



Universidad
Carlos III de Madrid



This document is published in:

Proceedings VTC Fall 2012 (IEEE Vehicular Technology Conference) (2012) pp. 1-5

DOI: 10.1109/VTCFall.2012.6399091

© 2012 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Resource Allocation between Feedback and Forward MIMO Links and Energy Consumption

Daniel Sacristán-Murga*, Antonio Pascual-Iserte^{†*}, and Víctor P. Gil Jiménez[‡]

*Centre Tecnològic de Telecomunicacions de Catalunya (CTTC), Spain

[†]Dept. of Signal Theory and Communications - Universitat Politècnica de Catalunya (UPC), Spain

[‡]Dept. of Signal Theory and Communications - Universidad Carlos III de Madrid, Spain

Email: daniel.sacristan@cttc.es, antonio.pascual@upc.edu, vgil@tsc.uc3m.es

Abstract—This work deals with the problem of radio resource allocation in a point-to-point multiple-input multiple-output (MIMO) communications system with feedback of channel state information. A total pool of radio resources (power and transmission time) is allocated among training, feedback, and data transmission phases in order to maximize system performance in terms of throughput. The energy consumption associated to the base band signal processing and decoding is also evaluated. This resource allocation problem is studied analytically in a two-way time division duplex (TDD) communications scenario with time correlated channels, where the effect of feedback errors and feedback delay is also taken into account.

Index Terms—MIMO systems, feedback communication, limited feedback, resource allocation, energy consumption.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems require channel state information (CSI) at both the transmitter and the receiver in order to optimize the performance [1].

The CSI is usually obtained at the receiver with the help of pilot symbols. In non-reciprocal channels, the CSI is then sent to the transmitter through a feedback link. There is a wide range of literature on the benefits of this scheme, and on the different techniques used to maximize the performance depending on the quality and accuracy of the CSI available at the transmitter. Such quality depends on the accuracy of the channel estimation at the receiver, the errors that result from the feedback transmission, and the delay due to the estimation and feedback processes.

In this paper we consider a two-way MIMO communication link with feedback, where two users communicate between each other following a time division duplex (TDD) scheme with given transceiver and feedback design criteria. Under these conditions, the accuracy of the CSI at the transmitter depends on: (i) the power and duration of the training phase devoted to channel estimation at the receiver, (ii) the power and duration of the feedback phase related to the quantization of such channel estimate, (iii) the errors produced in the feedback communication, and (iv) the delays associated to the CSI estimation and feedback transmission. The communication performance depends not only on the accuracy of the CSI at the transmitter, but also on the resources allocated to the data transmission phase. In this sense, a tradeoff exists, since if more resources are allocated to training and feedback, then the accuracy of the CSI increases, but less resources are available for transmission of data. The contribution of this paper is a formulation of such tradeoff taking into account

all the parameters associated to the radio resource allocation. An optimization problem is presented in terms of power and duration associated to the training, feedback, and data transmission phases. Furthermore, the effect of the tradeoff on the base band energy consumption is also analyzed.

A similar analysis was conducted in [2] for the case of frequency division duplexing (FDD), where a block fading model was assumed instead of the time variant model considered here. Furthermore, in [2] CSI symmetry was necessary, while in this paper it is not required. The work in [3] also studied the allocation tradeoff, but it was done using a simplified system model and without considering training nor feedback errors. Additionally, this paper introduces the issue of energy consumption in the base band [4], which is analyzed in terms of its variation with respect to the resource allocation.

The remainder of this paper is organized as follows. The system and signal models are described in section II. Section III presents the general resource allocation problem. The energy consumed in the base band is introduced and modeled in section IV. Finally, sections V and VI provide numerical simulations and conclusions, respectively.

II. SYSTEM AND SIGNAL MODELS

Let us consider a flat fading MIMO channel with 2 half duplex users that communicate between each other using a TDD scheme.¹ The propagation channels from user 1 and user 2 are denoted by $\mathbf{H}_1 \in \mathbb{C}^{N_2 \times N_1}$ and $\mathbf{H}_2 \in \mathbb{C}^{N_1 \times N_2}$, respectively, where $N_i, i = \{1, 2\}$, denotes the number of antennas of user i . The channel is modeled as a first-order Gauss-Markov process such that at time instant $n + 1$ the channel response matrix associated to user i is given by

$$\mathbf{H}_i(n+1) = \rho \mathbf{H}_i(n) + \sqrt{1 - \rho^2} \mathbf{N}_i(n), \quad (1)$$

where matrices $\mathbf{H}_i(0)$ and $\mathbf{N}_i(n), \forall n$ are independent and composed of i.i.d. zero-mean complex and circularly symmetric Gaussian entries with unit variance. Consequently, also the components of $\mathbf{H}_i(n)$ follow the same distribution. The time correlation factor ρ models the variability of the channel and depends on the Doppler frequency f_D caused by the movement of the users according to the Jakes' model, $\rho = J_0(2\pi f_D \tau)$ [5], where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, and τ corresponds to the channel instantiation interval.

