Insolvency institutions and efficiency. *

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Abstract

While there is a vast literature on optimal bankruptcy laws and, specifically, on the optimal allocation of control rights between debtors and creditors in corporate bankruptcy, little has been said about the role that alternative insolvency institutions may play in the design of the optimal insolvency framework. This paper attempts to fill this gap by modelling two insolvency institutions -a bankruptcy system and a foreclosure system- that firms and their creditors may use when dealing with financial distress. Firms choose between one or the other based on lenders' willingness to provide credit and the trade-off between two inefficiency costs, those from inefficient liquidations and those from productive inefficiencies caused by overinvestment in capital assets. The model's key result is that welfare is a non-monotonic function of creditor *control* rights in bankruptcy, implying that a *perfectly* "creditor-friendly" bankruptcy code (a code that always grants control of the distressed firms to creditors) is very inefficient. A second result is that welfare is higher when the bankruptcy system is *too* "creditor-friendly" (i.e., it ensures the provision of credit, but generates too many inefficient liquidations) than when it is too "debtor-friendly". Hence, if the optimal level of creditor *control* rights in bankruptcy cannot be ascertained in practice, it may be better to grant too much control of the bankruptcy process to creditors than too little, as the loss from undershooting that level is larger than that from overshooting it.

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1 Introduction

Many papers have analysed the optimal allocation of control rights between debtors and creditors in corporate bankruptcy. Creditor-friendly systems seem to promote the ex-ante provision of credit better than debtor-friendly ones because they deter borrower's moral hazard (La Porta *et al.*, 1997, Davydenko and Franks, 2008), but at the expense of generating more ex-post inefficient liquidations because creditors are inherently biased towards liquidating the firm (Hart, 2000). Debtor-friendly systems are supposed to induce greater innovation and ex-ante efficient risk-taking than creditor-friendly ones because they promote continuation upon failure (Acharya and Subramanian, 2009), but they may lead to excessive inefficient continuations. To solve this tension between ex-ante incentives and ex-post outcomes, mainly arising from the concave (convex) nature of creditors' (debtors') claims, some authors have proposed to allocate significant control rights to third parties, such as judges or insolvency practitioners, as long as they are able to make the correct decision regarding the reorganisation or liquidation of the distressed firm (Ayotte and Yun, 2009). By contrast, other authors have advocated a "contractualist" bankruptcy system that limits the discretion of courts to strict enforcement of contracts and, based on the evidence on floating charge financing[1] (Franks and Sussman, 2005, Djankov *et al.*, 2008), have proposed a debt structure with two classes of debt, one that gives control upon default to a large creditor and another fully dispersed debt class without control rights, which would remove the controlling creditor's liquidation bias (Gennaioli and Rossi, 2013).

However, the existing literature on optimal bankruptcy seems to assume that the bankruptcy system is the only insolvency *institution*, implying that, if private workouts fail -mainly because of coordination and asymmetric information problems[2]- bankruptcy is unavoidable. But this assumption does not seem to be warranted by the empirical evidence. Morrison (2008) documents

[1]A floating charge is a security interest in which the whole firm can be pledged as collateral, including working capital, intangibles and future cash flows.

[2]See, *inter alia,* Gilson *et al.* (1990) and Hart (2000).

that only twenty percent of distressed small[3] businesses file for bankruptcy in the U.S, while the rest use non-bankruptcy procedures. These procedures include both private workouts and *formal* debt enforcement procedures such as foreclosures, bulk sales[4] and assignments for the benefit of creditors.[5] García-Posada and Mora-Sanguinetti (2014) report that, while around 8,000 firms filed for bankruptcy in 2012 in Spain, there were more than 26,000 business foreclosures. A foreclosure is a non-bankruptcy procedure that is used worldwide,[6] even though its implementation differs across countries in several aspects. A foreclosure aims to recover the money owed to secured creditors by seizing the loan's collateral. It does not protect unsecured creditors, who must rely on separate insolvency proceedings to enforce their claims. In some countries the insolvent company can prevent creditors from foreclosing on its assets -an stay on the firm's assets- by filing for bankruptcy, while in others a bankruptcy filing does not stop foreclosure.[7] In some countries a foreclosure can be an entirely out-of-court procedure, a private contractual solution in which a receiver liquidates the company to maximise the recovery of the senior creditor,[8] whereas in others a court oversees foreclosure, although it is typically less involved than in bankruptcy.[9]

This paper's main contribution is to analyse the trade-offs faced by firms and their lenders in a context where an alternative debt-enforcement institution -the foreclosure system- exists, so that agents can "contract out" bankruptcy even when bargaining costs are too high to undertake a private workout. Specifically, credit contracts can be enforced under the bankruptcy system or under the foreclosure system. These two insolvency institutions differ in two crucial aspects: the

[3]Defined as those with less than 500 employees.

[4]In a bulk sale the debtor sells most or all of its business to a third party and distributes the proceeds to creditors.

[5]In an assignment for the benefit of creditors, the business assigns its assets to a trustee, who auctions them off and distributes the proceeds to creditors.

[6]Djankov *et al.* (2008) identify three basic insolvency procedures that are used around the world: (1) foreclosure by the senior creditor, (2) liquidation and (3) reorganisation, which often leads to subsequent liquidation if the reorganisation attempt fails. (2) and (3) are specific types of corporate bankruptcy (e.g., Chapter 7 and 11 in the U.S., respectively).

[7]In the latter case, as pointed out by Djankov *et al.* (2008), bankruptcy procedures may take place in parallel with foreclosure procedures.

[8]That was the case of Administrative Receivership in the U.K. before the entry into force of the Enterprise Act 2002, which abolished it.

[9]In the U.S. there are both types. The *judicial foreclosure* involves the sale of the mortgaged property, subject to auction, under the supervision of a court. The *non-judicial foreclosure* involves the sale of the property via a public auction by the mortgage holder without court supervision.

liquidation technology and the probability that creditors are granted control of the firm's assets following default, which measures the institution's degree of "creditor-friendliness". We assume that the liquidation technology of foreclosures is more efficient than that of bankruptcy procedures, in the sense of providing higher liquidation proceedings for the same liquidation value. We also assume that the probability that creditors get control of the distressed firm's assets is higher in foreclosure than in bankruptcy.

In that context, firms choose between one insolvency institution and the other based on lenders' willingness to provide credit and the trade-off between two inefficiency costs, those of ex-ante productive inefficiencies and those of ex-post inefficient liquidations. The trade-off occurs because firms may overinvest in capital to increase their assets' liquidation value, which reduces funding costs.[10] This overinvestment generates productive inefficiencies but, by increasing the liquidation value, it reduces the cost of inefficient liquidations (going concern value minus liquidation value).

All these effects can take place in both institutions, but with different frequency and/or magnitude. The foreclosure system, by always granting control of the firm's assets to creditors in the event of default, deters borrowers' moral hazard and promotes the provision of credit, while the bankruptcy system only does it as long as creditor *control* rights are sufficiently protected. But, because of the same reason, the foreclosure system leads to more inefficient liquidations than the bankruptcy system, as creditors are inherently biased towards liquidation. Finally, as the foreclosure system has a better liquidation technology than the bankruptcy system, it provides incentives to overinvestment in capital, which leads to lower liquidation costs, at the expense of greater productive inefficiencies.

The model's key result is that welfare is a non-monotonic function of the level of creditor *control* rights under bankruptcy. This result has a direct policy implication: a perfectly "creditor-friendly" bankruptcy code (a code that always grants control of the distressed firms to creditors) is very inefficient. In fact, it is so inefficient that agents would never use it, as the alternative insolvency institution, foreclosure, leads to the same number of inefficient liquidations but provides higher recovery rates to creditors and lower funding costs to firms. A second result is that welfare is

[10] The overinvestment in capital can take place in two different ways, the "intensive margin" and the "extensive margin". The intensive margin consists of exceeding the optimal proportion of capital for a given business project. The extensive margin consists of choosing projects that require a high proportion of capital assets over projects with higher productivity but a lower proportion of these assets. For simplicity, in the current model we only analyse the intensive margin, leaving the analysis of the extensive margin for further research.

higher when the bankruptcy system is too “creditor-friendly” (i.e., it ensures the provision of credit, but generates too many inefficient liquidations) than when it is too “debtor-friendly”. In other words, if the optimal level of creditor *control* rights in bankruptcy cannot be ascertained in practice, it may be better to grant too much control of the bankruptcy process to creditors than too little, as the loss from undershooting that level is larger than that from overshooting it. The intuition is that, as the optimal level of creditor control rights is the minimum that makes the credit contract feasible under bankruptcy, when creditor rights exceed the optimal level, the two insolvency institutions can be used, so that each firm chooses the one that maximises its payoffs. However, when creditor rights are lower than the optimal, only one institution, foreclosure, can be used. As bankruptcy and foreclosure are imperfect substitutes, if firms are heterogenous in some characteristic that makes them differ in their preferred insolvency institution (in this model, the marginal cost of overinvestment in capital), then a too “creditor-friendly” bankruptcy system may yield higher welfare.

A related result is the positive correlation between welfare and bankruptcy rates: when creditor rights are high (low), welfare is high (low) and bankruptcy rates are high (low). This theoretical prediction seems corroborated by the empirical evidence if we proxy welfare with per capita GDP (Claessens and Klapper, 2005; Celentani *et al.*, 2010). While there are a number of alternative explanations for the correlation between bankruptcy rates and per capita GDP, the mechanism analysed in this paper may explain *part* of such a correlation. Moreover, by showing that low bankruptcy rates may be associated with low levels of welfare, the model explains why policy-makers should be wary of the former, at variance with common wisdom: if they suspect that the bankruptcy law does not provide enough protection to creditors, then they should assess whether bankruptcy rates are “too low” (for instance, by international standards).

The rest of the paper is organised as follows. Section 2 presents the base model and solves for the optimal credit contracts under each insolvency institution, bankruptcy and foreclosure. Section 3 analyses the choice of insolvency institution by a representative firm. Section 4 generalises to a set of heterogenous firms in order to study the impact of the institutional design on welfare and bankruptcy rates. Section 5 concludes. Proofs of lemmas and propositions and other technical details are in Appendices A-F.

2 Model

2.1 General model: distress resolution via private workouts

In principle, individuals can deal with financial distress themselves -i.e., without the use of an insolvency institution- via a private workout. A firm and its creditors may write their own insolvency procedure by specifying as part of a debt contract what should happen in a default state.

Consider a three-period model[11] ($t = 0, 1, 2$) in which there is a wealthless firm manager and a perfectly competitive lender.[12] Both players are risk-neutral and there is no time discounting. Market interest rates are normalised to zero. The manager owns an investment project, which requires an initial outlay of $I > 0$ at $t = 0$ for the purchase of some productive assets.[13]

If the manager had I available (first-best), she would undertake the project. The project cash flows would be $\widetilde{\pi}_1 = \pi$ with probability θ or $\widetilde{\pi}_1 = 0$ with probability $1 - \theta$ at $t = 1$ and $\widetilde{\pi}_2 = \pi$ with probability ϕ or $\widetilde{\pi}_2 = 0$ with probability $1 - \phi$ at $t = 2$, where $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$ are independently distributed. We assume $I \leq \theta\phi\pi$ for lending to be feasible under very general circumstances.[14] If for some reason the project is liquidated, it yields proceedings equal to αl, where $0 \leq \alpha \leq 1$ captures the transaction costs incurred in liquidating the assets (i.e., the higher the α, the more efficient the liquidation tecnology) and l is the project's liquidation value.

We depart from the existing literature in the nature of the liquidation value l. While it has always been treated as an exogenous parameter, in this model it is an endogenous variable. The project can be undertaken with different combinations of productive assets, namely capital and labour, which determine the project's liquidation value. Specifically, the liquidation value l will be proportional to the share γ of the initial outlay I spent in the purchase of capital: $l = \gamma I$ where

[11]The model is an adaptation of Bolton and Scharfstein (1996) who use a model of nonverifiable cash flows to analyse the optimal debt structure as a function of the number of creditors.

[12]The assumption of perfect competition in the credit market is only made for analytical simplicity. In Supplement B there is a version of the model that relaxes this assumption, proving that the model's conclusions are robust to different degrees of competition in the credit market.

[13]In this set up we use a fixed-scale model, so we implictly assume away firm size from the analysis. In Supplement C we use instead a variable-scale model with constant returns to scale to extend the model's conclusions to a set up with heterogeneity in firm size.

[14]Specifically, that assumption ensures that financing is possible if contracting parties use the bankruptcy system (see section 2.2.1). However, in Supplement A that assumption is relaxed in order to explore other model's implications.

$0 \leq \gamma \leq 1$. Hence $l \in [0, I]$.

There is only a proportion of capital and hence a liquidation value l that is efficient from the point of view of the production process, as the rest of proportions lead to productive inefficiencies. Let us call this first-best liquidation value $\hat{l}$. Since there is a direct mapping between the proportion of the investment spent in capital γ and the liquidation value l, we can express the cost of productive inefficiencies as a function of l, i.e, $D(l)$, where $D(\hat{l}) = 0$. If the manager deviates from the optimal proportion of capital, then the cash flow at $t = 1$ would be $\widetilde{\pi}_1 = \pi - D(l)$ with probability θ or $\widetilde{\pi}_1 = 0$ with probability $1 - \theta$.

Let us assume $D(l) = n|l - \hat{l}|$ where $n > 0$. The parameter n is the marginal cost of over-investment,[15] which occurs if $l > \hat{l}$. Different investment projects may have different values of n.

Cash flows are observable to both parties but nonverifiable to a third party such as a court of law.[16] This assumption allows for moral hazard in the form of strategic default. By contrast, loan repayments, as well as the project's assets and the proceeds from the sale of liquidated assets, are verifiable. In this setting, credit contracts based on realised cash flows are not feasible because they cannot be enforced, but they can be based on repayments made by the firm, since the relevant judge or court can verify that the manager has defaulted and enforce the assets' repossession and subsequent liquidation. The threat to repossess the assets by the lender, thus depriving the manager from the project's cash flows at $t = 2$, provides the incentive for the manager to repay at $t = 1$.[17]

Within this framework let us analyse the following credit contract. In exchange for borrowing I at $t = 0$, the manager promises a repayment R at $t = 1$. If she repays R, she keeps control of the firm's assets throughout $t = 2$. If she does not repay, the lender assumes control of the assets with probability β. In such a case, the lender will always liquidate the firm because he will obtain $\alpha l \geq 0$ through the sale while obtaining zero if keeping it as a going concern, since he lacks

[15]The paper's key results do not change if we choose an alternative cost function where the marginal cost of overinvestment is not constant and independent of l, such as $D(l) = n\left(l - \hat{l}\right)^2$.

[16]This can result from direct expropriation of cash flows or from managerial perquisite consumption.

[17]The nonverifiability of cash flows also implies that long-term credit contracts (i.e., contracts payable at $t = 2$) are not feasible. Since the manager does not face any repossession threat at $t = 2$, she would always default and the lender could only recover $\alpha l \leq I$ of the loan.

the managerial skills to make the project generate any cash flow at $t = 2$. The assumption that the lender makes zero cash flows from managing the assets by himself is just a normalisation: the key point is that he gets more by selling the assets, which captures the classic idea that secured creditors are inherently biased toward liquidation.[18] Since the lender is perfectly competitive, the manager has all the bargaining power and she makes a take-it-or-leave-it offer to the lender $\{R, \beta, l\}$ at $t = 0$.

We assume that liquidations are *ex-post* inefficient, i.e., the liquidation value is lower than the continuation value: $\alpha l \leq \phi\pi \ \forall l$. This implies that, if the manager does not default at $t = 1$ -hence keeping control over the assets- she will want to continue the project throughout $t = 2$, therefore obtaining $\phi\pi$, instead of liquidating it herself, which would yield αl.

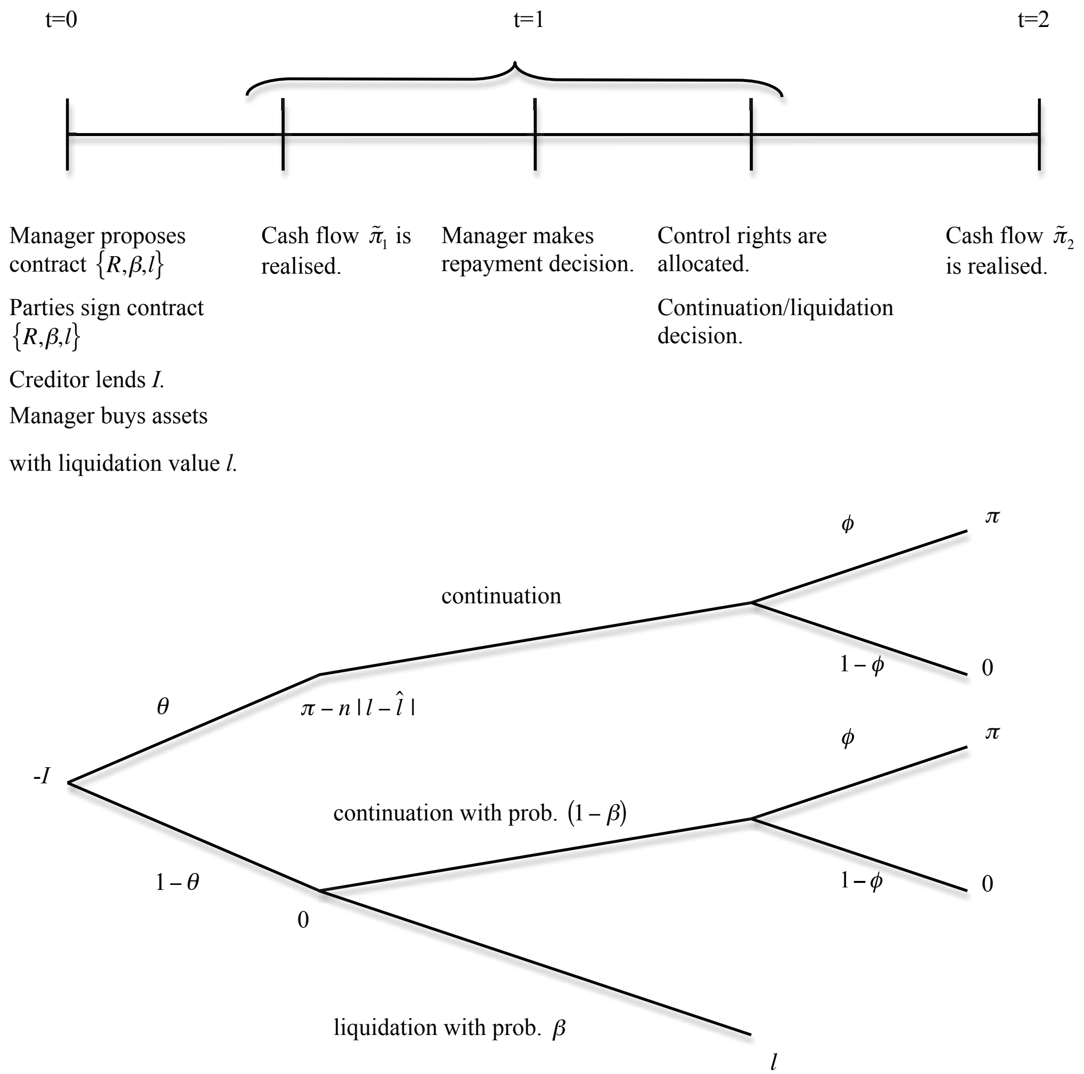
Notice that, since cash flows are nonverifiable, the manager can always choose whether to repay R when the cash flow at $t = 1$ is positive or to repay nothing and default (strategic default). If the cash flow at $t = 1$ is zero, then the firm must default on its debt. We abstract from renegotiation issues by assuming full commitment.[19] The model's timeline, as well as the cash flows contingent on the liquidation or continuation of the project -provided there is no strategic default- are shown in Figure 1[20].

