Towards Contextualizing Community Detection in Dynamic Social Networks

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Abstract. With the growing number of users and the huge amount of information in dynamic social networks, contextualizing community detection has been a challenging task. Thus, modeling these social networks is a key issue for the process of contextualized community detection. In this work, we propose a temporal multiplex information graph-based model to represent dynamic social networks: we consider simultaneously the social network dynamicity, its structure (different social connections) and various members' profiles so as to calculate similarities between "nodes" in each specific context. Finally a comparative study on a real social network shows the efficiency of our approach and illustrates practical uses.

Keywords: Temporal multiplex information graph · Dynamic social networks · Contextualized community detection · Modularity · Inertia · Similarity

1 Introduction

With the widespread use of online social networks in recent years, a huge number of users have become highly dynamic and continually seeking for new collaborators to form communities [1]. Yet, contextualizing community detection has been a challenging task. In fact, using context to find relevant communities and highly-connected-modules is crucial and hard mainly with dynamic networks of context-dependent individuals.

Therefore, as dynamic social network is a complex system [2], modeling it is a key issue for the process of contextualized community detection. In this setting, most widespread community detection approaches consider the social network as a graph and then analyze its structure with graph properties and algorithms built around its structure [3]. Even more, in some very influential works in the literature such as [4], the terms "graph" and "network" are used interchangeably [5]. In fact, graph is a powerful mathematical abstraction for representing

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entities (i.e., actors in social network) and their relationships [6]. However, each proposed graph-based model until recently is used to detect community in social network taking into account just one aspect: either the network structure or the similarity between nodes or network dynamicity. Furthermore, none of these existing models is including social context in its community construction.

Thus, in this work we propose a temporal multiplex information graph-based model for contextualized community detection in dynamic social networks. This model considers simultaneously the social network dynamicity, its structure (different social connections) and various members' profiles so as to calculate similarities between "nodes" in each specific context. In addition, a combined metric is defined in order to find relevant communities in the proposed graph.

The outline of this paper is as follows: in Section two, we present the definition of dynamic social networks and their characteristics. In Section three, we present an overview of different graph-based models used to deal with community detection in social networks. Then, we propose our graph-based model designed to be better suited for representing social networks. Finally, we report on an experiment designed to test how well different graphs allow community detection in particular contexts.

2 Communities and Contexts in Dynamic Social Networks

Social network can be generally defined as a group of individuals who are connected by a set of relationships [7]. For example, we can consider a research laboratory, illustrated by Fig. 1, as a social network. Within this network, there are different types of relationships between researchers. Indeed, the same members can be connected by a co-publication relationship on DBLP[1], a friendship on Facebook[2], or a professional relationship on Linkedin[3]. In addition, each member has a specific profile describing him in each social network: DBLP profile, Facebook profile and Linkedin profile.

Furthermore, a key characteristic of social networks is their continual change [7]. Real-world social networks such as the research laboratory are not always static. New nodes or new links could appear (new researchers could join the laboratory), and existing nodes or existing links could disappear (former members may leave the laboratory) [8]. Thus, dynamic social network can be defined as a succession of static social networks [9].

Besides, a common feature of dynamic social networks is community structure [10]. However, defining a community is quite a challenging task [3]. The most commonly used definition is that of [11]: "network communities are groups of vertices within a network that have a high density of within-group connections but a lower density of between-group connection".

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[1] http://dblp.uni-trier.de/.
[2] https://fr-fr.facebook.com/.
[3] https://www.linkedin.com/uas/login.