¹Note that, although the expressions in this paper consider only the TDD scheme, an equivalent derivation for FDD can be obtained straightforwardly. We considered only the case of TDD for the sake of clarity in the notation.

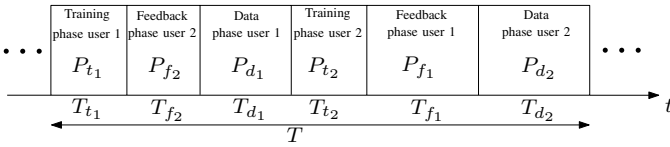


Fig. 1. TDD communication phases for one block of T channel uses.

The propagation channel model, which includes the effect of the RF chains with, for example, the high power amplifiers and the mixers, is not expected to be reciprocal due to the non-reciprocity of such RF chains. Consequently, feedback of the CSI is required.² The communication in each direction over a block of T channel uses is modeled as having 3 phases: a training phase, where one user sends pilot symbols that are used by the other user to estimate the channel; a feedback phase, where the estimated CSI is fed back to the other user; and the data transmission phase, where the CSI is used to design the precoder for the data transmission. The power and duration for each user and phase are denoted as depicted in Fig. 1, which also shows the structure of the communication scheme. The considered phase ordering minimizes the CSI delays at each side of the communication. However, other phase orderings could be considered without loss of generalization. Also, the notation could be adapted to the case of full-duplex FDD transmission just by changing the duration of the phases by their bandwidths. In the following, we describe in detail each phase.

A. Training phase and channel estimation

The channel estimation results in an estimate given by

$$\mathbf{H}_{e_i} = \mathbf{H}_i - \Delta_{e_i}; \quad \mathbf{H}_i = \mathbf{H}_{e_i} + \Delta_{e_i}, \quad (2)$$

where Δ_{e_i} is the channel estimation error. Using a minimum mean squared error (MMSE) estimator and an orthogonal training sequence, the channel estimation error Δ_{e_i} corresponding to user i is Gaussian distributed (when at least N_i channel uses are employed for the training phase) [6], where each element is i.i.d. with zero mean and variance $\sigma_{e_i}^2$ and independent from \mathbf{H}_{e_i} . The variance of the noise during the estimation process is related to the variance of the channel, with a proportionality factor that depends on the SNR during the estimation process for a unit transmission power, denoted as $\text{SNR}_{h_i} = \sigma_{n_i}^{-2}$, where $\sigma_{n_i}^2$ is the Gaussian noise power at the receiver when transmitting from side i to the other side. This leads to a variance of the estimation error given by [6]

$$\sigma_{e_i}^2 = \frac{\text{SNR}_{h_i}^{-1}}{1 + \frac{P_{t_i} T_{t_i}}{N_i}}. \quad (3)$$

B. Feedback phase

There are three factors that degrade the accuracy of the CSI when sent through the feedback link. First, feedback introduces a delay. Besides, the CSI has to be quantized before the feedback transmission introducing an error that depends on the quantization strategy employed. Also, there may be transmission errors during the feedback phase. In the following, we model in detail these three sources of error.

²Another possibility is to calibrate the RF chains, which is an expensive and technologically complex process that includes additional hardware at both transmitter and receivers and high quality RF chains, which increases the cost of the terminals significantly. Currently, such calibration is not considered in conventional terminals and, thus, has not been assumed in this paper.

