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Contracting Sequentially with Multiple Lenders: the Role of Menus

Andrea Attar* Catherine Casamatta† Arnold Chassagnon ‡ Jean Paul Décamps§

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Abstract

We study a credit market in which multiple lenders sequentially offer financing to a single borrower under moral hazard. We show that restricting lenders to post single offers involves a loss of generality: none of the equilibrium outcomes arising in this scenario survives if lenders offer menus of contracts. This result challenges the approach followed in standard models of multiple lending. From a theoretical perspective, we offer new insights on equilibrium robustness in sequential common agency games.

Keywords: Multiple Lending, Menus, Strategic Default, Common Agency, Bank Competition.

JEL Classification: D43, D82, G33.

1 Introduction

When firms negotiate with a bank, they commonly have access to a wide range of financial offers: loans of different size, maturity, interest rate and repayment schedule, with or without guarantees, or associated credit lines. The same applies to consumer credit: individuals have in general access to multiple sources of credit (personal loans, revolving credit facilities, overdraft, credit card...) from a given bank. Starting with the seminal work of Bester (1985), theoretical models of bank competition have rationalised the issuance of multiple financial contracts by a given bank as a device to screen different privately informed borrowers.[1] In this paper, we argue that allowing banks to post menus of financial contracts, instead of

*Università di Roma Tor Vergata, and Toulouse School of Economics (TSM-R-CNRS)
†Toulouse School of Economics, University of Toulouse Capitole (TSM-R)
‡Université de Tours and Paris School of Economics
§Toulouse School of Economics, University of Toulouse Capitole (TSM-R)

[1]See chapter 5 of Freixas and Rochet (2008) for an overview.

single offers, crucially affects equilibrium allocations even when there is complete information about the borrowers' characteristics.

Our starting point is that firms do not always have fixed-size investment needs, that they fulfil by borrowing from a single bank at a single time. In practice, a firm can choose variable investment amounts, at different periods, and borrow from several creditors.[2] This in turn can affect competition among banks. Indeed, when firms can borrow from different lenders who do not perfectly coordinate their offers, a creditor can be affected by his borrower's future debt issuances. If the borrower subsequently issues additional debt, defaulting on the initial debt becomes more likely, potentially inducing welfare losses.[3] In this paper, we explore how posting menus including several financial contracts can help creditors to protect themselves from this risk of debt dilution.

We consider a sequential version of the capital market described by Holmstrom and Tirole (1998), in which lenders sequentially make offers to a borrower, who then chooses an unobservable effort. We assume that contracts can be written on all observable variables: in particular, final repayments are a function of aggregate loans chosen by the entrepreneur. Restricting attention to a two-lender case, we consider two scenarios: first, lenders can only make single offers; then, they can post menus of financial contracts. We start by showing that, if the first lender is restricted to single offers, he cannot prevent being undercut by the second one. Thus, the first lender earns zero profit in any pure strategy equilibrium. We next show that the opportunity to choose menus of contracts crucially affects market equilibria. Indeed, by posting a menu of two non degenerate contracts, the first lender is able to prevent any profitable entry from the subsequent lender, therefore earning a monopolistic profit. As a consequence, none of the equilibrium outcomes arising when lenders are restricted to single offers survives the introduction of menus.

The literature on multiple lending has traditionally investigated two sets of issues. A first branch aims at understanding when it can be optimal for a firm to have several rather than a single lender. In these contexts, multiple lending can be an optimal response to the provision of monitoring activities (Winton (1995) and Park (2000)), firms' willingness to default (Bolton and Scharfstein (1996)), or informed lenders' ability to extract rents (Rajan (1992) and Berglof and von Thadden (1994)). Closer to our perspective, a second branch

[2] Multiple banking relationships are pervasive in credit markets: Ongena and Smith (2000) document that the average number of bank relationships is between 5 and 6 in their sample of 1129 European firms. This is also true for small firms: Guiso and Minetti (2010) report that among small and medium sized US firms, half of those that borrow at all have more than one lender. (See also Detragiache et al. (2000)). On consumer credit, Rysman (2007) reports that US households have on average 6.7 payment cards, and the 2013 Federal Reserve Bank of Boston Survey of Consumer Payment Choice finds that about 60% of US consumers have at least 2 credit cards.

[3] As an illustration of this effect, Asquith and Wizman (1990) and Warga and Welch (1993) report that bond value decreases following LBO announcements. Adams et al. (2009) and de Giorgi et al. (2017) document a large increase in the probability to default on existing loans after an (exogenous) approval of a new consumer credit.

considers that multiple lending can arise from lenders' impossibility to fully control borrowers' trades, as in Bizer and DeMarzo (1992), Parlour and Rajan (2001), Bennardo et al. (2015), Kahn and Mookherjee (1998), Bisin and Guaitoli (2004), Brunnermeier and Oehmke (2013), Castiglionesi and Wagner (2012), Castiglionesi et al. (2015), and Donaldson et al. (2017). The above multiple lending models typically assume complete information over the borrower's characteristics, and restrict lenders to post single offers. Our analysis suggests that lenders have instead an incentive to use menus of contracts to discipline their competitors.

