# Migration between platforms*

Gary Biglaiser†
Jacques Crémer‡
André Veiga§

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†University of North Carolina, Department of Economics, Chapel Hill, NC 27599-3305, USA; gbiglais@email.unc.edu

‡Toulouse School of Economics, Université Toulouse Capitole, 1 esplanade de l'Université, 31080 Toulouse Cedex 06, France; jacques.cremer@tse-fr.eu.com

§Department of Economics, Imperial College, London; a.veiga@imperial.ac.uk.

**Abstract**

We study incumbency advantage in markets with positive consumption externalities. Users of an incumbent platform receive stochastic opportunities to migrate to an entrant. They can accept a migration opportunity or wait for a future opportunity. In some circumstances, users have incentives to delay migration until others have migrated. If they all do so, no migration takes place, even when migration would have been Pareto-superior. This provides an endogenous micro-foundation for incumbency advantage. We use our framework to identify environments where incumbency advantage is larger.

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**JEL Classication Codes:** D85, L14, R23, L15, L16

# 1 Introduction

The utility of joining a telecommunications or a social media platform, buying a game console, or adopting an industry standard depends on who else has joined the platform, plays the same game, or uses the same standard. Users choose which platform to use, game to purchase, or standards to adopt based on their predictions of the number of users who will make the same choice. Economists and practitioners often believe that this makes entry difficult as users worry that others will not migrate from an incumbent to an entrant platform, even when the latter offers a superior product. This is easy to understand when there are important switching costs; it is more difficult to explain when incumbency advantage stems from network externalities, the issue we tackle in this paper.[1]

The topic has policy relevance as incumbency advantage forms the basis of many recent analyses and policy recommendations. For an example, consider the 2018 $7.5 billion acquisition by Microsoft of GitHub, a collaborative coding platform which, at the time of the acquisition, was used by 28 million developers working on 85 million projects.[2,3] GitHub is used by closed teams, but it is especially popular in the Open Source community which can use it at zero cost. The European Commission was concerned that incumbency advantage would impair the migration of users from GitHub to competing platforms, and that this would allow Microsoft to degrade its quality, perhaps by favouring Microsoft's own technologies. One can imagine that the Microsoft legal team acknowledged that the Commission's concerns would be valid if GitHub were a social network, but argued that its users are actually very sophisticated, well aware of the alternatives, and would surely migrate if the platform were degraded. Ultimately, the Commission approved the

[1] In most real world cases, there would be both switching costs and network externalities. As Crémer and Biglaiser (2012) argue, the interaction between the two phenomena is understudied.

[2] Our sources about this acquisition include the following pages, accessed on 9 July 2019:
http://europa.eu/rapid/press-release_IP-18-6155_en.htm, https://usefyi.com/github-history/, https://www.bloomberg.com/news/articles/2019-06-03/open-source-great-satan-no-more-microsoft-wins-over-skeptics, https://www.theverge.com/2018/6/18/17474284/microsoft-github-acquisition-developer-reaction.

[3] Actually, repositories. For discussion, see https://help.github.com/en/articles/about-repositories.

acquisition (see European Commission (2018), §§ 98-102).

We draw two lessons from this story. First, the Commission's decision seems reasonable and has been the object of little public criticism, but this seems to contradict the widespread belief that incumbency advantage is pervasive and important – that is something which needs explaining. Second, we know of no article in the economic literature that would have helped the Commission evaluate the claims made by Microsoft's legal team. There have been debates around the pervasiveness and size of incumbency advantage in the economy as a whole but, as we will discuss in section 2, very little theoretical work on the sources of incumbency advantage.

Explaining the relationship between network externalities and incumbency advantage is also an interesting analytical challenge. Consider, as we will, a situation in which all the users are initially on an incumbent platform and where they would all prefer to the status quo a *collective* migration to a new entrant platform. In the game in which each of the users chooses a platform, no-migration is a Pareto inferior equilibrium, but the one which the literature typically focuses on. This contradicts the assumption commonly made in other branches of economic theory where the Pareto superior equilibrium is often selected, without further discussion. To solve this quandary, we propose a new model of the migration process.

We make two main contributions. First, we develop a micro-founded model in which incumbency advantage emerges endogenously. Second, we study how incumbency advantage is determined by the migration process available to individuals. In our model, even if every consumer prefers a collective migration to the entrant platform, each consumer prefers that others migrate first to avoid spending time "alone" on the new platform. They are somewhat like pedestrians on the sidewalk of a street with slow traffic. They know that they can safely cross *en masse*, but each of them would prefer others to step on the road first.

To be more precise, we identify the circumstances under which no user initiates the migration process because, even if they believed that the others will migrate, accepting early migration opportunities implies giving up too much of the network value of the incumbent platform. In order to do this, we depart from Farrell and Saloner (1985), who assume that each user of the incumbent has a single opportunity to migrate. Instead, we assume that other opportunities will arise in the future. This yields very different dynamics and generates incumbency advantage even when Farrell and Saloner predict it would be absent. One of the main results of our paper is that, perhaps surprisingly, incumbency advantage is greater when users have multiple migration opportunities than only one (Proposition 4). Building on this foundation, we discuss how the stochastic process that generates migration opportunities, what we call the *migration process*, influences the existence of a migration equilibrium.

After a discussion of the literature in section 2, in section 3.1 we study the strategy of a single individual who receives opportunities to migrate from an incumbent platform (whose value is decreasing over time) to an entrant platform (whose value is increasing) and must decide which ones to accept. Under mild assumptions, she uses a "threshold" strategy, accepting all opportunities to migrate after a cutoff time. In section 3.2, we embed the individual's problem in an equilibrium model and study the existence of a "migration equilibrium", whose properties we explore in the remainder of Section 3. Our focus is on describing how the migration process itself, rather than the beliefs of the agents, can impede migration to a superior platform: when there are several equilibria, we select the one with migration. We show that incumbency advantage is larger when users have multiple rather than a single opportunity to migrate. We also show that, for low discount rates, incumbency advantage is invariant to the speed of the migration process and only depends on its shape.

In section 4, we conduct a sensitivity analysis on the fundamentals on our model and explore the characteristics of migration processes that affect the size of the incumbency advantage. For instance, we show that incumbency advantage increases when migration is more coordinated and is invariant to the speed of the migration process.

In section 5, we examine a parameterized setting where opportunities to migrate arise according to a combination of two natural and easily interpretable processes: a "word of mouth" process, where opportunities arise when a user of the incumbent platform randomly meets a user who has already migrated; and an "autonomous" process, where opportunities to migrate arise at a constant rate, for instance due to a constant flow of advertisements. This formulation provides a novel micro foundation for the "Bass diffusion model", one of the workhorses of the marketing literature (see Bass, 1969, 2004, among many others). We study how incumbency advantage varies with the weight of the two component processes. When word of mouth is the dominant of the two processes, migration only occurs when users prefer being alone on the entrant platform to sharing the incumbent platform with all the other users (*i.e.*, when it is a *dominant strategy* to migrate). When the autonomous component dominates, we show that there can be excessive or insufficient migration.

We then extend our model to explore other institutional features that can affect incumbency advantage. In Section 6.1, we show that incumbency advantage *increases* when users receive subsequent migration opportunities faster than the first one (for instance because users become more aware of the existence of the entrant platform). We demonstrate in Section 6.2 that the entrant can *decrease* incumbency advantage by committing to a capacity constraint, so that not all users can join it. Finally, we find that the possibility of multi-homing decreases, but does not eliminate, incumbency advantage. This provides some support to the policy recommendations that competition authorities should pay special attention to practices that hinder multi-homing (see, for instance, Crémer, de Montjoye and Schweitzer, 2019).

In Section 7, we examine the consequences of heterogeneity of users. We allow some users to prefer the entrant platform, while others actively dislike it. There can exist a staggered migration equilibrium where, initially, only those users who find the entrant platform most attractive migrate, while others wait until enough users have joined the entrant. If user preferences are sufficiently polarised, there exists an equilibrium where the different types of users settle on different platforms. In this case, the equilibrium can be inefficient since users do not internalize the network externalities they generate. Conclusions and paths for future research are presented in section 8.

# 2 Literature Review

We know of no econometric evidence of the size of incumbency advantage or of its determinant. On the other hand, there has been a vigorous discussion, often based on case studies, on the importance of lock-in. Arthur (1989) presents an early analysis of lock-in due to network effects while Levin (2013), among others, argue that it is unlikely. This article contributes to this debate by formalising an endogenous micro-foundation for incumbency advantage and studying how migration processes influence the level of incumbency advantage.

The article the most closely related to our work is Farrell and Saloner (1985). They consider a finite number of users who choose sequentially between two platforms, and show that users always coordinate on the superior platform. The last consumer who is given the choice to join the (Pareto) superior platform does so if the others have joined. The penultimate consumer, predicting that the last one will join, will herself join, and so forth. In contrast, our model allows for multiple opportunities to migrate, which significantly changes the model's predictions.

Farrell and Saloner also analyze a two-player model of incomplete information and show there can be excessive momentum or excessive inertia. Other authors also use imperfect information to explain incumbency advantage, in models where users sequentially must make a once and for all decision of which technology to use. Choi (1997) assumes that the quality of a technology becomes known to all users as soon as a single user adopts it. There will be less experimentation with new technologies than is optimal, because users fear being stranded by themselves once they adopt. Ochs and Park (2010) analyze an environment where a finite number of players differ in their how large a platform must be before they find it worthwhile to join. Each agent knows his own type, but there is aggregate uncertainty about the composition of the pool of players. They show that this uncertainty leads to inefficient adoption decisions.

Unlike the above papers, we assume a continuum of users and measurable strategies so that no single user can affect the decisions of the others. As in Farrell and Saloner (1985), adoption can also be inefficient in our setting, but the source of this inefficiency is not the "bandwagon" effect of early movers on later ones.

In the second part[4] of a follow-up paper, Farrell and Saloner (1986) analyze a model with network effects and two consumers who receive opportunities to switch according to a Poisson process. As in our model, users have multiple opportunities to switch. They find that there can be excessive inertia or excessive momentum relative to the efficient allocation. We allow for much more general migration processes and are able to characterize how the migration process affects the possibility of migration.

Ostrovsky and Schwarz (2005) analyze a model where there is uncertainty regarding the time at which a firm can adopt a new standard, and a free riding effect can induce the non-adoption of a Pareto dominant standard. By contrast, in our paper, there is uncertainty regarding when each agent will receive her next opportunity to migrate but the adoption decision of an individual agent does not affect other agent's decisions.

[4] In the first part, Farrell and Saloner use a model of successive choice by users to study what inefficiencies can stem from the presence of early adopters of an inferior technology.

Some papers explicitly examine the role of platform behavior in consumer adoption dynamics. Early papers include Katz and Shapiro (1992), where firms compete in price with entry of new consumers over time. Sakovics and Steiner (2012) study a model where a monopoly platform chooses the order in which to attract users and how much to subsidize each of them. Cabral (2019) studies a model of competition between platforms that adjust their prices dynamically. We abstract from strategic considerations by firms and focus on user decisions. Moreover, we also study circumstances where the order in which users join the platform is potentially based on the distribution of user heterogeneity, rather than chosen directly by a profit maximizing platform.

