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“Asymmetric information allocation to avoid coordination failure"

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Asymmetric information allocation to avoid coordination failure*

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Abstract

In the context of team production, this paper studies the optimal (deterministic and stochastic) information allocation that implements desired effort levels as the unique Bayesian equilibrium. We show that, under certain conditions, it is optimal to asymmetrically inform agents even though they may be ex ante symmetric. The main intuition is

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that informing the agents asymmetrically can be effective in avoiding "bad" equilibria, that is, equilibria with coordination failure.

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1 Introduction

In the context of the team-production problem of Holmstrom (1982), Winter (2004) finds that an asymmetric bonus contract may be optimal in uniquely implementing desired effort levels, even with homogeneous agents.[1] To understand its key intuition, imagine a principal who designs a bonus scheme to make two agents play high efforts in its unique equilibrium, and that the probability of successful outcomes is increasing and *supermodular* in their total efforts. To eliminate low-effort equilibria, the principal needs to offer a high-enough bonus scheme to one of the agents, say agent 1. On the other

[1] Asymmetric bonus contracts are usually suboptimal if the principal employs a wishful thinking in equilibrium selection. Unique implementation, on the other hand, may be a reasonable concern, for example, when lower effort levels significantly increase the probability of hazardous outcomes, and the principal does not have full control over equilibrium selection. For example, Kreps (1990) and Cronqvist, Low, and Nilsson (2007) argue that which equilibrium is to be played may be determined by the "corporate culture", which is usually difficult to fully control. Baliga and Sjostrom (1998) consider collusive behaviors in moral hazard. See Ma (1988), Arya, Glover, and Hughes (1997), and Winter (2004) for studies of optimal contracts that uniquely implement the desired effort choice under various assumptions. Winter (2004) is the closest to ours in that he considers the team-production model of Holmstrom (1982).

hand, given this, the principal does not need to offer such a high bonus scheme to agent 2. Because the success probability is supermodular in total effort, the fact that agent 1 plays a high effort makes agent 2's incentive constraint *less stringent*. Thus, the optimal bonus contract is asymmetric. Similar "divide-and-conquer" nature of optimal contracts also appear in other contexts. For example, see Segal (2003) in the context of bilateral contracting with externalities.[2]

Although the design of a bonus scheme is important in the optimal organization design, there are other important aspects as well. This paper studies the optimal information allocation in this context. More specifically, we consider a situation in which the probability of success is not only a function of total effort, but also a function of the (exogenously given) uncertain state. The principal decides which agent observes the realization of this state (as well as the bonus scheme).

Observing the realized state obviously affects this "informed" agent's effort incentive: better state realization makes him play a high effort with less bonus, and vice versa. However, even if an agent does not observe the state ("uninformed"), if he *knows* that the other agent observes the state, that knowledge could affect his incentive. For example, imagine the bonus level such that the informed agent plays a high effort in the good state. In view of the uninformed agent, this means that the informed agent will play a high effort *with some positive probability*. This fact along with the supermodularity of the success-probability function makes the uninformed agent's incentive constraint *less stringent*. Indeed, under certain parameter values,

[2]The term "divide-and-conquer" also appears in Segal (2003).

we find that such asymmetric information allocation is better than informing no agent or both agents.

In this sense, the optimal information allocation has features similar to the optimal bonus scheme in that treating homogeneous agents asymmetrically may be effective in avoiding bad-coordination equilibria, but there are some important differences too; for example, (i) providing information implies better incentive in the good state, but it also implies *worse* incentive in the bad state; in contrast, (ii) providing higher bonus *unambiguously* improves the agent's incentive. Because of this difference, the optimal information structure is asymmetric *only under certain parameter values*. Indeed, if the cost of allocating more information, that is, the deterioration of incentive in the bad state, is significant, then asymmetric information allocation is suboptimal. The main result of the paper establishes a clear connection between the optimal information allocation and the shape of the success-probability function.[3]

The paper is structured as follows. Section 2 introduces the model, and Section 3 studies the optimal bonus contract and information structure in a simple two-agent case with anonymous contracts. To highlight the main intuition, Section 3 only considers *deterministic* information allocation, that is, each agent is either fully informed of the realized state, or not informed

[3]In various contexts, it is observed that asymmetric information sometimes mitigates incentive problems or shrinks the set of equilibria (see, for example, Bergemann and Pesendorfer (2007) in optimal auction, Morris and Shin (2002) in coordination games, Schmitz (2006) and Goldlucke and Schmitz (2014) in pre-contracting investments). We make a step forward by characterizing the *optimal* information allocation in the team-production context.