From a theoretical standpoint, our findings can be interpreted in the light of the common agency literature, which analyses games in which principals compete through mechanisms in the presence of a single agent. Under complete information about the agent's characteristics, it is an established finding that restricting to single offers involves a loss of generality, i.e. there may exist additional equilibria supported by more sophisticated mechanisms.[4] A second, and possibly more relevant issue is to determine the *robustness* of equilibria in single offers, that is, whether they survive to unilateral deviations towards arbitrary mechanisms. In our model, none of such equilibria survives if a principal deviates to a menu of contracts. This is to be contrasted with the analysis of Peters (2003) for simultaneous games. In his Theorem 1, Peters (2003) shows that in simultaneous common agency games of complete information, every pure strategy equilibrium outcome of the single offer game is also an equilibrium outcome of the menu game. This result is often put forward to justify restricting attention to single offers in economic applications. Our analysis highlights that this does not extend naturally to sequential settings. Indeed, considering the game in which menus are allowed, there exists a best response of the first investor to the equilibrium strategy of his opponent that cannot be characterised through single offers. This suggests that further work is needed to identify robust equilibria in sequential common agency games.[5]

Overall, our results indicate that the financial instruments available to lenders are a key element to take into account when modelling competition in banking.

[4]The result, acknowledged as a failure of the revelation principle in common agency games, has been established by Martimort and Stole (2002), and Peters (2001) for games of simultaneous offers, and by Pavan and Calzolari (2009) for games of sequential offers. These theoretical works have constructed examples of equilibrium outcomes that can be supported by indirect mechanisms, but not by direct ones. Yet little intuition is provided to document the relevance of such equilibria in standard economic settings. We are among the first to show the key role of menus (which are indirect mechanisms) to support monopolistic rents in credit markets subject to moral hazard.

[5]The role of menus in sequential common agency is also considered by Calzolari and Pavan (2008) who show the existence of equilibrium outcomes supported by indirect mechanisms that cannot be reproduced by menus. That is, they provide instances in which indirect mechanisms allow to construct additional threats with respect to those made available by menus. In our context, menus are sufficiently powerful to secure a monopolistic profit to investor 1 who therefore does not need to rely on more sophisticated devices. Furthermore, Calzolari and Pavan (2008) do not investigate whether "simple" equilibrium outcomes survive in the menu game, which is the central issue of our paper.

The paper is organised as follows. Section 2 presents the model, section 3 analyses the game in which banks are restricted to single offers and Section 4 that in which banks post menus of contracts. Proofs are in the Appendix.

2 The model

We refer to the standard capital market model of Holmstrom and Tirole (1998), as reformulated by Attar et al. (2017). A risk-neutral entrepreneur has an endowment $A > 0$ and a variable size project: an investment of $I \in \mathbb{R}_+$ yields a verifiable cash flow GI if the project succeeds, and 0 if it fails. The probability of success π_e depends on the entrepreneur's binary unobservable effort $e = \{L, H\}$, with $\pi_H > \pi_L$. If the entrepreneur selects $e = L$, she receives a private benefit $B \in \mathbb{R}_+$ per unit invested. The project has a positive net present value if and only if the entrepreneur selects $e = H$, that is:

$$\pi_H G > 1 > \pi_L G + B. \tag{1}$$

As in Holmstrom and Tirole (1998), to ensure that the optimal (second-best) investment is finite, we assume that

$$0 < \pi_H G - \frac{\pi_H B}{\Delta\pi} < 1, \tag{2}$$

where $\Delta\pi = \pi_H - \pi_L$.

The entrepreneur is protected by limited liability and can raise funds from two competing investors. If she raises I and invests $I + A$, pays back R in case of success, and 0 in case of failure,[6] her net payoff is $U(I, R, H) = \pi_H\big(G(I + A) - R\big) - A$ if $e = H$, and $U(I, R, L) = \pi_L\big(G(I + A) - R\big) + B(I + A) - A$ if $e = L$. Her reservation utility, $U(0) = (\pi_H G - 1)A$, is strictly positive given (1). The expected profit of investor i when he provides I_i and obtains R_i in case of success, is $V_i = \pi_e R_i - I_i$ with $e \in \{L, H\}$.[7]

Lenders offer menus, that is, sets of financial contracts. Formally, a financial contract for lender i is $C_i = (I_i, R_i(.))$, where I_i is his investment, and $R_i(.) : \mathbb{R}_+ \to \mathbb{R}$ is the repayment he asks for when the project succeeds, as a function of the total cash flow.[8] The competition game unfolds as follows:

a) Investor 1 offers a menu of contracts M_1. Investor 2 observes M_1 and offers M_2.