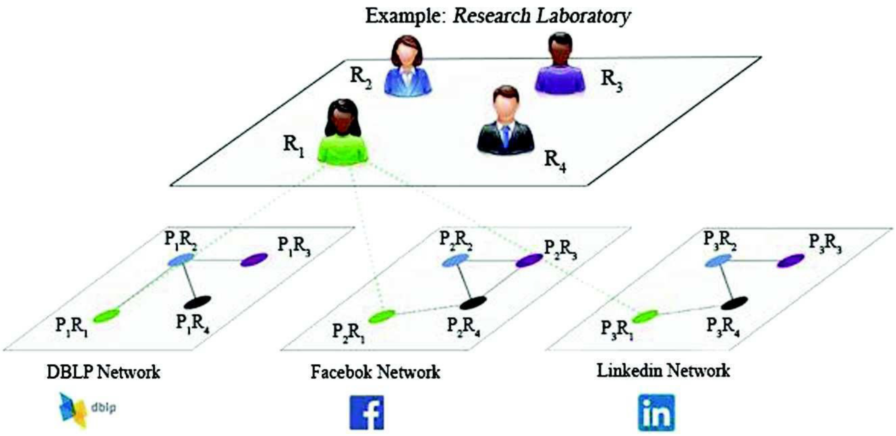

Fig. 1. Research laboratory as a social network

Despite its widespread use [3], this definition considers only the structural aspect of the community. This is why others community definitions based on vertex similarity are proposed. Indeed, [12] defines communities, calling them also clusters or modules, as groups of vertices that probably share common properties and/or play similar roles within the graph.

Inspired from these definitions, in this paper we propose to consider a community as a group of actors strongly connected and more weakly connected to the rest of the network and who have similar contexts. For example, spatial and temporal contexts can be treated to detect communities based on the availability of members.

As the notion of context is very large [13], several definitions of context have been proposed. From the diverse definitions of context, we adopt one of the most widely accepted and more formal [14] as proposed by [15]: "Context is any information that can be used to characterize the situation of an entity. An entity is a person, place, or object that is considered relevant to the interaction between a user and an application, including the user and applications themselves". In the case of community detection, a situation is defined as the image at a given time of the social network. Thus, community detection context is any information referring to this situation which is relevant to the community detection process. As illustrated in Fig. 2, and in order to design our community detection context model, we reuse the generic context model proposed by [14]. Therefore, we consider three context categories: Extrinsic Context, Interface Context and Intrinsic context. These correspond to the following interrelated elements in the triggering of the community detection process: the **who** question: user's profile attributes (Intrinsic Context) such as temporal context (time-bound), spatial context (related to the geographical location), emotional context (related to mood), user's activity, user's personality (collaboration degree), user's technical skills, or user's interests, etc., the **why** question: community detection need (Interface Context) and the **where** question: the environment itself (Extrinsic Context) [14].

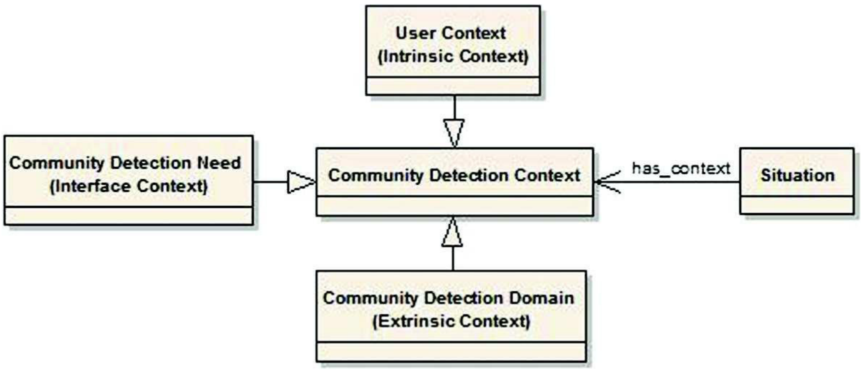

Fig. 2. Community detection context model

For example, a situation can be a member in the research laboratory who is seeking for collaborators to resolve a problem for a scientific research. For this situation, Table 1 shows different elements of community detection context.

Table 1. Community detection context example

Context categories	Information	Context element
Intrinsic context	Member in the research laboratory	User context
Interface context	Resolve a problem	Community detection need
Extrinsic context	Scientific research	Community detection domain

In the next section we will present a brief survey of graph-based models used for community detection in social networks.

3 Graph-Based Models for Community Detection in Social Networks

Community detection is presented by [16] as a partitioning networks technique into communities. In this context, a growing number of community detection methods have recently been published.

Most community detection algorithms lie in *monoplex graph* such as [11] and [17]. A monoplex graph, named also single layer graph [18], is defined as a tuple G = (V, E) where V is the set of vertices or nodes representing individuals and E is the set of edges that connect pairs of nodes. In Fig. 3(a), for example, we have represented a monoplex graph with four nodes and three edges (without specifying the weights). This may correspond to a portion of a research laboratory, where nodes represent researchers and edges represent the coauthoring relationships among these researchers. Weights can be used to represent the number of common publications.