1) *Delay error*: Since in the system model considered the channel is slowly time varying, there is an additional source of uncertainty in the channel estimation available for the transmitter design. This is due to the delay between the transmission of the training symbols and the use of the CSI at the transmitter. The CSI error Δ_{d_i} due to the delay for a given delay μ is described by the following equation:

$$\mathbf{H}_i(n) = \mathbf{H}_i(n - \mu) + \Delta_{d_i}(\mu). \quad (4)$$

We model \mathbf{H}_{e_i} and \mathbf{H}_i as the estimated and actual channels of user i in the middle of the training and data transmission phases, respectively. Consequently, we have that the delay between each acquisition of the channel estimate and the posterior use of this channel estimate in the data transmission phase is $\mu_{d1} = \frac{T_{t1}}{2} + T_{f2} + \frac{T_{d1}}{2}$ for user 1 and $\mu_{d2} = \frac{T_{t2}}{2} + T_{f1} + \frac{T_{d2}}{2}$ for user 2. Considering the same model for the delay in the CSI used during the feedback phase, we have that the delay between each acquisition of the channel estimate and the posterior use of this channel estimate for the design of the precoder in the feedback transmission is $\mu_{f1} = \frac{T_{t1}}{2} + T_{f2} + T_{d1} + T_{t2} + \frac{T_{f1}}{2}$ for user 1 and $\mu_{f2} = \frac{T_{t2}}{2} + T_{f1} + T_{d2} + T_{t1} + \frac{T_{f2}}{2}$ for user 2.

2) *Quantization error*: It has been shown in [7], [8] that the minimum necessary information for the design of the optimum linear precoder for the usual design criteria is contained in the channel Gram matrix defined as $\mathbf{R}_i = \mathbf{H}_i^H \mathbf{H}_i$ for user i ; therefore, in the following, quantization and feedback of the Gram matrix will be assumed, as done in [9], [10], [11]. This means that only a quantized version of the estimated channel Gram matrix \mathbf{R}_{e_i} , denoted by \mathbf{R}_{eq_i} , is available at user i . The quantization introduces a quantization error Δ_{q_i} to the estimated CSI, as modeled by the following equation:

$$\mathbf{R}_{eq_i} = \mathbf{R}_{e_i} + \Delta_{q_i}, \quad (5)$$

with $\mathbf{R}_{e_i} = \mathbf{H}_{e_i}^H \mathbf{H}_{e_i}$.

The quantization error depends also on the specific quantization scheme used. Since comparing different quantization schemes is not the focus of this paper, and for the sake of simplicity in the notation, we assume a uniform quantization of the real and imaginary parts of each element independently as is done for example in [10]. Since the matrix is Hermitian and, for user i , it has size $N_i \times N_i$, there are N_i^2 different real elements to be quantized (the real and imaginary parts of the m, n th element of the matrix, $\forall m < n$ and the real part of the N_i elements of the diagonal). We apply a uniform quantization to the real and imaginary parts using a quantization step ϵ_{q_i} , where $\epsilon_{q_i} = \frac{\gamma_i}{2^{q_i}}$, q_i is the number of bits, and γ_i is the dynamic range of the quantizer, that is fixed so that overflows in the quantization occur with a probability lower than 0.99. Since there are N_i^2 real elements to be quantized by the receiver, the total number of required quantization bits is given by $n_{b2} = q_1 N_1^2$ at user 2 and $n_{b1} = q_2 N_2^2$ at user 1.

3) *Transmission errors in the feedback link*: We will model the transmission errors in the feedback link through an outage probability p_{o_i} for the feedback link from user i . In the event of feedback error, the fed back CSI is incorrect and the achievable performance cannot be guaranteed and, therefore, since this

paper is focused on the evaluation of a lower bound of the worst-case performance, it will be assumed to be zero.

During the feedback phase the design of the precoder for feedback transmission is done according to the imperfect CSI available at the user generating such feedback, which corresponds to the estimated, quantized, and delayed channel Gram matrix. This means that, if single beamforming is considered for the feedback transmission,³ the precoding vector for the feedback transmission phase of user i , \mathbf{b}_{f_i} , is chosen as the eigenvector associated to the largest eigenvalue of the estimated, quantized, and delayed channel Gram matrix, denoted by $\mathbf{R}_{\text{edq}_i}^{(\mathbf{f})}$ and formulated in what follows in this subsection. We compute the estimated and delayed propagation matrix $\mathbf{H}_{\text{ed}_i}^{(\mathbf{f})}(n)$ for the feedback communication as

$$\mathbf{H}_i(n - \mu_{f_i}) \triangleq \mathbf{H}_{\mathbf{e}_i}(n - \mu_{f_i}) + \mathbf{\Delta}_{\mathbf{e}_i}(n - \mu_{f_i}); \quad (6)$$

