UNIVERSIDAD CARLOS III DE MADRID working papers

UC3M Working Papers
Statistics and Econometrics
15-14
ISSN 2387-0303
June 2015

Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-48

ON THE IMPORTANCE OF THE PROBABILISTIC MODEL IN IDENTIFYING THE MOST DECISIVE GAME IN A TOURNAMENT

Francisco Corona[a], Juan de Dios Tena[b] and Mike Wiper[a]

Abstract

Identifying the decisive matches in international football tournaments is of great relevance for a variety of decision makers such as organizers, team coaches and/or media managers. This paper addresses this issue by analyzing the role of the statistical approach used to estimate the outcome of the game on the identification of decisive matches on international tournaments for national football teams. We extend the measure of decisiveness proposed by Geenens (2014) in order to allow to predict or evaluate the decisive matches before, during and after a particular game on the tournament. Using information from the 2014 FIFA World Cup, our results suggest that Poisson and kernel regressions significantly outperform the forecasts of ordered probit models. Moreover, we find that although the identification of the most decisive matches is independent of the model considered, the identification of other key matches is model dependent. We also apply this methodology to identify the favorite teams and to predict the most decisive matches in 2015 Copa America before the start of the competition. Furthermore, we compare our forecast approach with respect to the original measure during the knockout stage.

Keywords: *Decisive game, Entropy, Poisson model, Kernel regression, Ordered probit model.*

[a] Department of Statistics, Universidad Carlos III de Madrid.
[b] Management School, University of Liverpool.

On the Importance of the Probabilistic Model in Identifying the Most Decisive Games in a Tournament

Francisco Corona[1], Juan de Dios Tena[2] and Michael P. Wiper[1]

[1]Department of Statistics, Universidad Carlos III de Madrid
[2]Management School, University of Liverpool

July 2016

Abstract

Identifying the decisive matches in international football tournaments is of great relevance for a variety of decision makers such as organizers, team coaches and/or media managers. This paper addresses this issue by analyzing the role of the statistical approach used to estimate the outcome of the game on the identification of decisive matches on international tournaments for national football teams. We extend the measure of decisiveness proposed by Geenens (2014) in order to allow to predict or evaluate the decisive matches before, during and after a particular game on the tournament. Using information from the 2014 FIFA World Cup, our results suggest that Poisson and kernel regressions significantly outperform the forecasts of ordered probit models. Moreover, we find that although the identification of the most decisive matches is independent of the model considered, the identification of other key matches is model dependent. We also apply this methodology to identify the favorite teams and to predict the most decisive matches in 2015 Copa America before the start of the competition. Furthermore, we compare our forecast approach with respect to the original measure during the knockout stage.

Keywords: Decisive game, Entropy, Poisson model, Kernel regression, Ordered probit model.

1 Introduction

International football competitions are events of a great social and economic interest. In particular, the World Cup, which takes place every four years, is the most widely viewed and followed sporting event in the world. Other competitions at continental level, such as the European Cup and the Copa America, have an important impact in the countries involved, where many people stop their usual daily activities when their teams are playing. Therefore, assessing game decisiveness in the competition is of great relevance for organizers, team coaches and media managers, as well as other interested parties.

The concept of decisiveness of a game has a long tradition in the sports economics literature, see for example Schilling (1994), Audas et al. (2002), Scarf and Shi (2008), Goossens et al. (2012), among many others. A highly insightful, critical discussion of this issue, as well as the presentation of a new indicator of game decisiveness that overcomes some of the most important drawbacks of these previous approaches can be found in Geenens (2014). Under Geenens' approach, a game can be considered as decisive if it has a significant impact on the whole tournament entropy instead of focusing only on the effect on the probabilities assigned to the different possible results of a single game. Although the evaluation of match decisiveness in a tournament depends on the probability model considered, this issue has not been properly explored.

Starting from Moroney (1956), many models for predicting football results have been developed. A number of approaches, stemming from Maher (1982) concentrate on predicting the scores of the individual teams in a match based on Poisson regression, see e.g. Dyte and Clarke (2000) and Suzuki et al. (2010). A recent and relevant contribution in this field was proposed by Groll et al. (2015) who, as we also do in this article,fit separate Poisson regression for goals scored and conceded in football World Cup competitions. Our approach shares some similarities with this paper although Groll et al. (2015) focuses on forecasting match results whereas our main interest is not on forecasting but on identifying decisive matches.

In contrast to the Poisson based models, alternative approaches have been developed to try to predict the result of a football match (win, draw or loss for a given team), instead of the goals scored. Important examples of models of this type are probit regression, see Kuypers (2000) and Scarf and Shi (2008) or kernel regression, Geenens (2014). In this article we shall also consider these models and compare them to the Poisson based approaches in terms of their predictions of match decisiveness.

This paper studies the identification of decisive matches in an international football tournament at different stages of the competition. Although having early access to this information is especially useful for a decision maker, its estimation requires considering a large combination of game fixtures which cannot be analytically treated in many instances. Two additional contributions of this paper to the existing literature are that, as far as we are aware, this is the first paper that evaluates the implications of the econometric model on the identification of decisive matches and check how this evaluation depends on the forecasting period. If model specification matters, the analysis of games should only be considered under the most accurate statistical framework. We compare a number of variants of the Poisson, probit and kernel based models which incorporate both Bayesian and classical estimation approaches. Also we allow the possibility of taking unobserved, heterogeneous effects for different groups of games into account. Additionally, the forecasts of the matches are carried out modifying the ability of the teams using a Canonical Correlation Analysis (CCA) maximizing the correlation between the FIFA rating with a new set of variables.

Our evaluation of the forecasting performance of the different models in WC2014 indicates that Poisson model and kernel regression significantly outperform the probit model. An advantage of Poisson regression models is that these are based on a much richer information set (goals scored and conceded, venue effect, etc.) and can implement tie-break criteria such as goal difference, as they take into account goals scored by each team, which cannot be accounted for by kernel or probit regression approaches. The selection of the forecasting model has important implications for the determination of decisive matches in the competition, above all, in the identification of matches of intermediate relevance. Additionally, the time information used in the forecast model has a very important effect. We also apply this methodology to identify the key matches and the favorites to win the 2015 Copa America (CA2015).

The rest of this article is structured as follows. The following Section presents the main groups of models we use to forecast football results. Then, in Section 3 we explain the concept of match decisiveness used in this article following from Geenens (2014) and we consider the measure for the forecasting case. The estimation of the different groups of models and a comparison of the forecasting performance follows in Section 4. We identify the most decisive games for the WC2014 and CA2015 under the different models in Section 5. Some concluding remarks follow in Section 6.

2 Probabilistic models for predicting football results

Here we briefly describe some of the most popular statistical models for predicting football results.

2.1 Poisson model

Poisson models have been successfully used in the sports literature for football results prediction. In particular, Dixon and Coles (1997) used Poisson regression to model results in the English Premier League from 1992 to 1995 and Dyte and Clarke (2000) modeled the 1998 FIFA World Cup using an approach which is similar to that which we outline below. Also, Suzuki et al. (2010) used Bayesian methods to predict the results of the 2006 Football World Cup using expert information.

Here, we consider a sample of K games, so that, the number of goals scored by team, T, against an opposing team, O, in game k, is Poisson distributed, $y_{T,k} \sim \text{Poisson}(\lambda_{T,k})$, with mean parameter, $\lambda_{T,k}$, for $k = 1, ..., K$, which follows the *log-linear* dependence relationship below:

$$\log(\lambda_{T,k}) = \beta_0 + \beta_{A_T} x_{A_T,k} + \beta_{A_O} x_{A_O,k} + \beta_{H_T} x_{H_T,k} + \beta_{N_T} x_{N_T,k}, \tag{1}$$

where $x_{A_T,k}$ represents the "ability" of team T, $x_{A_O,k}$ is the ability of the opposing team, $x_{H_T,k}$ indicates if team T plays at home and $x_{N_T,k}$ if they play at a neutral ground. The parameters β_{A_T}, β_{A_O}, β_{H_T} and β_{N_T} are coefficients that express the relationship between the explanatory variables and $\lambda_{T,k}$ and β_0 is a constant term. Equation (1) is called the *log link-function* and the parameters can be estimated by maximum likelihood estimation (MLE); see e.g. Winkelmann (2000), Hilbe (2014) for reviews of the literature on Poisson regressions. Henceforth, we denote this model by PO.

A Bayesian counterpart of this model can be defined by assuming a normal prior distribution for the regression coefficients as follows

$$\beta \sim \text{N}(\mu_\beta, V_\beta), \tag{2}$$

where $\beta = (\beta_0, \beta_{A_T}, \beta_{O_T}, \beta_{H_T}, \beta_{N_T})'$, μ_β is the prior mean and V_β is the prior variance. Estimation for this model is carried out generating a sample from the posterior parameter distribution using a random walk Metropolis algorithm; see Martin et al. (2011). We denote this model by BP.

Finally, we also consider a hierarchical Bayesian Poisson model (HBP) to account for the heterogeneity of the different games [1] by defining the following, mixed link-log function

$$\log(\lambda_{T,k_i}) = X_{T,k_i}\beta + \tilde{X}_{T,k_i} b_{T,k_i} + \varepsilon_{T,k_i}, \tag{3a}$$

$$b_{T,k_i} \sim \mathrm{N}_q(0, V_b), \tag{3b}$$

where the λ_{T,k_i}, and the random error, ε_{T,k_i}, distributed $\mathrm{N}_p(0, \sigma^2 I_{K_i})$, are vectors of games with length K_i, for $i = 1, \ldots, g$, where g is the number of formed groups or nestings. The matrix of covariates is $X_{T,k_i} = (X_{A_{T,k_i}}, X_{O_{T,k_i}}, X_{H_{T,k_i}}, X_{N_{T,k_i}})$ and β is normally distributed as in equation (2), measuring the fixed effects. On the other hand, the design matrix is $\tilde{X}_{T,k_i}$ and b_{T,k_i} measures subject-specific random effects. Note that this vector captures marginal dependence among the observations on the i^{th} unit. In practice, we assume $\tilde{X}_{T,k_i} = X_{T,k_i}$, so that we have $g =$ two groups of games corresponding to official competitions and friendlies respectively. We might reasonably expect differences between these two categories, as friendly games are often taken less seriously than competition games by the participating teams and are used, for example to try out new players, whereas in serious games, the strongest teams are generally selected. The hierarchical dependence is completed supposing $\sigma^2 \sim$ Inverse-Gamma$(\upsilon, 1/\delta)$ and $V_b \sim$ Inverse-Wishart(u, uU) as (semi-conjugate) priors. Estimation for this model is carried out by generating a sample from the posterior parameter distribution via Markov Chain Monte Carlo (MCMC) techniques following Chib and Carlin (1999).