Hałaburda, Jullien and Yehezkel (forthcoming) and Biglaiser and Crémer (forthcoming) allow firms to choose prices to attract consumers, but assume that all consumers make migration decisions after each round of price setting by firms, as do Fudenberg and Tirole (2000). These papers analyze the way in which dynamic competition between firms affects incumbency advantage, whereas we focus on the role of the behavior of consumers.

Finally, in a different line of inquiry Gordon, Henry and Murtoz (2018) study the way in which the graph theoretical shape of networks influence the spread of an innovation in a model with local externalities.

# 3 Model and equilibrium

## 3.1 One user choosing when to migrate

We will provide a micro-foundation in 3.2, but for now consider the problem of a single user of an incumbent platform $I$ who must decide if and when to migrate to an entrant platform $E$ — for simplicity we assume that migration is irreversible.[5]

At time $t \geq 0$ the utility of the user is $\widetilde{u}_I(t)$ on the incumbent platform and $\widetilde{u}_E(t)$ on the entrant platform. Because other users are migrating or due to other factors such as changes in the design of the platforms or in prices, we assume that the difference of utility $\widetilde{u}_E(t) - \widetilde{u}_I(t)$, is increasing

[5]The migration equilibrium we focus on below still exists if migration is reversible, but reversibility would introduce significant complexity and, we believe, a number of additional equilibria (where, for instance, individuals might follow each other back and forth across the platforms).

in $t$. From the perspective of the individual, the evolution of utilities over time is exogenous and unaffected by her actions. If there is migration, one would expect $\widetilde{u}_E(0) - \widetilde{u}_I(0) < 0$ and $\lim_{t \to +\infty} \widetilde{u}_E(t) - \widetilde{u}_I(t) > 0$, but these conditions are not necessary for the results of this section.

In any interval of time $[t, t + dt]$, a consumer in the incumbent platform receives an opportunity to migrate with probability $\widetilde{\mu}(t) \times dt$. The function $\widetilde{\mu}$ is called the *migration process* and plays a crucial role in the sequel. A key innovation of this article is to describe the effects of changes in $\widetilde{\mu}$. We assume that $\widetilde{\mu}(t) > 0$ for all $t$.

Our preferred interpretation is that the migration process $\widetilde{\mu}$ stems from a psychological (rather than a physical) process where, for instance, consumers think about or are reminded of the existence of the entrant platform at random times. This could be due, for example, to advertising by the entrant platform, word of mouth from other users who have already migrated, or simply because users remember at random times to re-optimize their choice of platform. One could think of consumers as being myopic in the sense of only thinking about migrating when they are given an opportunity (in all other dimensions, individuals are fully rational).

A strategy for the consumer is a measurable function $\phi(t) : \Re^+ \to [0, 1]$ which is interpreted as the probability that the agent accepts a migration opportunity that arises at time $t$. The consumer migrates during a "small" interval $[t, t+dt]$ if and only if she has the opportunity to do so, which happens with probability $\widetilde{\mu}(t) \times dt$, and if she accepts this opportunity, which she does with probability $\phi(t)$. Therefore, the probability of migration during $[t, t+dt]$ is $\phi(t) \times \widetilde{\mu}(t) \times dt$. If the agent is on the incumbent at time $t$, she still is on the incumbent at time $t + dt$ with probability $1 - \phi(t)\widetilde{\mu}(t)dt$. Therefore, the probability $\pi(t)$ that the consumer is on the incumbent at time $t$ satisfies

$$\pi(t + dt) = \pi(t) \times [1 - \widetilde{\mu}(t)\phi(t)dt] \tag{1}$$

which implies[6]

$$\pi(t) = \exp\left[-\int_0^t \widetilde{\mu}(\tau)\phi(\tau)d\tau\right]. \tag{2}$$

[6]From (1) and using $\pi(0) = 1$:

$$\frac{\pi(t + dt) - \pi(t)}{dt} = -\pi(t)\widetilde{\mu}(t)\phi(t) \Rightarrow \pi'(t) = -\pi(t)\mu(t)\phi(t) \Rightarrow \ln(\pi(t)) = -\int_0^t \widetilde{\mu}(\tau)\phi(\tau)d\tau.$$

Letting $r$ be the discount rate, the discounted utility of the user is

$$\int_0^\infty \Big[\widetilde{u}_I(t)\pi(t) + \widetilde{u}_E(t)(1-\pi(t)\Big] e^{-rt}dt$$
$$= \int_0^\infty \Big[\widetilde{u}_I(t) - \widetilde{u}_E(t)\Big]\pi(t)e^{-rt}dt + \int_0^\infty \widetilde{u}_E(t)e^{-rt}dt.$$

Since the second term does not depend on $\phi$, the user's problem is to choose a strategy $\phi$ which maximizes

$$\int_0^\infty [\widetilde{u}_I(t) - \widetilde{u}_E(t)]\pi(t)e^{-rt}dt$$

subject to (2).

The following proposition is a direct consequence of Proposition A.1 which can be found, along with its proof, in appendix A. It states that, once a user has started to accept migration opportunities with positive probability, then she will accept all future opportunities with probability one. We will call these strategies *threshold strategies*.

**Proposition 1.** *If $\widetilde{u}_E(0) - \widetilde{u}_I(0) \geq 0$, the user accepts all migration opportunities: $\phi^*(t) = 1$ for nearly all $t$. If $\lim_{t\to+\infty} \widetilde{u}_E(\infty) - \widetilde{u}_I(\infty) \leq 0$, the user accepts no migration opportunities: $\phi^*(t) = 0$ for nearly all $t$.*

*If $\widetilde{u}_E(0) - \widetilde{u}_I(0) < 0$ and $\lim_{t\to+\infty} \widetilde{u}_E(t) - \widetilde{u}_I(t) > 0$, there exists a unique $\overline{T} < \inf\{t : \widetilde{u}_E(t) - \widetilde{u}_I(t) \geq 0\}$ such that the user does not migrate before $\overline{T}$ and accepts all migration opportunities afterwards:*

$$\phi^*(t) = \begin{cases} 0 & \text{for nearly all } t < \overline{T}, \\ 1 & \text{for nearly all } t > \overline{T}. \end{cases}$$

*Moreover, $\overline{T}$ satisfies*[7]

$$\overline{T} = 0 \quad \text{and} \quad \int_{\overline{T}}^{+\infty} [\widetilde{u}_E(t) - \widetilde{u}_I(t)]\,\pi(t)\,e^{-rt}dt \geq 0, \tag{3a}$$

$$\text{or } \overline{T} > 0 \quad \text{and} \quad \int_{\overline{T}}^{+\infty} [\widetilde{u}_E(t) - \widetilde{u}_I(t)]\,\pi(t)\,e^{-rt}dt = 0, \tag{3b}$$

*where $\pi(t)$ is defined by (2).*

[7] It is tempting to interpret the integrals in (3a) and (3b) as the future discounted utility of the user. For instance, (3b) would state that if $\overline{T} > 0$, then the discounted utility of the user from $\overline{T}$ on is equal to 0. However, this is an artefact of the exponential function. As the proof in Appendix A makes clear, these integrals represent the *marginal* utility.

Once $\widetilde{u}_E(t) - \widetilde{u}_I(t) > 0$, the user will accept all migration opportunities ($\phi^* = 1$). She will start accepting migration opportunities sometime before $\widetilde{u}_E(t) = \widetilde{u}_I(t)$; if she waited until $\widetilde{u}_E(t) - \widetilde{u}_I(t) \geq 0$, then she would find herself on the incumbent platform with probability 1 at a time where the incumbent platform has lower value than the entrant platform. She prefers taking the risk of migrating when the entrant platform still yields slightly less utility than the incumbent platform.

To prove that $\phi^* = 1$ once migration has started, one shows that, if this were not the case, the user would be better off by waiting to start migrating and then accepting all migration opportunities later. She can do this in a way which increases the (expected) time she spends on the incumbent platform while $\widetilde{u}_E(t) - \widetilde{u}_I(t) < 0$ and at the same time keeping constant the probability that she is on the incumbent platform when it becomes positive.

One important corollary of Proposition 1 is that, for any $h(\cdot)$, there is a unique optimal strategy. As a consequence, similar users will all choose the same strategy and we exploit this fact in the equilibrium analysis that follows.

## 3.2 Equilibrium

We now embed the individual optimization problem into an equilibrium model. There is an incumbent platform $I$ and an entrant platform $E$. At time $t$, a mass $h(t)$ of users are members of the incumbent platform, while the entrant platform has $1 - h(t)$ users. At the outset, all users are on the incumbent platform: $h(0) = 1$.

Although some of our results are valid more generally, in the remainder of this paper (except in section 7) we assume that all users have the same utility function: if there is a mass $h$ of users on the incumbent and therefore a mass $1 - h$ on the entrant, the utility of the users of platform $I$ is $u_I(h, t)$ and those of platform $E$ is $u_E(1 - h, t)$. These utility functions are continuously differentiable and strictly increasing in their first arguments, so there are positive network externalities. Furthermore, we assume that

$$u_E(h, t) - u_I(1 - h, t)$$

is weakly increasing in $t$ for any $h$.

As in most of the literature on network externalities, we assume that there is no switching cost.[8] The lifetime discounted utility of a user who migrates at time $t = T$ is[9]

$$\int_0^T u_I(h(t), t)e^{-rt}\, dt + \int_T^{+\infty} u_E(1 - h(t), t)e^{-rt}\, dt.$$

The framework is quite flexible. For instance, the entry of a new platform in a market where none existed could be represented by assuming $u_I(h, t) \equiv 0$ for all $h(t), t$.

Let $h(t)$ be the measure of users on the incumbent platform at time $t$. As in 3.1, in any interval of time $[t, t + dt]$, each consumer on the incumbent platform is given an irreversible opportunity to migrate with probability $\mu(h(t), t) \times dt$. Therefore, a *migration path* $h(t)$ is *feasible* if and only if $h(0) = 1$ and

$$-\mu(h(t), t) \ \times h(t) \leq h'(t) \leq 0 \text{ for all } t.$$

Individual users cannot affect the aggregate migration path and will choose a strategy $\phi(\cdot)$ that maximizes

$$\int_0^\infty \Big[u_I(h(t), t)\pi(t) + u_E(h(t), t)(1 - \pi(t))\Big]e^{-rt}dt, \tag{4}$$

which, by the same reasoning that led to (2), implies

$$\pi(t) = \exp\left[-\int_0^t \mu\big(h(\tau), \tau\big)\phi(\tau)d\tau\right]. \tag{5}$$

Since each individual takes $h(t)$ as given, by Proposition 1, all users follow the same strategy $\phi$.

This enables us to write the following definition.

**Definition 1** (Equilibrium Migration Path)**.** *An equilibrium migration path is a path $h$ such that, taking $h(\cdot)$ as given, $\phi$ maximizes (4) subject to (5) and such that*

$$h'(t) = -h(t) \times \mu(h(t), t) \times \phi(t).$$

[8]See Farrell and Klemperer (2007) for a discussion of switching costs and Crémer and Biglaiser (2012) for a discussion of the way switching costs interact with network externalities.

[9]This essentially assumes that strategies are measurable in the sense that no single user can influence the migration of others.