[18]In this model secured creditors are biased toward liquidation because we assume away floating charge financing, as it is not allowed in many non-common law jurisdictions (Djankov *et al.*, 2008).

[19]As in Bolton and Scharfstein (1996), the debt contract is not renegotiation proof.

[20]A more general contracting space does not change the results. In the current setup we analyse probabilistic all-or-nothing liquidations, in which *all* assets are seized and sold by the creditor. In Bolton and Scharfstein (1996) probabilistic liquidations are combined with the possibility of partial liquidations, where only a fraction z of the assets is liquidated. However, they show that partial liquidations are never optimal, while probabilistic liquidations (i.e., $\beta < 1$) are optimal, so the former can be ruled out. We could also allow creditors to take control of the assets even if there is repayment with probability β_π, but it is easy to show that the solution for the optimal contract yields $\beta_\pi^* = 0$ (i.e., it is never optimal to liquidate when the manager repays). Finally, one could set a repayment schedule for both states of nature, i.e., R_π and R_0 but, since the manager is wealthless, we need to set the feasibility condition $R_0 \leq 0$, which it is easy to show that leads to $R_0^* = 0$.

Figure 1: model's timeline and cash flows (no strategic default)



2.1.1 Optimal contract

The optimal contract maximises the manager's expected utility EU -which is the project's expected profits- subject to the following constraints: (1) the manager does not default strategically (incentive compatibility); (2) the lender decides to provide credit (individual rationality); (3) since the manager is wealthless, the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature (first feasibility constraint); (4) the liquidation probability β lies in the interval $[0, 1]$ (second feasibility constraint). The maximisation problem is the following:

$$\max_{R,\beta,l} EU = \theta \left[\pi - n|l - \hat{l}| - R + \phi\pi\right] + (1-\theta)(1-\beta)\phi\pi$$

s.t.:

$$\pi - nl - R + \phi\pi \geq \pi - nl + (1-\beta)\phi\pi \tag{1}$$

$$\theta R + (1-\theta)\beta\alpha l \geq I \tag{2}$$

$$R \leq \pi - n|l - \hat{l}| \tag{3}$$

$$0 \leq \beta \leq 1 \tag{4}$$

Equation (1) shows the repayment decision of the manager. If she chooses to repay R, she keeps the control of the firm with certainty and obtains the expected cash flow $\phi\pi$ at $t = 2$. If instead she chooses to default, she repays nothing and keeps the cash flow at $t = 2$ with probability $(1 - \beta)$. An alternative interpretation comes from simplifying and rearranging (1):

$$\beta\phi\pi \geq R \tag{5}$$

Equation (5) shows that, for the contract to be incentive-compatible, the expected punishment for strategic default, $\beta\phi\pi$, must be greater or equal to the benefit from carrying out such a strategy, R. Proposition 1 summarises the optimal contract.

Proposition 1. *The optimal contract* $\{R^*, \beta^*, l^*\}$ *in the case of distress resolution via a private workout is:*

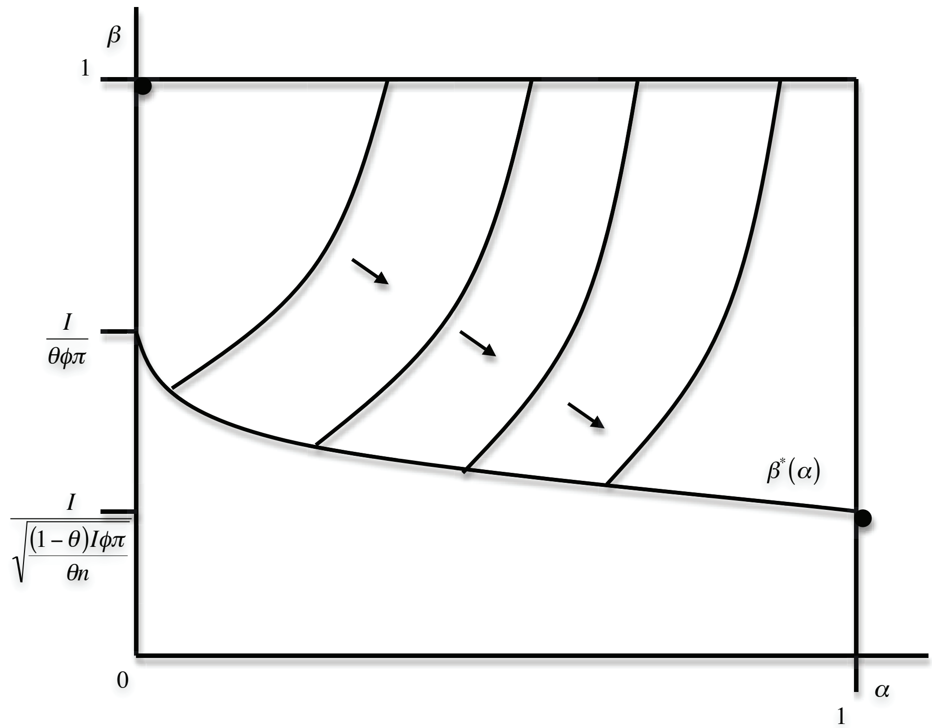
(i) If $0 < \alpha \leq 1$. $R^* = \phi\pi\beta^*, \beta^* = \frac{I}{\sqrt{\frac{I\phi\pi(1-\theta)\alpha}{\theta n}}}$, $l^* = \sqrt{\frac{I\phi\pi}{\theta(1-\theta)n\alpha}} - \frac{\theta\phi\pi}{(1-\theta)\alpha}$. *The contract is feasible if* $I \in \left[\frac{\theta^3\phi n\pi}{(1-\theta)\alpha}, \frac{(1-\theta)\alpha\phi\pi}{\theta n}\right]$.

(ii) If $\alpha = 0$. $R^* = \frac{I}{\theta}$, $\beta^* = \frac{I}{\theta\phi\pi}$, $l^* = \hat{l}$.

The optimal contract is tailored to the project's characteristics and, specifically, to the technological parameter n. First, notice that $\frac{\partial l^*}{\partial n} < 0$, i.e., the higher the marginal cost of overinvesting in capital n, the lower the optimal liquidation value l^*. Second, $\frac{\partial \beta^*}{\partial n} > 0$ and $\frac{\partial R^*}{\partial n} > 0$, i.e., projects with higher marginal cost of overinvesting in capital n need to offer a higher liquidation probabilityβ^* and a higher repayment R^* to the lender. The intuition is that increasing the liquidation value reduces the required liquidation probability and repayment because the lender expects to recover more in case of default but, as just explained, the optimal liquidation value decreases as the marginal cost of overinvesting in capital rises. The intuition of Proposition 1(ii) is straightforward. If $\alpha = 0$, then the liquidation proceedings αl are zero irrespective of the liquidation value l, so that overinvesting in capital ($l > \hat{l}$) does not reduce funding costs R but it reduces cash flows at $t = 1$ by $n|l - \hat{l}|$. In other words, since pledging more collateral is useless but costly, no overinvestment takes place: $l^* = \hat{l}$.

The main insights of the analysis are illustrated in Figure 2, which depicts welfare indiference curves in the space (α, β) for a given value of the technological parameter n. The lowest welfare is achieved in point $(\alpha = 0, \beta = 1)$, where the creditor always (inefficiently) liquidates in the event of default and obtains zero liquidation proceedings. From that point welfare increases as we move downwards and rightwards up to the point $(\alpha, \beta) = (1, \beta^*(1))$, where welfare is highest. In that point the liquidation probability is the minimum feasible value -i.e., the minimum value that deters strategic default, so that the lender is willing to provide credit and the probability of inefficient liquidations is minimised- and liquidation proceedings are the highest given a marginal cost of overinvesting in capital n. The unfeasible allocations lie below the curve $\beta^*(\alpha)$, where credit cannot be provided.

Figure 2: welfare indifference curves in the (α, β) space



2.2 Special cases: distress resolution via insolvency institutions

Although a firm and its creditors may write their own insolvency procedure by specifying as part of a debt contract what should happen in a default state, contract incompleteness makes private workouts often unfeasible in practice (Hart, 2000). For instance, writing such a contract may be difficult because the debtor may acquire new assets and creditors as time passes. Moreover, private workouts are often unfeasible due to coordination and asymmetric information problems (Gilson *et al*, 1990; Morrison, 2008 and 2009). In fact, the empirical evidence shows that firms rarely write such contracts[21] and that, by contrast, almost all countries have some form of state-provided

[21] A remarkable exception was Administrative Receivership in the U.K. Under Administrative Receivership, an important creditor -typically a bank- contracted with the debtor to be granted a "floating charge", which gave the creditor the right to appoint a receiver if the firm defaulted. The receiver would take charge of the firm and decide whether to liquidate it or keep it as a going concern. Franks and Sussman (2005) show that Administrative Receivership was best seen as a privately negotiated contract between a debtor and its creditors. However, Administrative Receivership was abolished in 2003 after the entry into force of the Enterprise Act 2002.

insolvency institution (Hart, 2000). In other words, an insolvency institution would offer an "off the shelf" procedure for distress resolution, i.e., one that the parties can use if they do not write their own.

In this section we model two insolvency institutions as providers of pre-specified credit contracts that the manager and the lender can use. The contracting game of section 2.1 can be implemented under two different institutions: the bankruptcy and the foreclosure systems. This means that the manager and the lender, when aggreeing on $\{R, \beta, l\}$, also choose which institution they will use in the event of default, so that contracts are signed and enforced under that institution. We use the subscripts B and F for the values of variables and parameters in bankruptcy and foreclosure, respectively.

If parties use bankruptcy, the lender takes control of the firm's assets with probability β_B in case of default, where $\beta_B \in [0, 1]$ is an *exogenous* parameter set by the bankruptcy code. As in Ayotte and Yun (2009) and in Acharya *et al.* (2011), we interprete β_B as the variable that measures the degree of "creditor-friendliness" of the bankruptcy law. Hence, the higher β_B, the higher the creditor *control* rights. In case of liquidation, the lender obtains $\alpha_B l_B$, where $\alpha_B \in [0, 1]$ captures the transaction costs incurred in liquidating the assets (litigation costs, length of the process, etc.) and, in turn, it measures the efficiency of the liquidation technology of bankruptcy.

If parties use foreclosure, the lender assumes control of the firm's assets with *exogenous* probability $\beta_F = 1$ in case of default, i.e., control is transferred to creditors with certainty if the firm does not repay the loan. In that case, the liquidation proceedings obtained by the lender are $\alpha_F l_F$, where $\alpha_F \in [0, 1]$.

These insolvency institutions differ in two crucial aspects: the liquidation technology and the probability that creditors are granted control of the firm's assets following default. We assume that the liquidation technology of foreclosures is more efficient than that of bankruptcy procedures, in the sense of providing higher liquidation proceedings for the same liquidation value: $\alpha_F > \alpha_B$. This seems a reasonable assumption, as most bankruptcy procedures, such as Chapter 11 and Chapter 7 in the U.S., are often critisized for being lengthy and costly, implying that direct and indirect costs consume an important share of the firm's asset value.[22] By contrast, foreclosures are gen-

[22] According to Bris *et al.* (2006) and their large sample of Chapter 11 and Chapter 7 cases, among firms with assets worth less than \$ 100,000, the median direct costs burn 23.2% of asset value. Among firms with assets worth between \$ 100,000 and \$ 1 million, the median direct costs consume 4.9% of asset value.

erally cheaper and faster (Morrison, 2008, Succurro, 2012, García-Posada and Mora-Sanguinetti, 2014), as they are well-defined quite standardised processes with a low degree of uncertainty about its final outcome, implying that they require less intervention by the court, insolvency administrators/trustees, accountants, lawyers, etc.[23] We also assume that the probability that creditors get control of the distressed firm's assets is (weakly) higher in foreclosure than in bankruptcy: $\beta_F \geq \beta_B$. This is a very realistic assumption, as by definition the creditor repossesses the collateral in a foreclosure, while in bankruptcy, depending on the procedure and the country, the debtor remains in control of the firm (e.g. "debtor in possession" in Chapter 11 in US, *concurso voluntario* in Spain), that control is transferred to creditors (e.g. administration in the U.K.) or to a court-appointed administrator (e.g. *redressement judiciare* in France, *concurso necesario* in Spain).

We again abstract from renegotiation issues by assuming full commitment. This assumption is quite plausible in the case under analysis. First, the foreclosure system does not provide any mechanism for debt restructuring: default triggers straight liquidation of the seized assets. Second, although many bankruptcy systems allow for reorganisations, the empirical evidence shows that most of the firms that file for bankruptcy end up liquidated (Celentani *et al.*, 2010).[24]

2.2.1 Optimal contracts under bankruptcy and optimal bankruptcy code

For the study of optimal contracts under bankruptcy it is necessary to analyse first the optimal bankruptcy code β_B^* , i.e., the liquidation probability that maximises welfare. A way to address the problem would be to assume that a welfare-maximising social planner chooses and announces β_B^* before $t = 0$, understanding how contracts $\{R_B, l_B\}$ will respond in equilibrium. However, for simplicity of exposition we assume that the bankruptcy code β_B is chosen by the manager at $t = 0$ and offered along with $\{R_B, l_B\}$ as part of an optimal contract that maximises his profits. The two perspectives are equivalent (Ayotte and Yun, 2009).

The manager's maximisation program is the same as the one for private workouts (section

[23]In fact, Djankov *et al.* (2008), in their survey of 88 countries, find that the efficiency of foreclosure rises when the senior creditor is allowed to take collateral in an out-of-court procedure.

[24]Nevertheless, the analysis of debt renegotiation inside bankruptcy, while beyond the scope of this paper, yields some interesting insights. See Tarantino (2013).

2.1.1.) and so are its solutions (Proposition 1). For simplicity, let us set the efficiency of the liquidation technology to zero, i.e., $\alpha_B = 0$. Since our ultimate aim is to analyse the determinants of the choice of insolvency institution, we only care about the relative values of the key parameters, so that the assumption $\alpha_B = 0$, which satisfies $\alpha_F > \alpha_B$ for any $\alpha_F > 0$, does not determine our conclusions.[25] We can also find the economy's welfare by first computing the manager's equilibrium utility and then making use of the fact that the lender is perfectly competitive, so its equilibrium utility is zero. We summarise the results in Proposition 2.

Proposition 2. *The optimal bankruptcy code β_B^* and the optimal contract under bankruptcy $\{R_B^*, l_B^*\}$ when $\alpha_B = 0$ are $\beta_B^* = \frac{I}{\theta\phi\pi}$, $R_B^* = \frac{I}{\theta}$, $l_B^* = \hat{l}$. Welfare is $W_B^* = (\theta + \phi)\,\pi - I - \frac{1-\theta}{\theta} I$.*

In order to analyse the inefficiencies that may arise under this contract it is useful to rewrite the economy's welfare as $W_B^* = (\theta + \phi)\,\pi - I - (1 - \theta)\,\beta_B^* \phi\pi$. The first two terms $(\theta + \phi)\,\pi - I$ express the project's net present value in the first-best, while the last term $-(1 - \theta)\,\beta_B^* \phi\pi$ is the expected cost of inefficient liquidations, which can be decomposed as the product of the probability of inefficient liquidations $(1 - \theta)\,\beta_B^*$ and the size of such inefficiency $\phi\pi$. With probability $(1 - \theta)$ the manager defaults and with probability β_B^* the lender takes control of the firm and liquidates its assets. Since the cash flow at $t = 2$, $\phi\pi$, is foregone, while the project yields zero liquidation proceeds regardless of the liquidation value l because $\alpha_B = 0$, the size of the inefficient liquidation is $\phi\pi$. Hence, by choosing $l_B^* = \hat{l}$ the manager avoids any cost of productive inefficiencies ($D(\hat{l}) = 0$) and the cost of inefficient liquidations is the maximum.

Furthermore, β_B^* is the minimum level of creditor control rights that deters strategic default by the manager and consequently makes the lender be willing to provide credit. As creditors liquidate the firm when they are granted control over it, β_B^* also minimises the probability that an inefficient liquidation occurs. In other words, if $\beta_B < \beta_B^*$, the contract cannot be signed under the bankruptcy institution, while if $\beta_B > \beta_B^*$ the contract can be signed but the likelihood of an

[25] Nevertheless, Appendix C shows that the model's key results do not change qualitatively when we allow for some positive credit recovery under bankruptcy, i.e., $0 < \alpha_B < \alpha_F$.

inefficient liquidation is not minimised, implying that welfare is lower than in the case of optimal bankruptcy code ($\beta_B = \beta_B^*$). Let us summarise these results in Lemma 3 and Proposition 4.

Lemma 3. *β_B^* is the minimum level of creditor rights in bankruptcy that makes the lender provide credit. If $\beta_B < \beta_B^*$, then the contract is not feasible under the bankruptcy institution.*

Proposition 4. *If $\beta_B > \beta_B^*$, then the contract is feasible under bankruptcy. In that case the optimal contract $\{R_B^{**}, l_B^{**}\}$ is given by $R_B^{**} = \frac{I}{\theta}, l_B^{**} = \hat{l}$. Equilibrium welfare is $W_B^{**} = (\theta + \phi)\pi - I - (1-\theta)\beta_B \phi \pi$*

2.2.2 Optimal contract under foreclosure

The analysis of the optimal contract under foreclosure differs from that under bankruptcy in two key points: $\alpha_F > 0$ and $\beta_M = 1$. $\alpha_F > 0$ makes the decision of overinvesting in capital non-trivial, since increasing the liquidation value l_F reduces funding costs R_F but at the expense of incurring in productive inefficiencies that reduce the cash flow at $t = 1$ by $n|l_F - \hat{l}|$. $\beta_F = 1$ maximises the likelihood of inefficient liquidations, since the firm will be liquidated with certainty following default, but it also maximises creditor protection and hence the incentives to lend.