However, with the emergence of Web 2.0 and digital networks, the concept of social networks has to be generalized to account for features describing the actors of the network and their relationships. This led to the definition of new concepts such *information graph* by [19]. Thus, [20] defines an information network as a graph where each node is described by data that can be structured or unstructured. It may include digital data in the form of a set or more commonly of a vector, textual data, or more generally of data of any type. For example, as showed in Fig. 3(b), a research laboratory can be considered as an information network where each researcher is described through a vector enumerating his name, his age and his job.

In some recent works of community detection, another issue arises around the *multiplexes graphs* in order to deal with the multiple aspects of relationships within social network. In this context, [21] defines a multiplex graph, named also multi-slice graph or multi-relational graph or multilayer graph [22], as a graph composed of a set of nodes of the same type, connected by different types of relations. Each layer contains the same set of nodes. But each layer corresponds to a different type of relationship. For example, as illustrated by Fig. 3(c), in the case of bibliographic networks, [23] defines it as a multiplex graph where nodes are authors and each layer corresponds to a different relationship: co-publication, co-citation, co-cited and co-participation in a conference.

All works cited above are interested in the community detection in static networks. However, real-world social networks are not always static. In this context, others graph-based models are proposed in order to consider social network dynamicity. For example, [24] and [9] consider that a *temporal graph* is a succession of static graphs (Fig. 3(d)), each of them representing the state of the complex network at a given time. As result, the temporal graph G on a set of snapshots $S = \{1, 2, \ldots, n\}$ is $G = \{G_1, G_2, \ldots, G_n\}$ with $G_i = (V_i; E_i)$ the snapshot i with nodes V_i and edges E_i.

Figure 3 shows an example of each type of graph.

Each graph-based model is used to detect community in social network taking into account just one aspect: either the network structure or the similarity between nodes or network dynamicity.

Thus, inspiring from these graph-based models and in order to contextualize community detection in dynamic social network, we need a new graph-based model which considers simultaneously the social network dynamicity, its structure (different social connections) and various members' profiles so as to calculate similarities between "nodes" in each specific context. This graph-based model will be described in the next section.

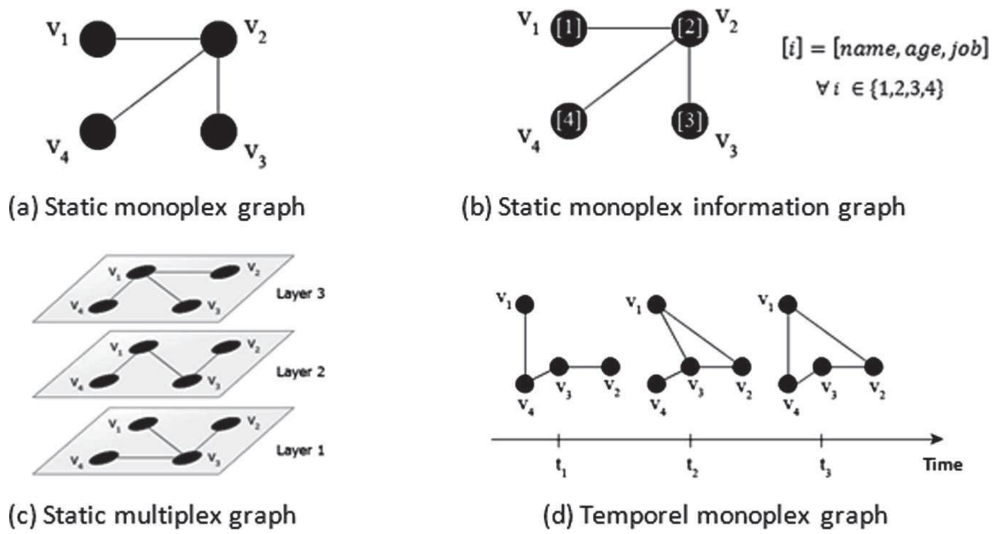

Fig. 3. Examples of graph-based models for community detection

4 Graph-Based Model for Contextualized Community Detection in Dynamic Social Networks

4.1 Temporal Multiplex Information Graph-Based Model Description

In this paper, our aim is to propose a graph-based model to represent dynamic social networks. This model aims to facilitate the contextualization of community detection within dynamic social networks.