$$\mathbf{H}_{\text{ed}_i}^{(\mathbf{f})}(n) \triangleq \mathbf{H}_{\mathbf{e}_i}(n - \mu_{f_i}) = \rho^{-\mu_{f_i}} \mathbf{H}_i(n) + \rho^{-\mu_{f_i}} \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}(n), \quad (7)$$

where $\mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}(n) = -\left(\rho^{\mu_{f_i}} \mathbf{\Delta}_{\mathbf{e}_i}(n - \mu_{f_i}) + \sqrt{1 - \rho^{2\mu_{f_i}}} \tilde{\mathbf{N}}_i(n)\right)$ has i.i.d. Gaussian elements with zero mean and variance $\sigma_{\text{ed}_i}^2 = 1 + \rho^{2\mu_{f_i}}(\sigma_{\mathbf{e}_i}^2 - 1)$, and $\tilde{\mathbf{N}}_i(n)$ is composed of i.i.d. Gaussian elements with zero mean and unit variance and is derived recursively based on the model presented in (1).

The estimated and delayed channel Gram matrix, prior to the quantization, is given by

$$\begin{aligned} \mathbf{R}_{\text{ed}_i}^{(\mathbf{f})}(n) &\triangleq \mathbf{H}_{\text{ed}_i}^{(\mathbf{f})H}(n) \mathbf{H}_{\text{ed}_i}^{(\mathbf{f})}(n) \\ &= \rho^{-2\mu_{f_i}} \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})H}(n) \mathbf{H}_i(n) + \rho^{-2\mu_{f_i}} \mathbf{H}_i^H(n) \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}(n) \\ &\quad + \rho^{-2\mu_{f_i}} \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})H}(n) \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}(n) + \rho^{-2\mu_{f_i}} \mathbf{R}_i(n), \end{aligned} \quad (8)$$

where $\mathbf{R}_i(n) = \mathbf{H}_i^H(n) \mathbf{H}_i(n)$. And, finally, the CSI available at user i for feedback transmission, which is the estimated, delayed, and quantized channel Gram matrix, is given by

$$\mathbf{R}_{\text{edq}_i}^{(\mathbf{f})}(n) \triangleq \mathbf{R}_{\text{ed}_i}^{(\mathbf{f})}(n) + \mathbf{\Delta}_{\mathbf{q}_i}(n). \quad (9)$$

From (8) and (9) it follows that:

$$\begin{aligned} \mathbf{R}_i(n) &= \rho^{2\mu_{f_i}} \mathbf{R}_{\text{edq}_i}^{(\mathbf{f})}(n) - \mathbf{H}_i^H(n) \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}(n) - \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})H}(n) \mathbf{H}_i(n) \\ &\quad - \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})H}(n) \mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}(n) - \rho^{2\mu_{f_i}} \mathbf{\Delta}_{\mathbf{q}_i}(n). \end{aligned} \quad (10)$$

The feedback SNR is formulated as $\text{SNR}_{f_i}(n) \triangleq P_{f_i} \text{SNR}_{h_i} \mathbf{b}_{f_i}^H \mathbf{R}_i(n) \mathbf{b}_{f_i}$. With the phase ordering considered in this paper and depicted in Fig. 1, \mathbf{b}_{f_i} is equal to the precoder used in the previous data transmission phase of the corresponding user. The temporal index n will be omitted from now on for clarity reasons. Consequently, SNR_{f_i} can be expressed as

$$\text{SNR}_{f_i} \triangleq P_{f_i} \text{SNR}_{h_i} \mathbf{b}_{f_i}^H \mathbf{R}_i \mathbf{b}_{f_i} = P_{f_i} \text{SNR}_{h_i} (A_{f_i} + B_{f_i} + C_{f_i} + D_{f_i}),$$

where A_{f_i} , B_{f_i} , C_{f_i} and D_{f_i} are defined and lower bounded

³Note that the single beamforming precoding strategy is considered here for simplicity, but any other precoding strategy could also have been used within the proposed model for the feedback transmission.