[6] Given the entrepreneur's limited liability, repayment is always zero if the project fails.

[7] Note that V_i includes investor i's *actual* payment R_i in case of success. This does not preclude default in case of success. For instance, if the entrepreneur borrows an amount I, promises $R_i(I)$ to investor i, but is only able to repay $R_i < R_i(I)$ in case of success, say, because the entrepreneur must also repay $R_j(I)$ to investor j and $R_i(I) + R_j(I) > G(I + A)$, then there is default.

[8] Equivalently, the repayment can be contingent on the aggregate investment.

b) Having observed M_1 and M_2, the entrepreneur chooses one contract in each menu and an effort level.[9]

c) The cash flow is realised and payments are made.

There is perfect information over investors' offers. A pure strategy for investor 1 is a menu M_1, and a pure strategy for investor 2 is a mapping that associates a menu M_2 to every M_1. A pure strategy for the entrepreneur associates to each array of menus the choice of one contract in each menu and an effort level. We assume that each menu only contains a finite number of elements, and we further require each $R_i(.)$ function to be lower semicontinuous on a compact set. These assumptions guarantee that every subgame admits an equilibrium. Throughout the paper we focus on pure strategy subgame perfect equilibria (SPE).

Given limited liability, the entrepreneur can trade contracts which involve conflicting prescriptions, and induce *strategic default*. This is the case if $I = I_1 + I_2$ is such that $R = R_1(I) + R_2(I) > G(I + A)$. Under strategic default, the entrepreneur chooses $e = L$, and obtains $B(I_1 + I_2 + A) - A$. Investors do not receive the contracted-upon repayment (R) but receive a share of the final cash flow proportional to their investment. Because of (1), investors collectively make negative profit. We denote Ψ the set of aggregate investment-repayment pairs $(I, R) \in \mathbb{R}^2_+$ such that the entrepreneur is indifferent between $e = H$ and $e = L$, and call it the incentive frontier. On Ψ, we denote (I^m, R^m) the monopolistic allocation that maximises the investors' profit subject to the entrepreneur's participation.[10] Last, in this model, we assume that

$$\pi_H < 2\pi_L, \tag{3a}$$

$$B > (\pi_H G - 1). \tag{3b}$$

3 The single offers game

We first consider the scenario in which investors can at most post one non-degenerate contract. In this single offers game, investor 2 successfully undercuts any profitable offer of investor 1, as illustrated by the following:

Proposition 1 *Investor 1 earns zero profit in any SPE. In addition, each profit level between zero and the monopolistic one for investor 2 can be supported at equilibrium.*

Intuitively, if investor 1 proposes a contract that grants him a strictly positive payoff, investor 2 can always select an investment I_2 and a repayment function $R_2(.)$ to undercut investor 1's offer. The repayment

[9] Menus always include the null contract $C_0 = (0, 0)$ to incorporate the entrepreneur's participation decisions in a simple way.

[10] Formally, $(I^m, R^m) \in \arg\max_{(I,R)} \pi_H R - I$ s.t. $U(I, R, H) \geq U(I, R, L)$ and $U(I, R, H) \geq U(0)$. In the solution, both constraints are binding and $(I^m, R^m) \in \Psi$.

is designed to discourage the entrepreneur from overborrowing by accepting both loans at a time. Investor 2 then appropriates all available rents providing exclusive financing to the entrepreneur, who optimally chooses $e = H$.

At equilibrium, investor 1 can therefore only be active by trading a zero-profit loan contract. We show that market equilibria can be constructed by letting the entrepreneur accept any zero-profit contract posted by investor 1, and complementing it with an additional offer of investor 2. The corresponding allocations are constrained (second-best) efficient and typically yield a strictly positive profit to investor 2.

4 Menus and sequential contracting

We then consider the general framework described in Section 2. In this menu game, the undercutting of investor 2 may be successfully prevented by the threat that investor 1 includes in his equilibrium menu. This threat takes the form of an additional, large investment contract, designed to be traded by the entrepreneur together with any deviating offer of investor 2. In case the entrepreneur overborrows, she is induced to choose $e = L$, which deters entry and supports a positive profit for investor 1 at equilibrium. Specifically, we have the following:

Proposition 2 *In any SPE, investor 1 earns the monopolistic profit $V_1^* = \pi_H R^m - I^m$, and investor 2 earns zero profit.*

