and Zakrajsek (2012) - is not necessarily counterfactual with the possibility that the credit crunch was triggered by a fall in credit demand due to the deterioration of households' balance sheets, as shown by Mian and Sufi (2009, 2011).

Asset Pricing Implications

What are the characteristics of asset prices implied by the model? Table 1.5 reports the equity premium, the market price of risk and the Sharpe ratio associated with the investment in housing in the five different economies.

Table 1.5 shows that overall the behavior of asset prices starkly differs across normal times and financial crises. The unconditional equity premium is very low in all the different economies, ranging from the 0.65% of the "RBC" model up to the 1.14% of the benchmark model. Instead, upon the realization of a financial crises, the premium skyrockets up

TABLE 1.5
ASSET PRICES

	RBC	Search Frictions	Stochastic Volatility	Search Frictions & Stochastic Volatility	
				Fixed LTV	Stochastic LTV
a. Unconditional					
Equity Premium	0.65%	0.72%	0.67%	0.88%	1.14%
Market Price of Risk	1.65%	1.93%	1.73%	2.31%	2.50%
Sharpe Ratio	0.15	0.21	0.16	0.23	0.28
b. Crisis Times					
Equity Premium	10.59%	10.96%	10.79%	11.63%	12.51%
Market Price of Risk	3.97%	4.17%	4.02%	4.60%	4.93%
Sharpe Ratio	0.34	0.46	0.38	0.50	0.57

Note: The "Equity Premium" refers to the difference between the return on housing and the fixed risk-free interest rate. The "Market Price of Risk" is the ratio between the unconditional standard deviation and the unconditional average of the stochastic discount factor of the family. The "Sharpe Ratio" denotes the ratio between unconditional average and the unconditional standard deviation of the excess return. "Unconditional" denotes the moments of the model average over all the states of nature. "Crisis Timesńń refer to the states in which aggregate credit growth drops by more than one standard deviation. The "RBC" refers to an economy with only level shocks to TFP and a perfectly liquid housing market. The "Search Frictions" refers to a RBC economy with a frictional housing market. The "Stochastic Volatility" refers to a RBC economy with stochastic volatility. The "Search Frictions & Stochastic Volatility" refers to a RBC economy with stochastic volatility and a frictional housing market. This economy is studied in two different cases. In the first one, "Fixed LTV", the LTV ratio is fixed at 100%. In the second one, "Stochastic LTV", the LTV ratio is endogenous and moves over time as a function of housing liquidity.

to 10.59% in the "RBC" economy and an even higher 12.51% under the benchmark economy. The same applies for the market prices of risk and the Sharpe ratio. For instance, the Sharpe ratio of the economy with search frictions in the housing market, stochastic volatility and stochastic LTV ratio is 0.28 unconditionally, and gets up to 0.57 amidst the occurrence of a financial crisis.

The asset pricing implications of the mode are then twofold. First, as in Bianchi and Mendoza (2013) and He and Krishnamurthy (2010), the non-linearities implied by the occasionally-binding borrowing constraint generate asymmetric movements in asset prices, which depend on whether the economy is experiencing a financial crisis. Second, the rare events in which the economy experiences a major drop in aggregate credit and a sharp rise in the excess returns help increasing the overall unconditional predictions of the model in terms of asset prices. Although the model still falls short in accounting for asset prices unconditionally, it is able to generate an excess return as high as 1.14% in the benchmark version.

Finally, Table 1.5 confirms that the search frictions in the housing market and especially the stochastic LTV ratio which depends on housing liquidity are important propagation channels of the TFP shocks. Indeed, the search frictions in the housing market increase the excess return by around 11%, while the interactions of search frictions and stochastic volatility further raises the excess return by around 58%. Importantly, the stochastic LTV ratio accounts for almost 23% of the overall unconditional equity premium. I deem the results to be an important contribution in and of itself: the interaction between funding liquidity and market liquidity is an important channel that could help standard production economies in accounting for the characteristics of asset prices.

1.5 Concluding Remarks

In this paper I show that financial crises - i.e., major credit crunches - can be triggered by real shocks. I consider a model where the exogenous source of variation is given by shocks to both the level and the volatility of TFP. In particular, I emphasize the role of shocks to the volatility of TFP as a source of financial instability, which generates periods of credit booms followed by deep busts.

The main propagation mechanism I propose is the presence of search frictions in the housing market. I show that in this environment the volatility shocks are propagated into the real economy by the liquidity of housing, which in the model is captured by search frictions. Moreover, as long as houses serve as collateral assets, the liquidity of the housing market determines households' maximum LTV ratio. LTV ratio can then be interpreted as liquidity discounts: households can access to a higher LTV ratio when the housing market is more liquid.

Search frictions in the housing market are crucial to let volatility shocks directly affect households' investment propensity in housing. In my model, the search frictions determine both partial irreversibilities (i.e., there is an endogenous bid-ask spread between the relevant house price of sellers and buyers) and adjustment costs in housing investment. Since housing investment is expensive to reverse, agents prefer a wait-and-see behavior in times of high uncertainty, which is eventually reflected in a lower investment. Therefore, changes in volatility drive the level of investment, and the higher the volatility, the lower the housing investment, the lower both the housing liquidity and households' LTV ratio.

Interestingly, in the model financial crises are characterized by deflationary spirals in both the house price and the LTV ratio, a novel mechanism which amplifies the magnitude of the credit crunch. These dynamics do not hinge on the presence of a financial sector: both the credit boom and the credit bust are entirely driven by changes in households' credit demand. Yet, the model generates dynamics in the LTV ratios which are observationally equivalent to a financial shock à la Jermann and Quadrini (2012). This evidence supports the findings of Mian and Sufi (2009, 2011), who point out that the deterioration of the balance sheet of the households, rather than the one of the financial intermediaries, has triggered the Great Recession.

The policy implications of this paper are twofold. First, these results warn policymakers in interpreting shifts in LTV ratios as entirely driven by changes in credit supply. Hence, a financial shock is not a smoking gun supporting the government intervention in the financial sector. Second, the liquidity of housing - rather than the house price - is the relevant variable that captures the condition of the housing cycle. In a companion paper, Rachedi (2014), I provide evidence showing that the liquidity crunch in 2005 predicts the fall in house prices and households' leverage during the Great Recession.

1.A Appendix: Data

1.A.1 Aggregate Volatility and Financial Crises

I build a panel of 20 developed countries from 1980 until 2013. Extending the panel back to the 60's or 70's does not alter the results because in those years the 20 developed countries under investigation experienced almost no financial crisis. The countries covered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States.

Financial Crises: I take the dates of financial crises from multiple sources, that is, Bordo et *al.* (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et *al.* (2015). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention.

Recessions: The dates of recessions are instead given by the OECD recession indicators. For the United States, I follow the dates provided by NBER. The dates of crises and recessions by country are reported in Table 1.A.1.

Total Factor Productivity: I take the series of TFP from the Penn World Tables 8.0. TFP is computed as the residual of real GDP minus the capital stock times the complement to one of the share of labour compensation on GDP minus the total level of labor force (employment times average annual hours worked by persons engaged) multiplied by the share of labour compensation. The nominal variables are normalised at constant 2005 national prices.

Stock Market Volatility: The measure of aggregate volatility is based on stock market returns. For each of the 20 countries of the panel, I consider the representative stock market index, I take daily returns and compute a measure of dispersion (either the variance or the interquantile range) within a period (either a year or a quarter). The stock market indexes are the following: MSCI for Australia, MSCI for Austria, MSCI for Belgium, TSX for Canada, MSCI for Denmark, MSCI for Finland, MSCI for France, DAX for Germany, ATHEX for Greece, MSCI for Ireland, MSCI for Italy, NIKKEI for Japan, MSCI for Netherlands, MSCI for Norway, MSCI for Portugal, MSCI for Spain, MSCI for Sweden, MSCI for Switzerland, FTSE for the UK, DJIA for the US. The source of the data is Datastream.

Credit to the Private Nonfinancial Sector: I take the series on private credit from the "Long Series on Total Credit and Domestic Bank Credit to the Private Nonfinancial Sector" of the Bank for International Settlements. For each country, I take the adjusted for breaks nominal quarterly series. I take the series in which the lending sector is any sector and the borrowing sector is the private nonfinancial sector. Real values are derived by dividing the credit series by the CPI. Annual observations are computed by averaging the quarterly values within a year.

Table 1.A.1
The Dates of Financial Crises and Recessions

	Financial Crises	Recessions
Australia	1989	1982, 1986, 1994, 2000, 2002, 2006, 2008, 2012
Austria	2008	1980, 1984, 1986, 1992, 1995, 2001, 2011
Belgium	2008	1980, 1984, 1991, 1995, 1998, 2001, 2011
Canada	2008	1980, 1986, 1989, 1995, 2001, 2003, 2012
Denmark	1987, 2008	1980, 1989, 1993, 1995, 1997, 2001, 2011
Finland	1991	1980, 1982, 1990, 1992, 1995, 2001, 2012
France	2008	1981, 1983, 1995, 1998, 2001, 2012
Germany	2008	1980, 1986, 1991, 1995, 2001, 2011
Greece	1991, 2009	1997, 2000, 2004
Ireland	2008	1982, 1985, 1991, 1997, 2001
Italy	1990, 2008, 2011	1980, 1986, 1996, 2001
Japan	1992	1982, 1986, 1997, 2001, 2004, 2008, 2012
Netherlands	2008	1980, 1986, 1991, 1995, 2000, 2011
Norway	1988	1980, 1991, 1996, 2001, 2008, 2012
Portugal	2008, 2011	1980, 1983, 1991, 1995, 2001
Spain	2008, 2011	1980, 1984, 1991, 1995, 2000
Sweden	1991, 2008	1980, 1984, 1996, 2000, 2011
Switzerland	2008	1982, 1986, 1994, 2000, 2002, 2006, 2008, 2012
United Kingdom	1984, 1991, 2007	1994, 1997, 2000, 2003, 2011
United States	2007	1980, 1981, 990, 2000

Note: The dates of financial crises come from Bordo et *al.* (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et *al.* (2015). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. The dates of recessions come from OECD recession indicators.

Gross Domestic Product: I take the series of real GDP for the United States from the Bureau of Economic Analysis, series ID *GDPC1*. For all the other countries, I take the series of nominal GDP from the "Main Economic Indicators" database of the OECD. I compute the real series by dividing the nominal GDP series by the CPI.
House Prices: Real house prices are mostly taken from the International House Price database of FED Dallas, which is borrowed from Mack and Martinez-Garcia (2011). For Austria, Greece and Portugal, I have taken the quarterly series of house prices from the Property Price Statistics of the Bank for International Settlements (BIS). For Austria, I consider the series of "Residential Property Prices, All Flats (Vienna), per square meter", for Greece I consider the series of "Residential Property Prices, All Flats (Other Cities), per dwelling", and for Portugal I consider the series. The real annual prices are taken by deflating with the according CPI series the nominal series, which has been aggregated at the annual level by taking the average over the four quarterly observations per year. For Portugal, I take the monthly series from the Property Price Statistics of the BIS, considering

the series of "Residential Property Prices, All Dwellings, per square meter". The annual series is computed by taking the average over the twelve observations per year.

1.A.2 SVAR: Volatility Shocks and the Housing Market

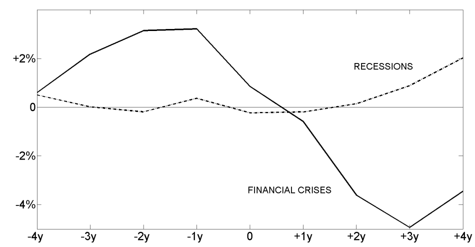
The VAR is estimated using with monthly data from January 1963 until December 2013 on the level of S&P 500 returns, an indicator of volatility, the Federal Funds Rate, the consumer price index, industrial production and three variables on the housing markets related to price, quantity and liquidity. Each series but the volatility indicator is taken in logarithm and detrended with a band-pass filter that removed frequencies below 18 months and above 96 months. The VAR includes a set of 12 lags.
S&P 500 returns: I take the logarithmic returns of the series of S&P 500 Stock Price Index provided by S&P Dow Jones Indices LLC.
Indicator of Volatility: The indicator of volatility is borrowed by Bloom (2009). The measure of volatility is an indicator function which equals one in the events in which the VIX index (or the volatility of daily returns within a month in case the VIX data is not available) is at least 1.65 standard deviations above its long run trend, as proxied by the HP-filtered trend.
Federal Funds Rate: The series is the Effective Federal Funds Rate provided by the Board of Governors of the Federal Reserve System. The FED-FRED indicator code is *FEDFUNDS*.
Consumer Price Index: The series is the Consumer Price Index for All Urban Consumers: All Items provided by the Bureau of Labor Statistics. The FED-FRED indicator code is *CPIAUCSL*.
Industrial Production: The series is the Industrial Production Index provided by the Board of Governors of the Federal Reserve System. The FED-FRED indicator code is *INDPRO*.
House Price: The series is the Median and Average Sales Prices of New Homes Sold provided by the Census Bureau. The series refers to new, single-family houses only. The FED-FRED indicator code is *MSPNHSUS*. In the robustness checks, I also use the series of the Conventional Mortgage Home Price Index provided by Freddie Mac, which starts in January 1975.
Quantity of Houses Sold: The series is the Number of Houses Sold provided by the Census Bureau. The series refers to new, single-family houses only. The FED-FRED indicator code is *HSN1F*.
Liquidity of the Housing Market: The series is the Monthly Supply of Home provided by the Census Bureau. The series refers to new, single-family houses only. The series indicates the expected time of the market of houses put up on sale. The FED-FRED indicator code is *MSACSR*.

1.B Appendix: Dynamics around Crises and Recessions

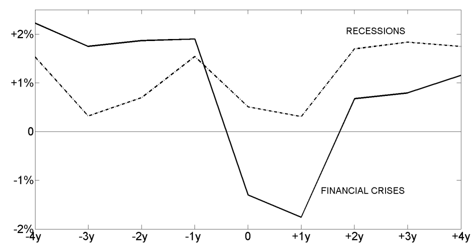
Figure 1.B.1 plots the dynamics around financial crises and recessions of credit growth, GDP growth, the house price growth and the level of the Solow residual. Panel (a) of Figure 1.B.1 shows that credit growth is much more volatility around financial crises than around recessions. Moreover, financial crises are preceded by a credit boom in which credit grows around 2% above trend. The trend is reversed upon the burst of the crisis, after which credit growth becomes highly negative. Instead, the dynamics around recessions do not present sizeable deviations from the long-run mean of credit growth. An analogous dynamics characterize also the GDP, the house price growth and the level of the Solow residual, as depicted in Panel (b), (c) and (d). This evidence supports the view of Reinhart and Rogoff (2009), Mendoza and Terrones (2012), Schularick and Taylor (2012), and Jorda et *al.* (2013, 2015) that financial crises are booms gone bust.

FIGURE 1.B.1
DYNAMICS AROUND CRISES AND RECESSIONS.

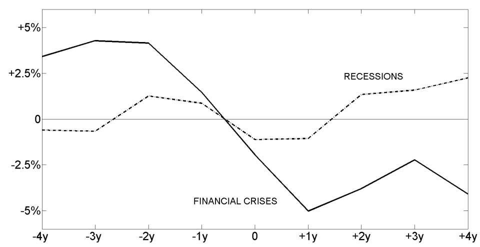
(a) Real Credit Growth



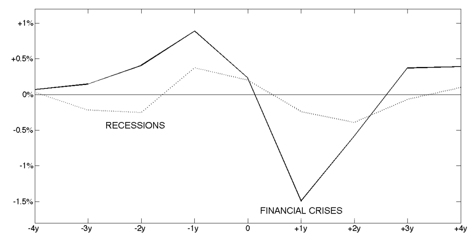
(b) Real GDP Growth



(c) Real House Price Growth



(d) Solow Residual



Note: The figure plots the median values of cross-country annual growth rates of real credit to the private non-financial sector (Panel a), real GDP growth rates (Panel b), real house price growth (Panel c) and the level of the Solow residual (Panel d) measured in log differences from the long-run mean - around recessions and financial crises (9 year window). The continuous line indicates the dynamics around financial crises, while the dashed line refers to recessions. The dates of financial crises are taken from Reinhart and Rogoff (2009). Recessions are derived from the OECD recession indicators.

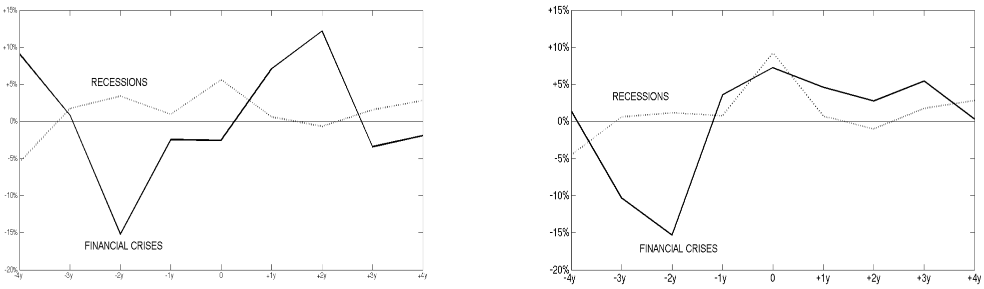
Figure 1.B.2 shows that the dynamics of volatility around financial crises and recessions are not altered when either computing volatility as the median values of the deviations of the Solow residual from the trend (instead of the mean as in Figure 1.1), or when excluding the recent financial crises episodes. Figure 1.B.3 shows that the VIX was well

below average over the three years preceding the financial crisis, and experience a surge raise at the beginning of 2007, well before the burst of the Great Recession. Over the three quarters preceding the financial crisis, the VIX has experienced a cumulative increase of around 50% from its beginning of 2007 level. This evidence suggests that a sudden volatility spike after a prolonged period of low volatility tends to lead to a financial crisis.

Figure 1.B.2

Different Measures of Aggregate Volatility.

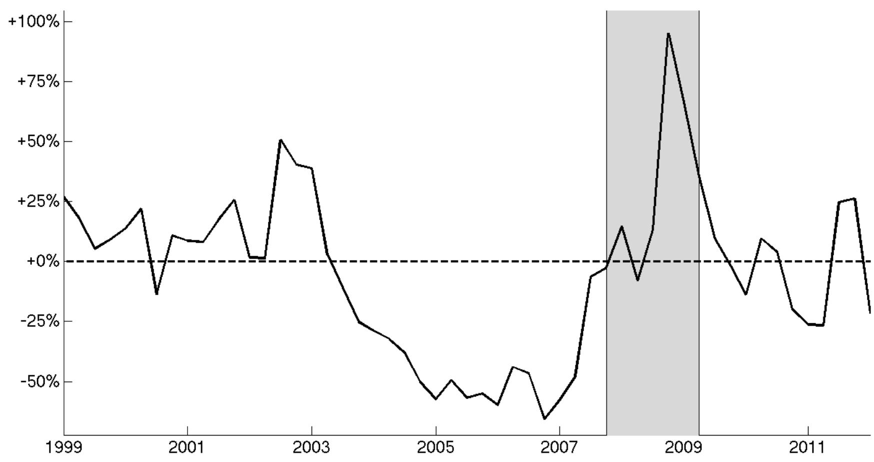
(a) Volatility of Solow Residual - Median (b) Volatility of Solow Residual - 1980 - 2006



Note: The figure plots the dynamics of aggregate volatility around financial crises and recessions (9 year window). The continuous line indicates the dynamics around financial crises, while the dashed line presents the dynamics around recession. In Panel (a) aggregate volatility is measured as the median values of the deviations from the trend of the stochastic volatility of countries' total factor productivity. In Panel (b) aggregate volatility is measured as the median values of the deviations from the trend of the stochastic volatility of countries' total factor productivity over the period 1980-2006, therefore excluding the recent financial crisis. The dates of financial crises are taken from Reinhart and Rogoff (2009). Normal recessions are derived from the OECD recession indicators.

Figure 1.B.3

The VIX and the Great Recession.



Note: The figure plots the changes in the quarterly VIX index, from January 1999 until December 2011. The series is defined as the percentage deviation from the long-run mean. The shadow area denotes the last financial crisis.

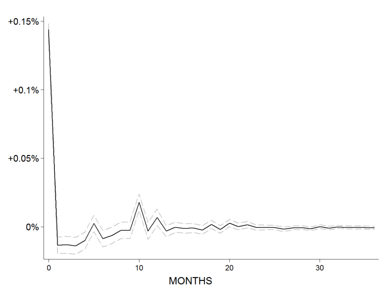
1.B.1 SVAR and the House Price

Figure 1.B.4 shows that the impulse response functions of the housing market variables does not change even when considering a different measure of the house price, that is, the CMHPI series from Freddie Mac.

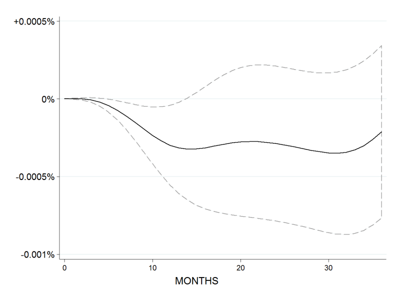
Figure 1.B.4

Volatility Shocks and the Housing Market.

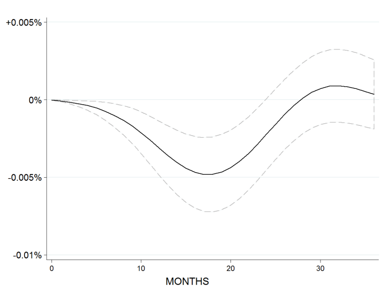
(a) Volatility



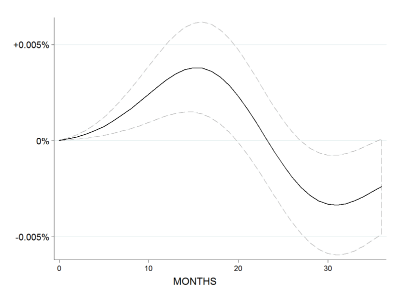
(b) House Price



(c) House Sales



(d) Time on the Market



Note: VAR estimated from January 1975 to December 2013. The dashed lines are 1 standard-error bands around the response to a volatility shock. The coordinates indicate percent deviations from the baseline.

1.C Appendix: Characterization of the Equilibrium

1.C.1 Definition of Decentralized Equilibrium

In this environment, a recursive decentralized equilibrium is defined by the individual value function $V(h, d; H, D, z, \sigma)$ and optimal policy functions $\left\{\hat{c}(h, d; H, D, z, \sigma), \hat{n}(h, d; H, D, z, \sigma), \hat{s}(h, d; H, D, z, \sigma), \hat{d}'(h, d; H, D, z, \sigma)\right\}$, pricing functions for occupied housing $q^{\text{mkt}}(H, D, z, \sigma)$, vacant housing $q^{\text{opt}}(H, D, z, \sigma)$ and labor $w(H, D, z, \sigma)$, probabilities of selling and buying a house $P^{\text{sell}}(H, D, z, \sigma)$ and $P^{\text{buy}}(H, D, z, \sigma)$, and a perceived law of motion for aggregate bond holdings $\Gamma_D(H, D, z, \sigma)$ and occupied housing $\Gamma_H(H, D, z, \sigma)$ such that:

1. Given the pricing functions $q^{\text{mkt}}(H, D, z, \sigma)$, $q^{\text{opt}}(H, D, z, \sigma)$ and $w(H, D, z, \sigma)$, the probability of selling and buying a house, $P^{\text{sell}}(H, D, z, \sigma)$ and $P^{\text{buy}}(H, D, z, \sigma)$, and the law of motions of aggregate bond holdings $\Gamma_D(H, D, z, \sigma)$ and aggregate occupied housing $\Gamma_H(H, D, z, \sigma)$, the families' problem is solved by $V(h, d; H, D, z, \sigma)$ and $\left\{\hat{c}(h, d; H, D, z, \sigma), \hat{n}(h, d; H, D, z, \sigma), \hat{s}(h, d; H, D, z, \sigma), \hat{d}'(h, d; H, D, z, \sigma)\right\}$.

2. The housing markets clear, the probability of buying a house is

$$P^{\text{buy}}(H, D, z, \sigma) = \frac{\hat{s}(h, d; H, D, z, \sigma)\left[(1-h)\hat{s}(h, d; H, D, z, \sigma)\right]^{1-\gamma}(1-h)^{\gamma}}{(1-h)\,\hat{s}(h, d; H, D, z, \sigma)},$$

the probability of selling a home is

$$P^{\text{sell}}(H, D, z, \sigma) = \frac{\left[(1-h)\hat{s}(h, d; H, D, z, \sigma)\right]^{1-\gamma}(1-h)^{\gamma}}{1-h},$$

where the prices of occupied and vacant housing are determined by Equation (1.24) and (1.8), respectively.

3. The labor market clears at the equilibrium wage $w(H, D, z, \sigma)$.

5. The perceived law of motion of aggregate bond holdings coincide with the actual one, that is, $\Gamma_D(H, D, z, \sigma) = \hat{d}'(h, d; H, D, z, \sigma)$.

6. The perceived law of motion of the aggregate stock of occupied houses coincide with the actual one: $\Gamma_H(H, D, z, \sigma) = (1-\psi)\left(h + P^{\text{buy}}(H, D, z, \sigma)\,\hat{s}(h, d; H, D, z, \sigma)(1-h)\right)$.

1.C.2 First Order Conditions

The first order conditions of the problem yield the optimal choices on the supply of working hours, the number of workers to hire, housing investment and borrowing:

$$w_t = \frac{U_{l_t}}{U_{c_t}} \tag{1.C.1}$$

$$z_t F_{n_t} = w_t\left[1 + \frac{\phi_t \nu}{U_{c_t}}\right] \tag{1.C.2}$$

$$\begin{aligned} q_t^{\text{mkt}} + \frac{2\kappa s_t}{P_t^{\text{buy}}(1-h_t)} &= \psi \mathbb{E}_t\left[\Lambda_{t+1}\left(P_{t+1}^{\text{sell}} q_{t+1}^{\text{mkt}} + \left(1 - P_{t+1}^{\text{sell}}\right) q_{t+1}^{\text{opt}}\right)\right] + \dots \\ &\dots + (1-\psi)\,\mathbb{E}_t\left[\Lambda_{t+1}\left(V_{t+1}^{H} + \frac{U_{h_{t+1}}}{U_{c_{t+1}}} + e^{z_{t+1}} F_{h_{t+1}} + \frac{\phi_{t+1}}{U_{c_{t+1}}} q_{,t+1}^{\text{opt}}\right)\right] \end{aligned} \tag{1.C.3}$$

$$U_{c_t} = \beta R \mathbb{E}_t\left[U_{c_{t+1}}\right] + \phi_t \tag{1.C.4}$$

where Y_{x_t} denotes the derivatives of the function $Y(\cdot)$ with respect the term x_t, and ϕ_t is the Lagrange multiplier associated to the borrowing constraint of the families.

The Equation (1.C.1) is the standard condition for the optimal labor supply. Instead, the optimal labor demand (1.C.2) is distorted by the presence of the Lagrange multiplier associated to the borrowing constraint ϕ_t. In the states in which the borrowing constraint binds, the multiplier ϕ_t is positive, and the shadow price of the borrowing constraint defines a wedge above the marginal cost. Hence, when a family is borrowing constrained, the cost of hiring labor force de-facto increases, forcing the families to reduce the number of workers hired and the overall level of production.

The Equation (1.C.3) represents the equilibrium conditions for the search effort on the frictional market. It stipulates that in equilibrium the overall cost of searching for a house equal its marginal gain. The cost is the sum of the searching cost and the house price. The gain is the sum of the production dividends, the utility services received from occupying the house, the extra amounts of resources obtained by relaxing the borrowing constraint with an additional unit of collateral and the continuation value of owning a house. This term also considers the event in which the member is hit by a mismatch shock and forced to sell the house.

Finally, the Equation (1.C.4) characterizes the optimal choices of bonds. Again, the borrowing constraint adds an extra-financing cost ϕ_t which increases the actual repayment cost. Therefore, in the states in which the borrowing constraint binds, households de-facto incur in an interest rate that is above the one charged by foreign investors.

1.D Appendix: Proofs

1.D.1 Equilibrium Borrowing Constraint

The derivation of the equilibrium borrowing constraint closely follows Bianchi and Mendoza (2013). The borrowing constraint arises in equilibrium as an incentive compatibility constraint which grounds on a limited enforceability of debt, that is, families lack of commitment to repay their debt. I consider the incentive compatibility constraint which yields zero expected profits for the lenders in case they seize families' collateral, and ensures that families do not default. I consider the following environment:

1. Loans are signed with lenders in a competitive environment;
2. Financial contracts are not exclusive;
3. There is no informational friction between lenders and families;
4. Families borrow during the second stage of each period of the model, that is, just after the realization of the shocks, and before production takes place;

5. Families lack of commitment in repaying the debt only during the first stage of the problem;

6. If families renege on their debt, the stock of occupied housing $h_{i,t}$ is seized by the lenders during the third stage, that is, defaulting families can still use their stock of occupied housing for production and enjoy its utility services;

7. Lenders immediately sell the liquidated housing to a real estate sector in the third stage;

8. The real estate sector consists of a continuum of real estate agencies;

9. Each family owns a diversified stake in the real estate sector;

10. There is free entry in the real estate sector, which is further perfectly competitive;

11. The real estate sector buys the liquidated houses from the lenders and puts them up on sale on the frictional market;

12. The real estate sector do not use the stock of liquidated houses either as a production input or as a collateral asset, and does not enjoy any utility service of housing;

13. After reneging on debt, families can immediately access again financial market at no penalty, and can purchase again its housing stock at competitive prices.

In this environment, in case a family defaults on its current level of debt, the lenders lose an amount of resources that equals $\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t}$, and gain $q_t^{\text{opt}} h_{i,t}$ from selling the liquidated housing to the real estate sector. Hence, in equilibrium lenders will not require a collateral value larger than $q_t^{\text{opt}} h_{i,t}$.

