Green strategies for Block Diagonalization - Network MIMO with Fairness

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***Abstract*—Future cellular networks will be more dense and heterogeneous. A typical deployment will be based on micro, pico and femto cells. Under this scenario, novel interference management techniques such as network MIMO will be mandatory. However, it is usual to combine them with power allocation strategies to maximize the sum of the rates or minimize the power consumption. Those strategies are unfair at user level in heterogeneous networks and do not provide a "green" measure. In this work we propose some more equitable alternatives to the classical power allocation schemes and a strategy based on a green metric. Finally we compare the performance, in user spectral efficiency and power consumption terms, of these strategies in micro and femto cell deployments.**

I. INTRODUCTION

Typically mobile communications are based in outdoor base stations (BS) covering a fairly large geographical area and serving a relatively large number of users. On the other hand, increased demand for high speed data services has resulted in the introduction of new technologies and standards like 3rd Generation Partnership Project (3GPP) High Speed Packet Access (HSPA) or Worldwide Interoperability for Microwave Access (WiMAX). Even with these standards users often experience poor coverage and reduced data throughputs. The growth in wireless capacity is exemplified by the observation from Martin Cooper "The wireless capacity has doubled every 30 months over the last 104 years". This increase was due to improvements from the use of wider spectrum, which can be divided into smaller slices, better modulation schemes and specially reducing the cell sizes and the distance between user and transmitter [1].

Therefore macro/micro cells reduce their coverage radius continuously to satisfy the users demand. From the operator point of view, this continued micro-ization of the cellular network results too expensive, actually only the energy cost represents a significant portion of operator overall expenditures (OPEX) [2]. In the last years, with the regular use of the data service, a decoupling between traffic and revenues has occurred because of this reason. At this point, a possible solution are femtocells. Femtocells are small base stations operating in the licensed cellular band and backhauled onto IP networks through conventional digital subscribers lines (DSL). They work in a short range, must be low cost and transmit at low power. They are meant to be placed anywhere at homes or in small offices by the users. Under this situation, the evolution to four generation (4G) services with standards as Long Term Evolution Advanced (LTE-A) presents a new paradigm where the wireless network requires more dense and heterogeneous deployments of base stations composed by macro, micro, pico, and femto cells [3].

However, heterogeneous networks present the interference management as a great challenge. If we consider a Frequency Reuse (FR) scheme such as that used in traditional mobile networks, a poor performance is obtained due to the high interference and their complex treatment [4]. From a game theoretical point of view, FR corresponds to a competitive solution. In [5] a comparison between competitive and cooperative solutions is developed. It is shown that the competitive case provides a suboptimal Nash equilibrium point, but when all base stations cooperate to achieve a global goal, the bit rate achieved corresponds to an optimal Nash equilibrium point. Therefore there is a need of new interference management schemes which are able to involve sets of base stations cooperating between them [6]. In this sense [7] propose to use a Coordinated Base Station Transmission (CBST) scheme for a generic cellular system. So it is possible to obtain a network MIMO where the set of base stations cooperate between them. The interference between these cells may be eliminated, provided that they all share the channel and information to be transmitted in the downlink, by cooperative encoding using Dirty Paper Coding (DPC). In [8] authors propose a Block Diagonalization (BD) CBST scheme, where the interference between users can be eliminated as base stations cooperate between them in a multiantenna system.

After BD process, a power allocation strategy is required. Typically the objective of this optimization problem is to maximize the sum of the rates subject to some available power constraints [8]. This strategy achieves a great performance in global terms, however, in a very heterogeneous deployment where the resources are distributed with high statistical variance, the sum rate strategy can be unfair at user level [9]. On the other hand, actually a "Green" approach is mandatory, energy efficiency in cellular networks is a growing concern for cellular operators to not only maintain profitability but also to reduce the overall environmental effects [2]. About 60% of the operator power usage corresponds to the base stations and currently there are more than 4 million base stations consuming an average of 1400 watts ($3200 per year) each one. The radio part of a BS represents the 80% of this consumption, therefore minimizing the global power consumption subject to a minimum achievable rate by the users could be a good strategy. However, this solution is also inefficient in heterogeneous networks composed by small cells, since users with better resources would be limited to the achievable rates by the users with worst channel conditions.