Therefore, we propose firstly to reuse a *multiplex graph* in order to represent different types of relationships. Then, as community detection is contextualized, we propose to reuse *information graph*. In fact, due to this type of graph we can represent various members' profiles so as to calculate similarities between "nodes" in each specific context. Finally, to represent dynamicity, we propose simply to add a time parameter. Indeed, we consider a dynamic multiplex social network as a succession of *static multiplex social networks*(where at each time step considered we have a constant number of nodes and edges).

To present the proposed graph-model in a formal way, we use the following notations:

- TMIG : Temporal Multiplex Information Graph
- MIG_i : a static Multiplex Information Graph at time t_i
- $N^{(i)}$: the set of Nodes of MIG_i
- $E_j^{(i)}$: The set of edges of MIG_i which represents edges between nodes of MIG_i linked by the same type of relationship j
- $IG_j^{(i)}$: Monoplex Information Graph which represents the layer j of MIG_i
- $P_j^{(i)}$: the set of Nodes of $IG_j^{(i)}$
- t_i : time

– w : the time window which depends on the initial social network considered. For example in the case of the research laboratory a time window can be equal to a year.

The Temporal Multiplex Information Graph, as illustrated by Fig. 4, is a succession of static multiplex information networks. So, a possible mathematical way to write it is as follows:

$$TMIG = \bigcup_{i} (MIG_i, t_i). \tag{1}$$

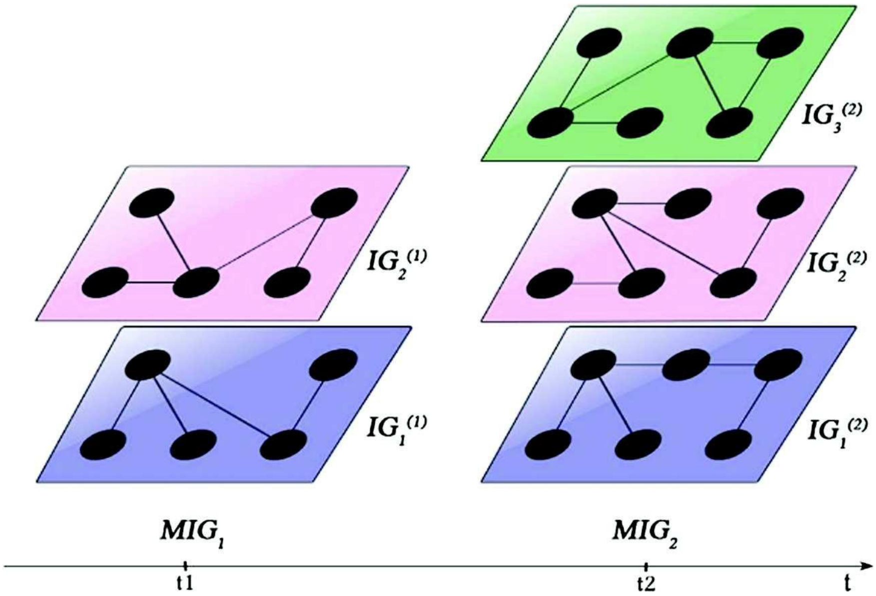

Fig. 4. Example of temporal multiplex information graph-based model

Each static Multiplex Information graph MIG_i at time t_i is characterized by a fixed number Nbr_i of nodes and a fixed number K_i of slices. Indeed, Nbr_i is the number of the social network members at time t_i; and K_i is the number of relationships types between them at time t_i.

$$MIG_i = \left\langle N^{(i)}, E_1^{(i)}, E_2^{(i)}, \ldots, E_{K_i}^{(i)} \right\rangle. \tag{2}$$

$$N^{(i)} = \left\langle N_1^{(i)}, N_2^{(i)}, N_3^{(i)}, \ldots, N_{Nbr_i}^{(i)} \right\rangle. \tag{3}$$

Each node $N_j^{(i)}$ is associated at the most K_i profiles $P_{j,l}^{(i)}$, $l \in \{1 \ldots K_i\}$ (we propose to use ontology to define and to model conceptually each user profile) and it is characterized by K_i weights $p_{j,l}^{(i)}(C)$, $l \in \{1 \ldots Ki\}$. Each weight depends on the community detection context denoted by C, and it represents similarities between profiles. If the node isn't associated at a profile (for example, if a member of the laboratory hasn't a Facebook account) its weight will be zero.