On the other hand, from a family perspective, the gain of defaulting equals $\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t}$ while its cost is $V_{i,t}^H h_{i,t}$, that is, the value that families attribute to the stock of housing seized by the lenders. Since $V_{i,t}^H h_{i,t} \geq q_t^{\text{opt}} h_{i,t}$, families will always decide to repay back their debt. Thus, the borrowing constraint

$$\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq q_t^{\text{opt}} h_{i,t}$$

ensures that lenders do not make ex-ante profits on a defaulting family and that families do not default in equilibrium. In this way, the real estate sector does not operate on an equilibrium path.

1.D.2 Proof of Proposition 1.

The loan-to-value ratio $\frac{q_t^{\text{opt}}}{q_{h,t}}$ equals

$$\frac{q_t^{\text{opt}}}{q_t^{\text{mkt}}} = P_t^{\text{sell}} + \left(1 - P_t^{\text{sell}}\right) \mathbb{E}_t \left[\Lambda_{t+1} q_{t+1}^{\text{opt}}\right]$$

In a steady-state equilibrium, the loan-to-value ratio equals

$$\frac{q^{\text{opt}}}{q^{\text{mkt}}} = P^{\text{sell}} + \left(1 - P^{\text{sell}}\right) \beta \frac{q^{\text{opt}}}{q_{\text{mkt}}} = \frac{P^{\text{sell}}}{1 - \left(1 - P^{\text{sell}}\right) \beta}$$

since $\Lambda_{t+1} = \beta \frac{U_{c_{t+1}}}{U_{c_t}}$, and $U_{c_{t+1}} = U_{c_t} = U_c$ along the steady-state. Thus, the derivative of the loan-to-value ratio with respect to a change in the current level of the liquidity of the frictional housing market, measured in terms of probability of selling a house is

$$\frac{\partial \frac{q^{\text{opt}}}{q^{\text{mkt}}}}{\partial P^{\text{sell}}} = \frac{1 - \beta}{\left[1 - \left(1 - P^{\text{sell}}\right) \beta\right]^2} > 0 \qquad \forall \beta \in (0,1),\ P^{\text{sell}} \in (0,1)$$

1.D.3 Proof of Proposition 2.

I use the equation of house price $q_{w,t}$ given by the condition (1.24) to characterize the expected equity premium associated to the investment in housing

$$\mathbb{E}_t \left[R_{t+1}^{ep}\right] = \mathbb{E}_t \left[R_{t+1}^h - R\right]$$

where $R_{t+1}^h = \frac{e^{z_{t+1}} F_{h_{t+1}} + q_{t+1}^{\text{mkt}}}{q_t^{\text{mkt}}}$ denotes the cum-dividend return on housing investment. The equity premium reads

$$\begin{aligned}
\mathbb{E}_t \left[R_{t+1}^{ep}\right] = \frac{1}{\mathbb{E}_t \left[\Lambda_{t+1}\right]} \Bigg\{ & \underbrace{\frac{\phi_t}{U_{c_t}}}_{\text{Collateral}} + \underbrace{\mathbb{E}_t \left[\Lambda_{t+1} \Delta q_{t+1}^{\text{mkt}} \left(1 - \frac{q_{t+1}^{\text{opt}}}{q_{t+1}^{\text{mkt}}}\right)\right]}_{\text{Search Frictions}} + \underbrace{\mathbb{E}_t \left[\Lambda_{t+1} \Omega_{t+1}\right]}_{\text{Mismatch \& Bargaining}} \\
& - \underbrace{\mathbb{E}_t \left[\Lambda_{t+1} \Delta q_{t+1}^{\text{mkt}} \frac{V_{t+1}^H}{q_{t+1}^{\text{mkt}}}\right]}_{\text{Continuation Value Match}} - \underbrace{\mathbb{E}_t \left[\Lambda_{t+1} \Delta q_{t+1}^{\text{mkt}} \frac{\phi_{t+1}}{U_{c_{t+1}}} \frac{q_{t+1}^{\text{opt}}}{q_{t+1}^{\text{mkt}}}\right]}_{\text{Collateral} \times \text{Search Frictions}} - \underbrace{\mathbb{C}_t \left[R_{t+1}^{ep}, \Lambda_{t+1}\right]}_{\text{Risk}} \Bigg\}
\end{aligned}$$

where

$$\Omega_{t+1} = \zeta\psi\mathbb{E}_t\left[\Lambda_{t+1}\Delta q^{\text{mkt}}_{t+1}\left(1 - P^{\text{sell}}_{t+1}\right)\left(1 - \frac{q^{\text{opt}}_{t+1}}{q^{\text{mkt}}_{t+1}}\right)\right] + \zeta\mathbb{E}_t\left[\Lambda_{t+1}\Delta q^{\text{mkt}}_{t+1}\left(1 - \frac{q^{\text{opt}}_{t+1}}{q^{\text{mkt}}_{t+1}}\right)\right] +$$
$$+ (1 - (1 - \psi)\,\zeta)\,\mathbb{E}_t\left[\Lambda_{t+1}\Delta q^{\text{mkt}}_{t+1}\frac{V^H_{t+1}}{q^{\text{mkt}}_{t+1}}\right] + (1 - (1 - \psi)\,\zeta)\,\mathbb{E}_t\left[\Lambda_{t+1}\Delta q^{\text{mkt}}_{t+1}\frac{\phi_{t+1}}{U_{c_{t+1}}}\frac{q^{\text{opt}}_{t+1}}{q^{\text{mkt}}_{t+1}}\right] +$$
$$+ (1 - (1 - \psi)\,\zeta)\,\mathbb{E}_t\left[\Lambda_{t+1}\Delta q^{\text{mkt}}_{t+1}\frac{z_{t+1}F_{h_{t+1}}}{q^{\text{mkt}}_{t+1}}\right] - \zeta\psi\mathbb{E}_t\left[\Lambda_{t+1}\Delta q^{\text{mkt}}_{t+1}\right]$$

The formula above highlights that the premium, and therefore the house price, depends on collateral values and search frictions. Indeed, in standard asset pricing conditions, the equity return depends only the level of risk, that is, the covariance between families' stochastic discount factor and the equity premium. Here, the equity premium is also increasing in the current Lagrange multiplier associated to the borrowing constraint ϕ_t and the search frictions as measured by the margin of the borrowing constraint. On one hand, when the borrowing constraint binds, the equity premium rises, and the house price q^{mkt}_t declines. Thus, borrowing constrained families that are forced to fire sales depress the current house price. On the other hand, when the future probability of selling houses in the frictional market decreases, tightening the borrowing margin, the equity premium rises and therefore the house price declines. So, a liquidity freeze lowers the house price. In either case, there is also an indirect effect. The high equity return in the states in which the borrowing constraint binds and the liquidity of the housing market is low tends to be associated by disproportionately higher levels of families' marginal utility of consumption. This comovement further depresses the house price.

CHAPTER 2

Asset Pricing With Heterogeneous Inattention

2.1 Introduction

Does the observed households' limited attention to the stock market quantitatively account for the bulk of asset prices? I address this question introducing an observation cost in a production economy with heterogeneous agents, incomplete markets and idiosyncratic labor income risk. In this environment inattention changes endogenously over time and across agents. I discipline the quantitative analysis by calibrating the observation cost to match the observed duration of inattention of the median household. I find that the presence of the observation cost improves the performance of the model, generating limited equity market participation, a realistic dynamics of consumption growth and countercyclical patters for both the stock returns volatility and the equity premium. Yet, inattention cannot account for the bulk of stock prices.

This paper studies the role of households' inattention by relaxing the assumption that agents are always aware of the state of the economy. Despite standard models postulate that households continuously collect information on the stock market and derive optimal consumption/savings plans, in the data we observe a different pattern. For example, Ameriks et *al.* (2003) show that households plan infrequently, and wealthy agents plan more often than poor ones. Alvarez et *al.* (2012) use data from two Italian surveys and find that the median household pays attention to the stock market every 3 months. Furthermore, there is a sizeable heterogeneity in inattention across households: 24% of agents observe the financial portfolios less than twice per year, whereas 20% of them do it more often than once per week. Finally, Rossi (2010), Da et *al.* (2011), Sichermann et *al.* (2012), and Andrei and Hasler (2015) find that the allocation of attention is time-varying, although the sign of the relation between inattention and financial returns is ambiguous.[1] This evidence has motivated a new strand of literature, which concentrates on infrequent planning and limited attention as potential solutions to the equity premium puzzle. A priori, these factors could improve the performance of standard models by increasing the

[1]Few other papers show that investors' allocation of attention affects stock prices and portfolio choices, e.g. Barber and Odean (2008), Brunnermeier and Nagel (2008), Della Vigna and Pollet (2009), Hirshleifer et *al.* (2009) and Mondria et *al.* (2010).

risk of holding stocks and implying a low correlation between consumption and equity returns. Nonetheless, the literature finds inconclusive results. Lynch (1996), Gabaix and Laibson (2002), Rossi (2010) and Chien et *al.* (2011, 2012) show that models embodying inattention or infrequent planning can account for the level and the dynamics of asset prices. Conversely, Chen (2006) and Finocchiaro (2011) find that although these features do increase the volatility of stock returns, they have no effects on the equity premium.

In this paper I evaluate whether the observed duration of households' inattention can account for the equity premium and the dynamics of asset prices. I develop a model that plugs the inattention of Reis (2006) in the environment of Krusell and Smith (1997, 1998). I consider a production economy with incomplete markets and heterogeneous agents, who incur in an observation cost whenever they collect information on the state of the economy and formulate a new plan for consumption and financial investment. This feature creates a trade-off: attentive households take better decisions, but also bear higher costs. As a result, households decide to plan at infrequent dates and stay inattentive meanwhile. Inattentive agents do not gather new information and follow by inertia pre-determined paths of consumption and financial investment. To discipline the role of infrequent planning, I calibrate the observation cost to match the actual duration of inattention for the median household, as estimated by Alvarez et *al.* (2012). This choice implies that the aim of the paper is not to use inattention to match asset prices, but rather to study its quantitative implications once observation costs are calibrated to the inattention observed in the data.

Looking at the results of the model, I find that inattention differs across agents and co-moves with financial returns. The level of inattention depends negatively on households' wealth - in line with the evidence of Ameriks et *al.* (2003) - because poor agents face disproportionately higher observation costs. The cyclicality of inattention depends on the marginal gain and the marginal cost of being attentive and actively investing in the stock market. Both forces are countercyclical, but they asymmetrically affect different agents. Poor households plan in expansions because they cannot afford the observation cost in bad times. Instead, wealthy agents plan in recession to benefit of the higher expected return to equity. Overall the level of inattention is countercyclical. Second, the participation to the equity market is limited because the observation cost is de facto a barrier to an optimal investment in stocks. In turn, limited participation implies a more realistic wealth distribution since only wealthy stockholders can benefit of the returns to equity. In the benchmark model, inattention impedes 27% of households to participate in the stock market and raises the Gini index of wealth by 56%. Third, the volatility of stock returns is high and countercyclical. The observation cost boosts the level of volatility because it acts as a capital adjustment cost. Indeed, inattentive agents cannot immediately adjust their financial positions to the realizations of the aggregate productivity shock. Furthermore, the limited participation in the equity market intensifies the inelasticity in the supply of capital. More interestingly, the countercyclical dynamics of inattention implies time-varying adjustment costs which are more stringent in bad times. As a result, the volatility of stock returns peaks in recessions. Inattention has two further effects on stock prices. On one hand, it generates a weak correlation between equity returns and consumption growth, through the slow dissemination of information across agents. On the other hand, it induces large variations in the excess returns. This second result is usually obtained through consumption habits or heteroskedastic consumption growth. Instead, here it is just the by-product of the observation cost, that concentrates the aggregate risk on a small measure of agents. At each point of time there are few attentive investors that trade stocks and bear the whole aggregate risk of the economy, commanding a higher return rate on equity. As long as the number of active investors shrinks down in recessions, stockholders require a higher compensation in bad times. This mechanism is amplified by the presence of inattentive agents, who create a residual aggregate risk by consuming too much in bad times and too little in good times. Such behavior forces attentive stockholders to switch their consumption away from times in which their marginal utility is high. In this respect, the model endogenizes the limited stock market participation and heterogeneity in trading technologies that Guvenen (2009) and Chien et *al.* (2011, 2012) take as exogenous to replicate the dynamics of asset prices. Fourth, in the benchmark model the equity premium is still around 1%. The price of risk is low because households react to the observation cost by becoming inattentive, accumulating savings and deleveraging out of stocks. These mechanisms explain why increasing the magnitude of the observation cost barely alters the Sharpe ratio. Finally, I find that the effects of inattention on asset prices crucially depend on the specification of the borrowing constraints. When they are loose enough, all households participate in the stock market following buy-and-hold positions, as pointed out in Chen (2006). Since there is no risk of hitting the borrowing constraint, agents can dilute the observation cost by trading more infrequently, and inattention does not affect asset prices.

2.2 Related Literature

This paper adds to the literature on the equity premium puzzle. Since the seminal paper of Mehra and Prescott (1985), many solutions have been proposed: long-run risk (Bansal and Yaron, 2004), consumption habits (Campbell and Cochrane, 1999), and limited stock market participation (Guvenen, 2009), among others. The emphasis of this paper is on households' inattention to the stock market. In the literature, households' inattention is usually achieved either by making agents gathering information and planning financial investment at discrete dates (e.g., Duffie and Sun, 1990; Lynch, 1996; Gabaix and Laibson, 2002; Chen, 2006; Reis, 2006 and Finocchiaro, 2011), or through learning with capacity constraints (as in Sims, 2003; Peng, 2005; Huan and Liu, 2007).[2] I follow the first strand of the literature because of my emphasis on the effects of inattention on agents' portfolio decisions. Indeed, I study a heterogeneous agent economy, where any household can react to the risk of inattention by modifying its portfolio. This feature avoids having a representative agent which in equilibrium holds anyway the market portfolio. Models featuring learning with capacity constraint can be extended to the case of heterogeneous agents and idiosyncratic shocks only by neglecting the existence of higher-order beliefs, as discussed in Porapakkarm and Young (2008).[3] Yet, Angeletos and La'O (2009) show that higher-order beliefs do play a crucial role in the dissemination of information across agents. Instead, models in which inattention is modeled as agents gathering information at infrequent times do not suffer of this problem and are therefore more tractable.

My paper differs from the literature on inattention on two main dimensions. First, I discipline the role of infrequent planning by calibrating the observation cost to match the actual duration of inattention for the median household. In this way, I can evaluate whether the observed level of inattention can quantitatively account for the dynamics of asset prices. Second, I identify the mechanisms tempering or amplifying the effects of the observation cost on stock prices. In this respect, this paper mirrors the analyses that Pijoan-Mas (2007) and Gomes and Michaelides (2008) carried out for habits and agents heterogeneity.

2.3 The Model

In the discrete-time economy there is a representative firm that uses capital and labor to produce a consumption good. On the other side, there is unit measure of ex-ante identical agents. Households are ex-post heterogeneous because they bear an uninsurable idiosyncratic labor income risk. Moreover, they face a monetary observation cost whenever collecting information on the states of the economy and choosing consumption and savings.

2.3.1 The Firm

The production sector of the economy constitutes of a representative firm, which produces a homogeneous consumption good $Y_t \in \mathbf{Y} \subset \mathbb{R}_+$ using a Cobb-Douglas production

[2]The notion of inattention is also closely tied to the concept of information acquisition, e.g. Grossman and Stiglitz (1980) and Peress (2004), and the one of uncertainty, see Veronesi (1999) and Andrei and Hasler (2015).

[3]When agents have imperfect common knowledge and differ in their information set, they need to forecast other agents' forecast, and so on so forth. In this case, equilibrium prices do not depend only on the infinite-dimensional distribution of agents across wealth, but also on the infinite-dimensional distribution of beliefs.

function

$$Y_t = z_t N_t^{1-\eta} K_t^{\eta} \tag{2.3.1}$$

where $\eta \in (0,1)$ denotes the capital income share. The variable $z_t \in \mathbf{Z} \subset R_+$ follows a stationary Markov process with transition probabilities $\Gamma_z(z', z) = \Pr\left(z_{t+1} = z' | z_t = z\right)$. The firm hires $N_t \in \mathbf{N} \subset \mathbb{R}_+$ workers at the wage w_t, and rents from households the stock of physical capital $K_t \in \mathbf{K} \subset \mathbb{R}_+$ at the interest rate r_t^a. Physical capital depreciates at a rate $\delta \in (0,1)$ after production. At every point of time, after the realization of the shock z, the firm chooses capital and labor to equate their marginal productivity to their prices, as follows

$$r_t^a = \eta z_t N_t^{1-\eta} K_t^{\eta-1} - \delta \tag{2.3.2}$$

$$w_t = (1-\eta) z_t N_t^{-\eta} K_t^{\eta}. \tag{2.3.3}$$

Both prices depend on the realization of the aggregate productivity shock z_t. I intentionally abstract from any adjustment cost to focus on inattention as the only source of slowly-moving capital, as in Duffie (2010).

2.3.2 Households

The economy is populated by a measure one of ex-ante identical households. They are infinitely lived, discount the future at the positive rate $\beta \in (0,1)$ and maximize lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)\, \mathrm{d}t \tag{2.3.4}$$

where $c_t \in \mathbf{C} \subset \mathbb{R}_+$ denotes consumption at time t. The utility function is a CRRA, $U(c) = \frac{c^{1-\theta}}{1-\theta}$, where θ denotes the risk aversion of households.

Idiosyncratic Shocks

As in Pijoan-Mas (2007), households bear an idiosyncratic labor income risk which consists of two components. First, agents are hit by a shock $e_t \in \mathbf{E} \subset \{0,1\}$, which determines their employment status.[4] A household has a job when $e_t = 1$ and is unemployed when $e_t = 0$. I assume that e_t follows a stationary continuous Markov process with transition probabilities

$$\Gamma_e(z, z', e, e') = \Pr\left(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z'\right). \tag{2.3.5}$$

The shock is idiosyncratic and washes out in the aggregate. Yet, its transition probabilities depend on the aggregate productivity shock. As a consequence, both the idiosyncratic

[4]The only purpose of the presence of an employment status shock is to relax the conditions governing the modeling of households' inattention.

uncertainty and the unemployment rate of the economy rise in recessions.[5] Second, when a household is given a job, it faces a further shock $\xi_t \in \ddot{} \subset \mathbb{R}_+$, which determines the efficiency units of hours worked. This shock is orthogonal to the aggregate productivity shock and follows a stationary continuous Markov process with transitional probabilities

$$\Gamma_\xi(\xi, \xi') = \Pr(\xi_{t+1} = \xi' | \xi_t = \xi). \tag{2.3.6}$$

When a household is unemployed, it receives a constant unemployment benefit $\bar{w} > 0$. Households' labor income l_t is then

$$l_t = w_t \xi_t e_t + \bar{w}(1 - e_t). \tag{2.3.7}$$

Market Arrangements

Households own the capital of the economy. Each agent holds $a_t \in \mathbf{A} \equiv [\underline{a}, \infty]$ units of capital, which are either rented to the firm or traded among households. Capital is risky and yields the rate r_t^a, as defined in (2). Agents can also invest in a one-period non-contingent bond $b_t \in \mathbf{B} \equiv [\underline{b}, \infty]$, which is in zero net supply. The bond yields a risk-free rate r_t^b. Households face exogenous borrowing constraints for both assets and cannot go shorter than $\underline{b}$ in the risk-free bond and $\underline{a}$ for the risky equity. When these values equal zero, no short position is allowed at all. I also consider a borrowing constraint $\underline{f}$ on the total financial portfolio $a_{t+1} + b_{t+1}$.

In this framework, markets are incomplete because agents cannot trade claims which are contingent on the realizations of the idiosyncratic shock. As long as the labor income risk cannot be fully insured, agents are ex post heterogeneous in wealth, consumption and portfolio choices.

Observation Cost

Agents incur in a monetary observation cost proportional to their labor income χl_t whenever acquiring information on the state of the economy and defining the optimal choices on consumption and savings. This cost is a reduced form for the financial and time opportunity expenditures bore by households to figure out the optimal composition of the financial portfolio. The observation cost induces the agents to plan infrequently and stay inattentive meanwhile. Planning dates are defined as dates $d_i \in \mathbf{D} \subset \mathbb{N}$ such that $d_{i+1} \geq d_i$ for any i. At a planning date d_i, households pay the cost χl_{d_i}, collect the information on the states of the economy and decide the next planning date d_{i+1}. Moreover, at planning dates, households decide the stream of consumption throughout the period of inattention $\left[c_{d_i}, c_{d_{i+1}-1}\right]$, and the investment in risky capital a_{d_i+1} and risk-free bonds b_{d_i+1}. Instead,

[5] I define such a structure for the employment shock following Mankiw (1986), who shows that a countercyclical idiosyncratic uncertainty accommodates a higher price of risk. Without such feature, incomplete markets would not affect the equity premium, as discussed in Krueger and Lustig (2010). Anyway, Storesletten et *al.* (2007) find that in the data labor income risk does peak in recessions.

at non planning dates, households are inattentive and follow the pre-determined plan for consumption set in the previous planning date. I assume that the financial portfolio of inattentive households is re-balanced every period to match the initial share of risky assets $\frac{a_{d_i}}{a_{d_i}+b_{d_i}}$.[6]

In the model, attentive households observe the states of the economy, while inattentive ones do not. These states include the realizations of the aggregate productivity and the idiosyncratic labor income shock. On one hand, it is reasonable to assume that agents are not fully aware of the actual realization of the aggregate shock.[7] On the other hand, inattentive agents cannot observe even their labor income. This condition is required to preserve the computational tractability of the model. Indeed, if households could also observe their stream of labor income, then they would always gather some new information. Hence, agents would make their decision on whether to be inattentive on a continuous basis. Furthermore, agents could infer the dynamics of the aggregate states by exploiting the correlation between aggregate productivity and labor earnings, implying an additional learning dynamics within the model. These features would inflate the states and the mechanisms of the model making it computationally infeasible. Nevertheless, to mitigate the assumption that households do not observe their labor income, I postulate that inattention breaks out exogenously when the employment status changes, from worker to unemployed or vice versa. Changes in employment status are interpreted as major events which capture the attention of agents and require them to change previous plans on consumption and savings. In such a case, households are forced to become attentive and pay the observation cost. This assumption implies that each household is always aware of its employment status. As a result, labor income is only partially unknown to inattentive agents.[8,9] I define one further condition on the behavior of inattention. To maintain the existence of credit imperfections, I postulate that inattention breaks out exogenously when agents are about to hit the borrowing constraints. In such a case, an unmodeled financial intermediary calls the attention of the agents which are forced to become attentive and pay the obser-

[6]This assumption, which is also made in Gabaix and Laibson (2002), Abel et *al.* (2007) and Alvarez et *al.* (2012), is consistent with the empirical evidence on weak portfolio re-balancing across households. For example, Ameriks and Zeldes (2004) study a ten-year panel of households and document that around 60% of them changed the composition of the portfolio at most once.

[7]For example, the statistics on the gross domestic product are released with a lag of a quarter.

[8]Unemployed inattentive agents are aware of their earnings while employed inattentive agents have an unbiased expectation about their labor income. Employed agents in the model are akin to workers who receive stochastic bonuses at infrequent dates during the year. Note that the observation cost is calibrated to imply a length of inattention for the median agent which equals a quarter. Therefore, the median agent does not gather full information about her labor income just for three months.

[9]Even if the correlation between labor income and the aggregate shock is set to zero, agents that observe their labor income every period will always have new flow of information upon which to update their optimal policies on consumption and investment. Accordingly, the problem of inattentive agents would depend on both the actual states of the economy and the beliefs on the states they cannot directly observe. As a result, the number of relevant states for the maximization problem would increase from six up to eleven. Under the calibration choices of this paper, the grid points of the value function iteration would go from the current 5 millions up to around 450 billions, making infeasible any computation algorithm.

vation cost. These two assumptions affect the outs from inattention. Indeed, a household that at time d_i decides not to observe the states of the economy until d_{i+1} will cease to be inattentive at the realized new planning date $\lambda(d_{i+1})$, which is the minimum between the desired new planning date d_{i+1} and the periods in which either the employment status of the household changes, $\{j \in [d_i, d_{i+1}) : \; e_j \neq e_{j-1}\}$, or the household is about to hit the borrowing constraint, $\left\{j \in [d_i, d_{i+1}) : \; b_{j+1} < \underline{b} \text{ or } a_{j+1} < \underline{a} \text{ or } \left(a_{j+1} + b_{j+1}\right) < \underline{f}\right\}$.

Value Function

To define the aggregate states of the households' problem, I introduce the distribution of the agents γ - defined over households' idiosyncratic states, the decisions of inattention, the portfolio choices, and the consumption path $\{\omega_t, e_t, \xi_t, d_t, a_t, b_t, c_t\}$ - which characterizes the probability measure on the σ-algebra generated by the Borel set $\mathcal{J} \equiv \dot{} \times \mathbf{E} \times \ddot{} \times \mathbf{D} \times \mathbf{A} \times \mathbf{B} \times \mathbf{C}$. Roughly speaking, γ_t keeps track of all the heterogeneity among agents. In this environment, γ_t is an aggregate state because prices depend on it. Krusell and Smith (1997, 1998) discuss how prices depends on the entire distribution of agents across their idiosyncratic states. The further addition of the duration of inattention across agents makes the prices to depend also on further objects, which are required to define the optimal behavior of inattentive agents at each point of time. Indeed, these objects signal active investors about the degree of the informational frictions in the economy. The distribution γ_t evolves over time following a law of motion

$$\gamma_{t+1} = H\left(\gamma_t, z_t, z_{t+1}\right) \tag{2.3.8}$$

The operator $H(\cdot)$ pins down the changes in the measure γ_t taking as given the initial value of γ_t itself, and the realizations of the aggregate shock z_t.

The structure of the problem should also take into account how the information is revealed to the agents. The state variables of this economy $x_t \equiv \{\omega_t, e_t, \xi_t; \; z_t, \gamma_t\}$ are random variable defined on a filtered probability space (X, F, P). X denotes the set including all the possible realizations of x_t, F is the filtration $\{F_t, \; t \geq 0\}$ consisting of the σ-algebra that controls how the information on the states of the economy is disclosed to the agents, and P is the probability measure defined on F. Hereafter, I define the expectation of a variable v_t conditional on the information set at time k as $\mathbb{E}_k[v_t] = \int v_t \mathrm{dP}(\mathrm{F_k}) = \int v(x_t)\, \mathrm{dP}(\mathrm{F_k})$. The state vector $P\left(v_t \middle| F_k\right) = P\left(v_t \middle| x_k\right)$ is a sufficient statistics for the probability of any variable v_t because of the Markov structure of x_t. The presence of observation costs and inattentive agents implies some measurability constraints on the expectations of households. Namely, a planning date d_i defines a new filtration $\mathcal{F}_s$ such that $\mathcal{F}_s = F_{d_i}$ for $s \in [d_i, \lambda(d_{i+1}))$. Hence, any decision made throughout the duration of inattention is conditional on the information at time d_i, because the household does not update its information set until the new planning date $\lambda(d_{i+1})$. Taking into account this measurability constraint, I write the agents' recursive problem as

$$V(\omega_t, e_t, \xi_t; z_t, \gamma_t) = \max_{d,\, [c_t, c_{\lambda(d)-1}],\, a_{t+1},\, b_{t+1}} \mathbb{E}_t \left[\sum_{j=t}^{\lambda(d)} \beta^{j-t}\, U\left(c_j\right) + \dots \right.$$

$$\left. \dots + \beta^{\lambda(d)-t}\, V\left(\omega_{\lambda(d)}, e_{\lambda(d)}, \xi_{\lambda(d)}; z_{\lambda(d)}, \gamma_{\lambda(d)}\right) \right] \tag{2.3.9}$$

$$\text{s.t.} \quad \omega_t + l_t(z_t, \gamma_t) = c_t + a_{t+1} + b_{t+1} \tag{2.3.10}$$

$$\omega_{\lambda(d)} = \left(a_{t+1} + b_{t+1}\right) \prod_{k=t+1}^{\lambda(d)} r^p_k(z_k, \gamma_k; \alpha_{t+1}) + \dots$$

$$\dots + \sum_{j=t+1}^{\lambda(d)-1} \left[\left(l_j - c_j\right) \prod_{k=j+1}^{\lambda(d)} r^p_k(z_k, \gamma_k; \alpha_{t+1}) \right] - \chi l_{\lambda(d)} \tag{2.3.11}$$

$$\gamma_{\lambda(d)} = H\left(\gamma_t,\, z^{\lambda(d)}\right) \tag{2.3.12}$$

$$a_{j+1} \geq \underline{a}, \qquad b_{j+1} \geq \underline{b}, \qquad \omega_{j+1} \geq \underline{\omega}, \qquad \forall j \in [t, \lambda(d) - 1) \tag{2.3.13}$$

$$\lambda(d) = \min_{j \in [t,d]} \left\{ d,\; e_j \neq e_{j-1},\; b_{j+1} < \underline{b},\; a_{j+1} < \underline{a},\; \left(a_{j+1} + b_{j+1}\right) < \underline{f} \right\} \tag{2.3.14}$$

where $r^p(z, \gamma; \alpha) = \alpha\left(r^a(z, \gamma) - r^b(z, \gamma)\right) + (1 + r^b(z, \gamma))$ denotes the total returns of the financial portfolio and $\alpha_t = \frac{a_{t+1}}{a_{t+1} + b_{t+1}}$ is the share of the financial portfolio invested in the risky asset at the planning date t. Equation (11) denotes the budget constraint of the agents, who use their wealth and labor income to consume and invest in the two assets. Equation (12) shows the evolution over time of total wealth, which depends on the consumption stream and the returns to investment throughout inattention. Note that the share of risky capital in the financial portfolio is kept constant at $\alpha_{t+1} = \frac{a_{t+1}}{a_{t+1} + b_{t+1}}$. Moreover, at the realised new planning date $\lambda(d)$ agents incur in the observation cost $\chi l_{\lambda(d)}$. Equation (13) defines the law of motion of the distribution of agents γ_t conditional on the history of aggregate shocks $z^{\lambda(d)}$. Finally, Equation (14) denotes the borrowing constraints faced by the households, whereas Equation (16) describes the new realised planning date $\lambda(d)$, which depends not only on the decision of the next planning date d, but also on the dynamics of the employment status, and the value of stock and bonds. In this environment, Reis (2006) shows that the measurability constraint holds as long as the optimal choices $\left\{d,\, \left[c_t, c_{\lambda(d)-1}\right],\, a_{t+1},\, b_{t+1}\right\}$ are made only upon the information given by $\left\{\omega_t, e_t, \xi_t; z_t, \gamma_t\right\}$.