In this work we develop alternative power allocation strategies for heterogeneous networks which achieve more fair solutions at user level than classical approaches as sum rate. Optimization problems under heterogeneous resources are typical in microeconomics, so first we develop power allocation schemes based in economic utility functions [10]. Finally, according with [2] we develop a strategy based on a green metric with the aim to minimize the ratio of energy consumption over effective system capacity. All these strategies are evaluated by simulations in micro and femto cell deployments, showing the user spectral efficiency and the average consumption per spectral efficiency results.

The remainder of this paper is structured as follows. In section II we describe the system model, in section III classical power allocation schemes are described and in section IV we develop alternative strategies. Section V shows the simulation scenarios and section VI shows and discusses the simulation results. Finally concluding remarks are made in section VII.

II. SYSTEM MODEL

The system model assumes M base stations or access points (AP) serving N user equipments (UE). Each BS is equipped with t transmit antennas and each UE has r receive antennas. Assuming narrow band transmission and that all propagation channels are frequency flat, including large scale effects with the distance and small scale multipath Rayleigh fading, the channel may be modeled by a $Nr \times Mt$ channel matrix $\mathbf{H}$, where each coefficient h_{ij} represents the path loss between a given transmit antenna at the BS j and a given receive antenna at the UE i. The received signal model is as follows

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is the received $Nr \times 1$ signal vector, $\mathbf{x}$ is the $Mt \times 1$ signal vector transmitted from the BSs, and $\mathbf{n}$ is the $Nr \times 1$ noise vector of Additive White Gaussian Noise (AWGN) components of zero mean and variance σ_n^2. We can write the channel matrix as

$$\mathbf{H} = \left[\ \mathbf{H}_1^{\mathrm{T}}\mathbf{H}_2^{\mathrm{T}}\ldots\mathbf{H}_N^{\mathrm{T}}\ \right]^{\mathrm{T}}. \tag{2}$$

Following the CBST approach we obtain $\mathbf{x}$ as follows

$$\mathbf{x} = \sum_{j=1}^{r} b_{1j}\mathbf{w}_{1j} + \sum_{j=1}^{r} b_{2j}\mathbf{w}_{2j} + \cdots + \sum_{j=1}^{r} b_{Nj}\mathbf{w}_{Nj} = \mathbf{W}\mathbf{b} \tag{3}$$

where each b_{ij} represents the j-th symbol for user i, transmitted with power P_{ij} and $\mathbf{w}_{ij}$ are the precoding vectors $\mathbf{w}_{ij} = \left[w_{ij}^{11},\ldots,w_{ij}^{1t},\ldots,w_{ij}^{kl},\ldots,w_{ij}^{Mt}\right]^{\mathrm{T}}$. The precoding matrices $\mathbf{W}_i = [\mathbf{w}_{i1},\ldots,\mathbf{w}_{ir}]$ will be obtained under a BD criteria to guarantee that the interference is canceled

$$\mathbf{H}_i\mathbf{W}_n = \ \mathbf{0} \text{ if } i \neq n. \tag{4}$$

If we define

$$\widetilde{\mathbf{H}}_i = \left[\mathbf{H}_1^{\mathrm{T}}\mathbf{H}_2^{\mathrm{T}}\ldots\mathbf{H}_{i-1}^{\mathrm{T}}\mathbf{H}_{i+1}^{\mathrm{T}}\ldots\mathbf{H}_N^{\mathrm{T}}\right]^{\mathrm{T}} \tag{5}$$

the condition (4) is obtained when $\mathbf{w}_{ij}$ lie in the null space of $\widetilde{\mathbf{H}}_i$. Let $\widetilde{l}_i$ be the rank of $\mathbf{H}_i$ and define the following singular value decomposition (SVD)

$$\widetilde{\mathbf{H}}_i = \widetilde{\mathbf{U}}_i\widetilde{\mathbf{S}}_i\left[\widetilde{\mathbf{V}}_i^{(1)} \quad \widetilde{\mathbf{V}}_i^{(0)}\right] \tag{6}$$

where $\widetilde{\mathbf{V}}_i^{(0)}$ holds the last $Mt - \widetilde{l}_i$ right singular vectors. If we define a second SVD

$$\mathbf{H}_i\widetilde{\mathbf{V}}_i^{(0)} = \mathbf{U}_i\begin{bmatrix}\mathbf{S}_i & 0\\ 0 & 0\end{bmatrix}\left[\mathbf{V}_i^{(0)} \quad \mathbf{V}_i^{(1)}\right] \tag{7}$$

