The economic determinants of risk-adjusted social discount rates*

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Abstract

In theory, the measurement of the social value creation of any investment project requires estimating its consumption-based CAPM beta in order to compute its associated risk-adjusted discount rate. In order to assist evaluators to perform this task, we link this social beta to the underlying technical and economic environment of the project, such as the price and income elasticities of the supply and demand for the flow of goods and services generated by the investment. When the consumers' willingness to pay and the variable production cost are Cobb-Douglas in aggregate income and quantity, the beta of the infrastructure has a flat term structure, and is positive for a normal good. But when the infrastructure has a limited capacity, the term structure of the beta is decreasing. Finally, as an illustration, we explain why an investment in a transfrontier trading infrastructure line should have a negative beta for the country that most often uses the line to export its cheaper good (such as electricity).

Keywords: Investment valuation, investment decision, CCAPM, risk-adjusted discount rate.

JEL codes: D61, D92, G11.

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1 Introduction

Although the ex ante evaluation of public policies has become a legal obligation in most countries of the Western world, operational rules to perform them often remain fuzzy when they exist. Moreover, these rules are usually second-best. The choice of the social discount rate, which is a key parameter when most benefits are spread into the future, is particularly controversial among economists. This was particularly apparent after the publication of the Stern Review (Stern (2007)), as discussed for example by Nordhaus (2007), Weitzman (2007) and Dasgupta (2008). The surveys performed by Weitzman (2001) and later on by Drupp et al. (2015) showed little agreement among expert respondents about which rate should be used to discount climate damages. This lack of consensus is also apparent in the attempt by the Interagency Working Group on Social Cost of Carbon (2015) to estimate the Social Cost of Carbon (SCC), since the report estimated it for different discount rates from 2.5% to 5%, far from the U.S. official social discount rate of 7%. Unsurprisingly, this generates estimations of the SCC that are far apart, from $11 to $56 per ton of CO_2. This confirms that "the choice of a discount rate over especially long periods of time raises highly contested and exceedingly difficult questions of science, economics, philosophy, and law" (Greenstone et al. (2013)).

It is also true that the theory has often been misleading. For example, the celebrated Arrow-Lind theorem (Arrow and Lind (1970)) prevailed over decades among public institutions on both sides of the Atlantic to support the use of a single discount rate to evaluate all public investment projects. The idea was that the mutualization capability of the public sector is so large that the risk of individual projects are washed out by diversification, so that it should evaluate them assuming risk-neutrality. But many economists and most evaluation experts overlooked the fact that Arrow and Lind implicitly assumed that the net benefit of the public projects are statistically independent from each other, so that consumption per capita is certain (Baumstark and Gollier (2014)). In reality, most investment projects, public or private, have benefits that are statistically related to aggregate consumption. This implies that diversification does not fully eliminate risk, and evaluators should be concerned with the impact of public actions on the collective risk eventually borne by the risk-averse citizens.

The right reaction to the fallacious interpretation of the Arrow-Lind theorem was provided by the development of the Consumption-based Capital Asset Pricing Model (CCAPM) by Rubinstein (1976), Lucas (1978) and Breeden (1979). In a Gaussian world, it justifies using a discounting system that combines three ingredients: A "risk-free" discount rate for projects whose net benefits are independent from aggregate consumption, a systematic risk premium, and a CCAPM cash-flow beta (later on referred to as the "beta").[1] The beta is specific to the project, and, potentially, to the maturity of the benefit under scrutiny. It is defined as the elasticity of the net social benefit of the project to a change in aggregate consumption. A positive beta means that the project contributes positively to the macroeconomic risk and

[1] If there is no serial correlation in the per-period growth rate of aggregate consumption and if the representative agent has constant relative risk aversion, then the risk-free discount rate and the systematic risk premium have a flat term structure, i.e., they should be independent of the maturity under consideration (Gollier (2002, 2016)). Countries like the United Kingdom and France currently use a non-flat term structure for social discount rates, an approach that can be justified by the persistence of shocks to aggregate consumption (Gollier and Weitzman (2010), Gollier (2012)). In fact, the three variables characterizing the efficient discounting system can have a non-flat term structure.

should be penalized accordingly by a larger risk-adjusted discount rate. On the contrary, a negative beta signals a project that hedges the macroeconomic risk, and its insurance value should be recognized by a reduced risk-adjusted discount rate. This project-specific risk-adjusted discount rate should in fact be equal to the risk-free rate plus the product of the project's beta by the systematic risk premium. This means that the risk premium determines the intensity of the penalization of a project due to its contribution to the macroeconomic risk.

The literature has mostly been focused on the debate about which risk-free rate and systematic risk premium should be used in this discounting system. This has been done in the context of the risk-free rate puzzle (Weil (1989)) and the equity premium puzzle (Mehra and Prescott (1985)) which state that the CCAPM predicts a risk-free rate that is too large and a systematic risk premium that is too large compared to the asset prices that prevailed during the last century of so. Many resolutions of these puzzles have been explored, in particular by allowing extreme events (Barro (2006)) or by disentangling risk aversion from the aversion to consumption fluctuations combined with adding long run risks in the modeling of the stochastic growth (Bansal and Yaron (2004)). But very little has been made to help evaluators in their quest of the project-specific betas.

Contrary to financial assets for which financial betas can easily be estimated by regressing the asset return on the market return, it is often the case that no data is available to estimate the risk profile of the net social benefit of specific public investment projects. Moreover, using the classical arbitrage argument to value them is complex because of the lack of private investments with a similar risk profile. This is partly due to the fact that the public sector should integrate the social and environmental externalities into the net social benefit of each project, something that is not systematically done in the private sector. One possibility is to simulate a model by using the Monte-Carlo method, as done for example by Dietz et al. (2017) to estimate the social beta of green investments aimed at reducing emissions of greenhouse gases. The difficulty is that the result heavily relies on the assumptions of the model and on the calibration of the distribution for the uncertain parameters. All this implies that estimating project-specific social betas remains a challenge.

The complexity of the estimation of social betas and the controversies associated to the social discounting system have induced most governments to use a much simplified system in which a single discount rate is used independent of the risk profile of the investment projects. The second-best solution is then to use a single "risk-adjusted" discount rate which is computed on the basis an average social beta in the economy. The United States have adopted such an approach by selecting a discount rate of 7%, which is assumed to be the average cost of capital in that country.[2] This is a catastrophe, because it implies a vast under-investment in the safest projects, and a vast over-investment in the riskiest ones. Moreover, this single discount rate for the public sector should induce a transfer of risk from the private sector.[3] Other countries such as the United Kingdom and Norway took the even simpler (and probably more inefficient) approach to estimate the social discount rate by using the (extended) Ramsey rule, which is useful to compute the risk-free discount rate. Thus, in the spirit of the fallacious interpretation of the Arrow-Lind theorem, these governments just

[2]See OMB (2003). A smaller discount rate of 3% could also be used for a sensitivity analysis.