2.3.3 Equilibrium

Definition of Equilibrium.

A competitive equilibrium for this economy is a value function V and a set of policy functions $\left\{g^c, g^h, g^b, g^a, g^d\right\}$, a set of prices $\left\{r^b, r^a, w\right\}$, and a law of motion $H(\cdot)$ for the measure of agents γ such that[10]:

- Given the prices $\left\{r^b, r^a, w\right\}$, the law of motion $H(\cdot)$, and the exogenous transition matrices $\left\{\Gamma^z, \Gamma^e, \Gamma^\xi\right\}$, the value function V and the set of policy functions $\left\{g^c, g^h, g^b, g^a, g^d\right\}$ solve the household's problem;
- The bonds market clears, $\int g^b \mathrm{d}\gamma = 0$;
- The capital market clears, $\int g^a \mathrm{d}\gamma = K'$;
- The labor market clears, $\int e\xi \mathrm{d}\gamma = N$;
- The law of motion $H(\cdot)$ is generated by the optimal decisions $\left\{g^c, g^h, g^b, g^a, g^d\right\}$, the transition matrices $\left\{\Gamma^z, \Gamma^e, \Gamma^\xi\right\}$ and the history of aggregate shocks z.

First-Order Conditions.

Gabaix and Laibson (2002) consider an environment where agents are exogenously inattentive for a given number of periods. In their model, the Euler equation for consumption holds just for the mass of attentive agents because inattentive households are off their equilibrium condition. Instead, here the Euler equations of both attentive and inattentive agents hold in equilibrium. Indeed, the Euler equation of an agent at a planning date t is a standard stochastic inter-temporal condition that reads

$$\mathbb{E}_t\left[M_{\lambda(d),t}\prod_{k=t+1}^{\lambda(d)}\left(\alpha_{t+1}\left(r_k^a(z_k,\gamma_k) - r_k^b(z_k,\gamma_k)\right) + (1 + r_k^b(z_k,\gamma_k))\right)\right] = 0 \qquad (2.3.15)$$

where $\lambda(d)$ denotes the next date in which the household will gather new information and define a new consumption/savings plan, and $M_{\lambda(d),t} = \beta^{\lambda(d)-t}\frac{U'\left(c_{\lambda(d)}\right)}{U'(c_t)}$ is the households' stochastic discount factor. This condition posits that the optimal share of stocks in the portfolio is the one which equalizes the expected discounted flow of returns from stocks and bonds throughout the period of inattention. The Euler equation is not satisfied with

[10]With an abuse of notation, I neglect the dependence of the value function, the policy functions, the set of prices and the law of motion of the measure of agents on the states of the households' problem.

equality for borrowing constrained agents. Instead, the Euler equation of an inattentive agent between time s and q, with $t < s < q < \lambda$ is deterministic and equals

$$M_{q,s} \prod_{k=s+1}^{q} \left(\alpha_{s+1} \left(r_k^a (z_k, \gamma_k) - r_k^b (z_k, \gamma_k) \right) + (1 + r_k^b (z_k, \gamma_k)) \right) = 0 \tag{2.3.16}$$

Inattentive agents do not gather any new information on the states of the economy and therefore they behave as if there were no uncertainty. Agents get back to the stochastic inter-temporal conditions as soon they reach a new planning date, and update their information set. Therefore, as agents alternate between periods of attention and inattention, they also shift from stochastic to deterministic Euler equations.

2.4 Calibration

The calibration strategy follows Krusell and Smith (1997, 1998) and Pijoan-Mas (2007). Some parameters (e.g., the risk aversion of the household) are set to values estimated in the literature, while others are calibrated to match salient facts of the U.S. economy. The idiosyncratic labor income risk is defined to target the cross-sectional distribution of labor income. It is important to have a realistic variation in labor income because the choice of inattention, and consequently all the effects of the observation cost on asset prices, depends on the budget of households. Then, the aggregate shock is calibrated to match the volatility of aggregate output growth, while the observation cost is defined to replicate the duration of inattention of the median household. Finally, despite I set one period of the model to correspond to one month, I report the asset pricing statistics aggregated at the annual frequency to be consistent with the literature.

The parameters set to values estimated in the literature are the capital share of the production function η, the capital depreciation rate δ, and the risk aversion of the household θ. I choose a capital share $\eta = 0.40$, as suggested by Cooley and Prescott (1995). The depreciation rate equals $\delta = 0.0066$ to match a 2% quarterly depreciation. The risk aversion of the household is $\theta = 5$, which gives an intertemporal elasticity of substitution of 0.2, at the lower end of the empirical evidence. Then, I set the constraint on financial wealth $\underline{f}$ to be minus two times the average monthly income of the economy, and households can reach this limit by short selling either bond or capital, that is, $\underline{b} = \underline{f}$ and $\underline{a} = \underline{f}$.[11] Finally, I calibrate the first parameter, the time discount rate of the household, to match the U.S. annual capital to output ratio of 2.5, and find $\beta = 0.9951$.

[11] In Guvenen (2009) the borrowing constraints equal 6 months of labor income. Instead, Gomes and Michaelides (2008) rule out any short sale. In Section 2.5.6, I evaluate how different values for the borrowing constraints might change the results of the model.

2.4.1 Aggregate Productivity Shock

I assume that the aggregate productivity shock follows a two-state first-order Markov chain, with values z_g and z_b denoting the realizations in good and bad times, respectively. The two parameters of the transition function are calibrated targeting a duration of 2.5 quarters for both states. The values z_g and z_b are instead defined to match the standard deviation of the Hodrick-Prescott filtered quarterly aggregate output, which is 1.89% in the data. These values are therefore model dependent, and vary with the specification of the environment.

2.4.2 Idiosyncratic Labor Income Shock

Employment Status. The employment shock e follows a two-state first-order Markov chain, which requires the calibration of ten parameters that define four transition matrices two by two. I consider the ten targets of Krusell and Smith (1997, 1998). I first define four conditions that create a one-to-one mapping between the state of the aggregate shock and the level of unemployment. That is, the good productivity shock z_g comes always with an unemployment rate u_g, and the bad one z_b with an unemployment rate u_b, regardless of the previous realizations of the shock. In this way, the realization of the productivity shock pins down the unemployment rate of the economy. The four conditions are

$$1 - u_g = u_g \Gamma_e \left(z_g, z_g, 0, 1\right) + \left(1 - u_g\right) \Gamma_e \left(z_g, z_g, 1, 1\right) \tag{2.4.17}$$

$$1 - u_g = u_b \Gamma_e \left(z_b, z_g, 0, 1\right) + (1 - u_b) \Gamma_e \left(z_b, z_g, 1, 1\right) \tag{2.4.18}$$

$$1 - u_b = u_g \Gamma_e \left(z_g, z_b, 0, 1\right) + \left(1 - u_g\right) \Gamma_e \left(z_g, z_b, 1, 1\right) \tag{2.4.19}$$

$$1 - u_b = u_b \Gamma_e (z_b, z_b, 0, 1) + (1 - u_b) \Gamma_e (z_b, z_b, 1, 1) \tag{2.4.20}$$

The level of the unemployment rate in good time and bad time are defined to match the actual average and standard deviation of the unemployment rate. I compute the two moments using data from the Bureau of Labor Statistics from 1948 to 2012, and obtain 5.67% and 1.68%, respectively. Under the assumption that the unemployment rate fluctuates symmetrically around its mean, I find $u_g = 0.0406$ and $u_b = 0.0728$. Two further conditions come by matching the expected duration of unemployment, which equals 6 months in good times and 10 months in bad times. Finally, I set the job finding probability when moving from the good state to the bad one as zero. Analogously, the probability of losing the job in the transition from the bad state to the good one is zero.

Unemployment Benefit. I set the unemployment benefit $\bar{w}$ to be 5% of the average monthly labor earning. Although different values of the benefit affect the lower end of the wealth distribution, they have no sizable effect on the asset pricing moments of the model.

Efficiency Units of Hour. The efficiency units of hour ξ follows a three-state first-order

Markov chain. The values of the shock and the transition function are calibrated to match three facts on the cross-sectional dispersion of labor earnings across households: the share of labor earnings held by the top 20% and the bottom 40% of households, and the Gini coefficient of labor earnings. The data, taken from Díaz-Gímenez et *al.* (2011), characterize the distribution of earnings, income and wealth in the United States in 2007. Table 2.4.1 reports the calibrated values and the transition function of the shock ξ, while Table 2.4.2 compares the three statistics on the distribution of labor earnings in the data and in the model.

TABLE 2.4.1

PARAMETERS SHOCK EFFICIENCY UNITS OF HOUR

	$\xi_1 = 15$	$\xi_2 = 4$	$\xi_3 = 1$
	$\Gamma_\xi(\xi_1, \cdot)$	$\Gamma_\xi(\xi_2, \cdot)$	$\Gamma_\xi(\xi_3, \cdot)$
$\Gamma_\xi(\cdot, \xi_1)$	0.9850	0.0025	0.0050
$\Gamma_\xi(\cdot, \xi_2)$	0.0100	0.9850	0.0100
$\Gamma_\xi(\cdot, \xi_3)$	0.0050	0.0125	0.9850

Note: The efficiency unit of hours ξ follows a first-order Markov chain with transition function Γ_ξ.

TABLE 2.4.2

THE DISTRIBUTION OF LABOR EARNINGS

Target	Model	Data
Share earnings top 20%	62.1%	63.5%
Share earnings bottom 40%	4.4%	4.2%
Gini index	0.57	0.64

Note: the data is from Díaz-Gímenez et *al.* (2011).

2.4.3 Observation Cost

The observation cost is calibrated to match the duration of inattention of the median household in a year, which Alvarez et *al.* (2012) estimate to be around 3 months. Accordingly, I set the fixed cost to $\chi = 0.029$. It amounts to 2.9% of households' monthly labor earnings. For example, if the average household earns an income of around $\$3,000$ per month, the cost equals \$87.

2.4.4 Computation of the Model

The computation of heterogeneous agent models with aggregate uncertainty are known to be cumbersome because the distribution γ, a state of the problem, is an infinite-dimensional object. I approximate γ using a finite set of moments of the distribution of aggregate capital K, as in Krusell and Smith (1997, 1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008), and the number of inattentive agents in the economy in every period ζ. On one hand, the approximation using a finite set of moments of aggregate capital K can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of γ. Nevertheless, this class of models generates almost linear economies, in which it is sufficient to consider just the first moment of the distribution of capital to have almost a perfect fit for the approximation. On the other hand, inattention adds a further term ζ, which signals active investors about the degree of the informational frictions in the economy. This condition adds a further law of motion upon which to find convergence. The presence of inattention implies one further complication. The decision of the agents on how long to stay inattentive requires the evaluation of their value function over a wide range of different time horizons. I report the details of the algorithm in the Appendix 2.A.

2.5 Results

I compare the results of the benchmark model with three alternative calibrations. In the first, the observation cost is zero and there is no inattention. In the second one, the observation cost is more severe and amounts to $\chi = 0.058$. Finally, I consider an economy in which agents are more risk averse, with $\theta = 8$. I calibrate each version of the model to match both the level of aggregate wealth and the volatility of aggregate output growth. Results are computed from a simulated path of $3,000$ agents over $10,000$ periods.

2.5.1 Inattention

The observation cost is calibrated to a 3 months duration of inattention for the median household. It turns out that such a cost prevents a third of agents from gathering information on the stock market. Table 2.5.3 shows that in the model, in any given month, the average fraction of inattentive agents in the economy equals 39%.

TABLE 2.5.3
INATTENTION

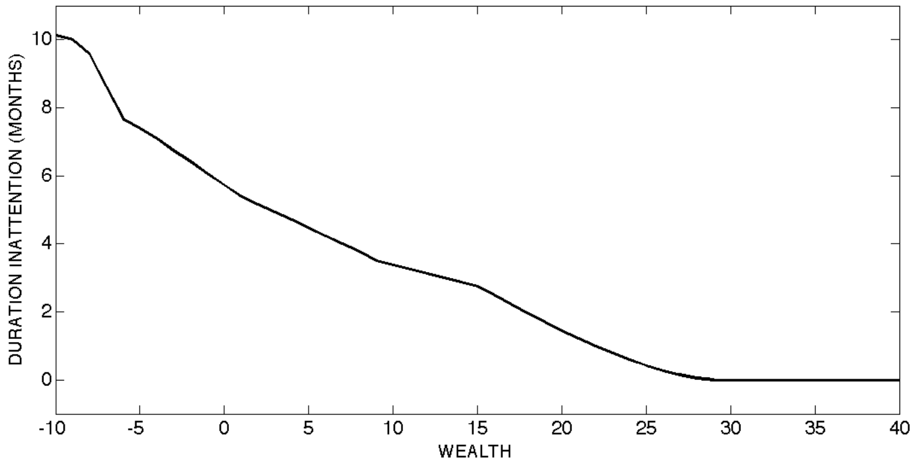
Inattention	χ = 0.024	χ = 0	χ = 0.048	θ = 8	Data
		A. Duration of inattention (months)			
Median	3.0	0	3.3	3.7	3.0
Median - good times	2.8	0	3.0	3.3	-
Median - bad times	3.2	0	3.5	4.0	-
75th percentile - good times	1.0	0	1.1	1.2	-
75th percentile - bad times	0.7	0	0.8	0.7	-
25th percentile - good times	5.5	0	5.8	6.2	-
25th percentile - bad times	6.0	0	6.2	6.6	-
		B. Fraction of inattentive agents			
Median	0.39	0	0.41	0.45	-
Median - good times	0.36	0	0.38	0.41	-
Median - bad times	0.42	0	0.44	0.49	-

Note: the variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. Good times denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z = z_b$. The fraction of inattentive agents are reported in percentage values. Data is from Alvarez et *al.* (2012).

Furthermore, Figure 2.5.1 shows that there is a negative correlation between wealth and inattention, in line with the empirical evidence of Ameriks et *al.* (2003) and Alvarez et *al.* (2012). There is also a sizable dispersion of inattention across agents, because poor agents cannot afford the observation cost and end up being more inattentive. For example, the wealthiest 20% of households observe the states of the economy every period, while the poorest 20% stay inattentive for 8 months on average. Such behavior implies that in the model inattention behaves as both a time-dependent and a state-dependent rule. Indeed, at each point of time households set a time-dependent rule, deciding how long to stay inattentive. Yet, when a household becomes wealthier, it opts for shorter periods of inattention. Thus, inattention looks as if it were conditional on wealth.[12]

FIGURE 2.5.1

OPTIMAL CHOICE OF INATTENTION



Note: the figure plots the policy function of inattention g^d as a function of wealth ω. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate shock is $z = z_g$ and the aggregate capital equals its mean.

When studying the dynamics of inattention over the cycle, I find that it depends on two forces. On one hand, the countercyclical equity premium induces agents to plan in recessions because the cost of inattention in terms of foregone financial returns is lower in good times. On the other hand, the severity of the observation cost fluctuates as a function of households' wealth. In recessions, agents are poorer and cannot afford the observation cost. The results point out that the former channel dominates in wealthy agents, whose inattention is pro-cyclical. For example, in the model the agents at the 75-th percentile of the wealth distribution are on average inattentive for 1 month in good times and 0.7 months in bad times. Instead, the direct cost of inattention affects relatively more poor agents, which prefer to plan in expansions. The agents at the 25-th percentile of the wealth distribution are on average inattentive for 5.5 months in good times and 6 months in bad times. Overall, inattention is countercyclical: both the duration of inattention for the median agent and the fraction of inattentive agents in the economy rise in recession. Such a result can also be interpreted as a foundation to the countercyclical dynamics of uncertainty. Indeed, the two concepts are intimately tied: when agents pay less attention to the states of the economy, the dispersion of their forecasts over future returns rises, boosting the level of uncertainty in the economy.

Increasing the size of the observation cost to $\chi = 0.048$ extends the duration of inattention for the median agent up to 3.3 months. Also a risk aversion of $\theta = 8$ does increase

[12] Reis (2006) labels this property of inattention as "recursive time-contingency". See Alvarez et *al.* (2012) and Abel et *al.* (2007, 2013) for further characterizations of the dynamics of inattention over time.

the duration of inattention, which goes up to 3.7 months. This last result is in line with the evidence provided by Alvarez et *al.* (2012), who show that more risk averse investors observe their portfolio less frequently. This outcome is the net result of two counteracting forces. Agents with a higher risk aversion changes their portfolio towards risk-free bonds, decreasing the need of observing the stock market. At the same time, more risk averse agents have a stronger desire for consumption smoothing, which induces them to keep track of their investments more frequently. In the model, the first channel offsets the second one, implying a longer duration of inattention for more risk averse agents.

2.5.2 Stock Market Participation

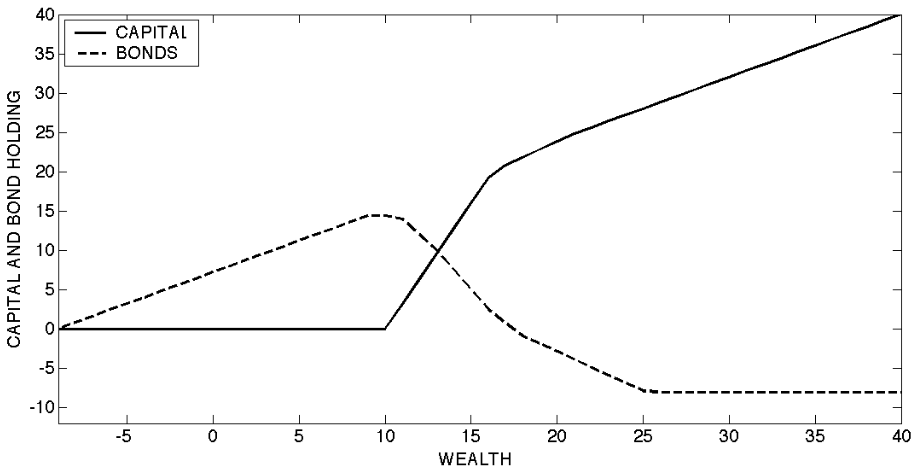
The observation cost induces a large fraction of households not to own any stock. As reported in Table 2.5.4, 26.6% of households do not participate to the equity market. Favilukis (2013) shows that in 2007 the actual share of stockholders equals 59.4%. Hence, the observation cost accounts for 44.8% of the observed number of non-stockholders. Unlike in Saito (1996), Basak and Cuoco (1998), and Guvenen (2009), here the limited participation does not arise exogenously. Indeed, in the economy without inattention virtually all households access the market. Therefore, the observation cost is de facto a barrier to the investment in stocks, as the fixed participation cost does in the environment of Gomes and Michaelides (2008). This result points out to a new rationale to the limited stock market participation: it is not just the presence of trading costs that matters, but also the fact that processing all the information required to invest optimally in the financial markets is not a trivial task at all. In addition, the model successfully predicts that stockholders are on average wealthier than non-stockholders. As Figure 2.5.2 shows, stockholders tend to be the wealthiest agents of the economy. For example, the poorest 7.3% of households do not hold any risky capital because they are the most inattentive agents of the economy. However, the model fails in reproducing the higher consumption growth volatility of stockholders with respect of non-stockholders. Mankiw and Zeldes (1991) find that the consumption growth of stockholders is 1.6 times as volatile than the one of non-stockholders. Instead, in the benchmark model the ratio of the consumption growth of stockholders over the one of non-stockholders equals 0.78. Indeed, stockholders turn out to be wealthy agents that are still able to self-insure their consumption stream, experiencing thereby a lower volatility than non-stockholders. I find that even higher observation costs and risk aversion cannot fully account for the observed participation rate and the higher consumption growth volatility of stockholders. Also Guvenen (2009) finds that a low participation rate is not enough to generate a higher volatility of consumption for stockholders, unless it is assumed that stockholders have a higher intertemporal elasticity of substitution than non-stockholders.

TABLE 2.5.4
PARTICIPATION TO THE STOCK MARKET

Variable	χ = 0.024	χ = 0	χ = 0.048	θ = 8	Data
% Stockholders	73.4	98.5	70.5	64.9	40.6
$\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$	0.78	0.37	0.80	0.88	1.6

Note: the variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. The ratio $\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$ compares the standard deviation of stockholders' consumption growth $\sigma(\Delta \log c_S)$ with the standard deviation of non-stockholders' consumption growth $\sigma(\Delta \log c_{NS})$. Data is from Mankiw and Zeldes (1991) and Favilukis (2013).

FIGURE 2.5.2
OPTIMAL PORTFOLIO CHOICES



Note: the figure plots the policy functions of investment in risky assets g^a (continuous line) and risk free bonds g^b (dashed line) as a function of wealth ω. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate shock is $z = z_g$ and the aggregate capital equals its mean.

2.5.3 The Distribution of Wealth

The observation cost spreads also the distribution of households' wealth ω_t. Table 2.5.5 reports that the Gini index equals 0.41 in the economy with no inattention. This value is exactly half the value of 0.82 that Díaz-Gímenez et *al.* (2011) find in the data. Indeed, the distribution is too concentrated around the median: there are too few poor and rich agents. This is no surprise. Krusell and Smith (1997, 1998) already discuss how heterogeneous agent models have a hard time to account for the shape of the wealth distribution. Yet, when I consider the observation cost of the benchmark model, the Gini coefficient goes up by 56% to 0.64. Inattention generates a more dispersed distribution through the limited participation in the stock market and the higher returns to stock. Poor agents cannot afford the observation cost and end up being more inattentive. Accordingly, they decide not to own any stock and give up the higher return to risky capital. The model describes well the wealth distribution at the 20-th, 40-th and 60-th quantiles, but it falls short in replicating the tails of the distribution. Increasing the size of the observation cost or the risk aversion of households improves just slightly the performance of the model.

Table 2.5.5
The distribution of wealth

% wealth held by	χ = 0.024	χ = 0	χ = 0.048	θ = 8	Data
20th percentile	3.3	5.5	2.9	2.5	1.1
40th percentile	6.6	14.2	6.3	5.7	4.5
60th percentile	16.7	31.6	16.1	14.3	11.2
90th percentile	51.8	29.4	53.0	59.3	71.4
Gini index	0.64	0.41	0.66	0.69	0.82

Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. Data is from Díaz-Gímenez et *al.* (2011).

2.5.4 Asset Pricing Moments

Stock and Bond Returns

The Panel A of Table 2.5.6 reports the results of the model on the level and the dynamics of stock returns, bond returns and the equity premium. First, I discuss the standard deviations because the observation cost triples the volatility of stock returns. In the benchmark model the standard deviation of returns is 6.68%, which is around a third of the value observed in the data, 19.30%. Nonetheless, without inattention the standard deviation would be just 2.21%. The observation cost boosts the volatility of returns because it acts as a capital adjustment cost. Indeed, inattention makes the supply of capital to be inelastic along two dimensions. On one hand, inattentive agents follow pre-determined path of capital investment and cannot adjust their holdings to the realizations of the aggregate shock. On the other hand, the limited participation in the equity market shrinks the pool of potential investors. As far as the volatility of the risk-free rate is concerned, I find a standard deviation of 3.57%, which is lower than its empirical counterpart, that equals 5.44%. Note that standard models usually deliver risk-free rates which fluctuate too much. For example, Jermann (1998) and Boldrin et *al.* (2001) report a standard deviation between 10% and 20%. The mechanism that prevents volatility to surge is similar to the one exploited by Guvenen (2009). Poor agents have a strong desire to smooth consumption, and their high demand of precautionary savings offsets any large movements in bond returns. Although in Guvenen (2009) the strong desire for consumption smoothing is achieved through a low elasticity of intertemporal substitution, here it is the observation cost that forces poor and inattentive agents to insure against the risk of infrequent planning. When looking at the level of the equity premium reported in Panel B of Table 2.5.6, I find that the model generates a wedge between stock returns and bond yields which is too low. It equals 0.93% while in the data it is 6.17%. Since the model does not suffer of the risk-free rate puzzle of Weil (1989), the weakness is entirely in the level of stock returns. In the model the average stock returns is 3.16%, around a third of the value observed in the data. Again, the observation cost goes a long way forward in explaining the equity premium, because the model with no inattention has a differential between stock and bond returns of 0.01%. Indeed, the limited participation in the stock market concentrates the entire aggregate risk of the economy on a smaller measure of stock-holders, who accordingly demand a higher compensation for holding equity. Furthermore, inattention exacerbates the curvature of the value function of the agents. Figure 2.5.3 - 2.5.4 show that the value function of agents in an inattentive economy is much more concave that in the absence of any observation cost. Moreover, the curvature of inattentive agents is much more responsive to aggregate conditions. Indeed, while the risk aversion of agents in attentive economies is rather constant along the cycle, the risk aversion of inattentive agents rises dramatically in recessions. As a result, inattention amplifies the risk associated to holding stocks, especially in bad times. These mechanisms explain why inattention generates an equity premium several orders of magnitude higher than in a model without observation costs. Yet, the improvements are not enough to explain the puzzle. Doubling the size of the observation cost does not yield any better result: the Sharpe ratio barely changes. So, observation costs should be unreasonably high to provide a premium as it is in the data. Only a higher risk aversion of $\theta = 8$ seems to deliver better asset pricing moments, with an average stock returns of 3.85% and a 0.18 Sharpe ratio which implies an equity premium of 1.25%. These results confirm the findings of Gomes and Michaelides (2008) and Guvenen (2009), in which limited participation in the stock market is not sufficient to imply a high equity premium. Both papers introduce heterogeneity in the intertemporal elasticity of substitution to increase the volatility of consumption growth of stockholders and generate a high price of risk.

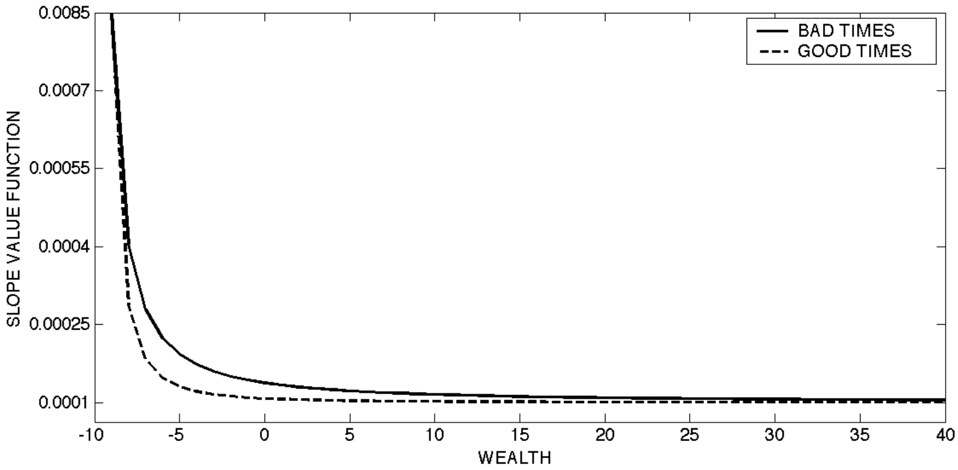
Table 2.5.6

Asset pricing moments

Variable	Moment	$\chi = 0.024$	$\chi = 0$	$\chi = 0.048$	$\theta = 8$	Data
		A. Stock and bond returns				
Stock return	Mean	3.16	1.13	3.37	3.85	8.11
	Std. dev.	6.68	2.21	6.86	7.08	19.30
Risk-free return	Mean	1.84	1.12	1.86	2.04	1.94
	Std. dev.	3.57	2.61	4.02	4.33	5.44
		B. Equity premium				
Equity premium	Mean	0.93	0.01	1.01	1.25	6.17
	Std. dev.	6.44	1.11	6.68	6.97	19.49
Sharpe ratio	Mean	0.14	0.01	0.15	0.18	0.32
		C. Cyclical dynamics				
Stock returns	Std. dev. - good times	6.52	0.55	6.79	6.94	-
	Std. dev. - bad times	6.64	0.55	7.91	7.16	-
Equity premium	Mean - good times	0.90	0.01	0.98	1.22	-
	Mean - bad times	0.96	0.01	1.06	1.31	-

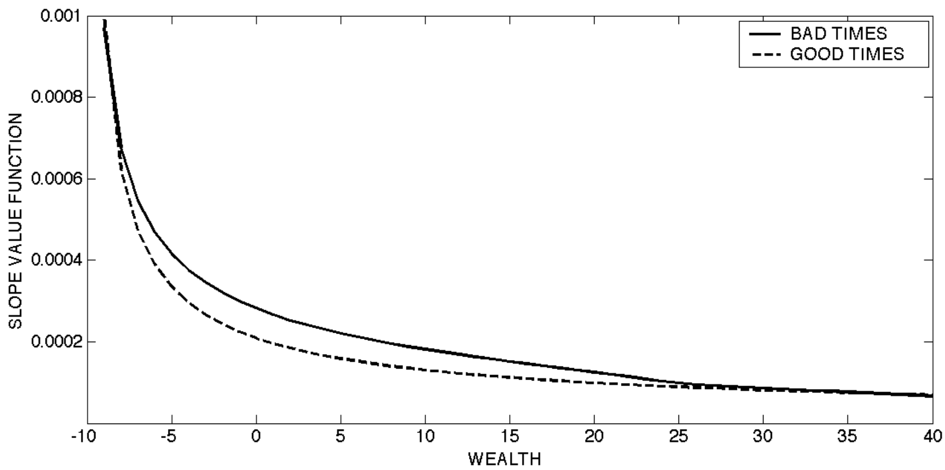
Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in expectation and reported in annualized percentage values. Annual returns are defined as the sum of log monthly returns. The equity premium is the $r^e = \mathbb{E}\left[r^a - r^b\right]$. The Sharpe ratio is defined as the ratio between the equity premium and its standard deviation. Good times denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z = z_b$. Data is from Campbell (1999) and Guvenen (2009).