To understand how menus help investor 1 secure a monopolistic profit, suppose first that he only offers the monopolistic contract $C^m = (I^m, R^m)$ (along with C_0). If the entrepreneur accepts C^m and does not borrow from investor 2, she obtains by definition $U(0)$. As Proposition 1 shows, investor 2 can then undercut C^m. Suppose that he offers $C_2' = (I^m, R_2'(I^m) = R^m - \varepsilon)$, with $\varepsilon > 0$ small, with the additional extreme penalty $R_2'(I) = G(I + A)$ if $I \neq I^m$. C_2' undercuts C^m because it allows the entrepreneur to borrow the same amount I^m at a lower rate $R^m - \varepsilon$. The issue is that the entrepreneur may choose to accept both contracts and overborrow. But if she accepts both C^m and C_2', the entrepreneur defaults because of the penalty included in C_2': she then obtains $B(2I^m + A) - A$, which is lower than $U(0)$ by (3a).[11] So the entrepreneur prefers to accept C_2' only, and investor 1 cannot obtain the monopolistic profit with this single offer.

Suppose next that investor 1 includes a second contract in his menu: $\hat{C}^m = (\hat{I}^m, \hat{R}^m(.))$, which is designed, if accepted, to be traded together with C_2'. This additional contract specifies a larger loan $\hat{I}^m >$

[11] To see this point, note that $I^m = A(\frac{G\Delta\pi}{B} - 1)$. The general argument is developed in the proof of Lemma 1 in the appendix.

I^m, and a larger repayment $\hat{R}^m(I_2' + \hat{I}^m) > R^m$. To fix ideas, C^m and $\hat{C}^m$ can be thought of as a loan offer coupled with a credit line. The loan amounts to I^m, and the credit line amounts to $\hat{I}^m - I^m$. Accepting C^m means that the entrepreneur borrows I^m and does not use the credit line; accepting $\hat{C}^m$ means that the entrepreneur draws on her credit line on top of borrowing I^m. We show that it is possible to construct an additional contract $(\hat{I}^m, \hat{R}^m(.))$ that prevents investor 2 from profitably undercutting with C_2': offering C_2' then induces the entrepreneur to overborrow and default. The use of menus therefore allows investor 1 to create a threat of overborrowing, which deters competition.[12]

Precisely, we show in the proof of Proposition 2 that setting the total available credit $\hat{I}^m$ such that

$$U(I^m, R^m, H) = B(\hat{I}^m + A) - A, \tag{4}$$

is sufficient to prevent *all* unilateral deviations of investor 2. Equation (4) requires the entrepreneur to be indifferent between her equilibrium payoff $U(I^m, R^m, H)$, and the maximal available payoff if she chooses $e = L$. In this nonexclusive context, (4) represents an aggregate incentive compatibility constraint for the entrepreneur. This constraint must be binding at equilibrium. Indeed, if $U(I^m, R^m, H) > B(\hat{I}^m + A) - A$, then investor 2 can earn a strictly positive profit by providing a "small" loan to the entrepreneur, without threatening strategic default.

Proposition 2 shows that only one allocation can be supported at equilibrium, with the first investor appropriating the whole surplus. Yet, equilibrium strategies are typically indeterminate. Indeed, there is a wide range of contracts $(\hat{I}^m, \hat{R}^m(.))$ that can be included in the equilibrium menu of investor 1 that successfully prevent investor 2's undercutting. The reason is that $\hat{I}^m$ is pinned down by (4), but almost any schedule $\hat{R}^m(.)$ such that $\hat{R}^m(\hat{I}^m) = G(\hat{I}^m + A)$ provides a credible threat. In this respect, an "extreme" choice could be to set $\hat{R}^m(I) = G(I + A)$ for each $I \geq 0$. As shown in the proof of Proposition 2, however, the result also obtains if $\hat{R}^m(.)$ is a constant equal to $G(\hat{I}^m + A)$: in this case, any deviation of investor 2 induces overborrowing and $e = L$, which makes the deviation not profitable.

While we highlight the importance for lenders to offer more than one contract, our analysis does not derive general predictions on the number of contracts to be issued at equilibrium. However, we show that investor 1 can secure the monopolistic profit by posting only one contract on top of C^m. In this case, our equilibrium menus have a natural interpretation. The menu posted by the first investor includes a contract, not traded at equilibrium, which offers a higher amount of investment than the one chosen at equilibrium.

[12] Note that the threat of overborrowing is of the same nature as the one used by investor 2 with the penalty included in C_2' above.