at all. Section 4 considers all (possibly stochastic) information allocations. Although this case is much harder, the optimal stochastic information structure is similar to the optimal deterministic information structure in that information is given in an asymmetric manner. Furthermore, they coincide under certain parameter values. Section 5 concludes the paper. Although we only consider a simple environment for brevity, the conclusion of this paper is robust in a number of extensions and generalizations, such as more than two agents, asymmetric bonus contracts, and asymmetric characteristics of agents. Interested readers may consult our working paper version Moriya and Yamashita (2016).

2 Model

We consider a team-production model with one manager (a principal) and two workers (agents) engaged in a project. Each worker $i = 1, 2$ simultaneously chooses an effort level $e_i \in \{0, 1\}$, which costs ce_i for $c > 0$. The profit of the project is $y \in \{S, F\}$ $(S > F)$. Let $p_\theta(x)$ denote the probability of success $(y = S)$, which depends on the agents' total effort $x = e_1 + e_2$ and task environment $\theta \in \{H, L\}$. We assume that $p_\theta(x)$ is increasing in x for any θ. The prior probability for each θ is $f_\theta \in (0, 1)$ with $f_H + f_L = 1$. Let $p_\phi(x) = f_H p_H(x) + f_L p_L(x)$ denote the mean success probability, given x.

The marginal productivity of effort, denoted by $\pi_\theta(x) \equiv p_\theta(x) - p_\theta(x-1)$, satisfies (i) $\pi_H(x) > \pi_L(x)$ for all x and (ii) $\pi_\theta(x) > \pi_\theta(x-1)$ for all x and θ. The first condition requires that the marginal productivity is always higher in state H than in state L, and the second condition requires that the agents' efforts are complementary; thus the agents' collaboration is important for success.

The main choice variable of the principal is the agents' information structure. In Section 3, we only consider deterministic information structures, which simply refers to how many agents are informed of θ before they make effort choices, represented by $m = 0, 1, 2$.[4] A possible interpretation is that the principal controls each agent's cost of acquiring information on θ (although he himself does not observe θ). Once the principal chooses an information structure $m = 0, 1, 2$, we assume that this structure becomes common knowledge among the agents. A bonus contract can be contingent only on the outcome, and is assumed to be the same for both agents:[5] each agent earns bonus $b \geq 0$ if $y = S$, while he earns 0 if $y = F$.

Let $s_i \in S_i = \{H, L, \phi\}$ represent agent i's information about the state. More specifically, (i) if he is informed, then $s_i = \theta$ for each θ, and (ii) if he is uninformed, then $s_i = \phi$ for each θ. Hence, agent i's strategy is to choose $e_i(s_i) \in \{0, 1\}$ for each s_i. Given bonus b and the agents' effort profile $e = (e_1, e_2) \in \{0, 1\}^2$, agent i's payoff in state θ is $u_i(e, \theta; b) = bp_\theta(e_1 + e_2) - ce_i$. Thus, given the information structure, the agents' strategy profile $\mathbf{e} = (e_i(s_i))_{i,s_i}$ is a (pure-strategy Bayesian) *equilibrium* if (i) for each i who

[4] In Section 4, we consider a general (stochastic) information structure.

[5] We restrict our attention to these symmetric bonus contracts for simplicity of analysis. As shown by Winter (2004), in general, this restriction is *with* loss of generality. Nevertheless, the qualitative features of the optimal information allocation (e.g., optimality of asymmetric information allocation under certain parameter values) do not essentially change even if we allow for asymmetric bonus contracts. See Remark 1 at the end of Section 3 and our working paper version Moriya and Yamashita (2016).

is informed: for each $\theta \in \{H, L\}$ and $e'_i \in \{0, 1\}$,

$$u_i(e_i(\theta), e_{-i}(s_{-i}), \theta; b) \geq u_i(e'_i, e_{-i}(s_{-i}), \theta; b),$$

and (ii) for each i who is uninformed: for each $e'_i \in \{0, 1\}$,

$$E[u_i(e_i(\phi), e_{-i}(s_{-i}), \theta; b)] \geq E[u_i(e'_i, e_{-i}(s_{-i}), \theta; b)].$$

As standard in the literature, we assume that S is sufficiently larger than F so that the principal's goal is to implement the *full-effort strategy profile*, that is, $\mathbf{e} = (e_i(s_i))_{i,s_i}$ such that $e_i(s_i) = 1$ for all i and s_i.