We obtain the win, draw and loss probabilities for team T against team O in game k, say $p_{W_T,k}$, $p_{D,k}$ and $p_{L_T,k}$ respectively, following the procedure given by Dyte and Clarke (2000) and Suzuki et al. (2010).

Following Dyte and Clarke (2000) and Suzuki et al. (2010), we can obtain the win, draw and loss probabilities, say $p_{W_T,k}$, $p_{D,k}$ and $p_{L_T,k}$ respectively for team T

[1] Note that the mean in the Poisson models also can be expressed as a function of the teams. We use this notation in terms of the games, k, for convenience.

against team O in game k as:

$$p_{W,k} = \sum_{i_T=1}^{\infty} \sum_{i_O=1}^{i_T-1} P(y_{T,k} = i_T) P(y_{O,k} = i_O), \tag{4a}$$

$$p_{D,k} = \sum_{i_T=1}^{\infty} P(y_{T,k} = i_T) P(y_{O,k} = i_T), \tag{4b}$$

$$p_{L,k} = \sum_{i_O=1}^{\infty} \sum_{i_T=1}^{i_O-1} P(y_{T,k} = i_T) P(y_{O,k} = i_O), \tag{4c}$$

where i_T and i_O are all possible scored goals for each team, where $P(y_{T,k} = i_T)$ and $P(y_{O,k} = i_O)$[2] represent the Poisson probabilities of goals scored (with $\lambda_{T,k}$ and $\lambda_{O,k}$ as means) for each team.

Alternative, Poisson regression based models could also be considered. Firstly, one possibility is to use a zero-inflated model to account for a possibly larger numbers of games where a team does not score than would be expected under Poisson regression. Secondly, we might expect that numbers of goals scored by the two opposing teams in a game to be independent. This might suggest applying a correlated regression model following e.g. McHale and Scarf (2011). In our later examples, some brief comments on these models are given, although in our examples, there does not seem to be any clear evidence that they improve on the simpler approaches introduced above.

2.2 Ordered Probit model

Ordered probit (OP) models have been used to model football results by Audas et al. (2002) and Tena and Forrest (2007) among others. Scarf and Shi (2008) use classical OP models to evaluate game decisiveness in the English Premier League. In contrast to the Poisson models, an OP model directly estimates the win, draw and loss probabilities in a game.

The OP model is defined as follows. Let $P_k = 1$ $(0, -1)$ represent the event that team T wins (draws, loses) a game against opponent O in game k. Then following Scarf and Shi (2008) the match outcome, P_k, is modelled as:

$$P_k = \begin{cases} 1(\text{win}) & \text{if} \quad c_1 + \varepsilon_k \leqslant \beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k}, \\ 0(\text{draw}) & \text{if} \quad c_{-1} + \varepsilon_k < \beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k} \leqslant c_1 + \varepsilon_k, \\ -1(\text{loss}) & \text{if} \quad \beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k} < c_{-1} + \varepsilon_k, \end{cases} \tag{5}$$

[2]Note that by the HBP model the probabilities must be p_{W_T,k_i}, p_{D,k_i} and p_{L_T,k_i}, such that we have a $K_i \times 3$ matrix of probabilities given by $P = (p_{W,k_i}, p_{D,k_i}, p_{L,k_i})'$.

where $x_{A_D,k} = x_{A_T,k} - x_{A_O,k}$, is the "ability difference" between the teams T and O in match k, β_{A_D} is its associated coefficient, and $x_{H_T,k}$ and β_{H_T} are defined as in the Poisson regression models described previously. $c_1 + \varepsilon_k$ is a random cut-off point for winning with fixed component, c_1, and a random component, $\varepsilon_k \sim \mathrm{N}(0,1)$ and $c_{-1} + \varepsilon_k$ is a random cut-off point for losing. Therefore, P_k, can be expressed as a multinomial distribution with three categories given by

$$p_{W,k} = \Pr(P_k = 1) = \Phi(\beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k} - c_1), \tag{6a}$$

$$\begin{aligned} p_{D,k} = \Pr(P_k = 0) = \Phi(\beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k} - c_{-1}) - \\ \Phi(\beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k} - c_1), \end{aligned} \tag{6b}$$

$$p_{L,k} = \Pr(P_k = -1) = 1 - \Phi(\beta_{A_D} x_{A_D,k} + \beta_{H_T} x_{H_T,k} - c_{-1}), \tag{6c}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. We can estimate the parameters via maximum likelihood; see McCullagh (1980). This model will be denoted by OP.

A Bayesian extension of OP can be obtained using expression (2) as prior distribution but now for $\beta = (\beta_{A_D}, \beta_{H_T})$. Additionally we assume that $c_{-1} \sim \mathrm{N}(0, a_0)$ and $c_1 = c_{-1} + c_r$ where $c_r \sim \mathrm{Gamma}(a_1, a_1)$ as prior distributions of the cut-off parameters where $a_1 > a_0$. Inference can be carried out using MCMC techniques, using the approach presented by Lancaster (2014). This model will be denoted by BOP.

2.3 Kernel regression

In our final approach, following the notation of Geenens (2014), we estimate the win, draw and loss probabilities $p_{W,k}$, $p_{D,k}$ and $p_{L,k}$ nonparametrically using kernel regression (KE), as follows:

$$\begin{pmatrix} \hat{p}(\chi)_{W,k} \\ \hat{p}(\chi)_{D,k} \\ \hat{p}(\chi)_{L,k} \end{pmatrix} = \frac{\sum_{k=1}^{K} \kappa\left(\frac{\chi - x_{A_D,k}}{b}\right) \begin{pmatrix} Z_k^{(W)} \\ Z_k^{(D)} \\ Z_k^{(L)} \end{pmatrix} + \sum_{k=1}^{K} \kappa\left(\frac{\chi + x_{A_D,k}}{b}\right) \begin{pmatrix} Z_k^{(W)} \\ Z_k^{(D)} \\ Z_k^{(L)} \end{pmatrix}}{\sum_{k=1}^{K} \kappa\left(\frac{\chi - x_{A_D,k}}{b}\right) + \sum_{k=1}^{K} \kappa\left(\frac{\chi + x_{A_D,k}}{b}\right)}, \tag{7}$$

where $Z_k = (Z_k^{(W)}, Z_k^{(D)}, Z_k^{(L)})'$ is the vector of 0/1 indicator variables such that $Z_k^{(W)} + Z_k^{(D)} + Z_k^{(L)} = 1$, κ represents a Gaussian weight function and b is the bandwidth selected according to the criterion of Wand and Jones (1995). The ability difference, $x_{A_D,k}$, is defined as in the ordered probit models and χ is a grid of ability differences.

Geenens (2014) uses the KE model for predicting the results of the 2012 Euro Cup and note that $-x_{A_D,k}$ represents the inverse of the ability difference, which guarantees the symmetry in the estimation of the probabilities. This model only depends on the ability difference, however in order to make it comparable with the other, previously mentioned specifications, we estimate two different models: one in which there are only home and away teams and a second, in which there are only neutral teams.

3 Measuring game decisiveness

Here, we consider the approach developed by Geenens (2014) based on the *entropy* principle. In this way, we can define the *most decisive game* of a competition as "the game that has most influence in the eventual victory in the tournament".[3]

To formally define this idea, let $p_{jht} = \mathrm{P}(V_{jh}|\xi_t)$ be the final victory probability of the team j at time h conditional on (pre tournament games and the history of all matches played in the tournament up to time), t, say ξ_t, for $t \in \{0, 1, ..., h\}$. In our case, the final victory probabilities depend on t, such that, we can estimate them at any time before, or during game h . For simplicity of notation, assuming there are N teams in the tournament and $t = h$; after the time h, the entropy is given by:

$$e_h = -\sum_{j=1}^{N} p_{jh} \log_2 p_{jh}. \tag{8}$$

Minimum entropy or maximum information occurs when some $p_{jh} = 1$ while the others are 0 so that the result of the game is certain. In this case, $e_h = 0$. On the contrary, maximum entropy is when all probabilities p_{jh} are equal to $1/N$ so that there is maximum uncertainty[4]. In the following, we use capital letters to refer to the random entropy, $E_h = (e^{(W_h)}, e^{(D_h)}, e^{(L_h)})$. Then, $e^{(W_h)}$, $e^{(D_h)}$ and $e^{(L_h)}$ are the resulting values of the tournament entropy when the match outcome, h, has not

[3]Bojke (2007) and Koning (2007) propose an alternative definition of game decisiveness applied to the English and the Dutch leagues respectively that evaluates the importance of a match not only on the probability of obtaining the final victory but also on intermediate targets such as the probability of promotion in the English 1 or the probability of qualifying for different European tournaments in the Dutch league. However, in our particular case, this approach is not considered as it is difficult and subjective to define these intermediate targets for national teams in international competitions where only the final gets a prize.

[4]Note that we follow Lesne (2014) in setting the base of the logarithm to 2 in (8) and it is shown there that this formula inherits who usual mathematical properties of the Shannon entropy (based on the natural logarithm). Geenens (2014) normalizes the entropy to measure between 0 and 1 and uses a logarithm base equal to the number of teams in the competition. However, as the author mentions, this fact is not important.

been observed, such that we consider the three possible outcomes.