where $V_i^{(1)}$ represents the first l_i singular vectors, being l_i the rank of $\mathbf{H}_i\widetilde{\mathbf{V}}_i^{(0)}$. The product $\mathbf{W}_i = \widetilde{\mathbf{V}}_i^{(0)}\mathbf{V}_i^{(1)}$ represents the transmission vectors that maximize the information rate for user i subject to canceling interference. Then we obtain for the user of interest

$$\mathbf{H}_i\mathbf{W}_n = \mathbf{U}_i\mathbf{S}_i \text{ if } i = n \tag{8}$$

where

$$\mathbf{S}_i = \mathrm{diag}\{\lambda_{i1}^{1/2}, \lambda_{i2}^{1/2}, \ldots, \lambda_{ir}^{1/2}\}. \tag{9}$$

Then the received signal is

$$\mathbf{y} = \begin{bmatrix} \mathbf{U}_1\mathbf{S}_1 & \mathbf{0} & \ldots & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_2\mathbf{S}_2 & \ldots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{U}_N\mathbf{S}_N \end{bmatrix}\mathbf{b} + \mathbf{n}. \tag{10}$$

Each user may independently rotate the received signal and decouple the different streams

$$\widetilde{\mathbf{y}} = \begin{bmatrix} \mathbf{U}_1^{\dagger} & \mathbf{0} & \ldots & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_2^{\dagger} & \ldots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{U}_N^{\dagger} \end{bmatrix}\mathbf{y} = \begin{bmatrix} \lambda_{11}^{1/2}b_{11}\\ \vdots\\ \lambda_{1r}^{1/2}b_{1r}\\ \vdots\\ \lambda_{Nr}^{1/2}b_{Nr} \end{bmatrix} + \widetilde{\mathbf{n}} \tag{11}$$

where the noise $\widetilde{\mathbf{n}}$ remains white with the same covariance because of the unitary transformation.

Under this strategy, the overall system boils down to a set of parallel non-interfering channels. Therefore the achievable rate of user i is

$$R_i = \sum_{j=1}^{r}\log_2\left(1 + \lambda_{ij}P_ij\right) \tag{12}$$

At this point a power allocation strategy is required to assign the P_{ij} values with the aim to optimize a global goal.

III. CLASSICAL POWER ALLOCATION STRATEGIES

A. *Maximize sum rate strategy*

This scheme maximizes an objective function composed by the sum of the rates of N users subject to M per base stations restrictions, where each base station $k = 1, \ldots, M$ has a maximum available power P_{maxBS}. Finally it can be formulated as

$$\texttt{maximize:} \quad \sum_{i=1}^{N} \sum_{j=1}^{r} \log_2 \left(1 + \lambda_{ij} P_{ij}\right) \tag{13}$$

$$\texttt{subject to:} \quad \sum_{l=1}^{t} \sum_{i=1}^{N} \sum_{j=1}^{r} P_{ij} \left| w_{ij}^{kl} \right|^2 \leq P_{maxBS} \tag{14}$$

Since logarithmic function is concave, and the sum operation preserves the concavity, the objective function is concave subject to linear constraints, therefore this problem can be solved by convex optimization methods. In [8] a strategy maximizing the sum rates is developed. This solution achieves a good performance in global rate terms applied to homogeneous cellular deployments and can be solved by suboptimal approaches as Waterfilling. However, this strategy results unfair in heterogeneous networks where λ_{ij} values are distributed with a high statistical variance [9]. Figure 1 shows the probability density function (pdf) of the spectral efficiency achievable per user from 15 to 6 active users in a deployment of 15 randomly distributed femtocells. When there are a number of users close to the number of base stations the distribution decreases from a maximum to zero. Therefore it is an unfair solution where most users achieve a rate close to zero while few users achieve a great throughput.