FIGURE 2.5.3
SLOPE OF THE VALUE FUNCTION - ATTENTIVE ECONOMY



Note: the figure plots the slope of agents' value function as a function of wealth ω. in an economy with observation cost $\chi = 0$. Good times (dashed line) and bad times (continuous line) denote periods in which the aggregate productivity shock is $z = z_g$ and $z = z_b$, respectively. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. Aggregate capital equals its mean.

FIGURE 2.5.4
SLOPE OF THE VALUE FUNCTION - INATTENTIVE ECONOMY



Note: the figure plots the slope of agents' value function as a function of wealth ω. in an economy with observation cost $\chi = 0.024$. Good times (dashed line) and bad times (continuous line) denote periods in which the aggregate productivity shock is $z = z_g$ and $z = z_b$, respectively. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. Aggregate capital equals its mean.

Cyclical Dynamics

Inattention generates countercyclical variations in stock returns volatility and the equity premium, as shown in Panel C of Table 2.5.6. Since the observation cost bites more strongly in recessions, there are very few active investors in the economy which implies that the quantity of capital is low and very responsive to the investment of the marginal attentive stockholder. Instead, when the observation cost goes to zero the volatility becomes acyclical. Therefore, in this setting the observation cost mimics the role of countercyclical uncertainty in Veronesi (1999), which induces the volatility to be asymmetric over the cycle, peaking in recessions. Also the equity premium is countercyclical and displays a sizable variation over the cycle. It equals 0.90% in good times and 0.96% in bad times. This result is in line with the empirical evidence on a positive risk-return trade-off.[13] Again, this dynamics is driven by inattention since the equity premium does not move over the cycle in the economy with no observation costs. Hence inattention generates countercyclical variations in the price of risk which are usually obtained through consumption habits and long-run risk.

The model implies one further successful prediction: both the level and the volatility of the excess returns can be predicted using the consumption-wealth ratio. This result is in line with the broad literature that provides evidence in favor of the predictability of stock returns, see for example Campbell (1991) and Cochrane (1991). Lettau and Ludvigson (2001) exploit the insight of Campbell and Mankiw (1989) on the cointegrating relationships between consumption growth and wealth growth to show that the consumption-wealth ratio does predict future stock returns. Basically, when the log consumption-wealth ratio increases, either the expected consumption grows less quickly, or future returns are expected to be high. Table VII shows that in the model the consumption-wealth ratio predicts the excess return at a four year horizon, despite there is no predictability at shorter frequencies. The picture on the volatility of the equity premium is completely reversed: the consumption-wealth ratio can predict it just at a horizon of one year, but not afterwards. When the observation cost equals zero, there is no predictability at all.

Consumption Growth

I report in Table 2.5.7 the prediction of the model on the dynamics of consumption growth. Panel A shows that inattention does not substantially increases the standard deviation of aggregate consumption growth, which keeps around 0.60 while in the data it equals 0.76. Indeed, despite inattention forces agents to sharp changes in consumption at planning dates, in a general equilibrium agents are aware of it and optimally respond to the observation cost by choosing even smoother consumption paths. Overall, these two counter-

[13] The evidence on the sign of the risk-return relationship is mixed. Lettau and Ludvigson (2010) shows that while the unconditional correlations are weakly negative, the conditional correlation provides evidence in favor of a strong positive relationship.

acting forces offset each other. Nonetheless, in the model attentive agents experience a slightly lower volatility of consumption growth than inattentive agents. Indeed, as long as inattentive agents and non-stockholders overlap, the counterfactual prediction that stockholders insure relatively better their stream of consumption turns into a lower volatility for attentive agents. More interestingly, the observation cost disconnects the movements in stock returns and aggregate consumption growth. For example, in the model without inattention, the correlation between stock returns and consumption on a quarterly basis is 0.77. This number falls to 0.38 in the benchmark model and further down to 0.31 with the higher risk aversion coefficient, getting closer to the empirical value of 0.22. Stock returns and consumption are not so correlated because inattentive agents do not react to changes in the current realizations of the productivity shocks, and in turn to current values of asset prices. The mechanism reminds of Lynch (1996), in which the the lack of synchronization across agents weakens the correlation between equity and consumption. There is also another newsworthy pattern that emerges out of Panel B. Attentive agents display higher than average correlations between consumption growth and stock returns. Indeed, as long as inattentive agents follows pre-determined path of consumption, they do not react to realizations of the aggregate shock and tend to consume too much in bad states and too low in good states. Such behavior generates an additional source of risk since attentive agents are forced to give up consumption in recessions, which are times in which their marginal utility of consumption is highest. As a result, they command a higher premium for clearing the goods market. This mechanism is akin to the one studied in Chien et *al.* (2011, 2012), where passive investors which do not re-balance their portfolio raise the risk bore by active investors. When looking at the dynamics of aggregate consumption growth, Panel C shows that the model delivers a series which is not i.i.d. As in Peng (2005), the frictions in the dissemination of information rationalizes the presence of predictability in consumption growth. One one hand, Hall (1978) posits that consumption growth paths formed by rational agents should be unpredictable. On the other hand, Campbell and Mankiw (1990) find the presence of serial dependence. In the model, the consumption growth paths of agents conditional on their information sets are unpredictable. Still, an econometrician - who can observe all the information of the economy which has not been updated by agents yet - can find evidence of sizable positive autocorrelations. Therefore, it is the different information set between the econometrician and the agents which determines or not the predictability of consumption. Finally, the model fails in generating a dynamics of aggregate consumption growth consistent with the data, because consumption growth is homoskedastic and too persistent. Indeed, a Lagrange Multiplier test rejects the presence of heteroskedasticity in the simulated series of consumption growth at any confidence level. Furthermore, in the model the first autocorrelation of consumption growth measured at the quarterly frequency is 0.31, while in the data it equals 0.20.

Table 2.5.7
Moments of aggregate consumption growth

	χ = 0.024	χ = 0	χ = 0.048	θ = 8	Data
		A. Standard deviations			
Average	0.63	0.61	0.64	0.67	0.76
Attentive agents	0.60	0.61	0.60	0.63	-
Inattentive agents	0.68	-	0.69	0.72	-
		B. Correlation with stock returns			
Average	0.38	0.77	0.36	0.31	0.22
Attentive agents	0.41	0.77	0.39	0.34	-
Inattentive agents	0.33	-	0.34	0.28	-
		C. Time-series dynamics			
Autocorrelation	0.31	0.09	0.33	0.37	0.20
ARCH Effects	No	No	No	No	Yes

Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in quarterly values. In the model, the series of consumption and output growth are derived taking the Hodrick-Prescott filter of the logarithm of the simulated series aggregated at the quarterly frequency. The correlation of attentive agents' consumption growth with aggregate output growth is computed pooling the average of the individual correlations of attentive agents over time. The same applies to the correlation of inattentive agents. The autocorrelation reports the persistence of an AR(1) model fitted to the series of consumption growth. ARCH effects are evaluated using a Lagrange Multiplier test upon the fit of a ARMA(1,1)-ARCH(1) model. If the test statistics is greater than the Chi-square table value, the null hypothesis of no ARCH effects is rejected. Data is from Campbell (1999) and Guvenen (2009).

2.5.5 Decomposing the Price of Risk

The observation cost is not enough to generate an equity premium as high as it is in the data, because in a general equilibrium households take it into account when making their optimal portfolio choices. As in Heaton and Lucas (1996), agents respond to the additional fluctuations in consumption due to inattention by reducing their exposure to aggregate risk. In this Section, I disentangle the four main mechanisms through which households' optimal reaction to the observation cost impedes the price of risk to rise: 1) adjustments in the duration of inattention; 2) switches across consumption and precautionary savings; 3) changes in the composition of the financial portfolio and 4) shifts in the set of the agents pricing stocks and bonds. To identify these channels, I follow Pijoan-Mas (2007) by comparing six different equilibria. First, I consider the economy with no observation cost. Second, I formulate an economy where the observation cost equals $\chi = 0.024$ and take as given the optimal choices of agents in the economy with no inattention. So, households suffer the observation cost but cannot react to it. Third, I compute a new equilibrium by allowing agents to react to the observation cost by setting their optimal duration of inattention. Fourth, I derive the model by allowing agents to modify the composition of the financial portfolio, but not the allocations in consumption and savings. As a result, households can adjust the share of risky assets in their portfolio, but cannot increase the buffer of precautionary savings. Fifth, I consider the benchmark model with observation costs where agents can entirely decide their optimal policies. Finally, I take the last equilibrium focusing only on households with interior solutions, which are eventually those pricing the two assets. Table 2.5.8 shows the coefficient of variation of the marginal value of wealth over different percentiles of its distribution under all these scenarios.

The price of risk peaks when moving from the economy without observation cost to one in which $\chi = 0.024$ and the agents are forced to follow the optimal policies of the economy without inattention. For the median households, it surges from 0.002 to 0.49, reaching even 0.74 on the right tail of the distribution. Since agents cannot modify their choices, they command a very high premium to bear the risk of being inattentive and owning stocks. The price of risk decreases as long as we allow agents to modify first their portfolio and then the whole set of choices (i.e., consumption/savings and the composition of the portfolio), and it eventually reaches 0.13 for the median agent that prices risk in the benchmark economy. I therefore uses the entries of Table 2.5.8 at the median to disentangle the different channels through which households' reaction reduces the prices of risk. The most important one is the reaction of consumption to the observation cost, which explains 51.2% of the fall in the price of risk. Indeed, households take into account the risk of being inattentive by increasing precautionary savings. The second most important channel is the adjustment in the duration of inattention, which decreases the price of risk by 20.9%. Households temper the effects of the observation cost by deciding to incur in it less often. The third channel is given by changes in the financial portfolio, which accounts for 18.6% of the difference in the price of risk. Inattentive agents shifts their portfolio towards the risk-free bond, to diversify away the risk of stocks. Finally, the changes in the set of agents pricing risk matter too, accounting for 9.3% of the difference in the price of risk between the two setups considered here. Indeed, in this environment the equilibrium prices are defined by the stochastic discount factor of the households with interior solutions for both bonds and stocks. As far as stockholders are wealthy, they can self-insure their stream of consumption, implying a low price of risk. If the volatility of consumption growth of stockholders were higher than the one of non-stockholders, the changes in the set of households pricing risk would have boosted the equity premium rather than tempering it.

Table 2.5.8
Price of risk under different scenarios

Economy / Percentiles	25th	50th	75th	95th
χ = 0	0.002	0.002	0.002	0.002
χ = 0.024 - Choices as in χ = 0	0.46	0.49	0.55	0.74
χ = 0.024 - Consumption and portfolio as in χ = 0	0.40	0.41	0.46	0.66
χ = 0.024 - Consumption as in χ = 0	0.33	0.34	0.37	0.48
χ = 0.024 - Optimal policies	0.16	0.17	0.20	0.31
χ = 0.024 - Pricing agents	0.12	0.13	0.16	0.20

Note: This Table reports the price of risk of wealth along some percentiles of its distribution under different economies. The variable χ defines the observation cost. The pricing agents have interior policy functions for both bonds and capital. The first equilibrium derives agents' price of risk in an environment without observation costs. The second equilibrium derives agents' price of risk in an environment with observation costs, taking as given agents' optimal policy function from the economy without observation costs. The third equilibrium derives agents' price of risk in an environment with observation costs by allowing agents to set their optimal duration of inattention, taking as given the policy functions of consumption and portfolio choices from the economy without observation costs. The fourth equilibrium derives agents' price of risk in an environment with observation costs by allowing agents to set the duration of inattention and the optimal share of risky assets, taking as given the policy function for consumption from the economy without observation costs. The fifth equilibrium derives agents' price of risk in an environment with observation costs by allowing agents to set all their optimal choices. The sixth equilibrium derives the agents' price of risk in an environment with observation costs by focusing on households with interior policy solutions for both bonds and capital.

2.5.6 The Role of Borrowing Constraints

Chen (2006) considers a Lucas-tree economy where heterogeneous agents face an observation cost, finding that the equity premium is zero and inattention does not prevent agents from owning stocks. Instead, in my model the equity premium is around 1% and the number of agents that do not participate on the equity market is substantial. In this Section I show that such contradictory results can be rationalized by the interaction between the observation cost and the borrowing constraints. In what follows, I compare three economies which differ only for the level of the borrowing constraints. The first one is the benchmark model, where the borrowing constraints equal minus two times the average monthly income. In the second case, I consider an economy in which agents cannot borrow at all while in the last set up the constraints are loose and equal minus four times the average monthly income of households.

TABLE 2.5.9
THE ROLE OF BORROWING CONSTRAINTS

Variable	Benchmark	Tight Constraints	Loose Constraints
	A. Inattentive economy - χ = 0.024		
% Stockholders	73.4	60.5	90.1
Gini index wealth	0.64	0.73	0.49
Equity premium	0.93	6.06	0.08
	B. Attentive economy - χ = 0		
% Stockholders	98.5	91.2	98.7
Gini index wealth	0.41	0.54	0.36
Equity premium	0.01	4.85	0.004

Note: The variable χ defines the observation cost. In the "Benchmark" model, borrowing constraints equal minus two times the average monthly income of households, that is, $\underline{f} = -2\mathbb{E}[l_t]$. The "Tight Constraints" model does not allow short sales, that is, $\underline{f} = 0$. In the "Loose Constraints" model borrowing constraints equal minus four times the average monthly income of households, that is, $\underline{f} = -4\mathbb{E}[l_t]$.

In Panel A of Table 2.5.9, I report the fraction of stockholders, the Gini coefficient of the distribution of wealth and the equity premium implied by these three economies. When agents cannot borrow at all, the stock market participation falls to 60.5%, further spreading the distribution of wealth, whose Gini index is 0.73, and the equity premium is 6.06%. These numbers match almost perfectly their empirical counterparts. Instead, in the economy with loose financial constraints, almost all households own stocks. Furthermore, the wealth distribution is more concentrated and the equity premium is around zero. This exercise highlights that the definition of the borrowing constraints changes starkly the results of the model. Most importantly, the interaction between borrowing constraints and the observation cost plays a non negligible role. Indeed, Panel B of Table 2.5.9 shows the result of the same exercise applied to an economy without observation cost, that is, $\chi = 0$. In this case, tight constraints do increase the equity premium but just up to 4.85%.

Therefore, borrowing constraints can generate a high price of risk not only per se, as pointed out in Pijoan-Mas (2007) and Gomes and Michaelides (2008), but also for their interaction with the observation cost.

2.6 Conclusion

A recent strand of literature studies the role of agents' infrequent planning and limited attention to the stock market on asset prices, finding inconclusive results. Although inattention unambiguously increases the wedge between stock and bond returns, it is not clear yet whether it can account for the equity premium puzzle. In this paper, I evaluate the quantitative performance of inattention on asset prices in a production economy with heterogeneous agents and uninsurable labor income risk. I consider a monetary observation cost which generates a level of households' inattention and infrequent planning which is endogenous, heterogeneous across agents and time-varying. To discipline the role of infrequent planning, I calibrate the observation cost to match the actual duration of inattention of the median household. I find that the observation cost improves the performance of the model over several dimensions. Inattention spreads the wealth distribution toward realistic values and induces households not to to hold stocks, pointing a new rationale for the limited stock market participation. Households do not own stocks because investing in equity is not a trivial task at all. Then, I show that inattention induces the volatility of stock returns to be high and countercyclical. It also generates sizable countercyclical variations in the equity premium. Indeed, on one hand the aggregate risk is concentrated on a small measure of agents. On the other hand, inattentive agents create a residual risk by consuming too much in recessions and too little in expansions. Thus, attentive agents that actively invest in stocks command a countercyclical compensation to bear such additional source of risk. Nevertheless, any effect of inattention on the dynamics of stock prices vanishes as long as borrowing constraints are loose enough. This result suggests that models featuring inattention should carefully take into account the imperfections credit markets to deliver predictions consistent with the data. Furthermore, the model fails in delivering a consumption growth for stockholders less volatile than the one of non-stockholders, and an aggregate consumption growth which is homoskedastic and too persistent. Also the equity premium is still too low, around 1%. Raising the observation cost barely alters the Sharpe ratio. Indeed, in such a case households reduce the equilibrium price of risk by extending the duration of their inattention, accumulating more precautionary savings and disinvesting out of stocks. Overall, although inattention improves the performance of the model, it cannot quantitatively account for the observed dynamics of stock prices and excess returns yet.

2.A Appendix: Computational Algorithm

This Section describes the steps and the details of the computational algorithm I used to numerically solve the model. The algorithm is an extension to the case of inattention of the standard heterogeneous agent model with aggregate uncertainty and two assets, which has been already implemented by Krusell and Smith (1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008).

It is well known that the numerical computation of heterogeneous agent model with aggregate uncertainty and two assets is very cumbersome. The reason is twofold. First, one of the endogenous aggregate state of the problem is given by the distribution of the agents over their idiosyncratic states γ, which is an infinite-dimensional object. Indeed, as noted by Krusell and Smith (1997), agents need to know the entire distribution γ in order to generate rational expectations on prices. To circumvent this insurmountable curse of dimensionality, the state space has to be somehow reduced. I approximate the entire distribution γ by a set of moments $m < \infty$ of the stock of aggregate capital K, as in Krusell and Smith (1997), and the number of inattentive agents in the economy in every period ζ_t. On one hand, the approximation with a finite set of moments of K can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of γ. As in previous studies, I find that $m = 1$ is enough to have an almost perfect approximation of γ. That is, the mean of aggregate capital $\bar{K}$ is a sufficient statistics that capture virtually all the information that agents need to forecast future prices. On the other hand, the variable ζ signals agents about the degree of informational frictions in the economy. Indeed, when every agent is attentive, the model shrinks down to the standard Krusell and Smith (1998). Instead, where there is a (non-negligible) measure of inattentive agent, which is the case at the core of my analysis, the model departures from the standard setting. As far as the presence of observation costs pin down different equilibria, and therefore different path of futures prices, agents are required to be aware of the extent of the frictions in the economy whenever taking their optimal choices on consumption and savings. Second, when extending the basic Krusell and Smith (1997) algorithm to the case of an economy with two assets, the market for bonds does not clear at all dates and states. Indeed, the total bondholdings implied by the model is almost a random walk. As far as total bondholdings experience large movements over time, it is not always possible to achieve the clearing of the market. I therefore follow the modified algorithm of Krusell and Smith (1998), where agents perceive the bond return as a state of the economy. The equilibrium bond return is then the one in which the bond return perceived by the agents and the one implied by the optimal decisions of the agents coincide. The presence of the observation cost adds a further complication. Agents have to decide their optimal duration of inattention. This step requires the derivation of the household's maximization procedure not just in one case (i.e., today vs. the future), but in a much wider set of alternatives. Indeed, the household can decide whether to be attentive today and tomorrow, whether to be attentive today and inattentive for the following period, or to be attentive today and inattentive for the following two periods, and so on and so forth. Accordingly, I define a grid over all the potential durations of inattention that agents can pick up, solve the model over each grid points and eventually take the maximum among the different value functions to derive the optimal choice of inattention.

The computation of the model requires the convergence upon six forecasting rules which predict the future mean of the stock of aggregate capital, the future price of the bond and the future number of inattentive agents for both the aggregate shocks z_b and z_g. The procedure yields a set of twenty two different parameters upon which to converge. This algorithm is very time-consuming and makes at the moment computationally infeasible any extension of the model that inflates either the mechanisms or the number of states. For example, the assumption that inattentive agents do not gather information about their idiosyncratic shocks is required by this computational constraint.

In what follows, I first describe the computational algorithm in Section 2.A.1. Then, I discuss the problem of the household given the forecasting rule on future prices in Section 2.A.2. Finally, Section 2.A.3 concentrates on the derivation of the equilibrium forecasting rules. I also show that the substitution of the entire distribution γ with the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents ζ yields an almost perfect approximation.

2.A.1 Algorithm

The algorithm works around nine main steps, as follows:

1. Guess the set of moments m_t of aggregate capital K_t upon which to approximate the distribution of agents γ_t;
2. Guess the functional forms for the forecasting rule of the set of moments m_t, the number of inattentive agents in the economy ζ_t and the risk-free return to bond r_t^b;
3. Guess the parameters of the forecasting rules;
4. Solve the household's problem;
5. Simulate the economy:
   (a) Set an initial distribution of agents over their idiosyncratic states ω, e and ξ;
   (b) Find the interest rate r^{b*} that clears the market for bonds. Accordingly, guess an initial condition $r^{b,0}$, solve the household's problem in which agents perceive the bond return $r^{b,0}$ as a state, and obtain the policy functions $g^c\left(\omega, e, \xi; z, m, \zeta, r^{b,0}\right)$, $g^b\left(\omega, e, \xi; z, m, \zeta, r^{b,0}\right)$, $g^a\left(\omega, e, \xi; z, m, \zeta, r^{b,0}\right)$ and $g^d\left(\omega, e, \xi; z, m, \zeta, r^{b,0}\right)$. Use the policy functions on bondholdings g^b to check whether the market clears,

that is, whether the total holdings of bond equals zero. If there is an excess of bond supply, then change the initial condition to $r^{b,1} < r^{b,0}$. If there is an excess of bond demand, then change the initial condition to $r^{b,1} > r^{b,0}$. Iterate until the convergence on the interest rate r^{b*} that clears the market.

(c) Derive next period distribution of agents over their idiosyncratic states ω, e and ξ using the policy functions $g^c\left(\omega, e, \xi; z, m, \zeta, r^{b*}\right)$, $g^b\left(\omega, e, \xi; z, m, \zeta, r^{b*}\right)$, $g^a\left(\omega, e, \xi; z, m, \zeta, r^{b*}\right)$ and $g^d\left(\omega, e, \xi; z, m, \zeta, r^{b*}\right)$ and the law of motions for the shocks z, e and ξ.

(d) Simulate the economy for a large number of periods T over a large measure of agents N. Drop out the first observations which are likely to be influenced by the initial conditions.

6. Use the simulated series to estimate the forecasting rules on m_t, ζ_t and r^b_t implied by the optimal decisions of the agents;

7. Check whether the coefficients of the forecasting rules implied by the optimal decisions of the agents coincide with the one guessed in step (3). If they coincide, go to step (8). Otherwise, go back to step (3);

8. Check whether the functional forms of the forecasting rule as chosen in step (2) give a good fit of the approximation of the state space of the problem. If this is the case, go to step (9). Otherwise, go back to step (2);

9. Check whether the set of moments m_k of aggregate capital K yields a good approximation of the distribution of agents γ. If this is the case, the model is solved. Otherwise, go back to step (1).

2.A.2 Household's Problem

I solve the household's problem using value function iteration techniques. I discretize the state space of the problem as follows. First, I guess that the first moment of aggregate capital and the number of inattention agents are sufficient statistics describing the evolution of the distribution of agents γ. Later on, I evaluate the accuracy of my conjecture. Then, I follow Pijoan-Mas (2007) by stacking all the shocks, both the idiosyncratic and the aggregate ones, in a single vector ϵ, which has 8 points: four points - one for unemployed agents and three different level for employed agents - for each aggregate shock z. For the wealth ω I use a grid of 60 points on a logarithmic scale. Instead, for the possible durations of inattention d, I use a grid of 30 points: the first 25 points are equidistant and goes from no inattention at all, 1 month of inattention until 2 years of inattention. The following four grid points are equidistant on a quarterly basis. In this respect, the assumption made in the model on when inattention breaks out exogenously are very helpful in the definition of the grid. Indeed, agents will not choose too long durations of inattention because they take into account the probability of being called attentive due to a change in their employment status or because they hit the borrowing constraints. For example, in the benchmark model the largest point of the grid yields a duration of inattention of 3 years. Yet, this choice is hardly picked up by households in the simulations done to solve the model. Without the two assumptions on the exogenous ending of inattention, then some households could theoretically be inattentive forever, which would require a wider grid for the choice variable d. Then, for the grids of the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents ζ I use 6 points since the value function does not display a lot of curvature along these dimensions. To sum up, any value functions is computed over a total of $518,400$ different grid points. Furthermore, the state space is inflated in the case of the problem in which households perceive the bond return as a state of the economy. I use a grid for r^b formed by 10 points, which yields a total of $5,184,000$ grid points. Decisions rules off the grid are evaluated using a cubic spline interpolation around along the values of wealth ω and a bilinear interpolation around the remaining endogenous state variables. Finally, the solution of the model is simulated from a set of $3,000$ agents over T=10,000 time periods. In any evaluation of the simulated series, the first $1,000$ observations are dropped out.

The household's problem used in step (4) of the algorithm modifies the standard structure presented in the text to allow for the approximation of the measure of agents μ with the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents ζ. Then, I postulate three forecasting rules (R_1, R_2, R_3) for aggregate capital K, the number of inattentive agents ζ and the return of the bond r^b, respectively. The household's problem reads

$$V(\omega_t, \epsilon_t, K_t, \zeta_t) = \max_{d,\, \left[c_t, c_{\lambda(d)-1}\right],\, a_{t+1},\, b_{t+1}} \mathbb{E}_t \left[\sum_{j=t}^{\lambda(d)} \beta^{j-t}\; U\left(c_j\right) + \ldots \right.$$

$$\left. \ldots + \beta^{\lambda(d)-t}\; V\left(\omega_{\lambda(d)}, e_{\lambda(d)}, \xi_{\lambda(d)}; z_{\lambda(d)}, \gamma_{\lambda(d)}\right) \right]$$

$$\text{s.t.} \quad \omega_t + l_t(\epsilon_t, K_t, \zeta_t) = c_t + a_{t+1} + b_{t+1}$$

$$\omega_{\lambda(d)} = \left(a_{t+1} + b_{t+1}\right) \prod_{k=t+1}^{\lambda(d)} r_k^p(\epsilon_k, K_k, \zeta_k; \alpha_{t+1}) + \ldots$$

$$\ldots + \sum_{j=t+1}^{\lambda(d)-1} \left[\left(l_j - c_j\right) \prod_{k=j+1}^{\lambda(d)} r_k^p(\epsilon_k, K_k, \zeta_k; \alpha_{t+1}) \right] - \chi l_{\lambda(d)}$$

$$K_{\lambda(d)} = R_1\Big(K_t, \zeta_t, [z_t, z_{\lambda(d)}]\Big), \qquad \zeta_{\lambda(d)} = R_2\Big(K_t, \zeta_t, [z_t, z_{\lambda(d)}]\Big)$$

$$r^b_{\lambda(d)} = R_3\Big(K_t, \zeta_t, [z_t, z_{\lambda(d)}]\Big)$$
$$a_{j+1} \geq \underline{a}, \qquad b_{j+1} \geq \underline{b}, \qquad \omega_{j+1} \geq \underline{\omega}, \qquad \forall j \in [t, \lambda(d) - 1)$$
$$\lambda(d) = \min_{j \in [t,d]} \left\{ d,\ e_j \neq e_{j-1},\ b_{j+1} < \underline{b},\ a_{j+1} < \underline{a},\ \left(a_{j+1} + b_{j+1}\right) < \underline{f} \right\}$$

Instead, in step (5b) of the problem the households perceive the return of the bond r^b as a state of the economy, as follows

$$V\left(\omega_t, \epsilon_t, K_t, \zeta_t, r^b_t\right) = \max_{d,\ \left[c_t, c_{\lambda(d)-1}\right],\ a_{t+1},\ b_{t+1}} \mathbb{E}_t \left[\sum_{j=t}^{\lambda(d)} \beta^{j-t}\ U\left(c_j\right) + \ldots \right.$$
$$\left. \ldots + \beta^{\lambda(d)-t}\ V\left(\omega_{\lambda(d)}, e_{\lambda(d)}, \xi_{\lambda(d)}; z_{\lambda(d)}, \gamma_{\lambda(d)}\right) \right]$$
$$\text{s.t.} \quad \omega_t + l_t\left(\epsilon_t, K_t, \zeta_t, r^b_t\right) = c_t + a_{t+1} + b_{t+1}$$
$$\omega_{\lambda(d)} = \left(a_{t+1} + b_{t+1}\right) \prod_{k=t+1}^{\lambda(d)} r^p_k\left(\epsilon_k, K_k, \zeta_k, r^b_k; \alpha_{t+1}\right) + \ldots$$
$$\ldots + \sum_{j=t+1}^{\lambda(d)-1} \left[\left(l_j - c_j\right) \prod_{k=j+1}^{\lambda(d)} r^p_k\left(\epsilon_k, K_k, \zeta_k, r^b_k; \alpha_{t+1}\right) \right] - \chi l_{\lambda(d)}$$
$$K_{\lambda(d)} = R_1\Big(K_t, \zeta_t, [z_t, z_{\lambda(d)}]\Big), \qquad \zeta_{\lambda(d)} = R_2\Big(K_t, \zeta_t, [z_t, z_{\lambda(d)}]\Big)$$
$$r^b_{\lambda(d)} = R_3\Big(K_t, \zeta_t, [z_t, z_{\lambda(d)}]\Big)$$
$$a_{j+1} \geq \underline{a}, \qquad b_{j+1} \geq \underline{b}, \qquad \omega_{j+1} \geq \underline{\omega}, \qquad \forall j \in [t, \lambda(d) - 1)$$
$$\lambda(d) = \min_{j \in [t,d]} \left\{ d,\ e_j \neq e_{j-1},\ b_{j+1} < \underline{b},\ a_{j+1} < \underline{a},\ \left(a_{j+1} + b_{j+1}\right) < \underline{f} \right\}$$

I use this problem to simulate the economy given the return to the bond r^b as a perceived state for the households. I follow Gomes and Michaelides (2008) by aggregating agents' bond demands and determining the bond return that clears the market through linear interpolation. This value is then used to recover the implied optimal decisions of the agents, which are then aggregated to form the aggregate variables that becomes state variables in the following time period.