[3]Notice however that many corporations also fail to adjust the discount rate for the risk profile of specific investment projects. As shown by Krueger et al. (2015), they tend to use their weighted-average cost of capital as their unique discount rate.

ignore the risk aversion of their citizens. France is the only country that uses a CCAPM discounting system in which evaluators must estimate the social beta of their project.[4]

This paper is a contribution to make the discounting methodology more user-friendly. One clear identified point of resistance is the difficulty to estimate project-specific betas. This is partly due to the absence of a literature in this domain. As far as we know, Breeden (1980) is the only exception. Breeden derives asset-specific CCAPM betas from a system of demand, and shows how to estimate them from the parameters of this demand system. For the sake of pedagogy, we use a partial equilibrium approach by focusing on each specific project and on its relation to aggregate consumption. We also aim at characterizing social betas from the characteristics of the supply and demand for the flow of goods or services that are generated by the public investment. From this work, the evaluators would be left with the easier and more classical task of estimating these parameters, such as the price and income elasticities of supply and demand.

2 The CCAPM discounting system

We consider an investment project which creates a market for a new non-durable product or service. It could be a new drug, or an infrastructure in transportation, telecommunication, health, or education for example. We want to measure the socioeconomic value of the flow of net benefits generated by this new market.[5] Once this investment project has been implemented and at any further date t, it costs $g(x; C_t, \theta_t)$ per capita to produce x units of the product. This cost is a function of the income per capita C_t that prevails at date t, and also of a set of other factors characterized by the vector θ_t, which is assumed to be statistically independent of C_t. We assume that g is increasing and weakly convex in x. The consumer's value (or willingness to pay) per capita associated to consuming x at date t is equal to $f(x; C_t, \nu_t)$. This consumer value is a function of the income per capita and of other factors that are characterized at date t by ν_t, which is assumed to be statistically independent of C_t. We assume that function f is increasing and weakly concave in x. All positive and negative externalities generated in the production and consumption processes are integrated into functions f and g.

We evaluate the investment project at date 0, which is the only date at which the project can be implemented. This means that there is no option value to delay the investment in this context. Seen from date 0, the future evolution of (C_t, ν_t, θ_t) is uncertain and is characterized by an exogenous joint stochastic process which is arbitrary at this stage of the analysis. We assume a flexible production technology, so that the optimal (or equilibrium) production and consumption x_t at any date conditional to (C_t, ν_t, θ_t) maximizes the net social benefit

$$B_t = B(C_t, \nu_t, \theta_t) = \max_x \quad f(x; C_t, \nu_t) - g(x; C_t, \theta_t). \tag{1}$$

[4] Norway was a precursor in this domain, but it reversed to a single discount rate in 2012 (Hagen et al. (2012)). Ironically, France is now contemplating the possibility to also reverse to a single discount rate, using the argument that France cannot be right alone.

[5] The global cost-benefit analysis of the project should also include the initial fixed cost of the project, which is not included in our analysis. This fixed cost, the potential uncertainties associated to it, and the measure of the cost of capital associated to its financing, raise important questions that are orthogonal to the questions that we address in this paper and should be treated independently.

Suppose that the project is scalable, and let s denote the scale of the project. Thus, the net benefit at date t is sB_t. We examine the impact of the implementation of this project on the intertemporal welfare function

$$W(s) = \sum_{t=0}^{+\infty} e^{-\delta t} Eu(C_t + sB_t), \tag{2}$$

where δ is the rate of pure preference for the present, u is the increasing and concave von-Neumann-Morgenstern utility function of the representative agent, and E is the expectation operator conditional to the information available at date 0. At that date, the value $V(s)$ of the flow of the net benefits generated by the project is such that the representative agent is indifferent between implementing the project or receiving $V(s)$ at date 0. This means that $u(C_0 + V(s)) = W(s)$. As is standard in the asset pricing literature and in the cost-benefit methodology, we assume that the project to be evaluated is marginal in the sense that its implementation does not modify the beliefs associated to the dynamics of (C_t, ν_t, θ_t). This is done by assuming that s tends to zero, and by measuring the social value $P = V'(0)$ of the project. This means that

$$P = \sum_{t=0}^{+\infty} e^{-r_t t} EB_t, \tag{3}$$

where the risk-adjusted discount rate of the project for maturity t is defined as follows:

$$r_t = \delta - t^{-1} \log \left(\frac{E\left[B_t u'(C_t)\right]}{u'(C_0) EB_t} \right). \tag{4}$$

We normalize C_0 to unity. In this paper, we use the standard calibration of the consumption-based CAPM for economic growth and risk preferences. It is summarized in the following assumption.

Assumption 1. *The stochastic process governing aggregate consumption C_t is a geometric Brownian motion with trend μ and volatility σ. Relative risk aversion is a constant γ.*

This implies that the distribution of log consumption $c_t = \log(C_t)$ is $N(\mu t, \sigma^2 t)$ and that $u'(C) = C^{-\gamma}$. In this standard CCAPM framework, the pricing of a risk-free benefit and of a claim on aggregate consumption is well-known under this assumption. The interest rate r^f is obtained by applying pricing equation (4) to $B_t = 1$, or more generally to any B_t that is statistically independent of C_t:

$$r^f = \delta - t^{-1} \log\left(E \exp(-\gamma c_t)\right) = \delta + \gamma\mu - 0.5\gamma^2\sigma^2. \tag{5}$$

The last equality is easy to obtain from the fact that when $c \sim N(\mu, \sigma^2)$, then $E\exp(kc)$ is equal to $\exp(k\mu + 0.5k^2\sigma^2)$.[6] When the net benefit of the project is a claim on aggregate consumption, i.e., when $B_t = C_t$, the risk-adjusted discount rate net of the interest rate defines the systematic risk premium π. From equations (4) and (5), we have that

$$\pi = \delta - t^{-1} \log \left(\frac{E \exp((1-\gamma)c_t)}{E \exp(c_t)} \right) - r^f = \gamma\sigma^2. \tag{6}$$

[6] This is a classical formula in finance theory. For a survey of its use in the discounting literature, see Gollier (2012).

Notice that in this standard framework, the interest rate and the systematic risk premium are independent of the maturity, i.e., their term structures are flat.