The optimal contract and equilibrium welfare under foreclosure are summarised in Proposition 5.

Proposition 5. *The optimal contract $\{R_F^*, l_F^*\}$ and equilibrium welfare W_F^* under foreclosure when $n \leq \frac{1-\theta}{\theta}\alpha_F$ are:*

$$l_F^* = I,\ R_F^* = \frac{I - (1-\theta)\alpha_F I}{\theta},\ W_F^* = (\theta + \phi)\pi - I - (1-\theta)[\phi\pi - \alpha_F I] - \theta n\left(I - \hat{l}\right).$$

When the marginal cost from overinvesting in capital n is low enough *vis-à-vis* the efficiency of the foreclosure's liquidation technology α_F $(n \leq \frac{1-\theta}{\theta}\alpha_F)$,[26] the marginal reduction in funding

[26] We set $n \leq \frac{1-\theta}{\theta}\alpha_F$ to restrict the attention to scenarios where the choice between foreclosure and bankruptcy depends on the marginal cost of overinvestment in capital. As the maximisation problem is linear, there are two

costs is higher than the marginal cost of productive inefficiencies, so the manager overinvests as much as possible, obtaining the maximum liquidation value $l_F^* = I$. An important consequence is highlighted by the equilibrium welfare: the cost of inefficient liquidations $[\phi\pi - \alpha_F I]$ is minimised by setting the maximum liquidation value I, but at the expense of maximising the (expected) cost of productive inefficiencies $\theta n|I - \hat{l}|$.

3 Choice of insolvency institution

The firm's manager chooses to sign the credit contract with the lender under the insolvency institution that maximises her expected utility, which equals total welfare because the lender is perfectly competitive, so he makes zero profits. Hence the analysis relies in the comparison of equilibrium welfare in each of the scenarios described by Propositions 2, 4 and 5 and in the conditions under which contracts are feasible (Lemma 3).

The choice of the insolvency institution depends on three exogenous parameters: the marginal cost of productive inefficiencies n, the efficiency of the liquidation technology of foreclosure relative to bankruptcy α_F and the level of creditor control rights in bankruptcy β_B. Figure 3 summarises the results. In this section we provide the intuition behind the findings, relegating the full characterisation to Appendix D.

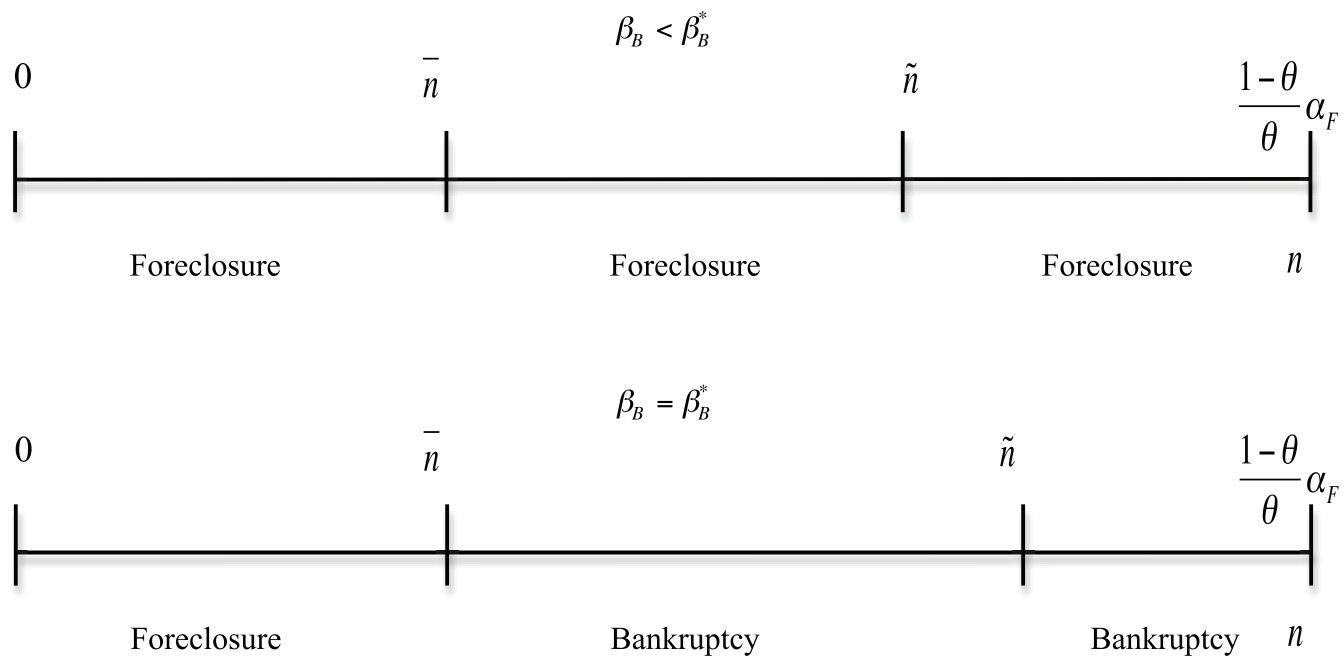
When creditor rights in bankruptcy are lower than the optimal ($\beta_B < \beta_B^*$) the firm's manager can only implement the project under the foreclosure institution, as any contract under bankruptcy is unfeasible (Lemma 3). When creditor rights in bankruptcy are greater than or equal to the optimal ($\beta_B \geq \beta_B^*$), both institutions are feasible, so the choice of the insolvency institution depends on the marginal cost from productive inefficiencies. Foreclosure is chosen for some sufficiently low values of n ($n \leq \bar{n}$ if $\beta_B = \beta_B^*$, $n \leq \widetilde{n}$ if $\beta_B > \beta_B^*$).[27] The intuition is straightforward: when the

corner solutions. When $n > \frac{1-\theta}{\theta}\alpha_F$ (the marginal cost from overinvesting in capital n is high *vis-à-vis* the efficiency of foreclosure's liquidation technology) $l_F^{**} = \hat{l}$, which means that the manager does not overinvest in capital. In that case, the choice of insolvency institution depends on the values of the creditor control rights in bankruptcy β_B, the efficiency of foreclosure's liquidation technology α_F and the first-best liquidation value $\hat{l}$. The model's main results, displayed in Section 4, are qualitatively the same. A detailed analysis of this scenario can be found in Appendix F.

[27] $\bar{n}$ and $\widetilde{n}$ are the values that make the manager's utility under foreclosure and under bankruptcy equal for

cost of overinvesting in capital is low enough, the manager chooses foreclosure because the gains from lower funding costs outweigh the costs of productive inefficiencies and the higher probability of liquidation in case of default. Notice also that, in the extreme case $\beta_B = 1$, bankruptcy is never chosen, as it has the same probability of inefficient liquidations as in foreclosure but a lower liquidation value.

Figure 3: Choice of insolvency institution.



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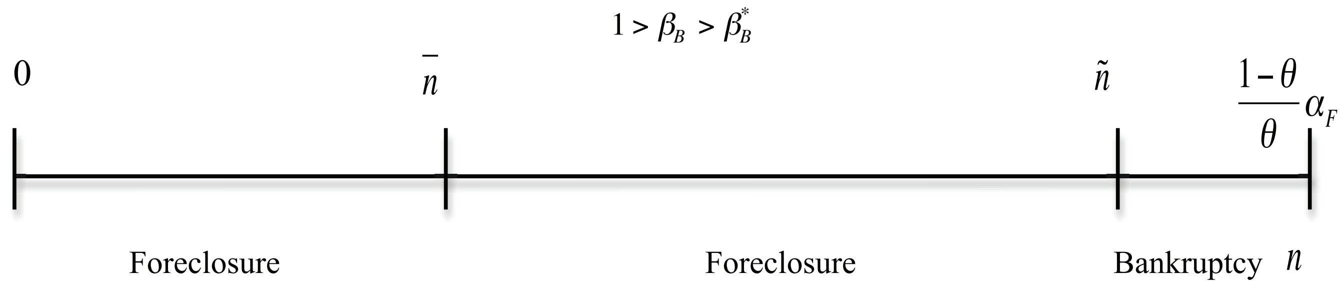
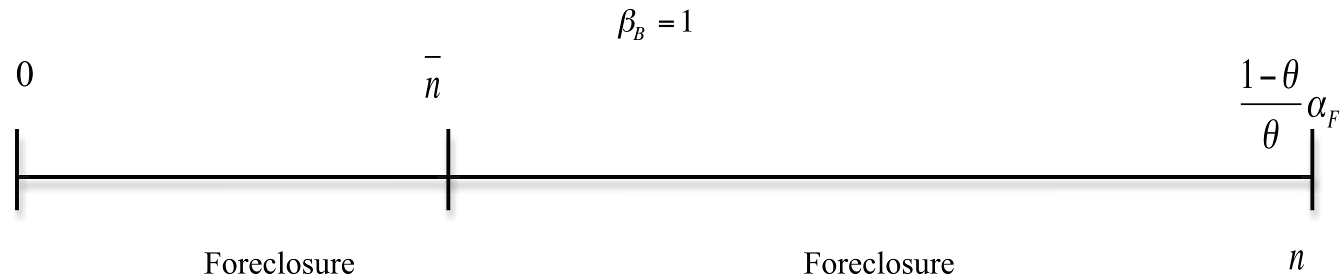
the optimal and the greater-than-optimal bankruptcy liquidation probability, respectively, i.e., $EU_F^* = EU_B^*$ if $n = \bar{n}$ and $\beta_B = \beta_B^*$; $EU_F^* = EU_B^{**}$ if $n = \widetilde{n}$ and $1 > \beta_B > \beta_B^*$, where $\bar{n} \equiv \min\left\{\frac{(1-\theta)\left[\frac{I}{\theta}+\alpha_F I-\phi\pi\right]}{\theta(I-\hat{l})}, \frac{1-\theta}{\theta}\alpha_F\right\}$ and $\widetilde{n} = \min\left\{\frac{(1-\theta)[\alpha_F I-\phi\pi(1-\beta_B)]}{\theta(I-\hat{l})}, \frac{1-\theta}{\theta}\alpha_F\right\}$.





As the contracts under bankruptcy and under foreclosure are special cases of the general contracting game analysed in section 2.1 (private workouts), Figures 4 and 5 illustrate the choice of insolvency institution in the space (α, β) for different values of the technological parameter n. Points B and F show the optimal contracts under bankruptcy and foreclosure, respectively. In Figure 4 n lies in the interval $(0, \bar{n}]$ and, accordingly, the welfare curve for foreclosure $\overline{W}_F$ is below that for optimal bankruptcy $\overline{W}_B$: welfare is higher if the contract is signed and enforced under the foreclosure institution. By contrast, in Figure 5 n lies in the interval $(\bar{n}, \frac{1-\theta}{\theta}\alpha_F]$, so the opposite occurs: the welfare curve for optimal bankruptcy is below the one for foreclosure.

Figure 4: welfare indifference curves in the (α, β) space with $0 < n \leq \bar{n}$

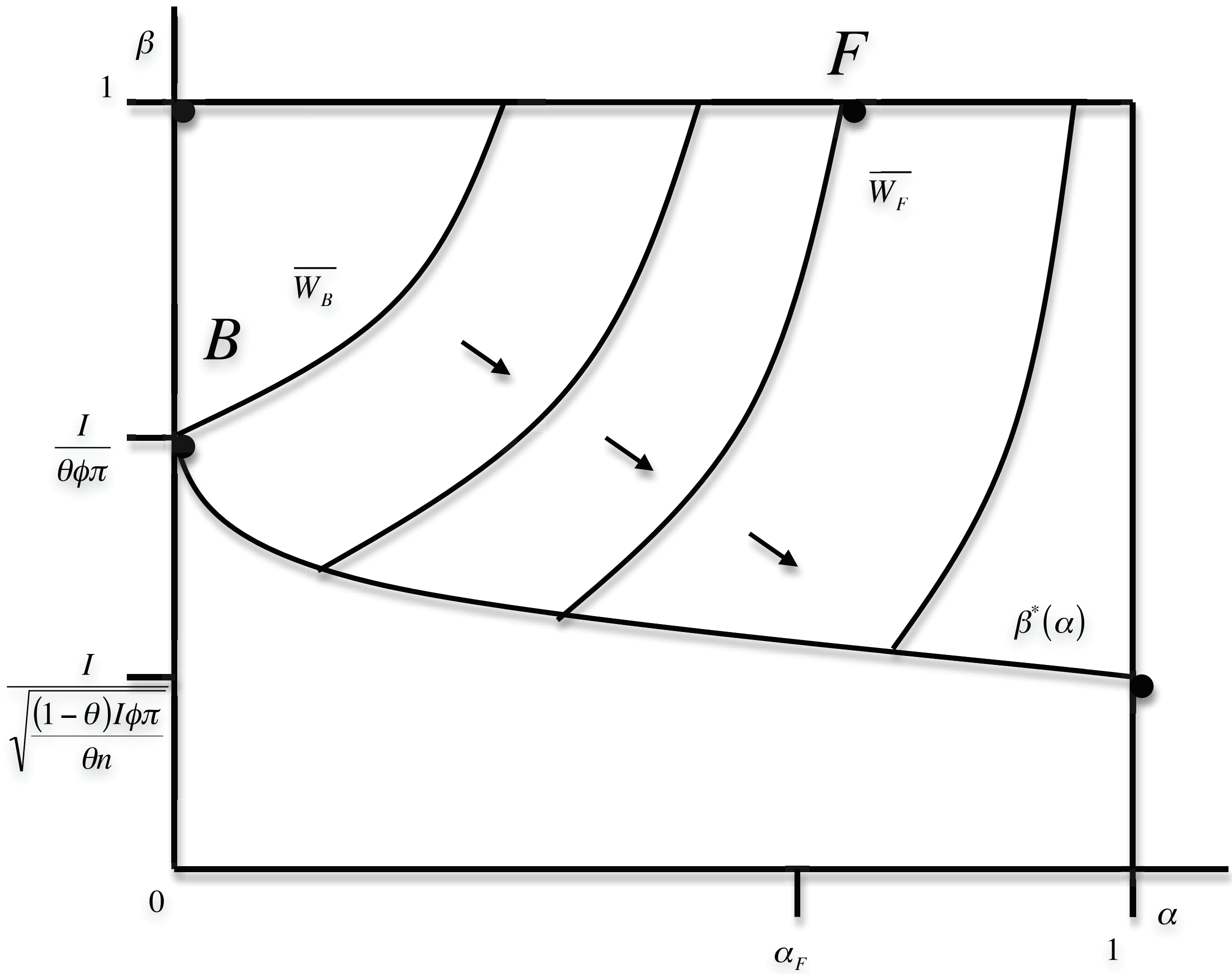
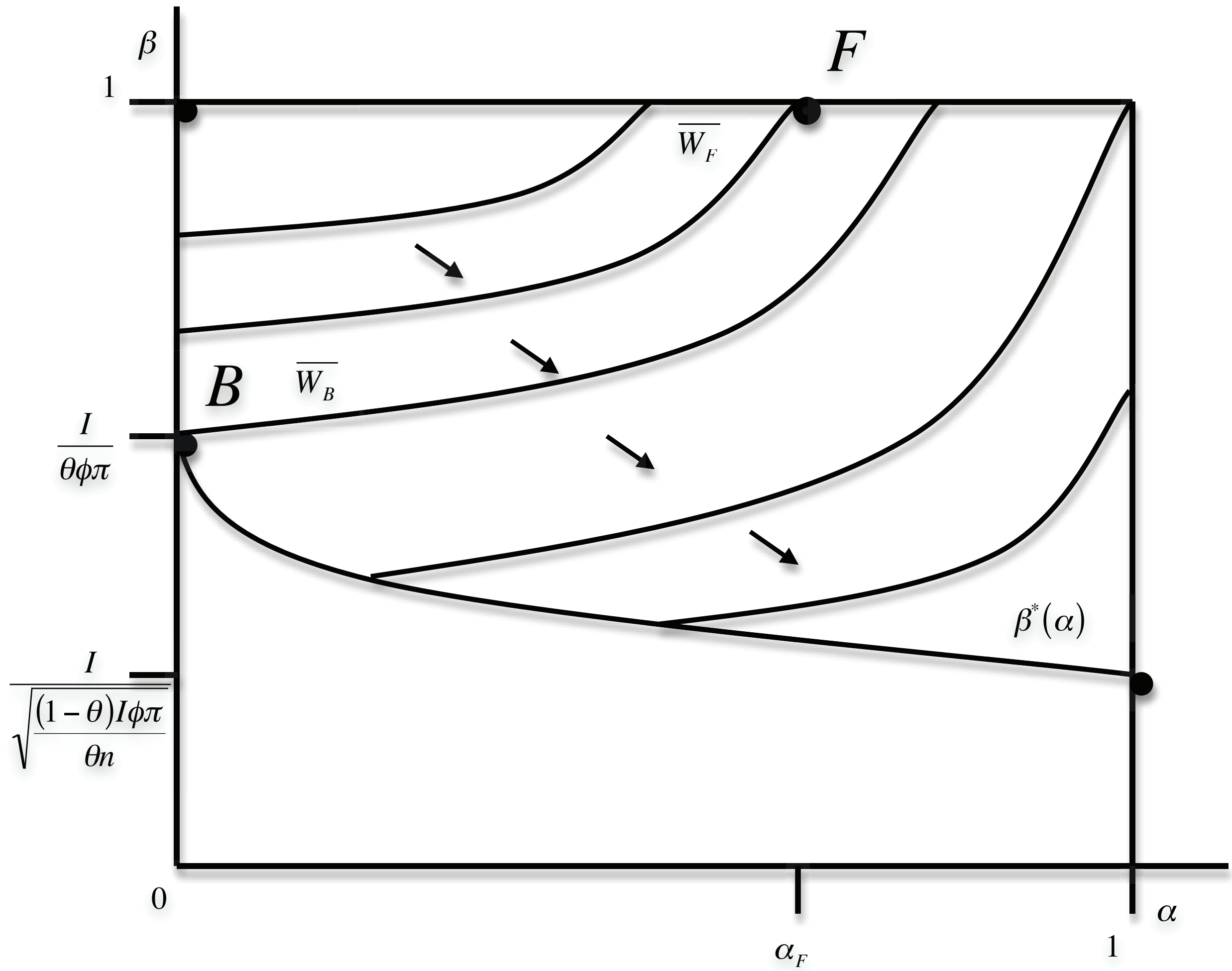


Figure 5: welfare indifference curves in the (α, β) space with $\overline{n} < n \leq \frac{1-\theta}{\theta}\alpha_F$



4 Aggregate welfare and bankruptcy rates

So far we have discussed the case of a single firm. For the analysis of aggregate welfare and bankruptcy rates -number of firms that file for bankruptcy divided by the stock of active firms- it is convenient to move to a set up of multiple heterogenous firm managers. Each manager can implement a project with the same net present value in the first best $((\theta + \phi)\pi - I)$ but that differs in the marginal cost of overinvesting in capital n. We consider a population of managers of measure N who are continuously and uniformly distributed with $n \in [N_1, N_2]$ and $N_2 - N_1 = N$.