Since consumers are ex-ante identical and follow the same strategy, they all have the same probability of being on the incumbent at any time $t$. Because the total mass of consumers is 1, by (5) we have

$$h(t) = \pi(t) = \exp\left[-\int_0^t \mu\big(h(\tau),\tau\big)\phi(\tau)d\tau\right].$$

We can now define *migration equilibria*:

**Definition 2** (Migration equilibria)**.** *A migration equilibrium is an equilibrium migration path $h(t)$ where a strictly positive mass of consumers migrate: $\lim_{t\to+\infty} h(t) < 1$.*

From Proposition 1 and the definition of migration equilibrium, follows Proposition 2 (whose proof is straightforward and therefore omitted).

**Proposition 2.** *In any migration equilibrium, all consumers use the same threshold strategy. There exists a $t_0$ such that $h(t) = 1$ for $t \le t_0$ and $h'(t) = -\mu\big(h(t),t\big) \times h(t)$ for all $t > t_0$.*

The following corollary is a direct consequence of Proposition 2 and plays an important role in the sequel.

**Corollary 1.** *There exists a migration equilibrium if and only if there exists a $t_0$ such that*[10]

$$\int_{t_0}^{+\infty} h(t)\Big[u_E\big(1-h(t),t\big) - u_I\big(h(t),t\big)\Big]e^{-r(t-t_0)}\,dt \ge 0 \tag{6}$$

*with*

$$h(t) = \begin{cases} 1 & \text{if } t < t_0, \\ 1 - \displaystyle\int_{t_0}^{t} \mu(h(\tau),\tau)h(\tau)\,d\tau & \text{if } t \ge t_0. \end{cases} \tag{7}$$

*If $t_0 > 0$, condition (6) must be satisfied as an equality.*

*If $\mu$, $u_I$ and $u_E$ are independent of $t$, there exists a migration equilibrium if and only if*

$$\int_0^{+\infty} h(t)\Big[u_E\big(1-h(t)\big) - u_I\big(h(t)\big)\Big]e^{-rt}\,dt \ge 0. \tag{8}$$

[10] Notice that (6) corresponds to (3a) and (3b) in Proposition 1 while Condition (8) below corresponds to (3a).

*with*

$$h(t) = 1 - \int_0^t \mu\big(h(\tau)\big) \times h(\tau)\, d\tau. \tag{9}$$

*If inequality (8) holds strictly, there is a unique migration equilibrium which starts at $t = 0$: $h(t) < 1$ for all $t > 0$.*

If there is a migration equilibrium, there must be a date $t = t_0$ after which users accept their first opportunity to migrate. A user who migrates at time $t = t_0$ has a discounted utility equal to

$$\int_{t_0}^{+\infty} u_E(h(t), t) e^{-r(t-t_0)}\, dt \tag{10}$$

where $h(t)$, defined by (7), is the mass of users on the incumbent platform if all users choose to migrate. If she chooses to wait for the next opportunity, given that every customer uses the same strategy, at any time $t \geq t_0$ she will be on the incumbent platform with probability $h(t)$ and on the entrant platform with probability $1 - h(t)$, which yields an expected utility of

$$\int_{t_0}^{+\infty} \Big[h(t) \times u_I(h(t), t) + (1 - h(t)) \times u_E(1 - h(t), t)\Big] e^{-r(t-t_0)}\, dt \geq 0. \tag{11}$$

Condition (6) states that (10) is greater than (11), and therefore that at $t_0$ users prefer migrating than waiting for the next opportunity. If (6) is a strict inequality with $t_0 > 0$, then a user who receives an opportunity to migrate just before $t_0$ would have strict incentives to accept it.

The proof that (8) is necessary and sufficient when the migration process and the utilities are independent of time is straightforward. If (8) is a strict inequality, at date $t = 0$ users prefer to migrate.[11] Then, there is a unique migration equilibrium where users accept all migration opportunities for $t \geq 0$. With time dependence, one would have to impose further conditions on $\mu$ and on the utilities to obtain uniqueness of the migration equilibrium.

If

$$\lim_{t \to +\infty} u_E(0, t) - u_I(1, t) > 0, \tag{12}$$

there exists some $\bar{t}$ such that for all $t \geq \bar{t}$ users would rather be alone on the entrant platform than with all the other users on the incumbent platform.

[11] This occurs when (8) is a strict inequality, *not* when $u_E(0) > u_I(1)$.

Therefore, migration is the unique equilibrium. In all other cases, no migration ($h(t) = 1$ for all t) is *an* equilibrium, although the focus of our inquiry is on the existence of migration equilibria.

In the case of time independence, (12) becomes $u_E(0) > u_I(1)$: migration at time $t = 0$ is a *dominant strategy* in the sense that users would choose to migrate whatever the migration path $h(\cdot)$. Similarly, when $u_E(1) < u_I(0)$, it is a dominant strategy not to migrate.[12]

Assuming that $\mu, u_E, u_I$ are independent of time, migration increases welfare if and only if

$$\int_0^{+\infty} h(t)u_I(h(t))e^{-rt}\,dt + (1-h(t)u_E(1-h(t))e^{-rt}\,dt \geq \int_0^{+\infty} u_I(1)e^{-rt}\,dt$$
$$\iff \int_0^{+\infty} u_E(1-h(t))e^{-rt}\,dt - \frac{u_I(1)}{1-r}$$
$$\geq \int_0^{+\infty} h(t)\Big[u_E\big(1-h(t)\big) - u_I\big(h(t)\big)\Big]e^{-rt}\,dt. \quad (13)$$

In the limit as $r \to 0$, migration increases welfare if $u_E(1) > u_I(1)$, and decreases welfare when $u_E(1) < u_I(1)$.[13] Comparing with the condition for existence of equilibrium in Corollary 1 one see that, in a migration equilibrium, there can be either excessive inertia or excessive migration (we provide an example in section 5).

It is not surprising that there can be too little migration as we have focused on the free rider problem faced by users. Perhaps less intuitively, there can be too much migration as we select the equilibrium most favorable to migration.

## 3.3 Linear Utilities

We will sometimes (but not always) consider linear utilities of the form

$$u_I(h) = b_I \times h,$$
$$u_E(1-h) = b_E \times (1-h) + k_E.$$

[12]We have defined directly migration equilibria in terms of the function $h(t)$ in order to shortcut the difficulties of defining a game theoretical equilibrium. Our focus on the existence of migration equilibria is a shortcut for the selection of beliefs favorable to the entrant.

[13]Formally, there exists an $\overline{r}$ such that (13) holds with a strict inequality for all $r \leq \overline{r}$.

This linear specification allows platforms to differ in the strength of network effects $(b_E, b_I)$ and/or in their "stand-alone" quality $k_E$ (without loss of generality, the stand-alone value of the incumbent is normalized to zero). Migration is a dominant strategy if $k_E > b_I$ , while not migrating is a dominant strategy if $k_E + b_E < 0$.

With linear utilities, Corollary 1 implies that there exists a migration equilibrium if and only if

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_{t_0}^{+\infty} h^2(t) e^{-rt} dt}{\int_{t_0}^{+\infty} h(t) e^{-rt} dt}, \tag{14}$$

with $h$ defined by (7) or (9).

The left-hand-side of (14) depends only on the preferences of the users. The right-hand-side of (14), which belongs to $(0, 1)$ because $h^2(t) \leq h(t)$, depends only on the migration process and is therefore a measure of the incumbency advantage associated with the migration path $h$.[14]

A migration equilibrium is more likely to exist the larger the quality advantage $k_E$ of the entrant. An increase in $b_E$ also makes migration more likely.[15] A proportional increase in the network effect parameters $b_E$ and $b_I$ always decreases the left hand side of (14) and thus makes a migration equilibrium less likely to exist. Intuitively, an overall increase in the strength of network effects increases the cost of early migration and therefore makes users less eager to start the migration process.

### *Stationarity and zero interest rate*

In the sequel, unless explicitly stated otherwise, we assume that the environment is stationary: the utilities $u_I$ and $u_E$ and the migration process $\mu$ are independent of $t$. Moreover, unless explicitly stated otherwise, $h(t)$ refers to the migration path described by (9) where all users accept the first opportu-

[14] If the left-hand-side of (14) is greater than 1, then there will be migration for any migration process $h$: this occurs when $k_E > b_I$ and migration is a dominant strategy. If the left-hand-side is negative, individuals will not migrate for any $h$: this occurs when $k_E + b_E < 0$ and not migrating is a dominant strategy.

[15] An increase in $b_E$ decreases the left-hand side of (14) if $b_I < k_E$ but, in this case, the left-hand side is greater than one so migration is a dominant strategy.

nity to migrate ($\phi(t) = 1$ for all $t$).[16] This implies

$$h'(t) = -h(t) \times \mu(h(t)) \iff h(t) = \exp\left[-\int_0^t \mu\big(h(\tau)\big) d\tau\right]. \tag{15}$$

There exists a migration equilibrium, with migration starting at time $t = 0$ if and only if (8) holds. If it holds strictly, there exists a unique migration equilibrium.

We will often focus on the case of $r = 0$. This can be interpreted as either migration not taking much time or users being very patient. The relevant integrals need not converge when $r = 0$.[17] Therefore, we will use the following definition.

**Definition 3.** *A property $\mathcal{P}(r)$ holds for $r = 0$ when there exists a $\bar{r} > 0$ such that $\mathcal{P}(r)$ holds whenever $r < \bar{r}$.*

# 4 Analysis

In this section, we use our basic model to explore the determinants of incumbency advantage. We first show that, for low values of $r$, speeding up the migration process does not affect the existence of a migration equilibrium. We then demonstrate that incumbency advantage is increased by the availability of more than one migration opportunity. Finally, we explore the influence of the shape of the migration path $h$ on incumbency advantage.

## 4.1 Speed of migration

Define an *acceleration* of the migration path $h$ as a migration path $\widetilde{h}$ such that $\widetilde{h}(t) = h(\alpha t)$ with $\alpha > 1$. Equivalently,[18] $\widetilde{\mu}(h) = \alpha \times \mu(h)$. As $\alpha$ becomes large, migration becomes faster. One might expect that an acceleration of the migration process reduces incumbency advantage as the first migrants

[16] We will relax these assumptions in Section 6.1 (where the arrival of migration opportunities depends explicitly on time) and in Section 7 (where some consumers delay the acceptance of migration opportunities).

[17] For instance in (14), the denominator and the numerator could become infinite as $r \to 0$.

[18] To see this, note that $\widetilde{h}(t) = h(\alpha t)$ satisfies $\widetilde{h}'(t) = -\widetilde{h}(t) \times \widetilde{\mu}(h(t))$ as $\widetilde{h}'(t) = \alpha h(\alpha t)$ and $\widetilde{h}(t) \times \widetilde{\mu}(h(t)) = h(\alpha t) \times \alpha\mu(h(\alpha t))$.

spend less time with few other users on the entrant platform. However, this intuition is wrong: for small $r$, the acceleration of the migration path does not affect in either way the possibility of migration: acceleration changes the benefits of migrating and the benefits of waiting in the same proportion.