We are interested in examining the determinant of the risk-adjusted discount rate for other investment projects. Based on the well-established tradition in finance, we define the maturity-specific consumption beta β_t of the investment project in such a way that the risk-adjusted discount rate r_t of the project for maturity t be equal to

$$r_t = r^f + \beta_t \pi. \tag{7}$$

In other words, the project's beta is defined as follows:

$$\beta_t = \frac{r_t - r^f}{\pi}, \tag{8}$$

where r_t, r^f and π are respectively defined by equations (4), (5) and (6). Given the interest rate r^f and the systematic risk premium π, the project's beta fully determines its discount rate. We hereafter characterize the determinants of consumption betas in different economic and technological environments. In the following proposition, we show that the beta of a project at any specific date is linked to the OLS regression of $\log(B_t)$ over $\log(C_t)$.

Proposition 1. *Suppose that there exists a pair* $(a_t, b_t) \in \mathbb{R}_+$ *such that* $\log(B_t) = a_t + b_t \log(C_t) + \varepsilon_t$*, with* $E[\exp(\varepsilon_t)|C_t] = 1$ *for all* C_t*. Then, under Assumption 1, the beta of the investment project for maturity* t *is* $\beta_t = b_t$*.*

Proof: Under the condition of this proposition, B_t equals $C_t^{b_t} \exp(a_t + \varepsilon_t)$. Because a_t is certain and $E[\exp(\varepsilon_t)|C_t]$ is independent of C_t, equation (4) implies that

$$r_t = \delta - t^{-1} \log \left(\frac{E C_t^{b_t - \gamma}}{E C_t^{b_t}} \right) = \delta - t^{-1} \log \left(\frac{E \exp((b_t - \gamma) c_t)}{E \exp(b_t c_t)} \right). \tag{9}$$

Because c_t is normally distributed with mean μt and variance $\sigma^2 t$, this implies that

$$r_t = \delta - t^{-1} \log \left(\frac{\exp\left((b_t - \gamma)\mu t + 0.5(b_t - \gamma)^2 \sigma^2 t\right)}{\exp\left(b_t \mu t + 0.5 b_t^2 \sigma^2 t\right)} \right). \tag{10}$$

This simplifies to

$$r_t = r^f + b_t \pi, \tag{11}$$

where r^f and π are defined respectively by equation (5) and (6). Plugging this into equation (8) yields $\beta_t = b_t$. ■

This means that if the logarithm of the net benefit is the sum of a linear function of log consumption plus a white noise, then the OLS estimator of the slope coefficient is the consumption beta of the project. This is the case if $B(C_t, \nu_t, \theta_t)$ is multiplicatively separable into a function $g(\nu_t, \theta_t)$ and a power function $C_t^{b_t}$ of aggregate consumption. Observe that this condition should be considered for each maturity in an independent manner from other maturities. If it happens that the slope coefficient of the OLS regression is the same at all maturities, then the term structure of the beta is constant. It makes sense in that case to refer to "the" consumption beta of the project. But in general, the relationship between aggregate consumption and the net benefit of the project is not loglinear, in which case the OLS regression yields a biased estimator of the true consumption beta.

In short, in the context described in Proposition 1, the CCAPM beta of a project is the income-elasticity of the net benefit of the project. In the remainder of this paper, we are interested in characterizing β_t to the characteristics of the project that are given by functions f and g. Before doing this, let us observe that a large social beta does not necessarily translate into a smaller present value. Indeed, if the net benefit B_t is proportional to C_t^β, the expected benefit EB_t is proportional to $\exp(\beta\mu + 0.5\beta^2\sigma^2)t$, which is increasing in β as long as β is larger than $-\mu/\sigma^2$. Thus, a larger income-elasticity of the net benefit implies a larger discount rate, but also a larger expected net benefit to be discounted, implying an ambiguous global effect.

3 The benchmark case: Cobb-Douglas

In this section, we consider the following consumer value function:

$$f(x; C, \nu) = \nu C^\rho \frac{x^{1-\alpha}}{1-\alpha}, \tag{12}$$

where ν is positive, α belongs to $[0, 1]$, and ρ belongs to $\mathbb{R}$. This implies that f is indeed positive, increasing and concave in x. It yields the following demand function:

$$x^d(p; C, \nu) = \nu^{\frac{1}{\alpha}} C^{\frac{\rho}{\alpha}} p^{-\frac{1}{\alpha}}. \tag{13}$$

We can thus interpret $\eta_p^d = -1/\alpha \leq -1$ as the price-elasticity of demand, whereas $\eta_c^d = \rho/\alpha \geq \rho$ is its income-elasticity. This implies that the income-elasticity ρ of the consumer willingness to pay is equal to the opposite of the ratio of the income-elasticity and the price-elasticity of demand.

We also assume that the variable cost function is as follows:

$$g(x; C, \theta) = \theta C^{\rho'} \frac{x^{1+\alpha'}}{1+\alpha'}, \tag{14}$$

where θ is positive, α' belongs to $\mathbb{R}_+$ and ρ' belongs to $\mathbb{R}$. It yields the following supply function:

$$x^s(p; C, \theta) = \theta^{-\frac{1}{\alpha'}} C^{-\frac{\rho'}{\alpha'}} p^{\frac{1}{\alpha'}}. \tag{15}$$

We can thus interpret $\eta_p^s = 1/\alpha'$ as the price-elasticity of supply, whereas $\eta_c^s = -\rho'/\alpha'$ is its income-elasticity.

Solving program (1) for the optimal production and consumption of the non-durable product yields

$$B(C_t, \nu_t, \theta_t) = \frac{(\alpha + \alpha')\nu_t^{\frac{1+\alpha'}{\alpha+\alpha'}} \theta_t^{\frac{\alpha-1}{\alpha+\alpha'}}}{(1-\alpha)(1+\alpha')} C_t^\beta, \tag{16}$$

with

$$\beta = \frac{\rho(1+\alpha') + \rho'(\alpha - 1)}{\alpha + \alpha'}. \tag{17}$$

We conclude that Proposition 1 can be applied to this benchmark case when (θ_t, ν_t) are jointly normally distributed. Namely, for this type of projects, the social beta has a flat term structure and this beta is linked to the income and price elasticity of supply and demand, as summarized in our next proposition, where equation (18) is a simple rewriting of equation (17).