Finally, there is a population of measure N of perfectly competitive lenders, each with an inital endowment of I, so that all the demand for credit by the managers can be met.

Figures 6 and 7, which depict aggregate welfare and bankruptcy rates as functions of the creditor rights under bankruptcy, show the main results. In this section we provide the intuition behind the findings, relegating the full characterisation to Appendix E.

Figure 6: welfare when $n \leq \frac{1-\theta}{\theta}\alpha_F \; \forall \; n \sim U[N_1, N_2]$

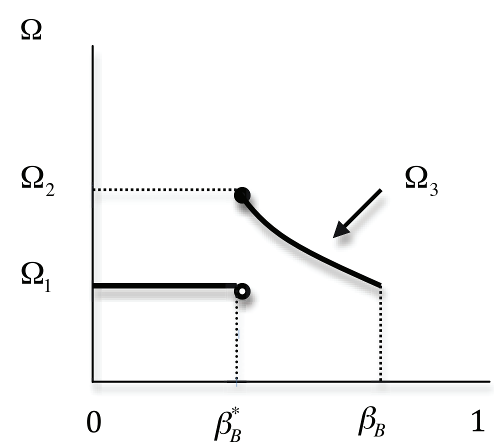
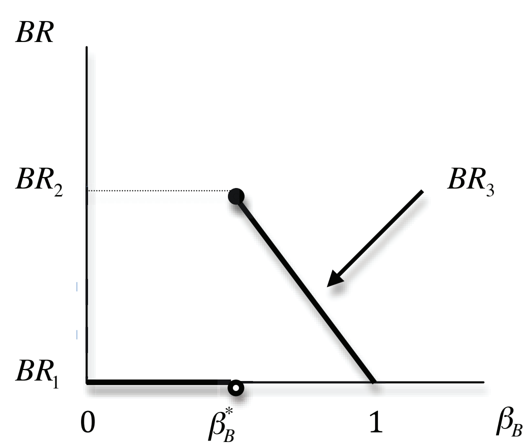


Figure 7: bankruptcy rates when $n \leq \frac{1-\theta}{\theta}\alpha_F \; \forall \; n \sim U[N_1, N_2]$



When creditor rights in bankruptcy are lower than the optimal ($\beta_B < \beta_B^*$) managers can only implement the project under foreclosure because any contract under bankruptcy is unfeasible, as stated in Lemma 3, implying that the bankruptcy rate is zero and welfare is at the low level Ω_1. When creditor rights in bankruptcy are equal to the optimal ($\beta_B = \beta_B^*$), some managers stick to foreclosure even though now bankruptcy is available, while those with projects with relatively higher costs of overinvesting in capital switch from foreclosure to bankruptcy, as they obtain a higher payoff with the latter. This is the case in which welfare and the bankruptcy rate are the highest (Ω_2 and BR_2, respectively). As creditor rights increase within the region $\beta_B > \beta_B^*$, less firms switch from foreclosure to bankruptcy, implying that welfare Ω_3 and bankruptcy rates BR_3 decrease. These results are formally stated in Proposition 6.

Proposition 6. *Welfare Ω is a non-monotonic function of the creditor control rights under bankruptcy β_B. It is maximum at their optimum level β_B^*, it is Ω_1 for $\beta_B < \beta_B^*$, it is decreasing in β_B for $\beta_B > \beta_B^*$ and it is Ω_1 for $\beta_B = 1$. The bankruptcy rate BR is a non-monotonic function of β_B. It is maximum at β_B^*, it is 0 for $\beta_B < \beta_B^*$, it is decreasing in β_B for $\beta_B > \beta_B^*$ and it is 0 for $\beta_B = 1$.*

The model's key result, that welfare is a non-monotonic function of the level of creditor control rights in bankruptcy, has a direct policy implication: a perfectly "creditor-friendly" bankruptcy code, i.e., one that always grants controls of the distressed firm to creditors ($\beta_B = 1$), is very inefficient. In fact, it is so inefficient that agents would never use it (i.e., the bankruptcy rate is zero) as the alternative insolvency institution, foreclosure, leads to the same number of inefficient liquidations but provides higher recovery rates to creditors and lower funding costs to firms.

A second result is that welfare is higher when the bankruptcy system is too "creditor- friendly", i.e., it ensures the provision of credit, but generates too many inefficient liquidations ($\beta_B > \beta_B^*$), than when it is too "debtor-friendly" ($\beta_B < \beta_B^*$). In other words, if the optimal level of creditor control rights in bankruptcy cannot be ascertained in practice, it may be better to grant too much control of the bankruptcy process to creditors[28] than too little, as the loss from undershooting that

[28]In practice, increasing creditor control rights can be achieved in different manners, such as letting creditors take over the bankrupt firm (e.g. administration in the U.K.), actively involving them in the appointment of the

level is larger than that from overshooting it.[29] The reason why welfare is higher when $\beta_B > \beta_B^*$ than when $\beta_B < \beta_B^*$ is that, when creditor rights exceed the optimal level, firms can choose between two different insolvency institutions, bankruptcy and foreclosure, in other to maximise their payoffs. However, when creditor rights are lower than the optimum, only one institution, foreclosure, can be used. As bankruptcy and foreclosure are imperfect substitutes and firms prefer one insolvency institution or the other depending on their marginal cost of overinvestment in capital, a too (but not perfectly) "creditor-friendly" bankruptcy system yields higher welfare.

For completeness, Figures 8 and 9 summarise welfare and bankruptcy rates when $n > \frac{1-\theta}{\theta}\alpha_F$, relegating its full characterisation to Appendix F. When $n > \frac{1-\theta}{\theta}\alpha_F$ (the marginal cost of over-investing in capital n is high vis-à-vis the efficiency of foreclosure's liquidation technology α_F) the manager does not overinvest in capital when using foreclosure, implying that the choice of insolvency institution does not depend on the marginal cost of overinvestment in capital. In that scenario the manager chooses foreclosure over bankruptcy if the gains from lower funding costs outweigh the higher probability of liquidation in case of default.[30]

As shown in Figure 8, the model's key result also holds in this scenario: welfare is a non-monotonic function of the creditor control rights in bankruptcy β_B. Specifically, welfare is Ω_4 for $\beta_B < \beta_B^*$, its maximum Ω_5 is achieved at $\beta_B = \beta_B^*$, it is $\Omega_6(\beta_B)$ with $\frac{\partial \Omega_6}{\partial \beta_B} < 0$ for the interval $(\beta_B^*, \overline{\beta}_B)$ and Ω_4 for the interval $\left[\overline{\beta_B}, 1\right]$. The policy implications are slightly different from the previous case. While it is still true that a *perfectly* "creditor-friendly" bankruptcy code ($\beta_B = 1$) is very inefficient, the same holds for *very* "creditor-friendly" codes ($\beta_B \geq \overline{\beta_B}$). Hence, welfare is

insolvency administrators (along the lines of the German and Italian systems) and allowing them to propose a liquidation plan -something that is forbidden in the French and Spanish bankruptcy codes, for instance- or even to impose it.

[29]To illustrate this idea more formally remember that, according to Proposition 2, the optimal level of creditor rights is $\beta_B^* = \frac{I}{\theta\phi\pi}$. Suppose that (I, θ, ϕ, π) are not parameters but random variables, so that β_B^* is also a random variable with expectation $E[\beta_B^*]$. If aggregate welfare $\Omega(\beta)$ was a symmetric function about β_B^*, then a risk-neutral legislator should set $\beta_B = E[\beta_B^*]$. However, as $\Omega(\beta_B > \beta_B^*) > \Omega(\beta_B < \beta_B^*)$, then the legislator must set $\beta_B = E[\beta_B^*] + \gamma$ with $\gamma > 0$ and $E[\beta_B^*] + \gamma < 1 = \beta_F$ to reduce welfare losses.

[30]Specifically, the choice of insolvency institution depends on the values of the creditor control rights in bankruptcy β_B, the efficiency of foreclosure's liquidation technology α_F and the first-best liquidation value $\hat{l}$. For the depiction of Figures 8 and 9 we have assumed $\beta_B^* \leq 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$ to rule out a trivial case. If the optimal level of creditor rights in bankruptcy is too high ($\beta_B^* > 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$), the manager always chooses foreclosure for any β_B, as the gains from using bankruptcy (less liquidations) are very low compared with the gains from using foreclosure (lower funding costs). In such a case, both welfare and bankruptcy rates are independent of β_B.

weakly higher under a too "creditor friendly" bankruptcy system than under a too "debtor friendly" one, implying that the loss from undershooting the optimal level of creditor rights *may* be larger than that from overshooting it. With respect to the bankruptcy rate, as shown in Figure 9, it is strictly positive for the interval $\left[\beta_B^{*}, \overline{\beta}_B\right)$ and zero elsewhere.

Figure 8: welfare when $n > \frac{1-\theta}{\theta}\alpha_F \ \forall \ n \sim U\left[N_1, N_2\right]$

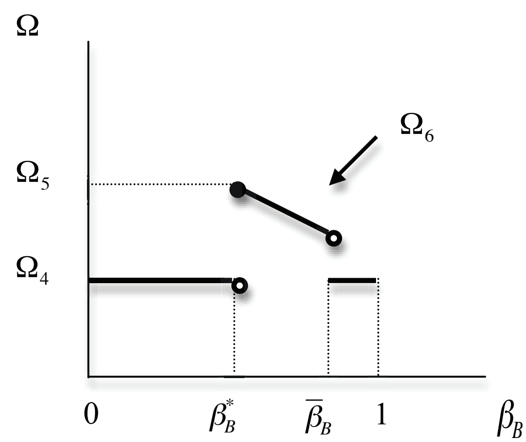
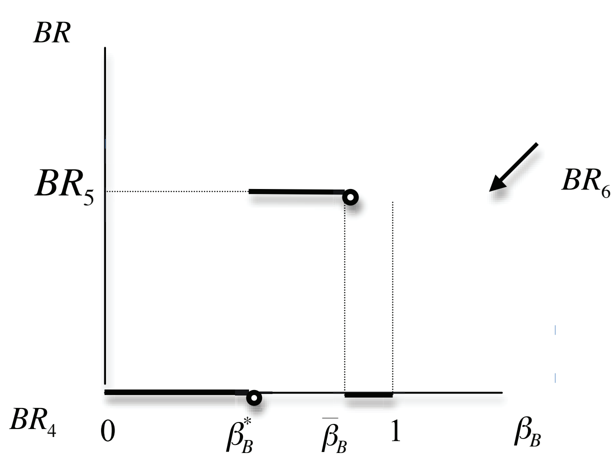


Figure 9: bankruptcy rate when $n > \frac{1-\theta}{\theta}\alpha_F \forall \ n \sim U\left[N_1, N_2\right]$



A related implication comes from the joint inspection of Figures 6 and 7 and Figures 8 and 9, which show a positive correlation between bankruptcy rates and welfare: when bankruptcy rates are high (low), welfare is high (low). This theoretical prediction seems corroborated by the empirical evidence if we proxy welfare with per capita GDP (Claessens and Klapper, 2005; Celentani *et al.*, 2010). While there are a number of alternative explanations for the correlation between bankruptcy rates and per capita GDP,[31] the mechanism analysed in this paper may explain *part* of such a correlation. Moreover, by showing that low bankruptcy rates may be associated with low levels of welfare, the model explains why policymakers should look at bankruptcy rates in a way that may seem counterintuitive: low bankruptcy rates are not always good news. More specifically, if they suspect that the bankruptcy law does not provide enough protection to creditors, then it makes sense to examine bankruptcy rates to determine if they are "too low" (for instance, by international standards).

5 Discussion

Many papers have studied whether creditor-friendly or debtor-friendly corporate bankruptcy systems perform better in several aspects such as ex-ante incentives (provision of credit, efficient risk taking) and ex-post outcomes (liquidation vs. reorganisation). However, the existing literature on optimal bankruptcy seems to assume that the bankruptcy system is the only insolvency *institution*, implying that, if private workouts fail -mainly because of coordination and asymmetric information problems- bankruptcy is unavoidable. But this assumption does not seem to be warranted by the empirical evidence, as many firms around the world resolve insolvency using other *formal* procedures such as foreclosures.

This paper aims to fill this gap by modeling two insolvency institutions -bankruptcy and foreclosure- that firms and their creditors may use when dealing with financial distress. Firms choose between one institution and the other based on lenders' willingness to provide credit and the trade-off between two inefficiency costs, those of ex-ante productive inefficiencies and those of ex-post inefficient liquidations. The trade-off occurs because firms may overinvest in capital to

[31] For instance, bureaucratic inefficiency and corruption may hamper both economic growth (Mauro, 1995; Sleiffer and Vishny, 1993) and the use of the bankruptcy system by distressed firms.

increase their assets' liquidation value, which reduces funding costs. This overinvestment generates productive inefficiencies but, by increasing the liquidation value, it reduces the cost of inefficient liquidations (going concern value minus liquidation value).

The model's key result is that welfare is a non-monotonic function of the level of creditor control rights in bankruptcy. This result has a direct policy implication: a *perfectly* "creditor-friendly" bankruptcy code, i.e., one that always grants controls of the distressed firm to creditors, is very inefficient. In fact, it is so inefficient that agents in the model never use it because the alternative insolvency institution, foreclosure, leads to the same number of inefficient liquidations but provides higher recovery rates to creditors and lower funding costs to firms.

A second result is that welfare is higher when the bankruptcy system is *too* "creditor- friendly", i.e., it ensures the provision of credit, but generates too many inefficient liquidations, than when it is *too* "debtor-friendly". The intuition is that, as the optimal level of creditor *control* rights is the minimum that makes the credit contract feasible under bankruptcy, when creditor rights exceed the optimal level, the two insolvency institutions can be used, so that each firm chooses the one that maximises its payoffs. However, when creditor rights are lower than the optimum, only one institution, foreclosure, can be used. As bankruptcy and foreclosure are imperfect substitutes and firms differ in their preferred insolvency institution, a too "creditor-friendly" bankruptcy system may yield higher welfare. Hence, if the optimal level of creditor control rights in bankruptcy cannot be ascertained in practice, it may be better to grant too much control of the bankruptcy process to creditors than too little, as the loss from undershooting that level is larger than that from overshooting it.

A related result is the positive correlation between welfare and bankruptcy rates: when creditor rights are high (low), welfare is high (low) and bankruptcy rates are high (low). This theoretical prediction seems corroborated by the empirical evidence if we proxy welfare with per capita GDP (Claessens and Klapper, 2005; Celentani *et al.*, 2010). While there are a number of alternative explanations for the correlation between bankruptcy rates and per capita GDP, the mechanism analysed in this paper may explain part of such a correlation. Moreover, by showing that low bankruptcy rates may be associated with low levels of welfare, the model warns that low bankruptcy rates, such as those observed in the countries in Southern Europe[32], may be the

[32]See Celentani *et al.* (2010) and García-Posada and Mora-Sanguinetti (2014).

symptom of a real malady. Hence, if policy makers suspect that the bankruptcy law does not provide enough protection to creditors, then they should assess whether bankruptcy rates are "too low" (for instance, by international standards).

This very stylised model can be extended in several dimensions that yield additional insights. First, we may allow for some positive credit recovery in bankruptcy, i.e., $0 < \alpha_B < \alpha_F$ (see Appendix C). In that case β_B^* depends on the technological parameter n, i.e., $\beta_B^*(n)$. Specifically, the higher the marginal cost of overinvesting in capital n, the more "creditor-friendly" the optimal bankruptcy code should be, i.e., $\frac{\partial \beta_B^*}{\partial n} > 0$. In practice, however, bankruptcy laws cannot be individually tailored to firm characteristics such as technology. Suppose that firms are homogeneous in all dimensions except in their n and that the same β_B must be applied to all firms. For simplicity, assume that there are only two firms, with n_H and n_L such as $n_H > n_L$ (high-cost and low-cost, respectively). As $\frac{\partial \beta_B^*}{\partial n} > 0$, $\beta_B^*(n_H) > \beta_B^*(n_L)$. If $\beta_B \geq \beta_B^*(n_H)$, then both firms can use bankruptcy, but it leads to too many inefficient liquidations for the low-cost firm. By contrast, if $\beta_B^*(n_H) > \beta_B \geq \beta_B^*(n_L)$, then only the low-cost firm can use bankruptcy, while the high-cost firm needs to use foreclosure, even though foreclosure is not well-suited to firms with high costs of overinvesting in capital. A social planner, when setting the bankruptcy code β_B, must take into account these trade-offs in order to maximise the total welfare created by the two firms.

A second extension is to include firm size in the model, as the available evidence shows that small firms rarely file for bankruptcy and rely in alternative debt-enforcements mechanisms (Claessens and Klapper, 2005; Morrison, 2008, 2009; García-Posada and Mora-Sanguinetti, 2014). While there are several potential explanations for this phenomenon (low coordination costs because of few creditors, low asymmetric information problems due to "relationship lending") probably the most uncontroversial observation is that bankruptcy procedures are expensive and that a substantial part of the costs are fixed. [33] Hence we may assume that the firm's project requires an initial outay $I \in (0, +\infty)$ and yields cash-flows $\pi I - n(l - \hat{l})$ (with $l = \gamma I$) at $t = 1$ and πI at $t = 2$. The probabilities of project success, as well as other features, are the same as in the basic model. The

[33] According to Bris *et al.* (2006) and their large sample of bankruptcies in the U.S., among firms with assets worth less than \$ 100,000, the median direct costs burn 23.2% of asset value. Among firms with assets worth between \$ 100,000 and \$ 1 million, the median direct costs consume 4.9% of asset value.

only other departure from that model is that the lender must pay a cost $K > 0$ for the bankruptcy procedure to take place. K is fixed, i.e., it does not depend on the firm's size I. Solving the model[34] it can be shown that $\frac{\partial \beta_B^*}{\partial I} < 0$, i.e., the larger the firm, the lower the optimal creditor control rights in bankruptcy. Suppose, for simplicity, only two firms with size I_S and I_L with $I_L > I_S$ (small and large, respectively). Since $\frac{\partial \beta_B^*}{\partial I} < 0$ then $\beta_B^* (I_L) < \beta_B^* (I_S)$. If the actual bankruptcy code β_B is $\beta_B^* (I_L) \leq \beta_B < \beta_B^* (I_S)$, then bankruptcy is only feasible for the large firm, implying that, while that firm can choose between bankruptcy and foreclosure to maximise its profits, the small firm has to obtain credit through the foreclosure system, even if it found more profitable to use bankruptcy (i.e., if its cost of overinvestment in capital n is quite high). By contrast, if the bankruptcy code β_B is $\beta_B \geq \beta_B^* (I_S) > \beta_B^* (I_L)$, then bankruptcy is also feasible for the small firm, implying that both firms are able to choose between bankruptcy and foreclosure to maximise its profits. Hence, a "too-creditor friendly" bankruptcy system ($\beta_B \geq \beta_B^* (I_S)$) allows for more insolvency options than a "too-debtor friendly" one ($\beta_B < \beta_B^* (I_S)$), in analogous fashion to the findings of the base model.