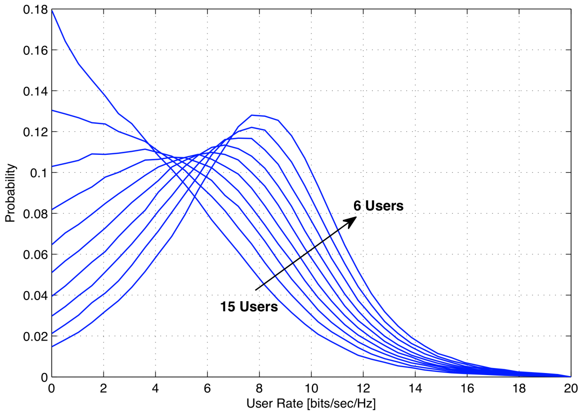

Fig. 1. User rate pdf in a femtocell deployment

B. *Minimize power consumption*

Another classical strategy is to minimize the global power consumption. In this case the objective is a linear function subject to a N minimum achievable rate per user $i = 1, \ldots, N$ constraints.

$$\texttt{minimize:} \quad \sum_{k=1}^{M} \sum_{l=1}^{t} \sum_{i=1}^{N} \sum_{r=1}^{r} P_{ij} \left| w_{ij}^{kl} \right|^2 \tag{15}$$

$$\texttt{subject to:} \quad \sum_{j=1}^{r} \log_2 \left(1 + \lambda_{ij} P_{ij}\right) \geq R_{min} \tag{16}$$

Since the sum of logarithmic functions is concave, its negative value is convex, so the following optimization problem is convex. Obviously the problem is subject to per base station restrictions, however in this case they are box constraints which are implicit in the R_{min} value. In this sense, note that the problem could be unfeasible if R_{min} is too great for the maximum available power constraints. To solve this condition, some strategies try to obtain the maximum minimum rate achievable by all users [11]. However this solution is also inefficient when the resources are heterogeneously distributed, because this rate is determined by the worst user.

IV. ALTERNATIVE POWER ALLOCATION STRATEGIES

Until now we have shown that classical strategies are unfair when they are applied in heterogeneous environments. In this section we propose some power allocation strategies to generate more fair rate distributions with a suitable energy efficiency. Note that all the following strategies are subject to the per base stations constraints (14).

A. *Maximize sum $\lambda_{ij} P_{ij}$*

First we propose to maximize the linear function $\lambda_{ij} P_{ij}$ with the aim to obtain a low complexity solution. As the per base station constraints are also linear, this problem can be solved quickly. The objective function is defined as follows

$$\texttt{maximize:} \quad \sum_{i=1}^{N} \sum_{j=1}^{r} \lambda_{ij} P_{ij} \tag{17}$$

B. *Maximize sum utility function*

Utility functions can be used to obtain more equitable distributions. We propose the utility function $U_\alpha(\omega)$ typically used in economic problems, which is defined by

$$U_\alpha(\omega) = \frac{\omega^\alpha - 1}{\alpha} \tag{18}$$

where $\alpha < 1$, $\alpha \neq 0$ to guarantee the properties of *non-satiation* (the first derivative $U'_\alpha(\omega) > 0$) and *risk-aversion* (the second derivative $U''_\alpha(\omega) < 0$). Note that these economic properties allow to ensure the convexity of our problem when the variable ω is convex. In our case ω_i represents the utility variable of each user $i = 1, \ldots, N$. Therefore, under this strategy it is necessary to maximize P_{ij} and ω_i variables subject to new capacity per user constraints. Finally the optimization problem is defined as follows

$$\texttt{maximize:} \quad \sum_{i=1}^{N} \frac{\omega_i^\alpha - 1}{\alpha} \tag{19}$$

$$\texttt{subject to:} \quad \omega_i \leq \sum_{j=1}^{r} \log_2 \left(1 + \lambda_{ij} P_{ij}\right) \quad i = 1, \ldots, N \tag{20}$$

C. *Maximize log sum rate*

Using the last utility function, users with high throughputs are more penalized in front of users with poor resources as variable α is close to 0. Writing ω^{α} as an exponential variable and using L'Hopital rule it can be demonstrated that the limit of U_{α} as $\alpha \to 0$ corresponds to the natural logarithmic function

$$\lim_{\alpha \to 0} \frac{\omega^{\alpha} - 1}{\alpha} = \lim_{\alpha \to 0} \frac{e^{\log(\omega)\alpha} - 1}{\alpha} = \log(\omega) \tag{21}$$

We propose a strategy whose goal is to maximize the logarithmic sum of the achievable rates per user. Since logarithmic and rate function are concave, maximizing the proposed objective function preserves the convex optimization requirements. Finally the strategy can be written as

$$\texttt{maximize:} \sum_{i=1}^{N} \log \left\{ \sum_{j=1}^{r} \log_2 \left(1 + \lambda_{ij} P_{ij}\ \right) \right\} \tag{22}$$