Benchmark: Full-effort strategy profile as one of the equilibria We first derive the optimal contract that makes the full-effort strategy profile *one of the equilibria* and observe that $m = 0$ (no information) is the optimal information structure.

With $m = 0$, the full-effort strategy profile is an equilibrium if $bp_\phi(n) - c \geq bp_\phi(n-1)$, or equivalently $b \geq \frac{c}{\pi_\phi(n)}$. With $m = 1$ or 2, the bonus must be sufficiently high for an informed agent to work in the low state, that is, $bp_L(n) - c \geq bp_L(n-1)$ or, equivalently $b \geq \frac{c}{\pi_L(n)}$. Therefore, $m = 0$ (no information) is the optimal information structure. Intuitively, this is because we must incentivize the agents for an *average* state under the no-information structure, whereas under any other information structure, we must incentivize the informed agent for *every* state.[6]

Unique implementation The contract to implement the full-effort strategy profile as *one of the equilibria* implicitly assumes that the agents would

[6] This argument is well-known in the literature. See Myerson (1983).

play the best equilibrium in view of the principal, even if there are multiple equilibria in the contract. However, in case failure of the project is extremely hazardous (e.g., accidents in a nuclear power plant, loss of a brand's long-term reputation, and so on), the principal may not want to follow such a wishful thinking. Rather, it may be more reasonable to require that the full-effort strategy profile is a *unique* equilibrium.

Given each information structure m, let $b_m \in \mathbb{R}_+$ denote the infimum level of bonus with which the full-effort strategy profile is a unique equilibrium.[7] We say that m is the optimal information structure if $b_m \leq b_{m'}$ for any other information structure m'.

Before the formal analysis, we first illustrate the main intuition in the following simple example.

Example 1. In the good state, each agent earns expected payoff 10 if both agents work, 7 if he shirks but the other works, 6 if he works but the other shirks, and 5 if both shirk. In the bad state, each agent earns 3 if both agents work, 4 if he shirks but the other works, 1 if he works but the other shirks, and 2 if both shirk.[8]

("good")	work	shirk
work	$(10, 10)$	$(6, 7)$
shirk	$(7, 6)$	$(5, 5)$

("bad")	work	shirk
work	$(5, 5)$	$(1, 4)$
shirk	$(4, 1)$	$(3, 3)$

Each state is equally likely, and hence, if no agent is informed about the state, then "both work" and "both shirk" are equilibria, which is not

[7] Winter (2004) calls it the *incentive-inducing contract* (for the full-effort strategy profile).

[8] One can interpret these numbers as the agents' expected utilities given an arbitrarily fixed bonus contract.

desirable for the principal. Informing both agents about the state is not desirable either, because "both shirk" is again an equilibrium in the bad state.

Nevertheless, informing *just one agent* can eliminate this bad coordination. Specifically, suppose that only agent 2 is informed about the state, whereas agent 1 is not (but agent 1 knows that agent 2 knows the state, and so on). First, if the state is good, then it is strictly dominant for agent 2 to work. *Given this*, it is now (iteratively) strictly dominant for agent 1 to work, as illustrated in the following table:

	work in both states	work only in good state
work	$(\frac{15}{2}, \frac{15}{2})$	$(\frac{11}{2}, 7)$
shirk	$(\frac{11}{2}, \frac{7}{2})$	$(5, \frac{9}{2})$

Finally, given that agent 1 works (in any state), it is (iteratively) strictly dominant for agent 2 to work *even in the bad state*. Therefore, the desired outcome that "both work in every state" is the unique strategy profile that survives iterative elimination of strictly dominated strategies.[9] $\square$

[9]In this example, one might think that (work,work) may be "selected" by the agents even if there are multiple equilibria because it is Pareto dominant for them. As a related point, one might also think that sequential action choices by the agents (if technologically possible) solve the problem because the "leader" agent chooses to work in order to essentially "select" the better equilibrium. However, this is just an artifact of our simple example. In general, Pareto domination and sequential action choices could lead to shirking behavior, while asymmetric information allocation could uniquely implement the full-effort strategy profile. See Moriya and Yamashita (2016).

3 Optimal deterministic information structure

We first characterize b_m for each information structure m, and then examine the optimal information structure.

