REJECTIONS OF ORTHOGONALITY IN RATIONAL EXPECTATIONS MODELS
Further Monte Carlo Results for an Extended Set of Regressors *

John W. GALBRAITH

McGill University, Montreal, Que., Canada H3A 2T7

Juan DOLADO

Institute of Economics and Statistics, Oxford 0X1 3UL, UK

Anindya BANERJEE

Jesus College, Oxford 0X1 3DW, UK

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It is well known that many rationality tests do not have the correct sizes if innovations in the explanatory series are correlated with the regressand and the explanatory series are substantially autocorrelated. We argue, by considering somewhat more general data generating processes and models, that the importance of the over-rejections may have been over-emphasized.

1. Introduction

Many rationality tests take the form of regression of a series of expectational errors on a set of random variables belonging to the information set upon which the expectations have been formed. It is well known that if the null of orthogonality between these series holds but if (i) innovations in some explanatory series are correlated with the regressand and (ii) the explanatory series are substantially autocorrelated, the usual t and F tests for coefficient significance do not have their nominal sizes. Hence the null of rationality may tend to be rejected more often than the nominal levels of the tests would suggest. This fact has been illustrated by Mankiw and Shapiro (1986) (referred to henceforth as MS) in a recent paper in this journal. They base their analysis on an extensive Monte Carlo study for the case in which the set of regressors contains only the first lagged level of a variable which belongs to the information set. MS therefore provide a set of critical values for rationality tests which, if generally applicable, would remove the problem of unknown test levels. Banerjee and Dolado (1987) have provided analytical approximations to those critical values by using Nagar type expansions for the moments of the t-statistic.

This paper makes two points by extending the set of regressors used in the orthogonality tests. First, we claim that the importance of the over-rejection effect may have been over-stated, because a number of small alterations to the data generating process (DGP) and model considered by MS yield true test levels considerably closer to the nominal ones. Second, for some interesting cases, the true test levels appear to vary substantially with small variations in the DGP. The revised critical values

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given by MS therefore cannot necessarily be used in more general circumstances; the appropriate critical values to use may depend sensitively upon a number of nuisance parameters [see, for example, Davidson and Hendry (1981) and Muellbauer (1983) for a discussion of these ideas in the context of the permanent income hypothesis of consumption].

The paper is organized as follows. Section 2 provides the DGP and the model. At this stage it should be noted that although we use a bivariate set of regressors, the results extend to more general cases. Section 3 contains results about the sizes of tests for parameter significance and their implications for interpretation of the outcomes. Section 4 concludes the paper.

2. The data generation process and models

MS consider the following problem. There exists a series $\{v_t\}$ which is postulated, under the null of rationality, to be a series of innovations relative to another stationary variable x_{t-1}. However, the series $\{v_t\}$ may be correlated with the innovation at time t, denoted ϵ_t, in that variable.

The *DGP* is therefore as follows:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \theta \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \nu_t \\ \epsilon_t \end{bmatrix}, \tag{1}$$

where $|\theta| < 1$; $\epsilon_t \sim \text{nid}(0, 1)$; $\nu_t \sim \text{nid}(0, 1)$; $\text{E}(\epsilon_t \nu_s) = \delta_{ts}\rho$. The test of rationality is conducted using the regression *model*:

$$y_t = \alpha_0 + \alpha_1 x_{t-1} + \omega_t. \tag{2}$$

The investigator runs this regression and checks for a significant test statistic on the estimate of α_1. MS provide correct Monte Carlo critical values for a range of values of θ and ρ, placing special emphasis on the cases in which θ takes values close to, but smaller than, unity; that is, when the series $\{x_t\}$ is borderline stationary.

We consider the following generalized DGP in order to address the points listed above:

$$\begin{bmatrix} y_t \\ x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \theta_{11} & \theta_{12} \\ 0 & 0 & \theta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{1t-1} \\ x_{2t-1} \end{bmatrix} + \begin{bmatrix} \nu_t \\ \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}, \tag{3}$$

where $|\theta_{ij}| < 1$ $(i, j = 1, 2)$; $\nu_t \sim \text{nid}(0, 1)$; $\epsilon_{it} \sim \text{nid}(0, 1)$; $\text{E}(\nu_t \epsilon_{is}) = \delta_{ts}\rho$; $\text{E}(\epsilon_{1t}\epsilon_{2s}) = 0$.