as shown next⁴ for user 1 (and similarly for user 2):

$$A_{f_1} \triangleq \rho^{2\mu_{f_1}} \mathbf{b}_{f_1}^H \mathbf{R}_{\text{edq}_1}^{(\mathbf{f})} \mathbf{b}_{f_1} = \rho^{2\mu_{f_1}} \lambda_{\max}\{\mathbf{R}_{\text{edq}_1}^{(\mathbf{f})}\}, \quad (11)$$

$$\begin{aligned} B_{f_1} &\triangleq -\mathbf{b}_{f_1}^H \left(\mathbf{H}_1^H \mathbf{\Delta}_{\text{ed}_1}^{(\mathbf{f})} + \mathbf{\Delta}_{\text{ed}_1}^{(\mathbf{f})H} \mathbf{H}_1 \right) \mathbf{b}_{f_1} \\ &\geq -2\sqrt{\epsilon_{\text{ed}_1} \lambda_{\max}\{\mathbf{R}_1\}}, \end{aligned} \quad (12)$$

$$\begin{aligned} C_{f_1} &\triangleq -\mathbf{b}_{f_1}^H \left(\mathbf{\Delta}_{\text{ed}_1}^{(\mathbf{f})H} \mathbf{\Delta}_{\text{ed}_1}^{(\mathbf{f})} \right) \mathbf{b}_{f_1} \\ &\geq -N_2 \epsilon_{\text{ed}_1}^2 \left(1 + \sqrt{N_1(N_1 - 1)} \right), \end{aligned} \quad (13)$$

$$\begin{aligned} D_{f_1} &\triangleq -\rho^{2\mu_{f_1}} \mathbf{b}_{f_1}^H \mathbf{\Delta}_{\mathbf{q}_1} \mathbf{b}_{f_1} \\ &\geq -\rho^{2\mu_{f_1}} \epsilon_{q_1} \left(\frac{N_1 - 1}{N_1} + \sqrt{\frac{N_1(N_1 - 1)}{2}} \right), \end{aligned} \quad (14)$$

where the Gaussian distributed error $\mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}$ is within the sphere of radius $\|\mathbf{\Delta}_{\text{ed}_i}^{(\mathbf{f})}\|_F \leq \sqrt{\epsilon_{\text{ed}_i}}$ with a probability p_{g_i} . Since this paper is focused on the evaluation of a lower bound of the worst-case performance, we will consider that when the error is out of this sphere, the system is in outage and the performance is zero. Note that by considering a lower bound for B_{f_i} and C_{f_i} independently we are dealing with a bound of the worst-case.

We consider that the system is in feedback outage when the achievable throughput of the feedback link is lower than the number of feedback bits to be transmitted. Consequently, the outage probability of the feedback sent by user i using a bandwidth W is given by

$$p_{o_i} = p(T_{f_i} W \log_2(1 + P_{f_i} \text{SNR}_{h_i} \mathbf{b}_{f_i}^H \mathbf{R}_i \mathbf{b}_{f_i}) < n_{b_i}). \quad (15)$$

C. Data transmission phase

In this phase the design of the precoder is carried out according to the imperfect CSI available at the transmitter, which corresponds to the estimated, quantized, and delayed channel Gram matrix. This means that the single beamforming precoding vector for the data transmission phase of user i , \mathbf{b}_{d_i} , is chosen⁵ as the eigenvector associated to the largest eigenvalue of the CSI available at the transmitter (the estimated, quantized, and delayed channel Gram matrix), which is denoted by $\mathbf{R}_{\text{edq}_i}^{(\mathbf{d})}$. Note that the CSI available at user i at the time of the data transmission, as well as the lower bound for SNR_d can be formulated following the same steps described for the CSI available at the time of feedback transmission in (6)-(14), only by replacing the indices f with d . The complete derivation is not reproduced again for space reasons.

III. PROBLEM STATEMENT

The objective is to optimize a generic cost function g that measures the system performance given a total frame length T and a total energy constraint per user, E_1 and E_2 , respectively.

⁴The derivations that result in the bounds from (11)-(14) are not included due to space constraints, however a similar derivation is explained in [3] for a more simplified problem.

⁵As in the feedback phase, a single beamforming precoding strategy is considered here for simplicity, but any other precoding strategy could also be considered within the proposed model.

This can be expressed mathematically as:

$$\max g \left(\{T_{t_i}, T_{f_i}, T_{d_i}, P_{t_i}, P_{f_i}, P_{d_i}, \epsilon_{ed_i}, q_i\}_{i=\{1,2\}} \right) \quad (16)$$

$$\text{s.t.} \quad T_{t_1} + T_{f_1} + T_{d_1} + T_{t_2} + T_{f_2} + T_{d_2} = T \quad (17)$$

$$T_{t_i} P_{t_i} + T_{f_i} P_{f_i} + T_{d_i} P_{d_i} = E_i; \quad i = \{1, 2\} \quad (18)$$

$$\epsilon_{ed_i} > 0; \quad i = \{1, 2\} \quad (19)$$

$$q_i \in \mathcal{N}; \quad i = \{1, 2\} \quad (20)$$

$$\text{w.r.t.} \quad \{T_{t_i}, T_{f_i}, T_{d_i}, P_{t_i}, P_{f_i}, P_{d_i}, \epsilon_{ed_i}, q_i\}_{i=\{1,2\}}. \quad (21)$$