Proposition 2. *Suppose that the consumers' willingness to pay and the variable cost of production of the service generated by the infrastructure are Cobb-Douglas. Then, under Assumption 1, the beta of the investment in this infrastructure is maturity-independent and equal to*

$$\beta = \frac{\eta_c^d(1+\eta_p^s) - \eta_c^s(1+\eta_p^d)}{\eta_p^s - \eta_p^d}. \tag{18}$$

In the special case in which costs are insensitive to the growth of aggregate consumption, i.e., when ρ' and η_c^s are zero, we obtain the following result:

$$\beta = \eta_c^d \frac{1+\eta_p^s}{\eta_p^s - \eta_p^d}. \tag{19}$$

Notice that because η_p^d is smaller than -1, the social beta for a normal good is increasing in the price-elasticity of supply. The net benefit of the project is more sensitive to aggregate income when the supply is more reactive to the increased demand generated by economic growth. In particular, we have that

$$\beta = \begin{cases} \rho, & \text{if } \eta_p^s = 0, \\ \eta_c^d \geq \rho, & \text{if } \eta_p^s \to +\infty. \end{cases} \tag{20}$$

When the price-elasticity of supply is zero, i.e., when the investment project generates a constant flow of goods or services at a zero marginal cost, the beta of the project is the income-elasticity of the consumer's willingness to pay. This is intuitive, since the net social benefit in this case coincides with the consumers' valuation of the good produced. Alternatively, when the price-elasticity of supply goes to infinity, i.e., when the marginal cost is constant, then the beta of the project is just equal to the income-elasticity of demand. Observe that in both cases, the beta of the investment project has the same sign as the income-elasticity of the demand for the good or service that it generates. This suggests that the social beta of public investment projects generating inferior (normal) goods be negative (positive). This reinforces the idea that there should be a valuation bonus for projects that generates goods that are most valuable for consumers in the worst-case scenarios. Some health and military infrastructures satisfy this property.

Because $1+\eta_p^d$ is negative, observe also from equation (18) that the social beta tends to be reduced if the variable production cost is positively correlated with aggregate consumption ($\rho' > 0$ and $\eta_c^s < 0$). This means that, everything else unchanged, one should favor investments that reduce the cost of producing goods particularly in the worst-case macroeconomic scenarios.

To sum up our results in this section, economic theory provides a simple description of the determinants of the social beta when the consumer's willingness to pay and the producers' cost function are representable by Cobb-Douglas functions of quantities and aggregate income. In that case, the social beta can directly derived from the income-elasticities and price-elasticities of the supply and demand for the flow of goods generated by the investment. When these elasticities are constant through time, the social beta of the project has a flat term structure.

4 Limited capacity

In this section, we maintain the assumption of a multiplicative consumer surplus as in (12). But we assume that the investment project is about creating an infrastructure with fixed maximum capacity K and a constant marginal cost below that capacity. This case corresponds to the following total cost function:

$$g(x;C,\theta)=\begin{cases}\theta x, & \text{if } x\leq K;\\ +\infty, & \text{if } x>K.\end{cases} \tag{21}$$

Observe that although the marginal cost below the capacity is constant at each date, its level can evolve stochastically through time. It yields the following net benefit:

$$B(C_t,\nu_t,\theta_t)=\begin{cases}\frac{\alpha}{1-\alpha}\nu_t^{\frac{1}{\alpha}}\theta_t^{\frac{\alpha-1}{\alpha}}C_t^{\frac{\rho}{\alpha}}, & \text{if } x^d(\theta_t;C_t,\nu_t)\leq K;\\ \nu_t C_t^{\rho}\frac{K^{1-\alpha}}{1-\alpha}-\theta_t K, & \text{if } x^d(\theta_t;C_t,\nu_t)>K,\end{cases} \tag{22}$$

where x^d is the demand function defined by equation (13). Notice that contrary to the previous case, the net benefit is not a power function of consumption, so that the CCAPM beta is more complex to estimate. Consider for example the case of a normal good ($\eta_c^d>0$) in a growing economy ($\mu>0$), so that the demand for the product or service generated by the infrastructure is expected to increase through time. Suppose also that the trend of growth of the marginal cost θ_t is small. We assume that the capacity of the infrastructure to be built is large enough to guarantee almost surely that the capacity constraint will not be binding in the short run. In that case, equation (22) implies that the logarithm of the net benefit is a linear function of log income plus a white noise, with a slope coefficient $\rho/\alpha=\eta_c^d$. This means that the beta of the project is equal to that income-elasticity of demand in the short run.

Consider alternatively very long maturities for which the demand is so large that the capacity constraint will almost surely be binding. In that case, equation (22) tells us that the beta of the project tends asymptotically to $\rho\leq\eta_c^d$.[7] For intermediate maturities, the project's beta goes down gradually from η_c^d to ρ. The term structure of the beta of the investment project is decreasing. It is due to the fact that at longer maturities, the supply is less likely to be reactive to changes in the demand, a phenomenon that reduces the income-elasticity of the net benefit as seen in the previous section. The exact formula to estimate the beta at each time horizon combines equations (4), (5) and (6), using equation (22) for B_t.

Proposition 3. *Suppose that the consumers' willingness to pay for the service generated by the infrastructure is Cobb-Douglas, and that the marginal cost of this service below the fixed capacity K of the infrastructure is constant but uncertain. Suppose also that the demand at*

[7] More precisely, the net benefit combines a gross benefit whose beta is ρ and a gross cost whose beta is zero. It is only when $\theta=0$ that the project's beta is ρ as soon as the capacity constraint is binding almost surely.

marginal cost pricing is expected to grow over time. Then, under Assumption 1, the beta of the investment is decreasing, going from η_c^d for maturities where the capacity constraint is almost surely not binding, to $\rho \leq \eta_c^d$ for maturities where the capacity constraint is almost surely binding.

We illustrate these results by the following numerical example. Consider an investment project to build an infrastructure with a capacity $K = 4$. We assume that the consumers' willingness to pay equals $f = 1.5Cx^{2/3}$. This implies a price-elasticity and an income-elasticity of demand equaling respectively -3 and 3. As stated earlier, we assume a geometric brownian motion for C_t, here with $C_0 = 1$, a trend $\mu = 0.02$ and a volatility $\sigma = 0.04$. We also assume an independent geometric brownian motion for θ_t, with $\theta_0 = 1$, a trend $\mu_\theta = 0$ and a volatility $\sigma_\theta = 0.001$. This implies in particular that the current demand for the service of the infrastructure at current marginal cost equals 1.[8] Finally, we assume that the rate of pure preference for the present is $\delta = 0$, and that relative risk aversion is $\gamma = 2$. In Figure 1, we have drawn the term structure of β_t in that case. It is obtained for each increment of 5-year maturity by Monte-Carlo simulation using 200 000 random draws of the pair (C_t, θ_t).