D. *Minimize sum Watt/bps ratio*

Finally we propose to optimize an energy efficient metric. According with [2] we try to minimize the Energy Consumption Rating (ECR). At this point it is necessary to remember that this type of metrics is required for measuring the degree of "greenness" in telecommunication networks. Therefore, it is necessary to define a power consumption to spectral efficiency ratio. In this case we use the log sum rate in the denominator because it is an efficient solution at user level. To preserve the convexity of this problem we apply a exponential function in the power consumption in the numerator, which is dissolved after applying the natural logarithm. Then our objective function is defined by

$$\texttt{minimize:} \sum_{i=1}^{N} \log \left\{ \frac{\exp \left(\sum_{k=1}^{M} \sum_{l=1}^{t} \sum_{r=1}^{r} P_{ij} \left| w_{ij}^{kl} \right|^2 \right)}{\sum_{j=1}^{r} \log_2 \left(1 + \lambda_{ij} P_{ij}\ \right)} \right\} \tag{23}$$

V. SIMULATION ENVIRONMENTS

The performance of the different power allocation strategies is analyzed through Monte-Carlo simulations in micro and femto cell deployments. In both scenarios, the user noise figures are 7 dB over a 5 MHz bandwidth, the user antenna gains are 0 dB and connector losses are 1 dB.

A. *Microcell Environment*

Microcell deployment is characterized by a high homogeneity. However, due to the cell radius micro-ization process and the users mobility, the load at base stations may be different from each other. We propose a 21 microcell deployment with a 300 m radius each one regularly distributed as we can see in Figure 2. Output power available in each micro Node B is 38 dBm. Finally the path losses between micro Node B and users ares calculated with the model ITU P.1411.

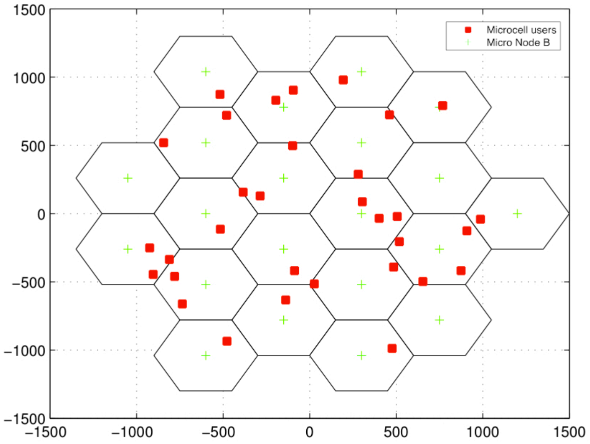

Fig. 2. Microcell scenario layout

B. *Femtocell Environment*

A femtocell scenario is characterized by a high density of nodes irregularly distributed with a short coverage radius. To simulate this heterogeneous distribution, in each iteration a quasi-randomly femtocell distribution subject to a separation distance between femtocells greater than 10 meters is generated. Therefore, the deployment avoids the femtocell "dead zone", areas where the quality of service is so poor as a result of interference that it is not possible to provide the demanded service. In our case there are fifteen femtocells operating in the same frequency. All BSs transmit 10 dBm maximum power. Finally we use the path loss model ITU-R P.1238 to calculate the propagation losses between femtocell access points (FAP) and femtocell user equipments (FUE). An example of our simulation layout is shown in Figure 3.

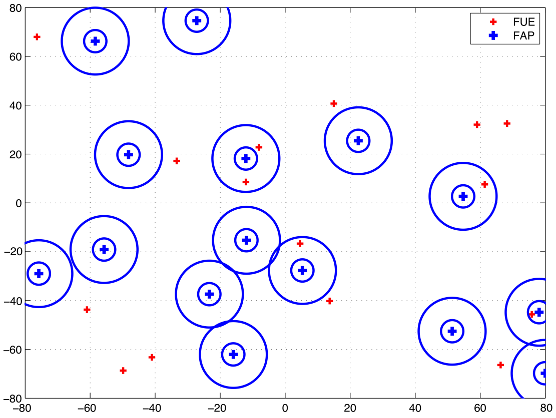

Fig. 3. Femtocell scenario layout

VI. NUMERICAL RESULTS

The performance of the proposed strategies is shown in this section. All these results have been obtained solving the optimization problems with CVX software [12]. First we analyze the spectral efficiency per user with the cumulative distribution function (CDF) after Monte-Carlo simulations in micro and femto cell deployments. Finally, the average power consumption versus achievable rate obtained is shown.