2.A.3 Equilibrium Forecasting Rules

I follow Krusell and Smith (1997, 1998) by defining log-linear functional forms for the forecasting rules of the mean of aggregate stock capital $\bar{K}$, the number of inattentive agents

ζ and the bond return r^b. Namely, I use the following law of motions:

$$\begin{aligned}
\log \bar{K} &= \alpha_0(z) + \alpha_1(z) \log \bar{K} + \alpha_2(z) \log \zeta \\
\log \zeta &= \beta_0(z) + \beta_1(z) \log \bar{K} + \beta_2(z) \log \zeta \\
r^b &= \gamma_0(z) + \gamma_1(z) \log \bar{K} + \gamma_2(z) \log \zeta + \gamma_3(z) \left(\log \bar{K}\right)^2 + \gamma_4(z) (\log \zeta)^2
\end{aligned}$$

The parameters of the functional forms depend on the aggregate shock z. Indeed, there is a set of three forecasting rule for each of the two realizations of the aggregate shock z, resulting in a total of six forecasting rules and twenty two parameters, upon which to find convergence.

I find the equilibrium forecasting rules as follows. First, I guess a set of initial conditions $\left\{\alpha_0^0(z), \alpha_1^0(z), \alpha_2^0(z), \beta_0^0(z), \beta_1^0(z), \beta_2^0(z), \gamma_0^0(z), \gamma_1^0(z), \gamma_2^0(z), \gamma_3^0(z), \gamma_4^0(z)\right\}$. Then, given such rules I solve the household's problem. I take the simulated series to then re-estimate the forecasting rules, which yields a new set of implied parameters $\left\{\alpha_0^1(z), \alpha_1^1(z), \alpha_2^1(z), \beta_0^1(z), \beta_1^1(z), \beta_2^1(z), \gamma_0^1(z), \gamma_1^1(z), \gamma_2^1(z), \gamma_3^1(z), \gamma_4^1(z)\right\}$. If the two sets coincide (up to a numerical wedge), then these values correspond to the equilibrium forecasting rules. Otherwise, I use the latter set of coefficients as a new initial guess.

For the benchmark specification of the model, I find the following equilibrium forecasting rules for $z = z_g$

$$\begin{aligned}
\log \bar{K} &= 0.101 + 0.976 \log \bar{K} - 0.249 \log \zeta \quad \text{with } R^2 = 0.993761 \\
\log \zeta &= -0.208 + 0.037 \log \bar{K} + 0.861 \log \zeta \quad \text{with } R^2 = 0.995890 \\
r^b &= 1.042 - 0.077 \log \bar{K} + 0.016 \log \zeta + \\
&\quad + 0.011 \left(\log \bar{K}\right)^2 + 0.006 (\log \zeta)^2 \qquad \text{with } R^2 = 0.998717
\end{aligned}$$

and the following equilibrium forecasting rules for $z = z_b$

$$\begin{aligned}
\log \bar{K} &= 0.084 + 0.986 \log \bar{K} - 0.240 \log \zeta \quad \text{with } R^2 = 0.994014 \\
\log \zeta &= -0.229 + 0.040 \log \bar{K} + 0.851 \log \zeta \quad \text{with } R^2 = 0.997081 \\
r^b &= 1.036 - 0.073 \log \bar{K} + 0.021 \log \zeta + \\
&\quad + 0.009 \left(\log \bar{K}\right)^2 + 0.009 (\log \zeta)^2 \qquad \text{with } R^2 = 0.998965
\end{aligned}$$

Note that the R^2 are all above 0.99. This result points out that approximating the distribution of agents γ with the first moment of aggregate capital $\bar{K}$ and the number of inattentive agents ζ implies basically no discharge of relevant information that agents can use to forecast future prices.

CHAPTER 3

Financial Development, Default Rates and Credit Spreads[1]

3.1 Introduction

We study the joint dynamics of corporate default and credit spreads from 1950 to 2012. We document that, over the last thirty years, defaults rates rose by 467% while credit spreads barely moved. We refer to this evidence as the diverging trend between rising default rates and constant credit spreads.

We provide statistical support for the presence of one structural break in the unconditional mean of default rates around 1984. This date splits the series of default rates in two samples with strikingly different characteristics. On one hand, during the 1950's and 1960's the US economy recorded almost no bankruptcies: the average default rate from 1950 to 1983 equals 0.3%. On the other hand, from the 1980's on we observe a dramatic rise in the number of defaults: the average number of corporate bankruptcies from 1984 to 2012 equals 1.7%. Hence, default rates have increased by 467% throughout the last thirty years. Conversely, the time series of credit spreads does not display any structural shift in its unconditional mean. The average credit spread over the period 1950-1983 records 91 basis points whereas the average spread from 1984 to 2012 amounts to 102 basis points. We run a battery of tests and show that this 11 basis points increase is not statistically significant.

At a first glance, it is hard to reconcile the different behavior of default rates and credit spreads. Anecdotal evidence would suggest the two time series to move together. The credit spread is a market measure of default risk and for this reason it should capture relevant information about default rates[1]. Therefore, such a steep rise in default rates should allegedly be mirrored by credit spreads. However it does not.

To understand this phenomenon, we propose an explanation based on a structural change in the supply side of credit. Although we acknowledge that changes in financial factors, such as shocks to liquidity or to the credit ratings, could account for this diverging trend, we provide a theory that is based just on fundamentals. We conjecture that the

[1]This Chapter is a joint work with Alessandro Peri.

[1]Longstaff et *al.* (2005) document that 71% of the Baa yield is explained by default risk.

reduction in the cost of borrowing due to the widely documented process of deregulation and innovation incurred by the financial sector in the 70s might explain this empirical evidence. Apropos, we construct a dynamic equilibrium model where two features, the development of credit markets and the limited enforceability of debt, can be accounted for the diverging trend between default rates and credit spreads. We model the development of credit markets in a reduced form, as an exogenous reduction of the fixed cost of borrowing. We find that financial development can explain 64% of the observed increase in average default rates and predict just a 2 basis points increase in the credit spreads. As a robustness check, our explanation quantitatively accounts for a number of trends that have characterized public firms over the last decades: the fall in the number of firms distributing dividends, the rise in the degree of dividend smoothing, and the increase in the idiosyncratic volatility of public firms.

In order to illustrate the model mechanism let us first discuss the implications of the main friction in our economy: limited enforceability of debt contracts. In the model there is a distribution of heterogeneous firms that can default on their debt. In such an event the credit intermediary seizes the assets of the firm. This environment generates endogenous borrowing constraints which depend on the level of capital of the firm, its idiosyncratic efficiency and the demanded amount of debt. In particular, firms with less collateral face tighter constraint because upon default credit intermediaries incur in higher losses. Secondly, less efficient firms face tighter borrowing constraints because they have a higher probability to default in the next period[2]. Finally, larger loans increase the probability of default, by raising the number of scenarios where the firm will not be able to repay its debt. Accordingly, the interest rate which is charged on the loan by the credit intermediary reflects these different determinants of the expected default cost. In conclusion, large or efficient firms can borrow more (or borrow the same quantity at a cheaper price) with respect to ceteris paribus smaller or less efficient ones. In addition to these features, we assume the presence of a fixed borrowing cost that further reduces the financing ability of all firms, hitting disproportionately small firms.

What happens with the development of the credit markets? What happens when fixed costs of borrowing are reduced? A reduction in the fixed cost of borrowing has both direct and indirect effects. The *direct effect* is straightforward and twofold. First of all, there is a reduction in credit rationing. Firms can now benefit from the possibility of accessing small amount of loans, before unfeasible because of the presence of a fixed borrowing cost[3]. Secondly, firms can either raise the same amount of debt at a cheaper price or, equivalently,

[2]Since the idiosyncratic shock is persistent, firms' actual status predicts their future productivity. If we assume independent idiosyncratic productivity shocks, the borrowing constraint would not depend on the actual efficiency of the firm.

[3]For example, suppose that before the credit market development a firm optimal loan (given the interest rate) was 100$ gross of the borrowing cost, and suppose that the cost of the borrowing process was 200$. The cost of the process is higher than the total amount of the loan required, therefore the firm would have not entered that contract. After financial development, there will be less firms constrained in this fashion.

access at the same price an higher amount of loan (just reallocating the resources before devoted to the payment of the borrowing cost to increase the amount of actual loan). The *indirect effect* is the result of the dynamic response of firms to the new environment. To understand it, we need to look more closely at the optimizing behavior of a firm in presence of endogenous borrowing constraints. In our model firms maximize the expected discounted value of the stream of dividends. The presence of endogenously convex loan price schedules makes the value function of the risk-neutral firms to be concave. For this reason, firms seek to smooth dividends against idiosyncratic shocks. Debt is a channel for doing so. Nonetheless the higher the fixed borrowing cost, the tighter the borrowing constraint, and, accordingly, the more difficult for the firm to use efficiently debt for this purpose. In order to partially overcome this obstacle firms try to build up physical capital. Physical capital is in fact the collateral against which firms can borrow at a cheaper price. The result is that firms which have been lucky in experiencing a raw of good productivity shocks tend to accumulate for precautionary reasons more physical capital than what might be motivated just by efficiency reasons. Therefore, the higher the level of fixed cost of borrowing, the higher the amount of physical capital devoted for this purpose. Indeed, the collateral value of capital decreases with the fixed cost of borrowing. Conversely, firms which have not been lucky/small firms struggle to optimally exploit profitable investment opportunities, given the high cost of debt. As a conclusion in this economy inefficiently large firms coexist with small firms which struggle to grow.

By reducing the fixed cost of borrowing, financial development significantly affects those dynamics. The reduction of borrowing costs eases firms' access to debt. Efficient small firms can finance more investment and grow, while inefficient firms can reduce their size without being penalized as much as before on their interest rates, due to the lack of collateral. Hence, inefficient large firms shrinks down their scale of operation. As a consequence, given the higher collateral value of capital, firms can borrow more debt for the same amount of capital, implying an increase in leverage. Together with a higher volatility of debt, this implies a higher volatility of leverage. Higher level and volatility of leverage boost the likelihood that firms end up in states of the world where they find optimal to default, pushing up the overall default rate of the economy.

Why the rise in default risk does not translate in an increase of the credit spreads? This question requires a quantitative answer, because the change in credit spreads is driven by two counteracting forces which exert their influence through three channels: the fixed cost of borrowing, the quantity of risk and the loss given default for the credit intermediaries. On the one hand, rising default rates increase the quantity of risk bore by credit intermediaries with the consequence that credit spreads have to rise too. On the other hand, there are two channels through which financial development reduces the credit spreads: the fixed cost of borrowing and the loss given default. First of all, ceteris paribus financial development reduces by construction the fixed cost of borrowing, and therefore the interest rate charged on the loan. The impact on the interest rate is stronger the higher is the expected probability of default of the firm, contributing to the reduction of the credit spread. Secondly, and more importantly from a quantitative point of view, financial development make less stringent the borrowing constraint by allowing firms to operate at a more efficient scale. As a result, the average size and profits increase, implying larger ex-ante liquidation values in case of default. This channel tempers the loss given default for the credit intermediaries, pushing down credit spreads. In the model, financial development makes default to rise from 0.3% to 1.2%. Yet, credit spreads rise just by 2 basis points because the higher default risk is offset by a 24% upsurge in the median expected recovery rate. The bulk of this increase comes from a boost in the profits of the firm, which go up by 21.73%. The median size of capital rises too, by 9.34%.

The model also predicts a number of trend that characterized public firms over the recent decades. First, we show that the reduction of the fixed credit costs changes firms' optimal decisions of dividend payout. After financial development firms are in fact more able to smooth dividends over time, and they can trade off this reduction in volatility with a decrease in the level of dividends. The reduction of the borrowing costs makes the measure of firms distributing dividends to shrink down by 34%. This number accounts for the 73% of the decline documented for the U.S. by Fama and French (2001). Furthermore, in the model firms also increase the degree of dividend smoothing by a magnitude which is remarkably close to the values estimated by Leary and Michaely (2011) on US public firms. Second, we study the volatility of firms' returns and sales. Indeed, Campbell et *al.* (2001), Comin and Mulani (2006), Comin and Philippon (2006) show the presence of a secular upward trend in the volatilities of firms. We suggest that this empirical evidence can be (at least partially) accounted for by financial development. Indeed, the model is able to reproduce a rise of 72 % in the volatility of sales and 67% for firms' returns.

3.1.1 Related Literature

This paper adds to the literature on the role of credit markets on firm dynamics. The seminal paper in this field is Cooley and Quadrini (2001), which augments the environment of Hopenhayn (1992) with financial frictions, namely an equity issuance and a bankruptcy deadweight loss. The authors present a model where the dynamics of firms, in terms of growth, job reallocation and exit, is negatively correlated with their initial size and age, as it is in the data. Following Cooley and Quadrini (2001), many papers attempted to understand qualitatively and quantitatively the role of financial frictions on firm characteristics, firm dynamics and the behavior of macroeconomic aggregates. Jermann and Quadrini (2012) show how the limited enforceability of firms' debt might generate endogenous borrowing constraints, which affects not only the dynamics of individual firms, but even the behavior of aggregate financial and real variables. Jermann and Quadrini (2006) use a similar model to show that financial development can be accounted for the rise in volatility of aggregate financial variables and the decline of the volatility of real economic activity. All these models share a common feature: despite firms are allowed to renege on their debt, there is no default in equilibrium. This result stems from the presence of an enforcement constraint, which binds in equilibrium, impeding the firms to default. Recently, few papers have relaxed this condition allowing for equilibrium default. Arellano et *al.* (2011) build a general equilibrium model which allows for equilibrium default, where financial frictions interact with increases in uncertainty at the firm level to generate a contraction in the economic activity. Khan et *al.* (2012) and Gomes and Schmid (2010a) instead use equilibrium default to show that credit shocks account for a sizable part of the business cycle fluctuations and generate recessions similar to the recent financial crisis of 2007-2009. Finally, Gomes and Schmid (2010b) develop a model with equilibrium default to explain the relationship between firms' leverage, book assets and stock prices.

Despite the different panorama of questions involved, these papers share the same idea on the role of equilibrium default. In all of them, equilibrium default is just a financial friction, valued in the extent in which is able to produce dynamics which are relevant for investigating phenomena other than default. In other words, it is the instrumental nature rather than the default phenomenon *per se* to be appraised. This paper reverses this logic. In particular it contributes on the literature investigating the phenomenon of corporate default *per se*. In so doing, we restrict our attention on the relationship between the magnitude of default in the economy (default rate) and the price of risk which is associated to it (credit spreads). From a modeling point of view, this paper builds on Arellano et *al.* (2012), despite the emphasis of the two paper is on completely different questions. In particular, Arellano et *al.* (2012) focus on the role of financial development on firm dynamics, showing how financial development reduces the differences in leverage and growth rates among large and small firms.

In conclusion, despite this paper is the first documenting the diverging trend between the rise in corporate default rates and constant credit spreads, the increase in default rate is not a new stylized fact. Among others, Campbell et *al.* (2008) show that corporate bankruptcies have increased by 150% or 300%, depending on how default is measured. A similar upward trend is instead found by Livshits et *al.* (2010) in the consumer bankruptcies in the United States. They show that, from 1970 to 2002, personal bankruptcies in the United States have increased by around 500%, which is analogous to the number we report on corporate bankruptcies.

3.2 Data

In this section we document the diverging trend between rising corporate default rates and constant credit spreads from the period 1950-2012. This empirical evidence builds upon the contribution of Giesecke et al. (2011) in the measurement of the average default rates and credit spreads of the economy.

3.2.1 Corporate Default Rates

We take the data on US public firms corporate default rates from the Moody's Analytics Default and Recovery Database. The data set covers the credit experiences of over 18,000 corporate issuers that sold long-term public debt at some time between 1920 and 2012[4]. On the one side, an appealing feature of the Moody's data is its broad definition of default which includes not only formal bankruptcy procedures (Chapter 7,Chapter 11) but also informal ones (distressed exchange[5]). We think this is the most relevant definition of default for our analysis, which focuses on economic consequences of default[6]. On the other side, this data set presents some drawbacks. First of all, the index made available to the public by Moody's is issuer-weighted, while a value-weighted index would be a more appropriate measure for gauging the economic impact of default. Second, the sample of firms in consideration (the denominator of the ratio from which we obtain the default rates) is the sample of firms rated by the rating agency, while we would need a broader measure of the public firms population. Third, this index is a global index, which includes not only US firms, contaminating the statistic with foreign default cycles. In conclusion, the index includes also financial and utilities firms, which we would like to depurate, given the different capital structure characteristics. A solution to these problems, would be to use the data of Giesecke et al. (2011), which shares similar properties to the one under study in this paper, and are cleaned of these shortcomings. Hence, until these better data become available, our results should be mostly viewed as suggestive.

Figure 3.2.2 displays the annual issuer-weighted corporate default rates for U.S. public firmsfrom 1950 to 2012. Visual inspection of the picture foregrounds a dramatic upsurge in default rates starting from the 1980's on, following a period of almost no default in the 1950's and 1960's. To test for the presence of a break in the data generating process behind the times series of corporate default rates we apply the Bai and Perron (1998)'s SupLR test statistics. This procedure checks the presence of multiple structural changes, occurring at random dates. The test statistic is obtained by running an OLS regression. We test for the existence of at most three break dates. Following Carvalho and Gabaix (2013), we assume that every date T lies in a range $[T_1, T_2]$, with $T_1 = 0.2n$ and $T_2 = 0.8n$, where n denotes the sample size. The choice of a 20% trimming parameter is recommended by Bai and Perron (2006) to reduce the size distortions which is present when allowing for serial correlation

[4] As of January 1, 2012 approximately 5,000 corporate issuers held a Moody's long-term bond, loan, or corporate family rating, see Moody's (2012, p. 16)

[5] A distressed exchange is one of three events which Moody's defines as a default for the purpose of its default rate statistics. It is any of the following two events: 1) the issuer can make a tender offer, agreeing to pay cash for all or a portion of an outstanding debt security, usually at a price above the trading price, but well below the face amount; 2) the issuer can make an exchange offer, through which an offer is made to substitute the current outstanding securities for a new package of securities which may include: cash, new bonds, stocks, other securities, or a combination thereof, see Moody's (2002).

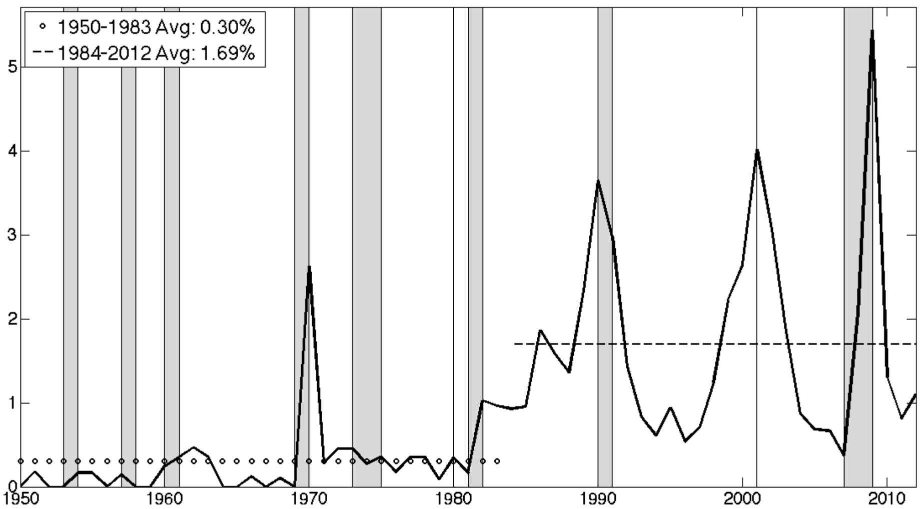
[6] Financial intermediaries care about the ultimate economic consequence of a delinquent loan, and not about the form of legal bankruptcy or informal default that the debt obligation might have turned into.

in the error.

As a data generator process for the default rates, we consider a first order autoregressive model plus a constant. We run two different test to check for a break in the constant and a joint break in the constant and in the autoregressive coefficient. Table 3.2.1 shows that we reject the null hypothesis of no break for both cases at a 5% significance level. Either case, the SupLR test statistic indicates the existence of a single break, which is estimated at 1983 and 1984. We find statistical evidence of these breaks even after controlling for lagged GDP growth rates and lagged stock returns volatility. These tests tell us that default rates *did* change their dynamics in the early 1980's. Hereafter, we follow the vast literature on the Great Moderation that indicates the existence of a break in the volatility of US GDP growth around 1984[7]. We will then compare two intervals of time, one going from 1950 to 1983 and the second from 1984 to 2012[8]. In Table 2 we report the mean values of the corporate default rates over the two intervals of time. Default rates rose from an average value of 0.3% during the period 1950-1983 up to 1.7% over the last thirty years. This corresponds to a 467% increase in average default rates. Surprisingly, this number almost equals the 500% increase in consumer bankruptcy documented over the same time period by Livshits et *al.* (2010).

FIGURE 3.2.1

CORPORATE DEFAULT RATE



Note: this graph plots the annual corporate default rate (in percentage points) in the United States from 1950 until 2012. The circle line plots the average annual corporate default rate from 1950 until 1983, while the dashed line plots the average annual corporate default rate from 1984 until 2012. Shaded area denotes recession.

[7]Among others, McConnell and Perez-Quiros (2000), Stock and Watson (2002), Carvalho and Gabaix (2013) and reference therein. In particular, Stock and Watson (2002) document a wide-spread decline in aggregate volatility, analysing the time series of 124 macro variables since 1960.

[8]The results of the paper do not change when considering 1983 as the break date.

TABLE 3.2.1
BREAK TEST FOR DEFAULT RATES

	$Default_t = a + \rho Default_{t-1} + \epsilon_t$	
	H_0: No Break in a	H_0: No Break in a and ρ
SupLR Stat	8.70	18.14
5% Critical Values	8.22	10.98
Null of No Break	Reject	Reject
Estimated Break Date	1983	1984

Note: this table reports the results of the Bai and Perron (1998) structural break test on annual default rates given by Moody's Default and Recovery Database. We assume that default rates follow an AR(1) process and we test two null hypotheses: either no break just in the constant or no break in both the constant and the autoregressive parameter. The table reports the test statistics (SupLR stat), the 5% Critical Values, whether the test reject or accepts the null hypothesis, and the estimated date of the break in case the null is rejected.

TABLE 3.2.2
AVERAGE DEFAULT RATES

1950-1983	1984-2012	Δ 1984-2012/1950-1983
0.3%	1.7%	+467%

Note: this table reports the average of annual default rates given by Moody's Default and Recovery Database, over two different periods, from 1950 until 1983, and from 1984 until 2012.

3.2.2 Corporate Credit Spreads

We measure the intensity of corporate default risk using the default rates of the public firms in the economy. Accordingly, we would need an analogous measure of the average price of bond risk. Unfortunately, such a series does not exist. As in Giesecke et al. (2011), we choose the series of spread of a hypothetical average bond, which is considered to be within the Aaa and the Baa credit rating. Therefore, we compute the spreads as the difference between Moody's Baa and Aaa Seasoned corporate all firms bond yields, and available at the FRED database[9]. Our implicit assumption is that the Baa bond proxies for the risky asset in the economy and the Aaa bond is the risk-free asset. We argue that this credit spread is the relevant measure for this analysis. First of all, Baa and Aaa rated corporate bonds belong to the investment-grade class. This class is the most representative form of corporate bond in terms of bond issuance[10] (supply side) and have peculiar liquidity properties[11]. The fact that both the risky asset and safe asset belong to the same class allows us to control (in the data) for common shift in the supply of liquidity for investment-grade bonds (demand side).

The reason why we preferred the Aaa corporate bond to the Treasury-Bill yield as a proxy of the risk-free rate is manifold. First of all, we study the relation between the dynamics of corporate default and the risk-based-differential in the firms cost of financing. Accordingly, an homogeneity argument would support the choice of a *firm* safe corporate bond yield as a proxy of the risk free rate. Moreover, our explanation of the joint dynamics is based on a structural break in the *firms* cost of financing. While we can empirically support that financial development has affected the *firms* cost of debt financing, we cannot claim the same for the government cost of debt financing. Therefore we would unsoundly model the impact on a leg of the credit spread, missing the fact that financial development affects both the cost of risky firm and *safe* firms. Secondly, we can safely affirm that Aaa corporate bonds and Treasury Bills are different securities. Apart from sharing the same rating class, they do not have much in common: they display different market microstructure, taxation, and they are exposed to different sources of risk[12]. All these aspects translate in an average credit spread between Aaa bonds and Treasury Bills amounts to 84 basis points (bp) over the period 1950-2012, which cannot be explained by a simple default risk story we are proposing here[13].

[9]For more information about the series, see Appendix 3.B.3.

[10]Investment grade bonds account for 2/3 for issuance volume in 1996, and more than 90% in 2006, see Bessembinder and Maxwell (2008, p.28)

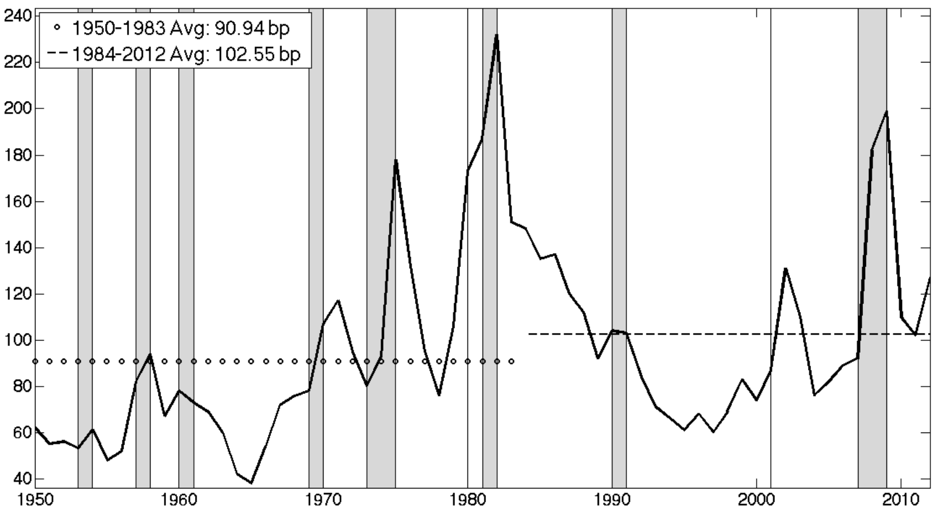
[11]"For regulated financial service firms, such as banks and life insurance companies, required reserves are greater for noninvestment grade bonds. Further, many financial institutions, including pension and mutual funds, face restrictions on amount of non-investment grade debt they can hold". Bessembinder and Maxwell (2008, p.5)

[12]First, Corporate bonds are (an order of measure) less liquid than Treasury bill. The Treasury bill Average Daily Trading Volume in the U.S. Bond Markets in 2001 amount to 297.9$ billions compared to only 17.9$ billions for the whole corporate bond sector. In 2006 the volumes of T-bill almost doubled (524.7$), while the volumes of corporate bonds raises only to 22.7$, see Bessembinder and Maxwell (2008, p. 29). Second, Corporate bond yields are subject to state taxation, while U.S. Treasury securities are exent. Longstaff (2011) shows that tax risk is an important determinant in the pricing of assets. Third, other than the common default risk and liquidity risk, sovereign bonds present a sizeable recovery rates risk. Due to the high uncertainty which characterise enforcement of international debt contracts sovereign bonds display a sizeable heterogeneity in the recovery rates. For example, the credit loss of the 1983-1986 debt restructuring in Argentina was 30%, while the one of the 2001-2002 crisis amounted to 72%. The other major sovereign default crisis, which involved Russia in the August of 1998, was characterized by a credit loss of 63%. On the contrary, recovery rates of Moody's Aa and Baa corporate bonds are stable around 40%, see Moody's (2012, p.26).

[13]Huang and Huang (2012) show that the expected Aaa-Treasury Bill spread should be around 1 bp, given

Figure ?? plots our series of credit spreads, measured in basis points. We can observe how credit spreads were low in the 1950's and 1960's, before peaking up to 232 bp in the 1982. From the 1980's on, credit spreads have been declining to values comparable to the one of 1950's and 1960's. Concomitant with the last financial crisis, credit spreads hike up to 199 bp in the 2009.

FIGURE 3.2.2
CORPORATE CREDIT SPREADS



Note: this graph plots the annual corporate credit spread (in basis points) in the United States from 1950 until 2012. The circle line plots the average annual corporate credit spread from 1950 until 1983, while the dashed line plots the average annual corporate credit spread from 1984 until 2012. Shaded area denotes recession.

As above, we apply the Bai and Perron (1998)'s test to check for structural changes in the credit spreads. Again, to comply with the assumption of the Bai-perron test, we proxy the credit spreads process as a first-order autoregressive model plus a constant, and - in conclusion - we test whether there is a break either just in the constant or both in the constant and the autoregressive parameter. Table 3 shows that we cannot reject the null of no break in either cases. Controlling for lagged GDP growth rates[14] and lagged stock returns volatility, testing for breaks using quarterly data and using the Chow test with 1983 or 1984 as pre-determined break date does not alter our finding[15]. Table 4 reports the mean values of the credit spreads over the two periods of interest, 1950-1983 and 1984-2012. From the 1950's to the 1970's, the average value of the credit spread was 91 bp. This average barely changed over the following three decades, reaching 102 bp. On

the 0.03% expected 5-year average cumulative credit loss of Aaa corporate bonds.