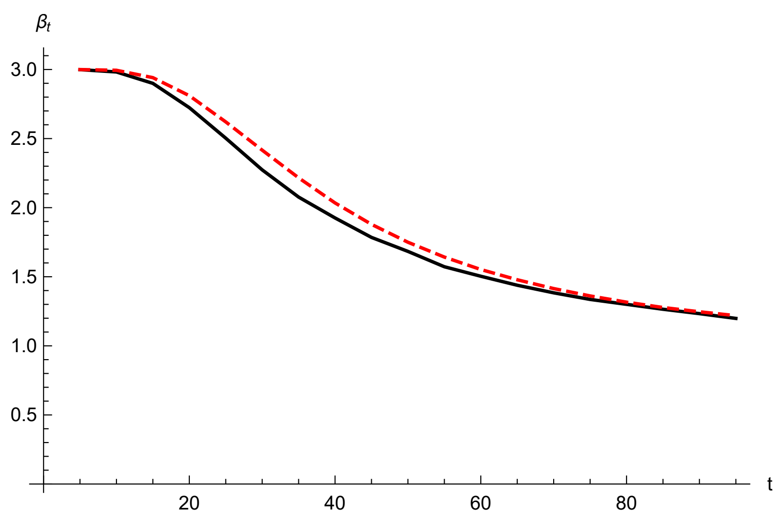

Figure 1: The term structure of the CCAPM cash-flow beta in the case of a capacity investment $K = 4$. We assume that $\alpha = 1/3$, $\rho = 1$, $\delta = 0$, $\gamma = 2$, $\log(\theta_t) \sim N(0, 0.001^2 t)$ and $\log(C_t) \sim N(0.02t, 0.04^2 t)$. The plain curve corresponds to the correct beta when using equation (8), whereas the dashed curve is obtained when approximating the beta as the regressor of $\log(B_t)$ over $\log(C_t)$.

Two remarks should be made at this stage. First, it can be very misleading to estimate "the" discount rate of the project by estimating the statistical log-relationship between the net benefit and aggregate income at short maturities. In our numerical example, benefits are disseminated over distant maturities, for which the socially desirable discount rate is much smaller than the short discount rate. Second, for intermediary maturities, the support of the distribution of the demand $x^d(\theta_t; C_t, \nu_t)$ contains K. As seen from equation (22), the

[8]Notice that, at maturity t, the log excess capacity $\log(K/x^d)$ is normally distributed with a mean of $\log(4) - 0.02t$ and a variance of $0.0144t$. This means in particular that the expected log excess capacity at marginal cost pricing is zero in 69 years.

logarithm of the net benefit B_t is not linear in the logarithm of aggregate income C_t. This is seen in Figure 2 in which we depicted the outcome of 10 000 random draws of the pair (C_{25}, θ_{25}) for a 25-year maturity, using the calibration as in the example of Figure 1. The plain curve corresponds the efficient beta with $B_{25} = kC_{25}^{\beta_{25}}$, with $k = EB_{25}/EC_{25}$, meaning that the uncertain benefit B_{25} represented by this cloud has the same value than a benefit which would have this deterministic relation with C_{25}. Notice that this beta fails to recognize the lower sensitivity of the net benefit to changes in aggregate consumption in the best states of nature. This compensates the underestimation of this sensitivity in intermediary states.

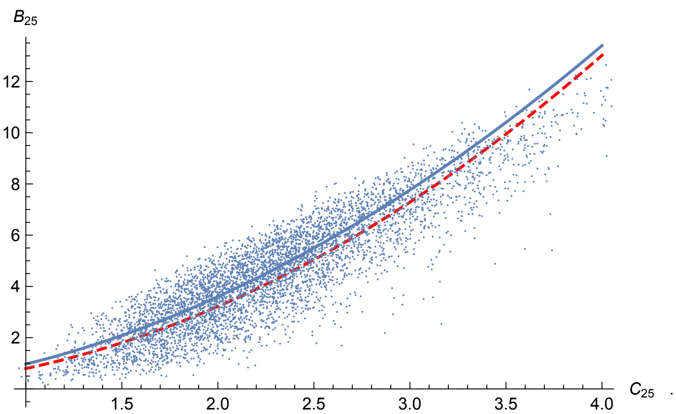

Figure 2: Monte-Carlo simulation of (C_{25}, B_{25}) for a 25-year maturity in the numerical example of capacity investment described in Figure 1. The plain curve describes the true beta, whereas the dashed curve is the best log-OLS fit.

As observed in Proposition 1, the beta of a project is equal to the regressor of $\log(B_t)$ over $\log(C_t)$ when these two random variables are jointly Normal. It is tempting to *define* the beta of a project in this way. But this should not be considered because it does not generate an efficient discounting system when using equation (11). This is illustrated by the two figures of this section. In Figure 2, we have drawn the best fit of the log-OLS for (C_{25}, B_{25}). This estimator is biased upwards. This is confirmed in Figure 1 in which we have depicted the term structure of the cash-flow beta of the capacity investment when using this econometric approach of the beta. The OLS beta tends to slightly overestimate the efficient beta at intermediary maturities. It is only at very short or long maturities that the econometric approach is unbiased. This is due to the fact that the probability of (not) being capacity constrained at those maturities is almost zero, so that the pair $(\log(C_t), \log(B_t))$ is almost joint Normal.

We now consider an investment that marginally increases the capacity K of the infrastructure. Using equation (22), we obtain that the net benefit in this case has the shape of a call option:

$$B(C_t, \nu_t, \theta_t) = \begin{cases} 0, & \text{if } \nu_t C_t^{\rho} K^{-\alpha} \leq \theta_t; \\ \nu_t C_t^{\rho} K^{-\alpha} - \theta_t, & \text{if } \nu_t C_t^{\rho} K^{-\alpha} > \theta_t, \end{cases} \tag{23}$$

Under assumption 1, the log of consumption at time t is a normal distribution with mean μt and variance $\sigma^2 t$. If we additionnally suppose that $\nu_t = \theta_t = 1$, the expected benefit $E[B(C)]$

can be rewritten as

$$K^{-\alpha}e^{\rho^2\sigma^2 t/2+\rho\mu t}\phi(\frac{\alpha\log K-\rho\mu t-\rho^2\sigma^2 t}{\rho\sigma\sqrt{t}})-\phi(\frac{\alpha\log K-\rho\mu t}{\rho\sigma\sqrt{t}})$$

where $\phi(z)=(2\pi)^{-1/2}\int_z^{+\infty}e^{-u^2/2}du$. When $t\to\infty$, we see that $E[B(C_t)]\sim K^{-\alpha}e^{(\rho^2\sigma^2/2+\mu\rho)t}$ and, similarly, $E[B(C_t)C_t^{-\gamma}]\sim K^{-\alpha}e^{((\rho-\gamma)^2\sigma^2/2+\mu(\rho-\gamma))t}$, so that $r_t\sim r_f+\rho\gamma\sigma^2/2$. This shows that the beta of this incremental project tends asymptotically to ρ when the demand for the infrastructure is expected to grow over time. Reversely, when K tends to ∞, since $E[B(C_t)C_t^{-\gamma}]<E[B(C_t)]K^{-\gamma\alpha/\rho}$, the risk-adjusted discount rate for a given maturity t tends to ∞. This reflects the fact that the probability of benefiting from the marginal increase of capacity tends to zero, and becomes very sensitive to economic growth. In other words, the expected benefit tends to zero whereas the corresponding beta tends to infinity.