3.1 $m = 0$ (no information)

With $m = 0$ where neither agent is informed, the optimal bonus contract that implements $\mathbf{e} = (1, 1)$ as *one of the equilibria* is $b = \frac{c}{\pi_\phi(2)}$. However, under this contract, not only $\mathbf{e} = (1, 1)$, but also $\mathbf{e} = (0, 0)$ is an equilibrium: if i chooses $e_i = 0$, then it is (strictly) optimal for j to choose $e_j = 0$.[10]

Therefore, the optimal bonus level that *uniquely* implements $\mathbf{e} = (1, 1)$ must be strictly greater than $\frac{c}{\pi_\phi(2)}$. Specifically, for $\mathbf{e} = (0, 0)$ not to be an equilibrium, we must have $b > \frac{c}{\pi_\phi(1)} (> \frac{c}{\pi_\phi(2)})$ so that an agent works even if the other agent does not work.

Now, given $b > \frac{c}{\pi_\phi(1)}$, a high effort is strictly dominant for each agent and, hence, $\mathbf{e} = (0, 0)$ and any other effort choice (except for $\mathbf{e} = (1, 1)$) cannot be an equilibrium. Therefore, we have the following lemma.

Lemma 1. $b_0 = \frac{c}{\pi_\phi(1)}$.

3.2 $m = 1$ (asymmetric information)

We now consider the asymmetric-information scenario $m = 1$, where only one of the agents (say, agent 2) is informed. We first state the result.

[10] Note that the convexity of the success probability function, p, plays a key role in this argument, as in Winter (2004).

Lemma 2. $b_1 = \max\left\{\frac{c}{\pi_H(1)}, \frac{c}{\pi_L(2)}, \frac{c}{f_H\pi_H(2)+f_L\pi_L(1)}\right\}$.

Proof. We first show that, if neither $(0,(0,0))$, $(0,(1,0))$, nor $(1,(1,0))$ is an equilibrium with bonus b, then

$$b > \bar{b} = \max\left\{\frac{c}{\pi_H(1)}, \frac{c}{\pi_L(2)}, \frac{c}{f_H\pi_H(2)+f_L\pi_L(1)}\right\}.$$

Indeed, first, to prevent $(0,(0,0))$ from being an equilibrium, we must have either $b > \frac{c}{\pi_\phi(1)}$, $b > \frac{c}{\pi_H(1)}$, or $b > \frac{c}{\pi_L(1)}$. Because $\frac{c}{\pi_H(1)} < \frac{c}{\pi_\phi(1)}, \frac{c}{\pi_L(1)}$, we obtain $b > \frac{c}{\pi_H(1)}$ as its necessary condition.

Given $b > \frac{c}{\pi_H(1)}$, to prevent $(0,(1,0))$ from being an equilibrium, we must have either $b > \frac{c}{f_H\pi_H(2)+f_L\pi_L(1)}$ or $b > \frac{c}{\pi_L(1)}$. Because $\frac{c}{f_H\pi_H(2)+f_L\pi_L(1)} < \frac{c}{\pi_L(1)}$, we obtain $b > \frac{c}{f_H\pi_H(2)+f_L\pi_L(1)}$ as its necessary condition.

Given $b > \max\{\frac{c}{\pi_H(1)}, \frac{c}{f_H\pi_H(2)+f_L\pi_L(1)}\}$, to prevent $(1,(1,0))$ from being an equilibrium, we must have $b > \frac{c}{\pi_L(2)}$.

Now we show that, conversely, $(1,(1,1))$ is uniquely implemented by any b such that $b > \bar{b}$. This completes the proof by establishing $b_1 = \bar{b}$. First, because $b > \frac{c}{\pi_H(1)}$, it is strictly dominant for the informed agent to make a high effort in state H. Given this, because $b > \frac{c}{f_H\pi_H(2)+f_L\pi_L(1)}$, it is (iteratively) strictly dominant for the uninformed agent to make a high effort. Given this, because $b > \frac{c}{\pi_L(2)}$, it is (iteratively) strictly dominant for the informed agent to make a high effort even in state L. Therefore, $(1,(1,1))$ is the unique strategy profile that survives iterative elimination of strictly dominated strategies, and hence, it is a unique equilibrium. $\square$

3.3 $m = 2$ (full information)

Finally, with $m = 2$, it is necessary to incentivize an agent to choose a high effort in *any* state even if the other agent does not work. Therefore, the bonus must be at least $\frac{c}{\pi_L(1)}$. Given such a bonus level, it is strictly dominant for each agent to choose a high effort in any state.