Nevertheless, the present analysis assumes away an important factor in the optimal design of bankruptcy laws and, especifically, its degree of "creditor friendliness": the efficient allocation of risk between risk neutral creditors and risk averse entrepreneurs and small firms. When the distressed business is a large limited liability firm, the assumption of risk-neutrality is quite plausible but, in the case of an entrepreneur who has unlimited liability, risk aversion seems more appropriate. The same may hold for small corporate firms because lenders often require personal guarantees or security in the form of a second mortgage on the owner's home, which wipes out the owner's limited liability (Berkowitz and White, 2004). As a consequence, those businesses often use *personal,* rather than corporate, bankruptcy. The optimal degree of "creditor friendliness" of personal bankruptcy mainly depends on the trade-off between two effects. On the one hand, creditor-friendly personal bankruptcy laws minimise the scope for moral hazard and they may in turn reduce the risk premium charged to entrepreneurs and facilitate their access to credit (the "credit supply effect"). On the other hand, debtor-friendly laws (i.e., those that allow a "fresh start" or set high exemption levels) provide partial insurance against business failure (the "insurance effect"), which may incentive risk-averse agents to undertake entrepreneurial activities. The available evidence suggests that the second effect dominates the first one [Fan and White (2003), Armour and Cumming (2008), Fossen

[34] See Supplement C.

(2014)], implying that at least some features of debtor-friendly bankruptcy laws are desirable in order to promote entrepreneurship.[35]

For future research, we could study whether some insolvency frameworks favour industries with low NPV but low cost of overinvesting in capital at the expense of deterring others with higher NPV but higher costs of overinvestment. By doing so we would analyse the "extensive margin" of the overinvestment in capital, rather than its "intensive margin", as done in the present model. This may have practical implications. For instance, according to Banco de España (2010) and Arce *et al.* (2013), in Spain the less productive sectors such as construction would have benefited from the strong credit growth between 1995 and 2007, one of the reasons being that those sectors produce assets that can be used as collateral on loans.[36]

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[35] However, the impact of personal bankruptcy laws on innovation is less clear. Cerqueiro *et al.* (2014) find that debtor-friendly laws hamper innovation due to the reduction in the credit supply to inventors and small firms. By contrast, Armour (2004) argues that too creditor-friendly laws may deter venture capital and, in turn, innovation, because he finds a negative correlation between the degree of "creditor friendliness" and venture capital, which attributes to the lower demand for venture capital by entrepreneurs in countries where personal bankruptcy is more creditor-friendly.

[36] Another paper close in spirit to these ideas is that of Araujo and Minetti (2011).

6 Appendix A: proof of $l \geq \hat{l}$

In solving all the paper's maximisation programs we have used the fact that $l \geq \hat{l}$, which simplifies $D(l)$ to $D(l) = n\left(l - \hat{l}\right)$.

The reason why $l \geq \hat{l}$ is the following. The manager may have incentives to deviate from the optimal proportion of capital, hence incurring in productive inefficiencies, if by doing so she increases the project's liquidation value l and consequently the recovery rate of the lender in the event of default, which decreases the required repayment R. This mechanism can be observed from the inspection of the lender's payoff function in the case of non-strategic default, recalling that he is perfectly competitive:

$$\theta R + (1-\theta)\,\beta\alpha l - I = 0$$

From the above equation one can see that a higher value of l yields, *ceteris paribus*, a lower value of R. Moreover, the manager would never choose a proportion of capital such that $l < \hat{l}$, because she would incurr in productive inefficiencies with cost $D(l) > D(l = \hat{l}) = 0$ and she would also have a higher funding cost R than if choosing $l = \hat{l}$. Thus, the only relevant cases are the ones in which $l \geq \hat{l}$.

7 Appendix B: proofs of lemmas and propositions

Proposition 1. *The optimal contract* $\{R^*, \beta^*, l^*\}$ *in the case of distress resolution via a private workout is:*

(i) If $0 < \alpha \leq 1$. $R^* = \phi\pi\beta^*, \beta^* = \frac{I}{\sqrt{\frac{I\phi\pi(1-\theta)\alpha}{\theta n}}}$, $l^* = \sqrt{\frac{I\phi\pi}{\theta(1-\theta)n\alpha}} - \frac{\theta\phi\pi}{(1-\theta)\alpha}$. *The contract is feasible if* $I \in \left[\frac{\theta^3\phi n\pi}{(1-\theta)\alpha}, \frac{(1-\theta)\alpha\phi\pi}{\theta n}\right]$.

(ii) If $\alpha = 0$. $R^* = \frac{I}{\theta}$,$\beta^* = \frac{I}{\theta\phi\pi}$, $l^* = \hat{l}$.

Proof. We use the result of Appendix A, according to which $l \geq \hat{l}$, so that $D(l)$ can be rewritten as $D(l) = n\left(l - \hat{l}\right)$. Let us start with the proof of (i). First, notice that the individual rationality constraint of the lender (2) is binding at the optimum because he is perfectly competitive. Suppose to the contrary that (2) is slack. In such a case lowering R would increase EU because $\frac{\partial EU}{\partial R} < 0$ and it would make (1) and (3) hold *a fortiori*. Second, notice that the incentive compatibility constraint of the manager (1) is also binding at the optimum. Suppose to the contrary that (1) is slack. In such a case we could lower β to $\beta' = \beta - \varepsilon$ where $\varepsilon > 0$. To keep (2) binding we need to increase R to $R' = R + \frac{1-\theta}{\theta}\alpha l\varepsilon$. The old utility is $V_0 \equiv \theta\left[\pi - n(l-\hat{l}) - R + \phi\pi\right] + (1-\theta)(1-\beta)\phi\pi$. The new utility is $V_1 \equiv \theta\left[\pi - n(l-\hat{l}) - \left(R + \frac{1-\theta}{\theta}\alpha l\varepsilon\right) + \phi\pi\right] + (1-\theta)\left[1-(\beta-\varepsilon)\right]\phi\pi$. The new utility is higher than the old utility because $V_1 - V_0 > 0 \iff -(1-\theta)\alpha l\varepsilon + (1-\theta)\phi\pi\varepsilon > 0 \iff \phi\pi > \alpha l$ which is true by assumption. Therefore, (1) cannot be slack at the optimum since there would be a pair β', R' that would increase the manager's utility without violating the lender's individual rationality constraint. Third, the first feasibility constraint (3) does not bind, so we can ignore it. To see that, rearrange (3): $R + n(l-\hat{l}) \leq \pi$. As lowering both R and l increases EU and it makes (3) hold *a fortiori,* (3) does not bind.[37] Combining (1) and (2) we can express the liquidation probability β and the repayment R as functions of the liquidation value l, i.e., $\beta(l) = \frac{I}{\theta\phi\pi+(1-\theta)\alpha l}$ and $R(l) = \frac{I\phi\pi}{\theta\phi\pi+(1-\theta)\alpha l}$. Plugging those expressions into the manager's utility and the remaining constraint (4) we have the following maximisation problem:

$$\max_{l} EU = \theta\left[\pi - n(l-\hat{l})\right] + \phi\pi - \frac{I\phi\pi}{\theta\phi\pi + (1-\theta)\alpha l}$$

s.t.:

$$I \leq \theta\phi\pi + (1-\theta)\alpha l \tag{6}$$

Since $I \leq \theta\phi\pi$ and $l \geq 0$ by construction, (6) is always satisfied for any value of l, so we can ignore it as well. We then face an unconstrained maximisation program, whose solutions are $l = -\frac{\theta\phi\pi}{(1-\theta)\alpha} \pm \sqrt{\frac{I\phi\pi}{\theta(1-\theta)n\alpha}}$. However, since $l \geq 0$, we can rule out the negative root, so the unique solution is $l^* = \sqrt{\frac{I\phi\pi}{\theta(1-\theta)n\alpha}} - \frac{\theta\phi\pi}{(1-\theta)\alpha}$. Plugging l^* into the above expressions for $\beta(l)$ and $R(l)$ we find $\beta^* = \frac{I}{\sqrt{\frac{I\phi\pi(1-\theta)\alpha}{\theta n}}}$ and $R^* = \phi\pi\beta^*$. By differentianting EU twice with respect to l we find $\frac{\partial^2 EU}{\partial l^2} < 0$,

[37] But we also need to check that, if (3) is not binding, i.e., $R + n(l-\hat{l}) < \pi$, the lender's individual rationality constraint (2) still holds. Rearrange (2): $R \geq \frac{1}{\theta}[I - (1-\theta)\beta\alpha l]$. Putting together the two conditions: $\frac{1}{\theta}[I - (1-\theta)\beta\alpha l] \leq R < \pi - n(l-\hat{l})$. This holds as long as $\pi - n(l-\hat{l}) > \frac{1}{\theta}[I-(1-\theta)\beta\alpha l]$, which can be rearranged to $\pi - \frac{I}{\theta} + n\hat{l} \geq l\left[n - \frac{1-\theta}{\theta}\beta\alpha\right]$. A sufficient (but not necessary) condition for that is $n \leq \frac{1-\theta}{\theta}\beta\alpha$; in that case, as $I \leq \theta\phi\pi$ and $l \geq 0$ the condition is always satisfied because its RHS≤ 0 and its LHS≥ 0.

i.e, the function is concave and $EU(l^*)$ is its maximum. Finally, for the contract to be feasible we need: $0 \leq \beta^* \leq 1$ (I); $l^* \geq 0$ (II); $R^* \geq 0$ (III). While any parameter value satifies (III), we need $I \leq \frac{(1-\theta)\alpha\phi\pi}{\theta n}$ to make (I) hold and $I \geq \frac{\theta^3\phi n\pi}{(1-\theta)\alpha}$ to make (II) hold. Putting these two conditions together, what we need is $I \in \left[\frac{\theta^3\phi n\pi}{(1-\theta)\alpha}, \frac{(1-\theta)\alpha\phi\pi}{\theta n}\right]$ (which is not an empty set as long as $n < \frac{1-\theta}{\theta^2}\alpha$).

For the proof of (ii), we again make use of the fact that the individual rationality constraint of the lender (2) and the incentive compatibility constraint of the manager (1) are binding at the optimum. Because $\alpha = 0$, l disappears from (2), implying that (1) and (2) make up a system of two equations and two unknowns, R and β, whose solutions are $\beta^* = \frac{I}{\theta\phi\pi}$ and $R^* = \frac{I}{\theta}$. Plugging those expressions into the manager's utility and the remaining constraints we have the following maximisation problem:

$$\max_{l} EU = \theta\left[\pi - n(l - \hat{l})\right] + \phi\pi - \frac{I}{\theta}$$

s.t.:

$$\left[\pi - n(l - \hat{l})\right] - \frac{I}{\theta} \geq 0 \tag{7}$$

$$I \leq \theta\phi\pi \tag{8}$$

Since $\frac{\partial EU}{\partial l} < 0$ and a low l makes (7) easier to hold, we have a corner solution: $l^* = \hat{l}$. Plugging $l^* = \hat{l}$ into (7) and rearranging it becomes $I \leq \theta\pi$. As $I \leq \theta\phi\pi$, (8) is satisfied by construction while (7) is satisfied because $0 < \phi < 1$. Finally, for the contract to be feasible we need: $0 \leq \beta^* \leq 1$ (I); $l^* \geq 0$ (II); $R^* \geq 0$ (III). Any parameter value satifies (II) and (III) and (I) always holds as we have assumed $I \leq \theta\phi\pi$.

□

Lemma 3. *β_B^* is the minimum level of creditor rights in bankruptcy that makes the lender provide credit. If $\beta_B < \beta_B^*$, then the contract is not feasible under the bankruptcy institution.*

Proof. In the proof of Proposition 1 we found that the manager's incentive-compatibility constraint (1) was binding at the optimum. As the manager's maximisation problem for bankruptcy is the same as the one for private workouts when $\alpha = 0$, the constraint is: $\beta_B^*\phi\pi = R_B$. If $\beta_B < \beta_B^*$, then $\beta_B\phi\pi < R_B$, i.e., the incentive-compatibility constraint is violated. The only way we could make the constraint hold again would be lowering R_B. However, in the proof of Proposition 1 we found that the individual rationality constraint of the lender (2) was also binding at the optimum, so that $R_B^* = \frac{I}{\theta}$ is both the optimal and the minimum feasible repayment and we cannot lower R_B below that value without violating the constraint.

□

Proposition 4. *If $\beta_B > \beta_B^*$, then the contract is feasible under the bankruptcy institution. In that case the optimal contract $\{R_B^{**}, l_B^{**}\}$ is given by $R_B^{**} = \frac{I}{\theta}, l_B^{**} = \hat{l}$. The equilibrium welfare is $W_B^{**} = (\theta + \phi)\pi - I - (1-\theta)\beta_B\phi\pi$*

Proof. We use the result of Appendix A, according to which $l \geq \hat{l}$, so that $D(l)$ can be rewritten as $D(l) = n\left(l - \hat{l}\right)$. The optimal contract under non-optimal bankruptcy ($\beta_B > \beta_B^*$) maximises the manager's expected utility EU_B subject to the following constraints: (9) the manager's incentive compatibility; (10) the lender's individual rationality; (11) the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature.

$$\max_{R_B, l_B} EU_B = \theta\left[\pi - n(l_B - \hat{l}) - R_B + \phi\pi\right] + (1-\theta)(1-\beta_B)\phi\pi$$

s.t.:

$$\beta_B\phi\pi \geq R_B \tag{9}$$

$$\theta R_B \geq I \tag{10}$$

$$R_B \leq \pi - n(l_B - \hat{l}) \tag{11}$$

First, notice that the lender's individual rationality constraint (10) is binding at the optimum. Suppose to the contrary that (10) is slack. In such a case lowering R_B would increase EU_B because $\frac{\partial EU_B}{\partial R_B}<0$ and it would make (9) and (11) hold *a fortiori*. Hence $R_B^{**} = \frac{I}{\theta}$. Since $\frac{\partial EU}{\partial l_B} < 0$ and lowering l_B makes (11) hold *a fortiori,* we have $l_B^{**} = \hat{l}$. As shown in the proof of Lemma 3, $\beta_B^*\phi\pi = R_B$, which implies that $\beta_B > \beta_B^*$ makes the incentive compatibility constraint be slack. Plugging R_B^{**} into that constraint we have $\beta_B\phi\pi > \frac{I}{\theta}$. Rearranging it we find a constraint for the initial outlay I: $I < \beta_B\theta\phi\pi$. Another constraint for I arises from plugging R_B^{**} and l_B^{**} into (11) and rearranging: $I \leq \theta\pi$. Since $0 \leq \beta_B \leq 1$ and $0 < \phi < 1$, if $I < \beta_B\theta\phi\pi$ holds, then

$I \leq \theta\pi$ must also hold, so we can ignore the latter. To see that $I < \beta_B \theta\phi\pi$ holds recall that we have $\beta_B > \beta_B^* = \frac{I}{\theta\phi\pi}$, which satisfies the constraint by construction. Finally, plugging $R_B^{**} = \frac{I}{\theta}$ and $l_B^{**} = \hat{l}$ into the manager's expected utility and using the fact the lender breaks even we find $W_B^{**} = (\theta + \phi)\,\pi - I - (1-\theta)\,\beta_B\phi\pi$.

□

Proposition 5. The optimal contract $\{R_F^*, l_F^*\}$ and the equilibrium welfare W_F^* under foreclosure when $n \leq \frac{1-\theta}{\theta}\alpha_F$ are:
$l_F^* = I$, $R_F^* = \frac{I-(1-\theta)\alpha_F I}{\theta}$, $W_F^* = (\theta+\phi)\,\pi - I - (1-\theta)\,[\phi\pi - \alpha_F I] - \theta n(I - \hat{l})$.

Proof. We use the result of Appendix A, according to which $l \geq \hat{l}$, so that $D(l)$ can be rewritten as $D(l) = n\left(l - \hat{l}\right)$. The optimal contract maximises the manager's expected utility EU_F subject to the following constraints: (12) manager's incentive compatibility; (13) lender's individual rationality; (14) the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature (feasibility constraint).

$$\max_{R_F, l_F} EU_F = \theta\left[\pi - n(l_F - \hat{l}) - R_F + \phi\pi\right]$$

s.t.:

$$\pi - n(l_F - \hat{l}) - R_F + \phi\pi \geq \pi - n(l_F - \hat{l}) \quad (12)$$

$$\theta R_F + (1-\theta)\alpha_F l_F \geq I \quad (13)$$

$$R_F \leq \pi - n(l_F - \hat{l}) \quad (14)$$

First, the manager's incentive compatibility constraint (12) is not binding at the optimum. Too see this, simplify and rearrange (12) to obtain $R_F \leq \phi\pi$. Since $\frac{\partial EU_F}{\partial R_F} < 0$ and by lowering R_F (14) holds *a fortiori,* one would like to decrease R_F as much as possible to increase EU_F, which implies that (12) is not binding. But we also need to check that, if (12) is not binding, i.e., $R_F < \phi\pi$, the lender's individual rationality constraint (13) still holds. Rearrange (13): $R_F \geq \frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F]$. Putting together the two conditions: $\frac{1}{\theta}\,[I - (1-\theta)\,\alpha_F l_F] \leq R_F < \phi\pi$. This holds as long as $\phi\pi > \frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F]$, which is true because $I \leq \theta\phi\pi$ and $l_F \geq 0$. Second, the feasibility constraint (14) is not binding at the optimum. To see this, rearrange (14) to obtain $R_F + n(l_F - \hat{l}) \leq \pi$. Since $\frac{\partial EU_F}{\partial R_F} < 0$ and $\frac{\partial EU_F}{\partial l_F} < 0$ and by lowering R_F and l_F (14) holds *a fortiori,* one would like to decrease R_F and l_F as much as possible to increase EU_F, which implies that (14) is not binding. But we also need to check that, if (14) is not binding, i.e., $R_F + n(l_F - \hat{l}) < \pi$, the lender's individual rationality constraint (13) still holds. Rearrange (13): $R_F \geq \frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F]$. Putting together the two conditions: $\frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F] \leq R_F < \pi - n(l_F - \hat{l})$. This holds as long as $\pi - n(l_F - \hat{l}) > \frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F]$, which can be rearranged to$\pi - \frac{I}{\theta} + n\hat{l} \geq l_F \left[n - \frac{1-\theta}{\theta}\alpha_F\right]$. Because $n \leq \frac{1-\theta}{\theta}\alpha_F$, $I \leq \theta\phi\pi$ and $l_F \geq 0$ the previous condition is always satisfied, as its RHS≤ 0 and its LHS≥ 0. Third, the lender's individual rationality constraint (13) is binding at the optimum. Suppose to the contrary that (13) is slack. In such a case lowering R_F and l_F would increase EU_F, since $\frac{\partial EU_F}{\partial R_F} < 0$ and $\frac{\partial EU_F}{\partial l_F} < 0$, and it would make (12) and (14) hold *a fortiori*. Now rearrange (13) to find the repayment cost as function of the liquidation value: $R_F = \frac{I-(1-\theta)\alpha_F l_F}{\theta}$. Plugging this expression into EU_F and knowing that (12) and (14) are not binding, we have the following unconstrained maximisation program:

$$\max_{l_F} EU_F = \theta\left[\pi - n(l_F - \hat{l}) - \tfrac{I-(1-\theta)\alpha_F l_F}{\theta} + \phi\pi\right]$$

Differentiating EU_F with respect to l_F we have $\frac{\partial EU_F}{\partial l_F} = -\theta n + (1-\theta)\,\alpha_F$. The sign of the derivative depends on the relative value of n *vis-à-vis* α_F. As we have imposed $n < \frac{1-\theta}{\theta}\alpha_F$, we have $\frac{\partial EU_F}{\partial l_F}>0$ and a corner solution: $l_F^* = I$. Plugging l_F^* into (13) we find $R_F^* = \frac{I-(1-\theta)\alpha_F I}{\theta}$. Plugging those solutions into the manager's expected utility and using the fact that the lender breaks even we find the equilibrium welfare.

