This document is published in:

IEEE Communications Letters 17 (2013) 6-June, pp. 1244-1247

DOI: 10.1109/LCOMM.2013.043013.122822

Robust Timing Synchronization for Multicarrier Systems Based on RST Invariance

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***Abstract*—In this paper, a blind robust timing synchronization method, based on invariance properties and cyclostationarity, for multicarrier systems such as Orthogonal Frequency Division Multiplexing (OFDM) is proposed and evaluated. Its results outperform the state-of-the-art for blind methods, specially in hard wireless channels where our proposal is highly superior. It even surpasses the performance of most of non-blind (pilot-aided) methods, while at the same time, it gets the advantages of energy and bandwidth saving of blind proposals. Moreover, since this technique does not require the use of pilots, it can be easily applied to either packet-based or broadcasting systems.**

***Index Terms*—OFDM systems, timing synchronization, invariant transform, cyclic prefix.**

I. Introduction

NOWADAYS, multicarrier systems such as Orthogonal Frequency Division Multiplexing (OFDM) have received a great attention for multipath wireless environments. However, it is well-known that the performance of these systems requires adequate time synchronization [1], so that the design of robust techniques for timing synchronization is mandatory in order to obtain a good performance. Indeed, there is an extensive literature dealing with this problem. These techniques can be grouped into two categories, namely, Pilot-Symbol Aided (PSA) and Non-Pilot-Symbol-Aided (NPSA), the latter also denoted as blind.

PSA approaches show a better performance when estimating the timing offset [2], specially in frequency-selective channels. However, they loose in data rate due to the need of using pilots. Moreover, there is an energy expenditure since these techniques typically send Pseudo Noise (PN) sequences or duplicated structures [3], [4] for synchronization purposes that, however, do not convey useful data.

On the other hand, NPSA schemes, also known as blind methods, are based on the analysis of the received signal mainly taking into account the cyclostationarity properties due to the insertion of the cyclic prefix [5], [6] and thus, pilots are not required to be inserted for the synchronization task. However, those blind methods are severely affected by a frequency-selective channel, and therefore, they typically demonstrate a poorer performance than PSA ones.

This work has been partly funded by the Spanish national projects GRE3N-SYST (TEC2011-29006-C03-03) and COMONSENS (CSD2008-00010).

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In this paper we present a novel blind technique for time synchronization that overcomes the disadvantages of previous NPSA methods and, at the same time, it even outperforms most PSA techniques in severe frequency-selective channels. Our proposal relies on a descriptor of the invariance that exploits the cyclostationarity of the cyclic prefix, what provides robustness against frequency-selective channels. It is based on the properties of an *invariant transform*, commonly used in digital watermarking for images [7]. This transform translates the original signal into an invariant domain where the embedded information is robust against attacks on the image such as Rotation, Scaling, or Translation (RST). These distortions are the main effect of a communication channel and thus, we propose it to be used over the received signal for the timing synchronization problem. When the received signal is translated into this RST invariant domain, the cyclic prefix and the last samples of the OFDM symbol remain being a copy without channel distortion. This key fact significantly eases timing synchronization.

The organization of this paper is as follows. In Section II, the system model is presented and then, the general procedure for time synchronization is explained. The RST *invariant transform* is described in Section III, and the proposed technique by using a descriptor of the invariance is developed in Section IV. Finally, the results and conclusions are provided in Section V and Section VI, respectively.

Notation: Letters in uppercase and lowercase refer to a signal in the frequency domain and time domain, respectively, bold face to vectors, the operator $\otimes$ denotes the convolution and conjugate is denoted by the superscript *.

II. System Model

In an OFDM system, the signal can be obtained by applying the Inverse Discrete Fourier Transform (IDFT) to the modulated complex symbol X_k at k-th subcarrier as

$$x(n) = \frac{1}{N} \sum_{k=1}^{N} X_k e^{j2\pi kn/N} \,, \quad (1)$$

where N is the number of subcarriers and X_k is the data to be sent at k-th subcarrier, with $k, n = \{0, 1, ..., N-1\}$ as the frequency and time indices, respectively. Next, the last samples are appended at the beginning of the OFDM symbol, what is denoted as Cyclic Prefix (CP), obtaining $\mathbf{x}_i \in \mathbb{C}^{1\times(N+L_{cp})}$, with L_{cp} as the number of samples of the cyclic prefix and i the OFDM symbol index

$$\mathbf{x}_i = [\ x(N-L_{cp}) \ ... \ x(N-1) \ x(0) ... \ x(N-1)]. \ 1 \quad (2)$$