**Proposition 3.** *When $r = 0$ an acceleration of the migration process does not affect the existence of a migration equilibrium: condition (8) holds if and only if it holds for $\widetilde{h}(t) = h(\alpha t)$ whatever $\alpha \geq 1$.*

*Proof.* Assume that (8) holds for $h$ for all $r < \bar{r}$. Let $\alpha > 1$ and $\widetilde{h}(t) = h(\alpha t)$. Then, by the change of variable $u = t/\alpha$, (8) holds with $h$ replaced by $\widetilde{h}$ for all $r < \alpha\bar{r}$. □

An analogous result holds when $r$ is not small. By a similar change of variables, one can easily show that if condition (8) holds for a migration process $h$ and discount rate $r > 0$, it also holds if the process is accelerated to $\widetilde{h}(t) = h(\alpha t)$ and the discount rate set to $\widetilde{r} = \alpha r$.

## 4.2 Multiple migration opportunities

In the introduction and in Section 2, we argued that a crucial difference between our framework and that of Farrell and Saloner (1986) is the possibility of other migration opportunities in the future after a user refused one. We also argued this always increases incumbency advantage. Proposition 4 formalises this intuition.

Suppose, as Farrell and Saloner do, that users have a single opportunity to migrate: if they reject it, they remain on the incumbent platform forever after. The incentives to migrate are lowest at time 0 when a) the discounted utility after accepting migration is $\int_0^{+\infty} u_E(1 - h(t))e^{-rt}dt$ and b) the discounted utility after rejecting it is $\int_0^{+\infty} u_I(h(t))e^{-rt}dt$. Therefore, a migration equilibrium exists if and only if

$$\int_0^{+\infty} [u_E(1 - h(t) - u_I(h(t))]e^{-rt}\, dt \geq 0. \tag{16}$$

This implies the following result.

**Proposition 4.** *Whenever there exists a migration equilibrium with multiple migration opportunities, there exist one with a single migration opportunity: condition (16) holds whenever (8) does.*

*Proof.* Assume that (8) holds and $u_E(1-h(0)) < u_I(h(0))$ (otherwise the result is trivial). There exists $\bar{t}$ such that $u_E(1-h(\bar{t})) = u_I(h(\bar{t}))$ with $h(\bar{t}) > 0$. Because the function $h$ is decreasing we have

$$\int_0^{+\infty} h(t)\Big[u_E(1-h(t)) - u_I(h(t))\Big]e^{-rt}\,dt \\ \leq h(\bar{t})\int_0^{+\infty}[u_E(1-h(t) - u_I(h(t))]e^{-rt}\,dt,$$

and therefore (8) implies (16). □

When users have multiple opportunities to migrate, they have incentives to reject early migration opportunities to avoid being on the entrant platform when it has few adopters. A "take it or leave it" offer favors migration due to the fear of being left behind on the incumbent platform.[19]

## 4.3 Coordination increases incumbency

To pursue our inquiry further, it is useful to define the following notation. For any functions $g$, $g_1$ and $g_2$ from $\Re_+$ into $\Re_+$, define expectation, variance and covariance under the exponential density $re^{-rt}$, as follows:

$$\mathbb{E}[g] \stackrel{\text{def}}{=} \int_0^{+\infty} g(t)re^{-rt}\,dt.$$

Similarly,

$$\begin{aligned}\mathbb{V}[g] &\stackrel{\text{def}}{=} \int_0^{+\infty}\Big(g(t) - \mathbb{E}[g]\Big)^2 re^{-rt}\,dt \\ &= \mathbb{E}[g^2] - E[g]^2\end{aligned}$$

and

$$\begin{aligned}\mathbb{C}\text{ov}[g_1, g_2] &\stackrel{\text{def}}{=} \int_0^{+\infty}\Big(g_1(t) - \mathbb{E}[g_1]\Big)\Big(g_2(t) - \mathbb{E}[g_2]\Big)re^{-rt}\,dt \\ &= \mathbb{E}[g_1g_2] - \mathbb{E}[g_1]\mathbb{E}[g_2],\end{aligned}$$

This implies $\mathbb{V}[g] = \mathbb{C}\text{ov}[g, g]$.

[19] The discussion surrounding equation (17) below provides more intuition on this issue.

We can rewrite (14) as

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_0^{+\infty} h^2(t)e^{-rt}dt}{\int_0^{+\infty} h(t)e^{-rt}dt} = \mathbb{E}[h] + \frac{\mathbb{V}[h]}{\mathbb{E}[h]}$$

which provides a simple interpretation for the effect of coordination on incumbency advantage. The term $\mathbb{V}[h]$ captures how coordinated is the migration process $h(t)$. Large values of $\mathbb{V}[h]$ means that $h(t)$ tends to take values close to 1 and 0: large masses of users migrate in a coordinated way. If users foresee an opportunity for a large coordinated migration, they will have a greater incentive to reject early opportunities, as waiting gives them a large probability to migrate alongside a large number of other users in the future, with minimal loss of utility. Therefore, migration processes with episodes of large coordinated migration are associated with higher incumbency advantage.

To make this statement more precise, let $\widetilde{h}(t) = h(t) + \gamma(t)$, where the function $\gamma$ is not uniformly equal to 0 and satisfies the following two properties: a) $E[\gamma] = 0$ and b) there exists a $\bar{t}$ such that $\gamma(t) \geq 0$ if $t \leq \bar{t}$ and $\gamma(t) \leq 0$ if $t \geq \bar{t}$. Obviously, $E[\widetilde{h}] = E[h]$ and[20] $\mathbb{V}[\widetilde{h}] > \mathbb{V}[h]$: the migration path $\widetilde{h}$ is less favorable to migration than the path $h$. Figure 1 illustrates migration processes with similar $\mathbb{E}[h(t)] \approx 1/2$ but different values of $\mathbb{V}[h(t)]$. Because $\mu = -h'/h$, we see that a large $\mathbb{V}[h]$ (*cf.* the red curve) is associated with a small $\mu$ for small and large values of $h$ and a larger $\mu$ for intermediate values of $h$.

To obtain further intuition notice that, when the utility functions are not linear, we can rewrite (8) as

$$\mathbb{E}\Big[u_E(1 - h(t)) - u_I(h(t))\Big] \geq -\frac{\mathbb{C}\text{ov}\big[h(t), u_E(1 - h(t)) - u_I(h(t))\big]}{\mathbb{E}[h(t)]} > 0, \quad (17)$$

[20] Indeed,

$$\begin{aligned}\mathbb{V}[\widetilde{h}] &= \mathbb{E}[h^2] + \mathbb{E}[\gamma^2] + 2\mathbb{E}[\gamma h] - \mathbb{E}[\widetilde{h}]^2 \\ &= \Big(\mathbb{E}[h^2] - \mathbb{E}[\widetilde{h}]^2\Big) + \mathbb{E}[\gamma^2] + 2\Big[\int_0^{\bar{t}} \gamma(t)h(t)re^{-rt}\,dt + \int_{\bar{t}}^{+\infty} \gamma(t)h(t)re^{-rt}\,dt\Big] \\ &\geq \Big(\mathbb{E}[h^2] - \mathbb{E}[h]^2\Big) + \mathbb{E}[\gamma^2] + 2h(\bar{t})\int_0^{+\infty} \gamma(t)re^{-rt}\,dt > \mathbb{V}[h].\end{aligned}$$

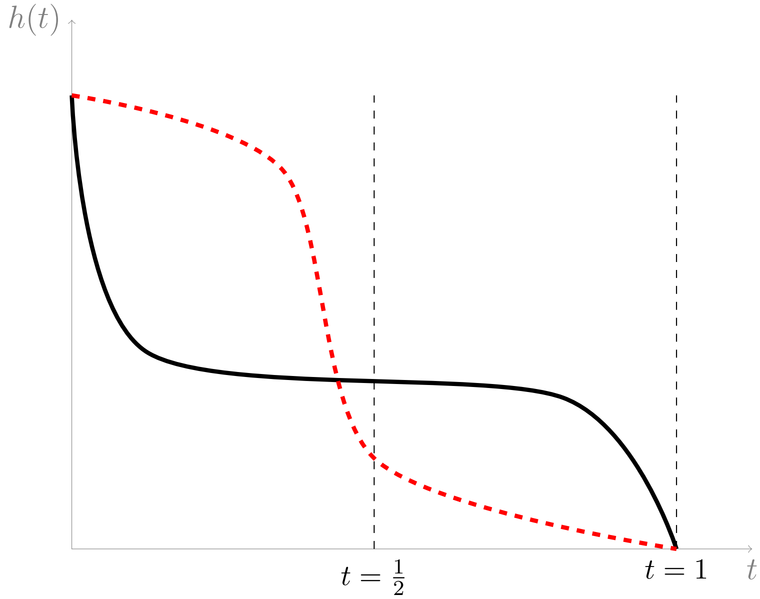


Figure 1: A function with a large $\mathbb{V}[h(t)]$ (in red, dashed) and small $\mathbb{V}[h(t)]$ (in black).

where the second inequality is a consequence of the fact that $h$ is a decreasing function of $t$ while $u_E(1 - h(t)) - u_I(h(t))$ is increasing.

Equation (17) has the same left hand side as (16). Its middle term therefore provides a measure of how strong the incentives to migrate in a model with single opportunities to migrate have to be for a migration equilibrium to exist when individuals actually have multiple opportunities. Because of the presence of the covariance term, improving the utility on the entrant network early in the process while keeping $\mathbb{E}[h(t)]$ constant makes migration more likely.

# 5 Autonomous vs. Word of Mouth Migration

We now specialize the model and assume that migration stems from the mixture of two easily interpretable basic processes. We think of the first as stemming from something like advertising or, more generally, from "one to many" forms of communications: the frequency at which users see advertisements or other form of information, and hence are reminded of the presence of the entrant platform, is constant over time. More formally, during any "small" interval of time of length $dt$ every user on the incumbent platform has a probability $s \times dt$ of being given the opportunity to migrate. We call this the *autonomous* migration process, since $\mu = s$ is independent of both $h(t)$ and $t$.[21]

In the second process, *word of mouth*, users learn about the new platform via pairwise meetings with other users who have already migrated. Formally, in an interval of time of length $dt$, any user meets another user with probability $a \times dt$. Assuming pairs of meetings are equally probable, each user on the incumbent platform has a probability $a \times (1 - h(t)) \times dt$ of meeting a user who belongs to the entrant platform.

In the case of programmers potentially affected by a degradation of the quality of GitHub, users would presumably learn about alternative platforms from online news sources or bulletin boards. The migration process would be closer to autonomous than to word of mouth.

We combine these two processes into the overall migration process[22]

$$\mu(h(t)) = s + a(1 - h(t)) = a(\sigma - h(t)).$$

where the parameter $\sigma = (s + a)/a \in (1, +\infty)$ captures the relative importance of $s$, the autonomous component of the migration process. For $\sigma \to 1$, word of mouth is dominant.[23] For $\sigma \to \infty$, the autonomous component dominates.

[21] This is the process assumed, for instance, by Farrell and Saloner (1986).

[22] This is the same equation used to define the Bass diffusion process (Bass, 1969, first equation on p. 217). However, our interpretation is different. Bass defines two types of users: a) innovators who "decide to adopt an innovation independently of the decisions of other agents in a social system" and b) adopters who "are influenced ... by the pressures of the social system" (Bass, 1969, p. 216). In our model, all the agents are influenced by the actions of the other agents. Furthermore, all users are identical but each can be reminded of the entrant platform in two distinct ways. Most importantly, we provide a more explicit linkage between our 'diffusion' equation and the way in which agents learn about the new opportunities.

