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Asymptotic Analysis of Multiuser-MIMO Networks with Battery-Constrained Receivers

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***Abstract*—In this paper, we present an asymptotic analysis of the behavior of a network where the mobile terminals are considered to be battery-powered devices provided with energy harvesting capabilities. The asymptotic analysis is based on a multiuser MIMO resource allocation strategy where the battery status of the mobile terminals are considered explicitly in the proposed allocation policy. We provide some numerical results and analytic expressions of the expected value of the data rates and the battery levels for different decoding power consumption models when convergence is attained.**

I. Introduction

In the last years, there has been a considerable expansion of wireless networks jointly with a continuous increase of the number of users. This expansion and the fact that newer applications require higher data rates, involve a need for a substantial increase of system capacity. In wireless networks, this capacity increase is technically challenging since the resources to be shared among users are limited. At the same time, in order to be more efficient, cell radii coverage are becoming shorter (such as in picocells and femtocells in cellular environments). Due to such short distances between transmitters and receivers, the radiated powers can be comparable or even lower than the powers consumed by the radio frequency (RF) chains and the baseband stages [1], [2].

On the other hand, it is also important to emphasize that one of the current limiting factors of today's technology is the short lifetime of the batteries. Due to such short lifetimes, the high data rates needed by the terminals entail situations where the users run out of battery noteworthy fast. In wireless sensors networks this can be a serious issue, since such sensors are placed in locations that cannot be accessed to replace their batteries. In cellular environments, the telecommunication providers has put a lot of attention on providing good services with enhanced coverage, but this will not be translated into an added value if the users cannot make use of them due to battery limitations.

Within this framework, the current work on energy harvesting (a technological solution to collect energy from the environment to recharge the batteries) is emerging [3], [4]. This suggests that new strategies for allocating the radio resources should be developed, considering explicitly such battery-related aspects, i.e., the harvesting capabilities and the battery status of the terminals.

In classical precoder design strategies for multi-antenna multi-input multi-output (MIMO) systems, a given objective function is optimized subject to some constraints typically considering only the radiated power. In our previous works [5], [6], we considered the design of a resource allocation policy in a multiuser (MU) broadcast system incorporating the idea that the nodes are battery-limited devices provided with energy harvesting capabilities and considering explicitly the power needed for decoding the received data. Thus, the information concerning the battery levels played an explicit role and had an impact on the design. The goal of this paper is to study the asymptotic behavior of such strategies and obtain analytic expressions for the data rates and the battery levels when the allocation algorithm is in steady state (i.e., temporal convergence has been attained), providing an insight into the inherent trade-offs of the parameters involved in the execution of the algorithm.

The remainder of this paper is organized as follows. In section II, we describe the system model. Section III presents a review of the resource allocation strategy under study. Section IV addresses the different asymptotic behaviors encountered in such policy and section V provides an analytic characterization of it. In section VI, we present some numerical results and, finally, conclusions are drawn in section VII.

II. System Model

Let us consider a set of K users indexed by $k \in \mathcal{K} \triangleq \{1, \ldots, K\}$. We focus on a MU-MIMO broadcast scenario where the k-th receiver has n_{R_k} antennas and the base station (BS) has n_T antennas. We index frames by $t \in \mathcal{T} \triangleq \{1, \ldots, T\}$ with a duration of T_f seconds each. We consider that the channels remain constant within a frame and change between consecutive frames. All the receivers in the system have a battery whose level decreases accordingly when the user receives and decodes data. We will assume throughout the paper that the battery size is infinite for all users, i.e., $C_{\max}^k = \infty$, $\forall k$. The terminals are also able to recharge their batteries by means of collecting energy dynamically from the environment with the help of a harvesting source. Let $E_h^k(t)$ be the energy harvested in Joules by the k-th user during the t-th frame. Accordingly, the battery level of the k-th user at the beginning of the $t+1$-th frame is, thus, denoted as

$$C_k(t+1) = C_k(t) - T_f P_{\text{tot},k}(R_k(t)) + E_h^k(t), \qquad (1)$$

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where $R_k(t)$ is the rate allocated to such user during the t-th frame and $P_{\text{tot},k}(R_k(t))$ is the power spent for decoding the message. The power consumed by the receiver $P_{\text{tot},k}(R_k(t))$ is modeled as the power consumed by the front-end P_c^{rx} plus the power consumed by the decoding stage $P_{\text{dec},k}(R_k(t))$: $P_{\text{tot},k}(R_k(t)) = P_{\text{dec},k}(R_k(t)) + P_c^{rx}$. Note that the decoding power $P_{\text{dec},k}(R_k(t))$ (and, thus, also the global power $P_{\text{tot},k}(R_k(t))$) depends increasingly on the rate R_k of the communication. In [5], the authors presented different models for $P_{\text{dec},k}(R_k(t))$, but, for the sake of generality, we consider it as a general function for the moment in this paper.