In practice, the option to obtain a larger loan is embedded in many loan contracts. For instance, when banks initiate a relationship with a customer, they do not set a priori how much they will lend to that customer over the course of their commercial relationship. Instead, their credit committees define an internal credit limit, e.g. a maximum credit exposure, with each client.[13] In our interpretation, $\hat{I}^m$ can be thought of as an internal credit limit, to the extent that it is announced to the borrower. Banks also grant lines of credit to their clients,[14] which secure additional funding at a predetermined rate: the borrower can draw on a credit line without asking for further approval from the bank. Drawing on a credit line is equivalent to accepting the contract $\hat{C}^m$ in our context. There is a large literature explaining why such optional additional loans can efficiently cope with future liquidity needs. Several empirical studies report however that firms more likely to face liquidity needs are less likely to rely on lines of credit, suggesting that liquidity provision might not be the only motivation for the supply of credit lines.[15]

One implication of our model is that one should observe more credit lines that are not drawn upon in situations in which nonexclusivity is a prevalent friction of credit markets. Such a friction is more likely to bind in sectors in which assets are more scalable, or in periods when investment opportunities are more numerous. Going back to the interpretation in terms of menus, another way to illustrate the results of the model would be to see if banks tend to offer a larger variety of credit facilities when they are more likely to face nonexclusivity frictions: to do so, one would need to observe the entire supply of loan offers, and not only those loans that are accepted by firms. This underlines the need for databases on the entire supply of bank loans, similar to those available in the insurance industry.[16]

5 Conclusion

In this paper, we highlight that whether lenders can offer multiple or single contracts affects their ability to control firms' borrowing policy, and the resulting competition outcome. In addition, the equilibrium menus offered by lenders feature similarities with actual lending practices, which sheds light on the strategic role of credit lines: lenders offer loans with credit lines to discourage future debt issuance and protect lenders' rents. This provides new insights on the role of credit lines in capital markets.

[13]See Degryse et al. (2016) for an empirical analysis of the impact of multiple lending on banks' internal credit limit.

[14]Sufi (2009) finds that more than 80% of firm-year observations in his $1996-2003$ sample had a credit line, while Acharya et al. (2014) find that 68% of firm-year observations had one for the period $2002-2011$.

[15]See Sufi (2009) or Acharya et al. (2014). We complement that literature by providing another reason why banks offer lines of credit, namely to increase their market power.

[16]See for instance Cawley and Philipson (1999) who exploit offered price schedules for life insurance from the CompuLife Quote Software.

Appendix

PROOF OF PROPOSITION 1

The proof establishes the following two Lemmas.

Lemma 1 *Investor 1 earns zero profit in any SPE.*

Proof. Suppose, by contradiction, that there is a SPE in which investor 1 earns a positive profit. Denote $(I_i^*, R_i^*(.))$ the contract that the entrepreneur trades with investor $i = 1, 2$ at equilibrium, with $I_1^* > 0$, $I_2^* \geq 0$, and $I^* = I_1^* + I_2^*$. The entrepreneur's equilibrium payoff $U^* = U\big(I_1^* + I_2^*, R_1^*(I^*) + R_2^*(I^*), H\big)$ must be such that

$$U^* \geq \underline{U}(I_1^*, R_1^*(I_1^*), H), \tag{5}$$

where $\underline{U}(I_1^*, R_1^*(I_1^*), H) = \max\{U(0), U\big(I_1^*, R_1^*(I_1^*), H\big), U(I_1^*, (G - \frac{B}{\Delta\pi})(I_1^* + A), H)\big)\}$. Indeed, U^* is necessarily larger than the reservation payoff $U(0)$, and than $U\big(I_1^*, R_1^*(I_1^*), H\big)$, the payoff corresponding to investing $I_1^* + A$ and choosing $e = H$. Also, given that $I_1^* \leq I^*$, and since $e = H$ is chosen at equilibrium, we have

$$U(I_1^*, (G - \frac{B}{\Delta\pi})(I_1^* + A), H) \leq U(I^*, (G - \frac{B}{\Delta\pi})(I^* + A), H) \leq U^*, \tag{6}$$

which implies (5).[17] For a given $(I_1^*, R_1^*(I_1^*))$, denote $(\bar{I}_2, \bar{R}_2) \in \Psi$ the investment-repayment such that $U\big(\bar{I}_2, \bar{R}_2, H\big) = \underline{U}(I_1^*, R_1^*(I_1^*), H)$. Existence and uniqueness of $(\bar{I}_2, \bar{R}_2)$ are guaranteed by the continuity and linearity of $U(I, R, H)$. It also follows from the definition of $\underline{U}(I_1^*, R_1^*(I_1^*), H)$ that $\bar{I}_2 \geq I_1^*$.

We then turn to investors' equilibrium profit. By assumption, $V_1^* = \pi_H R_1^*(I^*) - I_1^* > 0$, and $V_2^* = \pi_H R_2^*(I^*) - I_2^* \geq 0$. Thus, we have $V_2^* < V_1^* + V_2^* \leq \min\{(\pi_H G - 1)I^m, \pi_H \bar{R}_2 - \bar{I}_2\}$, where the first inequality follows from $V_1^* > 0$, and the second one from $U^* \geq \underline{U}(I_1^*, R_1^*(I_1^*)) = U(\bar{I}_2, \bar{R}_2, H)$.