[23] We must have $\sigma > 1$ for migration to occur. If $\sigma = 1$, the migration process $\mu = a(1 - h)$ is purely "word of mouth." In this case, the initial condition $h(0) = 1$ implies $h'(t) = 0$ for ll $t$.

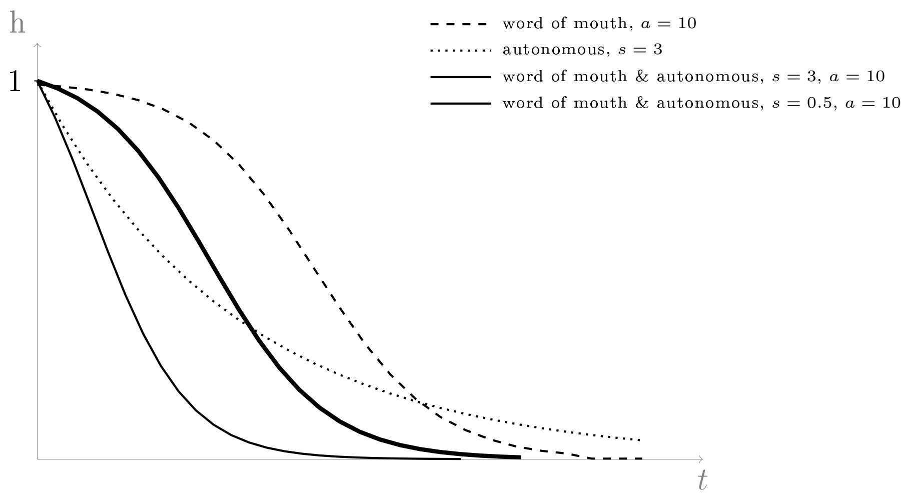


Figure 2: Migration paths as a function of the autonomous parameter $s$ and the word of mouth parameter $a$.

Then, (15) implies[24]

$$h(t) = \frac{\sigma}{1 + (\sigma - 1)e^{\sigma a t}}. \tag{18}$$

Figure 2 illustrates $h(t)$ for different values of $s$ and $a$.

We show in Appendix B.1 that when $r = 0$ the right hand side of (14) becomes

$$\frac{\int_0^{+\infty} h^2(t)dt}{\int_0^{+\infty} h(t)dt} = \sigma - \frac{1}{\ln \sigma - \ln(\sigma - 1)}, \tag{19}$$

which is decreasing in $\sigma$, as illustrated in Figure 3 and proved in Appendix B.2. Therefore, with linear utilities, the set of parameters $(k_E, b_E, b_I)$ for which a migration equilibrium exists, expands as $\sigma$ increases, *i.e.* as the autonomous component of migration becomes relatively more prominent.

[24] (18) implies $h(0) = 1$ and

$$h'(t) = -\frac{\sigma \times (\sigma - 1) \times \sigma a e^{\sigma a t}}{(1 + (\sigma - 1)e^{\sigma a t})^2} = -\frac{(\sigma - 1) \times \sigma a e^{\sigma a t}}{1 + (\sigma - 1)e^{\sigma a t}} h(t) = -a(\sigma - h(t))h(t).$$

As $\sigma \to 1$, the word of mouth component of the the migration process dominates and the right hand side of (19) converges to 1. In this case, a migration equilibrium exists when $k_E \geq b_I$ (migration is a dominant strategy). Since migration is efficient whenever $k_E + b_E > b_I$, there exists regions of the parameter space where migration is socially desirable but no migration equilibrium exists. Intuitively, with $\sigma \approx 1$, migration relies almost entirely on word of mouth which, given the initial condition of no participation in the entrant platform, will leave early migrants enjoying very low network externalities. This suggests that, in this setting, an entrant has incentives to "jump start" the market by engaging in activities which increase $\sigma$ such as advertising.

At the other extreme, as $\sigma \to \infty$ the word of mouth component vanishes and the right hand side of (19) converges to 1/2. This is illustrated in Figure 3. Then, a migration equilibrium exists if and only if $k_E \geq (b_I - b_E)/2$. Migration is socially desirable if $k_E \geq b_I - b_E$, so there can be insufficient or excessive migration. If $b_I - b_E < 0$ (the entrant has stronger network externalities), there is insufficient migration: for $k_E \in [b_I - b_E, (b_I - b_E)/2]$, migration is socially desirable but not an equilibrium. On the other hand, if the incumbent has stronger network externalities ($b_I - b_E > 0$), there can be excessive migration: for $k_E \in [(b_I - b_E)/2, b_I - b_E]$, a migration equilibrium exists even though migration is not socially desirable.

The case of $b_E = b_I = b$ and $\sigma \to \infty$ constitutes an important benchmark which we use below, especially in Section 7. In this case, the strength of network externalities is equal on both platforms, and migration opportunities arise solely through the autonomous process (effectively, the migration process is $\mu = s$). A migration equilibrium exists if and only if migration is socially efficient: consumers, efficiently, migrate to the entrant if and only if $k_E > 0$, for any value of the "autonomous" parameter $s$.

# 6 Other determinants of incumbency advantage

Our basic model can be extended in a number of ways to explore how the environment influences incumbency advantages. First, it is natural to assume that, once users have been made aware of the entrant platform, they consider migration more frequently. We show that this increases incumbency advantage. Second, we demonstrate that incumbency advantage decreases when possibly for strategic reasons, the entrant has limited capacity and cannot ac-

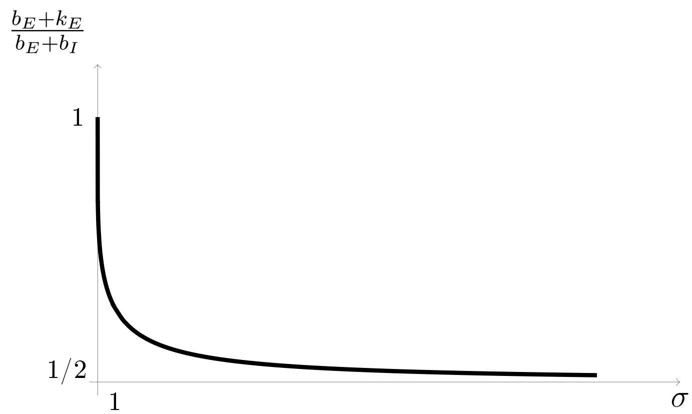


Figure 3: The cutoff $(b_E + k_E)/(b_E + b_I)$ as a function of $\sigma$, as described in (19). Notice that the function converges to 1 as $\sigma \to 1$.

commodate all possible users. Finally, we show that multi-homing decreases, but does not eliminate, incumbency advantage.[25]

## 6.1 Two speeds

It seems plausible that a user of the incumbent platform who has refused to migrate will think more often of the possibility of migrating than a user who has not yet been made aware of the existence of the entrant platform. This increases incumbency advantage, modulo the added assumption described in footnote 27.

We begin by making the extreme assumption that users who have refused their first migration opportunity continuously keep in mind the possibility of moving, and therefore can decide to migrate instantly at any subsequent time.[26] In this setting, a user who is offered the opportunity to move at time 0 would be better off waiting until enough other users have migrated that the (instantaneous) utilities on both platforms are equal. Therefore,

[25] As the environments which we consider in this section and in the next differ from the environment in which we define equilibrium, to be totally rigorous we would have to update the theory of sections 3.1 and 3.2. In the interest of brevity, we will not do so and simply study the circumstances under which migration can start at time 0.

[26] Equivalently, these individual compute the best time to migrate and set an alarm to remind themselves to do so.

there exists a migration equilibrium if and only if migrating is a dominant strategy. We formalize this in the following Proposition.[27]

**Proposition 5.** *If consumers can migrate at any time after their first migration opportunity, a migration equilibrium where migration begins at time 0 exists if and only if migration is a dominant strategy, i.e., if* $u_E(0) > u_I(1)$.

We continue our analysis by assuming that subsequent opportunities arrive faster than the first, but not infinitely fast. For simplicity, the basic migration process is autonomous: $\mu(h) = s$. After refusing a first opportunity users of the incumbent platform receive additional opportunities to migrate according to an accelerated autonomous process $\mu(h) = \alpha s$. We are mostly interested in the case of $\alpha > 1$, but the derivations are valid for any $\alpha$. With linear utilities, a user who migrates at $t = 0$ and expects others to follow obtains a benefit equal to

$$\int_0^\infty [b_E(1 - h(t)) + k_E]e^{-rt}dt.$$

The density function of the time of the next opportunity is $e^{-ast}/as$. Therefore, a user who waits for her next opportunity[28] to migrate will at time $t$ be on the incumbent platform with probability $e^{-\alpha st}$ and on the entrant platform with probability $1 - e^{-\alpha st}$. Her discounted utility is

$$\int_0^{+\infty} \Big[e^{-\alpha st}(b_I h(t)) + (1 - e^{-\alpha st})(b_E(1 - h(t)) + k_E)\Big] e^{-rt}\, dt$$
$$= \int_0^\infty \Big[b_E(1 - h(t)) + k_E\Big] e^{-rt}dt + \int_0^{+\infty} e^{-\alpha st}\Big[-(b_E + k_E) + (b_I + b_E)h(t)\Big] e^{-rt}dt.$$

[27] We assume away "delayed migration" equilibria where the users who receive early migration opportunities coordinate on all moving at some date $t^* > 0$. If $u_E(1) > u_I(1)$ and if all users eventually learn about the existence of the entrant, the users who have learned about the existence of the entrant by some large enough $t^*$ would be better off, collectively and individually, if they migrated simultaneously. We eliminate these types of equilibria by assuming that the entrant platform cannot survive if it has no clients for any interval of time, so that migration must begin at $t = 0$ or not at all.

[28] It is straightforward to prove that it is not optimal to wait for a later opportunity.

As, by (15), $h(t) = e^{-st}$, there exists a migration equilibrium if this last term is positive, *i.e.*, if

$$0 \leq -\frac{b_E + k_E}{\alpha s + r} + \frac{b_I + b_E}{\alpha s + s + r}.$$

**Proposition 6.** *With linear utilities and first opportunities arising according to the autonomous migration process of parameter $s$ and future opportunities according to the autonomous process of parameter $\alpha \times s$ with $\alpha > 1$, a migration equilibrium exists if and only if*

$$\frac{b_I - k_E}{b_E + k_E} \leq \frac{s}{\alpha s + r}. \tag{20}$$

Since the right-hand side of (20) is decreasing with $\alpha$, the incumbent is more likely to keep its market position if the users can migrate more frequently once they become aware of the existence of the entrant.

## 6.2 Capacity constraints

So far we have assumed that the entrant has the capacity to service all users. We now assume that the entrant has maximum capacity of $1 - \kappa < 1$. We show that the fear of being left behind on the incumbent platform increases incentives to migrate. Thus, it could be in an entrant's best interest to reduce its capacity in order to kick start migration.