The purpose of the CP is to maintain the orthogonality and avoid Inter Symbol Interference (ISI) in a wireless channel. However, at the receiver, there will exist an uncertainty on the timing transmission τ, and so

$$\breve{x}(n) = x(n-\tau) \ . \tag{3}$$

Our goal is to estimate τ. Taking into account the transmission of several symbols, we can define

$$\tilde{\mathbf{x}} = [\breve{\mathbf{x}}_1 \ \breve{\mathbf{x}}_2 \ \breve{\mathbf{x}}_3 \ \dots \ \breve{\mathbf{x}}_M] \tag{4}$$

where M is the number of transmitted OFDM symbols. The signal in (4) is transmitted through a frequency-selective channel and Additive White Gaussian Noise (AWGN) is added. The received signal is then

$$\mathbf{r} = \tilde{\mathbf{x}} \otimes \mathbf{h} + \mathbf{w}, \tag{5}$$

where $\mathbf{r} \in \mathbb{C}^{1\times M(N+L_{cp})}$, $\mathbf{h} \in \mathbb{C}^{1\times L_{ch}}$ is the Channel Impulse Response (CIR) with L_{ch} taps, and $\mathbf{w} \in \mathbb{C}^{1\times M(N+L_{cp})}$ refers to the AWGN vector.

A. Timing Synchronization

As mentioned above, time acquisition is a key issue for good performance. Most of the algorithms for synchronization are PSA and follow the basic idea proposed by Schmidl and Cox in [3]. This technique assumes a repeated structure as a preamble and then, a metric based on the correlation is performed in order to obtain a peak that determines the time offset (τ). This metric (M_{Sch}) [3] is given by,

$$M_{Sch}(d) = \frac{|P_{Sch}(d)|^2}{R_{Sch}(d)^2} \tag{6}$$

with $P_{Sch}(d) = \sum_{m=0}^{L-1} r^*(d+m) \ \ r(d+m+L)$ and $R_{Sch}(d) = \sum_{m=0}^{L-1} |r(d+m+L)|^2$, where L is the window length over which the metric is evaluated, $r(n)$ denotes the n-th received sample given in matrix notation by (5), and d is the sample index where the metric is calculated. For this metric, the preamble sent by the transmitter is composed of two identical halves in time domain with the pattern $[A \ A]$, where A represents samples of length $L = N/2$, generated by an IFFT operation over a collection of $N/2$ modulated data of a Pseudo Noise (PN) sequence. On the other hand, in [8], a different pattern helps getting a much better accuracy when estimating the time offset, $[A \ \ A \ -A \ -A]$, by using the same metric but now with A of length $L = N/4$ samples, $P_{Minn}(d) = \sum_{u=0}^{1}\sum_{m=0}^{L-1} r^*(d+2Lu+m) r(d+2Lu+m+L)$ and $R_{Minn}(d) = \sum_{u=0}^{1}\sum_{m=0}^{L-1} |r(d+2Lu+m+L)|^2$ instead.

A similar idea is used in NPSA methods such as in Beek's *et al* proposal [5], where the correlation is performed between the samples of the received signal in two windows, whose lengths are L_{cp}, separated N samples apart so that, there will experiment a peak when samples of cyclic prefix and the last part of the symbol fall into these windows, respectively

$$C_{Beek}(d) = \sum_{m=d}^{d+L_{cp}-1} r(m) r^*(m+N). \tag{7}$$

Indeed, according to [5], this is the Maximum Likelihood (ML) metric.

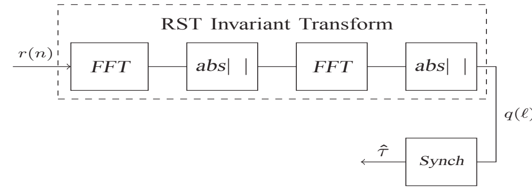

Fig. 1. Block diagram at the receiver for RST invariance-based synchronization, where $q(\ell)$ is the received signal in the invariant domain and $\hat{\tau}$ is the timing offset estimation.

Although all these metrics exhibit a well defined maximum peak when the signal is not distorted by a noisy channel, they usually severely degrade when the signal goes through a frequency-selective channel because the channel causes signal translation, rotation and attenuation, in addition to AWGN.

III. RST Invariant Transform

The idea behind the application of an RST *invariant transform* is to make the signal robust against the rotation, translation and scale caused by the channel. The most common scenario where the RST *invariant transform* is proposed is in digital watermarking, where a hidden signal is embedded in a domain called rotational, scale and translational invariant, so that the information cannot be lost because of distortions in the original image such as linear, rotational or scale changes. Therefore, the attacks on digital watermarking are equivalent to a wireless communication problem where the message goes through a channel which is composed of distortions, interference and noise such as those mentioned above.