According to the definition of decisive game, we are interested in the game which most changes the entropy during the competition. Therefore, we propose the following measures for $h = 1, \dots, n$:

$$d_{t,h} = \mathbf{E}(|E_h - E_{h-1}||\xi_t) \quad \text{for} \quad t = 0, \dots, h-2, \tag{9a}$$

$$d_{h-1,h} = \mathbf{E}(|E_h - e_{h-1}||\xi_{h-1}), \tag{9b}$$

$$d_{h,h} = |e_h - e_{h-1}|. \tag{9c}$$

These measures are the forecast versions of those proposed by Geenens (2014) and show how e_h or E_h change given the history of previously played matches, which can include time h of the actual tournament. The main difference of these equations defined in (9) with respect to Geenens (2014) is that our decisiveness measure depends on t, i.e., the history of games which determines the estimation of the final victory probabilities. Computationally, this require a different estimation procedure for each case, which is described in subsection 4.2. In other words, the measures given by (9) indicate the absolute variation of the uncertainty in the final victory possibilities in the tournament as a consequence of the result in a single match, using information before, during and after time h of the competition. Explicitly, the calculation when $t = h - 1$ is as follows:

$$d_{h-1,h} = |e^{(W_h)} - e_{h-1}|\mathbf{P}(W_h|\xi_{h-1}) + |e^{(D_h)} - e_{h-1}|\mathbf{P}(D_h|\xi_{h-1}) + |e^{(L_h)} - e_{h-1}|\mathbf{P}(L_h|\xi_{h-1}),$$

where $e^{(W_h}$, $e^{(D_h)}$ and $e^{(L_h)}$ are the components of the random entropy vector previously defined in time h for team T given the information up to time $h-1$, so that, $\mathbf{P}(W_h|\xi_{h-1})$, $\mathbf{P}(D_h|\xi_{h-1})$ and $\mathbf{P}(L_h|\xi_{h-1})$ represent the win, draw and loss probabilities for time h. When $t = h$, we already know which games will be played and, therefore, we do not need to estimate random entropies and in this case we write d_h instead of $d_{h,h}$.

Note that $d_{t,h}$ is most sensitive when the teams at the time t have similar probabilities and high chances of final victory in the tournament or when the final result at time h is a surprise. Additionally, we expect that games in a decisive stage (final game of the group or knockout stage) have more impact in the change of the entropy. Interestingly, the proposed measure only depends on the probabilistic model so that matches that, in principle, do not attract the focus of the media could have a great impact on the probability of success of the other teams in the tournament.

4 Tournament prediction: 2014 FIFA World Cup

In this Section we present the tools and methods used for the prediction of the WC2014 for each one of the models described in Section 2.

4.1 Basic model and modified FIFA rating

The WC2014 took place at several venues across Brazil from 12 June - 13 July 2014, with 32 national teams competing in a total of 64 matches. Following FIFA rules, the traditional World Cup format consists of two rounds: a group stage and a knockout stage. The group stage is carried out by dividing the 32 teams into eight groups of four, where the members of each group compete among themselves in a round-robin tournament. The two highest finishing teams in each group advance to the knockout stage. Teams are awarded three points for a win and one for a draw. The tie-break rules are a) greatest number of points obtained in all group matches, b) goal difference in all group matches and c) greatest number of goals scored in all group matches. There are extra tie break rules, but in the simulation of WC2014 we do not consider them[5]. In the knockout stage there are four rounds (round of 16, quarter-finals, semi-finals, and the final), with the losing team eliminated at each stage.

To estimate the parameters of the models described in Section 2, we use information from $K = 821$ games played during the year before the WC2014. For the difference of ability measures, we consider the FIFA/Coca-Cola World Ranking[6] according to the points obtained at the time of the game. For the estimation of the BOP model, the difference in ability is scaled so that the ability difference has zero mean with unit variance, which is necessary to allow for convergence in the estimation, for more details, see Lancaster (2014).

Table 1 shows the estimation results for the models and we can observe that PO and BP present similar results, unlike HBP, where the results vary slightly given that we consider the heterogeneity between official games. In particular, (examining the signs of the model coefficients) it can be observed that playing at home is more advantageous than playing at a neutral ground. As this is a log-linear relationship, the marginal effect on the mean parameter $\lambda_{T,k}$ of the home effect is 0.49 while the neutral effect is 0.28 (PO model). The estimated signs of the model parameters are also as expected in the ordered probit model. In particular, there is a slightly

[5]See Article 41 of the Regulations, 2014 FIFA World Cup Brazil, downloadable in `http://resources.fifa.com/mm/document/tournament/competition/01/47/38/17/regulationsfwcbrazil2014_update_e_neutral.pdf`.

[6]`http://www.fifa.com/fifa-world-ranking/`

positive relationship in the difference of teams abilities and a strong positive effect of the home coefficient.

Table 1 about here

We also fitted the zero-inflated model, outlined at the end of Section 2.1, to this data set. In terms of significance the traditional PO model gave better results given than the zero-inflated model as in this second case, only the ability measures are significant at 10% while in the PO model, all coefficients are significant at 1% with the exception of the intercept. The adjusted R^2 values are very similar, 0.83 for the PO model and 0.82 for the zero-inflated model. More importantly, the sample proportion of teams scoring zero goals in the FIFA World Cup matches was around 33%, and the mean number of goals scored per team was 1.27. The simple probability of observing zero events in a Poisson (1.27) model is around 28% which is relatively similar to the observed proportion and does not suggest much evidence in favor of a zero inflated model.

In order to evaluate the independence assumption inherent in the basic Poisson model, we also calculated linear (Kendall) correlations for nine ability groups of games in our sample size following a suggestion of McHale and Scarf (2011). The results indicate that only in a one group, between the games where the teams have ability differences between -328 and -180 (T with respect to O), does there appear to be a significant correlation coefficient of 0.24 and p value of 0.01. Therefore, there seems to be little evidence of correlation overall and this suggests that it is reasonable to use the standard Poisson regression model.

Figure 1 plots the estimation results of the KE considering the home and neutral effects. The top panel presents the estimation when the teams played as local (and visitor) in a total of 593 games and, we can see how the win probability increases as the difference in FIFA rating increases. The counterpart is the loss probability. The draw probability increases when the difference in FIFA rating tends to zero. The bottom panel shows the results considering the 228 games played in neutral venues and it is interesting given that the draw probability increases if the difference in FIFA rating is weakly large. For example, in November 11, 2013, Argentina vs Ecuador had a FIFA rating of 1,266 and 862 respectively, the venue was New York, USA, and the final result was 1-1. Note that the difference in FIFA rating is 404. On the other hand, the game played between Norway and Poland in January 18, 2014 with venue Abu Dhabi, where the FIFA rating were of 558 and 461 respectively (i.e. a difference of 97 points), the final result was 0-3. This can imply that when the ability between the teams is weakly large, the weaker team takes a defensive strategy while if the ability difference is similar, the teams play with a more offensive system. Note that the home effect for the win and loss probabilities are almost indistinguishable with respect to the neutral venue case.

Figure 1 about here

FIFA rankings might not include all relevant information about national teams abilities. They have been criticised, for example by McHale and Davies (2007), for not giving more weight to recent information on past results. Moreover, the elaboration of this index requires the use of subjective elements to compare games from different national tournaments.

We make use of this index as it allows to compare our paper with two recent and relevant contributions to our research that also consider official rankings for teams based on past performance. In particular, Groll et al. (2015) and Geenens (2014) make use FIFA and the UEFA rankings as ability measures for the WC2014 and the UEFA Euro 2012 respectively. However, we also complement this variable with additional information from the betting market in the forecasting exercise, an additional ability measure, the current market value of the national teams and the historical behavior in the competition. Specifically, we use the expected payout by bookmakers of bet365[7] (19 May 2014), FIFA rating (June 2014), ELO rating[8] (June 2014), market value of the national teams[9] (June 2014), historical percentage (1930-2010)[10] (Reached points)/(Possible points).

Our new measure is obtained using CCA which is an exploratory statistical method to highlight correlations between two data sets acquired in the same experimental units; such that we calculate a new variable that maximizes the correlation between the linear combinations, a and b, of the FIFA rating (Y) and the rest of variables (X) respectively. The optimization problem is as follows

$$\rho_1 = \max_{a,b} \mathrm{cor}(Xa, Yb) \quad \text{subject to} \quad \mathrm{Var}(Xa) = \mathrm{Var}(Yb) = 1.$$

For more details see Leurgans et al. (1993) or Gonzalez et al. (2008), among many others. In this way we normalize each one of the variables previously described and the first canonical variable respect to FIFA rating is used to build the ability

[7] `http://www.bet365.com`

[8] `http://www.eloratings.net/world.html`

[9] `http://www.transfermarkt.com`

[10] `http://www.fifa.com/worldfootball/statisticsandrecords/tournaments/worldcup/alltimerankings.html`

measure.

Note that Dyte and Clarke (2000) manually adjust the FIFA rating to obtain more accurate predictions for the 1998 FIFA World Cup and Zeileis et al. (2014) generate a "log-ability" measure using information from 22 betting houses for the WC2014. Our alternative measure can be interpreted as the linear combination of variables that has the highest correlation with the FIFA rating.[11]

4.2 Tournament simulation

Koning et al. (2003) present an excellent survey about the implications of simulation models for football championships. Here we consider simulation of the tournament is carried out using 5,000 replicates of the competition for the models considered in Section 2. For PO, BP and HBP it is possible to generate two Poisson random variables for every game, and simulate the results of the entire tournament, see Dyte and Clarke (2000), Suzuki et al. (2010), among many others. For the OP and KE models we use the win, draw and loss probabilities in each game, k, randomly taking a possible result. An advantage of Poisson models is that we can implement the tie-breaker criteria commented in the previous subsection. For the other cases, when teams finish level on points at the top of a group, we randomly select the team to continue to the knockout stage. Explicitly, for each the PO model in each replication we estimate the ranking in each group, considering, for the Poisson regression models the traditional tie-breaker criteria: points, difference of goals and goals to favour. For the OP and KE models we consider the expected values of the points given by $\Pr_{\text{points},k} = 3P_{W,k} + P_{D,k}$. As a final tie-break criterion, random selection of a team is used when two (or more) teams are tied according to the previously commented criteria. In the knockout stage, we only considered the probability of continuing in the tournament, by splitting the draw probability equally between the teams equally. This corresponds to assuming that in the case of a draw, then, for example, there is a penalty shoot out where both teams have an equal probability of success.