The capacity constraint stops migration at $t = T$ such that $1 - h(T) = 1 - \kappa$ (we are assuming that $\lim_{t \to +\infty} h(t) = 0$).[29] Then, by the same reasoning that leads to (4), the utility of a user who does not migrate at time 0 is

$$\int_0^T \big[h(t)u_I(h(t)) + (1 - h(t))u_E(1 - h(t))\big]e^{-rt}\,dt \\ + \int_T^\infty [\kappa u_I(\kappa) + (1 - \kappa)u_E(1 - \kappa)]e^{-rt}dt.$$

Comparing this to her utility if she migrated,

$$\int_0^T u_E(1 - h(t))e^{-rt}\,dt + \int_T^\infty u_E(1 - \kappa)e^{-rt}dt,$$

[29]Formally, this implies that the capacity-constrained model is a special case of the model of Section 3, with $\mu(h) = 0$ for $h$ small enough.

the test for the existence of a migration equilibrium is changed from (8) to

$$\int_0^T h(t)(u_E(1-h(t)) - u_I(h(t))e^{-rt}\,dt + \int_T^\infty \kappa(u_E(1-\kappa) - u_I(\kappa))e^{-rt}dt \geq 0. \tag{21}$$

We assume that $u_E(1) - u_I(0) > 0$, and that the derivative of $h \times [u_E(1-h) - u_I(h)]$ for $h = 0$ is strictly positive. Hence for $\kappa$ small enough and therefore $T$ large enough

$$\kappa(u_E(1-\kappa) - u_I(\kappa)) > h(t)[u_E(1-h(t)) - u_I(h(t))]$$

for all $t \geq T$ and (21) is easier to satisfy than (8). This yields the following proposition.

**Proposition 7.** *A small reduction in capacity by the entrant makes more likely the existence of a migration equilibrium: the set of utility functions $(u_I, u_E)$ such that a migration equilibrium exists with a capacity constraint (strictly) contains the set of utility functions such that a migration equilibrium exists without one.*

Therefore, an entrant might be able to initiate the migration process by committing, if it can, to accept a limited number of users.

## 6.3 Multi-homing

It is common for users to participate simultaneously in multiple platforms (multi-homing). We show that this decreases, but does not eliminate, incumbency advantage.

Suppose that once a user receives a migration opportunity, she has three options: a) continue single-homing on the incumbent; b) multi-home on both platforms; or c) single-home on the entrant. A multi-homing user can choose at any time to abandon the incumbent platform and switch to single-homing on the entrant platform.[30]

Let the utility of a consumer single-homing on the incumbency and entrant platforms be, respectively, $u_I(h(t)) = u(h(t))$ and $u_E(1-h(t)) = u(1-h(t)) + k_E$. A multi-homing user is connected to all other consumers, so her net benefits are

$$u_M = u(1) + k_E - c. \tag{22}$$

[30] Consistent with our assumption that migration is irreversible, we assume that a user cannot return to single-homing on the incumbent after multi-homing.

A multi-homing user is connected to a mass 1 of consumers and therefore obtains utility $u(1)$, in addition to the entrant platform's utility advantage $k_E$. On the other hand, multi-homing also imposes a cost $c > 0$, which can reflect either the fact that the consumer must divide her limited time between the two platforms, or that there is some loss of enjoyment by multitasking on both platforms.[31] We assume that $c$ is small enough that it is worthwhile paying it when $h = 1$: $u(1) - c \geq u(0)$.

Consumers prefer multi-homing to single-homing when

$$u(1) + k_E - c \geq u(1 - h(t)) + k_E \iff u(1) - u(1 - h(t)) - c \geq 0. \quad (23)$$

The left-hand-side of (23) is monotonically decreasing in $t$ and, by assumption, positive for $t = 0$. Therefore, there exists a $\bar{t}$ such that the inequality holds for $t \in [0, \bar{t}]$, and it is reversed for $t > \bar{t}$. Multi-homing is preferred early on, while there is still a significant mass of users only reachable through the incumbent platform ($t \in [0, \bar{t}]$). Once a sufficient mass of users is multi-homing, the advantage of being connected to the incumbent platform becomes lower than the cost of multi-homing. At that point ($t = \bar{t}$), users choose to single-home on the entrant.

Multi-homing increases the utility of a user who migrates at time 0 by

$$\int_0^{\bar{t}} (u(1) - u(1 - h(t)) - c)e^{-rt} dt. \quad (24)$$

The additional benefit of delaying migration at date $t = 0$ with the possibility of migrating at a future date and multi-homing if the date is less than $\bar{t}$ is

$$\int_0^{\bar{t}} (1 - h(t))[u(1) - u(1 - h(t)) - c]e^{-rt} dt. \quad (25)$$

Since for $t \in [0, \bar{t}]$, we have $u(1) - u(1 - h(t)) - c \geq 0$ and $1 - h(t) < 1$, at time 0, a user gains more from migration when she can multi-home.

**Proposition 8.** *If multi-homing is possible, a migration equilibrium is more likely (i.e., exists for lower values of $k_E$) than if multi-homing is impossible.*

[31] An alternative assumption would be that multi-homing brings only part of the stand alone benefits of belonging to the entrant platform, so that (22) would become $u_M = b + \alpha k_E - c$, with $\alpha \in (0, 1)$. This would lead to similar results as having the multi-homing cost be $\hat{c} = c + (1 - \alpha)k_E$.)

In a report written for the European Commission, Crémer, de Montjoye and Schweitzer (2019) argue that dominant firms should be asked to justify the use of policies that deter multi-homing. This proposal was made on the basis of an intuition similar to that of this section: multi-homing decreases incumbency advantage, and a dominant firm should be allowed to discourage it only when this has clear pro-competitive consequences (as it sometimes does, for reasons not analyzed in this paper).

# 7 Heterogeneous users

Up to this point, all users share the same preferences. We now allow for user heterogeneity and study its effect on incumbency advantage. Our main takeaways are: 1) equilibria can have delayed or no migration by a sub-set of users and 2) users can inefficiently segregate themselves across different platforms.

We assume that the utility of all the users is $bh$ on the incumbent platform. Utility on the entrant platform is

$$\begin{cases} b(1-h)+k_E & \text{for a mass } p_H \text{ of } \textit{eager} \text{ users,} \\ b(1-h)+k_L & \text{for a mass } p_L = 1-p_H \text{ of } \textit{reluctant} \text{ users.} \end{cases}$$

We call $\overline{k} = p_H k_H + p_L k_L$ the average value of the quality difference of the entrant platform. Migration opportunities arise solely based on the autonomous process, so $\mu(h) = s > 0$ for all $h$. There is no discounting: $r = 0$.

## 7.1 Migration equilibria with heterogeneous users

We focus on the "maximal-migration equilibria", that is, those equilibria in which the greatest number of users migrate and do so as early as possible. In these equilibria, eager users (if they migrate) accept migration opportunities for all $t \geq 0$, and reluctant users (if they migrate) accept all migration opportunities for all $t \geq T_L$ for some some $T_L \geq 0$. The equilibrium migration path is

$$h(t) = \begin{cases} p_H e^{-st} + (1-p_H) & t \in [0, T_L], \\ p_H e^{-st} + (1-p_H)e^{-s(t-T_L)} & t \geq T_L. \end{cases} \tag{26}$$

We obtain the following proposition, which is illustrated by Figure 4.

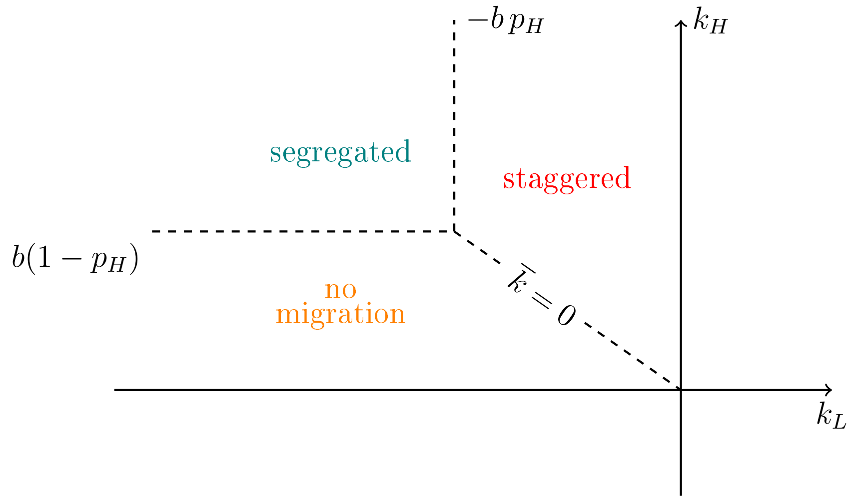


Figure 4: Types of equilibria with heterogeneous users.

**Proposition 9.** *The maximal-migration equilibria satisfy:*

- *If and only if $\overline{k} > 0$ and $bp_H > -k_L$, the maximal migration equilibrium is a "staggered" equilibrium where eager users accept all migration opportunities and reluctant users accept all migration opportunities for $t \geq T_L$, where $T_L$ is defined by*

$$p_H(1 - e^{-sT_L}) = -k_L/b. \tag{27}$$

- *If and only if $k_H > (1 - p_H)b$ and $bp_H \leq -k_L$, the maximal migration equilibrium is a "segregated" equilibrium where eager users accept all migration opportunities and reluctant users never migrate.*

- *In all other cases, there exists no migration in any equilibrium.*

For reasons similar to those discussed after Corollary 2, in any maximal-migration equilibrium, eager types accept all migration opportunities. In a staggered migration equilibrium, reluctant types will start accepting migration opportunities at $t = T_L$, which is the instant at which they derive the same utility by migrating immediately or by waiting for the next opportunity.

*Proof of Proposition 9.* As $k_H > 0$, if it is expected that all users will migrate, by the same reasoning as in the case of homogeneous users in the autonomous

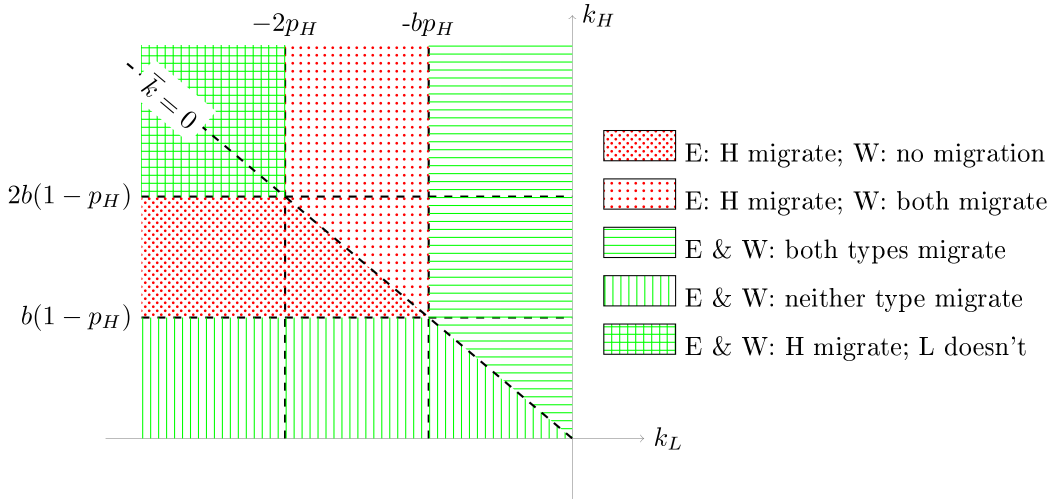


Figure 5: **Equilibria and welfare with 2 types of users.** The legend should be read as follows. E and W indicate respectively the Equilibrium and Social welfare maximising configurations. For instance, the first line shows that, for the relevant configuration of parameters, only the eager consumers migrate whereas it would be socially optimal not to have any migration at all. The bottom line indicates that, in equilibrium, eager consumers migrate while reluctant consumers do not, and this is socially optimal.

case, eager users find it optimal to migrate starting at time 0. The value of $T_L$ is computed in the Appendix's Lemma C.1, while the identification of the staggered and segregated equilibria are conducted in Lemma C.2. As one would expect from the analysis of the autonomous migration process with homogeneous users, in a segregated equilibrium, eager users migrate if and only if it is efficient for them to do so knowing that reluctant users will not migrate, *i.e.*, if the quality benefit of the entrant platform is greater than the loss of the externality benefits stemming from the absence of the reluctant users. □

## 7.2 Welfare with Heterogenous Users

We now discuss the relationship between equilibrium and efficiency in the model with preference heterogeneity. Since $r = 0$, the welfare lost during the migration process itself is ignored. Instead, we focus on long run welfare once "almost" all individuals have made their migration decisions, that is

$$\begin{cases} b & \text{without migration,} \\ b + p_H k_H + (1 - p_H)\, k_L & \text{if all users migrate,} \\ b\,(1 - p_H)^2 + p_H\,(b p_H + k_H) & \text{if only eager users migrate.} \end{cases}$$

The social desirability of migration can therefore be described as follows.