Motivated by the results in digital watermarking and its relationship with communication theory, we apply this tool to the time synchronization problem, where the signal has to be robust against the channel as if it were a hidden message to be sent. The RST *invariant transform* described in [9] makes use of the Fast Fourier Transform (FFT) properties and the Mellin Transform (MT). First, in order to obtain invariance with respect to the scale, we apply an FFT and the magnitude is extracted. Next, to acquire invariance on rotation and translation, the MT is used. In our case, since the signal is one dimensional (and not two dimensional as an image), the MT reduces to another FFT and magnitude operations, avoiding the log-polar transformation. After that, we get the signal in the invariant domain $q(\ell)$, with ℓ the invariant domain index. The process can be mathematically described as in (8) and it can be graphically seen in Fig. 1, where $q(\ell)$ is the received signal in the RST invariant domain, and therefore with the CP unaltered. Eventually, timing estimation can be extracted from this invariant domain.

$$q(\ell) = \left| \sum_{m=1}^{N} \left| \sum_{n=1}^{N} r(n)\, e^{-j2\pi nm/N} \right| e^{-j2\pi m\ell/N} \right| . \tag{8}$$

IV. Proposed Scheme

Since the OFDM symbol has a cyclic prefix (a copy of the last samples at the beginning of the OFDM symbol), we can leverage on this property to accomplish timing synchronization. The received samples, when transformed into the RST invariant domain, *i.e.,* $q(\ell)$, will experiment invariance to Rotation, Scaling and Translation operations that may have suffered through the channel. This means that same samples before transmission produce same results in the invariant domain. Thus, since the samples in the cyclic prefix are the same as those at the end of the OFDM symbol, a metric evaluated in the RST invariant domain will produce an almost undistorted result, only affected by the transformed noise. It should be noted that this method does not introduce any changes at the transmitter side. Moreover, this scheme provides several advantages such as the translation and scale invariance useful for timing synchronization. An interesting property of the proposed scheme is that it exploits the cyclostationarity provided by the cyclic prefix to perform robust synchronization. Thus, we do not need to send a preamble or pilot signal because the signal $q(\ell)$ is robust against distortions and thus, the metric can be evaluated by only using the last part of the symbol and the CP. The following metric, an adaptation of the one in [5] to the real-valued nature of signal $q(\ell)$, is proposed

$$C(d) = \sum_{m=0}^{\frac{N+L_{cp}}{2}-1} q(d+m)q\left(d+m+\frac{N+L_{cp}}{2}\right) . \quad (9)$$

This is indeed, the ML estimation adapted to a real-valued signal. Moreover, the obtained correlation (9) improves as the length of the OFDM symbol increases while maintaining the length of the cyclic prefix fixed. Due to the absolute value operations, only the use of M-PSK modulations is explored.

In order to capture the time offset (τ) in the middle of the plateau, an average can be calculated, after taking the correlation of the signal $q(\ell)$, in the following way.

$$\tau_i = \max_d \frac{1}{L_{cp}} \sum_{d=0}^{L_{cp}-1} C(d). \quad (10)$$

Since our proposal provides a timing estimation OFDM symbol by OFDM symbol, the final estimation can be improved by averaging the time estimation over several symbols.

$$\hat{\tau} = \frac{1}{N_a} \sum_{i=0}^{N_a-1} \tau_i, \quad (11)$$

where N_a refers to the number of OFDM symbols being averaged. This average is not mandatory but, as it can be seen in figures 2 through 4, with a small number of averaged symbols, the accuracy of the estimation is highly increased. Although averaging introduces delay and memory requirements, these are very low (with 3 symbols is enough) while final performance is improved in more than 5 dB.

This turns out to be very useful, particularly in broadcast systems, where there are no preambles and transmission is continuous.

V. RESULTS

In this section we show the results obtained when comparing conventional PSA schemes described in [4], [8], [10], and NPSA proposals such as in [6], [11] with respect to our proposed blind scheme. We have considered an OFDM system with a BW of 2MHz, N=256, QPSK modulation and $L_{cp} = 16$. The channel has an impulsive response given by the coefficients h_r, $r = \{0, \cdots, L_{ch}-1\}$, with $L_{ch} = 15$, with a power delay profile as an exponential form $exp\left(\frac{-\tau_r}{\beta}\right)$, where τ_r refers to the delay spread time and β is a constant set to the value of 4. 50000 realizations have been simulated for each scheme. It is important to highlight here that, since it is a blind method, our proposal presents an energy efficiency gain.