We summarize the simulation procedure for the entropy computation as follows:

- When $0 \leq t < h-2$, we carry out 5,000 replicates and estimate e_0. We estimate the forecast $\mathbf{P}(W_h|\xi_t)$, $\mathbf{P}(D_h|\xi_t)$ and $\mathbf{P}(L_h|\xi_t)$ for a fixed $0 \leq t < h-2$. For $h = 1$, We estimate the random entropy E_1 and $d_{t,1}$ using 5,000 replicates. We repeat the procedure for $h = 2, \ldots, n$ considering 5,000 replicates in each h.

[11] The proposed modification of the FIFA rating improves the estimation of the initial final victory probabilities given that, they are more correlated with the final positions of the WC2014 than the standard rating. Moreover, our models give statistically equal results with respect to Table 6 and Table 9 from Groll et al. (2015).

- When $t = h - 1$, we carry out 5,000 replicates and estimate e_0. For $h = 1$, we estimate the forecast $\mathbf{P}(W_1|\xi_0)$, $\mathbf{P}(D_1|\xi_0)$ and $\mathbf{P}(L_1|\xi_0)$, the random entropy E_1 and $d_{0,1}$ using 5,000 replicates. We update the forecasts and estimate e_{h-1}, the random entropy E_h and $d_{h-1,h}$ using 5,000 replicates in each h for $h = 2, \ldots, n,$.
- When $t = h$, we carry out 5,000 replicates and estimate e_0. Given the result in $h = 1$, we estimate e_1 also using 5,000 replicates and estimate d_1. We repeat the procedure for $h = 2, \ldots, n$.

Note that $0 \leq t < h - 2$ and we wish to evaluate the importance of a future and unknown game of the schedule competition (for example, a particular game of the knock-out stage), the calculation of the entropies only considers the case when the game may happen according to the probabilities estimated for each model. In other words, it is possible that we cannot evaluate the decisive measure of unexpected results in games which do not occur in the simulations. Geenens (2014) considered an alternative using exact probabilities of the schedule for all possible combinations of games. However the simulations carried out in this work typically produce the more likely scenarios in the competition and more realistic knock-out games.

Table 2 presents the final victory probabilities for each national team before starting the WC2014 and we note that all models indicate that the favorites to win were Brazil, Spain, Germany and Argentina. Note also how the championship winning probability co-varies with the ability measure. Moreover, we can see that the results of the KE model slightly differ from those of the other models in terms of the final victory probability. This fact is attributed to the situation explained in subsection 4.1, i.e., that the home effect seems not to have a strong impact. Note that this table presents p_{j0} but is constructed with $\mathbf{P}(W_h|\xi_0)$, $\mathbf{P}(D_h|\xi_0)$ and $\mathbf{P}(L_h|\xi_0)$. The accuracy of the models are carried out game by game and it is not necessarily the most accurate model that is the best predictor of the winner of the WC2014.

Table 2 about here

It is also necessary to compare the quality of the forecasts provided by the different models. To do this, we apply the logarithmic scoring rule (LSR) as suggested by Bickel (2007)[12]. In order to compare the predictive quality of two different fore-

[12]See e.g. Boero et al. (2011) for a definition of the logarithmic score. This rule is used to compare the quality of probability forecasts under different models. The preferred model should have the lowest logarithmic score.

casting methods, we also adapt the Wald-type statistic given by Boero et al. (2011); see also Giacomini and White (2006). For a sample size of n games, we construct the test statistic

$$T = n \left(n^{-1} \sum_{h=1}^{n} m_h \Delta L_h \right)' \Omega^{-1} \left(n^{-1} \sum_{h=1}^{n} m_h \Delta L_h \right) = n \Upsilon' \Omega^{-1} \Upsilon, \tag{10}$$

where m_h is a vector of test functions, ΔL_h is the difference in the values of the logarithmic scores of the two models in the game h and $\Omega = n^{-1}\Upsilon\Upsilon'$, is a matrix that consistently estimates the variance of Υ. Under the null hypothesis that both models are equally good predictors, it is known that T tends to χ_1^2 as $n \to \infty$, which gives the "unconditional" test of equal performance, introduced in Boero et al. (2011). We can conclude that a particular model (A) outperforms another (B) when we reject the null hypothesis and the area of the density of $\Delta L = (\Delta L_{1A} - \Delta L_{1B}, \ldots, \Delta L_{nA} - \Delta L_{nB})'$ indicating the mass of the distribution is more inclined to left or right. For example, if ΔL is a loss function between models A and B, if its density mass is leaning to left, model A outperforms model B.

Table 3 presents the results of the LSR for each game for each model. The bold letters show the games considered by the estimation of the Wald-test. We observed that the different statistical models did not provide significantly different forecasts in many games. For example, all models forecast that Brazil should beat Cameroon in the group stage round. Therefore, in a second step we asses the performance of prediction models in a group of key games, i.e., the 23 games involving tournament favorites where the predicted result did not occur, according to the betting house `bet365`, plus 9 randomly selected games to give 50% of the total games in the tournament. Brazil vs Mexico is the game with highest LSR for the PO, BP and OP models and Spain vs Chile for the KE and finally, Costa Rica vs England for the BOP. On the other hand, for all models the game with the lowest LSR is Cameroon vs Brazil.

Table 3 about here

Table 4 shows the results of the estimates of the Wald-test to compare different pairs of models. We can observe that the PO model Poisson regression models (PO and BP) and the KE model outperform the OP model ordered probit models and the hierarchical model in terms of predictive ability.

Table 4 about here

Table 5 we present the results of the model selection procedure described previously. For example, comparing the PO and OP model, more than half (54%) of the mass of the distribution is on the left, indicating that PO outperforms here. One reason for the worse performance of OP models may be that these do not take into account that many World Cup games are played in neutral venues, a situation that is accounted for by the Poisson models. An extra-advantage of Poisson regression is that we can also predict the number of goals, generating more explicit information about the tournament.

Table 5 about here

These results impact on the entropy given by the expression (8) because the differences in the probabilities for each method directly affect the uncertainty distribution of the final victory possibilities. To analyze these implications we carry out Tukey's HSD (honest significant difference) test for the mean of e_h according to each method, where the null hypothesis indicates that the difference of means is equal, see Miller (1981). Table 6 shows that the Poisson regression models have similar outcome uncertainty as measured by the entropy coefficient, as do the HBP and the Ordered Probit models, however only 5 out of 15 models are statistically equal at 1%, (6 out of 15 at 5% and 7 out of 15 at 10%) clearly indicating that there are differences in the uncertainty according to each method.[13]

Table 6 about here

5 Identifying and predicting decisive games

In this Section we use the definition of decisive games described in Section 3 to determine the most decisive matches in the WC2014 and also for the CA2015 from 12 June to 4 July 2015. According to the definition of decisive games, for the WC2014 we consider the observed entropy, d_h, based on all games played, while for the CA2015 we use $d_{0,h}$ and $d_{t,h}$, that are a predictive measures of game decisiveness before the competition starts and during the tournament.

[13]This test only implies that the average entropy for each model is equal or different. If the means of the entropies are unequal, the models have different central values of the estimated entropy. This does not necessarily indicate different decisiveness measures in all games for each model.

5.1 2014 FIFA World Cup: identifying decisive games

The WC2014 was won by Germany and prior to the tournament, Brazil, Spain Germany and Argentina were predicted to be the most probable tournament winners, so, it might be reasonably expected that games involving these teams would cause the biggest changes in the entropy of the championship distribution. Furthermore, those games that help these teams to advance in the tournament may be decisive matches. On the other hand in the later rounds of the competition, we would expect that the remaining teams' ability levels would be similar, increasing the uncertainty in predicting match results. Table 7 shows the estimates of d_h for each of the models used and we can observe how games in the knockout stage present higher decisiveness values as we expected. Although, it seems obvious that games at the later stage of the competition are more decisive. This type of analysis allows us to compare decisiveness among games in the same stage of the tournament. In bold, we illustrate the maximum entropy games for each model and it can be noted that in all cases, these are games with at least one of the top ten teams.

Table 7 about here

For the PO model the most decisive game is Brazil vs Germany, followed by Netherlands vs Argentina, Brazil vs Colombia, Brazil vs Chile and Spain vs Netherlands. These results would appear very natural. Brazil vs Germany resulted in a famous (and unexpected) 1-7 victory for Germany, see games[14], Netherlands vs Argentina was the other semi-final and the next three games, all from the knockout stage, were decisive in determining the progression (or not) of some of the pre-tournament favorites, and in particular, Spain vs Netherlands was the first surprise of the WC 2014 resulting in a loss for Spain, the winner of the previous World Cup. Another interesting game is Spain vs Chile, that lead to the elimination of Spain from the tournament and to Chile reaching the knockout stage.

The results for the KE model are very similar in that the most decisive games are Brazil vs Germany, Netherlands vs Argentina, Brazil vs Colombia, Brazil vs Chile, France vs Germany and Argentina vs Belgium. Also, for BP and BOP the most decisive game is the Netherlands vs Argentina and for the OP is Brazil vs Germany.

Note that under all models, the top two decisive games, not necessarily in that order, are the semi-final encounters Brazil vs Germany and Netherlands vs Argentina. Furthermore, the PO and KE models all classify the matches Brazil vs Chile and Brazil vs Colombia in third and fourth places (in different orders accord-

[14]`http://en.wikipedia.org/wiki/Brazil_v_Germany_(2014_FIFA_World_Cup)`

ing to the individual model). Therefore, we might conclude that the model used is not very important for identifying the *ex post* most decisive games, but is influential in identifying games of relatively high influence in determining the outcome of the tournament. Figure 2 illustrates this issue.Although the final or semi finals could be deemed as decisive even without using any sophisticated analysis, the chosen econometric specification is relevant to estimating the decisiveness order of other games for which their relevance is not so clear a priori. Figure 2 illustrates important discrepancies in decisiveness rankings under different econometric specifications.