In a given frame, the transmitter needs to allocate a given finite amount of resources among the users. The precoder and the power to be assigned to each user are computed according to the technique presented in our previous work [5], [6]. In the next section, we will present a summary of this resource allocation strategy.

The main idea of this paper is to study and characterize analytically and numerically the asymptotic behavior of the resource allocation strategy presented in [5], [6]. In those papers, we only focused on the transient period of the allocation strategy, but for long transmissions, it is even more important to study the asymptotic steady-state behavior once temporal convergence has been attained. In that sense, we will characterize the asymptotic behavior of the data rate and the battery levels of the users in the system.

III. Summary of the Resource Allocation Strategy

In this section, we provide a summary of the main ideas of the resource allocation based on [5] whose asymptotic behavior will be studied later in this paper. The signal model for the received signals for the k-th user at the n-th time instant within the t-th frame is

$$\mathbf{y}_k(n,t) = \mathbf{H}_k(t)\sum_{j=1}^{K}\mathbf{B}_j(t)\mathbf{x}_j(n,t) + \mathbf{n}_k(n,t), \qquad (2)$$

where $\mathbf{y}_k(n,t) \in \mathbb{C}^{n_{R_k}\times 1}$ is the received vector, $\mathbf{x}_k(n,t) \in \mathbb{C}^{n_{S_k}\times 1}$ is the Gaussian data vector, $\mathbf{H}_k(t) \in \mathbb{C}^{n_{R_k}\times n_T}$ is the MIMO channel matrix from the BS to the k-th user, and $\mathbf{B}_k(t) \in \mathbb{C}^{n_T\times n_{S_k}}$ is the precoder matrix of user k, being n_{S_k} the number of streams. The transmit covariance matrix for user k is $\mathbf{Q}_k(t) = \mathbf{B}_k(t)\mathbf{B}_k(t)^H$ assuming that $\mathbb{E}\left[\mathbf{x}_k(n,t)\mathbf{x}_k(n,t)^H\right] = \mathbf{I}_{n_{S_k}}$. Finally, $\mathbf{n}_k(n,t) \in \mathbb{C}^{n_{R_k}\times 1}$ is the Gaussian noise with $\mathbb{E}\left[\mathbf{n}_k(n,t)\mathbf{n}_k(n,t)^H\right] = \sigma^2\mathbf{I}_{n_{R_k}}$.

Since the terminals are battery-limited, the resource allocation strategy will only allow each user to spend (at frame t) a given fraction of the available battery level for decoding the message, which is formulated as:

$$E_g^k(t) = \alpha_k C_k(t), \qquad 0 \le \alpha_k \le 1, \qquad (3)$$

which, in turn, implies that

$$T_f \cdot \left(P_{\text{dec},k}(R_k(t)) + P_c^{rx}\right) \le E_g^k(t). \qquad (4)$$

Note that α_k is a constant whose value impacts on the final performance as discussed in [6]. The value of α_k can be optimized. Such value depends on the harvesting intensity, among other parameters [6]. The previous relation (4) can also be written in terms of an equivalent upper bound on the maximum data-rate, $R_k(t) \le R_{\max,k}(C_k(t))$, where

$$R_{\max,k}(C_k(t)) = P_{\text{dec},k}{}^{-1}\left(\frac{E_g^k(t)}{T_f} - P_c^{rx}\right). \qquad (5)$$

Now, we formulate the resource allocation strategy as the following convex optimization problem (where we have omitted the time dependence t to simplify the notation):

$$\begin{aligned}
&\underset{\{R_k\},\{\mathbf{Q}_k\}}{\text{maximize}} && \sum_{k=1}^{K} R_k && (6)\\
&\text{subject to} && C1: \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Q}_k) \le P_T &&\\
& && C2: -\log_2\det\left(\mathbf{I} + \frac{\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H}{\sigma^2}\right) + R_k \le 0, \forall k &&\\
& && C3: R_k \le R_{\max,k}(C_k), \quad \forall k &&\\
& && C4: \mathbf{H}_k\mathbf{Q}_j\mathbf{H}_k^H = 0, \quad \forall k, \forall j, k \ne j &&
\end{aligned}$$

where P_T represents the maximum total radiated power at the BS. Notice that $C4$ forces the precoder design to follow a block-diagonalization (BD) strategy [7]. The obtained precoder matrix is given by

$$\mathbf{B}_k = \mathbf{V}_k\mathbf{P}_k^{1/2} \in \mathbb{C}^{n_T\times n_{S_k}}, \qquad (7)$$

where $\mathbf{V}_k = \tilde{\mathbf{V}}_k^{(0)}\mathbf{V}_k^{(1)}$, being $\tilde{\mathbf{V}}_k^{(0)}$ and $\mathbf{V}_k^{(1)}$ two matrices obtained from the BD procedure (see [5] and [7] for more details) and $\mathbf{P}_k$ is a diagonal matrix with entries