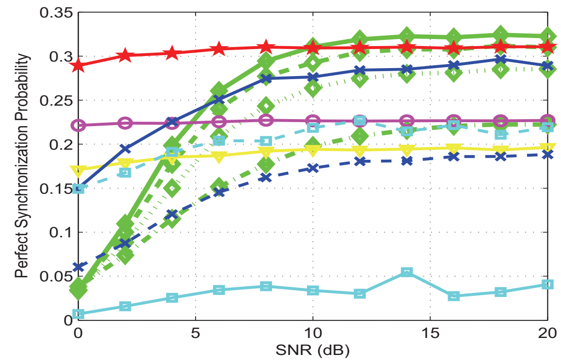

Fig. 2. Perfect Synchronization Probability comparison. Legend is the same as in Fig.3, where numbers in brackets indicate the number of OFDM symbols that have been averaged (N_a).

The energy efficiency E_{ff} is given by (12) where L_p is the length of the preamble in number of samples, E_s the energy per complex symbol and N_s the number of data OFDM symbols after the preamble.

$$E_{ff} = \frac{N_s(N+L_{cp})E_s}{L_pE_s + N_s(N+L_{cp})E_s}. \quad (12)$$

This ratio approaches to one as the length of the preamble is reduced or the number of symbols in the packet increases to infinity, whereas the efficiency for NPSA schemes is always 1. In traditional systems, where the preamble consists of a set of two OFDM symbols, *i.e.,* $L_p = 2\,(N+L_{cp})$, the energy efficiency is given by $E_{PSA} = \frac{N_s}{N_s+2}$. For schemes with only one preamble as in [8], [10], $E_{PSA} = \frac{N_s}{N_s+1}$, which is a poor efficiency for especially continuous broadcast systems.

For the performance evaluation and comparison, the Perfect Synchronization Probability (or Lock-in probability) $P_r\,(\hat{\tau} = L_{cp})$ is used as the primary measurement. Additionally, we also provide results for the Mean Squared Error (MSE) traditionally used as a quality indicator for algorithms. However, MSE does not take into account when estimated timing point $\hat{\tau}$ is out of the Safe Region[1]. For this reason, we also use the Probability of Timing Failure as defined in [12]

$$P_{tf} = P_r^{(m)}\left(|\hat{\tau} - E\{\hat{\tau}\}| \geq m\right). \quad (13)$$

Fig. 2 shows the comparison between our *invariant transform*-based proposal and the PSA methods given in [4], [8], [10] and the NPSA in [6], [11]. As it can be seen, our method outperforms them as the SNR becomes high and only averaging over a reduced number of OFDM symbols. It can be observed that above SNR= 10 dB our proposal is superior to all the other methods, including the PSA ones, but also with

[1]Region defined in the values of $\hat{\tau}$ where estimation errors cause neither ISI nor Inter Carrier Interference (ICI).

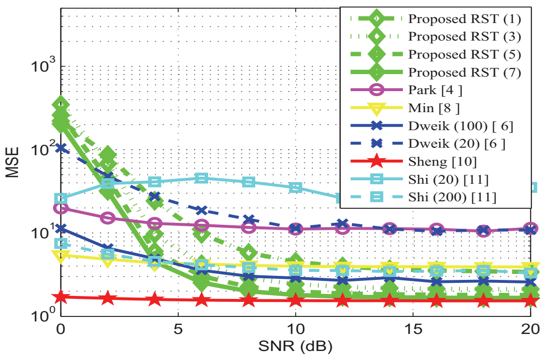

Fig. 3. MSE comparison. Numbers in brackets indicate the number of OFDM symbols that have been averaged (N_a).

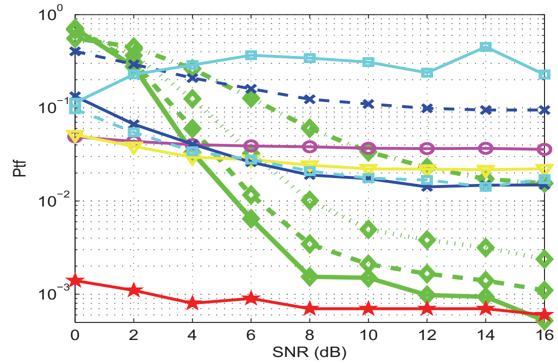

Fig. 4. Probability of Timing Failure (m = 5). Legend is the same as Fig. 3, where numbers in brackets are the number of OFDM symbols being averaged.

a considerable saving of energy, as explained before. Furthermore, our proposal is more robust against harder frequency-selective channels than the other schemes since it provides better performance in hard channels, *i.e.,* with more taps. Besides, the result here is the perfect synchronization goal. If we allow a small timing error of up to two samples (commonly used in a real implementation because the misalignments can be absorbed by the channel estimation), our proposal obtains nearly 90% synchronization in hard wireless channels without expending energy and bandwidth in preambles.