We choose as cost function g the worst-case average two-way achievable communication rate with single beamforming.⁶

Therefore we have that, considering a block length of T time instants ($n = 1, \dots, T$) as depicted in Fig. 1 and a bandwidth W , the average throughput for user 1 is given by

$$g_1 = \frac{T_{d_1}}{T} W \mathbb{E}_{\mathbf{H}_1, \mathbf{H}_2} \left\{ (1 - p_{o_2}) \log_2 \left(1 + P_{d_1} \text{SNR}_{h_1} \mathbf{b}_{d_1}^H \mathbf{R}_1^{(d)} \mathbf{b}_{d_1} \right) \right\},$$

where $\mathbf{R}_1^{(d)}$ is the channel Gram matrix of the channel from user 1 during the data transmission phase, i.e., \mathbf{R}_1 at time instant $n = T_{t_1} + T_{f_2} + \frac{T_{d_1}}{2}$, $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation, and p_{o_2} is the probability of outage in the feedback from user 2 given by

$$p_{o_2} = p \left(T_{f_2} W \log_2 \left(1 + P_{f_2} \text{SNR}_{h_2} \mathbf{b}_{f_2}^H \mathbf{R}_2^{(f)} \mathbf{b}_{f_2} \right) < n_{b_2} \right), \quad (22)$$

with $n_{b_2} = q_1 N_1^2$ and $\mathbf{R}_2^{(f)}$ corresponding to the channel from user 2 during the feedback transmission phase, i.e., \mathbf{R}_2 at time instant $n = T_{t_1} + \frac{T_{f_2}}{2}$.

Following an equivalent development for user 2 results in expressions for the average throughput of user 2, g_2 , and the probability of outage in the feedback from user 1, p_{o_1} .

Observe that, in the transmission model described in Fig. 1, the design of the transceiver for the data transmission phase of each user is performed with the same CSI as the design of the transceiver for the following feedback transmission phase from the same user. This means that, if the same transceiver architecture/design criterion is considered for both the transmission of data and feedback, then the resulting transceiver for the data transmission phase and the following feedback transmission phase of each user is the same, i.e., $\mathbf{b}_{f_1} = \mathbf{b}_{d_1}$ and $\mathbf{b}_{f_2} = \mathbf{b}_{d_2}$. Note that if a different phase ordering is considered, this could be adapted accordingly.

We will optimize a lower bound of the total two-way communication rate defined as $g = g_1 + g_2$. An analytical optimization of this problem is extremely complex and, in the simulations section, a numerical optimization is performed in which the tradeoff can be clearly observed.

IV. ENERGY CONSUMPTION IN THE BASE BAND

The signal processing and decoding required at the receiver to process the received signals also requires a relevant amount of energy [4], which has not been included in the formulation of (16)-(20). At the transmitter this effect is not so important because the computational complexity is lower and the energy

⁶The worst-case average achievable communication rate is chosen as an example of cost function since it is frequently used to measure system performance. However, other cost functions such as packet error rate or SNR could also be considered following the same procedure described in this paper.

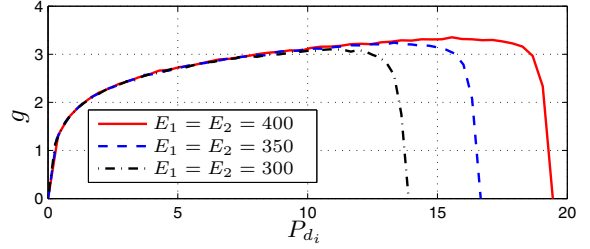


Fig. 2. Power allocation between training and data transmission phases.

consumption can be assumed negligible [4]. The consumption at the receiver depends greatly on the specific hardware used and is usually modeled in other works such as [4] as an exponential function of the communication rate.

In this paper we will study the energy required for the base band signal processing given the optimization problem presented in section III. For this purpose the energy consumed in the base band of user i is modeled as:

$$E_{bb_i} = T_{d_j} c_1 c_3^{c_2 R_j}, \quad (23)$$

where the constants c_1, c_2 and c_3 are decoder specific, and R_j is the instantaneous transmission rate during user's j data transmission phase lower bounded by $R_j = g_j \frac{T}{T_{d_j}}$.