We next show that there exists a pure strategy for investor 2 yielding him a profit $V_2' > V_2^*$. For each offer $(I_1, R_1(.))$ of investor 1, let investor 2 post the single offer $(I_2, R_2^{\varepsilon}(.))$ where I_2 is such that $\frac{\pi_H}{\Delta\pi}B(I_2 + A) - A = \underline{U}(I_1, R_1(I_1), H)$, and where $R_2^{\varepsilon}(I) = G(I + A)$ for $I \neq I_2$, and $R_2^{\varepsilon}(I_2) = (G - \frac{B}{\Delta\pi})(I_2 + A) - \varepsilon$, with $\varepsilon > 0$.[18] Observe that, if investor 1 posts $(I_1^*, R_1^*(.))$, the strategy above

[17] If a investment-repayment pair (I, R) belongs to the incentive frontier Ψ, it must be that $R = (G - \frac{B}{\Delta\pi})(I + A)$, that is, $U(I, (G - \frac{B}{\Delta\pi})(I + A), H) = U(I, (G - \frac{B}{\Delta\pi})(I + A), L)$. It follows that, since $e = H$ in any SPE, the equilibrium repayment $R_1^*(I^*) + R_2^*(I^*) = R^*(I^*)$ must be smaller than $(G - \frac{B}{\Delta\pi})(I^* + A)$, otherwise the borrower would choose $e = L$. This guarantees that the second inequality in (6) is satisfied.

[18] $\underline{U}(I_1, R_1(I_1), H)$ and I_2 are defined in the same way as $\underline{U}(I_1^*, R_1^*(I_1^*), H)$ and $\bar{I}_2$.

prescribes investor 2 to set $I_2 = \bar{I}_2$. We now show that, following this strategy, investor 2 successfully undercuts $(I_1^*, R_1^*(.))$ by inducing the entrepreneur to invest only $I_2 + A$ and to select $e = H$, which yields investor 2 a profit strictly above the equilibrium one.

Indeed, if the entrepreneur selects $e = H$, her (unique) optimal choice is to raise I_2 only: $R_2^{\varepsilon}(.)$ is such that if the entrepreneur raises $I_2 + I_1$, she optimally chooses $e = L$. Furthermore, since $U(\bar{I}_2, \bar{R}_2 - \varepsilon, H) = U(\bar{I}_2, \bar{R}_2, H) + \pi_H \varepsilon > \underline{U}(I_1^*, R_1^*(I_1^*), H)$, the entrepreneur strictly prefers to raise I_2 only, rather than to raise I_1^* only, or to raise nothing.[19] It remains to show that the entrepreneur optimally chooses $e = H$ at the deviation stage. Since $U(\bar{I}_2, \bar{R}_2 - \varepsilon, H) > U(\bar{I}_2, \bar{R}_2 - \varepsilon, L)$ by construction, we only have to consider the alternative situation in which she chooses $e = L$ and raises $I_1^* + \bar{I}_2$. In this case, given $R_2^{\varepsilon}(.)$, the entrepreneur strategically defaults and gets

$$B(I_1^* + \bar{I}_2 + A) - A < B\left(\frac{\pi_L}{\Delta\pi} + 1\right)(\bar{I}_2 + A) - A = \underline{U}(I_1^*, R_1^*(I_1^*)) < U(I_2, R_2^{\varepsilon}(I_2), H), \qquad (7)$$

where the first inequality follows from $(3a)$ and from $\bar{I}_2 + A \geq \bar{I}_2 \geq I_1^*$. Thus, given $(I_1^*, R_1^*(.))$, there is a strategy for investor 2 inducing a unique continuation equilibrium in which the entrepreneur chooses $e = H$ and only raises funds from him. Investor 2's corresponding profit is $V_2' = \min\{(\pi_H G - 1)I^m, \pi_H \bar{R}_2 - \bar{I}_2\} - \pi_H \varepsilon > V_2^*$. This contradicts the assumption that $((I_1^*, R_1^*(I^*), (I_2^*, R_2^*(I^*))$ is an equilibrium allocation.

We next show that any profit between zero and the monopolistic one for investor 2 can be supported at equilibrium.