Also in this Fig. 2, the benefits of averaging are clearly highlighted. It can be seen that the more symbols are averaged, the better the synchronization is. However, there is a trade-off between the number of averaged symbols and the delay (and thus the memory) allowed by the system. Moreover, by using our proposal, a sliding window for averaging can be used and therefore, this would improve the performance over broadcasting (usually continuous) systems. An interesting conclusion is that averaging only over 3 OFDM symbols is enough to improve performance in 5 dB.

Another quality measurement is the MSE, which is shown in Fig. 3, where the MSE for all the previous evaluated schemes is shown. It can be observed that our proposal, averaging only over 3 OFDM symbols, outperforms all the other methods from SNR$=8$ dB except the one in [10], which is PSA. In any case, differences with respect to this scheme above SNR$=11$ dB are negligible.

Finally, in Fig. 4, the Probability of Timing Failure for $m=5$ is plotted for the different proposals showing that our proposal is superior to the other methods above 5 dB of SNR, except the PSA proposal by *Sheng et al.* [10], although, we obtain a saving in energy compared to it since ours is blind.

VI. Summary

We have proposed a blind method based on a RST *invariant transform* that outperforms the state-of-the-art of blind (NPSA) methods, and even most of PSA time synchronization techniques while, at the same time, it obtains the energy efficiency of NPSA methods. Besides, as blind scheme, it can be applied to every receiver and system since it does not rely on a specific transmission structure (such as preambles or pilots). We have proven that our proposal is robust against frequency-selective channels and moreover, the more frequency-selective the channel is, the more improvement in performance with respect to existing methods there is. Since it does not need a preamble, it can be used for either packet-based or broadcasting systems, ameliorating both in energy and efficiency aspects.

Acknowledgment

The authors would like to thank the anonymous reviewers for their helpful comments that have improved the quality of the paper.

References

[1] R. V. Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Artech House Publishers, 2000.

[2] J. M. Páez-Borrallo, S. Zazo, and M. J. F.-G. García, "A new time-frequency synchronization scheme for OFDM-TDMA systems," in *Proc. 1999 IEEE Vehicular Technology Conference*, pp. 2408–2412.

[3] T. M. Schmidl and D. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol. 45, pp. 1613–1621, 1997.

[4] B. Park, H. Cheon, C. Kang, and D. Hong, "A novel timing estimation method for OFDM systems," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 239–241, May 2003.

[5] J. J. V. de Beek, M. Sandell, and P. O. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. Signal Process.*, vol. 45, pp. 1800–1805, 1997.

[6] A. Al-Dweik, S. Younis, A. Hazmi, C. Tsimenidis, and B. Sharif, "Efficient OFDM symbol timing estimator using power difference measurements," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 509–520, Feb. 2012.

[7] M. Álvarez Rodríguez and F. Pérez-González, "Analysis of pilot-based synchronization algorithms for watermarking of still images," *Signal Process.: Image Commun.*, vol. 17, pp. 611–633, 2002.

[8] H. Minn, M. Zeng, and V. Bhargava, "On timing offset estimation for OFDM systems," *IEEE Commun. Lett.*, vol. 4, no. 7, pp. 242–244, 2000.

[9] D. Zheng, Y. Liu, J. Zhao, and A. el Saddik, "A survey of RST invariant image watermarking algorithms," *ACM Computing Surveys*, vol. 39, no. 2, 2007.

[10] B. Sheng, J. Zheng, X. You, and L. Chen, "A novel timing synchronization method for OFDM systems," *IEEE Commun. Lett.*, vol. 14, no. 12, pp. 1110–1112, Dec. 2010.

[11] M. Shi, Y. Bar-Ness, and W. Su, "Revisiting the timing and frequency offset estimation based on cyclostationarity with new improved method," *IEEE Commun. Lett.*, vol. 13, no. 7, pp. 537–539, Jul. 2009.

[12] S. Muller-Weinfurtner, "On the optimality of metrics for coarse frame synchronization in OFDM: a comparison," in *Proc. 1998 IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, pp. 533–537.