V. SIMULATIONS

In this section the performance of the resource allocation is analyzed numerically for different scenarios. The following parameters are considered in the simulations: $\rho = 0.9999$, $N_1 = 3$, $N_2 = 3$ antennas, a normalized bandwidth $W = 1$, ϵ_{ed_i} such that estimation plus delay error for the data transmission phase is within the sphere of radius $\sqrt{\epsilon_{ed_i}}$ with probability $p_{g_i} = 0.7$, ϵ_{q_i} such that there is no quantization overflow in 99% of the cases, $T = 60$, and a total energy constraint per user of $E_1 = 400$ and $E_2 = 400$, respectively.

A. Computation of ϵ_{q_i} and the dynamic range of the quantizer

For the computation of ϵ_{q_i} we consider a system with $T_{t_1} = T_{t_2} = 2$, $P_{t_1} = P_{t_2} = 25$, $\text{SNR}_{h_1} = \text{SNR}_{h_2} = 10$, and $q_1 = q_2 = 4$ quantization bits per element (i.e., $n_{b_1} = n_{b_2} = 32$ bits). The numerical simulations averaged over 80000 channel realizations show that, in order to have a maximum overflow of 1%, the dynamic range of the quantizer is $\gamma_1 = \gamma_2 = 13.827$, which corresponds to $\epsilon_{q_1} = \epsilon_{q_2} = 0.8642$.

B. Tradeoff between feedback and data transmission energy

In this subsection the allocation of energy between the feedback and data transmission phases is studied in the scenario considered in the previous simulations and with a fixed training phase power $P_{t_1} = P_{t_2} = 25$. Fig. 2 shows the system performance g as a function of the power dedicated to the data transmission. Note that if no power is dedicated to data transmission then the performance is zero, and also if all the power is dedicated to data transmission and nothing is used for the transmission of feedback then there is feedback outage and the performance is also zero.

As shown in Fig. 2, for the case of $E_1 = E_2 = 400$, the optimum allocation is achieved with $P_{d_1} = P_{d_2} = 14.78$, which corresponds to $P_{f_1} = P_{f_2} = 8.396$. This results in a total energy used in the feedback phase of each user

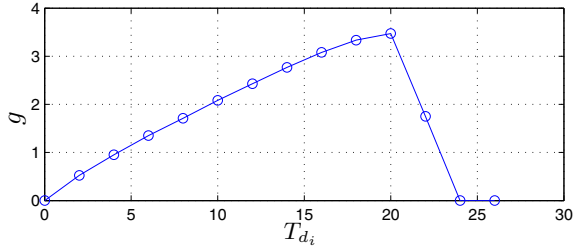


Fig. 3. Time allocation between training and data transmission phases.

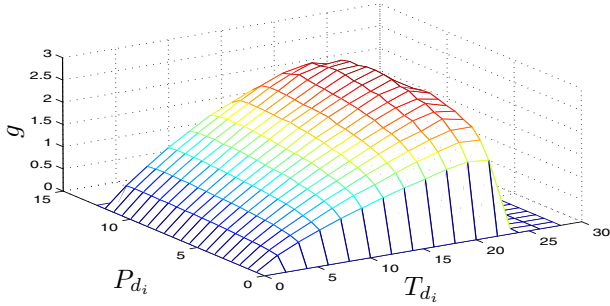


Fig. 4. Joint time and power allocation between training and data transmission phases.

of $P_{f_i} T_{f_i} = 83.96$, and a total energy used in the data transmission phase of each user of $P_{d_i} T_{d_i} = 266.04$.

C. Tradeoff between feedback and data transmission duration

In this subsection the energy allocation between phases is fixed: $P_{t_i} T_{t_i} = 50$, $P_{f_i} T_{f_i} = 83.96$, and $P_{d_i} T_{d_i} = 266.04$. Furthermore, the duration of the training phase is set to $T_{t_1} = T_{t_2} = 2$. In this setup the allocation of time between feedback and training phases is considered, with the additional constraint that $T_{f_1} = T_{f_2}$ and $T_{d_1} = T_{d_2}$. Numerical simulations were conducted and the result is represented in Fig. 3.