Lemma 2 *For any $V_2^* \in [0, \pi_H I^m - R^m]$, there exists a SPE in which investor 2's profit is exactly V_2^*.*

Proof. Take any $V_2^* \in [0, \pi_H I^m - R^m]$. Let $(I^*, R^*) \in \Psi$ be the investment-repayment pair such that $\pi_H R^* - I^* = V_2^*$. Let also I_1^* be the investment level such that $U(I_1^*, I_1^*/\pi_H, H) = (\pi_H G - 1)(I_1^* + A) = U(I^*, R^*, H) \geq U(0)$. Consider the following strategies for investors:

1. Investor 1 posts $\{(0,0), (I_1^*, R_1^*)\}$, with $R_1^* = I_1^*/\pi_H$.

2(i). If investor 1 posts $\{(0,0), (I_1^*, R_1^*)\}$, investor 2 posts $\{(0,0), (I_2^*, R_2^*(\cdot))\}$ with

$$I_2^* = I^* - I_1^* \qquad R_2^*(I_1^* + I_2^*) = R^* - R_1^* \equiv R_2^* \qquad R_2^*(I) = G(I + A)\, \forall I \neq I_1^* + I_2^*. \qquad (8)$$

[19]The result obtains since, by construction, $\underline{U}(I_1^*, R_1^*(I_1^*), H) = \max\big(U(I_1, R_1(\cdot), H), U(0)\big)$.

2(ii). If investor 1 does not post $\{(0,0),(I_1^*, R_1^* = I_1^*/\pi_H)\}$, then investor 2 posts $\{(0,0),(I^*, R^*)\}$ where R^* is a constant.

Observe that $U(I^*, R^*, H) = U(I_1^*, R_1^*(I_1^*), H) \geq U(I_1^*, (G - \dfrac{B}{\Delta\pi})(I_1^* + A))$, where the last inequality follows from (2), which guarantees that (I^*, R^*) coincides with the pair $(\bar{I}_2, \bar{R}_2)$ defined in the proof of Lemma 1.

We show that these strategies are part of a SPE in which the entrepreneur invests $I^* = I_1^* + I_2^*$ and earns $U^* = U(I^*, R^*, H)$, with $R^* = R_1^* + R_2^*(I^*)$. Investor 2 earns V_2^*, which is the maximal profit available to investors when the entrepreneur's payoff is fixed to be U^*, given that she chooses $e = H$.

See first that, given the offers above, if the entrepreneur selects $e = H$, then she optimally invests $I_1^* + I_2^*$. Indeed, given $R_2^*(.)$, she optimally chooses $e = L$ whenever she trades with investor 2 only. In addition, since $U^* = U(I_1^*, R_1^*(I_1^*), H)$, the entrepreneur does not strictly prefer to trade with investor 1 only. We next show that $e = H$ is an optimal effort choice on the equilibrium path. Three cases must be considered. If the entrepreneur raises I_1^* only, choosing $e = L$ yields $U(I_1^*, I_1^*/\pi_H, L) < U(I_1^*, I_1^*/\pi_H, H) = U^*$. If the entrepreneur raises $I_1^* + I_2^*$, choosing $e = L$ yields $U(I^*, R^*, L) = U(I^*, R^*, H)$. If she raises I_2^* only, she necessarily defaults and obtains $B(I^* - I_1^* + A) - A$. One therefore has

$$B(I^* - I_1^* + A) - A < B(I^* + A) - A < \frac{\pi_H}{\Delta\pi} B(I^* + A) - A = U(I^*, R^*, H),$$

where the second inequality follows from $\pi_L < \pi_H$, and the equality is implied by $(I^*, R^*) \in \Psi$. This guarantees the optimality of $e = H$ on the equilibrium path.

We next show that none of the investors can profitably deviate. Consider first investor 2. Given $(I_1^*, R_1^*(\cdot))$, he can profitably deviate only by granting the entrepreneur a payoff strictly above $U(I_1^*, R_1^*(I_1^*), H)$. However, given that (I^*, R^*) belongs to the incentive frontier Ψ, any such deviation necessarily yields a profit smaller than V_2^* to investor 2.

Finally, suppose that investor 1 posts a contract $(I_1, R_1(\cdot)) \neq (I_1^*, R_1^*(\cdot))$; then, recalling that $(I^*, R^*) = (\bar{I}_2, \bar{R}_2)$, the arguments developed in Lemma 1 can be used to show that investor 2 undercuts investor 1 granting the entrepreneur a payoff of $U(I_1, R_1(I_1), H)$. The equilibrium strategy of the entrepreneur can be constructed by letting her trade with investor 2 only, whenever she is indifferent between several options. This in turn blocks any profitable deviation of investor 1. ■

PROOF OF PROPOSITION 2

Consider the menus (M_1^*, M_2^*), with $M_1^* = \{(0,0), (I^m, R^m(.)), (\hat{I}^m, \hat{R}^m(.))\}$, and $M_2^* = \{(0,0)\}$. Let $R^m(I) = R^m$ for each $I \geq 0$, and recall that $I^m = A(\frac{G\Delta\pi}{B} - 1)$, $R^m = GI^m$ and $U(I^m, R^m, H) = U(0)$. In addition, let $\hat{I}^m$ be such such that $U(I^m, R^m, H) = B(\hat{I}^m + A) - A$, and $\hat{R}^m(I) = G(\hat{I}^m + A)$ for each $I \geq 0$. That is, $\hat{I}^m$ is such that the entrepreneur obtains the same utility if she borrows I^m, selects $e = H$, and obtains $G(I^m + A) - R^m$ in case of success, as if she borrows $\hat{I}^m$, selects $e = L$, and obtains only his private benefit $B(\hat{I}^m + A)$. Observe that both schedules $R^m(.)$ and $\hat{R}^m(.)$ are constant over the aggregate investment I.