Proposition 4. *Suppose that the consumers' willingness to pay for the service generated by the infrastructure is Cobb-Douglas with $\nu_t=1$, and that the marginal cost of this service below the fixed capacity K of the infrastructure is equal to 1. Suppose also that the service is a normal good ($\rho\geq 0$). Under Assumption 1, the beta of a marginal increment in the capacity infrastructure has the following properties:*

- *Assuming a positive trend of growth, its term structure tends to ρ asymptotically.*
- *It tends to infinity with K.*

To illustrate this, let's consider an investment in supplementary marginal capacities that should be required to meet the next years growth of demand. For short maturities, the net expected utility of the project is very low - since there is a very low probability that these additional capacity will be used. The additional capacity is very large with respect to the actual need, so that the beta of the project is very high at short horizon and tends, according to the previous proposition, to ρ for large horizon. In the left part of Figure 3 thereafter, we have drawn this term structure with the same assumption as in figure 1, but for a incremental investment from $K=4$ to $K=4.1$. Since the usage of such investment is low when the beta is high, one may wonder whether this term structure has a strong impact on the total expected value of the project. In the right figure thereafter, we compare the net present value of yearly expected benefit when using either the exact term structure of the beta, or its limit value $\beta=\rho=1$. We see that it has indeed a very significant impact, since it decreases the expect value by about 20% when the additional infrastructure induces the highest discounted benefit.

5 Transfrontier trade infrastructure

We consider now an investment that creates an infrastructure to trade goods between two countries. It could be a transport infrastructure such as a railway infrastructure, or a transfrontier electricity connection. This infrastructure is particularly useful to transfer a tradable good from the country where the equilibrium price of this good is low to the other country where it is larger. For the sake of illustration, let us consider a concrete example, where two countries exchange electricity through a high-voltage electricity line. In that case, when a

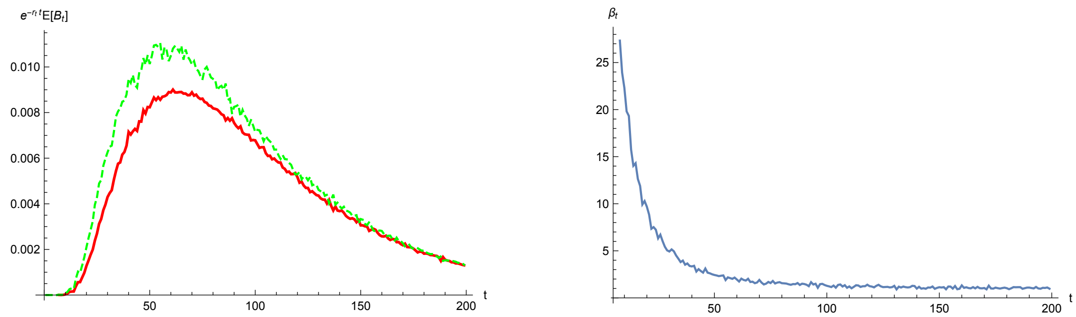

Figure 3: Term structure of the beta for of a capacity investment from $K = 4$ to 4.1 (right), and its impact of net present values of benefit (left). This latter graph compares the expected discounted benefits according to whether the exact term structure for the beta is used, or its limit value $\beta = 1$.

country has an instantaneous surplus of zero-marginal-cost energy, it may use the transfrontier electricity connection to export this energy to the other country. The occurrence of such an event depends upon weather conditions, as well as upon local demand. As a result, an exporting country that slips into recession will consume less electricity and might then be able to exploit more the transfrontier connection to its own benefit.

We use the same notation as before by adding a subscript $i = 1, 2$ to represent the two countries. In autarky, the welfare of country i is given by

$$V_i = \max_{x_i} f_i(x_i, C_i) - g_i(x_i, \theta_i). \tag{24}$$

Let us consider a trade infrastructure of size K whose flow of net benefits is equally shared by the two countries. The social surplus generated by the infrastructure is then defined as follows:

$$\begin{aligned} S(K) \quad = \quad & \max_{x_1,x_2,y_1,y_2} \sum_{i\in\{1,2\}} \left(f_i(x_i, C_i) - g_i(y_i, \theta_i)\right) \quad - \quad \sum_{i\in\{1,2\}} V_i \\ \text{s.t.} \quad & y_1 + y_2 = x_1 + x_2 \\ & |y_1 - x_1| \leq K, \end{aligned}$$

where x_i and y_i denote respectively the local consumption and local production that may be exchanged. We are interested in measuring the social benefit of marginally increasing the capacity of the transfrontier infrastructure. Let's first assume that country 1 has a comparative advantage in terms of production costs or that country 2's demand is higher, so that this infrastructure is used to transfer goods from country 1 to country 2, and $p_2 > p_1$ where p_i is the equilibrium price in country i. We have $x_i = y_i + (-1)^i K$ and, by the envelop theorem, the social surplus of increasing marginally K is equal to $p_2 - p_1$. It's the productivity gain generated by the investment, which marginally transfers the production of the good from the high marginal cost country to the low marginal cost one. For simplicity, we assume here that the sharing of the social surplus between the two countries is such that country i receives share k_i, with $k_1 + k_2 = 1$. The general formula that gives the social benefit of the project accruing to country i is then

$$B_i = k_i \left|p_2 - p_1\right|. \tag{25}$$

Equation (25) states that the benefit of the extension of the infrastructure accruing to country i just corresponds to its share of its global value creation.