$$p_{i,k}^\star = \left(\frac{1-\beta_k^\star}{\mu^\star\ln(2)} - \frac{\sigma^2}{\lambda_{i,k}^2}\right)^+, \quad i = 1,\cdots,n_{S_k}, \qquad (8)$$

where $(x)^+ = \max(0,x)$, β_k and μ are Lagrange multipliers [8], and $\lambda_{i,k}^2$ is the eigenvalue of the equivalent channel gain after applying BD. In [5], we proposed an optimum iterative algorithm for computing the powers given in (8). Finally, the resulting data rate of user k is

$$R_k^\star = \sum_{i=1}^{n_{S_k}}\log_2\left(1 + \frac{1}{\sigma^2}p_{i,k}^\star\lambda_{i,k}^2\right). \qquad (9)$$

So far we have presented the resource allocation strategy that is to be applied at a given frame. However, as commented before, we consider a transmission throughout T consecutive frames. As a consequence, the BS has to solve the allocation problem (6) T times and perform an update of the batteries according to (1), in order to compute the value of $R_{\max,k}$ needed for the constraint $C3$. The time-evolving algorithm considering the allocation strategy and the updates is presented in Table I. In the next section, we will characterize the asymptotic temporal behavior of such algorithm.

IV. Asymptotic Analysis

In this section we will study the asymptotic behavior of the rates and the batteries of the users considering that the transmission is sufficiently long to attain convergence. The values of the data rates and the battery levels at each particular frame are obtained from algorithm in Table I. Notice that, since the battery levels depend upon the harvesting process, which is a stochastic process, and the rates are obtained as the solution of problem (6), where some of the constraints

TABLE I
RESOURCE ALLOCATION AND BATTERY UPDATE

1:	**set** $t = 1$ and initial energies:
	$E_g^k(t) = \alpha_k C_k(t)$, $\forall k$
2:	**solve** optimization problem (6) and obtain:
	$R_k^\star(t), \mathbf{B}_k^\star(t)$, $\forall k$
3:	**set** battery level according to data rate used and harvesting:
	$C_k(t+1) = C_k(t) - T_f P_{\text{tot},k}(R_k(t)) + E_h^k(t)$, $\forall k$
4:	**update** energies and maximum rates of users:
	$E_g^k(t+1) = \alpha_k C_k(t+1)$, $\forall k$
	$R_{\max,k}(t+1) = P^{-1}_{\text{dec},k}\left(\frac{E_g^k(t+1)}{T_f} - P_c^{rx}\right)$, $\forall k$
5:	**set** $t = t+1$ and go to step 2

are given by $R_{\max,k}$, (and, therefore, depend on the battery levels), then, both the battery levels and the rates are also stochastic processes.

In this work and in our previous work [6], we consider the harvesting to be stationary and ergodic. If fact, we model the harvesting as a discretized energy packet arrival process, where each energy packet contains a finite amount of Joules. This behavior can be modeled by means of a Bernoulli process (or, in a more general sense, through a Markov chain [9]). Therefore, the values of $E_h^k(t)$ over frames are i.i.d., with only two possible outcomes at a given frame, i.e., $E_h^k(t) \in \{0, E_k\}$, where the outcomes 0 and E_k are obtained with probability $1 - p_k$ and p_k, respectively, being E_k the amount of Joules contained in an energy packet. However, as we will show later, even if the harvesting is stationary, that does not imply that the batteries are stationary. In any case, we are not interested in the stationarity of the process, but we seek to characterize the mean convergence[1] of the rates and the batteries. Given that, let us present the following result:

Lemma 1. *In the steady state regime (i.e., as $t \to \infty$), we have that $\lim_{t\to\infty} \mathbb{E}[R_k(t)] = \varphi_k$, where φ_k is a constant such that $0 \le \varphi_k < \infty$.*

Intuition behind the proof: The idea behind the proof is to note that the rates $R_k(t)$ are the solution of the optimization problem (6) where there is a maximum power constraint, $C1$, that implies that $R_k(t) < \infty$ and, therefore, $\mathbb{E}[R_k(t)] < \infty$. Moreover, as the rates depend on the harvesting process, which is stationary, $\mathbb{E}[R_k(t)]$ will not oscillate with time (see section VI). A more formal proof of the convergence is out of the scope of this paper. ∎

Thus, from previous lemma, we will assume throughout the paper that the rates converge in mean. Of course, the instantaneous value of $R_k(t)$ depends on the current channel and battery level and, thus, $R_k(t)$ will show some random fluctuations throughout time. Unfortunately, the mean convergence of the battery depends upon several parameters, not just the radiated power and the harvesting. As a consequence, there is not a simple relation between the convergence of the rates and the batteries. Before analyzing the battery convergence, let us present another interesting result:

Lemma 2. *The stochastic variable $R_{\max,k}(t)$ can only converge to the allocated rates, $\lim_{t\to\infty} \mathbb{E}[R_{\max,k}(t)] = \lim_{t\to\infty} \mathbb{E}[R_k(t)]$, or diverge $\lim_{t\to\infty} \mathbb{E}[R_{\max,k}(t)] = \infty$.*

Proof: We develop the proof for one particular user k, but it can be extended to the rest of the users. Let us consider that the rate has converged to a given constant, $\mathbb{E}[R_k(t)] = \varphi$. Let us assume that there is a gap between $\mathbb{E}[R_{\max,k}(t)]$ and $\mathbb{E}[R_k(t)]$, i.e., $\mathbb{E}[R_{\max,k}(t)] - \mathbb{E}[R_k(t)] = \kappa$, where $\kappa > 0$ (notice that a negative value of κ is not possible due to constraint $C3$). Then, since $\mathbb{E}[R_{\max,k}(t)]$ is constant, it implies that the battery has also converged, i.e., $\mathbb{E}[C_k(t+1)] = \mathbb{E}[C_k(t)]$. Hence, from (1), we have that $\mathbb{E}[T_f P_{\text{tot},k}(R_k(t))] = \mathbb{E}[E_h^k(t)]$, but since $\mathbb{E}[R_k(t)] < \mathbb{E}[R_{\max,k}(t)]$, constraint $C3$ is not active and the optimum data rates are the ones obtained from the classical water-filling policy. However, as the rates computed from the water-filling have no relation with the harvesting process, the probability of such event is 0, which leads to a contradiction. Thus, $\mathbb{E}[R_{\max,k}(t)]$ may decrease until convergence with $\mathbb{E}[R_k(t)]$ or diverge. ∎

Based on the same principle presented in the previous lemma, we are able to define three regions of different asymptotic behaviors that are based on the battery convergence. Let us assume that the algorithm is already in steady state and that the rates have converged, i.e., $t \to \infty$. Then, we define the following three regions:

Definition 1. (Region 1 - R1) *This region is defined such that $\mathbb{E}[T_f P_{tot,k}(R_k(t))] < \mathbb{E}[E_h^k(t)]$, $\forall k$. In such case, the batteries keep increasing as t increases, i.e., $\mathbb{E}[C_k(t+1)] > \mathbb{E}[C_k(t)]$ and, thus, $\lim_{t\to\infty} \mathbb{E}[C_k(t)] = \infty$, $\forall k$. In fact, $\lim_{t\to\infty} \mathbb{E}[R_{\max,k}(t)] = \infty$, and, as a consequence, the rates $R_k(t)$ are only limited by the transmission power P_T. This means that the optimum rates are obtained by means of classical water-filling policy given by $R_k^\star = \sum_{i=1}^{n_{S_k}} \log_2\left(c\frac{\lambda_{i,k}^2}{\sigma^2}\right)^+$, where c fulfills $\sum_{i,k}\left(c - \frac{\sigma^2}{\lambda_{i,k}^2}\right)^+ = P_T$. However, in reality, the batteries have a finite size and, in this case, they would grow until they reach their maximum capacity.*

Definition 2. (Region 3 - R3) *This region is defined such that $\mathbb{E}[T_f P_{tot,k}(R_k(t))] = \mathbb{E}[E_h^k(t)]$, $\forall k$, which means that all the batteries have converged to a finite value. As expressed in Lemma 2, the batteries may increase or decrease until $\mathbb{E}[R_{\max,k}(t)]$ converges to $\mathbb{E}[R_k(t)]$. This is the most interesting region since it captures the behavior of a battery-limited network.*

Let us present the following result concerning region 3:

Lemma 3. *Let the harvesting intensity be finite, i.e., $\mathbb{E}[E_h^k(t)] < \infty$ and let the algorithm be in steady state ($t \to \infty$). If $P_T \to \infty$, then $\lim_{t\to\infty} \mathbb{E}[R_k(t)] = \lim_{t\to\infty} \mathbb{E}[R_{\max,k}(t)]$.*

Proof: The proof follows directly from Lemma 2 and Definition 2. ∎

Definition 3. (Region 2 - R2) *This region is an intermediate region where $\mathbb{E}[T_f P_{tot,k}(R_k(t))] < \mathbb{E}[E_h^k(t)]$ may be true for some users and $\mathbb{E}[T_f P_{tot,k}(R_k(t))] = \mathbb{E}[E_h^k(t)]$ may be true for other users. Hence, some users will experience a battery divergence, while others will experience a battery convergence. This means that, within this region, there are users behaving as being in R1 and others in R3.*

The three regions are represented in Fig. 1 and Fig. 2. The setting of the simulation is: $T = 400$ for which we observe that the rates have already converged, the decoder power consumption model is linear with constant $\nu_k = 15000$ (see (10) in section V), $\alpha_k = 0.1$, the harvesting intensity is

[1] Throughout the paper, we will say indistinctly mean convergence and convergence but, formally speaking, we refer to mean convergence if not stated otherwise.