We now show that (M_1^*, M_2^*) are part of a SPE in which the entrepreneur raises I^m from investor 1 who therefore earns a monopolistic profit. Given (M_1^*, M_2^*), it is a best reply for the entrepreneur to select the contract (I^m, R^m) in M_1^* and to choose $e = H$. That is, she trades the monopolistic allocation (I^m, R^m). Since investor 1 earns a monopolistic profit, he has no incentive to deviate. It is then sufficient to show that there is no profitable deviation for investor 2 either. Any such deviation can, without loss of generality, be represented by a simple menu $M_2' = \{(I_2', R_2'(.)), (0,0)\}$, with $I_2' > 0$, which includes only one non degenerate contract.[20] Following the deviation, two situations must be considered:

1. The deviation is designed so that the entrepreneur borrows from both investors: she chooses $(I_2', R_2'(.))$ in M_2', and (I^m, R^m) in M_1^*, and obtains the allocation $\left(I^m + I_2', R^m + R_2'(I^m + I_2')\right)$. A necessary condition for this deviation to be profitable, is that $\dfrac{R_2'(I^m + I_2')}{I_2'} > \frac{1}{\pi_H}$ which, given (2) and since $R^m = GI^m = \left(G - \dfrac{B}{\Delta\pi}\right)(I^m + A)$, implies that

$$R^m + R_2'(I^m + I_2') > \left(G - \frac{B}{\Delta\pi}\right)(I^m + A + I_2') \tag{9}$$

That is, when choosing $\left(I^m + I_2', R^m + R_2'(I^m + I_2')\right)$, the entrepreneur selects $e = L$. See that this makes the deviation non profitable. Indeed, for the deviation to be profitable given $e = L$, one should have $\dfrac{R_2'(I^m + I_2')}{I_2'} > \frac{1}{\pi_L}$. Then, given (1), we get $\dfrac{R_2'(I^m + I_2')}{I_2'} > G + \dfrac{B}{\pi_L}$, which implies that

$$U\left(I^m + I_2', R^m + R_2'(I^m + I_2'), L\right) < U\left(I^m, R^m, L\right) = U^*.$$

This in turn guarantees that the deviating contract would not be chosen at the deviation stage. Similarly,

[20]This is a direct implication of the robustness result derived in Theorem 1 of Peters (2003). Indeed, given M_1^*, any deviation to a more sophisticated menu of contracts $M_2^{''}$ can be reproduced by directly offering only one non degenerate contract which corresponds to the optimal choice of the entrepreneur in $M_2^{''}$.

since $\hat{R}^m = G(\hat{I}^m + A)$, the entrepreneur selects $e = L$ if she accepts $(\hat{I}^m, \hat{R}^m)$ together with $(I_2', R_2'(.))$, rendering the original deviation non profitable.

2. The deviation is designed so that the entrepreneur borrows from investor 2 only. The borrower chooses $(I_2', R_2'(.))$ in M_2', and $(0, 0)$ in M_1^*. In this case, she earns $U(I_2', R_2'(I_2'), H)$ by choosing $e = H$. Yet, by choosing $e = L$, she obtains $B(\hat{I}^m + I_2' + A) - A$. We can hence write

$$\begin{aligned} U(I_2', R_2'(I_2'), H) - (B(\hat{I}^m + I_2' + A) - A) &< U(0) + (\pi_H G - 1)I_2' - (B(\hat{I}^m + I_2' + A) - A) \\ &= (\pi_H G - 1 - B)I_2' < 0. \end{aligned} \tag{10}$$

The first inequality follows from the fact that, for the deviation to be profitable, it must be that $\pi_H R_2'(I_2') - I_2' > 0$, which implies that $U(I_2', R_2'(I_2'), H) < U(I_2', \frac{1}{\pi_H} I_2', H) = U(0) + (\pi_H G - 1)I_2'$. The equality comes from the fact that $U(0) = U(I^m, R^m, H) = B(\hat{I}^m + A) - A$. This highlights the role of $\hat{I}^m$: the second contract offered by investor 1 prevents investor 2 from successfully undercutting investor 1's monopoly offer. The last inequality follows from (3b) and from $I_2' > 0$, and shows that $e = H$ is not an optimal effort choice which implies that investor 2's profit is negative. This finally establishes that the monopolistic allocation for investor 1 is supported at equilibrium, and concludes the proof. ■

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