$$N_j^{(i)} = \left\{ (P_{j,l}^{(i)}, p_{j,l}^{(i)}(C)); l \in \{1 \ldots K_i\} \right\}; \forall j \in \{1 \ldots Nbr_i\}. \tag{4}$$

Each set of edges $E_j^{(i)}$ can be written as follows:

$$E_j^{(i)} = \left\{ (P_{x,j}^{(i)}, P_{y,j}^{(i)}, v_{x,y}^{(i,j)}); x, y \in \{1 \dots Nbr_i\} \right\}; \forall j \in \{1 \dots k_i\}. \tag{5}$$

$v_{x,y}^{(i,j)}$ is the weight of the edge between $P_{x,j}^{(i)}$ and $P_{y,j}^{(i)}$. This weight represents the structural similarity between the two nodes. For example, for a co-publishing relationship, structural similarity may be the number of published papers.

Each monoplex information graph $IG_j^{(i)}$, can be written in the form given by (6).

$$IG_j^{(i)} = \left\langle P_j^{(i)}, E_j^{(i)} \right\rangle. \tag{6}$$

With $P_j^{(i)}$ is:

$$P_j^{(i)} = \left\{ P_{l,j}^{(i)}; l \in \{1 \dots Nbr_i\} \right\}; \forall j \in \{1 \dots K_i\}. \tag{7}$$

4.2 Contextualized Community Detection Algorithm

In order to contextualize community detection at a given time t_i, we propose to define a new combined metric Q_{TMIG}. This metric given by (8) is a function of two parameters: the context noted C and the time t_i. Inspired from [25], the proposed metric Q_{TMIG} is based on a weighted combination of two components that must be maximized simultaneously:

$$Q_{TMIG}(C, t_i) = \alpha(C) M_{MIG_i}(C) + (1 - \alpha(C)) I_{MIG_i}(C). \tag{8}$$

The first component concerns the degree of social interactions between individuals based on the assumption that people who frequently socialize (have interactions between them) are more likely to collaborate together. It relies on the structural quality, thus we propose to define a new contextualized modularity noted $M_{MIG_i}(C)$ and given by (9). This contextualized modularity is based on the modularity M of Newman for weighted graphs.

$$M_{MIG_i}(C) = \sum_{j=1}^{K_i} \beta_j(C) M_{IG_{i,j}}. \tag{9}$$

The second component concerns the attribute similarity. Thus, we propose to reuse the notion of inertia $I_{MIG_i}(C)$ [26]. Inertia is a metric that permits to measure the dispersion of a weighted cloud (a set of nodes where each node has a weight). In order to calculate this inertia, we propose to define for each node $N_l^{(i)}$ a global weight $p_l^{(i)}(C)$ as:

$$p_l^{(i)}(C) = \sum_{j=1}^{K_i} \gamma_j(C) p_{l,j}^{(i)}(C). \tag{10}$$

$\alpha(C)$ is a dynamic weighting factor where $0 < \alpha(C) < 1$ that can be changed. This factor is related to the community detection context C. If we want for example to obtain equitability between modularity proportion and inter-classes inertia proportion, we can set $\alpha(C)$ to 0.5.

The coefficients $\beta_j(C)$ and $\gamma_j(C)$ are related to the community detection context C too. And they must be chosen so that:

$$\sum_{j=1}^{K_i} \beta_j(C) = 1. \tag{11}$$

$$\sum_{j=1}^{K_i} \gamma_j(C) = 1. \tag{12}$$

Finally, in order to maximize this contextualized combined quality, which is a NP-hard (non-deterministic polynomialtime hard) optimization, we use a computational optimization technique (i.e. Particle Swarm Optimization) as proposed in the community detection approach of [25].