Figure 2 about here

In the following subsection we use the PO and KE models (which were selected as the best performing over WC2014) to predict the CA2015 and the most decisive games in this tournament.

5.2 2015 Copa America: predicting which games will be decisive

Here, we used the same parameter estimation and team ability estimation procedure as for WC2014, but now incorporating into the database information up to March 23, 2015, a total of 1,442 matches. Table 8 denotes the predictive percentage probabilities of final victory for each team for both models and we can observe how Argentina, Brazil, Colombia and Chile are the favorites to win the competition. Note how for the PO model the home effect is so important that it gives Chile the same probability of final victory as Colombia. A different conclusion was obtained from the KE model, which as for the WC2014, gives less advantage to the home team in the tournament than the PO model.

Table 8 about here

It is natural to expect that matches involving the tournament favorites would be the most decisive games. Table 9 presents the predictions for the most decisive matches under both models. For PO the most decisive match is Argentina vs Uruguay "el Clásico de Rio de la Plata" which involves the two teams with most Copa America victories (14 and 15 respectively) followed by Colombia vs Peru, then Brazil vs Venezuela and at more distance Argentina vs Jamaica and Mexico vs Bolivia. In these last two cases, the estimated decisiveness measures are very similar to those of the games Chile vs Ecuador and Brazil vs Colombia which might also be expected to be important in determining the final tournament result.

Table 9 about here

It might seem surprising that the game Brazil vs Colombia is not identified as one of the top five decisive games by the PO (or KE) model. One interpretation of this is that both teams are expected to qualify for the second round regardless of the result in this game and we can note that the appearance of Colombia vs Peru and Brazil vs Venezuela can be interpreted as suggesting that an upset in these games would probably lead to one of the tournament favorites failing to qualify for the second round.

For the KE model, Colombia vs Peru gains the highest decisiveness ranking (0.0732) with a large difference to the second ranked game, Argentina vs Uruguay (0.0396). A natural interpretation of this is that probably, Colombia has a large chance of being eliminated if they lose this game (they also play Brazil in the qualifying group) and that if they do, they would have good chances of a final victory. The third to fifth ranked games are Colombia vs Venezuela, Mexico vs Ecuador and Argentina vs Paraguay, all with similar rankings to Argentina vs Uruguay match and very closely followed by Chile vs Bolivia, just outside the top five. Other matches are ranked much lower in decisiveness. Note that under the KE model Colombia is the third favorite to win the competition and similar to the situation for the PO model, an upset in the match against Peru could well lead to their elimination from the tournament.

The same two matches, Argentina vs Uruguay and Colombia vs Peru are classified as the *ex ante*, top two decisive games by both models (although in different orders) although there are differences for games of intermediate decisiveness, similar to the results for the WC2014.

To exemplify the difference between our forecast approach with respect to the Geenens' procedure in order to identify the decisive games, we compute decisiveness measures during the competition using two ways: 1) estimating the probabilities of the schedule competition using $0 < t \leq h-1$ and 2) the probabilities of $\mathbf{P}(W_h|\xi_n)$, $\mathbf{P}(D_h|\xi_n)$ and $\mathbf{P}(L_h|\xi_n)$. In other words, we estimate the probabilities of the decisiveness measures using forecasts of the matches results as we have proposed, and additionally, we compute the probabilities of the schedule competition using the information of the CA2015, similar to the empirical application for the UEFA Euro 2012 carried out by Geenens (2014). Both approaches are implemented for the PO model on the knock-out stage.

Table 10 shows the results for the forecast approach for $t = 18, \ldots, 24$. In the quarter-finals the most decisive game is Argentina vs Colombia for $t = 18, \ldots, 20$ and in semis this is Chile vs Peru for $t = 18, \ldots, 22$. The most decisive game in the knock-out stage is always Chile vs Argentina.

Table 10 about here

In Table 11, using the estimation of the probabilities with information from after the competition, for $t = 18$, Chile vs Uruguay is the most decisive game in the quarters and Argentina vs Colombia for $t = 19, 20$. In the semi-finals Chile vs Peru is the most decisive game for $t = 18, \ldots, 22$.

Table 11 about here

Obviously, when we use these models to forecast future results, the games that are predicted to be the most decisive are those involving teams with higher *prior* chances of final victory. Otherwise, when we fit the model to the set of games in the competition after this has finished, the most decisive games are those involving the eventual finalists.

6 Concluding remarks

In this paper, we analyzed the way in which the identification of decisive matches in international tournaments such as the 2014 FIFA World Cup and the 2015 Copa de America depends on the statistical approach used to estimate the outcome of the game. In terms of forecasting we found that Poisson models and kernel regression are not significantly different and that they both outperformed ordered probit models on these data sets.

Based on 5,000 replicates of the 2014 FIFA World Cup we observed that the *ex-post* identification of the first two most decisive matches does not depend on the model used, but that identification of other key matches varies substantially according to the model considered. In this aspect, the key matches selected by the Poisson and kernel regression models seem to be most in line with what we would expect from a football viewpoint, whereas the ordered probit models generate some more unexpected results. In a similar way, from a predictive viewpoint, both Poisson and kernel regression models suggest that the same two games will be the most decisive in the 2015 Copa America although decisiveness rankings lower down differ between models.

One interesting area for further study would be to try to identify when the estimated decisiveness scores for different games indicate that one game is significantly more important than another, or when similar decisiveness scores suggest that matches are of approximately equal decisiveness.

Acknowledgments

We are very greatful to Ruud H.Koning for helpful comments to an early version of this paper.

References

Audas, R., S. Dobson, and J. Goddard (2002). The impact of managerial change on team performance in professional sports. *Journal of Economics and Business 54*(3), 633–650.

Bickel, J. E. (2007). Some Comparisons among Quadratic, Spherical, and Logarithmic Scoring Rules. *Decision Analysis 4*(2), 29–65.

Boero, G., J. Smith, and K. F. Wallis (2011). Scoring rules and survey density forecast. *International Journal of Forecasting 27*, 379–393.

Bojke, C. (2007). The impact of post-season play-off systems on the attendance at regular season games. in J. Albert and R.H. Koning (eds.). *Statistical Thinking in Sports, Chapter 11*, 179–202.

Chib, S. and B. Carlin (1999). On MCMC Sampling in Hierarchical Longitudinal Models. *Statistics and Computing 9*, 17–26.

Dixon, M. J. and S. G. Coles (1997). Modelling Association Footbal Scores and Inefficiencies in the Footabll Betting Market. *Journal of the Royal Statistical Society C 46*(2), 265–280.

Dyte, D. and S. R. Clarke (2000). A ratings based Poisson model for World Cup soccer simulation. *Journal of the Operational Research Society 51*, 993–998.

Geenens, G. (2014). On the decisiveness of a game in a tournament. *European Journal of Operational Research 232*, 156–168.

Giacomini, R. and H. White (2006). Tests of conditional predictive anility. *Econometrica 74*, 1545–1578.

Gonzalez, I., P. G. P. Martin, S. Dejean, and A. Bacioni (2008). CCA: An R Package to Extend Canonical Correlation Analysis. *Annals of Operations Research 23*(12), 1–14.

Goossens, D., J. Beliën, and F. C. R. Spieksma (2012). Comparing league formats with respect to match importance in belgian football. *Annals of Operations Research 191*(1), 223–240.

Groll, A., G. Schauberger, and G. Tutz (2015). Prediction of a major international soccer tournaments based on team-specific regularized Poisson regression: An aplication to the FIFA World Cup 2014. *Journal of Quantitative Analysis in Sports 11*(2), 97–115.

Hilbe, J. (2014). *Modeling Count Data*. United States of America: Cambridge University Press.

Koning, R. (2007). Post-season play and league design in dutch soccer. in P. Rodríguez, S. Késenne, and J. García (eds.). *Governance and Competition in Profesisonal Sports Leagues, Oviedo: Ediciones de la Universidad de Oviedo*, 191–215.

Koning, R., M. Koolhaas, G. Renes, and G. Ridder (2003). A simulation model for football championships. *European Jouranl of Operational Research 142*(2), 268–276.

Kuypers, T. (2000). Information and efficiency: an empirical study of a fixed odds betting market. *Applied Economics 32*, 1353–1363.

Lancaster, T. (2014). *An Introduction to Modern Bayesian Econometrics*. United States of America: Blackwell Publishing.

Lesne, A. (2014). Shannon entropy: a rigorous notion at the crossroads between probability, information theory, dynamical systems and statistical physics. *Mathematical Structures in Computer Science 25*(3).

Leurgans, S. E., R. A. Moyeed, and B. W. Silverman (1993). Canonical Correlation Analysis when the Data are Curves. *Journal of the Royal Statistical Society B 55*(3), 725–740.

Maher, M. J. (1982). Modelling association football scores. *Statistica Neerlandica 36*, 109–118.

Martin, A., K. Quinn, and J. Park (2011). MCMCpack: Markov Chain Monte Carlo in R. *Journal of Statistical Software 43*(9), 1–29.

McCullagh, P. (1980). Regression Models for Ordinal Data. *Journal of the Royal Statistical Society. Series B (Methodological) 42*(2), 109–142.

McHale, I. and S. Davies (2007). *Statistical analysis of the FIFA world rankings in R. Koning and J. Albert (eds.), Statistical thinking in sport.* Chapman and Hall.

McHale, I. and P. Scarf (2011). Modelling the dependece of goals scored by oppsing teams in international soccer matches. *Statistical Modeling 11*(3), 219–236.

Miller, R. G. (1981). *Simultaneous Statistical Inference.* New York: Springe.

Moroney, M. J. (1956). *Facts from Figures.* UK: Penguin.