D. Joint optimization of feedback and data transmission

In this subsection only the training phase is fixed in advance, with $T_{t_1} = T_{t_2} = 2$ and $P_{t_1} = P_{t_2} = 25$, while both the power and duration of the feedback and data transmission phase are optimized. The result of the simulations is represented in Fig. 4. Note that the previous two subsections are contained in this figure as cuts in the y-axis and x-axis.

E. Energy consumed in the base band

This subsection evaluates the energy consumed in base band, $E_{bb} = E_{bb_1} + E_{bb_2}$, versus the power allocated to the data transmission phase in the considered scenario following the model presented in section IV. For the sake of simplicity, the decoder specific constants considered are $c_1 = c_2 = 1$ and $c_3 = 2$. Note that, since the simulations consider normalized values, the shape of the resulting curve is more relevant than the particular values obtained. The result of the numerical simulations performed is represented in Fig. 5. It can be observed that the resource allocation that maximizes the performance is also very demanding in terms of E_{bb} .

VI. CONCLUSIONS

This work presented an analysis of the resource allocation between the data and the feedback links of a MIMO communication system and the associated base band energy

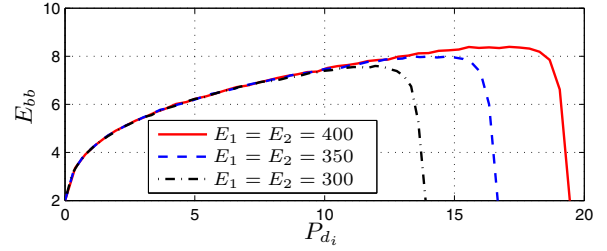


Fig. 5. Energy consumed in base band as a function of power allocated to the data transmission phase.

consumption. It is shown that, since resources for the feedback transmission come at a cost of resources for the data transmission, there is an optimum resource allocation strategy that maximizes system throughput. A natural and very interesting extension of this work would be to consider the energy consumed in the base band as part of the optimization problem. This could be implemented by modifying the constraint in (18) and leads to an even more challenging problem.

ACKNOWLEDGEMENTS

The research leading to these results has received funding from the European Cooperation in Science and Technology under project COST Action IC0902, from the Spanish Ministry of Economy and Competitiveness (Ministerio de Economía y Competitividad) under projects TEC2011-29006-C03-02 (GRE3N-LINK-MAC) and TEC2011-29006-C03-03 (GRE3N-SYST), and from the Catalan Government under grant 2009 SGR 891.

REFERENCES

- [1] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, Jun. 2003.
- [2] C. K. Au-Yeung and D. J. Love, "Optimization and tradeoff analysis of two-way limited feedback beamforming systems," *IEEE Trans. on Wireless Communications*, vol. 8, no. 5, pp. 2570–2579, May 2009.
- [3] D. Sacristán-Murga, A. Pascual-Iserte, and P. Tradacete, "Resource allocation between feedback and forward links: Impact on system performance and CSI," in *Proc. 19th European Signal Processing Conference (EUSIPCO)*, Aug. 2011, pp. 824–828.
- [4] P. Rost and G. Fettweis, "On the transmission-computation-energy tradeoff in wireless and fixed networks," in *Proc. IEEE Globecom'10*, Dec. 2010, pp. 1394–1399.
- [5] R. Steele and L. Hanzo, Eds., *Mobile Radio Communications*, 2nd ed. John Wiley & Sons, 1999.
- [6] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple antenna wireless links?" *IEEE Trans. on Information Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.
- [7] M. Payaró and D. P. Palomar, "On optimal precoding in linear vector Gaussian channels with arbitrary input distribution," in *Proc. IEEE International Symposium on Information Theory (ISIT'09)*, Jul. 2009, pp. 1085 – 1089.
- [8] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Trans. on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sep. 2003.
- [9] F. A. Monteiro and I. J. Wassell, "Recovery of a lattice generator matrix from its gram matrix for feedback and precoding in MIMO," in *Proc. 4th International Symposium on Communications, Control and Signal Processing (ISCCSP)*, Mar. 2010.
- [10] C.-B. Chae, D. Mazzarese, N. Jindal, and R. W. Heath, "Coordinated beamforming with limited feedback in the MIMO broadcast channel," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1505–1515, Oct. 2008.
- [11] D. Sacristán-Murga, M. Payaró, and A. Pascual-Iserte, "Transceiver design framework for multiuser MIMO-OFDM broadcast systems with channel gram matrix feedback," *accepted for publication in IEEE Trans. on Wireless Communications*, 2012.