5 Experimentation: Research Laboratory Case Study

To experiment the temporal multiplex information graph-based model, we choose part of the computer sciences research laboratory RIADI[4] as a dynamic social network that contains 155 members. As the example in Sect. 2, we consider three relationships: co-publication relationship on DBLP(represented by 3582 edges), friendship on Facebook (represented by 1253 edges), and professional relationship on Linkedin (represented by 261 edges). Thus, this social network will be represented by the proposed temporal multiplex information graph-based model. Each node represents a researcher, and it is associated at three profiles: DBLP profile, Facebook profile and Linkedin profile. All these data are collected manually. Then, to calculate the weight of each node, we define two different situations:

Situation 1. The first situation is: a researcher has a problem in the Java development. He is looking for help to resolve his problem. For this situation, we will assign the context C1 : we choose to set $\beta_1(C1)$ to 0.5, $\beta_2(C1)$ to 0(as we will not consider Facebook Graph for professional problem), $\beta_3(C1)$ to 0.5, $\gamma_1(C1)$ to 0.5, $\gamma_2(C1)$ to 0 and $\gamma_3(C1)$ to 0.5. For similarities, we will consider only technical skills and the availability of each researcher (spatial and temporal context).

Situation 2. The second situation is: a researcher is looking for car sharing to go home. For this situation, we will assign the context C2 : we choose to set $\beta_1(C2)$ to 0.25, $\beta_2(C2)$ to 0.5, $\beta_3(C2)$ to 0.25, $\gamma_1(C2)$ to 0, $\gamma_2(C2)$ to 1 and $\gamma_3(C2)$ to 0. For similarities, we will consider the spatial context of the participants (current location) and home address.

[4] www.riadi.rnu.tn.

For the two contexts, we choose to set $\alpha(C)$ to 0.5 so as to obtain equitability between modularity proportion and inter-classes inertia proportion and we apply the contextualized community detection approach using this graph-based model in each specific context.

Then, in order to evaluate the proposed model, we propose to compare its results with the results of other graph-based models for community detection. Thus, we choose to represent the research laboratory as a monoplex graph (nodes represent researchers and edges represent collaborative relationship) in order to apply the well-known Louvain community algorithm proposed in [17].

Then, we represent the considered social network as a monoplex information graph (nodes represent researchers and each one has a weight which represents the similarity between this node profile and the problem holder profile; and edges represent collaborative relationship) in order to apply the combined community detection approach of [25]. Finally, the research laboratory is represented by a multiplex graph in order to apply the Generalized-modularity optimization approach of [27].

The comparison between these approaches, given in Table 2, is based on the detected community size, the number of jointly detected members in order to know if the model will take into account the context, and the modularity and the inter-classes inertia proportion of the detected community given separately in each context.

Table 2. Comparison between qualities of the detected community

Graph-based model type	Community detection algorithm	Detected community modularity M		Detected community Inertia I		Detected community Size S		Number of jointly detected member
		M(C1)	M(C2)	I(C1)	I(C2)	S(C1)	S(C2)	
Monoplex graph	Louvain algorithm [17]	0.15	0.10	0.41	0.20	11	11	11
Monoplex Information graph	Combined community detection algorithm [25]	0.19	0.14	0.86	0.56	33	33	33
Multiplex graph	Generalized modularity optimization [27]	0.22	0.19	0.65	0.48	18	18	18
Temporal multiplex information graph	Contextualized community detection algorithm	0.27	0.23	0.96	0.98	26	8	3

For the selected example and as illustrated by Table 2, using the temporal multiplex information graph-based model gives better results than the three other models. Indeed, the three models detect the same community in both cases (Context 1 and Context 2). However, thanks to the proposed graph-based model, it is possible to contextualize community detection in dynamic social network as we obtain different and better results for each specific context. These results may suggest that the monoplex graph and the monoplex information graph lead to a great loss of information about the heterogeneous nature of links in social network. For the multiplex graph model, it gives important results for modularity but it does not deal with similarities between users. Furthermore, none of these models take into account the community detection context.

At this stage of work, we just detect community at instant t_i. Nonetheless, the temporal aspect in this model will be used to study community resilience using its productivity and its maturity during the evolution of the social network.

6 Conclusion and Further Research

This paper has proposed a temporal multiplex information graph-based model for contextualized community detection in dynamic social networks. To do so, we have considered simultaneously the social network dynamicity, the network structure (different social connections) and various members' profiles so as to calculate similarities between "nodes" in each specific context. Then, to contextualize community detection, we have proposed a new combined metric adapted for multiplex graph which combines the network structure (social connections) and profiles homophily (similarities). Finally, we have tested the proposed graph-based model with other models. The experimentation showed that using temporal multiplex information graph-based model gives better results for contextualized community detection in dynamic social networks.

In future works, we aim to test the performance and the scalability of the proposed graph-based model for community detection in other contexts using real large-scale dynamic social networks or benchmarks.

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