A. *Spectral efficiency*

Microcell environment

Figure 4 shows the throughput per user of the proposed strategies in a microcell environment, where each base station is equipped with $t = 2$ antennas, and there are 42 active users. Under this deployment still there is a low density of base stations and the BD resources λ_{ij} are not characterized by a high heterogeneity. Therefore the CDFs of the strategies are similar, except for the sum $\lambda_{ij}P_{ij}$ case, and it is possible to ensure a spectral efficiency above 3 bits/sec/Hz at the 50-th percentile. However, in Table I it is shown that the alternative strategies achieve a variance reduction without a high penalty in the global or mean rate. The variance is reduced from 4.722 using sum Rate to 3.433 and 3.212 with sum Log Rate and sum ECR respectively, with about 4.5% penalty in mean rate.

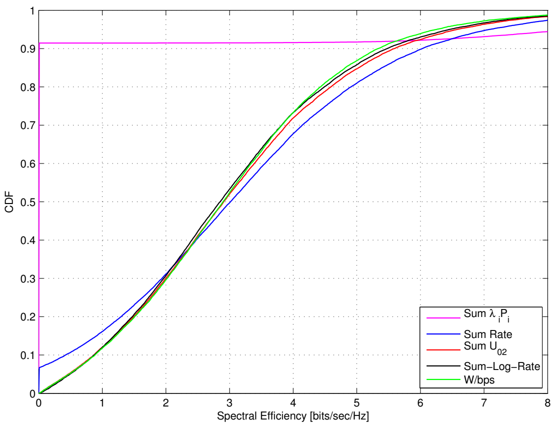

Fig. 4. CDFs of achievable throughput per user. Microcell scenario, 42 users

Femtocell environment

Next step is to analyze the spectral efficiency per user of these strategies in a femtocell scenario when FAPs and FUEs are equipped with one antenna $t = r = 1$. Under this scenario the λ_{ij} distribution is very heterogeneous and classical strategies result unfair since many users achieve rates close to zero [9].

Figure 5 shows the CDF when there are 15 active users. First we can see that sum rate criterion generates about 60% of users with throughputs close to zero. If the proposed alternatives strategies are applied, it is reduced to between 30% and 40% of users achieve null rates, so these strategies are more fair. On the other hand a low penalty is obtained in mean rate. As we can see in Table I the percentage of users with throughputs close to zero decreases greatly when there are only 10 active users as we can see in Figure 6. In this case about 18% of users achieve null rates, while alternative strategies allow to solve this problem completely and only few users, less than 1%, obtain zero throughput.