Scarf, P. A. and X. Shi (2008). The importance of a match in a tournament. *Computers and Operations Research 35*, 2406–2418.

Schilling, M. F. (1994). The importance of a game. *Mathematics Magazine 67*, 282–288.

Suzuki, A. K., L. E. B. Salasar, J. G. Leite, and F. Lozada-Neto (2010). A Bayesian approach for predicting match outcomes: The 2006 (Association) Football World Cup. *Journal of the Operational Research Society 61*, 1530–1539.

Tena, J. D. and D. Forrest (2007). Within-season dismissal of football coaches: Statistical analysis of causes and consequences. *European Journal of Operational Research 181*(1), 362–373.

Wand, M. P. and M. C. Jones (1995). *Kernel smoothing.* London: Chapman and Hall.

Winkelmann, R. (2000). *Econometric Analysis of Count Data.* Berlin: Springer-Verlag.

Zeileis, A., C. Leitner, and K. Hornik (2014). Home victory for Brazil in the 2014 FIFA World Cup. *Woriking Papers in Economics and Statistics. University of Innsbruck 2014*(17), 1–18.

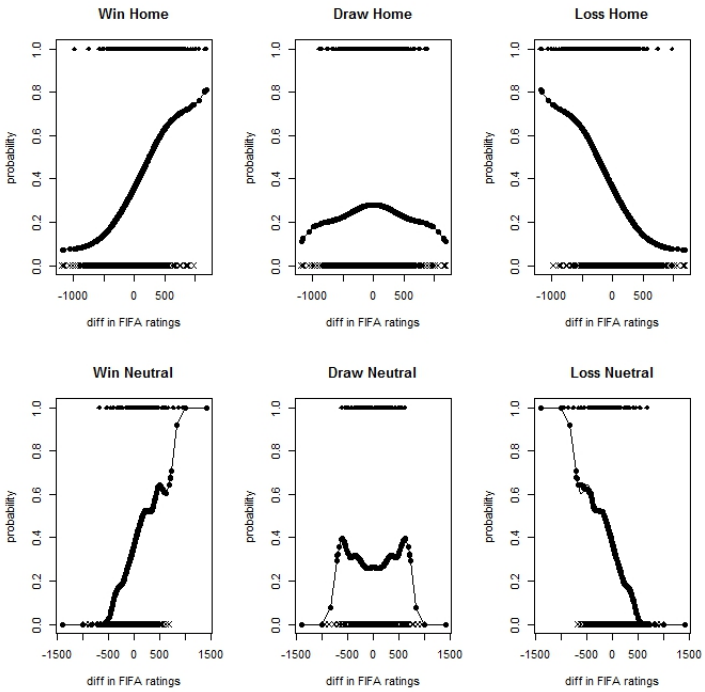

Figure 1: KE estimates of the $\hat{p}(\chi)_W$, $\hat{p}(\chi)_D$ and $\hat{p}(\chi)_L$ probabilities as function of differences of FIFA ratings through of 821 games and 821 "pseudo-games". The top panel shows the 1,186 games between "home" and "visitor" teams. The bottom panel shows the 456 games between "neutral" teams. The horizontal points indicates the observed results: 0 when the "Team T" loses and 1 when the "Team T" wins.

Table 1: Parameter estimates of the Poisson models and ordered probit models. For the BP, HBP and BOP we consider the median of the posterior distributions.

Poisson models										
Model	β_0	β_{A_T}	β_{A_O}	β_{H_T}	β_{N_T}	b_0	b_{A_T}	b_{A_O}	b_{H_T}	b_{N_T}
PO	-0.0784	0.0012	-0.0011	0.3839	0.2184					
BP	-0.0769	0.0012	-0.0011	0.3830	0.2164					
HBP	-0.1386	0.0011	-0.0011	0.4013	0.2573	0.0005	0.0001	0.0000	0.0003	-0.0002

Ordered probit models				
Model	β_{A_D}	β_{H_T}	c_1	c_{-1}
OP	0.002	0.305	-0.465	0.348
BOP	0.683	0.323	0.118	0.844

Table 2: Prior win probabilities for each model, p_{j0}, for the 2014 FIFA World Cup using 5,000 replicates. The teams are ordered according to the final position in the competition.

Final pos.	Team	Ability	PO	BP	BHP	OP	BOP	KE
1	Germany	1312.15	0.1338	0.1314	0.1380	0.1216	0.2562	0.1600
2	Argentina	1203.86	0.0786	0.0748	0.0692	0.0762	0.0944	0.1006
3	Netherlands	1149.2	0.0298	0.0368	0.0308	0.0268	0.0130	0.0408
4	Brazil	1421.82	0.3868	0.3956	0.4244	0.4152	0.3144	0.1870
5	Colombia	1042.54	0.0198	0.0196	0.0176	0.0202	0.0092	0.0314
6	Belgium	910.93	0.0078	0.0108	0.0074	0.0090	0.0046	0.0158
7	France	989.15	0.0174	0.0148	0.0158	0.0136	0.0052	0.0268
8	Costa Rica	743.2	0.0008	0.0008	0.0010	0.0004	0.0000	0.0018
9	Chile	1036.75	0.0128	0.0096	0.0108	0.0126	0.0024	0.0212
10	Mexico	910.07	0.0050	0.0062	0.0046	0.0054	0.0018	0.0118
11	Switzerland	916.99	0.0094	0.0092	0.0056	0.0114	0.0022	0.0156
12	Uruguay	1030.87	0.0162	0.0156	0.0148	0.0168	0.0056	0.0302
13	Greece	898.01	0.0048	0.0038	0.0042	0.0048	0.0012	0.0080
14	Algeria	632.79	0.0006	0.0000	0.0000	0.0002	0.0000	0.0006
15	United States	969.5	0.0124	0.0100	0.0084	0.0104	0.0076	0.0176
16	Nigeria	801.91	0.0020	0.0026	0.0014	0.0024	0.0000	0.0048
17	Ecuador	913.9	0.0054	0.0080	0.0054	0.0064	0.0022	0.0134
18	Portugal	1078.35	0.0278	0.0282	0.0232	0.0194	0.0504	0.0304
19	Croatia	874.37	0.0032	0.0048	0.0040	0.0038	0.0000	0.0044
20	Bosnia and Herzegovina	845.47	0.0036	0.0044	0.0026	0.0040	0.0004	0.0056
21	Ivory Coast	884.12	0.0064	0.0046	0.0034	0.0030	0.0016	0.0094
22	Italy	992.63	0.0112	0.0114	0.0122	0.0104	0.0014	0.0224
23	Spain	1366.74	0.1566	0.1498	0.1600	0.1684	0.1994	0.1700
24	Russia	735.42	0.0130	0.0120	0.0080	0.0088	0.0096	0.0164
25	Ghana	937.52	0.0010	0.0004	0.0006	0.0010	0.0004	0.0028
26	England	1089.38	0.0296	0.0306	0.0238	0.0250	0.0164	0.0406
27	South Korea	673.9	0.0008	0.0004	0.0004	0.0004	0.0000	0.0016
28	Iran	760.96	0.0016	0.0024	0.0006	0.0004	0.0002	0.0046
29	Japan	780.85	0.0016	0.0010	0.0016	0.0014	0.0002	0.0024
30	Australia	728.43	0.0000	0.0004	0.0000	0.0006	0.0000	0.0010
31	Honduras	651.08	0.0002	0.0000	0.0002	0.0000	0.0000	0.0006
32	Cameroon	566.13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004

Table 3: LSR for each model for the schedule of the 2014 FIFA World Cup. Bold letters indicate the games considered for the Wald-test. h represents the order of each game. Games 1 - 48 are the round stage. Games 49 - 62 are of knockout stage.