[14]Gomes and Shmid (2010a) investigates the endogenous link between macro aggregates and credit spreads.

[15]We apply the Chow test to quarterly data using as a pre-determined data any quarter between 1982:1 and 1984:1, and in all cases we reject the presence of a break at the 5% significance level.

Table 3.2.3

Break Test for Credit Spreads

	$Spread_t = a + \rho Spread_{t-1} + \epsilon_t$	
	H_0: No Break in a	H_0: No Break in a and ρ
SupLR Stat	5.68	5.62
5% Critical Values	8.22	10.98
Null of No Break	Accept	Accept
Estimated Break Date	-	-

Note: this table reports the results of the Bai and Perron (1998) structural break test on annual credit spreads, measured as the difference (in basis points) between Moody's Baa and Aaa bond yields. We assume that credit spreads follow an AR(1) process and we test two null hypotheses: either no break just in the constant or no break in both the constant and the autoregressive parameter. The table reports the test statistics (SupLR stat), the 5% Critical Values, whether the test reject or accepts the null hypothesis, and the estimated date of the break in case the null is rejected.

the ground of the evidence provided by the break tests, we interpret this 11 bp increase as not statistically significant. To check whether these 11 basis points are economically significant, we follow Giesecke et *al.* (2011) by using a back-to-the-envelope to estimate the average annual credit losses, assuming 50% recovery rate[16]. We find that a 1.4% increase in default rates should have pushed up credit spreads by 70 bp instead of the 11 bp observed in the data. Secondly, despite default rates have increased to a record number of 5.45% in the recent financial crises, the credit spread reached a peak of 199 bp. Yet, the global maximum over the over-all period in consideration, equals 232 bp and was reached in 1982 with a default rate of *only* 1.16%. This over period max-max comparison provides further economical support of our claim that something has structurally changed in the dynamics of default rates and credit spreads. As a conclusion, the change in average credit spreads over the two periods is insignificant from both an economic and a statistical point of view.

[16]The back-to-the-envelope estimate multiplies the physical probability of default to the expected loss upon default. We consider a 50% recovery rate, which is the average senior unsecured recovery rates on investment grade bond for the period 1982-2011, see Moody's (2012, p. 9). As a result, a 1% default rate translates into 50 basis points.

Table 3.2.4
Average Credit Spreads

1950-1983	1984-2012	Δ 1984-2012/1950-1983
91 bp	102 bp	+11 bp

Note: this table reports the average of annual credit spreads, measured as the difference (in basis points) between Moody's Baa and Aaa bond yields, over two different periods, from 1950 until 1983, and from 1984 until 2012.

3.2.3 Diverging Trend

In summary, starting from the early 1980's default rates rose by 467% while credit spreads kept constant. We refer to this evidence as the diverging trend between default rates and credit spreads. Longstaff et *al.* (2005) find that default risk explains 71% of the Baa bond yields. Therefore, a 467% increase in default rates should come at a neat rise in the credit spreads. In addition, even if actual average default rates in both periods are low in absolute value, such a steep increase in default rates should be mirrored in credit spreads for two reasons. First, Almeida and Philippon (2007) show that the risk-adjusted cost of default is four-five times larger than what the physical bankruptcy rates would suggest. Indeed, default is more likely to occur in bad times, which makes risk-averse agents to care more about financial distress than is suggested by physical credit losses. Second, though the average default in the last thirty years equals 1.7%, now financial distress has become more likely for the median firm too. In this sense, the rise in default rates cannot be diversified away and should, therefore, be translated in the pricing of debt.

3.2.4 Financial Development

In this paper we quantitatively investigate how financial development can affect average default rates and credit spreads, by influencing the economic decision of all the firms. Following the seminal papers of King and Levine (1993) and Rajan and Zingales (1998), a vast literature attempted to study the interaction between financial development and the real economy, an idea that actually traces back to Schumpeter (1911).

We focus on the process of deregulation and innovation that characterized the financial sector during the 1970s. This decade saw the introduction, among others, of ATMs, phone transfers for savings balances at commercial banks, the International Banking Act, the modification on the Regulation Q on the banking system, the Financial Institutions Regulatory and Interest Control Act, the Electronic Fund Transfer Act, the 1979 Bankruptcy Reform Act, NOW (negotiable order of withdraw) accounts, the securitization of debt collateralization, the introduction of the Securities Protection Act and the introduction of Asset Backed Securities (ABS), which has recently became the first source of funding for U.S. corporate firms, undertaking corporate bonds.

The deregulation in the financial sector has improved the access to credit for corporate firms, especially the small ones, and decreased the cost of external financing. Nowadays firms can borrow more and cheaper than 30 years ago. This view is supported by the empirical evidence provided by Jayaratne and Strahan (1996) and Demyanyk et *al.* (2007), among others. Accordingly, in our analysis we will model financial development in a reduced form, as an exogenous reduction in the fixed costs of borrowing.

3.3 The Model

3.3.1 Environment

In the economy there are two types of agents: firms and credit intermediaries. Firms have decreasing returns to scale production technologies and experience in each period a persistent idiosyncratic productivity shock and an i.i.d stochastic fixed cost of operation. They are run by risk neutral managers which maximize the expected discounted stream of dividends. Firms articulate in two types: incumbents and entrants. At each point of time, there is a distribution of heterogeneous incumbents, which are defined as the producing firms of the economy. Incumbents finance investment and dividends using internal and external funds: retained profits, new equity issuance and one-period non-contingent loans from the credit intermediaries. Incumbents can renege on their obligations and default. The presence of default risk generates endogenous borrowing constraints for the firms and makes loans' interest rates to be firm-specific. Less efficient firms and/or firms with less collateral face tighter borrowing constraints and access to loans at higher interest rates than more efficient firms and/or firms with more collateral.

Every period a mass of firms enters the economy and starts the production with a time-to-build lag. Entrants solve a problem identical to the incumbents with the difference that they resort uniquely to external funds.

There is also a competitive financial sector. Each financial intermediary offers a menu of loan sizes and interest rates to firms wherein each loan makes zero expected profits. When a firm defaults, creditors can seize its assets and profits net of a liquidation loss.

3.3.2 Firms

In the economy there are two types of firms: incumbents and entrants. Henceforth we denote with the i subscript an incumbent firm, while e stands for an entrant firm. We omit the subscript when the distinction is not necessary.

Firms use capital $k \in \mathbf{K} \subset \mathbb{R}_+$ to produce an homogeneous consumption good $y \in \mathbf{Y} \subset \mathbb{R}_+$ using a decreasing returns to scale technology[17,18]

$$y = xk^{\alpha} \tag{3.3.1}$$

where $\alpha \in (0,1)$ captures the degree of concavity of the production function and x is an uninsurable idiosyncratic shock. The idiosyncratic productivity $x \in \mathbf{X} \subset \mathbb{R}_+$ follows a first-order Markov processes whose transition function is $p_x(x'|x)$. In each period firms incur in a stochastic fixed cost of operation. The operating profits before interest and depreciation are defined as:

$$\pi = xk^{\alpha} - \chi \tag{3.3.2}$$

where $\chi \in \chi \subset \mathbb{R}_+$ is the i.i.d. fixed cost of operation drawn from the cumulative distribution $H(\chi)$. This shock is intended to create a link between negative cash flows and the firms' decision of going bankrupt. Without this feature, firms would always have non-negative profits. However, in the data defaulting firms experience negative profits. Physical capital depreciates at a rate $\delta \in (0,1)$ and accumulates with the law of motion

$$k' = (1-\delta)k + i \tag{3.3.3}$$

where k and k' denotes, respectively, the actual and next period stock of physical capital and i is the capital investment.

Entrants finance dividends and investment with one-period non-contingent loans and new equity issuance. Incumbents can resort, in addition, to retained profits. Because of limited enforceability, firms can renege on their debt. Then, loan contracts depend on those firms' characteristics that are informative about the default probability and the loss given default. When an incumbent firm defaults, it partially meets its obligations with the creditors. In such a case, the firm is liquidated and the creditors seize both its profits and undepreciated capital

$$L(k,x) = \max\{(1-\psi)\,(\pi + (1-\delta)k)\,,0\} \tag{3.3.4}$$

suffering a liquidation clearance loss $\psi \in (0,1)$. The recovery rate is then $L(k,x)/b$, where b refers to the firm outstanding debt.

Every period, after observing the realization of the shocks, incumbents choose whether to enter into a one-period non-contingent loan contract. A contract formalizes in a 4-tuple $\left(x_i, k_i', l_i', r_i'\right)$, which delivers a loan l_i' whose repayment value is $b_i' = \left(1 + r_i'\right) l_i'$, to

[17]Diminishing returns to scale at the firm-level may be explained with the span of control models of Rosen (1982) and Lucas (1978).

[18]Decreasing returns to scale technologies and perfect competition prevent the most productive firms from taking over the market completely and allow for the existence of heterogeneity in equilibrium. Since firms can be replicated, returns to scale are constant at the aggregate level.

firms with idiosyncratic efficiency x_i and future stock of physical capital k_i'. Contracts $\left(x_i, k_i', l_i', r_i'\right)$ belong to a set of debt schedules $\Omega\left(x_i, k_i', l_i'\right)$. This specification highlights the dependence of interest rates on three firms' *key characteristics*: 1) the productivity, 2) the size of assets and 3) the size of the loan. If the actual productivity is high, next period productivity is more likely to be high[19]. This decreases the probability of default and the interest rates. Similarly, firms with more capital have a larger collateral and therefore lower interest rates. Finally, larger loans increase the probability of default, implying a higher interest rate. It is worth noticing that the future levels of capital and outstanding debt are chosen at the same time, and they jointly determine the interest rate required by the credit intermediaries. Entrants face the same loan contracts with the difference that their debt schedules and interest rates do not depend on the idiosyncratic shock x_e.

Firms issue new equity when their dividends d are negative. The equity issuance comes at an additional proportional cost, $\gamma > 1$. The total cost of distributing dividends $d \in \mathbb{R}$ is then

$$g(d) = d\mathbb{I}_{\{d\geq 0\}} + (\gamma d)\mathbb{I}_{\{d<0\}}$$

where $\mathbb{I}_{\{y\}}$ is an indicator function that takes value 1 when y is true. The implication of the issuance cost is twofold. It prevents firms from distributing dividends and raising equity at the same time and it does not allow firms to issue as much equity as they need to circumvent the financial frictions due to bonds' limited enforceability[20]. Then, the equity issuance cost makes firms to prefer the use of retained profits and debt to equity, in accordance with the pecking order theory.

Firms can also save in the market portfolio of corporate bonds ($l' < 0$). Since the idiosyncratic uncertainty washes out in the aggregate, the gross return on the market portfolio of corporate bonds is the risk-free rate $1 + r_F$. Thus, the repayment value $b \in \mathbf{B} \subset \mathbb{R}$, is

$$b = \left([1 + r]\mathbb{I}_{\{l>0\}} + [1 + r_F]\mathbb{I}_{\{l<0\}}\right) l$$

Incumbents

An incumbent begins with an amount of net-wealth $\omega_i \in \mathbf{W} \equiv \mathbf{X} \times \mathbf{K} \times \chi \times \mathbf{B} \subset \mathbb{R}^3_+ \times \mathbb{R}$, which is a by-product of its holdings of physical capital and outstanding debt, that is

$$\omega_i = \pi_i + (1 - \delta)k_i - \left([1 + r_i]\, \mathbb{I}_{\{l_i>0\}} + [1 + r_F]\, \mathbb{I}_{\{l_i<0\}}\right) l_i$$

At each point of time, there is a large measure λ of incumbents, which are defined as the set of firms that either were operating or entered in the previous period. λ is the probability

[19]This argument holds as long as firms' idiosyncratic productivity shocks are persistent. In case of i.i.d productivity shocks the interest rate would not depend anymore on the *actual* x, as it happens for the fixed cost shock χ.

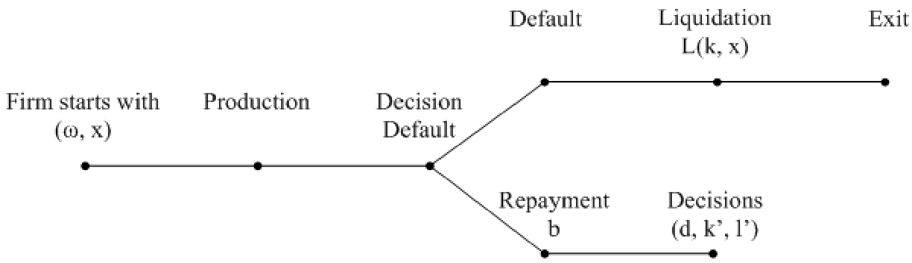
[20]The presence of the equity issuance cost and the bankruptcy deadweight loss make the Modigliani and Miller (1958) theorem not to hold in this framework.

measure over (ω_i, x_i), defined on the Borel algebra $\mathcal{J}$ generated by the open subset of the product space $\mathbf{J} = \mathbf{W} \times \mathbf{X} \subset \mathbb{R}^4_+ \times \mathbb{R}$.

We assume that an incumbent first observes the realization of the idiosyncratic productivity shock x_i and the stochastic fixed cost of operation χ_i, and then produces. At this point, each firm maximizes the expected present value of future profits in a two stage decision problem. First, a firm decides whether to default or not. The default implies the exit of the firm and an outside opportunity of not operating equals zero. Therefore, firms default whenever their continuation value is negative. Second, if the firm does not default, it finances the entire value of its outstanding liabilities $(1 + r_i)l_i$, and decides the amount of dividends to distribute d_i, the new level of physical capital k_i', and the new level of debt l_i', given the debt schedules $\left(x_i, k_i', l_i', r_i'\right) \in \Omega\left(k_i', l_i', x_i\right)$. Figure 3.3.3 summarizes the timing of the model.

FIGURE 3.3.3

TIMING OF THE MODEL



The states of the economy for an incumbent firm are, therefore, (ω_i, x_i). The incumbents' problem can be written as

$$V_i(\omega_i, x_i) = \max_{\phi_{D,i} \in \{0,1\}} (1 - \phi_{D,i}) V_i^c(\omega_i, x_i) \tag{3.3.5}$$

where $\phi_{D,i} = \phi_D(\omega_i, x_i)$ is an indicator function that takes value $\phi_{D,i} = 1$ in case of default, and $V_i^c(\omega_i, x_i)$ denotes the continuation value of an incumbent firm which does not default,

$$V_i^c(\omega_i, x_i) = \max_{d_i, k_i', l_i'} d_i + \beta \mathbb{E}_{H(\chi_i'), x_i' | x_i} \left[V_i(\omega_i', x_i') \right] \tag{3.3.6}$$

$$\text{s.t. } g(d_i) = \omega_i + l_i' - k_i'$$

$$\omega_i' \equiv \omega_i'(k_i', l_i', x_i', \chi_i') = \pi_i' + (1 - \delta)k_i' - \left(\left[1 + r_i'\right] \mathbb{I}_{\{l_i' > 0\}} + \left[1 + r_F\right] \mathbb{I}_{\{l_i' < 0\}} \right) l_i' \tag{3.3.7}$$

$$\left(x_i, k_i', l_i', r_i'\right) \in \Omega(k_i', l_i', x_i) \tag{3.3.8}$$

where: 1) β denotes the subjective time discounting rate of the firm's manager; 2) $\mathbb{E}_{H(\chi_i'), x_i' | x_i}$ denotes the expected value over the independent processes of χ_i' and x_i', where the realization of x_i' is conditional on x_i; 3) equation (3.3.7) denotes the law of motion of firm's net worth. ω_i' is a random process which inherits the first-order Markov property from the idiosyncratic productivity shock x_i', augmented by the independent i.i.d process of χ_i'. Formally ω_i' follows the transition function $p(x_i'|x_i)H(\mathrm{d}\chi_i')$[21].

Analogously to the analysis of entry and exit in Hopenhayn (1992), we can describe the optimal default policy as a threshold on the idiosyncratic productivity shock. Here the definition of the threshold is complicated by the dependence of the continuation value of the incumbents on both[22] the idiosyncratic productivity shock and the wealth. Using the weakly increasing property of the continuation value on both arguments, Khan et *al.* (2012) prove that for each level of ω_i there exists a schedule $\underline{x}_i = x(\omega_i)$ such that a firm with net-wealth ω_i defaults if and only if its productivity is lower than $\underline{x}_i$. Such a threshold $\underline{x}_i$ is defined as the value of x wherein $V^c(\omega_i, \underline{x}_i) = 0$.

Likewise, it might be shown that when firms default, their net-wealth is negative. Intuitively, when the net-wealth is non-negative, a firm is always able to pay back the debt without resorting to any additional external fund. In turn, this implies that the liquidation value (3.3.4) which the creditors seize out of defaulted firms is always less than the due repayment values of the loans. This result guarantees that creditors always incur in a loss when firms default.

Before concluding, it is worth noticing that the negative net-wealth is a necessary but not sufficient condition for the firm to default. Indeed, a firm with negative net-wealth can find optimal not to exit and decide to issue equity and roll over debt to fund its operations.

Entrants

The model features exogenous entry. At each point of time there is a mass Ξ_t of firms which enters in the economy, merely substituting the measure of firms which default. Production takes place with a lag, as a time-to-build restriction. The entrants begin the period with an amount of physical capital k_e. They then decide the amount of dividends to distribute d_e, the new level of physical capital k_e', and debt l_e' (or savings if $l_e' < 0$) with $(k_e', l_e', r_e') \in \Omega\left(k_e', l_e'\right)$ to maximize the expected present value of future profits. The entrants can also decide to raise equity at the proportional cost γ. Once entrants have solved for their optimal choices, they draw next-period idiosyncratic shock x_e' from a cumulative distribution $G\left(x_e'\right)$. The state of the economy for an entrant is then k_e. Hence,

[21] Where we assume that the pdf of $H(\chi_i')$ exists and it is atomless.
[22] In Hopenhayn (1992), the continuation value depends only on the productivity shock.

the entrants' problem can be written as

$$V_e(k_e) = \max_{d_e, k'_e, l'_e} d_e + \beta \mathbb{E}_{H(\chi'_e), G(x'_e)} \left[V_i(\omega'_e, x'_e) \right] \tag{3.3.9}$$

$$\text{s.t. } g(d_e) = k_e + l'_e - k'_e$$

$$\omega'_e \equiv \omega'_i(k'_e, l'_e, x'_e, \chi'_e) = \pi'_e + (1-\delta)k'_e - \left(\left[1 + r'_e\right] \mathbb{I}_{\{l'_e > 0\}} + \left[1 + r_F\right] \mathbb{I}_{\{l'_e < 0\}} \right) l'_e \tag{3.3.10}$$

$$\left(k'_e, l'_e, r'_e\right) \in \Omega(k'_e, l'_e) \tag{3.3.11}$$

3.3.3 Credit Intermediaries

In the economy there is a competitive financial sector which lends to firms (or borrows from firms, in case they save). The credit intermediaries offer a menu of loan sizes and interest rates, wherein each loan makes zero profits. For each loan the intermediaries have to pay a *fixed cost* ζ. As suggested by Arellano et *al.* (2012), the fixed credit cost can be interpreted as any financial intermediation cost that creditors incur when issuing a loan, as costs to obtain information about firms' default probability and overhead costs. The higher the value of ζ, the larger the costs firms incur in borrowing from the credit intermediaries, the less developed is the financial sector of the economy. Following Arellano et *al.* (2012), we consider this cost as a proxy for financial development.

The credit intermediaries price firms bonds by defining debt schedules which contingent on firm characteristics. The latter captures the probability of default and the amount of insurance in case of default. Formally, the credit intermediary offers a set of incumbent-specific contracts $\left(x_i, k'_i, l'_i, r'_i\right) \in \Omega\left(k'_i, l'_i, x_i\right)$ which read: in absence of arbitrage opportunities, the incumbent-specific (x_i, k'_i) interest rate r'_i associated to a required amount of loan l'_i is defined by the zero profit (break-even) condition of the intermediaries $\Omega\left(k'_i, l'_i, x_i\right)$:

$$\left(l'_i + \zeta\right)(1 + r_F) = \mathbb{E}_{H(\chi'_i), x'_i | x_i} \left[(1 - \phi_{D,i})(1 + r'_i) l'_i + \phi_{D,i} L(k'_i, x'_i) \right] \tag{3.3.12}$$

where ζ denotes the fixed cost of borrowing and $L(k'_i, x'_i)$ is the liquidation value of the firm in case of default, as defined in (3.3.4). In case of an entrant, the mapping is identical but for the expectation, which is taken unconditionally over the idiosyncratic shock x_e.

The availability and the interest rates of each loan depend on the default risk, on the amount of insurance provided by the expected liquidation value and on the borrowing costs ζ. While the first two channels generate endogenous borrowing constraints which are firm specific, the presence of fixed credit costs limits all firms access to credit. As pointed out above, the fixed cost has a further asymmetric effect: small and less efficient firms suffer disproportionately more from it.

3.4 Characterization of the Equilibrium

3.4.1 Definition of Equilibrium

A recursive equilibrium in this economy is given by the optimal choices of the incumbents $(\phi_{D,i}, k_i', l_i', d_i)$, optimal choices for the entrants (k_e', l_e', d_e), an exogenous risk-free rate r_F and the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, $\left(k_e', l_e', r_e'\right)$, such that:

1. given the exogenous risk-free rate r_F, the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, $\left(k_e', l_e', r_e'\right)$ satisfy the zero ex-ante profit condition of the credit intermediary (3.3.12)[23], for any choice of (k_i', l_i') and (k_e', l_e');

2. given the exogenous risk-free rate r_F and the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, the incumbent firms choose $(\phi_{D,i}, k_i', l_i', d_i)$ to maximize their problem described in (3.3.5);

3. given the exogenous risk-free rate r_F and the firm-specific contracts $\left(k_e', l_e', r_e'\right)$, the entrant firms choose (k_e', l_e', d_e) to maximize their problem described in (3.3.9);

4. given the exogenous risk-free rate r_F, the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, $\left(k_e', l_e', r_e'\right)$, the optimal choices of the incumbents $(\phi_{D,i}, k_i', l_i', d_i)$ and the optimal choices for the entrants (k_e', l_e', d_e), the law of motion of the distribution of firms is given by

$$\begin{aligned}\lambda(\omega_i', x_i') = &\int \left(1 - \phi_{D,i}\right) Q\left(\left(\omega_i', x_i'\right), (\omega_i, x_i)\right) H(\chi')p_x\left(x_i'|x_i\right) \lambda(\mathrm{d}\omega_i, \mathrm{d}x_i)\\ &+ \int \phi_{D,i} Q_e\left(\omega_e', x_e'\right) H(\chi_e')G\left(x_e'\right) \lambda(\mathrm{d}\omega_i, \mathrm{d}x_i)\end{aligned}$$

where $Q\left(\left(\omega_i', x_i'\right), (\omega_i, x_i)\right)$ denotes a transition functions such that

$$Q\left(\left(\omega_i', x_i'\right), (\omega_i, x_i)\right) = \begin{cases} 1, & \text{if } \omega_i'(\omega_i, x_i) = \omega_i',\ \ x_i'(x_i) = x_i' \\ 0, & \text{if otherwise} \end{cases}$$

The same applies to $Q_e(\omega_e', x_e')$.

3.4.2 The Role of Financial Development

In this section we investigate analytically the effect of changes in the borrowing cost ζ. The idea is to isolate in the simplest framework the effects of financial development on the

[23]Recall the variant of the zero ex-ante profit condition of the credit intermediary (3.3.12) for the entrant requires to use in the expectation the unconditional distribution $G(x_e')$ to determine the next period idiosyncratic shock.

borrowing constraints, and the leverage in the economy. We consider a simple economy without uncertainty, where there is a continuum of firms which are born with different idiosyncratic productivity $\bar{x}_i$, henceforth constant. Incumbents do not suffer stochastic fixed cost of operation. Entrants are endowed with no capital and debt. Output is produced using capital, which fully depreciate each period. Firms cannot save or issue equity. The following Propositions follow the results in Arellano et *al.* (2012). The interested reader can refer to Appendix 3.A for the detailed proofs.

Proposition 3.4.1. *In this economy there is a unique equilibrium which is characterized as follows. In equilibrium:*

1. *The policy functions of the firms are constant, $(\phi^*_{D,i}, k'^*_i, l'^*_i, d^*_i)$*
2. *Firms do not default. $\phi^*_{D,i} = 0$*
3. *Firms can borrow at the risk free rate, corrected for the fixed cost of borrowing. In formula:*
$$(1 + r^*_i)l^*_i = (1 + r^*)l^*_i + (1 + r^*_i)\zeta \tag{3.4.13}$$
4. *Firms demand capital up to equalize their marginal product to the risk-free interest rate. Let me name this level of capital as the first-best level of capital, $k_{fb,i}$:*
$$k_{fb,i} = \left(\frac{\alpha \bar{x}_i}{1 + r_i}\right)^{\frac{1}{1-\alpha}} \tag{3.4.14}$$
Notice there is a one-onto-one increasing relationship between $k_{fb,i}$ and $\bar{x}_i$.
5. *Firms are subject to endogenous borrowing constraints.*

The endogenous borrowing constraints arise from the necessity of making *incentive compatible* for the firms not to default. In particular:

Proposition 3.4.2. *A no defaulting equilibrium strategy for a firm i is sustained for level of debt $l^*_i \in [0, l_{D,i}]$, where $l_{D,i}$ represent the equilibrium firm-specific debt limit and it is defined as:*
$$l_{D,i} = \frac{1 + r_i - \alpha}{r_i \alpha} k_{fb,i} - \frac{(1 + r_i)}{r_i}\zeta \tag{3.4.15}$$
We name this level of debt, firm specific debt-limit, and it is defined as the level of debt for which it is incentive compatible for the firm not to default.

The endogenous nature of the debt-limit rationalizes the label endogenous borrowing constraint. Proposition 3.4.3 investigates the sensitivity of $l_{D,i}$ to financial development and (through the optimal choice of capital), to the level of idiosyncratic productivity of the firms.

Proposition 3.4.3. *In equilibrium:*

- $\frac{\partial l_{D,i}}{\partial \zeta} = -\frac{1+r_i}{r_i} < 0$*: the debt-limit is increasing in the level of financial development. The lower the level of financial development (the higher is* ζ*), the lower is the level of debt for which the firm is indifferent whether to default or not.*
- $\frac{\partial l_{D,i}}{\partial k_{fb,i}(\bar{x})} = \frac{1+r_i-\alpha}{r_i\alpha} > 0$*: the debt-limit is increasing in the optimal choice of capital, which depends uniquely on the original idiosyncratic efficiency. Then, the higher is the idiosyncratic productivity, the higher is the debt limit.*

In equilibrium the leverage evaluated at the debt-limit can be expressed as:

$$lev_i = \frac{l_{D,i}}{k_{fb,i}} = \frac{1 + r_i - \alpha}{r_i\alpha} - \frac{(1 + r_i)}{r_i}\frac{\zeta}{k_{fb,i}} \tag{3.4.16}$$

Similarly to what we have just done, Proposition 3.4.4 explores the sensitivity of the leverage, lev_i, to financial development.

Proposition 3.4.4. *In equilibrium:*

- $\frac{\partial lev_i}{\partial \zeta} = -\frac{1+r_i}{r_i}\frac{1}{k_{fb,i}} < 0$*: the leverage is strictly increasing in the level of financial development. The more developed is the credit intermediation (the lower is the* ζ*), the higher is the equilibrium leverage of the firms.*
- $\frac{\partial lev_i}{\partial k_i^{fb}(\bar{x})} = \frac{1+r_i}{r_i}\frac{\zeta}{(k_{fb,i})^2} > 0$*: the leverage is strictly increasing in the amount of capital. The higher is the productivity of a firm, the higher is the optimal level of capital, the higher is the equilibrium leverage.*

3.5 Quantitative Analysis

In this section we study the quantitative implications of financial development on the joint behavior of default rates and credit spreads and on other relevant dynamics of the US economy. To that end, we compute two equilibria whose parameters differ only for the value of the fixed borrowing costs. The first equilibrium is calibrated to proxy the behaviour of some relevant facts of the US economy over the period 1950-1983. The second equilibrium approximates the US economy over the period 1984-2012; it takes as given the estimated and calibrated parameters from the first period *but* for the fixed costs of borrowing, which are lowered. In line with the calibration strategy adopted by Buera and Shin (2013), we discipline this cut by matching the higher leverage of the firms over the period 1984-2012[24]. As mentioned above, the decline in the fixed cost of borrowing is a

[24]Buera and Shin (2013) studies the role of financial frictions on the so-called miracle economies. In their calibration, the authors pin down the exogenous size of financial development by matching the evolution of external finance to GDP ratios in the data.

reduced form way of modelling financial development[25]. This modelling strategy allows us to isolate and study the implications of financial development, and to test whether it might have been a relevant structural explanation of the diverging trend observed in the data.

3.5.1 Calibration

We calibrate the model over the period 1950-1983. In the model, one period corresponds to one year.

Uncertainty in the Economy

In order to proceed we need to impose more structure on the stochastic properties of the uncertainty in the economy: the idiosyncratic productivity shock and the stochastic fixed cost of operation. The idiosyncratic productivity shock of the incumbents follows an AR(1) process, such that

$$x_t = \rho_x x_{t-1} + e_t, \quad e_t \sim N(0, \sigma_e^2) \tag{3.5.17}$$

In the context of the calibration, we transform (3.5.17) into a discrete-state Markov chain, with 2 points in the support, using the standard Tauchen (1986) algorithm. Then, we assume the distribution $G(x)$ from which the entrants draw their first realization of the idiosyncratic shock is a Pareto distribution with exponent c. The choice of such a distribution is in accordance with the empirical evidence that the firms' size distribution is very heavy tailed, see Gabaix (2011) among others. Finally, we assume that the stochastic costs of operation follows an i.i.d. process, where $H(\chi)$ is modeled as a Bernoulli random variable which takes the value χ with probability p_χ and the value 0 with probability $1-p_\chi$.