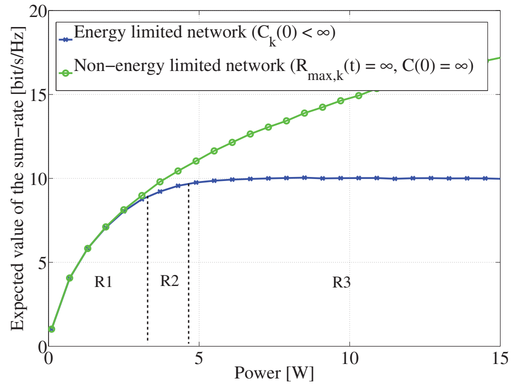

Fig. 1. Aggregated expected value of the rates $\lim_{t\to\infty} \sum_k \mathbb{E}[R_k(t)]$.

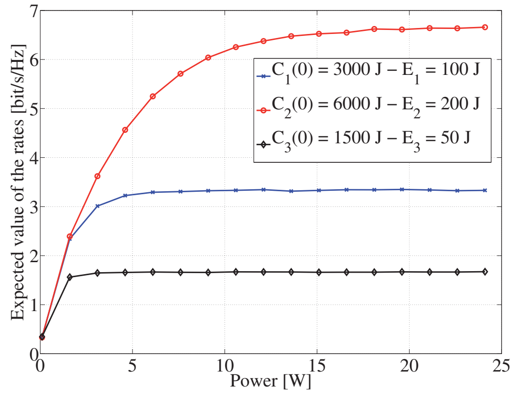

Fig. 3. Expected value of the rates with users with different harvesting.

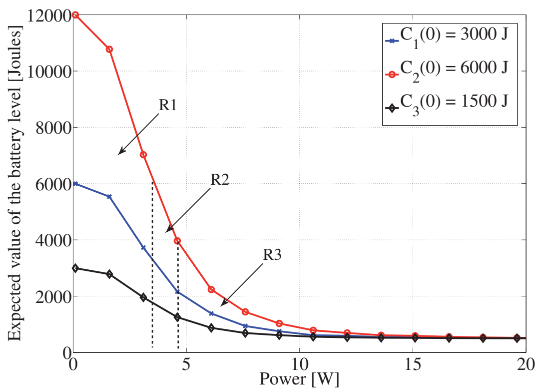

Fig. 2. Expected value of the individual batteries.

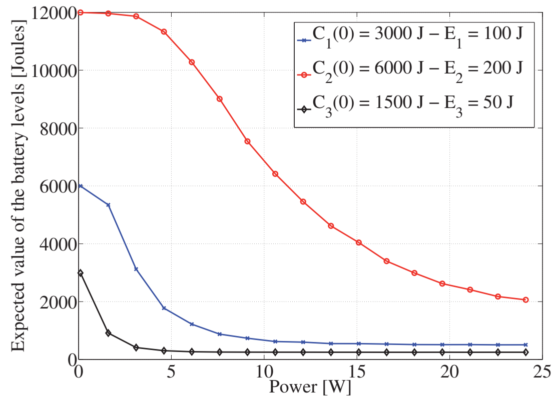

Fig. 4. Expected value of the batteries with users with different harvesting.

$p_k = 0.5$, the energy packet is $E_k = 100$ Joules, and there are three users with initial batteries $C_1(0) = 3000$ J, $C_2(0) = 6000$ J, and $C_3(0) = 1500$ J [5]. The channel matrices are generated randomly with i.i.d. entries distributed according to $\mathcal{CN}(0, 1)$. In Fig. 1, the green curve represents a system where the initial batteries are infinite, i.e., $R_{\max,k}(t) = \infty$, $\forall t, k$. (classical water-filling policy). The blue curve results from the application of the algorithm from Table I. As we can see, in R1 the optimum expected rates are the same as the ones obtained from classical water-filling, which means that the network is limited by the radiated power and not by the energy available at the batteries of the receivers. Fig. 2 depicts the evolution of the expected value of the batteries. Notice that the batteries in R1 should diverge, as stated in Definition 1, but as the number of simulated frames is $T = 400$, the obtained battery levels are finite.

From the definitions of the regions, we see that the thresholds between regions depend on the harvesting of the users. In the previous two figures we considered that the users were provided with the same energy harvesting source. Fig. 3 and Fig. 4 depict the expected value of the rates and the batteries where users are provided with different energy harvesting sources. By considering different harvesting sources among users (different energy packet sizes E_k), the three regions have been modified accordingly.