h	Stage	Team T	Team O	PO	BP	HBP	OP	BOP	KE
1	A	**Brazil**	**Croatia**	0.62	0.60	0.51	0.33	0.18	0.82
2	A	Mexico	Cameroon	1.14	1.16	1.11	0.93	0.89	1.17
3	B	**Spain**	**Netherlands**	2.28	2.32	2.26	2.64	2.18	2.55
4	B	Chile	Australia	1.22	1.20	1.15	0.99	1.02	1.16
5	C	Colombia	Greece	1.52	1.51	1.44	1.41	1.79	1.38
6	D	**Uruguay**	**Costa Rica**	2.50	2.50	2.54	3.07	2.66	2.77
7	D	**England**	**Italy**	2.08	2.03	1.99	2.32	1.52	2.09
8	C	**Ivory Coast**	**Japan**	1.57	1.57	1.57	1.50	2.06	1.46
9	E	**Switzerland**	**Ecuador**	1.77	1.80	1.78	1.79	2.64	1.70
10	E	France	Honduras	1.14	1.15	1.12	0.91	0.95	1.14
11	F	Argentina	Bosnia and Herzegovina	1.11	1.14	1.06	0.87	0.89	1.11
12	G	**Germany**	**Portugal**	1.31	1.34	1.24	1.18	1.35	1.28
13	F	**Iran**	**Nigeria**	2.11	2.12	2.21	1.96	2.55	2.27
14	G	**Ghana**	**United States**	1.34	1.37	1.29	1.28	0.39	1.29
15	H	Belgium	Algeria	1.28	1.28	1.19	1.03	1.19	1.20
16	A	**Brazil**	**Mexico**	3.08	3.07	3.34	3.73	4.56	2.62
17	G	**Russia**	**South Korea**	2.22	2.21	2.40	2.29	2.30	2.22
18	B	Australia	Netherlands	1.00	1.03	0.93	0.88	0.13	1.07
19	B	**Spain**	**Chile**	2.64	2.61	2.64	3.11	3.06	2.92
20	A	Cameroon	Croatia	1.19	1.18	1.16	1.10	0.27	1.19
21	C	Colombia	Ivory Coast	1.50	1.48	1.43	1.30	1.76	1.36
22	D	**Uruguay**	**England**	1.96	1.97	1.90	1.96	3.13	1.96
23	C	**Japan**	**Greece**	2.19	2.14	2.24	2.06	3.07	2.28
24	D	**Italy**	**Costa Rica**	2.41	2.41	2.40	2.90	2.32	2.61
25	E	Switzerland	France	1.66	1.65	1.62	1.81	0.82	1.56
26	E	Honduras	Ecuador	1.29	1.33	1.25	1.21	0.34	1.22
27	F	Argentina	Iran	0.97	0.97	0.88	0.70	0.63	1.04
28	G	**Germany**	**Ghana**	2.83	2.81	3.07	3.25	4.22	1.99
29	F	**Nigeria**	**Bosnia and Herzegovina**	1.89	1.91	1.88	1.95	3.01	1.93
30	H	Belgium	Russia	1.89	1.87	1.83	1.82	2.96	1.90
31	H	**South Korea**	**Algeria**	1.91	1.88	1.90	2.13	1.22	1.91
32	G	**United States**	**Portugal**	2.17	2.13	2.33	1.99	2.88	2.27
33	B	**Netherlands**	**Chile**	1.57	1.58	1.49	1.48	1.98	1.44
34	B	Australia	Spain	0.67	0.69	0.63	0.52	0.02	0.92
35	A	Cameroon	Brazil	0.29	0.29	0.22	0.15	0.00	0.64
36	A	Croatia	Mexico	1.76	1.73	1.69	1.81	0.95	1.64
37	D	**Italy**	**Uruguay**	1.71	1.72	1.65	1.84	0.96	1.64
38	D	**Costa Rica**	**England**	2.36	2.36	2.58	2.37	4.99	2.11
39	C	Japan	Colombia	1.29	1.30	1.21	1.26	0.36	1.15
40	C	**Greece**	**Ivory Coast**	1.77	1.81	1.75	1.73	2.61	1.79
41	F	Nigeria	Argentina	1.04	1.03	0.97	0.88	0.14	1.11
42	F	**Bosnia and Herzegovina**	**Iran**	1.65	1.62	1.57	1.54	2.23	1.46
43	E	Honduras	Switzerland	1.26	1.29	1.23	1.24	0.33	1.17
44	E	**Ecuador**	**France**	2.12	2.12	2.27	2.05	2.69	2.24
45	G	**Portugal**	**Ghana**	1.16	1.15	1.08	0.89	0.91	1.20
46	G	**United States**	**Germany**	1.13	1.14	1.05	1.06	0.21	1.14
47	H	South Korea	Belgium	1.36	1.33	1.28	1.27	0.40	1.24
48	H	**Algeria**	**Russia**	2.28	2.28	2.43	2.43	4.41	2.17
49	R16	Brazil	Chile	0.55	0.55	0.50	0.33	0.33	0.69
50	R16	Colombia	Uruguay	1.36	1.38	1.36	1.29	2.18	1.42
51	R16	Netherlands	Mexico	0.95	0.95	0.94	0.76	1.00	0.84
52	R16	Costa Rica	Greece	1.70	1.72	1.74	1.73	3.62	1.85
53	R16	France	Nigeria	1.05	1.03	1.02	0.88	1.20	0.95
54	R16	**Germany**	**Algeria**	0.42	0.42	0.37	0.23	0.08	0.38
55	R16	Argentina	Switzerland	0.91	0.89	0.85	0.65	0.84	0.79
56	R16	Belgium	United States	1.49	1.48	1.54	1.49	2.68	1.60
57	QF	France	Germany	0.84	0.83	0.81	0.76	0.14	0.74
58	QF	Brazil	Colombia	0.55	0.57	0.52	0.35	0.33	0.72
59	QF	Argentina	Belgium	0.87	0.88	0.85	0.66	0.77	0.78
60	QF	**Netherlands**	**Costa Rica**	0.72	0.74	0.68	0.51	0.46	0.62
61	SF	**Brazil**	**Germany**	2.01	2.02	2.08	2.47	1.74	1.67
62	SF	Netherlands	Argentina	1.27	1.29	1.27	1.36	0.64	1.17
63	3P	**Brazil**	**Netherlands**	2.51	2.48	2.59	3.05	2.82	2.03
64	1P	Germany	Argentina	1.19	1.19	1.18	1.05	1.58	1.08

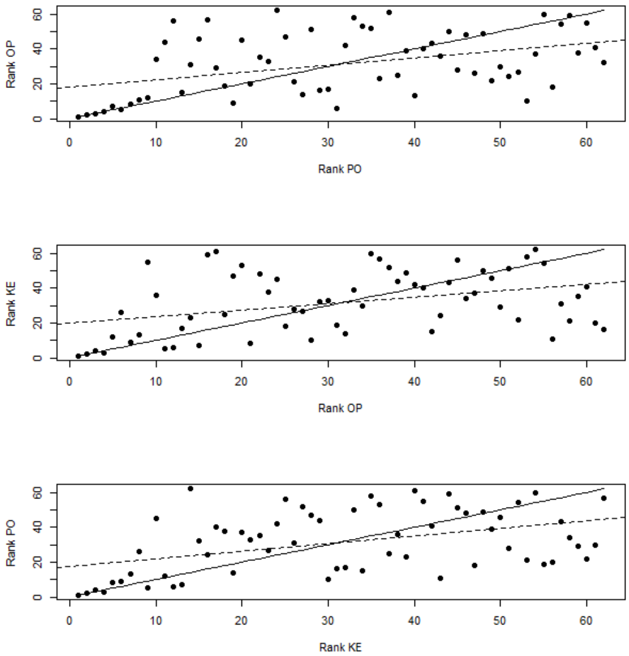

Figure 2: Scatter plot between the Rankings of the decisiveness measures $d_{h,h}$ of the WC2014 for each pair of models. The black line indicates an angle of 45 degrees and the dotted line the equation line obtained by linear regression.

Table 4: Wald-tests for the LSR for each pairs of models.

Models	LSR (Wald stat.)	Outperformance
PO–BP	0.14	-
PO–HBP	1.75	-
PO–OP	3.36*	PO
PO–BOP	5.38**	PO
PO–KE	1.01	-
BP–HBP	1.68	-
BP–OP	3.31*	BP
BP–BOP	5.39**	BP
BP–KE	0.98	-
HBP–OP	2.02	-
HBP–BOP	5.43**	HBP
HBP–KE	1.73	-
OP–BOP	2.90*	OP
OP–KE	3.42*	KE
BOP–KE	5.41**	KE

** 5% Sig.
*10% Sig.

Table 5: Density area for each different model according to Wald test.

Model A	Model B	Area A	Area B
PO	OP	0.538	0.462
PO	BOP	0.622	0.378
BP	OP	0.529	0.471
BP	BOP	0.620	0.380
HBP	BOP	0.618	0.382
OP	BOP	0.580	0.420
OP	KE	0.412	0.588
BOP	KE	0.391	0.609

Table 6: Tukey's HSD (honest significant difference) test for the means of e_h for each pairs of models. We present the difference of means, the lower and upper intervals and the p value of the test.

Models	Difference	Lower	Upper	P value
HBP-BOP	0.024	-0.013	0.061	0.427
OP-BOP	0.041	0.005	0.078	0.017
PO-BOP	0.059	0.022	0.095	0.000
BP-BOP	0.061	0.024	0.097	0.000
KE-BOP	0.192	0.156	0.229	0.000
OP-HBP	0.017	-0.019	0.054	0.752
PO-HBP	0.035	-0.002	0.072	0.075
BP-HBP	0.037	0.000	0.073	0.050
KE-HBP	0.169	0.132	0.205	0.000
PO-OP	0.017	-0.019	0.054	0.753
BP-OP	0.019	-0.017	0.056	0.663
KE-OP	0.151	0.114	0.188	0.000
BP-PO	0.002	-0.035	0.039	1.000
KE-PO	0.134	0.097	0.171	0.000
KE-BP	0.132	0.095	0.169	0.000

Table 7: Decisiveness measure, d_h, for each game and each model in the 2014 FIFA World Cup. Bold letters indicate the top ten most important games according to each model. Games 35- 48 are played at the same time. Games 49 - 62 are from the knockout stage.