Lemma 3. $b_2 = \frac{c}{\pi_L(1)}$.

3.4 Optimal information structure

We now compare the three information structures discussed above. Recall that $b_0 = \frac{c}{\pi_\phi(1)}$ is the bonus level that incentivizes an agent to work *in the average state*, whereas $b_2 = \frac{c}{\pi_L(1)}$ is the bonus level that incentivizes an agent to work *in any state*, even if the other agent does not work. We have $b_0 < b_2$, i.e., informing no agent is better than informing both agents.

The difference between $m = 0$ and $m = 1$ depends on the parameter values.

Proposition 1. $b_1 \leq b_0$ if and only if

$$\frac{\pi_L(2) - \pi_L(1)}{\pi_H(1) - \pi_L(1)} \geq f_H.$$

Proof. Observe that

$$b_0 \geq b_1 \quad \Leftrightarrow \quad \frac{c}{\pi_\phi(1)} \geq \frac{c}{\pi_L(2)} \quad \Leftrightarrow \quad \frac{\pi_L(2) - \pi_L(1)}{\pi_H(1) - \pi_L(1)} \geq f_H,$$

where the first equivalence is because we always have $\frac{c}{\pi_\phi(1)} \geq \frac{c}{\pi_H(1)}$ and $\frac{c}{\pi_\phi(1)} \geq \frac{c}{f_H \pi_H(2) + f_L \pi_L(1)}$. □

The inequality in the statement implies that asymmetric information is more likely to be optimal as (i) the "effort complementarity effect" on the production function measured by $\pi_L(2) - \pi_L(1)$ becomes greater, and (ii) the "state effect" measured by $\pi_H(1) - \pi_L(1)$ becomes smaller. This is because (i) if the effort complementarity effect is more important, then the concern of potential coordination failure is greater, and therefore, the benefit of asymmetric information allocation becomes greater. On the other hand, (ii) if the state effect is more important, then it is costly to incentivize an informed agent in the low state, and therefore, informing no agent is likely to be better.

Remark 1. In a similar team-production context but without state uncertainty, Winter (2004) shows that an asymmetric bonus contract outperforms any symmetric bonus contract (Winter (2004) also allows for more than two agents). Despite this result, in this paper, we focus on symmetric bonus contracts in order to simplify the analysis. Whether our result qualitatively changes or not with an asymmetric bonus contract as in Winter (2004) is a natural question.

In our working paper version Moriya and Yamashita (2016), we examine several extensions and generalizations, including those with asymmetric bonus contracts and more than two agents. We find that the qualitative feature of the main result is basically robust to them, although the analysis becomes more involved and richer. More specifically, (i) with more than two agents, we need to consider how many agents should be informed, rather than simply compare "no information" and "asymmetric information". However, the infimum bonus level given each information structure is derived from a similar iterative-elimination argument, and the optimal number of informed agents is similarly determined by the relative magnitudes of the state effects and effort complementarity effects. (ii) With asymmetric bonus contracts, again the basic logic does not change, but the analysis becomes much more complicated: even with two agents, we need to consider who should be informed *and* who should be paid more, and different configurations could admit different orders of iterative elimination. Nevertheless, in Moriya and Yamashita (2016), we show that asymmetric information allocation is optimal under a similar but *smaller* set of parameter values. Although the intuition is essentially the same (i.e., optimality of asymmetric information allocation occurs when the effort complementarity effects are more important than the state effects), a smaller set of such parameter values suggests that asymmetric bonus contracts and asymmetric information allocations are *(imperect) substitutes* in view of the principal.

4 Optimal general (stochastic) information structure

In the previous sections, we consider only *deterministic* information allocations, that is, each agent is either perfectly informed of θ or not informed at all. However, in some cases, the principal may have more flexibility in terms of precision of information provided to the agents. Furthermore, in case neither agent is fully informed, the correlation between the agents' signals could be an important variable to control too. Therefore, in this section, we study the optimal general (possibly stochastic) information structure.