Estimated Parameters

Table 3.5.5 reports the estimated parameters and the source whence they are taken. Following Gomes and Schmid (2010b) and Arellano et *al.* (2012), we set the parameters governing the decreasing returns to scale of the firms' production function to $\alpha = 0.65$[26]. The depreciation rate of capital is set to 10% per year. The risk-free interest rate is set to $r_F = 0.04$ according to the actual value of the annual real interest rate in the United States from 1950 to 1983. As in Arellano et *al.* (2012), we set the subjective discount rate parameter to $\beta = 0.9605$ per year. This value is slightly lower than its frictionless equilibrium value of $\frac{1}{1+r_F}$, as a proxy of the impatience of the risk neutral manager. This

[25]Other examples of models where financial development is modeled as an exogenous reduction of the economy's financial frictions are Buera et *al.* (2011) and Arellano et *al.* (2012).

[26]This value is on the lower bound of reasonable parameters for the Cobb-Douglas production function

TABLE 3.5.5
ESTIMATED PARAMETERS

	A. Firms	
$\alpha = 0.65$	Production Function Returns to Scale	Arellano et *al.* (2012)
$\delta = 0.10$	Capital Depreciation	Arellano et *al.* (2012)
$\psi = 0.30$	Bankruptcy Deadweight Loss	Gomes and Schmid (2010a)
$r_F = 0.04$	Real Risk-Free Interest Rate	Data (see Appendix B.4)
$\beta = 0.96$	Time Discounting Parameter	Arellano et *al.* (2012)
$\rho_x = 0.80$	Idiosyncratic Shock Persistence	Foster et *al.* (2008)
$\gamma = 0.35$	Equity Issuance Cost	Cooley and Quadrini (2001)
	B. Incumbents	
$p_\chi = 0.06$	Probability Operational Cost	Armenter and Hnatkosvka (2012)
	C. Entrants	
$c = 2$	Pareto Exponent	Axtell (2001)

Note: this table reports the values, the description and the source of the estimated parameters.

feature supplements the absence of nontax deducibility of interest rate payments in providing the firms with an incentive to borrow. As in Gomes and Schmid (2010a) we set the bankruptcy deadweight loss to $\psi = 30\%$[27]. The value of the equity issuance costs $\gamma = 0.35$ is borrowed from Cooley and Quadrini (2001). We take the value of the autoregressive parameter $\rho_x = 0.80$, from Foster et *al.* (2008), who estimate the production function and the Solow residual at the firm level. In line with the evidence of Axtell (2001) and Gabaix (2011) that the distribution of firm size is heavy tailed, we choose a Pareto exponent of 2. Finally, we set the probability of receiving a positive operational cost to 6% to match the transition rate from positive to negative cash flows for US public firms estimated by Armenter and Hnatkosvka (2012).

[27] Warner (1977) estimates that the direct and indirect costs associated to corporate bankruptcy equal 30% of the book value of the firm.

Calibrated Parameters

There are four parameters to be calibrated: 1) σ_e, the standard deviation of the idiosyncratic productivity shock; 2) χ, the magnitude of the stochastic costs of operation; 3) ζ, the borrowing fixed cost; and 4) k_e, the physical capital endowment of an entrant. Accordingly, we need (at least) four targets. Consistently with the structural break observed in the data, we calibrate our model economy to statistics computed over the period 1950-1983. In principle, these targets should capture relevant information about the process driving the default phenomenon in the US economy. In this spirit, we choose: 1) the average debt to asset ratio, b_i/a_i, where the assets in the model are given by $a_i = x_i k_i^{\alpha} + (1-\delta)k_i$; 2) the cross-sectional standard deviation of the average ratio of debt over asset; 3) the average default rate from 1950 to 1983; and 4) the growth rate of entrants. Apart from the last target - which is taken from Arellano et *al.* (2012) - the statistics are computed using Compustat data. Appendix 3.B describes in the detail the construction of the data.

Results of the calibration. Table 3.5.6 reports the value of the calibrated parameters while Table 3.5.7 compares the targets in the data with the one computed in the model. The stochastic operational cost is calibrated to $\chi = 9$. In relative terms, it represents 9% of the assets of the median firm. As far as the targets are concerned, on one hand, the average debt to asset ratio in the model is slightly overestimated at 0.29, compared to the actual value in the data of 0.24. In the model, firms borrow a little too much. On the other hand, the cross-sectional standard deviation of the ratio is much closer to the actual one, with a value of 0.17 compared with the 0.16 in data. The average default rate is perfectly matched: in the model the 0.3% of the firms defaults, as it is in the data. This is a successful matching since it is well known that models with equilibrium default struggle in providing quantitatively reasonable amount of default in the economy. As instance, in Arellano et *al.* (2012), the average default rate implied by the model is zero. Conversely, this model does not suffer the same weakness. The reason of this relevant difference rests on the introduction of the stochastic fixed cost of operation. Finally, we perfectly match the growth rate of the entrants.

3.5.2 Results: Financial Development and the Diverging Trend

We study the quantitative implications of financial development on the dynamics of default rates and credit spreads. To that end, we exogenously cut from 0.6 to 0.4 the fixed borrowing costs calibrated in the first equilibrium. We discipline this reduction by matching the higher ratio of total debt to asset observed over the period 1984-2012. This structural change leads a median firm leverage ratio of 0.36 in the second equilibrium, close to the 0.33 measured in the data. To confirm the plausibility of the magnitude of these values, note that the ratio of the fixed borrowing costs over the loan value for the median firm is 0.5% in the first equilibrium and 0.13% in the second equilibrium. Those figures are in line with the results in Altinkilic and Hansen (2000), which study a panel of 628 industrial

TABLE 3.5.6

CALIBRATED PARAMETERS

Parameter	Description
$\sigma_e = 0.65$	Standard Deviation Idiosyncratic Productivity Shock
$\chi = 9$	Stochastic Operational Cost
$\zeta = 0.60$	Borrowing Fixed Costs
$k_e = 145$	Capital Entrants

Note: this table reports the values and the description of the calibrated parameters.

TABLE 3.5.7

CALIBRATION TARGETS

Target	Data	Model
Average Debt to Asset Ratio	0.24	0.29
Standard Deviation of Debt to Asset Ratio	0.16	0.17
Average Default Rate	0.3%	0.3%
Growth Rate Entrants	0.95%	0.95%

Note: this table reports the targets of the calibration exercise. The Debt to Asset Ratio is computed in the model as the total amount of debt over the sum of firm's profits and undepreciated capital, and in the data as the book value of assets over the sum of long-term debt and debt in current liabilities.

firms from 1990 until 1997 and find that the fixed cost of debt issuance for public debt equals on average around 0.1% of the debt principal. As a byproduct, this result provides a robustness check of the consistency of our calibration. Hereafter, we refer to the (first) equilibrium with borrowing fixed costs of $\zeta_1 = 0.6$ as the *pre*-1984 steady-state, and the (second) equilibrium with borrowing fixed costs of $\zeta_2 = 0.4$ as the *post*-1984 steady-state.

Table 3.5.8
Predictions of the Model

Moment		*pre-1984*	*post*-1984	Δ *post*-1984/*pre*-1984
Fixed Borrowing Cost	Model	0.6	0.4	−33%
	Data	-	-	-
Median Debt to Asset Ratio	Model	0.29	0.36	24%
	Data	0.24	0.33	37%
Aggregate Default Rate	Model	0.3%	1.2%	300%
	Data	0.3%	1.7%	467%
Aggregate Credit Spread	Model	75bp	77bp	2bp
	Data	91bp	102bp	11bp
Median Expected Recovery Rate	Model	37%	46%	24%
	Data	–	–	–

Note: this table reports the results of the model in two equilibria. The first one, labeled as "*pre*-1984", denotes the version of the model with high fixed costs of borrowing, $\zeta = 0.6$. The second one, labeled as "*post*-1984", denotes the version of the model with low fixed costs of borrowing, $\zeta = 0.4$. The Median Debt to Asset Ratio is computed as the median value among firms' total amount of debt over the sum of firm's profits and undepreciated capital. The Median Expected Recovery Rate defines the median across firms of the sum of firms' profits and undepreciated capital over the total amount of debt over the states in which the expected probability of default is positive.

Table 3.5.8 reports the quantitative predictions of the model and the effects of financial development on default rates and credit spreads. The model successfully explains the dramatic rise in default rates. The *post*-1984 steady-state is characterized by an average default rate of 1.2%, implying a 300% increase between the two periods. Therefore, financial development accounts for the 64% of the total increase of default rates observed since the early 1980's. This result uniquely stems from the *interplay* between financial development and the stochastic fixed cost of operation. On one hand, the reduction in the fixed cost of borrowing tempers the non-linearities of the value function of the firms[28], making the debt a cheaper source of financing. As a result, efficient firms can finance through debt investment and build a more efficient size; conversely, inefficient firms can disinvest part of their capital, without being penalized as much as before due to the lack of collateral. Then, the reduction of the fixed cost of borrowing increases both the average level and volatility of firms' debt. On the other hand, given the higher collateral value of capital, firms can borrow more debt against the same amount of capital, implying an increase in leverage. Together with a higher volatility of debt, this implies a higher volatility of leverage. Indeed, the leverage of the median firm goes up from 0.29 to 0.36, while its volatility rises from 0.17 to 0.20. Thence, it becomes more likely that firms end up in states of the world where they find optimal to default, pushing up the overall default rate of the economy.

Yet, the idiosyncratic productivity shock alone is not sufficient to imply the default of the firms because of the persistency property of the idiosyncratic productivity shock process and the forward-looking nature of the borrowing constraint. Indeed, the persistent nature of the shock makes it highly predictable. A fortiori since the shock is highly persistent, the intermediary anticipates that a low-efficient firm will keep being inefficient in the next period, and, therefore, curtails the amount of loans for which the firm might find tempting to default. Intuitively, this is the reason at the base of the failure experienced by equilibrium default models in providing default in equilibrium. On the other hand, the rare event (small probability) and unpredictable (i.i.d) nature of the stochastic operational cost provides a modeling expedient for introducing a significant amount of unpredictable uncertainty in the economy, which eventually produces defaults in equilibrium.

On the other side of the picture, Table 3.5.8 reports a 2 bp increase in the credit spread, in line with the empirical evidence. With respect to the level, in both equilibria average credit spreads are around 20 bp *lower* than the real ones, which are around 90 bp. This result is not a surprise. Traditionally, macro-models have been having hard time in provide quantitatively reasonable credit spreads. Indeed, Chen et *al.* (2009) stress how models which are not able to provide sizable equity premium would never be able to predict the right amount of credit spreads, linking the equity premium puzzle to what they call the credit spread puzzle. In order to overcome these difficulties and match the level of credit spreads, we should add aggregate uncertainty in the model, as in Chen (2010). In this way, we would add a countercyclical default, countercyclical price of risk and procyclical liquidation values, which, in turn would deliver sizable credit spreads.

[28] In particular, it decreases the marginal utility cost of increasing debt, where utility is measured in terms of expected discounted feature profits.

Table 8 shows that the model *does* predict the dramatic rise in default rates and the constancy of credit spreads. What is the rationale for this result? The answer to this question is purely quantitative, and stems from the magnitude of three counteracting effects that financial development has on credit spreads. To clarify this point, let us restate Equation (3.3.12) as

$$l_i'(1+r_F) + \underbrace{\zeta(1+r_F)}_{\text{Fixed Cost Channel}} = \mathrm{E}_{H(\chi_i'),x_i'|x_i}\Big[\underbrace{(1-\phi_{D,i})(1+r_i')l_i' + \phi_{D,i}}_{\text{Default Risk Channel}} \underbrace{L(k_i', x_i')}_{\text{Insurance Channel}}\Big]$$

When pricing a debt, the credit intermediaries evaluate the fixed cost of issuing a loan (*fixed cost channel*), the probability of default of the firm (*default risk channel*) and the amount of insurance provided by the liquidation value in case of default (*insurance channel*).

As seen before, financial development increases (on average) the probability of endogenous default. Therefore, in absence of any form of insurance in case of default, credit spreads would have to increase, tracking monotonically the rise in default rates observed in the data. Nonetheless, there are two channels through which financial development reduces the credit spreads: the fixed cost of borrowing and the loss given default. First of all, financial development reduces by construction the fixed cost of borrowing, and therefore the interest rate charged on the loan. The impact on the interest rate is stronger the higher is the expected probability of default of the firm, contributing to the reduction of the credit spread. Secondly, financial development increases (on average) the liquidation value. Because of the reduction in the financial frictions in the economy, firms behave more optimally (literally, firms are less constrained in their optimization decisions) and increases their size of operation. As a consequence, firms (on average) produce higher profits and have a larger size. To attach some numbers on these dynamics, in the model firms' median profit and median size increase, respectively, by 21.73% and 9.34% when passing from the *pre*-1984 to the *post*-1984 equilibrium. Both these two components enter the definition of the liquidation value $L(k_i', x_i')$, and increase the insurance component of the credit spread. Indeed, the expected recovery rate hikes up by 24%, from a value of 37% in the first equilibrium to a value of 46% in the second one. Ergo, financial development produces (on average) a dramatic increase in the liquidation value which offsets the increase in the probability of default, producing just a 2 bp increase in the credit spreads in the second equilibrium.

3.5.3 Further Results

In the model, financial development is able to account for a number of trends - other than the rise in default rates - which characterized public firms over the last decades. Namely, the model gives relevant predictions on the number of firms distributing dividends, the way firms decide to smooth these dividends over time, and the level of firms' volatility.

Dividend Payout.

First of all, the way public firms pay dividends has substantially changed over time. As reported by Fama and French (2001), the number of publicly traded non-financial non-utility firms distributing dividends was 66.5% in 1978 compared to only 20% in 1999. The decline in percentage of dividend payers is attributed with equal importance to both a tilt of the publicly traded population towards firms with characteristics of firms that have never payed (low earnings, strong investment and small size), and to a general lower propensity to pay. Our model adds insights to both these channels. Secondly, Leary and Michaely (2011) document a steady and substantial increase in the degree of dividends smoothing over the past century. In the model, financial development account quantitatively for these changes in the dynamics of dividends.

As explained above, although the objective function of the firms is linear, due to the presence of endogenous borrowing constraints (and price schedules) their value functions are strictly concave. Therefore, when deciding the value of dividends to distribute, firms trade off level with volatility. In the model - as in the data - the firms distributing dividend are large firms. In the first equilibrium, credit frictions are tight and firms cannot smooth dividends as they wish. This extra volatility is then compensated by a higher level of dividends. When financial development tempers the credit frictions of the firms, they become better able at insuring dividends from the effects of the persistent idiosyncratic shocks. Hence large firms change their payout policy by increasing the smoothing of dividends while reducing their average level. At the same time, financial development allows for the presence of an higher number of small firms, therefore further reducing (through the distribution) the ratio of dividend payers. As shown in Table 9, financial development makes the number of firms distributing dividends to shrink down by 34%. This mechanism explains the 73% of the decline in the number of firms paying dividends observed in the data.

Meanwhile even the degree at which firms smooth dividends rise substantially. For measuring the degree of dividend smoothing, we follow Lintner (1956) by estimating the following regression

$$\Delta d_{i,t} = d_{i,t} - d_{i,t-1} = \alpha + \beta_1 d_{i,t-1} + \beta_2 y_{i,t} + \epsilon_{i,t}$$

where $d_{i,t}$ denotes the dividend of the i-th firm in time t, while $y_{i,t}$ is the value of the firm's sales. We then estimate the speed of adjustment of dividends by $-\hat{\beta}_1$. When this value equals 0, dividends are perfectly smoothed and follow a random walk. As reported in Table 9, in our model financial development makes the estimated speed of adjustments to decline from 0.43 to 0.22, which implies a substantial increase in the degree of firms' dividend smoothing. These values provided by the model are remarkably close the estimates of

Leary and Michaely (2011), which find an estimated speed of adjustment of about 0.3 during the 1960's and 1970's and about 0.2 for the most recent years. Moreover, the theory we propose is corroborated by (one of) the main findings in Leary and Michaely (2011). After testing several extant explanations of the smoothing motive[29], the authors finds smoothing to be prevalent among firms that appear to have the least constrained access to external capital and highest dividend levels, which have all the characteristics of the large firms described in our model.

Firm Volatility.

Campbell et *al.* (2001) provide evidence on an upward trend in the volatility of public firms' return, which has more than doubled from the 1960's to the late 1990's. Comin and Mulani (2006) and Comin and Philippon (2006) complement this finding by documenting an increase in idiosyncratic volatility of firm's *real* variables, such as real sales and employment. All these papers conjecture the origin of such trends and suggest that increased competition, R&D innovations, changes in the corporate governance of the firms and the institutionalization of equity ownership could have spurred the volatility of firms.
Here we show that financial development could have been another source of such steep increases of volatility. Actually, in the model the rise in the volatility of firms' sales and returns is the other side of the coin of the evidence reported above on dividends. Indeed, firms achieve a higher degree of dividend smoothing by increasing the volatility of their debt, which in turn spurs the fluctuations in investment, and eventually firms' sales and returns. We compute our measure of volatilities as in Comin and Mulani (2006), as

$$\sigma(x_{i,t}) = \sqrt{\frac{\sum_{\tau=t-4}^{t+5}\left(x_{i,\tau} - \bar{x}_i\right)^2}{10}}$$

where $\bar{x}_{i,}$ is the average of the variable $x_{i,t}$ between the periods $t-4$ and $t+5$. In what follows, we compute the volatility of two variables:firms' sales $y_{i,t}$ and firms' cum dividend returns $ret_{i,t} = \frac{V(\omega_{i,t},x_{i,t})+d_{i,t}}{V(\omega_{i,t-1},x_{i,t-1})}$. The bottom of Table 9 shows that, in the model, the median volatility of sales rise from a value of 0.14 to 0.24, and the one of returns from 0.12 to 0.20. As a further robustness check, one of the implications of our theory is that the increase in volatility is pervasive across *all* the quantiles of the size distribution, as observed by Comini and Mulani (2004). Therefore, financial development causes an increase in the volatility of firms' sales and returns which equals 72% and 67%, respectively.

[29]Theory based on asymmetric information, agency considerations external financing costs and tax planning.

TABLE 3.5.9
RESULTS ON DIVIDEND PAYOUT AND FIRM VOLATILITY

Variable	*pre*-1984	*post*-1984	Δ *post*-1984/*pre*-1984
A. Dividend Payout Policy			
N. Firm Paying Dividend	58.2%	38.4%	-34%
Degree Speed of Adjustment	0.43	0.22	-49%
B. Firm Volatility			
Median Sales Volatility	0.14	0.24	72%
Median Stock Return Volatility	0.12	0.20	67%

Note: this table reports the results of the model in two equilibria. The first one, labeled as "*pre*-1984", denotes the version of the model with high fixed costs of borrowing, $\zeta = 0.6$. The second one, labeled as "*post*-1984", denotes the version of the model with low fixed costs of borrowing, $\zeta = 0.4$. The Degree Speed of Adjustment defines the degree at which firms smooth dividends over time. We take the simulated data of the model, run the regression $\Delta d_{i,t} = d_{i,t} - d_{i,t-1} = \alpha + \beta_1 d_{i,t-1} + \beta_2 y_{i,t} + \epsilon_{i,t}$, and consider $-\hat{\beta}_1$ as the estimate of the speed of adjustment.

3.6 Conclusion

In this paper we document a diverging trend between default rates and credit spreads in the US economy over the last 60 years. On one hand, we find evidence in favour of the presence of one structural break in the unconditional mean of default rates in 1984. This date splits the series of default in two samples with very different characteristics. Indeed, the average corporate default rate rose from an average of 0.3%, during the period 1950-1983, to a value of 1.7% over the period 1984-2012. On the other hand, the average credit spreads barely moved, recording a 11 basis point increase. We run a battery of tests to show that this movement in credit spreads is statistically insignificant. Therefore, over the last three decades, default rates experienced a 467% increase, while credit spreads kept constant. Hence, nowadays corporate bankruptcies are more and more frequent than thirty years ago, but this came at no effect on the average borrowing cost.

We present a dynamic equilibrium model with heterogeneous firms where the development of credit markets and limited enforceability of debt contracts can be accounted for the diverging trend between rising default rates and constant credit spreads. We model the development of credit markets through an exogenous reduction of fixed costs of borrowing, as a reduced form for the development of the U.S. financial system during the

1970's and 1980's. The predictions of the model are quantitatively appealing. Financial development accounts for the 64% of the rise in the default rates, which is accompanied by an increase in credit spreads of just 2 basis points. Indeed, the reduction in the fixed borrowing costs make debt cheaper: Firms can access larger loans to invest more in capital and grow up in size. At the same time, firms that become inefficient can disinvest without being as penalized as before in their interest rates, due to the lack of collateral. So, the volatility of investment goes up, just because debt becomes more volatile too. Hence, financial development increases the level of debt, its volatility and makes eventually default more likely. On the other hand, credit spreads barely move because the insurance channel due to the financial development prevails on the default risk channel. Indeed, if on one side the cut in the borrowing fixed costs increases (on average) the endogenous probability of default in the economy, on the other side it reduces (on average) the wedge between the actual firms' optimal choices and the frictionless ones. As a consequence, financial development increases both the median size of capital by 9.34% and median profits by 21.73%. The upsurge in the expected liquidation value of the firms offsets the dramatic rise in default rates, leaving the credit spreads unchanged.

Furthermore, we show that in the model financial development can account for a number of trends - other than the increase in default rates - that characterized public firms in the last thirty years. First, we show that the reduction of the fixed credit costs changes firms' optimal decisions of dividend payout. Since firms are now better able to smooth dividends over time, they can trade off this reduction in volatility with a decrease in the level of dividends. As a result of financial development, the measure of firms distributing dividends shrinks down by 33%. This number accounts for the 73% of the decline documented for the U.S. by Fama and French (2001). Furthermore, in the model the median degree of dividend smoothing increases of a magnitude which is remarkably close to the values estimated by Leary and Michaely (2011). Second, we study the volatility of firms' returns and sales. Indeed, Campbell et *al.* (2001), Comin and Mulani (2006), Comin and Philippon (2006) show the presence of a secular upward trend in the volatilities of firms. We suggest that this empirical evidence can be (at least partially) accounted for by financial development. Indeed, the model is able to reproduce a 72% in the volatility of sales and a 67% for firms' returns.

3.A Appendix: The Role of Financial Development

Assumptions 1. Let us assume the following:

1. $\delta = 1$: full depreciation. This assumption implies that capital is not anymore a state.
2. $\psi = 1$: full clearance loss which implies no liquidation value.
3. $x_i = \bar{x}_i$: each firm initially experiences (is endowed with) a different idiosyncratic shock, henceforth constant. For example, firms are born of a particular type, where the type captures different levels of productivity.
4. Firms do not suffer the stochastic fixed cost of operation.
5. Firms cannot issue equity.
6. Entrants are endowed with no debt and no capital.
7. The subjective time discounting parameter β equals $\frac{1}{1+r_F}$.

Under these assumptions, operating profits and the firm net-worth reduces to:

$$\pi_i = x_i k_i^{\alpha}$$
$$w_i = \pi_i - (1 + r_i) l_i$$

Let us now characterize the equilibrium properties of this economy. Since there is no uncertainty in the model, in equilibrium the firms optimal policies are constant, (ϕ_D^*, k'^*, l'^*, d^*). In principle there are two putative equilibria where firms choose with probability one to default ($\phi_D^* = 1$) or not to default ($\phi_D^* = 0$). On one hand, we can show that the defaulting equilibrium is not an equilibrium for the firm. The proof is trivial and is made by contradiction.

Proof. Let us assume that a firm defaults in equilibrium; this means that it exits without distributing any dividend. This strategy violates the profit maximizing condition. We can find a feasible strategy that delivers an higher payoff. In particular not defaulting, producing at the first best level of capital each period, distributing the rest in dividend (without using debt) is a feasible strategy with constant policies which provides an higher payoff. □

On the other hand we can show that the No-Defaulting equilibrium holds only for a bounded set of loans. Despite the proof is more involved, the intuition is straightforward. An increase in the accorded level of debt increases the incentive for the firm to deviate from the equilibrium policy of not defaulting, distributing a big dividend in the current period and defaulting the period later. In particular in what follows we will show that there exists a threshold value of debt, $l^*_{D,i}$ above which firms will find optimal to deviate from the no default equilibrium strategy. Let us prove it.

Proof. The proof articulates as follows:

1. Proposition 3.A.1 states the zero-profit condition of the intermediary under Assumptions 3.A
2. We plug the result of Proposition 3.A.1 in the problem of the incumbents and we find the optimal policy of capital
3. We define the incentive compatibility constraint of the firm given the above policy functions
4. We define from the incentive compatibility constraint of the firm the debt-limit

Proposition 3.A.1. *Under Assumptions 3.A, the equilibrium zero profit condition* $(l^*_i, r^*_i) \in \Omega(l^*_i, \bar{x}_i; \lambda)$ *can be rewritten as:*

$$(1 + r^*_i)l^*_i = (1 + r^*)l^*_i + (1 + r^*)\zeta \tag{3.A.1}$$

Proof. The credit intermediary zero profit condition reduces to:

$$\begin{aligned} l'_i + \zeta &= \frac{\mathrm{E}_{x'|x}\left[(1 - \phi_{D,i})(1 + r_i)l'_i + \phi_{D,i}L(k')\right]}{1 + r} \\ l'_i + \zeta &= \frac{(1 - \phi_{D,i})(1 + r'_i)l'_i + \phi_{D,i}L(k')}{(1 + r)} \\ l'_i + \zeta &= \frac{(1 - \phi_{D,i})(1 + r'_i)l'_i}{(1 + r)} \end{aligned}$$

The first step is a result of the absence of idiosyncratic uncertainty while the second one comes from the absence of liquidation value. Since we showed before that the firm does not default in equilibrium, the equilibrium level of debt obeys:

$$l^*_i + \zeta = \frac{(1 + r^*_i)l^*_i}{(1 + r^*)} \tag{3.A.2}$$

which concludes the proof.

□

Hence the firms in equilibrium can borrow at the risk free rate, corrected for the fixed cost of borrowing. This result stems from the fact that firms do not default in equilibrium. However, this result does not imply that firms can borrow as much as they want. This point is made clear following the proof.

In order to derive further insights on the firms optimal behaviors let's analyze the problem of the incumbents. Under Assumptions 3.A it reduces to:

$$
\begin{aligned}
V_i = \max_{\{k_i', l_i'\}} \; & d_i + \frac{1}{1+r} V_i(x_i'; \lambda') \\
\text{s.t} \quad & d_i = \bar{x}_i k_i^{\alpha} - k_i' + l_i' - (1 + r_i) l_i \\
& \left(l_i', r_i'\right) \in \Omega(l_i', \lambda)
\end{aligned}
$$

Substituting the zero profit condition (3.A.1):

$$
\begin{aligned}
V = \max_{\{k_i', l_i'\}} \; & d_i + \frac{1}{1+r} V_i(x_i'; \lambda') \\
\text{s.t} \quad & d_i = \bar{x}_i k_i^{\alpha} - k_i' + l_i' - (1 + r) l_i - (1 + r)\zeta
\end{aligned}
$$

The first order necessary conditions reads:

$$
\begin{aligned}
rCl1 = & \quad \frac{\alpha \bar{x}_i (k_i')^{\alpha - 1}}{1 + r} & (k_i) \\
1 = & \quad \frac{1 + r}{1 + r} & (b_i)
\end{aligned}
$$

from which we obtain:

$$
rCl1 + r = \quad \alpha \bar{x}_i (k_i')^{\alpha - 1} \quad (k_i)
$$

Hence in equilibrium firms can equal their marginal product to the risk free rate, choosing the first best level of capital:

$$
k^* = k_{fb,i} = \left(\frac{\alpha \bar{x}_i}{1 + r} \right)^{\frac{1}{1-\alpha}} \tag{3.A.3}
$$

Notice that the firms still suffer a dead-weight loss due to the fix-cost of borrowing ζ, but this burden does not affect the optimal choice of capital which is taken at the margin. At this point we can define the firm incentive compatibility constraint as the feasible set of policies strategies for which the firm does not want to default. In equilibrium:

$$
\Phi(\omega_i^*, \bar{x}_i; \lambda) = \left\{ (d_i^*, l_i^*, k_i^*) \in \mathbb{R}^2 \times \mathbb{R}_+ : \omega_i^* + d_i^* + l_i^* - k_i^* \geq 0 \right\} \tag{3.A.4}
$$

Given the monotonicity properties of the firms value function there exists a firm specific

debt limit $l_{D,i}$, such that any accorded level of debt higher than this debt limit, will provide an incentive for the firm to deviate and default. This interpretation rationalizes the label endogenous borrowing constraint.
The debt limit is defined as the level of debt for which the optimal policy functions deliver a zero-net worth:

$$\bar{x}_i k_{fb,i}^{\alpha} - k_{fb,i} + l_{D,i} - (1+r) l_{D,i} - (1+r)\zeta = 0$$

which simplifies to:

$$\bar{x}_i k_{fb,i}^{\alpha} - k_{fb,i} - r l_{D,i} - (1+r)\zeta = 0 \tag{3.A.5}$$

Proposition 3.A.2. *A no defaulting equilibrium strategy for a firm i is sustained for level of debt* $l_i^* \in [0, l_{D,i}]$, *where* $l_{D,i}$ *represent the equilibrium firm-specific debt limit and it is defined as:*

$$l_{D,i} = \frac{1+r-\alpha}{r\alpha} k_{fb,i} - \frac{(1+r)}{r}\zeta \tag{3.A.6}$$

Proof. Substituting (3.A.3)

$$\begin{aligned}
l_{D,i} &= \frac{\bar{x}_i k_{fb,i}^{\alpha} - k_{fb,i} - (1+r)\zeta}{r} \\
&= \frac{\bar{x}_i \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} - (1+r)\zeta}{r} \\
&= \frac{\bar{x}_i^{-\frac{1}{\alpha-1}} \left(\frac{1+r}{\alpha}\right)^{\frac{1}{\alpha-1}+1} - \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} - (1+r)\zeta}{r} \\
&= \frac{\left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} \left[\frac{1+r}{\alpha} - 1\right] - (1+r)\zeta}{r} \\
&= \frac{\left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} \left[\frac{1+r-\alpha}{\alpha}\right] - (1+r)\zeta}{r} \\
&= \frac{\left[\frac{1+r-\alpha}{\alpha}\right] \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}}}{r} - \frac{(1+r)\zeta}{r} \\
&= \frac{1+r-\alpha}{r\alpha} k_{fb,i} - \frac{1+r}{r}\zeta
\end{aligned}$$

□

Now, we can easily derive the following derivatives:

$$\begin{aligned}
rCl\frac{\partial l_{D,i}}{\partial \zeta} &= & -\frac{1+r_i}{r_i} < 0 \\
\frac{\partial l_{D,i}}{\partial k_{fb,i}} &= & \frac{1+r_i-\alpha}{r\alpha} > 0
\end{aligned}$$

reported in Proposition 3.4.3. First, the debt-limit is increasing in the level of financial development. The lower the level of financial development (the higher is ζ), the lower is the level of debt for which the firm is indifferent whether to default or not. Second, the debt-limit is increasing in the optimal choice of capital, which, in this context, is function uniquely of the original idiosyncratic shock. The higher is the idiosyncratic shock (quality of production), the higher is the debt limit, i.e. the threshold of debt for which a firm is indifferent whether to default or not.