The generalization of the DGP in (1) consists in the inclusion of an extra variable x_{2t} which is independent of both y_t and x_{1t}. This latter assumption is made in order to consider the more realistic case where regressors in the information set are not generated by univariate AR(1) processes. The corresponding test of orthogonality is conducted using the extended regression model: [1]

$$y_t = \beta_0 + \beta_1 x_{1t-1} + \beta_2 x_{2t-1} + \omega_t, \tag{4}$$

and testing, either separately or jointly, the null hypothesis

$$\text{H}_0: \beta_1 = \beta_2 = 0.$$

[1] This corresponds to a test of semi-strong efficiency.

Table 1

Case	θ_{11}	θ_{22}	θ_{12}	x_{1t}	x_{2t}	NC	Slope [a]
A	$\ll 1.000$	0.999	0.000	I(0)	NI(1)	No	–
B	0.999	$\ll 1.000$	0.000	NI(1)	I(0)	No	–
C	$\ll 1.000$	0.999	$\neq 0.000$	NI(1)	NI(1)	Yes	$(1-\theta_{11})^{-1}\theta_{12}$
D	0.999	$\ll 1.000$	$\neq 0.000$	NI(1)	I(0)	No	–
E	0.999	0.999	0.000	NI(1)	NI(1)	No	–
F	0.999	0.999	$\neq 0.000$	NI(2)	NI(1)	No	–

[a] Co-integrating parameter, for case C.

The simulations were carried out using 1000 replications, [2] in the parameter space $T \times \Xi_{11} \times \Xi_{12} \times \Xi_{22} \times \rho$ where $T = \{120\}$, [3] $\Xi_{11} = \{0, 0.9, 0.999\}$, $\Xi_{12} = \{0, 0.1, 0.9\}$, $\Xi_{22} = \{0, 0.9, 0.999\}$; $\rho = \{0, 1\}$. Before commenting on the results, it is interesting to specify the taxonomy of cases displayed in table 1 which will help us to interpret the outcomes.

In table 1, $\ll 1.00$ denotes non-borderline stationary processes, (i.e., $|\theta_{ii}| \leq 0.9$). NI denotes nearly integrated processes [see Phillips and Ouliaris (1986)] and NC denotes nearly co-integrated processes [see Granger and Engle (1987)]. In fact, for the representative sample size chosen, according to the MS results, the borderline case is practically indistinguishable from the unit root. Therefore, in all cases, except C, the order of integration of the right-hand side in (4) is effectively different from that of the left-hand side. Unwarranted reliance on distributions that are correct only asymptotically would lead to incorrect inference about the individual or joint significance of the regressors. In case C, the regressors are co-integrated and hence the particular linear combination $\{x_{1t} - (1-\theta_{11})^{-1}\theta_{12}x_{2t}\}$ has an asymptotic normal distribution. This fact was first conjectured by Sims (1978) and recently proved formally by Phillips and Ouliaris (1986). Cases A and B (respectively C and D) are symmetric in the sense that θ_{11} and θ_{22} have been interchanged for a given zero (respectively non-zero) value of θ_{12}. Similarly, E and F are symmetric with respect to θ_{12}. Finally, when θ_{11} and θ_{22} take the value of, say, 0.9, cases A and B (respectively C and D) tend towards case E (respectively F), illustrating the properties of the tests where one of the roots is a mild borderline case and the other is a strong borderline case.

3. Results

Table 2 presents the results for the parameter space described above, where the different cases have been combined according to the symmetry considerations previously discussed. Each cell contains the true sizes of the t and F statistics. These are based on the critical values for the nominal five percent level given by the ordinary asymptotic distribution. Each value of the size has a 95% confidence interval of approximately ± 1.4 percentage points (for $p = 0.05$; variance $= p(1-p)/N$ where p is the true test level).

An interpretation of the results is as follows. When $\rho = 0$, the rejection rates are correct, irrespective of the order of integration of x_{1t} and x_{2t}, given that ϵ_{it} and ν_t are independent (see tables 1 and 2 in MS). When $\rho = 1$, in those cases (B, D, E) where x_{1t} is NI(1) and non-co-integrated with x_{2t}, both the t-test of β_1 and the F-test reject a true null hypothesis significantly more times

[2] Initial values were chosen from the corresponding unconditional multivariate normal distributions.

[3] Similar results were obtained for $T = 200$.