V. Analytic Characterization

In the previous section, we studied the asymptotic analysis of the algorithm in Table I. We presented under what conditions the rates and the batteries converge and we discussed the different asymptotic behaviors that are possible in these networks. In addition, in this section we characterize the asymptotic behavior analytically for a concrete decoding power consumption model. This would allow us to determine what the values of the rate and the battery level would be as a function of the initial conditions, the harvesting capabilities, the transmitted power, and the value of α_k.

As we discussed before, in R1 the batteries diverge and the rates are the ones obtained from classical water-filling policy, regardless of the decoder being used. Therefore, we only have to characterize R3, as R2 is a combination of R1 and R3 with an intermediate behavior.

In our previous works, we considered two different models for the decoder consumption function $P_{\text{dec},k}(R_k(t))$: a linear model and an exponential model. The motivations behind these models can be found in [5]. In the following, we derive some expressions for the asymptotic values and for the specific decoding model.

A. Linear decoder consumption model

The linear decoder consumption can be modeled as

$$P_{\text{dec},k}(R_k(t)) = \nu_k R_k(t), \tag{10}$$

where ν_k models the decoder efficiency. Let us present some interesting asymptotic results concerning this model.

Lemma 4. *The expected value of the data rate in convergence, $\lim_{t\to\infty} \mathbb{E}[R_k(t)]$, does not depend on α_k or on the initial battery level $C_k(0)$.*

Proof: since we are in R3, the battery has converged as $t \to \infty$, i.e., $\mathbb{E}[C_k(t+1)] = \mathbb{E}[C_k(t)]$. Then, $\mathbb{E}[T_f P_{\text{tot},k}(R_k(t))] = \mathbb{E}[E_h^k(t)]$ holds. By just replacing the model introduced before, we end up with

$$\begin{aligned} \lim_{t\to\infty} \mathbb{E}[R_k(t)] &= P_{\text{dec},k}^{-1}\left(\frac{\mathbb{E}[E_h^k(t)] - P_c^{rx} T_f}{T_f}\right) \\ &= \frac{p_k E_k - P_c^{rx} T_f}{\nu_k T_f}, \end{aligned} \tag{11}$$

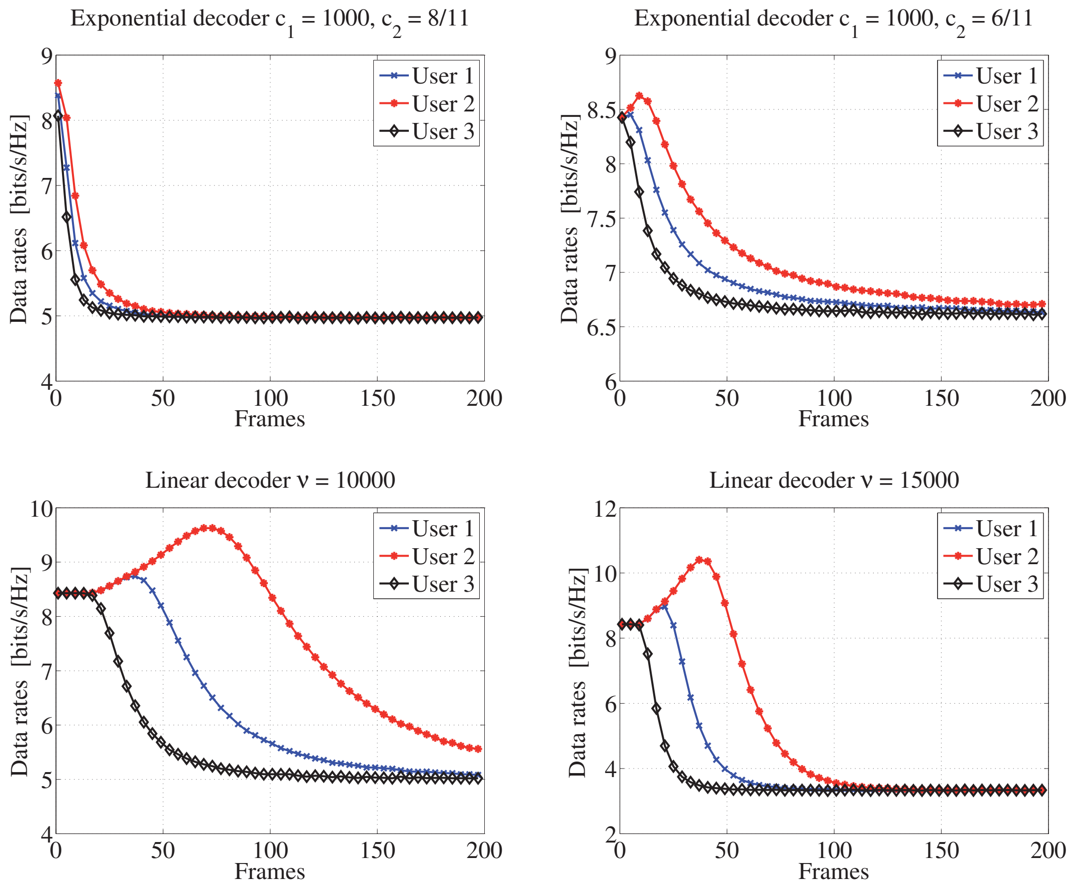

Fig. 5. Evolution of the data rates for different type of decoder power consumption models.