□

Proposition 6. *Welfare Ω is a non-monotonic function of the creditor control rights under bankruptcy β_B. It is maximum at their optimum level β_B^*, it is Ω_1 for $\beta_B < \beta_B^*$, it is decreasing in β_B for $\beta_B > \beta_B^*$ and it is Ω_1 for $\beta_B = 1$. The bankruptcy rate BR is a non-monotonic function of β_B. It is maximum at β_B^*, it is 0 for $\beta_B < \beta_B^*$, it is decreasing in β_B for $\beta_B > \beta_B^*$ and it is 0 for $\beta_B = 1$.*

Proof. See Apendix E. □

8 Appendix C: analysis when $\alpha_B > 0$

The aim of this section is to show that the model's results do not change qualitatively when we relax the assumption $\alpha_B = 0$ and we allow for some positive credit recovery under bankruptcy, i.e., $\alpha_B > 0$. Specifically, under bankruptcy the level of overinvestment in capital is (weakly) lower, i.e., $l_B^* \leq l_F^*$, as well as the probability of inefficient liquidations, i.e., $\beta_B^* \leq \beta_F^*$.

8.1 Optimal contract under bankruptcy and optimal bankruptcy code

In the baseline model, where $\alpha_B = 0$, the optimal bankruptcy code β_B^* does not depend on the marginal cost from productive inefficiencies n. If $\alpha_B > 0$ this is no longer the case. The maximisation program to find the optimal bankruptcy code β_B^*, together with the optimal contract $\{R_B^*, l_B^*\}$, is the following:

$$\max_{R_B,\beta_B,l_B} EU_B = \theta \left[\pi - n(l_B - \hat{l}) - R_B + \phi\pi\right] + (1-\theta)(1-\beta_B)\phi\pi$$

s.t.:

$$\pi - n(l_B - \hat{l}) - R_B + \phi\pi \geq \pi - n(l_B - \hat{l}) + (1-\beta_B)\phi\pi \tag{15}$$

$$\theta R_B + (1-\theta)\beta_B\alpha_B l_B \geq I \tag{16}$$

$$R_B \leq \pi - n(l_B - \hat{l}) \tag{17}$$

$$0 \leq \beta_B \leq 1 \tag{18}$$

$$l_B \geq \hat{l} \tag{19}$$

As the above program is identical to that of the base model (distress resolution via private workouts), we refer the reader to the proof of Proposition 1. The results are summarised in Proposition C1.

Proposition C1. *The optimal bankruptcy code β_B^* and the optimal contract $\{R_B^*, l_B^*\}$ when $\alpha_B > 0$ are:*
$\beta_B^* = \frac{I}{\sqrt{\frac{I\phi\pi(1-\theta)\alpha_B}{\theta n}}}$; $R_B^* = \phi\pi\beta_B^*$; $l_B^* = \sqrt{\frac{I\phi\pi}{\theta(1-\theta)n\alpha_B}} - \frac{\theta\phi\pi}{(1-\theta)\alpha_B}$. *The contract is feasible if* $I \in \left[\frac{\theta^3\phi n\pi}{(1-\theta)\alpha_B}, \frac{(1-\theta)\alpha_B\phi\pi}{\theta n}\right]$.

A couple of remarks are worth making regarding the optimal bankruptcy code β_B^*. First, β_B^* depends on the technological parameter n, which implies that the bankruptcy code is firm/industry specific. Second, the higher the marginal cost of overinvesting in capital n, the more "creditor-friendly" the optimal bankruptcy code should be (i.e., $\frac{\partial \beta_B^*}{\partial n} > 0$). In practice, however, bankruptcy laws cannot be individually tailored to firm characteristics such as technology. Suppose, as in section 4, that firms are homogeneous in all dimensions except for their marginal cost of overinvesting in capital n and that the same β_B must be applied to all firms. For simplicity of exposition, assume that there are only two firms, with n_H and n_L such as $n_H > n_L$ (high-cost and low-cost, respectively). As $\frac{\partial \beta_B^*}{\partial n} > 0$, $\beta_B^*(n_H) > \beta_B^*(n_L)$. If $\beta_B > \beta_B^*(n_H)$, then both firms can use the bankruptcy system. By contrast, if $\beta_B^*(n_H) > \beta_B > \beta_B^*(n_L)$, then only the low-cost firm can use the bankruptcy system, while the high-cost firm needs to use foreclosure. In setting the bankruptcy code β_B a social planner must maximise the total welfare created by the two firms. It is straightforward to show that one of two outcomes is optimal from a social planner's perspective, either $\beta_B = \beta_B^*(n_H)$ (both firms can use bankruptcy, but it leads to too many inefficient liquidations for the low-cost firm) or $\beta_B = \beta_B^*(n_L)$ (only the low-cost firm can use bankruptcy, which is its optimal bankruptcy procedure).[38]

Comparing bankruptcy and foreclosure, we find that $\beta_B^* \leq \beta_F = 1$, i.e., the liquidation probability is lower in bankruptcy than in foreclosure. Too see this notice that the optimal liquidation probability can be expressed as a function of the optimal liquidation value: $\beta_B^* = \frac{I}{\theta\phi\pi + (1-\theta)\alpha_B l_B^*}$. Since $I \leq \theta\phi\pi$ by assumption, $l_B^* \geq \hat{l} > 0$ and $\alpha_B > 0$, then $\beta_B^* < 1 = \beta_F$. With respect to the level of overinvestment in capital under bankruptcy, it is (weakly) lower than under foreclosure by construction, as $l_B^* \in \left[\hat{l}, I\right]$ (the higher n, the lower l_B^*) while we know that $l_F^* = I$ when $n \leq \frac{1-\theta}{\theta}\alpha_F$ (Proposition 5) and that $l_F^{**} = \hat{l}$ when $n > \frac{1-\theta}{\theta}\alpha_F$ (Appendix F).

[38] A similar conclusion is found in Ayotte and Yun (2009), although they study the heterogeneity in the project's NPV and the choice between bankruptcy and private workouts.

8.2 Optimal contracts under non-optimal bankruptcy code

8.2.1 Optimal contracts

The maximisation program to find the optimal contract for any bankruptcy code such that $\beta_B > \beta_B^*$ is the following:

$$\max_{R_B, l_B} EU_B = \theta \left[\pi - n(l_B - \hat{l}) - R_B + \phi\pi \right] + (1-\theta)(1-\beta_B)\phi\pi$$

s.t.:

$$\beta_B \phi\pi \geq R_B \tag{20}$$

$$\theta R_B + (1-\theta)\beta_B \alpha_B l_B \geq I \tag{21}$$

$$R_B \leq \pi - n(l_B - \hat{l}) \tag{22}$$

$$l_B \geq \hat{l} \tag{23}$$

This maximisation program is the generalisation of that for the foreclosure institution (see proof of Proposition 5) in which the liquidation probability β_B is not necessarily 1. Thus we follow the same strategy to solve it. The optimal contracts are summarised in Proposition C2.

Proposition C2. *The equilibrium contracts under bankruptcy for any bankruptcy code such that $\beta_B > \beta_B^*$ are:*

a) For $n \leq \frac{1-\theta}{\theta}\beta_B\alpha_B$: $l_B^* = I$, $R_B^* = \frac{I-(1-\theta)\beta_B\alpha_B I}{\theta}$.

b) For $n > \frac{1-\theta}{\theta}\beta_B\alpha_B$: $l_B^{**} = \hat{l}$, $R_B^{**} = \frac{I-(1-\theta)\beta_B\alpha_B \hat{l}}{\theta}$.

8.2.2 Overinvestment: bankruptcy vs. foreclosure

As displayed in Proposition C2, unlike in the case with $\alpha_B = 0$, there is overinvestment in capital for some values of n. However, *such overinvestment occurs for less (lower) values of n than in the case of foreclosure, i.e., there is less overinvestment.*

In the case of bankruptcy: $l_B^* = I$ for $n \leq \frac{1-\theta}{\theta}\beta_B\alpha_B$ and $l_B^{**} = \hat{l}$ for $n > \frac{1-\theta}{\theta}\beta_B\alpha_B$. In the case of foreclosure: $l_F^* = I$ for $n \leq \frac{1-\theta}{\theta}\alpha_F$ (see Proposition 5) and $l_F^{**} = \hat{l}$ for $n > \frac{1-\theta}{\theta}\alpha_F$ (see Appendix F). The fact that $\beta_B \leq 1$ and $\alpha_B < \alpha_F$ implies $\frac{1-\theta}{\theta}\beta_B\alpha_B < \frac{1-\theta}{\theta}\alpha_F$, which means that overinvestment under bankruptcy ($l_B^* = I$) takes place for less (lower) values of n than under foreclosure ($l_F^* = I$).

The intuition is straightforward. Overinvesting in capital reduces funding costs at the expense of costs of productive inefficiencies. The funding costs, as a function of the liquidation value, are $R_B = \frac{I-(1-\theta)\beta_B\alpha_B l_B}{\theta}$ under bankruptcy and $R_F = \frac{I-(1-\theta)\alpha_F l_F}{\theta}$ under foreclosure. Since $\frac{\partial R_B}{\partial l_B} = -\frac{1-\theta}{\theta}\beta_B\alpha_B$ while $\frac{\partial R_F}{\partial l_F} = -\frac{1-\theta}{\theta}\alpha_F$ and $\alpha_B < \alpha_F$, $\beta_B \leq 1$, the marginal benefit of overinvesting in capital (i.e., the marginal reduction in funding costs) is higher in the case of foreclosure, while its marginal cost, n, is the same for both institutions. The same intuition applies, *a fortiori*, to the case of optimal bankruptcy, as $\beta_B^* < \beta_B$.

9 Appendix D: choice of insolvency institution when $n \leq \frac{1-\theta}{\theta}\alpha_F$

9.1 $\beta_B < \beta_B^*$

If $\beta_B < \beta_B^*$ then the project cannot be undertaken under bankruptcy, as expressed in Lemma 3, so it will be carried out under foreclosure.

9.2 $\beta_B = \beta_B^*$

If $\beta_B = \beta_B^*$ then the manager chooses foreclosure iff $W_F^* \geq W_B^*$ which, by simple algebraic manipulation of the equilibrium welfares in Propositions 2 and 5, amounts to:

$$n \leq \frac{(1-\theta)\left[\frac{I}{\theta}+\alpha_F I-\phi\pi\right]}{\theta(I-\hat{l})}$$

As we are in the scenario $n \leq \frac{1-\theta}{\theta}\alpha_F$ and $\frac{(1-\theta)\left[\frac{I}{\theta}+\alpha_F I-\phi\pi\right]}{\theta(I-\hat{l})}$ may be greater than $\frac{1-\theta}{\theta}\alpha_F$ for sufficiently high values of I (especifically, if $I \geq \theta\phi\pi - \theta\alpha_F\hat{l}$) the value of n that makes the manager indifferent between foreclosure and bankruptcy is $\bar{n} \equiv \min\left\{\frac{(1-\theta)\left[\frac{I}{\theta}+\alpha_F I-\phi\pi\right]}{\theta(I-\hat{l})}, \frac{1-\theta}{\theta}\alpha_F\right\}$. In other words, the manager chooses foreclosure if $n \in [0,\bar{n}]$ and bankruptcy if $n \in \left(\bar{n}, \frac{1-\theta}{\theta}\alpha_F\right]$.

For $\bar{n}$ to be feasible we need to check that $\bar{n} \geq 0$. As $\frac{1-\theta}{\theta}\alpha_F > 0$ by construction, we only need to check that $\frac{(1-\theta)\left[\frac{I}{\theta}+\alpha_F I-\phi\pi\right]}{\theta(I-\hat{l})} \geq 0$. That inequality is equivalent to $\alpha_F \geq \frac{\phi\pi}{I} - \frac{1}{\theta}$, which can be rewritten, for its interpretation, as $\alpha_F I \geq (1-\beta_B^*)\,\phi\pi$, i.e., the liquidation proceedings in foreclosure are greater than the expected continuation cash-flows in optimal bankruptcy. Recalling that $0 < \alpha_F \leq 1$, we need to check that $\frac{\phi\pi}{I} - \frac{1}{\theta} \leq 1$, so there may exist a sufficiently high $\alpha_F \leq 1$

that satisfies $\alpha_F \geq \frac{\phi\pi}{I} - \frac{1}{\theta}$. As $\frac{\phi\pi}{I} - \frac{1}{\theta} \leq 1$ is equivalent to $I \geq \frac{\theta\phi\pi}{1+\theta}$, the *necessary* condition for $\alpha_F \geq \frac{\phi\pi}{I} - \frac{1}{\theta}$ to hold is $I \geq \frac{\theta\phi\pi}{1+\theta}$. In sum, $\bar{n}$ is feasible if $\alpha_F \geq \frac{\phi\pi}{I} - \frac{1}{\theta}$ and $I \geq \frac{\theta\phi\pi}{1+\theta}$.

9.3 $1 > \beta_B > \beta_B^*$

If $\beta_B > \beta_B^*$ then the manager chooses foreclosure iff $W_F^* \geq W_B^{**}$ which, by simple algebraic manipulation of the equilibrium welfares in Propositions 4 and 5, is equivalent to:

$$n \leq \frac{(1-\theta)[\alpha_F I - \phi\pi(1-\beta_B)]}{\theta(I-\hat{l})}$$

As we are in the scenario $n \leq \frac{1-\theta}{\theta}\alpha_F$ and $\frac{(1-\theta)[\alpha_F I - \phi\pi(1-\beta_B)]}{\theta(I-\hat{l})}$ may be greater than $\frac{1-\theta}{\theta}\alpha_F$ for sufficiently high values of β_B (e.g. $\beta_B = 1$), the value of n that makes the manager indifferent between foreclosure and bankruptcy is $\widetilde{n} = \min\left\{\frac{(1-\theta)[\alpha_F I - \phi\pi(1-\beta_B)]}{\theta(I-\hat{l})}, \frac{1-\theta}{\theta}\alpha_F\right\}$. In other words, the manager chooses foreclosure if $n \in [0, \widetilde{n}]$ and bankruptcy if $n \in \left(\widetilde{n}, \frac{1-\theta}{\theta}\alpha_F\right]$. It can be shown that $\widetilde{n} \geq \bar{n}$, i.e., there are more values for which the manager chooses foreclosure over bankruptcy when bankruptcy is not optimal, simply because $W_B^{**} \leq W_B^*$.

For $\widetilde{n}$ to be feasible we need to check that $\widetilde{n} \geq 0$, so the interval $[0, \widetilde{n}]$ for which foreclosure is chosen is not empty. As $\frac{1-\theta}{\theta}\alpha_F > 0$ by construction, we only need to check that $\frac{(1-\theta)[\alpha_F I - \phi\pi(1-\beta_B)]}{\theta(I-\hat{l})} \geq 0$. That inequality is equivalent to $\alpha_F I \geq (1-\beta_B)\phi\pi$, i.e., the liquidation proceedings in foreclosure are greater than the expected continuation cash-flows in non-optimal bankruptcy. Rewriting that expression as $\alpha_F \geq \frac{\phi\pi}{I}(1-\beta_B)$, we need to check that $\frac{\phi\pi}{I}(1-\beta_B) \leq 1$, so there may exist a sufficiently high $\alpha_F \leq 1$ that satisfies the condition. Since $\frac{\phi\pi}{I}(1-\beta_B) \leq 1$ is equivalent to $\beta_B \geq 1 - \frac{I}{\phi\pi}$, we need to check that its RHS is lower than 1 for a sufficiently high $\beta_B \leq 1$ to be able to satisfy it. Notice that it is always the case because $\frac{I}{\phi\pi} > 0$. In sum, $\widetilde{n}$ is feasible if $\alpha_F \geq \frac{\phi\pi}{I}(1-\beta_B)$.

9.4 $\beta_B = 1$

If $\beta_B = 1$ then the manager always chooses foreclosure because $W_F^* \geq W_B^{**}$ if $n \leq \widetilde{n}$ and $\widetilde{n} = \frac{1-\theta}{\theta}\alpha_F$ when $\beta_B = 1$.