Given that the principal has flexibility in terms of information allocation, our problem is related to the literature on Bayesian persuasion (e.g., Kamenica and Gentzkow (2011)) and incomplete-information or Bayes correlated equilibrium (e.g., Forges (1993) and Bergemann and Morris (2013)). However, these studies usually consider the problem of implementing a strategy profile of interest as *one of the equilibria* by some feasible information structures, and some techniques (most importantly, a version of revelation principle) crucially hinge on such presumptions. On the other hand, our problem is to *uniquely* implement the full-effort strategy profile, and as shown in the previous section, the key driving force for our problem comes from the concern of eliminating "bad equilibria". Thus, we cannot straightforwardly apply those known techniques to our problem.[11] Rather, our argument hinges on the well-known property of the supermodular games (Topkis (1979), Vives (1990), Milgrom and Roberts (1990)) that the lowest-effort equilibrium is characterized by iterative elimination of strictly dominated strategies "from below". Because of this property, our goal is essentially to find a feasible information structure (and a bonus level) such that only the full-effort strategy profile survives this iterative elimination procedure.

A stochastic information allocation is given by (S_1, S_2, μ), where each S_i denotes the message space for agent i, and $\mu : \Theta \to \Delta(S_1 \times S_2)$ is such that, for each θ, $\mu(s_1, s_2|\theta)$ represents the probability of sending $s_i \in S_i$ to each agent i in state θ. For simplicity, we only consider finite and full-support information structures in the sense that (i) each S_i is finite, and (ii) for all i

[11] Indeed, as we show below, the optimal information structure may involve an infinite number of messages/signals even though there are only two states and two actions. In this sense, our result below provides a counterexample for a revelation principle in the context of unique implementation.

and s_i, there exists some θ and s_{-i} such that $\mu(s_i, s_{-i}|\theta) > 0$.

Our goal is to identify the infimum bonus level, denoted by b^*, with which the high-effort profile is the unique equilibrium outcome regardless of the true state, by carefully designing a stochastic information structure.

Theorem 1. $b^* = \max\{\frac{c}{\pi_H(1)}, \frac{2c}{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))}\}$.

To provide some intuition before the formal proof, we first consider the case where

$$\frac{c}{\pi_H(1)} \geq \frac{2c}{f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))},$$

so that the statement reduces to $b^* = \frac{c}{\pi_H(1)}$.[12] This case holds if the "state effects" represented by $\pi_H(x) - \pi_L(x)$ for each $x = 1, 2$ are relatively small compared to the "effort effects" represented by $\pi_\theta(2) - \pi_\theta(1)$ for each θ. In this case, even if an agent is informed that the state is H, he may not have enough incentive to work if the other agent does not work. On the other hand, once such a "never-work" strategy profile is eliminated, it is relatively less costly to eliminate all the other "intermediate" strategy profiles. Thus, we obtain $b^* = \frac{c}{\pi_H(1)}$.

The other case with

$$\frac{c}{\pi_H(1)} \leq \frac{2c}{f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))}$$

holds if the state effects are large relatively to the effort effects. In this case, an agent who is informed that the state is likely to be H has a large incentive to work, and hence, the constraint that $b^* \geq \frac{c}{\pi_H(1)}$ does not bind. On the

[12] Recall that this is the amount of bonus necessary to incentivize an agent in state H if the other agent does not work.

other hand, an agent who is informed that the state is likely to be L does not have enough incentive to work, even if he knows that the other agent works. To avoid this problem, the optimal information structure achieves the following two properties simultaneously, by constructing a long chain of messages: given each message (except for the "first" and "last" messages), (i) each agent is made unaware that the state is L (even if it is so); and (ii) each agent is confident that the other agent works with a sufficiently high probability.

Proof. We first show that

$$b^* \geq \max\left\{\frac{c}{\pi_H(1)}, \frac{2c}{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))}\right\}.$$

Consider an arbitrary information structure (S_1, S_2, μ), and a bonus level b with which the high-effort profile is the unique equilibrium outcome regardless of the true state. Then, there exists some agent i and his message $s_i \in S_i$ such that

$$b \cdot \frac{\sum_{s_{-i},\theta} \mu(s_i, s_{-i}|\theta) f_\theta \pi_\theta(1)}{\sum_{s_{-i},\theta} \mu(s_i, s_{-i}|\theta) f_\theta} \geq c,$$

that is, agent i works given s_i even if agent $-i$ does not work. Such message must exist, because otherwise, it is an equilibrium where no one plays a high effort given any message. This inequality then implies that

$$b\pi_H(1) \geq c.$$

Let $S^1 \subseteq S_1$ denote the set of agent 1's messages such that, when $s_1 \in S^1$, agent 1 works even if agent 2 does not work. As shown above, S^1 is nonempty.[13] Let $T^1 = S^1$.