h	Stage	Team T	Team O	PO	BP	HBP	OP	BOP	KE
1	A	Brazil	Croatia	0.0330	0.0392	0.0487	0.0533	0.0514	0.0018
2	A	Mexico	Cameroon	0.0026	0.0462	0.0232	0.0510	0.0399	0.0429
3	B	**Spain**	**Netherlands**	**0.1119**	**0.0943**	**0.1259**	**0.1109**	**0.1555**	**0.1145**
4	B	Chile	Australia	0.0004	0.0402	0.0041	0.0203	0.0282	0.0148
5	C	Colombia	Greece	0.0189	0.0492	0.0013	0.0224	0.0216	0.0088
6	D	Uruguay	Costa Rica	0.0606	0.0047	0.0112	0.0150	0.0404	0.0212
7	D	England	Italy	0.0360	0.0259	0.0821	0.0575	0.0052	0.0437
8	C	Ivory Coast	Japan	0.0041	0.0519	0.0308	0.0262	0.0120	0.0064
9	E	Switzerland	Ecuador	0.0017	0.0366	0.0041	0.0249	0.0137	0.0119
10	E	**France**	**Honduras**	0.0398	0.0365	**0.0836**	0.0270	0.0033	0.0023
11	F	Argentina	Bosnia and Herzegovina	0.0176	0.0666	0.0627	0.0594	0.0014	0.0559
12	G	**Germany**	**Portugal**	0.0816	0.0246	0.0381	0.0166	**0.0864**	0.0125
13	F	Iran	Nigeria	0.0387	0.0320	0.0291	0.0139	0.0017	0.0193
14	G	Ghana	United States	0.0045	0.0441	0.0754	0.0393	0.0087	0.0113
15	H	**Belgium**	**Algeria**	0.0044	0.0157	0.0146	**0.0869**	0.0084	0.0193
16	A	Brazil	Mexico	0.0132	0.0638	0.0241	0.0121	0.0031	0.0281
17	H	**Russia**	**South Korea**	**0.0267**	0.0969	0.0233	0.0070	0.0423	0.0038
18	B	**Australia**	**Netherlands**	0.0560	0.0122	**0.0850**	0.0346	0.0000	0.0225
19	B	**Spain**	**Chile**	**0.1108**	0.0811	**0.0993**	**0.1521**	**0.2079**	0.0999
20	A	Cameroon	Croatia	0.0077	0.0014	0.0229	0.0129	0.0345	0.0103
21	C	Colombia	Ivory Coast	0.0033	0.0410	0.0537	0.0004	0.0401	0.0146
22	D	Uruguay	England	0.0045	0.0060	0.0117	0.0375	0.0815	0.0363
23	C	Japan	Greece	0.0096	0.0027	0.0305	0.0383	0.0054	0.0284
24	D	**Italy**	**Costa Rica**	0.0098	0.0234	0.0318	0.0370	0.0213	**0.1042**
25	E	Switzerland	France	0.0153	0.0058	0.0198	0.0170	0.0325	0.0430
26	E	Honduras	Ecuador	0.0273	0.0092	0.0175	0.0025	0.0608	0.0484
27	F	Argentina	Iran	0.0611	0.0447	0.0350	0.0321	0.0575	0.0519
28	G	Germany	Ghana	0.0174	0.0561	0.0240	0.0211	0.0516	0.0129
29	F	Nigeria	Bosnia and Herzegovina	0.0010	0.0156	0.0012	0.0044	0.0052	0.0051
30	H	Belgium	Russia	0.0098	0.0623	0.0446	0.0133	0.0001	0.0070
31	H	South Korea	Algeria	0.0067	0.0430	0.0564	0.0340	0.0073	0.0215
32	G	**United States**	**Portugal**	0.0798	**0.0931**	0.0874	**0.0029**	0.0711	0.1027
33	B	**Netherlands**	**Chile**	0.0298	0.0693	**0.0922**	**0.1278**	**0.0917**	0.0404
34	B	Australia	Spain	0.0332	0.0135	0.0683	0.0560	0.0134	0.0025
35	A	**Cameroon**	**Brazil**	0.0469	0.0145	0.0397	**0.0877**	0.0224	0.0044
36	A	Croatia	Mexico	0.0603	0.0729	0.0312	0.0029	0.0242	0.0246
37	D	**Italy**	**Uruguay**	0.0258	**0.0896**	0.0619	0.0091	0.0276	0.0464
38	D	Costa Rica	England	0.0255	0.0265	0.0038	0.0421	0.0036	0.0174
39	C	**Japan**	**Colombia**	**0.0939**	0.0278	0.0426	0.0272	0.0220	0.0280
40	C	Greece	Ivory Coast	0.0536	0.0024	0.0227	0.0492	0.0428	0.0100
41	F	Nigeria	Argentina	0.0025	0.0175	0.0347	0.0044	0.0243	0.0015
42	F	Bosnia and Herzegovina	Iran	0.0025	0.0083	0.0242	0.0024	0.0296	0.0206
43	E	Honduras	Switzerland	0.0388	0.0271	0.0454	0.0291	0.0168	0.0152
44	E	Ecuador	France	0.0402	0.0198	0.0336	0.0468	0.0145	0.0062
45	G	Portugal	Ghana	0.0343	0.0118	0.0189	0.0093	0.0020	0.0065
46	G	United States	Germany	0.0143	0.0245	0.0422	0.0262	0.0097	0.0038
47	H	South Korea	Belgium	0.0068	0.0408	0.0066	0.0461	0.0284	0.0089
48	H	**Algeria**	**Russia**	0.0387	0.0612	0.0492	0.0461	**0.0943**	**0.1182**
49	R16	**Brazil**	**Chile**	**0.3289**	**0.2906**	**0.3137**	**0.3018**	**0.1006**	**0.2361**
50	R16	Colombia	Uruguay	0.0208	0.0366	0.0133	0.0387	0.0132	0.0528
51	R16	**Netherlands**	**Mexico**	0.0699	0.0681	0.0692	0.0560	0.0060	**0.1245**
52	R16	Costa Rica	Greece	0.0388	0.0079	0.0329	0.0000	0.0479	0.0606
53	R16	France	Nigeria	0.0446	0.0407	0.0213	0.0152	0.0416	0.0040
54	R16	Germany	Algeria	0.0001	0.0370	0.0456	0.0304	0.0557	0.0765
55	R16	**Argentina**	**Switzerland**	**0.1071**	0.0735	0.0503	**0.0994**	0.0340	0.0913
56	R16	Belgium	United States	0.0287	0.0462	0.0132	0.0197	0.0711	0.0637
57	QF	**France**	**Germany**	**0.0995**	**0.1146**	0.0722	0.0671	**0.0187**	0.2074
58	QF	**Brazil**	**Colombia**	**0.2281**	**0.2450**	**0.2677**	**0.2855**	**0.0981**	**0.2418**
59	QF	**Argentina**	**Belgium**	**0.0962**	**0.0878**	0.0650	0.0656	**0.1310**	**0.1269**
60	QF	Netherlands	Costa Rica	0.0234	0.0093	0.0084	0.0003	0.0495	0.0495
61	SF	**Brazil**	**Germany**	**0.3959**	**0.3785**	**0.3187**	**0.3682**	**0.4219**	**0.5700**
62	SF	**Netherlands**	**Argentina**	**0.3930**	**0.4004**	**0.3901**	**0.3670**	**0.4286**	**0.3965**

Table 8: p_{j0} for the 2015 Copa America for PO and KE models.

Team	Rating	PO	KE
Argentina	1501.99	0.3080	0.3000
Bolivia	416.20	0.0000	0.0018
Brazil	1489.78	0.2630	0.2600
Chile	1111.95	0.1590	0.1012
Colombia	1316.10	0.1640	0.1536
Ecuador	955.10	0.2900	0.5380
Jamaica	315.76	0.0000	0.0000
Mexico	1025.88	0.0370	0.0642
Paraguay	432.45	0.0000	0.0006
Peru	642.74	0.0000	0.0072
Uruguay	1003.04	0.0390	0.0550
Venezuela	538.00	0.0010	0.0026

Table 9: Decisiveness measure, $d_{0,h}$, for PO and KE models in the 2015 Copa America (group stage). Bold numbers indicate the top five most important games according to each considered model.

h	Group	Team T	Team O	PO	KE
1	A	Chile	Ecuador	0.0239 (6)	0.0297 (7)
2	A	Mexico	Bolivia	**0.0241 (5)**	0.0291 (8)
3	B	Uruguay	Jamaica	0.0202 (9)	0.0265 (12)
4	B	Argentina	Paraguay	0.0200 (10)	0.0363 (6)
5	C	Colombia	Venezuela	0.0051 (18)	**0.0369 (3)**
6	C	Brazil	Peru	0.0183 (12)	0.0227 (14)
7	A	Ecuador	Bolivia	0.0200 (11)	0.0212 (15)
8	A	Chile	Mexico	0.0180 (13)	0.0147 (17)
9	B	Paraguay	Jamaica	0.0144 (16)	0.0282 (10)
10	B	Argentina	Uruguay	**0.0554 (1)**	**0.0396 (2)**
11	C	Brazil	Colombia	0.0235 (7)	0.0287 (9)
12	C	Peru	Venezuela	0.0122 (17)	0.0137 (18)
13	A	Mexico	Ecuador	0.0148 (15)	**0.0366 (4)**
14	A	Chile	Bolivia	0.0208 (8)	**0.0363 (5)**
15	B	Uruguay	Paraguay	0.0168 (14)	0.0266 (11)
16	B	Argentina	Jamaica	**0.0270 (4)**	0.0242 (13)
17	C	Colombia	Peru	**0.0520 (2)**	**0.0732 (1)**
18	C	Brazil	Venezuela	**0.0366 (3)**	0.0170 (16)

Table 10: Decisiveness measures of the games of the knock-out stage for PO model.

h	Round	Team 1	Team 2	$d_{18,h}$	$d_{19,h}$	$d_{20,h}$	$d_{21,h}$	$d_{22,h}$	$d_{23,h}$	$d_{24,h}$
19	QF	Chile	Uruguay	0.2330	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	QF	Bolivia	Peru	0.0345	0.0350	0.0000	0.0000	0.0000	0.0000	0.0000
21	QF	Argentina	Colombia	0.2820	0.2669	0.2683	0.0000	0.0000	0.0000	0.0000
22	QF	Brazil	Paraguay	0.0677	0.0677	0.0618	0.0668	0.0000	0.0000	0.0000
23	SF	Chile	Peru	0.4360	0.4161	0.4364	0.4074	0.1701	0.0000	0.0000
24	SF	Argentina	Paraguay	0.0470	0.0539	0.0593	0.0413	0.0997	0.1097	0.0000
25	1P	Chile	Argentina	0.9666	0.9630	0.9631	0.9607	0.9689	0.9682	0.9648

Table 11: Decisiveness measures of the games of the knock-out stage for PO model using the estimated probabilities of the schedule tournament with information after of the competition.

h	Round	Team 1	Team 2	$d_{18,h}$	$d_{19,h}$	$d_{20,h}$	$d_{21,h}$	$d_{22,h}$	$d_{23,h}$	$d_{24,h}$
19	QF	Chile	Uruguay	0.5678	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	QF	Bolivia	Peru	0.0466	0.0651	0.0000	0.0000	0.0000	0.0000	0.0000
21	QF	Argentina	Colombia	0.1309	0.1236	0.1549	0.0000	0.0000	0.0000	0.0000
22	QF	Brazil	Paraguay	0.1145	0.0825	0.0975	0.0929	0.0000	0.0000	0.0000
23	SF	Chile	Peru	0.2161	0.2118	0.2180	0.2023	0.3669	0.0000	0.0000
24	SF	Argentina	Paraguay	0.0220	0.0335	0.0365	0.0073	0.1804	0.1710	0.0000
25	1P	Chile	Argentina	0.8135	0.8003	0.8100	0.8052	0.7977	0.8200	0.8169