10 Appendix E: full characterisation of welfare and bankruptcy rates when $n \leq \frac{1-\theta}{\theta}\alpha_F$

If $\beta_B < \beta_B^*$ no project can be undertaken under bankruptcy, as expressed in Lemma 3, so all of them are carried out under foreclosure. Hence the corresponding bankruptcy rate $BR_1 \equiv BR\left(\beta_B < \beta_B^*\right)$ is zero and the aggregate welfare Ω_1 is computed using Proposition 5:

$$\Omega_1 \equiv \Omega\left(\beta_B < \beta_B^*\right) = \int_{N_1}^{N_2} W_F^* dn = \int_{N_1}^{N_2} \left\{ \left(\theta + \phi\right)\pi - I - \left(1-\theta\right)\left[\phi\pi - \alpha_F I\right] - \theta n(I - \hat{l}) \right\} dn$$

If $\beta_B = \beta_B^*$ we know from the analysis in section 3 that the projects with $n \in [N_1, \bar{n}]$ are undertaken under foreclosure and the projects with $n \in (\bar{n}, N_2]$ are implemented under bankruptcy. The corresponding bankruptcy rate is $BR_2 \equiv BR\left(\beta_B = \beta_B^*\right) = \left(1-\theta\right)\frac{N_2 - \bar{n}}{N}$, since a proportion $\frac{N_2 - \bar{n}}{N}$ of managers use the bankruptcy system and default with probability $(1-\theta)$. The aggregate welfare Ω_2 is computed using propositions 2 and 5:

$$\Omega_2 \equiv \Omega\left(\beta_B = \beta_B^*\right) = \int_{N_1}^{\bar{n}} W_F^* dn + \int_{\bar{n}}^{N_2} W_B^* dn = \int_{N_1}^{\bar{n}} \{\left(\theta + \phi\right)\pi - I - \left(1-\theta\right)\left[\phi\pi - \alpha_F I\right] - \theta n(I - \hat{l})\} dn +$$
$$+ \int_{\bar{n}}^{N_2} \{\left(\theta + \phi\right)\pi - I - \tfrac{1-\theta}{\theta} I\} dn$$

If $\beta_B > \beta_B^*$ we know from the analysis in section 3 that the projects with $n \in [N_1, \widetilde{n}]$ are undertaken under foreclosure and the projects with $n \in (\widetilde{n}, N_2]$ are implemented under bankruptcy. The corresponding bankruptcy rate is $BR_3 \equiv BR\left(\beta_B > \beta_B^*\right) = \left(1-\theta\right)\frac{N_2 - \widetilde{n}}{N}$, since a proportion $\frac{N_2 - \widetilde{n}}{N}$ of managers use the bankruptcy system and default with probability $\left(1-\theta\right)$. The aggregate welfare Ω_3 is computed using propositions 4 and 5:

$$\Omega_3 \equiv \Omega\left(\beta_B > \beta_B^*\right) = \int_{N_1}^{\tilde{n}} W_F^* dn + \int_{\tilde{n}}^{N_2} W_B^{**} dn =$$
$$= \int_{N_1}^{\widetilde{n}} \left\{ \left(\theta + \phi\right)\pi - I - \left(1-\theta\right)\left[\phi\pi - \alpha_F I\right] - \theta n(I - \hat{l}) \right\} dn + \int_{\widetilde{n}}^{N_2} \{\left(\theta + \phi\right)\pi - I - \left(1-\theta\right)\beta_B \phi\pi\} dn$$

These results are represented in figures 6 and 7 (see section 4 of the main text).

Proposition 6. *Welfare Ω is a non-monotonic function of the creditor control rights under bankruptcy β_B. It is maximum at their optimum level β_B^*, it is Ω_1 for $\beta_B < \beta_B^*$, it is decreasing in β_B for $\beta_B > \beta_B^*$ and it is Ω_1 for $\beta_B = 1$. The bankruptcy rate BR is a non-monotonic function of*

β_B. It is maximum at β_B^*, it is 0 for $\beta_B < \beta_B^*$, it is decreasing in β_B for $\beta_B > \beta_B^*$ and it is 0 for $\beta_B = 1$.

Proof. The results summarised in Proposition 6 come from the analysis of the values for welfare and the bankruptcy rate that were found in the first part of this Appendix: $\{\Omega_1, \Omega_2, \Omega_3\}$ and $\{BR_1, BR_2, BR_3\}$. To prove that $\Omega(\beta_B)$ is a non-monotonic function of β_B^* it is sufficient to prove that $\Omega_1 < \Omega_3 < \Omega_2$ when $\beta_B < 1$. To see that $\Omega_1 < \Omega_3$ (i), rewrite $\Omega_1 = \int_{N_1}^{\tilde{n}} W_F^* dn + \int_{\tilde{n}}^{N_2} W_F^* dn$ and compare it with $\Omega_3 = \int_{N_1}^{\tilde{n}} W_F^* dn + \int_{\tilde{n}}^{N_2} W_B^{**} dn$. Because $W_F^* < W_B^{**}$ when $n > \widetilde{n}$ (see Appendix D) we have $\int_{\tilde{n}}^{N_2} W_F^* dn < \int_{\tilde{n}}^{N_2} W_B^{**} dn$. To see that $\Omega_3 < \Omega_2$ (ii), rewrite $\Omega_3 = \int_{N_1}^{\bar{n}} W_F^* dn + \int_{\bar{n}}^{\tilde{n}} W_F^* dn + \int_{\tilde{n}}^{N_2} W_B^{**} dn$ and compare it with $\Omega_2 = \int_{N_1}^{\bar{n}} W_F^* dn + \int_{\bar{n}}^{N_2} W_B^* dn$. Because $W_F^* < W_B^*$ when $n > \bar{n}$ (see Appendix D) and $W_B^{**} < W_B^*$, we have $\int_{\bar{n}}^{\tilde{n}} W_F^* dn + \int_{\tilde{n}}^{N_2} W_B^{**} dn < \int_{\bar{n}}^{N_2} W_B^* dn$. Putting together (i) and (ii) we have $\Omega_1 < \Omega_3 < \Omega_2$. To prove that $BR(\beta_B)$ is a non-monotonic function of β_B^* it is sufficient to prove that $BR_1 < BR_3 < BR_2$ when $\beta_B < 1$, which is straightforward to show as $\widetilde{n} > \overline{n}$. To prove that $\Omega\left(\beta_B < \beta_B^*\right) = \Omega\left(\beta_B = 1\right) = \Omega_1$ if $\beta_B = 1$, remember that $\widetilde{n} \equiv \min\left\{\frac{(1-\theta)[\alpha_F I - \phi\pi(1-\beta_B)]}{\theta(I-\hat{l})}, \frac{1-\theta}{\theta}\alpha_F\right\}$, which becomes $\widetilde{n} \equiv \frac{1-\theta}{\theta}\alpha_F$ if $\beta_B = 1$, implying that $[\widetilde{n}, N_2]$ is an empty set and $\Omega_3 = \int_{N_1}^{N_2} W_F^* dn = \Omega_1$. Same reasoning applies to prove that $BR\left(\beta_B < \beta_B^*\right) = BR\left(\beta_B = 1\right) = 0$. Using simple calculus it can be shown that $\frac{\partial\Omega_3}{\partial\beta_B} = (1-\theta)\,\phi\pi\left(\widetilde{n} - N_2\right)$, which is negative because $\widetilde{n} \leq N_2$, as $\widetilde{n} \equiv \min\left\{\frac{(1-\theta)[\alpha_F I - \phi\pi(1-\beta_B)]}{\theta(I-\hat{l})}, \frac{1-\theta}{\theta}\alpha_F\right\}$ and $N_2 \leq \frac{1-\theta}{\theta}\alpha_F$ by construction. In the case of the bankruptcy rate, $\frac{\partial BR_3}{\partial\beta_B} \leq 0$ because $BR_3 = (1-\theta)\,\frac{N_2-\widetilde{n}}{N}$ and $\frac{\partial\widetilde{n}}{\partial\beta_B} \leq 0$.

□

11 Appendix F: analysis when $n > \frac{1-\theta}{\theta}\alpha_F$

11.1 Optimal contract and equilibrium welfare under foreclosure when $n > \frac{1-\theta}{\theta}\alpha_F$

Proposition F1. *The optimal contract and equilibrium welfare under foreclosure when $n > \frac{1-\theta}{\theta}\alpha_F$ is:*

$l_F^{**} = \hat{l}$, $R_F^{**} = \frac{I-(1-\theta)\alpha_F\hat{l}}{\theta}$, $W_F^{**} = (\theta+\phi)\,\pi - I - (1-\theta)\left[\phi\pi - \alpha_F\hat{l}\right]$.

Proof. We use the result of Appendix A, according to which $l \geq \hat{l}$, so that $D(l)$ can be rewritten as $D(l) = n\left(l - \hat{l}\right)$. The optimal contract maximises the manager's expected utility EU_F subject to the following constraints: (24) manager's incentive compatibility; (25) lender's individual rationality; (26) the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature (feasibility constraint); (27) the liquidation value must be greater than or equal to the first-best value.

$$\max_{R_F, l_F} EU_F = \theta\left[\pi - n(l_F - \hat{l}) - R_F + \phi\pi\right]$$

s.t.:

$$\pi - n(l_F - \hat{l}) - R_F + \phi\pi \geq \pi - n(l_F - \hat{l}) \tag{24}$$

$$\theta R_F + (1-\theta)\alpha_F l_F \geq I \tag{25}$$

$$R_F \leq \pi - n(l_F - \hat{l}) \tag{26}$$

$$l_F \geq \hat{l} \tag{27}$$

First, we know from the proof of Proposition 5 (Appendix B) that the manager's incentive compatibility constraint (24) is not binding at the optimum and the lender's individual rationality constraint (25) is binding at the optimum. Second, the feasibility constraint (26) is not binding at the optimum. To see this, rearrange (26) to obtain $R_F + n\left(l_F - \hat{l}\right) \leq \pi$. Since $\frac{\partial EU_F}{\partial R_F} < 0$ and $\frac{\partial EU_F}{\partial l_F} < 0$ and by lowering R_F and l_F (26) holds *a fortiori,* one would like to decrease R_F and l_F as much as possible to increase EU_F, which implies that (26) is not binding. But we also need to check that, if (26) is not binding, i.e., $R_F + n\left(l_F - \hat{l}\right) < \pi$, the lender's individual rationality constraint (25) still holds. Rearrange (25): $R_F \geq \frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F]$. Putting together the two conditions: $\frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F] \leq R_F < \pi - n\left(l_F - \hat{l}\right)$. This holds as long as $\pi - n\left(l_F - \hat{l}\right) \geq \frac{1}{\theta}[I - (1-\theta)\,\alpha_F l_F]$, which can be rearranged to $\pi - \frac{I}{\theta} + n\hat{l} \geq l_F\left[n - \frac{(1-\theta)}{\theta}\alpha_F\right]$. If $n > \frac{(1-\theta)}{\theta}\alpha_F$, then the condition is satisfied if $l_F \leq \frac{\pi - \frac{I}{\theta} + n\hat{l}}{n - \frac{1-\theta}{\theta}\alpha_F}$. As (25) binds, rearranging it we find the repayment cost as function of the liquidation value: $R_F = \frac{I - (1-\theta)\alpha_F l_F}{\theta}$. Plugging this expression into EU_F, we have the following constrained maximisation program: $\max_{l_F} EU_F = \theta\left[\pi - nl_F - \frac{I-(1-\theta)\alpha_F l_F}{\theta} + \phi\pi\right]$ s.t. $l_F \leq \frac{\pi - \frac{I}{\theta} + n\hat{l}}{n - \frac{1-\theta}{\theta}\alpha_F}$. As $\frac{\partial EU_F}{\partial l_F} < 0$ iff $n > \frac{1-\theta}{\theta}\alpha_F$, the constraint on l_F does not bind either. Hence we have the following unconstrained maximisation program:

$$\max_{l_F} EU_F = \theta \left[\pi - n \left(l_F - \hat{l} \right) - \frac{I - (1-\theta) \alpha_F l_F}{\theta} + \phi\pi \right]$$

Differentiating EU_F with respect to l_F we have $\frac{\partial EU_F}{\partial l_F} = -\theta n + (1-\theta)\alpha_F$. As $\frac{\partial EU_F}{\partial l_F}<0$ iff $n > \frac{1-\theta}{\theta}\alpha_F$, we have a corner solution: $l_F^{**} = \hat{l}$, $R_F^{**} = \frac{I-(1-\theta)\alpha_F \hat{l}}{\theta}$. Plugging those solutions into the manager's expected utility and using the fact that the lender breaks even we find the equilibrium welfare: $W_F^{**} = (\theta + \phi)\pi - I - (1-\theta)\left[\phi\pi - \alpha_F \hat{l}\right]$.

□

11.2 Choice of insolvency institution when $n > \frac{1-\theta}{\theta}\alpha_F$

11.2.1 $\beta_B < \beta_B^*$

If $\beta_B < \beta_B^*$ the project cannot be undertaken under bankruptcy, as expressed in Lemma 3, so it is carried out under foreclosure.

11.2.2 $\beta_B = \beta_B^*$

If $\beta_B = \beta_B^*$ the manager chooses foreclosure iff $W_F^{**} \geq W_B^*$. This is equivalent to $I \geq \theta\phi\pi - \theta\alpha_F \hat{l}$, which can be rewritten as $\beta_B^* \geq 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$ because $\beta_B^* = \frac{I}{\theta\phi\pi}$. That condition is feasible because we have assumed $\alpha l \leq \phi\pi \; \forall l$ and $\alpha_F > 0$ and $\hat{l}>0$. Foreclosure is chosen if the optimal creditor control rights in bankruptcy β_B^*, the efficiency of foreclosure's liquidation technology α_F and the first-best liquidation value $\hat{l}$ are high enough. The intuition is that, as β_B^* increases, bankruptcy leads to more inefficient liquidations, losing its appeal relative to foreclosure, where there is always liquidation ($\beta_M = 1$). Likewise, as α_F and $\hat{l}$ increase, the cost of inefficient liquidations in foreclosure $\left[\phi\pi - \alpha_F \hat{l}\right]$ decreases, making it more appealing relative to bankruptcy, where the cost of such an inefficiency is the maximum: $\phi\pi$.

11.2.3 $1 > \beta_B > \beta_B^*$

If $1 > \beta_B > \beta_B^*$ the manager chooses foreclosure iff $W_F^{**} \geq W_B^{**}$, which is equivalent to $\beta_B \geq 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$. Foreclosure is chosen for more values of the creditor control rights in bankruptcy β_B the higher the efficiency of its liquidation technology α_F and the higher the first-best liquidation value $\hat{l}$.

11.2.4 $\beta_B = 1$

If $\beta_B = 1$ the manager chooses foreclosure iff $W_F^{**} \geq W_B^{**}$, which is equivalent to $1 \geq 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$, which is always satisfied because $\alpha_F > 0$ and $\hat{l}>0$.

11.3 Welfare and bankruptcy rates when $n > \frac{1-\theta}{\theta}\alpha_F$

If $\beta_B < \beta_B^*$ projects cannot be undertaken under bankruptcy, as expressed in Lemma 3, so they are carried out under foreclosure. Hence the corresponding bankruptcy rate BR_4 is 0 and the aggregate welfare Ω_4 is computed using the equilibrium welfare W_F^{**} that is displayed in Proposition E1:

$$\Omega_4 = \int_{N_1}^{N_2} W_F^{**} dn = \int_{N_1}^{N_2} \{(\theta + \phi)\, \pi - I - (1-\theta) \left[\phi\pi - \alpha_F \hat{l}\right]\} dn$$

If $\beta_B = \beta_B^*$ we know from the analysis of Section 10.2 that the project is undertaken under bankruptcy if $I < \theta\phi\pi - \theta\alpha_F\hat{l}$ and implemented under foreclosure if $I \geq \theta\phi\pi - \theta\alpha_F\hat{l}$. The corresponding bankruptcy rate is $BR_5 = (1-\theta)$ if $I < \theta\phi\pi - \theta\alpha_F\hat{l}$ (all managers use the bankruptcy system and default with probability $(1-\theta)$) and 0 otherwise. The aggregate welfare Ω_5 is computed using Proposition 2 and Proposition F1:

$\Omega_5 = \int_{N_1}^{N_2} W_B^* dn = \int_{N_1}^{N_2} \{(\theta + \phi)\, \pi - I - \frac{1-\theta}{\theta} I\} dn$ if $I < \theta\phi\pi - \theta\alpha_F$; $\int_{N_1}^{N_2} W_F^{**} dn = \int_{N_1}^{N_2} \{(\theta + \phi)\, \pi - I - (1-\theta) \left[\phi\pi - \alpha_F \hat{l}\right]\} dn$ *otherwise.*

If $\beta_B > \beta_B^*$ we know from the analysis of Section 10.2 that the project is undertaken under bankruptcy if $\beta_B < \overline{\beta_B} \equiv 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$ and implemented under foreclosure if $\beta_B \geq \overline{\beta_B}$. The corresponding bankruptcy rate is $BR_6 = (1-\theta)$ if $\beta_B < 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$ and 0 otherwise. The aggregate welfare Ω_6 is computed using Proposition 4:

$\Omega_6 = \int_{N_1}^{N_2} W_B^{**} dn = \int_{N_1}^{N_2} \{(\theta + \phi)\,\pi - I - (1-\theta)\,\beta_B \phi\pi\} dn$ if $\beta_B^* < \beta_B < 1 - \frac{\alpha_F \hat{l}}{\phi\pi}$; $\int_{N_1}^{N_2} W_F^{**} dn = \int_{N_1}^{N_2} \{(\theta + \phi)\,\pi - I - (1-\theta)\left[\phi\pi - \alpha_F \hat{l}\right]\} dn$ *otherwise.*

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SUPPLEMENTS (NOT FOR PUBLICATION)

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1 Supplement A: analysis without restrictions in the value of the initial outlay

In the baseline model we have assumed $I \leq \theta\phi\pi$ to ensure that financing is possible under the bankruptcy system, as the optimal bankruptcy code is $\beta_B^* = \frac{I}{\theta\phi\pi}$ and the non-optimal one is $\beta_B > \beta_B^*$. In this supplement we remove that assumption to see whether credit can still be provided and under which conditions.

If $I > \theta\phi\pi$, then β_B^* and β_B would be greater than 1: hence bankruptcy is unfeasible. The intuition comes from analysing the manager's incentive-compatibility constraint:

$$\beta_B \phi\pi \geq R_B \tag{1}$$

Plugging $R_B^* = \frac{I}{\theta}$ into the above equation we obtain:

$$\beta_B \phi\pi \geq \frac{I}{\theta} \tag{2}$$

If the initial outlay is too high (i.e., $I > \theta\phi\pi$) then the contract is not incentive-compatible, as the maximum expected punishment from defaulting strategically, $\max\{\beta_B\phi\pi\} = \phi\pi$, is lower than the benefit from carrying out such a strategy, $R_B^* = \frac{I}{\theta}$.

By contrast, in the case of foreclosure, the incentive compatibility constraint of the manager is not binding at the optimum, as shown in the proof of Proposition 5 (see Appendix B). Hence, the only restriction on the value of I comes from the fact that the equibrium welfare must be weakly positive (otherwise, the agents would not sign the contract).

From Proposition 5 we know that welfare is $W_F^* = (\theta + \phi)\pi - I - (1-\theta)[\phi\pi - \alpha_F I] - \theta n(I - \hat{l})$ when $n \leq \frac{1-\theta}{\theta}\alpha_F$. $W_F^* \geq 0$ is equivalent to $I \leq \frac{\theta(1+\phi)\pi + \theta n \hat{l}}{[1+\theta n - (1-\theta)\alpha_F]}$. As the RHS of that inequality[1] is greater than $\theta\phi\pi$, projects with $\left[\theta\phi\pi, \frac{\theta(1+\phi)\pi + \theta n \hat{l}}{[1+\theta n - (1-\theta)\alpha_F]}\right]$ are feasible under foreclosure when $n \leq \frac{1-\theta}{\theta}\alpha_F$.