**Proposition 10.** *No migration is optimal if* $\overline{k} < 0$ *and* $k_H < 2b(1 - p_H)$*.*

- *All users migrating is optimal if* $\overline{k} > 0$ *and* $k_L > -2bp_H$*.*
- *Only eager users migrating is optimal if* $k_H > 2b(1 - p_H)$ *and* $k_L < -2bp_H$*.*

Figure 5 illustrates Proposition 10 and contrasts socially optimal behaviour with the equilibrium behaviour described in Proposition 9. The three green shaded areas in the figure illustrate regions of the parameter space when a migration equilibrium exists and it is the welfare maximising outcome.

First, if $k_H/|k_L|$ is large, there exists a migration equilibrium where both types migrate. This is socially desirable since the mild aversion of reluctant users is not enough to justify the loss in network externalities that would result from segregation.[32] Second, if $k_L$ is very negative and $k_H$ not too large, so that $\overline{k} < 0$, a migration equilibrium does not exist. In this case, migration is also socially undesirable because preferences are, overall, in favor of the incumbent and the mild preferences of eager types for the entrant are not enough to justify segregation. Third, if preferences are sufficiently polarised (both $|k_L|$ and $k_H$ large), there exists a migration equilibrium where only eager users migrate. This is socially optimal because each type has an extreme preference for a different platform.

In the red regions of Figure 5, the equilibrium outcome is not socially desirable. The inefficiency is always due to excessive segregation: types $k_H$ migrate and types $k_L$ do not, but it would be optimal for all users to be in the same platform since this maximises network externalities. If $|k_L|$ is much greater than $k_H$, the socially optimal outcome is for all types to remain on the incumbent platform. If $k_H$ is much larger than $|k_L|$, it is optimal for all users to migrate. The excessive migration of the eager users arises because they do not take into account the negative externalities they impose on the reluctant users.

[32] Migration is staggered, but since we are considering the limit as $r \to 0$, this delay does not affect long run welfare.

# 8 Conclusions and paths for future research

There are many extensions of our model which could be worth exploring, for instance imperfect information about the quality of the entrant platform. Also, we have used a simplified description of the timing of the migration decisions. If the agents belonged to a more structured network, the decisions of their "neighbors" would prompt each user to decide whether or not to migrate. Kempe, Kleinberg and Tardos (2015) have studied the diffusion of an innovation in a network, where the agents are represented as nodes in a graph. However, they assume exogenous rules for adoption. For instance, in their "linear threshold model", an agent adopts the innovation if a sufficient number of his neighbors do.[33] It would be interesting to study such a model in the context of migration between platforms, with a more solid game theoretical basis. However, Kempe et al. show that the problem is computationally difficult even without this complication. Thinking of the proper representation of the bounded rationality of agents for such decisions would be of great interest.

We have assumed that consumers act strategically, but that platforms do not. This was done to focus on how the consequences of the migration process on migration incentives. There are many ways to model competition among platforms, and we believe that our framework can be used as a building block for this analysis. For instance, in reality entrants choose the quality of their platform. This would be natural in many settings with network externalities for firms to compete in quality and not prices, such as social media platforms where platform revenues are generated through advertising. If this does not affect a consumers' migration opportunities, then clearly the entrant would choose the minimum quality level that induces early consumers to migrate. Platforms can also affect the migration opportunities of consumers. For example, they can choose the rate at which consumers see ads and hence affect the migration process — there could be interesting links to the marketing literature.

Another direction is to allow platforms to choose prices. This brings up various interesting economic issues. Whether platforms can use history dependent prices — in the sense of charging different prices for new and "old"

[33] This is a very simplified description of that model. Actually each agent exerts a (exogenous) weight on the decision by his neighbors to imitate this adoption of the innovation. An agent adopts the innovation if the sum of these weights for his neighbors who have accepted the innovation is large enough.

consumers — has been discussed in the literature (see, for instance, Cabral, 2019). The amount of information available to online platforms raises other possibilities. For instance, platforms have information about the social graph of users and prices could be made dependent on the number of their contacts who have migrated. Closer in spirit to our model, platforms often know the information available to users, and in particular they can know whether users have seen ads for the new platform or have read reviews of its features. Charging higher prices to consumers who have turned down previous opportunities to migrate might help mitigate the free riding phenomena which we have discussed in this paper (although we suspect that the same data would also provide information about the willingness to pay of the consumer, which would also be relevant for pricing).

Finally, in reality platforms are not as perfectly substitutable as they are in our model, and multi-homing is often used to take advantage of different functionalities. For instance, the same people might communicate through e-mail or through WhatsApp depending on the nature of the communication. To study this issue, one would need to think, instead of migration of users, about migration of communications.

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# Appendix

## A Proof of Proposition 1

In order to prove Proposition 1 we show the following, more general, result. Let $v : \Re^+ \to \Re$ be a continuous differentiable decreasing function with $v(0) > 0$ and $\lim_{t\to+\infty} v(t)$ finite. Let $g : \Re^+ \to (0, 1]$ be continuous and strictly decreasing with $g(0) = 1$. Let $\mu : \Re^+ \to \Re^{++}$ be a function from into $\Re^{++}$ with $\mu(t) > 0$ for all $t$.

(Note, the fact that $g$ is always strictly positive ensures that migration will last forever — the same results would hold with migration ending in finite time.)

Let $\phi^*$ be a solution of the problem

$$\max_{\phi:\Re^+\to[0,1]} \int_0^{+\infty} v(t)\pi(t;\phi)e^{-rt}\,dt,$$
$$\text{subject to } \pi(t;\phi) = g\left(\int_0^t \mu(\tau)\phi(\tau)\,d\tau\right).$$

We show the following two Propositions.

**Proposition A.1.** *There exists* $\overline{T} \in [0, +\infty) \bigcup \{+\infty\}$, *with* $\overline{T} \leq \inf\{t : v(t) \leq 0\}$ *such that* $\phi^*(t)$ *is equal to 0 on* $[0, \overline{T})$ *and to 1 on* $(\overline{T}, +\infty)$.

**Proposition A.2.** *If the function g is twice differentiable and concave, then a necessary and sufficient condition for* $\overline{T}$ *to be optimal is*

$$\overline{T} \times \int_{\overline{T}}^{+\infty} v(t)g'\left(\int_0^t \mu(\tau)\phi(\tau)\,d\tau\right)e^{-rt}\,dt = 0. \quad \text{(A.1)}$$

Proposition A.1 is a direct consequence of Lemmas A.1 to A.4. The proof of Proposition A.2 is presented after these lemmas.

**Lemma A.1.** *If* $v(t) > 0$ *for all t, then* $\phi^*(t) = 0$ *for nearly all t.*

*Proof.* For all $\phi$ strictly greater than 0 on a measurable interval, $\pi(t;\phi) < 1$ for all $t$ greater than some $t'$ and therefore $\int_0^{+\infty} v(t)\pi(t;\phi)e^{-rt}\,dt < \int_0^{+\infty} v(t)e^{-rt}\,dt$, which is attainable with $\phi(t) = 0$ for all $t \geq 0$. □

From now on, we assume $\lim_{t\to+\infty} v(t) < 0$.

**Lemma A.2.** *If $v(T) < 0$, then $\phi^*(t) = 1$ for nearly all $t \geq T$.*

*Proof.* Because $v$ is decreasing, $v(t) < 0$ for all $t \geq T$.

Assume that we did not have $\phi^*(t) = 1$ for nearly all $t \geq T$. For any $t$ in some interval $[t_1, t_2]$ with $T \leq t_1 < t_2$ we would have $\phi^*(t) < 1$. Let $\widetilde{\phi}(t) = \phi^*(t)$ for $t \leq T$ and equal to 1 for $t > T$. Then, $\pi(t; \widetilde{\phi}) = \pi(t; \phi^*)$ for $t \leq T$, $\pi(t; \widetilde{\phi}) \leq \pi(t; \phi^*)$ for $t \geq T$ and $\pi(t; \widetilde{\phi}) < \pi(t; \phi^*)$ for $t > t_1$. This would imply

$$\int_0^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt = \underbrace{\int_0^{t_1} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt}_{=\int_0^{t_1} v(t)\pi(t;\phi^*)e^{-rt}\,dt} + \underbrace{\int_{t_1}^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt}_{>\int_{t_1}^{+\infty} v(t)\pi(t;\phi^*)e^{-rt}\,dt}$$
$$> \int_0^{+\infty} v(t)\pi(t;\phi^*)e^{-rt}\,dt,$$

which establishes the contradiction. □

Because $v$ is decreasing and continuous, it is equal to zero on an interval $[\underline{T}^0, \overline{T}^0]$, with, of course, maybe, $\underline{T}^0 = \overline{T}^0$.

**Lemma A.3.** *For nearly all $t > \underline{T}^0$, $\phi^*(t) = 1$.*

*Proof.* If $\underline{T}^0 = \overline{T}^0$, the lemma is a direct consequence of lemma A.2. Assume therefore that we have $\underline{T}^0 < \overline{T}^0$.

Let $\widetilde{\phi}(t) = \phi^*(t)$ for $t \leq \underline{T}^0$ and to 1 for $t > \underline{T}^0$. Clearly, $\pi(t; \widetilde{\pi}) = \pi(t; \pi^*)$ for $t \leq \underline{T}^0$. For $t > \underline{T}^0$, we have

$$\begin{aligned}\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau &= \int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau \\ &= \int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau \\ &\geq \int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\phi^*(\tau)\,d\tau,\end{aligned}$$

which implies, because $g$ is decreasing, $\pi(t; \widetilde{\phi}) \leq \pi(t; \pi^*)$ with a strict inequality if $\phi^*(t)$ is not nearly always equal to 1 for $\tau \in (\underline{T}^0, t)$.