□

Let us now define the leverage at the debt limit as:

$$\frac{l_{D,i}}{k_{fb,i}} = \frac{1 + r_i - \alpha}{r_i \alpha} - \frac{1 + r_i}{r_i} \frac{\zeta}{k_{fb,i}} \tag{3.A.7}$$

From which we can easily derive the following derivatives:

$$\begin{aligned} rCl\frac{\partial \frac{l_{D,i}}{k_{fb,i}}}{\partial k_i^{fb}} &= \frac{1 + r_i}{r_i} \frac{\zeta}{(k_{fb,i})^2} > 0 \\ \frac{\partial \frac{l_{D,i}}{k_{fb,i}}}{\partial \zeta} &= -\frac{1 + r_i}{r_i} \frac{1}{k_{fb,i}} < 0 \end{aligned}$$

reported in Proposition 3.4.4. First, the leverage is strictly increasing in the amount of capital. The higher is the productivity of a firm, the higher is the optimal level of capital, the higher is the equilibrium leverage. Second, the leverage is strictly increasing in the level of financial development. The more developed is the credit intermediation, the lower is ζ, the higher is the equilibrium leverage of the firms.

3.B Appendix: Data

3.B.1 Firm Characteristics.

Data on firms characteristics are taken from Compustat, fundamental annual data from 1950 to 2006. Compustat includes public firms listed on the three US exchanges, NYSE, AMEX, and Nasdaq, with a non-foreign incorporation code.

Following Covas and DenHaan (2011), we exclude: 1) American Depository Receipts (ADRs) - securities created by U.S. banks to permit a U.S.-based trading of stocks listed on foreign exchanges; 2) financial firms (SIC classification between 6000 and 6999); 3) utilities (SIC classification between 4900 and 4949); 3) firms involved in major mergers (Compustat footnote code AB[30]); 4) firms with missing value for the book value of assets.

Entrants are defined as firms which are showing up on Compustat for the first time. The assets, ($a \equiv k_{it} + xk_{it}^{\alpha}$), in the model, are computed as the book value of assets (Compustat data item 6 - mnemonic AT).
The total debt, (b_{it}) in the model, is computed as long-term debt (item 9 , mnemonic) plus debt in current liabilities (item 34, mnemonic), since there is no distinction among the two in the present model.

3.B.2 Default Rate.

Data on corporate default rates are taken from Giesecke et *al.* (2011).

3.B.3 Credit Spreads.

Data on credit spreads are taken from the St Louis FRED. Credit spreads are computed using the monthly seasonally not-adjusted Moody's Seasoned Corporate Bond Yield, from 1950 to 2012. The series follows an investment bond that acts as an index of the performance of all bonds given a specific rating by Moody's Investment Firm. Annual series are constructed by averaging the monthly percent bond yields. The spread is then computed as the difference of the natural logarithm of BAA and AAA bond yields.

3.B.4 Ex-post Real Risk Free Interest Rate.

The ex-post real risk free interest rates are computed using data on 1) the Treasury Constant Maturity Rate bill, from the St Louis FRED, and 2) the Personal Consumption Expenditures (PCE) Chain-type Price Index, from the Bureau of Economic Analysis.
The ex-post real risk free interest rate is computed as the difference between the three-month Treasury bill rate minus the realized inflation in the subsequent quarter.
We use the three-month Treasury Constant Maturity Rate, at monthly frequency. We then build annual data averaging (equal weights) the monthly rates.
For the inflation, we use the seasonally adjusted quarterly rate of the Personal Consumption Expenditures (PCE) Chain-type Price Index. The annualized growth for PCE deflator is computed by taking 400 times the first differences of the natural logs of the PCE deflator. The series of ex-post real interest rate so constructed goes from 1950 to 1985.

3.B.5 GDP growth.

We take the data on real GDP from St Louis FRED. We compute the GDP growth from 1949 until 2011 as the log difference of the raw GDP data. As a robustness check, we also

[30] Compustat assigns a footnote AB to total sales if sales increase by more than 50 percent in response to a merger or an asset acquisition.

use HP-filtered data ($\lambda = 6.25$).

3.B.6 Stock Returns Volatility.

We take daily data on stock returns from CRSP. We compute the annual volatility from 1949 until 2011 by computing the standard deviation of returns within a year. Since the measure is computed over non-overlapping spans of time, the measurement errors are uncorrelated and do not bias the estimates of volatility.

3.C Appendix: Computational Algorithm

The computation adopts the discrete choice method. Grids on bond and capital consist of 200 grid points.
The computational algorithm articulates as follows:

1. It starts with the guess of: 1) the continuation value function of the incumbents; 2) the default policy function of the incumbents; 3) the debt schedules of the incumbents. In line with Arellano et *al.* (2012) the initial guess of the debt schedules is the risk free interest rate.

2. It iterates over the continuation value function of the incumbents in the fixed point algorithm till convergence.

3. The implied continuation value function is used for updating, through the optimal default decision rule (3.3.5), the default policy function. Clearly the convergence of the value function implies the convergence of the default policy functions, but not *viceversa.*

4. The implied default policy functions are used to update the endogenous probability of default, which in turns is used for updating the feasible correspondence Ω-set and, therefore, the debt schedules (3.3.12).

5. Points 2, 3, 4 are iterated till convergence of the debt schedules.

Technical Details:

- The levels of tolerance for the convergence of the value function and of the debt schedules are set to 1e-6.

- The grids are controlled not to be binding in equilibrium.

- The statistics reported in Table 3.5.8 are obtained using the (ergodic) distribution at period T=1000, obtained simulating 15000 firms over 1000 periods, .

- The Tauchen (1986) algorithm truncates the $\pm \inf$ values of the support of the normal distribution at $\pm 20\sigma_{log(x)}$.

Bibliography

Abel, A., J. Eberly, and S. Panageas. 2007. Optimal Inattention to the Stock Market. *American Economic Review: Papers and Proceedings*, 97, 244-9.

Abel, A., J. Eberly, and S. Panageas. 2013. Optimal Inattention to the Stock Market with Information Costs and Transaction Costs. *Econometrica*, 81, 1455-81.

Almeida, H., and T. Philippon. 2007. The Risk-Adjusted Cost of Financial Distress. *Journal of Finance*, 62, 2557-86.

Altinkilic, O., and R. Hansen. 2000. Are There Economies of Scale in Underwriting Fees? Evidence of Rising External Financing Costs. *Review of Financial Studies*, 13, 191-218.

Alvarez, F., L. Guiso, and F. Lippi. 2012. Durable Consumption and Asset Management with Transaction and Observation Costs. *American Economic Review*, 102, 2272-300.

Ameriks, J., A. Caplin, and J. Leahy. 2003. Wealth Accumulation and the Propensity to Plan. *Quarterly Journal of Economics*, 118, 1007-47.

Ameriks, J., and S. Zeldes. 2004. How Do Household Portfolio Shares Vary with Age? *Working Paper.*

Andrei, D., and M. Hasler. 2015. Investors' Attention and Stock Market Volatility. *Review of Financial Studies*, 28, 33-72.

Angeletos, G.-M., and J. La'O. 2009. Incomplete Information, Higher-Order Beliefs, and Price Inertia. *Journal of Monetary Economics*, 56, 19-37.

Arellano, C., Y. Bai, and P. Kehoe. 2011. Financial Markets and Fluctuations in Uncertainty. *Working Paper.*

Arellano, C., Y. Bai, and J. Zhang. 2012. Firm Dynamics and Financial Development. *Journal of Monetary Economics*, 59, 533-49.

Armenter, R., and V. Hnatkovska. 2012. The Macroeconomics of Firms' Savings. Working Paper.

Axtell, R. 2001. Zipf Distribution of U. S. Firm Sizes. *Science*, 293, 1818-20.

Bai, J., and P. Perron. 1998. Estimating and testing linear models with multiple structural changes. *Econometrica*, 66, 47-78.

Bansal, R., D. Kiku, I. Shaliastovich, and A. Yaron. 2014. Volatility, the Macroeconomy and Asset Prices. *Journal of Finance*, 69, 2471-2511.

Bansal, R., and A. Yaron. 2004. Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance*, 59, 1481-1509.

Barber, B., and T. Odean. 2008. All That Glitters: The Effect of Attention and News on the Buying Behavior of Individual and Institutional Investors. *Review of Financial Studies*,

21, 785-818.
Barro, R. 2006. Rare Disasters and Asset Markets in the Twentieth Century. *Quarterly Journal of Economics*, 121, 823-66.
Basak, S., and D. Cuoco. 1998. An Equilibrium Model with Restricted Stock Market Participation. *Review of Financial Studies*, 11, 309-41.
Bessembinder, H., and W. Maxwell. 2008. Markets: Transparency and the Corporate Bond Market. *Journal of Economic Perspectives*, 22, 217-34.
Bianchi, J., and E. Mendoza. 2013. Optimal Time Consistent Macroprudential Policy. *NBER Working Paper*, 19704.
Bianchi, J. 2011. Overborrowing and Systemic Externalities in the Business Cycle. *American Economic Review*, 101, 3400-26.
Bloom, N. 2009. The Impact of Uncertainty Shocks. *Econometrica*, 77, 623-85.
Boldrin, M., L. Christiano, and J. Fisher. 2001. Habit Persistence, Asset Returns, and the Business Cycle. *American Economic Review*, 91, 149-66.
Bordo, M., B. Eichengreen, D. Klingebiel, and M. Peria. 2001. Is the Crisis Problem Growing More Severe? *Economic Policy*, 16, 51-82.
Born, B., and J. Pfeifer. 2013. Policy Risk and the Business Cycle. *CESifo Working Paper*, 4336.
Brunnermeier, M., and S. Nagel. 2008. Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-Evidence on Individuals' Asset Allocation. *American Economic Review*, 98, 713-36.
Brunnermeier, M., and L. Pedersen. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies*, 22, 2201-38.
Brunnermeier, M., and Y. Sannikov. 2013. A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104, 379-421.
Buera, F., J. Kaboski, and Y. Shin. 2011. Finance and Development: A Tale of Two Sectors. *American Economic Review*, 101, 1964-2002.
Buera, F., and Y. Shin. 2013. Financial Frictions and the Persistence of History: A Quantitative Exploration. *Journal of Political Economy*, 121, 221-72.
Caballero, R. 1991. On the Sign of the Investment-Uncertainty Relationship. *American Economic Review*, 81, 279-88.
Caprio, G., and D. Klingebiel. 2003. Episodes of Systemic and Borderline Financial Crises. *Working Paper.*
Campbell, J. 1991. A Variance Decomposition for Stock Returns. *Economic Journal*, 101, 157-79.
Campbell, J. 1999. Asset Prices, Consumption, and the Business Cycle. In J. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*. Amsterdam: North-Holland.
Campbell, J., and J. Cochrane. 1999. By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy*, 107, 205-51.
Campbell, J., J. Hilscher, and J. Szilagyi. 2008. In Search of Distress Risk. *Journal of*

Finance, 63, 2899-939.

Campbell, J., M. Lettau, B. Malkiel, and Y. Xu. 2001. Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk. *Journal of Finance*, 56, 1-43.

Campbell, J., and G. Mankiw. 1989. Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence. In O. Blanchard and S. Fischer (eds.), *NBER Macroeconomics Annual*, 185-216.

Campbell, J., and G. Mankiw. 1990. Permanent Income, Current Income and Consumption. *Journal of Business and Economic Statistics*, 8, 265-279.

Carvalho, V., and X. Gabaix. 2013. The Great Diversification and Its Undoing. *American Economic Review*, 103, 1697-1727.

Chen, H. 2006. Can Information Costs Explain the Equity Premium and Stock Market Participation Puzzles? *Working Paper.*

Chen, H. 2010. Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure. *Journal of Finance*, 65, 2171-212.

Chen, L., P. Collin-Dufresne, and R. Goldstein. 2009. On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle. *Review of Financial Studies*, 22, 3367-409.

Chien, Y., H. Cole, and H. Lustig. 2011. A Multiplier Approach to Understanding the Macro Implications of Household Finance. *Review of Economic Studies*, 78, 199-234.

Chien, Y., H. Cole, and H. Lustig. 2012. Is the Volatility of the Market Price of Risk due to Intermittent Portfolio Re-Balancing? *American Economic Review*, 102, 2859-96.

Christiano, L., R. Motto, and M. Rostagno. 2014. Risk Shocks. *American Economic Review*, 104, 27-65.

Cochrane, J. 1991. Explaining the Variance of Price-Dividend Ratios. *Review of Financial Studies*, 5, 243-80.

Comin, D., and S. Mulani. 2006. Diverging Trends in Macro and Micro Volatility. *Review of Economics and Statistics*, 88, 374-83.

Comin, D., and T. Philippon. 2006. The Rise in Firm-Level Volatility: Causes and Consequences. *NBER Macroeconomics Annual 2005*, 20, 167-228.

Cooley, T., and E. Prescott. 1995. Economic Growth and Business Cycles. In T. Cooley (ed.), *Frontiers of Business Cycle Research*. Princeton: Princeton University Press.

Cooley, T., and V. Quadrini. 2001. Financial Markets and Firm Dynamics. *American Economic Review*, 91, 1286-310.

Covas, F., and W. den Haan. 2011. The Cyclical Behavior of Debt and Equity Finance. *American Economic Review*, 101, 877-99.

Da, Z., J. Engelberg, and P. Gao. 2011. In Search of Attention. *Journal of Finance*, 66, 1461-99.

Davis, M., and J. Heathcote. 2007. The Price and Quantity of Residential Land in the United States. *Journal of Monetary Economics*, 54, 2595-620.

Davis, M., and F. Ortalo-Magné. 2011. Household Expenditures, Wages, Rents. *Review of Economic Dynamics*, 14, 248-61.

Della Vigna, S., and J. Pollet. 2009. Investor Inattention and Friday Earnings Announcements. *Journal of Finance*, 64, 709-49.

Del Negro, M., G. Eggertsson, A. Ferrero and N. Kiyotaki. 2011. The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities. *Federal Reserve Bank of New York Staff Reports*, 520.

Demyanyk, Y., C. Ostergaard, and B. Sorensen.2007. U.S. Banking Deregulation, Small Businesses, and Interstate Insurance of Personal Income. *Journal of Finance*, 62, 2763-801.

Diaz, A., and B. Jerez. 2013. House Prices, Sales, And Time On The Market: A Search-Theoretic Framework. *International Economic Review*, 54, 837-72.

Díaz-Giménez, J., A. Glover, and J.-V. Ríos-Rull. 2011. Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update. *Federal Reserve Bank of Minneapolis Quarterly Review*, 34, 2-31.

Duffie, D. 2010. Asset Price Dynamics with Slow-Moving Capital. *Journal of Finance*, 65, 1237-67.

Duffie, D., and T.-S. Sun. 1990. Transactions Costs and Portfolio Choice in a Discrete-Continuous-Time Setting. *Journal of Economic Dynamics and Control*, 14, 35-51.

Fama, E., and K. French. 2001. Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay? *Journal of Financial Economics*, 60, 3-43.

Favilukis, J. 2013. Inequality, Stock Market Participation, and the Equity Premium. *Journal of Financial Economics*, 107, 740-59.

Favilukis, J., S. Ludvigson, and S. Van Nieuwerburgh. 2013. The Macroeconomic Effects of Housing Wealth, Housing Finance and Limited Risk Sharing in General Equilibrium. *NBER Working Paper*, 15988.

Fernández-Villaverde, J., and J. Rubio-Ramírez. 2007. Estimation Macroeconomic Models: A Likelihood Approach. *Review of Economic Studies*, 74, 1059-87.

Fernández-Villaverde, J., P. Guerrón-Quintana, J. Rubio-Ramírez, and M. Uribe. 2011. Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review*, 101, 2530-61.

Finocchiaro, D. 2011. Inattention, Wealth Inequality and Equilibrium Asset Prices. *Journal of Monetary Economics*, 58, 146-55.

Fisher, I. 1933. A Debt-Deflation Theory of Great Depressions. *Econometrica*, 1, 337-57.

Fostel, A., and J. Geanakoplos. 2008. Leverage Cycles and the Anxious Economy. *American Economic Review*, 98, 1211-44.

Foster, L., J. Haltiwanger, and C. Syverson. 2008. Reallocation, Firm Turnover and Efficiency: Selection on Productivity or Profitability. *American Economic Review*, 98, 394-425.

Gabaix, X. 2011. The Granular Origins of Aggregate Fluctuations. *Econometrica*, 79, 733-72.

Gabaix, X., and D.Laibson. 2002. The 6d Bias and the Equity Premium Puzzle. In B. Bernanke and K. Rogoff (eds.), *NBER Macroeconomics Annual*, 16, 2341-78.

Geanakoplos, J. 2010. The Leverage Cycle. *NBER Macroeconomics Annual*, 1 - 65.

Genesove, D., and L. Han. 2012. Search and Matching in the Housing Market. *Journal of Urban Economics*, 72, 31-45.

Giesecke, K., F. Longstaff, S. Schaefer, and S. Strebulaev. 2011. Corporate Bond Default Risk: A 150-Year Perspective. *Journal of Financial Economics*, 102, 233-50

Gilchrist, S., and E. Zakrajsek. 2012. Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, 102, 1692-720.

Godsill, S., A. Doucet, and M. West. 2004. Monte Carlo Smoothing for Nonlinear Time Series. *Journal of the American Statistical Association*, 99, 156-68.

Gomes, F., and A. Michaelides. 2008. Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. *Review of Financial Studies*, 21, 415-48.

Gomes, J., and L. Schmid. 2010a. Equilibrium Credit Spreads and the Macroeconomy. *Working Paper.*

Gomes, J., and L. Schmid. 2010b. Levered Returns. *Journal of Finance*, 65, 467-94.

Guvenen, F. 2009. A Parsimonious Macroeconomic Model for Asset Pricing. *Econometrica*, 77, 1711-50.

Gourio, F. 2012. Disaster Risk and Business Cycles. *American Economic Review*, 102, 2734-66.

Greenwood, J., Z. Hercowitz, and G. Huffman. 1988. Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review*, 78, 402-17.

Grossman, S., and J. Stiglitz. 1980. On the Impossibility of Informationally Efficient Markets. *American Economic Review*, 70, 393-408

Hall, R. 1978. Stochastic Implications of the Life Cycle Permanent Income Hypothesis: Theory and Evidence. *Journal of Political Economy*, 86, 971-87.

He, Z., and A. Krishnamurthy. 2013. Intermediary Asset Pricing. *American Economic Review*, 103, 732-70.

Heaton, J., and D. Lucas. 1996. Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing. *Journal of Political Economy*, 104, 443-87.

Hirshleifer, D., S. Lim and S. Teoh. 2009. Driven to Distraction: Extraneous Events and Underreaction to Earnings News. *Journal of Finance*, 64, 2289-325.

Hopenhayn, H. 1992. Entry, Exit, and Firm Dynamics in Long Run Equilibrium. *Econometrica*, 60, 1127-50.

Hosios, A. 1990. On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies*, 57, 279-98.

Huan, L., and H. Liu. 2007. Rational Inattention and Portfolio Selection. *Journal of Finance*, 62, 1999-2040.

Huang, J., and M. Huang. 2012. How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk? *Review of Asset Pricing Studies*, 2, 153-202.

Iacoviello, M. 2005. House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review*, 95, 739-64.

Jayaratne, J., and P. Strahan. 1996. The Finance-Growth Nexus: Evidence from Bank Branch Deregulation. *Quarterly Journal of Economics*, 111, 639-70.

Jermann, U. 1998. Asset Pricing in Production Economies. *Journal of Monetary Economics*, 41, 257-75.

Jermann, U., and V. Quadrini.2006. Financial Innovations and Macroeconomic Volatility. *NBER Working Paper*, 12308.

Jermann, U., and V. Quadrini. 2012. Macroeconomic Effects of Financial Shocks. *American Economic Review*, 102, 238-71.

Jorda, O., M. Schularick, and A. Taylor. 2013. When Credit Bites Back. *Journal of Money, Credit and Banking*, 45, 3-28.

Jorda, O., M. Schularick, and A. Taylor. 2015. Sovereigns versus Banks: Credit, Crises, and Consequences. *Journal of the European Economic Association*, forthcoming.

Justiniano, A., and G. Primiceri. 2008. The Time Varying Volatility of Macroeconomic Fluctuations. *American Economic Review*, 98, 604-41.

Justiniano, A., A. Tambalotti, and G. Primiceri. 2014. The Effects of the Saving and Banking Glut on the U.S. Economy. *Journal of International Economics*, 92, 52-67.

Khan, A., T. Senga, and J. Thomas. 2012. Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity. *Working Paper*.

King, R., and R. Levine. 1993. Finance and Growth: Schumpeter Might be Right. *Quarterly Journal of Economics*, 108, 713-37.

Kiyotaki, N., and J. Moore. 1997. Credit Cycles. *Journal of Political Economy*, 105, 211-48.

Kiyotaki, N., and J. Moore. 2012. Liquidity, Business Cycles, and Monetary Policy. *NBER Working Paper*, 17934.

Krueger, D., and H. Lustig. 2010. When is Market Incompleteness Irrelevant for the Price of Aggregate Risk (and when is it not)? *Journal of Economic Theory*, 145, 1-41.

Krusell, P. and A. Smith. 1997. Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns. *Macroeconomics Dynamics*, 1, 387-422.

Krusell, P., and A. Smith. 1998. Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106, 867-96.

Laeven, L., and F. Valencia. 2012. Systemic Banking Crises Database: An Update. *IMF Working Paper*, 12/163.

Leary, M. and R. Michaely. 2011. Determinants of Dividend Smoothing: Empirical Evidence. *Review of Financial Studies*, 24, 3197-249.

Lettau, M., and S. Ludvigson. 2001. Consumption Aggregate Wealth and Expected Stock Returns. *Journal of Finance*, 56, 815-49.

Lettau, M., and S. Ludvigson. 2010. Measuring and Modeling Variation in the Risk-Return Tradeoff. In Y. Ait-Sahalia and L. Hansen (eds.), *Handbook of Financial Economet-*

rics. Oxford: North Hooland.

Lintner, J. 1956. Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes. *American Economic Review*, 46, 97-113.

Liu, Z., P. Wang, and T. Zha. 2013. Land-Price Dynamics and Macroeconomic Fluctuations. *Econometrica*, 81, 1147-84.

Livshits, I., J. MacGee, and M. Tertil. 2010. Accounting for the Rise in Consumer Bankruptcies. *American Economic Journal: Macroeconomics*, 2, 165-93.

Longstaff, F., S. Mithal, and E. Neis. 2005. Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market. *Journal of Finance*, 55, 2213-53.

Longstaff, F. 2011. Municipal Debt and Marginal Tax Rates: Is There a Tax Premium in Asset Prices? *Journal of Finance*, 66, 721-51.

Lucas, R. 1978. On the Size Distribution of Business Firms. *Bell Journal of Economics*, 9, 508-23.

Lynch, A. 1996. Decision Frequency and Synchronization across Agents: Implications for Aggregate Consumption and Equity Return. *Journal of Finance*, 51, 1479-97.

Mankiw, G. 1986. The Equity Premium and the Concentration of Aggregate Shocks. *Journal of Financial Economics*, 17, 211-9.

Mankiw, G., and S. Zeldes. 1991. The Consumption of Stockholders and Nonstockholders. *Journal of Financial Economics*, 29, 97-112.

McConnell, M., and G. Perez-Quiros. 2000. Output Fluctuations in the United States: What Has Changed Since the Early 1980's? *American Economic Review*, 90, 1464-76.

Mehra, R., and E. Prescott. 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics*, 15, 145-61.

Mendoza, E. 2010. Sudden Stops, Financial Crises and Leverage. *American Economic Review*, 100, 1941-66.

Mendoza, E., and V. Quadrini. 2010. Financial Globalization, Financial Crises and Contagion. *Journal of Monetary Economics*, 57, 24-39.

Mendoza, E., and M. Terrones. 2012. An Anatomy of Credit Booms and Their Demise. *NBER Working Paper*, 18379.

Mian, A., and A. Sufi. 2009. The Consequences of Mortgage Expansion: Evidence from the U.S. Mortgage Default Crisis. *Quarterly Journal of Economics*, 124, 1449-96.

Mian, A., and A. Sufi. 2011. House Prices, Home Equity-Based Borrowing, and the U.S. Household Leverage Crisis. *American Economic Review*, 101, 2132-56.

Minsky, H. 1992. The Financial Instability Hypothesis. *Working Paper*, 74, Levy Economics Institute.

Modigliani, F., and M. Miller. 1958. The Cost off Capital, Corporate Finance and the Theory of Investment. *American Economic Review*, 48, 261-97.

Mondria, J., T. Wu, and Y. Zhang. 2010. The Determinants of International Investment and Attention Allocation: Using Internet Search Query Data. *Journal of International Eco-*

nomics, 82, 85-95.

Moody's Investor Service. 2012. Corporate Default and Recovery Rates, 1920-2011. *Technical Report.*

Moody's Investor Service. 2013. Rating Symbols and Definitions. *Technical Report.*

Ngai, R., and S. Tenreyro. 2014. Hot and Cold Seasons in the Housing Market. *American Economic Review*, 104, 3991-4026.

Peng, L. 2005. Learning with Information Capacity Constraints. *Journal of Financial and Quantitative Analysis*, 40, 307-29.

Peress, J. 2004. Wealth, Information Acquisition, and Portfolio Choice. *Review of Financial Studies*, 17, 879-914

Pijoan-Mas, J. 2007. Pricing Risk in Economies with Heterogeneous Agents and Incomplete Markets. *Journal of the European Economic Association*, 5, 987-1015.

Porapakkarm, P., and E. Young. 2008. Information Heterogeneity in the Macroeconomy. *Working Paper.*

Rajan, R., and L. Zingales. 1998. Financial Dependence and Growth. *American Economic Review*, 88, 559-86.

Reinhart, C., and K. Rogoff. 2009. *This Time is Different: Eight Centuries of Financial Folly.* Princeton: Princeton University Press.

Reis, R. 2006. Inattentive Consumers. *Journal of Monetary Economics*, 53, 1761-800.

Rosen, S. 1982. Authority, Control and the Distribution of Earnings. *Bell Journal of Economics*, 13, 311-23.

Rossi, A. 2010. Towards a Theoretical Explanation of Time-Varying Trading. *Working Paper.*

Saito, M. 1996. Limited Participation and Asset Prices. *Working Paper.*

Schmitt-Grohe, S., and M. Uribe. 2007. Optimal Simple and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics*, 54, 1702-25.

Schularick, M., and A. Taylor. 2012. Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises. *American Economic Review*, 102, 1029-61.

Schumpeter, J. 1911. *A Theory of Economic Development.* Cambridge: Harvard University Press.

Sichermann, N., G. Loewenstein, D. Seppi, and S. Utkus. 2012. To Look or not to Look: Financial Attention and Online Account Logins. *Working Paper.*

Sims, C. 2003. Implications of Rational Inattention. *Journal of Monetary Economics*, 50, 665-90.

Stock, J., and M. Watson. 2002. Has the Business Cycle Changed, and Why? *NBER Macroeconomics Annual*, 17, 159-218.

Storesletten, K., C. Telmer, and A. Yaron. 2007. Asset Pricing with Idiosyncratic Risk and Overlapping Generations. *Review of Economic Dynamics*, 10, 519-48

Tauchen, G. 1986. Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economics Letters*, 20, 177-81.

Ungerer, C. 2013. Monetary Policy, Hot Housing Markets and Leverage. *Working Paper.*

Veronesi, P. 1999. Stock Market Overreaction to Bad News in Good Times: A Rational Expectations Equilibrium Model. *Review of Financial Studies*, 12, 975-1007.

Warner, J. 1977. Bankruptcy Costs: Some Evidence. *Journal of Finance*, 32, 337-47.

Warnock, F., and V. Warnock. 2009. International Capital Flows and US Interest Rates. *Journal of International Money and Finance*, 28, 903-19.

Weil, P. 1989. The Equity Premium Puzzle and the Risk-free Rate puzzle. *Journal of Monetary Economics*, 24, 401-21.

Wheaton, W. 1990. Housing Investment in the United States. *Journal of Political Economy*, 98, 1270-92.