We examine the beta of the marginal investment project from the point of view of the exporting country 1. Suppose first that shocks to the two countries are idiosyncratic. Assuming that the good is normal, a positive shock on country 1's growth increases demand in that country, and thus the price p_1, whereas it has no impact on p_2. Therefore, the net benefit for the exporting country 1 is the difference between two flows, one with a zero beta, and the other with a positive beta. This implies a negative beta for this exporting country. The intuition of this result is simple: When the exporting country faces a positive shock to its aggregate income, its equilibrium price goes up, thereby reducing the price gap between the two countries, at least in expectation. This reduces the social value of the infrastructure. Symmetrically, the beta of the infrastructure is positive for the importing country.

Proposition 5. *Consider two countries in which the willingness to pay and the cost function are described by equation (26), with $\rho_i > 0$. Suppose that the two countries face idiosyncratic macroeconomic uncertainties, and that the infrastructure is almost surely used to expert from one specific country to the other. Then, the beta of the social value of a transfrontier trading infrastructure is negative for the exporting country, and positive for the importing one.*

In most cases, the trade infrastructure can be used to transfer goods in both directions. This is the case when the price gap alternates in sign across different states in nature. For instance, in the case of a high-voltage electricity line, depending on weather events and time of the year, a country may export energy some days, and import energy the rest of the time through the cross-border infrastructure. The value at date t of the investment is equal to

$$B_t = \Sigma \varrho_{t,\theta} B_{t,\theta} = \Sigma \varrho_{t,\theta} \frac{\partial S_{t,\theta}}{\partial K},$$

where $S_{t,\theta}$ is the social surplus of country 1 at date t when the weather state θ occurs. We denote $\varrho_{t,\theta}$ for the probability of that state. We are interested to measure the income-elasticity of this benefit B_t. To give the intuition, if the estimation with an OLS-regressor of the logarithm was correct, the beta would be exactly the weighted average of the expected value of the beta associated to each climatic event, that is $\beta = \Sigma \varrho' \beta_{t,\theta}$ where $\beta_{t,\theta} = dLn(B_{t,\theta})/dLn(w)$, for the modified weight $\varrho'_t = \varrho_{t,\theta} B_{t,\theta} / \Sigma \varrho_{i,\theta'} B_{i,\theta'}$. This suggests that Proposition 5 can be generalized when the direction of the flow of trades is uncertain. In that case, it is the expected beta measured as explained above that determines the beta to be used for the evaluation of the project. In practice, this method is biased because the distribution of the log of social benefits is not normal, and because its relation with log consumption is not linear.

In the following numerical example, we show that the same ideas continue to hold, in which the country which is expected to predominantly use the line as an exporter should use a negative beta to evaluate that trading infrastructure project. We assume that the two countries have the same constant price elasticities of demand and supply, and that the marginal cost is independent of income, so that the consumer surplus and the cost function are given by

$$f_i(x_i, C_i) = C_i^{\rho_i} \frac{x_i^{1-\alpha}}{1-\alpha} \qquad g_i(y_i, \theta_i) = \theta_i \frac{y_i^{1+\alpha'}}{1+\alpha'}, \tag{26}$$

We suppose that the traded good is normal ($\rho_i > 0$). Suppose as in the other examples of this paper that $\gamma = 2$ and $\delta = 0$. Suppose also that $\rho_1 = \rho_2 = \alpha' = 1$, $\alpha = 1/3$. We assume further that $(C_{1t}, C_{2t}, \theta_{1t}, \theta_{2t})$ follows independent geometric brownian processes with $\mu_{C_1} = \mu_{C_2} = 0.02$, $\sigma_{C_1} = 0.04$, $\sigma_{C_2} = 0.01$, $\mu_{\theta_1} = \mu_{\theta_2} = 0$ and $\sigma_{\theta_1} = \sigma_{\theta_2} = 0.001$. One key element is that although the initial aggregate consumption levels $C_{10} = C_{20} = 1$ are the same, country 1 has a smaller initial variable cost, as we assume that $\theta_{10} = 1$ and $\theta_{20} = 2$. This implies that, initially, $p_{10} = 1$ and $p_{20} = 2$, so that country 1 is exporting to country 2.

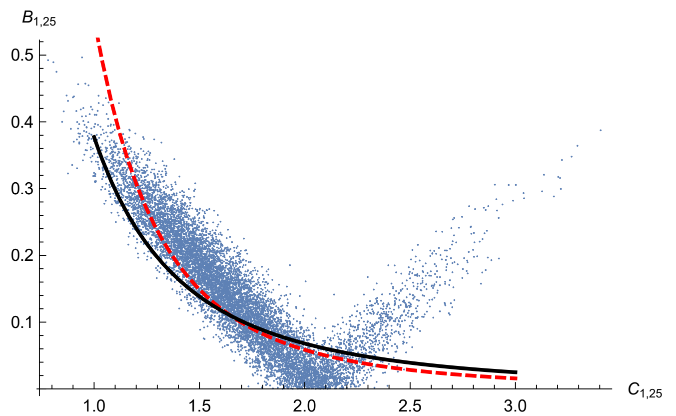

Figure 4: Monte-Carlo simulation of (C_{1t}, B_{1t}) for a $t = 25$ years maturity for the initially exporting country 1. The plain curve describes the true beta ($\beta = -2.47$) of that country for that maturity, whereas the dashed curve is the best log-OLS fit ($\beta^{OLS} = -3.25$). We assume $\mu_{C_1} = \mu_{C_2} = 0.02$, $\sigma_{C_1} = 0.04$, $\sigma_{C_2} = 0.01$, $\mu_{\theta_1} = \mu_{\theta_2} = 0$, $\sigma_{\theta_1} = \sigma_{\theta_2} = 0.001$, $C_{1,0} = C_{2,0} = 1$, $\theta_{1,0} = 1$, $\theta_{2,0} = 2$. and $k_1 = k_2 = 0.5$.

We see in Figure 4 that the statistical relationship between the aggregate consumption in country 1 and the net benefit of the infrastructure for that country is well fit by a negative exponential on the left of this figure, where economic growth in that country is low enough to preserve its exporting advantage. This implies the negative beta for this country. However, it fails to recognize that for larger growth rates of C_1, the demand and the local price in that country will grow faster, transforming the country into an importer. This explains why the relationship between aggregate consumption in country 1 and its net benefit of the infrastructure becomes positive. For longer maturities, the initial comparative advantage of country 1 is more likely to disappear because of the uncertain growth in local demands and costs. This tends to bring the beta of the infrastructure of country 1 closer to zero. This is suggested in Figure 5, which describes the term structure of the cash-flow beta from the point of view of country 1. It is obtained by Monte-Carlo simulations of 200 000 random draws at each maturity.