Therefore

$$\begin{aligned}\int_0^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt &= \int_0^{\underline{T}^0} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt + \int_{\underline{T}^0}^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt \\ &\geq \int_0^{\underline{T}^0} v(t)\pi(t;\pi^*)e^{-rt}\,dt + \int_{\underline{T}^0}^{+\infty} v(t)\pi(t;\pi^*)e^{-rt}\,dt \\ &= \int_0^{+\infty} v(t)\pi(t;\pi^*)e^{-rt}\,dt\end{aligned}$$

with a strict inequality if $\phi^*(t)$ is not nearly always equal to 1, which proves the result. □

**Lemma A.4.** *There exist a $\overline{T} \in [0,\underline{T}^0]$ such that $\phi^*(t)$ is equal to 0 for nearly all $t \in [0,\overline{T}]$ and to 1 for nearly all $t \in [\overline{T},\underline{T}^0]$.*

*Proof.* For $T \leq \underline{T}^0$ let $h(T) \overset{\text{def}}{=} \int_T^{\underline{T}^0} \mu(\tau)d\tau$. The function $h$ is continuous and decreasing on $[0,\underline{T}^0]$ and satisfies

$$h(0) = \int_0^{\underline{T}^0} \mu(\tau)d\tau \geq \int_0^{\underline{T}^0} \mu(\tau)\,\pi(\tau;\phi^*)d\tau \geq 0 = h(\underline{T}^0).$$

Therefore there exists $\overline{T}$ such that $h(\overline{T}) = \int_0^{\underline{T}^0} \mu(\tau)\pi(\tau;\phi^*)d\tau$.

Let $\widetilde{\phi}$ be defined by

$$\widetilde{\phi}(t) = \begin{cases} 0 & \text{for } t \leq \overline{T}, \\ 1 & \text{for } t \in (\overline{T},\underline{T}^0], \\ \phi^*(t) & \text{for } t \geq \underline{T}^0. \end{cases}$$

This implies

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau \leq \int_0^t \mu(\tau)\phi^*(\tau)\,d\tau \text{ for } t \in [0,\overline{T}],$$

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau = \underbrace{\int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau}_{=\int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau} - \underbrace{\int_t^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau}_{\geq\int_t^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau} \leq \int_0^t \mu(\tau)\phi^*(\tau)\,d\tau$$

$$\text{for } t \in [\overline{T},\underline{T}^0],$$

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau = \int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau = \int_0^t \mu(\tau)\phi^*(\tau)\,d\tau$$

$$\text{for } t \geq \underline{T}^0.$$

Because $g$ is decreasing, this implies

$$\widetilde{\pi}(t) = \pi^*(t) \text{ for } t \geq \underline{T}^0$$

when $v(t)$ is negative, and

$$\widetilde{\pi}(t) \geq \pi^*(t) \text{ for } t \leq \underline{T}^0$$

when $v(t)$ is positive, with a strict inequality if $\phi^*(t) \neq \widetilde{\phi}(t)$ on a subset of $[0, \underline{T}^0]$ of measure greater than 0 and proves the lemma and therefore the proposition. □

*Proof of Proposition A.2.* By Proposition A.1 the optimal $\overline{T}$ is solution of

$$\max_{\overline{T} \geq 0} \int_0^{\overline{T}} v(t) e^{-rt} \, dt + \int_{\overline{T}}^{+\infty} v(t) g \left( \int_{\overline{T}}^t \mu(\tau) \, d\tau \right) e^{-rt} dt.$$

After elimination of two terms which cancel out, the derivative of the maximand of this expression is equal to

$$\int_{\overline{T}}^{+\infty} \left[ v(t) g' \left( \int_{\overline{T}}^t \mu(\tau) \, d\tau \right) \times \left( -\widetilde{\mu}(\overline{T}) \right) \right] e^{-rt} \, dt$$
$$= -\widetilde{\mu}(\overline{T}) \int_{\overline{T}}^{+\infty} \left[ v(t) g' \left( \int_{\overline{T}}^t \mu(\tau) \right] d\tau \right) e^{-rt} \, dt. \quad \text{(A.2)}$$

By assumption $\widetilde{\mu}$ is strictly positive, we have therefore proved that condition (A.1) is a necessary condition. To see that it is a sufficient condition, note that

$$\frac{d}{d\overline{T}} \left[ \int_{\overline{T}}^{+\infty} v(t) \, g' \left( \int_{\overline{T}}^t \mu(\tau) \, d\tau \right) e^{-rt} \, dt \right]$$
$$= -v(\overline{T}) g'(0) e^{-r\overline{T}} - \widetilde{\mu}(\overline{T}) \int_{\overline{T}}^{+\infty} g'' \left( \int_{\overline{T}}^t \mu(\tau) \, d\tau \right) e^{-rt} \, dt.$$

The first term is positive because $v(\overline{T}) > 0$ on the relevant range and $g$ is decreasing. So is the second term when $g$ is concave. Hence, the derivative of the second term of the right hand side of (A.2) is negative, which implies that the derivative is negative everywhere if it is for $\overline{T} = 0$ and cannot be equal to 0 more than once. □

# B Proofs for Section 5

## B.1 Proof of (19)

We first derive an expression for $\int_0^{+\infty} h(t)\,dt$. Because

$$\frac{d}{dt}\left[\sigma t - \frac{\ln[1+(\sigma-1)e^{\sigma a t}]}{a}\right] = \sigma - \frac{(\sigma-1)\sigma a e^{\sigma a t}}{a(1+(\sigma-1)e^{\sigma a t})} = h(t),$$

we have

$$\int_0^{+\infty} h(t)\,dt = \left[\sigma t - \frac{\ln[1+(\sigma-1)e^{\sigma a t}]}{a}\right]_0^{+\infty}. \tag{B.3}$$

Also

$$\begin{aligned}
&\lim_{t\to+\infty}\left[\sigma t - \frac{\ln(1+(\sigma-1)e^{\sigma a t})}{a}\right] \\
&\quad = \lim_{t\to+\infty}\left[\sigma t - \frac{\ln[(\sigma-1)e^{\sigma a t}]}{a} - \ln\left(1+\frac{1}{(\sigma-1)e^{\sigma a t}}\right)\right] \\
&\quad = \lim_{t\to+\infty}\left[\sigma t - \frac{\ln(\sigma-1)}{a} - \sigma t - \frac{1}{(\sigma-1)e^{\sigma a t}}\right] = -\frac{\ln(\sigma-1)}{a}
\end{aligned}$$

and

$$\sigma t - \frac{\ln[1+(\sigma-1)e^{\sigma a t}]}{a}\bigg|_{t=0} = -\frac{\ln\sigma}{a}.$$

Therefore, from (B.3)

$$\int_0^{+\infty} h(t)\,dt = \frac{\ln\sigma - \ln(\sigma-1)}{a}.$$

We now compute $\int_0^{+\infty} h^2(t)\,dt$. Note that $h'(t) = -\mu(h(t)) \times h(t)$ implies $h^2(t) = h'(t)/a + \sigma h(t)$ and therefore

$$\begin{aligned}
\int_0^{+\infty} h^2(t)\,dt &= \frac{[h(t)]_0^{+\infty}}{a} + \sigma \int_0^{+\infty} h(t)\,dt \\
&= \frac{-1}{a} + \sigma\frac{\ln\sigma - \ln(\sigma-1)}{a} = \frac{\sigma(\ln\sigma - \ln(\sigma-1)) - 1}{a}.
\end{aligned}$$

## B.2 The right hand side of (19) is decreasing in $\sigma$

The derivative of the right hand side of (19) with respect to $\sigma$ is

$$1 + \frac{\frac{1}{\sigma} - \frac{1}{\sigma - 1}}{(\ln \sigma - \ln(\sigma - 1))^2} = 1 - \frac{1}{\sigma(\sigma - 1)(\ln \sigma - \ln(\sigma - 1))^2} > 0,$$

where the inequality is a consequence of the fact that, by strict concavity of the function ln, we have

$$\ln \sigma - \ln(\sigma - 1) < \left.\frac{\partial \ln}{\partial \sigma}\right|_{\sigma = \sigma - 1} \times (\sigma - (\sigma - 1)) = \frac{1}{\sigma - 1}.$$

# C Proofs for Section 7

The two lemmas in this appendix assume the hypotheses of Section 7.

**Lemma C.1.** *If eager users begin migrating at time 0 and reluctant users begin migrating at time $t \geq T_L > 0$, $T_L$ satisfies (27).*

*Proof.* Under the hypotheses of the lemma, for $t \geq T_L$, a reluctant user is on the incumbent platform with probability $e^{-s(t-T_L)}$. Migrating at time $T_L$ yields the same utility than waiting for the next opportunity; therefore

$$\begin{aligned}
&\int_{T_L}^{+\infty} [b(1 - h(t)) + k_L] e^{-rt}\, dt \\
&\quad = \int_{T_L}^{+\infty} \Big[ e^{-s(t-T)} bh(t) + (1 - e^{-s(t-T)})[b(1 - h(t)) + k_L] \Big] e^{-rt}\, dt \\
&\qquad = \int_{T_L}^{+\infty} [b(1 - h(t)) + k_L] e^{-rt}\, dt \\
&\qquad\qquad + \int_{T_L}^{+\infty} e^{-s(t-T)} [2bh(t) - b - k_L] e^{-rt}\, dt.
\end{aligned}$$

This implies $\int_{T_L}^{\infty} e^{-s(t-T)} [2bh(t) - b - k_L] e^{-rt}\, dt = 0$ and therefore, by (26) and taking the limit as $r \to 0$,

$$\frac{k_L + b}{s} = \frac{b}{s} \left[ (1 - p_H) + e^{-sT} p_H \right],$$

which implies (27). □

**Lemma C.2.** *Eager users migrate at time $t = 0$ if*

$$\begin{cases} k_H \geq -(1-p_H)k_L/p_H & \text{when reluctant users begin migrating at } T_L < +\infty, \\ k_H \geq b\,(1-p_H) & \text{otherwise.} \end{cases}$$

*Proof.* An eager user migrates at time 0 rather than wait for the next opportunity if

$$\int_0^{+\infty} [b(1-h(t)) + k_H]e^{-rt}\,dt \geq \int_0^{+\infty} \Big[e^{-st}bh(t) + (1-e^{-st})[b(1-h(t)) + k_H]\Big]e^{-rt}\,dt$$

$$\iff \int_0^{+\infty} [b + k_H]e^{-(r+s)t}\,dt \geq \int_0^{+\infty} 2be^{-(r+s)t}h(t)\,dt$$

Using (26), this is equivalent to

$$\frac{k_H + b}{2b(s+r)} \geq \int_0^{T_L} \Big[e^{-(r+2s)t}p_H + e^{-(r+s)t}(1-p_H)\Big]\,dt + \int_{T_L}^{+\infty} \Big[e^{-(r+2s)t}p_H + e^{-(r+s)(t-T_L)}(1-p_H)\Big]\,dt.$$

As $r \to 0$ and using (27), this condition is equivalent to $k_H/b \geq (1 - p_H)(1-e^{-sT_L}) = -(1-p_H)k_L/(bp_H)$. This completes the proof for the case $T_L < +\infty$.

The result for $T_L = +\infty$ follows trivially. It is equivalent to the fact that for purely autonomous migration process and $r \to 0$, migration takes place if and only if it is efficient. □