Table 2
Rejection frequencies (%) at nominal 5% level. [a]

Case	ρ	θ_{11}	θ_{22}	θ_{12}	$t(\beta_1 = 0)$	$t(\beta_2 = 0)$	$F(\beta_1 = \beta_2 = 0)$
A.1	1.000	0.000	0.999	0.000	5	5	5
A.2	0.000	0.000	0.999	0.000	5	5	4
A.3	1.000	0.900	0.999	0.000	10 *	7 *	8 *
A.4	0.000	0.900	0.999	0.000	6	5	4
B.1	1.000	0.999	0.000	0.000	32 *	5	23 *
B.2	0.000	0.999	0.000	0.000	5	5	5
B.3	1.000	0.999	0.900	0.000	40 *	7 *	30 *
B.4	0.000	0.999	0.900	0.000	5	4	4
C.1	1.000	0.000	0.999	0.100	5	4	4
C.2	0.000	0.000	0.999	0.100	5	4	5
C.3	1.000	0.000	0.999	0.900	5	5	4
C.4	0.000	0.000	0.999	0.900	5	5	5
D.1	1.000	0.999	0.000	0.100	34 *	5	22 *
D.2	0.000	0.999	0.000	0.100	5	4	4
D.3	1.000	0.999	0.000	0.900	17 *	7 *	10 *
D.4	0.000	0.999	0.000	0.900	5	5	5
E.1	1.000	0.999	0.999	0.000	42 *	14 *	31 *
E.2	0.000	0.999	0.999	0.000	5	4	4
F.1	1.000	0.999	0.999	0.100	5	5	4
F.2	0.000	0.999	0.999	0.100	4	4	4
F.3	1.000	0.999	0.999	0.900	4	4	4
F.4	0.000	0.999	0.999	0.900	4	4	4

[a] Each cell in the last three columns represents actual rejection rates at the five percent nominal critical values;
* denotes a significant deviation from the nominal size.

than the nominal size requires. Note that, as A.1 reflects, actual and nominal test levels coincide if θ_{11} is well inside the unit circle. E.1 in table 2 highlights the fact that non-stationary features in both regressors lead to wrong inferences in both t-ratios. Case C, in which the series are co-integrated, conforms to the nominal size, even for low values of the co-integrating slope.

The most interesting result arises from case F, in which both series are positively integrated of different orders. Where $\rho = 1$, it is difficult to distinguish at this sample size between co-integrated series (C.1, C.3) and non-co-integrated series (F.1, F.3) of different strictly positive orders [see Banerjee et al. (1986)]. It is particularly interesting to compare the latter cases with E.1. In case F.3, where $\theta_{12} = 0.9$, the rejection rates are very close to the nominal rates; even for F.1 with $\theta_{12} = 0.1$, rates are much closer to the nominal values than for $\theta_{12} = 0$ (E.1). Thus the appropriate critical value is very sensitive to a small variation in the DGP.

Finally, A.3 and B.3, which tend to E.1 as $\theta_{11} \to 1$, illustrate how roots of 0.9 are still difficult to distinguish from unit roots, even for a sample size which is larger than those typically found in applied macroeconomic research.

It is also worth emphasizing a modification of the model used in the regression test from (4) to

$$y_t = \beta_0 + \beta_1 x_{1t-1} + \beta_2 x_{1t-2} + \omega_t. \tag{5}$$

This extends the MS model by inclusion of an extra lag on the explanatory variable x_1. We now find that the actual test levels conform very nearly to the nominal levels for almost all values in the parameter space $\Xi_{11} \times \Xi_{12} \times \Xi_{22} \times \rho$. As an example, consider the worst case for model (4), given as E.1 in table 2; the analogous entry for model (5) is

[E.1′] 1.000 0.999 0.999 0.000 7 3 26. [4]

Clearly the nominal levels are much better guides to the true levels of t-statistics than where only one lagged value of the explanatory is present.

4. Concluding remarks

We have illustrated how the presence of more than one regressor in the orthogonality conditions [such as our eq. (4)] which characterize rationality tests, poses some difficulties for the recommendation of an uncritical use of the MS results. Notably, in some instances, the incorporation of a new regressor brings the actual sizes of the t and F statistics closer to the nominal ones. Hence the extent of the over-rejection of rationality in empirical work may have been over-stated. Moreover, the fact that critical values vary substantially with the addition of new regressors belonging to the DGP renders any adjustment of the nominal levels hazardous.

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[4] Note that the large number of rejections on the F-statistic disappears when the constant is deleted from the model.