Let $S^2 \subseteq S_2$ denote the set of agent 2's messages such that, with $s_2 \in S^2$, agent 2 works if agent 1 works whenever $s_1 \in S^1$. By the logic above, S^2 must be nonempty. Let $T^2 = S^2$.

Inductively, for each $k > 2$ odd, let $S^k \subseteq S_1 \setminus T^{k-2}$ denote the set of agent 1's messages such that, with $s_1 \in S^k$, agent 1 works if agent 2 works whenever $s_2 \in T^{k-1}$. Then, let $T^k = S^k \cup T^{k-2}$. Similarly, for each $k > 2$ even, let $S^k \subseteq S_2 \setminus T^{k-2}$ denote the set of agent 2's messages such that, with $s_2 \in S^k$, agent 2 works if agent 1 works whenever $s_1 \in S^{k-1}$. Then, let $T^k = S^k \cup T^{k-2}$.

For each k, we have

$$\begin{aligned} & b[f_H(\mu(T^{k-1}, S_k|\theta)\pi_H(2) + \mu(\neg T^{k-1}, S_k|\theta)\pi_H(1)) \\ & +f_L(\mu(T^{k-1}, S_k|\theta)\pi_L(2) + \mu(\neg T^{k-1}, S_k|\theta)\pi_L(1))] \\ \geq \; & c[f_H(\mu(T^{k-1}, S_k|\theta) + \mu(\neg T^{k-1}, S_k|\theta)) \\ & +f_L(\mu(T^{k-1}, S_k|\theta) + \mu(\neg T^{k-1}, S_k|\theta))], \end{aligned}$$

where $\mu(A|\theta) = \sum_{s \in A} \mu(s|\theta)$ for each $A \subseteq S_1 \times S_2$, and $\neg T^{k-1}$ denotes the complement of set T^{k-1}.

The sum of the left-hand sides for all k is

$$b[f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))],$$

and the sum of the right-hand sides for all k is $2c$. Thus,

$$b[f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))] \geq 2c.$$

[13] It is *some* agent i in the discussion above, but without loss of generality, we let this agent be agent 1.

Next, we show that

$$b^* \leq \max\left\{\frac{c}{\pi_H(1)}, \frac{2c}{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))}\right\}.$$

First, if

$$\frac{c}{\pi_H(1)} \geq \frac{2c}{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))},$$

then the deterministic asymmetric information structure in our paper is optimal (Lemma 2). Hence, in the following, we assume

$$\frac{c}{\pi_H(1)} \leq \frac{2c}{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))},$$

or equivalently,

$$2\pi_H(1) \leq f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1)),$$

and we show that

$$b^* \leq \frac{2c}{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))}.$$

Define g_H by

$$g_H = \frac{\frac{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))}{2} - \pi_L(1)}{\pi_H(1)-\pi_L(1)},$$

and let $g_L = 1 - g_H$. We have $g_H > f_H$ because

$$\begin{aligned}
& \frac{\frac{f_H(\pi_H(2)+\pi_H(1))+f_L(\pi_L(2)+\pi_L(1))}{2} - \pi_L(1)}{\pi_H(1)-\pi_L(1)} - f_H \\
= \quad & \frac{f_H(\pi_H(2)-\pi_H(1))+f_L(\pi_L(2)-\pi_L(1))}{2(\pi_H(1)-\pi_L(1))} \\
> \quad & 0.
\end{aligned}$$

Fix arbitrary $M \in \mathbb{N}$, and let $\varepsilon = \frac{f_H f_L}{g_H f_L + 2M g_L f_H}$ and $\alpha = \frac{g_H f_L - g_L f_H}{g_L f_H (2M+1)}$. Note that $\varepsilon, \alpha > 0$, and that $\varepsilon, \alpha \to 0$ as $M \to \infty$. Consider the following information structure (S_1, S_2, μ): $S^1 = \{s_1, s_3, \ldots, s_{2M+1}\}$, $S^2 = \{s_2, s_4, \ldots, s_{2M+2}\}$, and μ is given as follows. For each $k = 1, 2, \ldots, 2M+1$,

$$\begin{aligned}
\mu(s_k, s_{k+1}|H) &= \frac{g_L \varepsilon}{f_L}, \\
\mu(s_k, s_{k+1}|L) &= \frac{g_L \varepsilon (1+\alpha)}{f_L}, \\
\mu(s_1, s_{2M+2}|H) &= \left(\frac{g_H}{f_H} - \frac{g_L}{f_L} \right) \varepsilon,
\end{aligned}$$

and $\mu(s, \tilde{s}|\theta) = 0$ for any other combination of $(s, \tilde{s}, \theta) \in S_1 \times S_2 \times \Theta$.