and this concludes the proof. ∎

In case that the users are provided with the same harvesting source and the same decoder, they end up with the same expected rate. Then, the expected sum rate in convergence is just $\lim_{t\to\infty} \mathbb{E}[\mathrm{SR}(t)] = \lim_{t\to\infty} \mathbb{E}\left[\sum_{k=1}^{K} R_k(t)\right] = \frac{K(pE-P_c^{rx}T_f)}{\nu_k T_f}$. Now, let us present a result concerning the value of the battery:

Lemma 5. *The expected value of the battery in convergence,* $\lim_{t\to\infty} \mathbb{E}[C_k(t)]$, *does not depend on the initial battery level* $C_k(0)$.

Proof: as we are in R3, $\mathbb{E}[R_k(t)] = \mathbb{E}[R_{\max,k}(t)]$ holds. From Lemma 4 and (5), we obtain the value of the battery in convergence:

$$\lim_{t\to\infty} \mathbb{E}[C_k(t)] = \frac{\mathbb{E}[E_h^k(t)]}{\alpha_k} = \frac{p_k E_k}{\alpha_k}, \tag{12}$$

which concludes de proof. ∎

Interestingly, the value of the battery in convergence only depends on the harvesting source and the value of α_k. Thus, the total battery reduction from initial conditions considering the whole transmission would be $\sum_{k=1}^{K} C_k(0) - \sum_{k=1}^{K} \frac{p_k E_k}{\alpha_k}$.

B. Exponential decoder consumption model

The exponential decoder consumption can be modeled as

$$P_{\mathrm{dec},k}(R_k(t)) = c_{1k} \mathrm{e}^{c_{2k} R_k(t)}, \tag{13}$$

where c_{1k} and c_{2k} model the decoder efficiency. Let us present some interesting asymptotic results concerning this model.

Lemma 6. *The expected value of the data rate in convergence,* $\lim_{t\to\infty} \mathbb{E}[R_k(t)]$, *does not depend on the initial battery level* $C_k(0)$.

Proof: since we are in R3, the battery has converged, i.e., $\mathbb{E}[C_k(t+1)] = \mathbb{E}[C_k(t)]$. Then, $\mathbb{E}[T_f P_{\mathrm{tot},k}(R_k(t))] = \mathbb{E}[E_h^k(t)]$ holds, which proves that the final rate does not depend on the initial battery value. In addition, we can find an upper-bound for the final average rate. Since $P_{\mathrm{dec},k}(R_k(t))$ is a convex function, we can apply Jensen's inequality[2] [10] and obtain the following upper bound on the expected rate:

$$\begin{aligned} \mathbb{E}[R_k(t)] &\leq P_{\mathrm{dec},k}^{-1}\left(\frac{\mathbb{E}[E_h^k(t)] - P_c^{rx}T_f}{\nu_k T_f}\right) \\ &\leq \frac{1}{c_{2,k}} \ln\left(\frac{p_k E_k - P_c^{rx}T_f}{c_{1,k}T_f}\right), \end{aligned} \tag{14}$$

which concludes the proof. ∎

Unfortunately, we can not provide an exact analytic result for the value of the battery in convergence if the receivers' decoding consumption is modeled with the exponential function.

VI. Simulation Results

In this section we present some numerical results that support the analytic results derived in the previous sections. For the simulations, we consider a scenario with a BS with $n_T = 6$ antennas and $K = 3$ receivers with $n_{R_k} = 2$ antennas each. The front-end power consumption at the receiver is $P_c^{rx} = 0.2$ W. The maximum available power at the BS is $P_T = 5$ W. We assume a normalized noise

[2]Jensen's inequality states that if X is a random variable and φ is a convex function, then $\varphi(\mathbb{E}[X]) \leq \mathbb{E}[\varphi(X)]$.