We applied this methodology to a concrete example, the investment in an additional cross-border electricity connection between France and Spain. This was done based on data provided by the French electricity transmission system operator (RTE). Uncertainty on demand and supply is captured by 14 climatic scenarios, giving for each hour of year 2030 consumers' demand and available renewable energy. This takes also into account the available production capacity of different types of non-renewable energy (gas, hard coal, light oil,

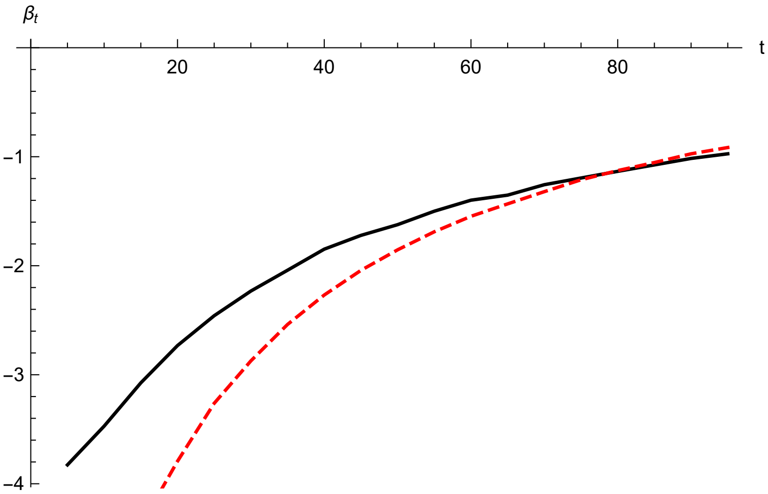

Figure 5: The term structure of the CCAPM cash-flow beta in the case of the small transfrontier trade infrastructure from the point of view of the initially exporting country 1, assuming the same parameters as in Figure 4. The plain curve corresponds to the correct beta when using equation (8), whereas the dashed curve is obtained when approximating the beta as the regressor of $\log(B_{1t})$ over $\log(C_{1t})$.

	France	Spain
Welfare analysis	-1.17	0.42

Table 1: Beta of a marginal investment to increase the size of the cross-border link.

lignite and nuclear) in both countries. The analyzed scenario within the Ten-Year Network Development Plan designed by the community of European Network of Transmission System Operators for Electricity (ENTSO-E), and is a conservative one with about 40% renewable energy[9]. We estimate the beta of the benefit characterized by equation (25) for a project that increases marginally the existing cross-border connection between France and Spain. Results are presented in Table 1. As expected, the beta of the project is negative for the exporting country (France), and positive for the importing one (Spain).

A refinement of this tool would be to take into account the impact of a marginal investment in a cross-border link on the terms of trade. It can be quite significant in the previous example because electricity is an homogeneous good that is sold in bulk at a price equal to its marginal production cost. Consequently, a slight change in the cross-border transport infrastructure can trigger a jump from a price equal to the zero marginal cost of renewable energy to the non-zero marginal cost of fossil energy. It will induce a gain of trade for the exporting country, and an equivalent loss for the importing country. An empirical evaluation using RTE's data shows that the impact can be positive or negative at very specific time horizons, and close to zero elsewhere depending on the expected volume of energy produced in each

[9]This scenario corresponds more precisely to vision 1 of the Ten-Year Network Development Plan, released in 2014 by ENTSO-E.

countries.[10] We do not consider that in our evaluation of electricity betas because it does not affect global welfare, since it is a zero-sum transfer between countries, and also because this effect is very transient, as we just explained, and very sensitive to assumptions with respect to profit sharing.

A second refinement would be to take into account the correlation between macroeconomic cycles, which is significantly positive between France and Spain - cf. for instance Agresti and Mojon (2001) for an evaluation of this correlation. In that case, the social benefit $0.5|p_2 - p_1|$ is the sum of a negative-beta term and a positive-beta term, and betas are closer to zero than in the idiosyncratic case estimated in the previous figure. Reversely, in the presence of asymmetric shocks, a negative shock in the exporting country reduces the local price whereas price would rise on the other side of the border if a positive shock were affecting at the same time the importing country. This boosts the net benefit of the infrastructure even more than when shocks are idiosyncratic.

All this matters a lot as far as an incomplete monetary union is concerned. Indeed, the above example concerns the euro area, which remains quite distant from an optimal monetary zone in the sense of Mundell. We know since Eichengreen (1992) that it is composed of countries experiencing significant asymmetric shocks, which traditional adjustment mechanisms via labor mobility are not sufficient to mitigate.[11] The preceding analysis shows that an investment in a cross-border infrastructure can bring to each country a benefit strongly negatively correlated to its economic activity, and thus play an attenuating role in the event of an asymmetric shock. Other similar investments may have a strong positive beta, and the tools presented here are a valuable way to distinguish and prioritize investments with this factor in mind.

6 Conclusion

Discounting remains a crucial, controversial and complex ingredient of modern investment theory. It traditionally serves two purposes by penalizing cash flow for being both delayed and risky. These two dimensions are taken care of by two independent parameters, the risk-free rate and the systematic risk premia, for which a large literature exists that elaborates on their level and term structure. A full discounting system also requires a methodology for determining the weight beta accrued to the risk component of the project-specific discount rate. In the simplest case in which the net benefit of the investment project under scrutiny and aggregate consumption are jointly log-normal, this "cash-flow" beta is just the income-elasticity of the net benefit. We show in this paper that this situation prevails when the investment project is to build an infrastructure that produces a good or service whose consumers' willingness to pay and variable production costs are Cobb-Douglas in income and quantity. We showed how to derive the cash-flow beta of such an investment from the price and income elasticities of supply and demand functions. We developed the idea that the beta is increasing in the income-elasticity of demand. When the variable cost function is insensitive to aggregate consumption, the beta is positive for a normal good, and negative for

[10] More precisely, the marginal cost curve of electricity can be described by a step function, so that the impact on trade will be zero when the expected volume of electricity is located on the flat part of this curve.

[11] Whether the level of synchronization of business cycle has significantly increased or not since then is still an open debate, cf. De Haan et al. (2008) for a survey of that empirical literature.

an inferior good, as is intuitive. But this is a very special case. If for example, there exists a limited capacity for the infrastructure to produce a normal good, we showed that the term structure of the beta is decreasing.

We assumed in this paper take-it-or-leave-it-forever investment opportunities, thereby leaving aside the problem of real option values. Because the future benefits of an investment in a capacity infrastructure generally depend upon whether this capacity will be expanded or not when the capacity constraint will become binding, the problem of estimating the beta cannot be separated from the problem of estimating real option values. We leave this important question for further research.

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