For this to implement the full-effort strategy profile, the bonus must satisfy

$$\begin{aligned}
b(g_H \pi_H(1) + g_L(1+\alpha)\pi_L(1)) &\geq (g_H + g_L(1+\alpha))c, \\
b(\frac{f_H}{f_L}(\pi_H(2) + \pi_H(1)) + (1+\alpha)(\pi_L(2) + \pi_L(1))) &\geq 2(\frac{f_H}{f_L} + 1 + \alpha)c,
\end{aligned}$$

or equivalently $b \geq b(M)$, where

$$b(M) = \max \left\{ \frac{(g_H + g_L(1+\alpha))c}{g_H \pi_H(1) + g_L(1+\alpha)\pi_L(1)}, \frac{2(f_H + f_L(1+\alpha))c}{f_H(\pi_H(2) + \pi_H(1)) + f_L(1+\alpha)(\pi_L(2) + \pi_L(1))} \right\}.$$

Taking the limit as $M \to \infty$, we have

$$b(M) \to \frac{2c}{f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))}.$$

Therefore, for any $\delta > 0$, there exists $M > 0$ such that

$$b(g_H, M) < \frac{2c}{f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))} + \delta,$$

which implies that

$$b^* \leq \frac{2c}{f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1))}.$$

□

The iterative elimination logic exploited in the construction of the optimal information structure is reminiscent of the contagion arguments in email games (Rubinstein (1989)) and in global games (Carlsson and van Damme (1993), Frankel, Morris, and Pauzner (2003)). Interestingly, the contagious information structures considered as *instances* in those papers prove to be *optimal* in our context.[14] It may be interesting to study whether similar information structures become optimal (in certain senses) in other contexts, in particular in certain class of supermodular games.

As a final corollary, we observe that the deterministic asymmetric information allocation considered in the previous section continues to be optimal under certain parameter values, even if we allow for stochastic information allocations.

Corollary 1. If $b_1 = \frac{c}{\pi_H(1)}$, or equivalently, if

$$\pi_H(1) \leq \min\left\{\pi_L(2), f_H\pi_H(2) + f_L\pi_L(1)\right\},$$

then $b^* = b_1 (= \frac{c}{\pi_H(1)})$, that is, the deterministic information allocation in Section 3 is optimal among all stochastic information allocations.

Proof. We have

$$\begin{aligned} f_H\pi_H(1) + f_L\pi_H(1) &\leq f_H\pi_H(1) + f_L\pi_L(2) \\ \pi_H(1) &\leq f_H\pi_H(2) + f_L\pi_L(1). \end{aligned}$$

[14]Kajii and Morris (1997) observe that, in the context of robust prediction in games, a similar (though somewhat different) contagious information structure plays a crucial role in their *critical-path* theorem. It is an open question as to whether there is some formal link between our result and theirs.

Adding up both sides, we obtain

$$2\pi_H(1) \quad \leq \quad f_H(\pi_H(2) + \pi_H(1)) + f_L(\pi_L(2) + \pi_L(1)),$$

and thus, the previous theorem implies $b^* = \frac{c}{\pi_H(1)}$. $\square$

5 Conclusion

This paper considers a team-production model with state uncertainty. When the principal's goal is to uniquely implement desired effort choices, we show that, under certain conditions, asymmetrically informing the agents is the optimal information allocation. In this sense, by allocating information asymmetrically, it becomes less costly to avoid badly coordinated equilibria. As the degree of effort complementarity increases, asymmetric information allocation tends to improve. On the other hand, informing an agent is always costly in that this agent must be incentivized even in a low state, which is the fundamental difference from other "divide-and-conquer" papers in the related literature (e.g., Winter (2004) in the team-production context), where asymmetric bonus contracts unambiguously improve over symmetric ones.

While we show the robustness of this main intuition in a number of extensions and generalizations in the working paper version Moriya and Yamashita (2016), further research is necessary for a more comprehensive understanding of desirable information allocation in organizations. We believe that the analysis in this paper can serve as a useful benchmark for future research.

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