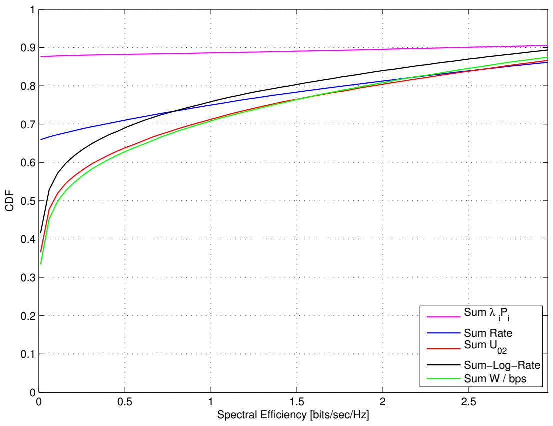

Fig. 5. CDFs of achievable throughput per user. Femtocell scenario, 15 users

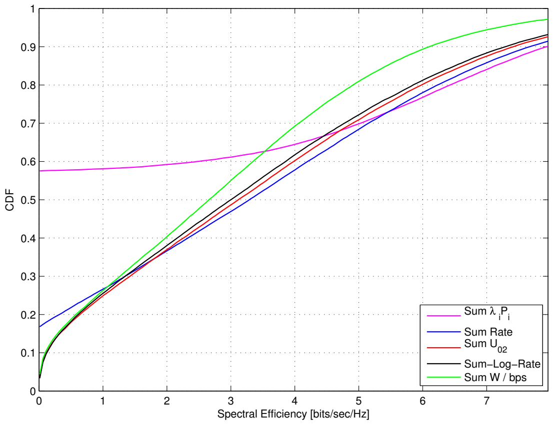

Fig. 6. CDFs of achievable throughput per user. Femtocell scenario, 10 users

B. *Power Consumption*

Microcell environment

Figure 7 shows the average power consumption versus achievable spectral efficiency in a microcell deployment. First we can see that sum $\lambda_{ij}P_{ij}$ strategy results inefficient. If we consider sum Rate, sum $U_{0.2}(\omega)$ and sum Log Rate strategies, we conclude that they obtain a similar energy performance. However, sum W/bps strategy achieves better results in power consumption terms, and as we can see in Table I. It is also a fair solution at user rate level without a high penalty in mean throughput with respect to maxize Sum Rate strategy.

TABLE I

ACHIEVABLE THROUGHPUTS STATISTICAL RESULTS, MEAN AND VARIANCE

Strategy	**Sum $\lambda_{ij}P_{ij}$**		**Sum Rate**		**Sum $U_\alpha(\omega)$**		**Sum Log Rate**		**Sum ECR**	
Deployment	mean	variance	mean	variance	mean	variance	mean	variance	mean	variance
Microcell 42 UEs, t = 2	0.762	6.671	**3.214**	4.722	3.133	3.596	3.075	3.433	3.066	3.212
Femtocell 15 UEs, t = 1	0.643	3.977	**1.138**	3.977	1.0469	3.263	0.865	2.727	1.002	2.782
Femtocell 10 UEs, t = 1	2.687	12.38	**3.585**	8.905	3.5	7.948	3.415	7.732	2.926	5.433

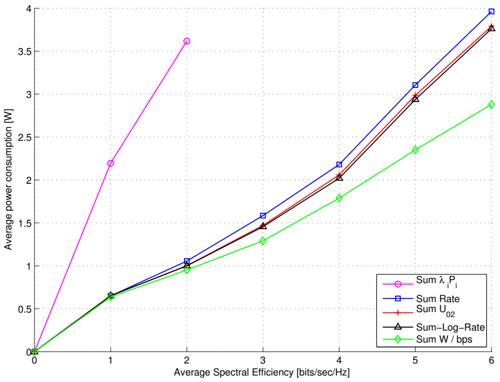

Fig. 7. Power consumption vs spectral efficiency. Microcell scenario, $N = 42$

Femtocell environment

Finally the performance in average power consumption is shown in Figure 8 when there are 15 active users. Again sum Rate, sum $U_{0.2}(\omega)$ and sum Log Rate achieve a similar performance which are better than the obtained with sum $\lambda_{ij}P_{ij}$. Sum W/bps achieves a great improvement with respect to other strategies. And as in the microcell case, according to Table I, we can consider sum W/bps an fair solution at user rate level without a great penalty in mean throughput per user.

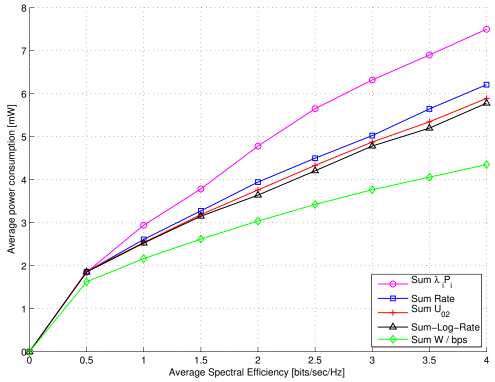

Fig. 8. Power consumption vs spectral efficiency. Femtocell scenario, $N = 15$

VII. CONCLUSIONS

In this work we have analyzed the performance of the classical power allocation strategies, concluding that they provide unfair solutions at user rate level in very heterogeneous networks. Then we have proposed some alternative strategies, including one of the them based on green metrics with the aim to minimize a $Watt/bps$ ratio. All of these strategies are subject to maximum available power constraints. With the objective functions defined, all the optimization problems are convex, so they can be solved by convex methods as CVX. We have evaluated all these power allocation alternatives in micro and femto cell deployments, concluding that they achieve more fair solutions at user level than classical approaches. In this sense, a reduction of the variance between user rates is obtained without a great penalty in mean with respect to sum rate strategy. Finally, after an average power consumption versus average spectral efficiency analysis we conclude that sum W/bps strategy offers a good trade off between achievable rate, global power consumption and fairness at user level.

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