From Appendix E we know that welfare is $W_F^{**} = (\theta + \phi)\pi - I - (1-\theta)\left[\phi\pi - \alpha_F \hat{l}\right]$ when $n > \frac{1-\theta}{\theta}\alpha_F$. $W_F^{**} \geq 0$ is equivalent to $I \leq \theta\phi\pi + \theta\pi + (1-\theta)\alpha_F \hat{l}$. As the RHS of that inequality is greater than $\theta\phi\pi$, projects with $I \in \left[\theta\phi\pi, \theta\phi\pi + \theta\pi + (1-\theta)\alpha_F \hat{l}\right]$ are feasible under foreclosure when $n > \frac{1-\theta}{\theta}\alpha_F$.

[1] Notice that, as $0 < n \leq \frac{1-\theta}{\theta}\alpha_F$, the denominator of the RHS is positive and lower than 1.

Hence, for projects with high values of the initial outlay I -and, correspondingly, low NPV- only one insolvency institution, foreclosure, is feasible.

2 Supplement B: analysis when the lender is not perfectly competitive

In the main model it has been assumed that the lender is perfectly competitive. However, this may not be a realistic assumption in some countries, especially in the case of some bank-based oriented financial systems if the banking sector is highly concentrated. In such a case firms are very dependent on banking credit -as most companies have limited access to capital markets- and competition among banks is not strong enough to enable firms to retain all the profits from the projects. The aim of this supplement is to show a version of the model that relaxes the assumption of perfect competition, proving that the model's conclusions are robust to different degrees of competition in the credit market. Specifically, it will be shown that, as in the main model, the level of overinvestment in capital under bankruptcy is lower than under foreclosure, i.e., $l_B^* < l_F^*$, as well as the probability of inefficient liquidations, i.e., $\beta_B^* \leq \beta_F^*$.

For the sake of simplicity let us assume, like in the baseline model, $\alpha_B = 0$. We depart from perfect competition by assuming that the lender has some bargaining power. Following Suárez and Sussman (2007) we assume that the manager makes a take-it-or-leave-it offer to the lender with probability λ and the lender makes the offer with probability $1 - \lambda$. Since we have already solved the contracts for the case where the manager makes the offer in the main model, what we need to do is to solve those contracts for the other case and then combine the results. The main findings are summarised in Propositions B1, B2 and B3.

Proposition B1. *The optimal bankruptcy code* β_B^* *and the optimal contract under bankruptcy* $\{R_B^*, l_B^*\}$ *are* $\beta_B^* = \lambda\left(\frac{I}{\theta\phi\pi} - 1\right) + 1$, $R_B^* = \lambda\frac{I}{\theta} + (1-\lambda)\phi\pi$, $l_B^* = \hat{l}$.

Proof. Let us first find the solution for the case where the lender makes a take-it-or-leave-it offer to the manager. The optimal contract and the optimal bankruptcy code maximise the lender's expected utility Π_B subject to the following constraints: (3) the manager does not default strategically (incentive compatibility); (4) the manager decides to undertake the project (individual rationality); (5) since the manager is wealthless, the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature (first feasibility constraint); (6) the liquidation probability β_B lies in the interval $[0, 1]$ (second feasibility constraint); (7) the liquidation value must be greater than or equal to the first-best value. Formally, the maximisation problem is the following:

$$\max_{R_B,\beta_B,l_B} \Pi_B = \theta R_B - I$$

s.t.:

$$\beta_B \phi\pi \geq R_B \tag{3}$$

$$\theta \left[\pi - n(l_B - \hat{l}) - R_B + \phi\pi\right] + (1-\theta)(1-\beta_B)\phi\pi \geq 0 \tag{4}$$

$$R_B \leq \pi - n(l_B - \hat{l}) \tag{5}$$

$$0 \leq \beta_B \leq 1 \tag{6}$$

$$l_B \geq \hat{l} \tag{7}$$

To solve this problem, first notice that, if (5) holds, then (4) must hold, so that we can ignore the latter. To see this rearrange (4): $R_B \leq \frac{1}{\theta}\left[\left(\pi - n(l_B - \hat{l}) + \phi\pi\right) + (1-\theta)(1-\beta_B)\phi\pi\right]$. The RHS of (4) is larger than the RHS of (5) because $\theta < 1$, $\phi\pi > 0$ and $\beta_B \leq 1$. Now notice that, since $\frac{\partial \Pi_B}{\partial R_B} > 0$, the lender chooses $l_B^* = \hat{l}$ to increase the RHS of (5) as much as possible. Plugging $l_B^* = \hat{l}$ into (5) and rearranging (3) as $R_B \leq \beta_B\phi\pi$ we see that, if (3) holds, then (5) must hold, so we can ignore the latter. (3) must be binding at the optimum because, if it was slack, we could raise R_B to increase Π_B. Now plug $R_B = \beta_B\phi\pi$ into (6) and rearrange to obtain $R_B \leq \phi\pi$. Since $\frac{\partial \Pi_B}{\partial R_B} > 0$, the previous constraint must be binding at the optimum, so we get $R_B^* = \phi\pi$. Then it follows that $\beta_B^* = 1$. Finally, let us check that the lender's utility is non-negative: $\Pi_B^* = \theta R_B^* - I = \theta\phi\pi - I \geq 0$ which is true because $I \leq \theta\phi\pi$ by assumption.

The optimal bankruptcy code and the optimal contract under bankruptcy are just weighted averages of those where the manager has all the bargaining power (shown in Proposition 2) and those where the lender has all the bargaining power (just shown), where the weights are the probabilities of each scenario, λ and $1-\lambda$.

□

Proposition B2. *The optimal contract* $\{R_B^{**}, l_B^{**}\}$ *under (non-optimal) bankruptcy is* $R_B^{**} = \lambda\frac{I}{\theta} + (1-\lambda)\beta_B\phi\pi$, $l_B^{**} = \hat{l}$.

Proof. Let us first find the solution for the case where the lender makes a take-it-or-leave-it offer to the manager. The optimal contract maximises the lender's expected utility Π_B subject to the following constraints: (8) the manager does not default strategically (incentive compatibility); (9) the manager decides to undertake the project (individual rationality); (10) since the manager is wealthless, the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature (feasibility constraint); (11) the liquidation value must be greater than or equal to the first-best value. Formally, the maximisation problem is the following:

$$\max_{R_B, l_B} \Pi_B = \theta R_B - I$$

s.t.:

$$\beta_B \phi \pi \geq R_B \tag{8}$$

$$\theta \left[\pi - n(l_B - \hat{l}) - R_B + \phi\pi\right] + (1-\theta)(1-\beta_B)\phi\pi \geq 0 \tag{9}$$

$$R_B \leq \pi - n(l_B - \hat{l}) \tag{10}$$

$$l_B \geq \hat{l} \tag{11}$$

The solution of the problem follows the same steps as the previous maximisation program, yielding $R_B^{**} = \beta_B \phi \pi$ and $l_B^{**} = \hat{l}$. Let us check that the lender's utility is non-negative: $\Pi_B^{**} = \theta R_B^{**} - I = \theta \beta_B \phi \pi - I \geq 0$. This can be rewritten as $\beta_B \geq \frac{I}{\theta \phi \pi}$. Remember that $\beta_B^* = \frac{I}{\theta \phi \pi}$ is the optimal bankruptcy law when the manager has all the bargaining power (Proposition 2) and that the non-optimal bankruptcy code in that scenario is $\beta_B > \beta_B^*$ (Proposition 4). Hence the non-optimal bankruptcy code is weakly higher than $\frac{I}{\theta \phi \pi}$ regardless of who has the bargaining power.

The optimal contract under (non-optimal) bankruptcy $\{R_B^{**}, l_B^{**}\}$ is just the weighted averages of R_B^{**} and l_B^{**} where the manager has all the bargaining power (shown in Proposition 4) and where the lender has all the bargaining power (just shown), where the weights are the probabilities of each scenario, λ and $1 - \lambda$.

□

Proposition B3. *The optimal contract under foreclosure when* $n \leq \frac{1-\theta}{\theta} \alpha_F$ *is:*
$R_F^* = \lambda \frac{I-(1-\theta)\alpha_F I}{\theta} + (1-\lambda)\phi\pi$, $l_F^* = \lambda I + (1-\lambda)\left[\frac{\pi}{n}(1-\phi) + \hat{l}\right]$.

Proof. Let us first find the solution for the case where the lender makes a take-it-or-leave-it offer to the manager. The optimal contract maximises the lender's expected utility Π_F subject to the following constraints: (12) the manager does not default strategically (incentive compatibility); (13) the manager decides to undertake the project (individual rationality); (14) since the manager is wealthless, the repayment cannot exceed the cash flow at $t = 1$ in the good state of nature (feasibility constraint); (15) the liquidation value must be greater than or equal to the first-best value. Formally, the maximisation problem is the following:

$$\max_{R_F, l_F} \Pi_F = \theta R_F + (1-\theta)\alpha_F l_F - I$$

s.t.:

$$\phi\pi \geq R_F \tag{12}$$

$$\theta\left[\pi - n(l_F - \hat{l}) - R_F + \phi\pi\right] \geq 0 \tag{13}$$

$$R_F \leq \pi - n(l_F - \hat{l}) \tag{14}$$

$$l_F \geq \hat{l} \tag{15}$$

To solve this problem, first notice that, if (14) holds, then (13) must hold, so that we can ignore the latter. Now rearrange (14) as $R_F + n(l_F - \hat{l}) \leq \pi$. (14) is binding at the optimum because, if it was slack, we could increase R_F and/or l_F to increase Π_F. (12) is also binding because $\frac{\partial \Pi_F}{\partial R_F} > 0$, so we obtain $R_F^* = \phi\pi$. Plugging R_F^* into (14) and rearranging we get $l_F^* = \frac{\pi}{n}(1-\phi) + \hat{l}$. Because $\pi > 0$, $n > 0$ and $0 < \phi < 1$ we have $l_F^* > \hat{l}$, i.e., (15) is satisfied but not binding.

The optimal contract under foreclosure $\{R_F^*, l_F^*\}$ is just the weighted averages of R_F^* and l_F^* where the manager has all the bargaining power (shown in Proposition 5) and where the lender has all the bargaining power (just shown), where the weights are the probabilities of each scenario, λ and $1-\lambda$.

□

Now let us compare the equilibrium liquidation values under (optimal and non-optimal) bankruptcy (Propositions B1 and B2) with those in foreclosure (Proposition B3). In bankruptcy those are $l_B^* = l_B^{**} = \hat{l}$ while in foreclosure it is $l_F^* = \lambda I + (1-\lambda)\left[\frac{\pi}{n}(1-\phi) + \hat{l}\right]$ with $n \leq \frac{1-\theta}{\theta}\alpha_F$. Since $l_F^* > \hat{l}$, we conclude that the level of overinvestment in capital under bankruptcy is lower than under foreclosure[2].

With regards to the equilibrium liquidation probabilities, recall that in foreclosure $\beta_F = 1$ by assumption. Under optimal bankruptcy $\beta_B^* = \lambda\left(\frac{I}{\theta\phi\pi} - 1\right) + 1 \leq 1$ since $I \leq \theta\phi\pi$. Under non-optimal bankruptcy $0 < \beta_B \leq 1$. Hence the probability of inefficient liquidations under bankruptcy is (weakly) lower than under foreclosure.

3 Supplement C: bankruptcy rates and size

Although the available evidence is rather limited, it seems that small firms, when financially distressed, use the bankruptcy system much less than large companies in the same situation. Claessens and Klapper (2005) find a negative correlation between bankruptcy rates and firm size in their sample of 35 countries. Morrison (2008, 2009)

[2] In the baseline model we have set $n \leq \frac{1-\theta}{\theta}\alpha_F$ to exclude cases where there is no overinvestment in capital under mortgage. But, for completeness, the equilibrium contract for $n > \frac{1-\theta}{\theta}\alpha_F$ is shown in Appendix F. Using that result and the one in this section we obtain $R_F^{**} = \lambda\frac{I-(1-\theta)\alpha_F\hat{l}}{\theta} + (1-\lambda)\phi\pi$, $l_F^{**} = \lambda\hat{l} + (1-\lambda)\left[\frac{\pi}{n}(1-\phi) + \hat{l}\right]$ for $n > \frac{1-\theta}{\theta}\alpha_F$. As $l_F^{**} > \hat{l}$ the conclusion is the same in that scenario: the level of overinvestment in capital under bankruptcy is lower than under foreclosure.

documents this fact in the US and García-Posada and Mora-Sanguinetti (2014) do the same in Spain. Hence, it may be worth analysing the role of firm size to see whether the main conclusions of our base model change or not.

There are several reasons why small firms may file less for bankruptcy. Small firms usually have few creditors and they may have engaged in "relationship lending" with their main bank, hence reducing coordination and asymmetric information problems that are solved best under a bankruptcy procedure (Gilson *et al.*, 1990, Hart, 2000). Personal, rather than corporate bankruptcy laws may apply to small firms (Fan and White 2003, Berkowitz and White, 2004). But probably the most uncontroversial observation is that bankruptcy procedures are expensive and that a substantial part of the costs are fixed. According to Bris *et al.* (2006) and their large sample of bankruptcies in the U.S., among firms with assets worth less than \$ 100,000, the median direct costs burn 23.2% of asset value. Among firms with assets worth between \$ 100,000 and \$ 1 million, the median direct costs consume 4.9% of asset value.

In the baseline set up we used a fixed-scale model, so we implictly assumed away firm size from the analysis. To analyse the role of size let us use instead a variable-scale model with constant returns to scale. Specifically, the project requires an initial outay $I \in (0, +\infty)$ and yields cash-flows $\pi I - n(l - \hat{l})$ (with $l = \gamma I$) at $t = 1$ and πI at $t = 2$. The probabilities of project success, as well as other features, are the same as in the baseline model. The only other departure from the benchmark model is that we assume the lender must pay a cost $K > 0$ for the bankruptcy procedure to take place. K is fixed, i.e., it does not depend on the firm's size I, which intends to capture the fact that a substantial part of the bankruptcy costs may be fixed. From a theoretical viewpoint, this assumption is related to that of costly state verification models[3] where the lender must pay an audit cost K to verify the project's income -which can be interpreted as a bankruptcy process (Tirole, 2006). However, in our model the project's assets, rather than its cash flows, are the ones that become perfectly verifiable if the audit/bankruptcy is carried out. Finally, as in the benchmark model, we abstract from renegotiation issues by assuming perfect commitment: the lender always pays K in the event of default.

3.1 Analysis with $\alpha_B = 0$

The model's main point can be easily made with the particular case of $\alpha_B = 0$. This means that the lender pays $K > 0$ to be able to trigger liquidation (with probability β_B) and hence deter strategic default. The maximisation program becomes:

[3]See Townsend (1979) for the seminal paper in this literature.

$$\max_{R_B,\beta_B,l_B} EU_B = \theta\left[\pi I - n(l_B - \hat{l}) - R_B + \phi\pi I\right] + (1-\theta)(1-\beta_B)\phi\pi I$$

s.t.:

$$\beta_B \phi\pi I \geq R_B \tag{16}$$

$$\theta R_B + (1-\theta)(-K) \geq I \tag{17}$$

$$R_B \leq \pi I - n(l_B - \hat{l}) \tag{18}$$

$$0 \leq \beta_B \leq 1 \tag{19}$$

$$l_B \geq \hat{l} \tag{20}$$

The model solution, which follows the same rationale as in the benchmark model, yields an optimal contract $\left\{R_B^* = \frac{I}{\theta} + \frac{1-\theta}{\theta}K; l_B^* = \hat{l}\right\}$ under the optimal bankruptcy code $\beta_B^* = \frac{1}{\theta\phi\pi} + \frac{1-\theta}{\theta\phi\pi}\frac{K}{I}$.[4] Hence: $\frac{\partial\beta_B^*}{\partial I} < 0$, which means that, the larger the firm, the lower the optimal creditor control rights in bankruptcy. The intuition of this result comes from inspecting the manager's incentive-compatibility constraint, which binds in equilibrium: $\beta_B\phi\pi I = R_B^*$. As I increases, while the punishment from defaulting $\beta_B\phi\pi I$ increases proportionally, the savings from defaulting $R_B^* = \frac{I}{\theta} + \frac{1-\theta}{\theta}K$ increase less than proportionally because K is fixed, allowing for a lower optimal bankruptcy code β_B^*. In other words, since the risk premium that the lender charges to the manager is fixed, large firms find their financing cheaper relative to their profits, reducing the incentives to default.

Now let us remember that β_B^* is also the minimum liquidation probability that makes the credit contract feasible under bankruptcy (Lemma 3). Suppose the simplest case, two firms with size I_S and I_L with $I_L > I_S$ (small and large, respectively). Since $\frac{\partial\beta_B^*}{\partial I} < 0$ then $\beta_B^*(I_L) < \beta_B^*(I_S)$. If it turns out that the actual bankruptcy code β_B is $\beta_B^*(I_L) < \beta_B < \beta_B^*(I_S)$, then bankruptcy will only be feasible for the large firm ("narrow-scope bankruptcy", in the terminology of Ayotte and Yun, 2009)[5], implying that, while that firm will be able to choose between bankruptcy and foreclosure to maximise its profits, the small firm will have to obtain credit through the foreclosure system, even if it found more profitable to use the bankruptcy system (i.e., in cases where the cost of overinvestment in capital n is quite high). By contrast, if the bankruptcy code β_B is $\beta_B \geq \beta_B^*(I_S) > \beta_B^*(I_L)$, then bankruptcy will also be feasible for the small firm ("wide-scope bankruptcy"), implying that both firms will be able to choose between bankruptcy and foreclosure to maximise its profits. Hence, we can conclude that a "too-creditor friendly" bankruptcy system ($\beta_B \geq \beta_B^*(I_S)$) allows for more insolvency options than a "too-debtor friendly" one ($\beta_B < \beta_B^*(I_S)$), in analogous fashion as what we found with our base model.

[4]For the optimal bankruptcy code to be feasible ($0 \leq \beta_B^* \leq 1$) two parametric assumptions are required: $I \geq \frac{(1-\theta)K}{\theta\phi\pi-1}$ (H1) and $\theta\phi\pi > 1$ (H2). (H1) means that the scale I is large enough to offset the fixed cost K; (H2) means that the project is profitable enough per unit of investment.

[5] Ayotte and Yun (2009) carry out a similar analysis, but with a fixed-scale model where firms differ in their startup cost and can "contract-out" bankruptcy.