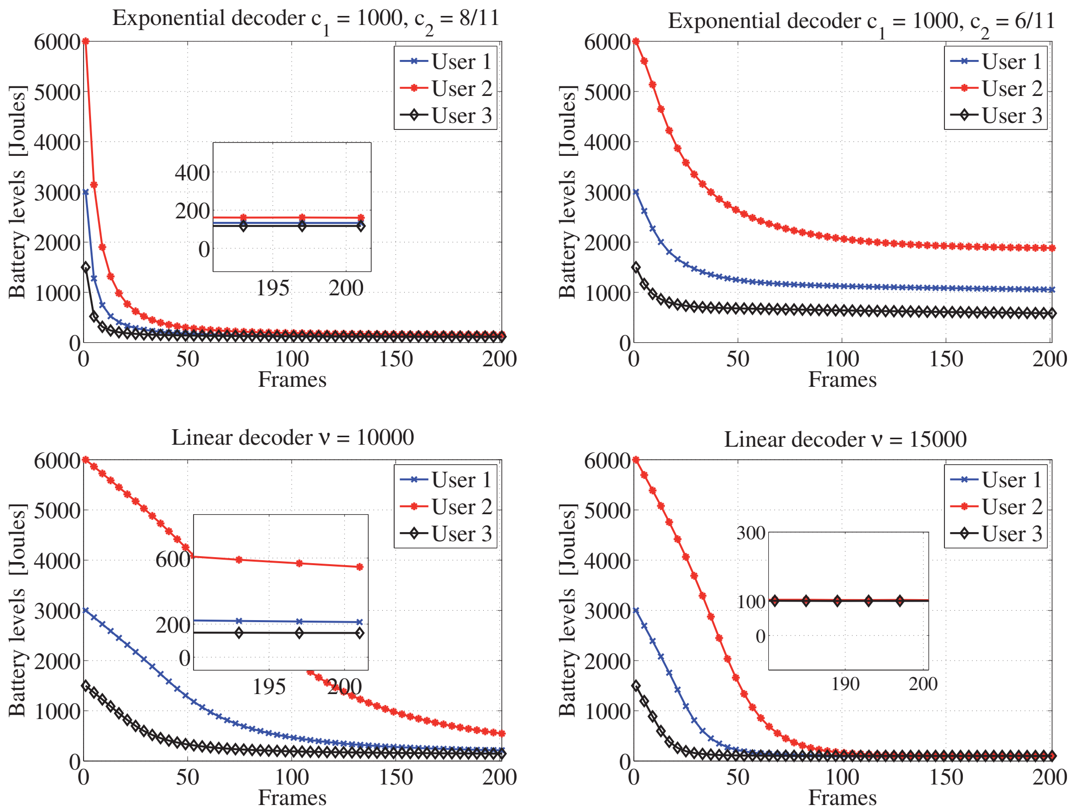

Fig. 6. Evolution of the battery levels for different type of decoder power consumption models.

power given by $\sigma^2 = 1$ W and a frame duration equal to $T_f = 1$ ms. For simplicity, the probability of energy packet arrival is $p_k = 0.5, \forall k$, the energy packet size is $E_k = 100$ J (all users are provided with the same energy harvesting source) and $\alpha_k = 0.5, \forall k$. This configuration yields to a battery-limited scenario, i.e., all users lie in region 3. The channel matrices are generated randomly with i.i.d. entries distributed according to $\mathcal{CN}(0,1)$. The initial battery levels are 6000, 3000, and 1500 Joules. The type of decoder and the decoder efficiencies are denoted on the title of the figures. All figures are averaged over 1000 channel and 1000 harvesting realizations.

Fig. 5 shows the evolution of the data rates. As we can see, for a given number of frames, the convergence time depends on the specific decoder and the decoder efficiency. If we compute the asymptotic expected rate using equation (11) for the linear decoder with the corresponding simulation parameters, we obtain $\lim_{t\to\infty} \mathbb{E}[R_k(t)] = 5$ bits/s/Hz and $\lim_{t\to\infty} \mathbb{E}[R_k(t)] = 3.3$ bits/s/Hz for the efficiencies $\nu_k = 10000$ and $\nu_k = 15000$, respectively. We are able to verify that result from the corresponding figures. Focusing on the exponential decoder, if we compute the upper bound given by (14), we obtain $\lim_{t\to\infty} \mathbb{E}[R_k(t)] \leq 5.3$ bits/s/Hz and $\lim_{t\to\infty} \mathbb{E}[R_k(t)] \leq 7$ bits/s/Hz, so there is approximately 0.3 bits/s/Hz of difference with the value obtained in the figures.

Fig. 6 presents the evolution of the battery levels for the same decoders and decoder efficiencies. We can also verify the analytic result presented in (12) for the linear decoder, which is $\lim_{t\to\infty} \mathbb{E}[C_k(t)] = 100$ J.

VII. Conclusions

In this paper, we have presented an asymptotic analysis of the behavior of a network where the mobile terminals have been considered to be battery-powered devices provided with energy harvesting sources. The asymptotic analysis has been based on a resource allocation strategy presented in a previous work. In such work, the key point was that the battery status of the users was taken into account explicitly in the proposed policy. In this paper, we have obtained some analytic expressions for the asymptotic values of the data rates and the batteries that the resource allocation policy is able to provide for sufficiently long transmissions under some harvesting conditions and the specific decoder power consumption.

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