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Andrea Attar, Eloisa Campioni and Gwenaël Piaser

Toulouse School of Economics

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Andrea Attar* Eloisa Campioni† Gwenaël Piaser ‡

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Abstract

We study competing mechanism games in which principals simultaneously design contracts to deal with several agents. We show that principals can profit from privately communicating with agents by generating incomplete information in the continuation game they play. Specifically, we construct an example of a complete information game in which none of the (multiple) equilibria in Yamashita (2010) survives against unilateral deviations to mechanisms involving private communication. This also contrasts with the robustness result established by Han (2007). The role of private communication we document may call for extending the standard construction of Epstein and Peters (1999) to incorporate this additional element.

*Toulouse School of Economics and University of Rome Tor Vergata, e-mail: andrea.attar@tse-fr.eu

†University of Rome Tor Vergata, e-mail: eloisa.campioni@uniroma2.it

‡Ipag Business School Paris, e-mail: piaser@gmail.com

1 Introduction

We study competing mechanism games: principals compete through mechanisms in the presence of multiple agents. Such a strategic scenario has become a reference framework to model competition in a large number of market settings.[1]

As first pointed out by McAfee (1993) and Peck (1997), the equilibrium allocations derived in these contexts crucially depend on the set of mechanisms made available to principals. For example, letting agents communicate to principals additional information on top of their "exogenous" type typically supports additional allocations at equilibrium.[2] This raises the issue of identifying a class of mechanisms allowing to reveal all information available to agents, including that generated by the mechanisms posted by principals. In an important contribution, Epstein and Peters (1999) have introduced a communication device that incorporates this market information. In their general construction, a mechanism for each principal requires each agent to send messages from a *universal* type space. The corresponding set of equilibrium allocations may be very large: Yamashita (2010) shows that a subset of such mechanisms, i.e. the *recommendation* mechanisms, is sufficient to derive a folk-theorem-like result. In a recommendation mechanism, a principal commits to post a particular direct mechanism if all but one agent recommend him to do so. Recommendation mechanisms hence allow to construct a flexible system of punishments: following a unilateral deviation of a given principal, agents can coordinate to select, amongst his opponents' decisions, those inducing the most severe punishment to the deviator.[3] As a result, any incentive compatible allocation yielding each principal a payoff above a given threshold can be supported at equilibrium if there are at least three agents.

The present work reconsiders this indeterminacy result from a traditional mechanism design perspective. That is, we evaluate the strategic role of a principal privately communicating with agents as in the canonical Myerson (1982) analysis. Standard approaches to model competing mechanisms disregard this possibility, by restricting each principal to communicate by *publicly* committing to take some decisions given the messages that agents *privately* send him. Yet, to the extent that he cannot directly contract on his opponents' mechanisms, a single principal may in principle gain by sending private signals to agents so to correlate their behaviors with all principals' decisions.

Specifically, we focus on a simple class of *complete information* competing mechanism

[1]See Peters (2014) for a recent survey.

[2]This result, which has been documented in single agent contexts by Martimort and Stole (2002) and Peters (2001), is often acknowledged as a failure of the revelation principle in games with multiple principals.

[3]This logic extends to multiple agency the role played by menus and latent contracts under common agency (Martimort and Stole, 2002; Peters, 2001).

games, in which agents' actions are contractible. In such a scenario, we construct an example that explicitly characterizes the set of (multiple) equilibrium allocations supportable by recommendation mechanisms. In a next step, we show that none of the corresponding equilibria survives against a deviation to a well chosen mechanism in which the principal correlates his decisions with the private signals he sends to agents. Since signals are private, the deviation induces an incomplete information game between agents. In the example, only one agent becomes perfectly informed of the decision selected by the deviating principal, while the others remain uninformed. This difference crucially affects the unique continuation equilibrium of the agents' effort game, generating a joint probability distribution over agents' efforts and principals' decisions that cannot be reproduced without private signals.

The result suggests that the equilibrium allocations characterized by allowing only agents to privately communicate through (possibly) large message spaces may not be robust against unilateral deviations toward mechanisms incorporating principals' private communication.

In this respect, we provide novel insights for applications. Economic models of competing mechanisms under complete information restrict attention to situations in which only pay-for-effort contracts are posted. That is, a principal does not ask for messages, and takes decisions contingent on agents' observable efforts which, in this context, identifies a direct mechanism. This is, for instance, the approach followed by Prat and Rustichini (2003) to model lobbying and vertical restraints among market interactions. An implication of our analysis is that this restriction is problematic once principals are allowed to design more sophisticated mechanisms. This stands in contrast with the result of Han (2007) who shows the robustness of equilibria supported by pay-for-effort contracts against unilateral deviations to mechanisms that only allow for agents' private communication.

A crucial feature for our result is the presence of many agents. In single-agent environments, any correlation between the agent's actions and his opponents' decisions, induced by any principal's deviation to a mechanism with signals, can be reproduced using mechanisms without signals.

Several folk-theorem results have been recently established in the competing mechanism literature. Generalizing the approach of Yamashita (2010), Peters and Troncoso-Valverde (2013) show that, under complete information, the outcomes supported at equilibrium by recommendation mechanisms correspond to those implementable in the mechanism design setting of Myerson (1979). Their analysis is casted in an abstract setting in which every player has commitment power, and a player's decisions are only enforced if all other players send him the same message. We consider, instead, the simpler context in which only a given subset of players is able to commit, as postulated in most economic applications, and analyze the robustness of pure strategy equilibria to the introduction of private communication for principals. A different strategy is followed by Kalai et al. (2010), Peters and Szentes (2012), Peters (2015), and Szentes (2015) who provide different attempts at modeling contractible contracts. These works show that by posting contracts that *directly* refer to each other, instead of asking agents to report their observed mechanisms, a principal may successfully deter his opponents deviations. In these settings, a folk theorem obtains even if no communication takes place after mechanisms are posted, which severely limits the strategic role of agents and therefore the power of the deviations we exploit.

This note is organized as follows: Section 2 introduces the competing mechanism model, Section 3 presents our example, Section 4 provides a general discussion, and Section 5 concludes.

2 The model

We study competing mechanism games of complete information in which the agents' actions are observable. There is a finite number $J \geq 2$ of principals dealing with a finite number I of agents. Each agent $i = 1, 2, \cdots, I$ takes an effort e^i from a finite set E^i, with $e = \left(e^1, \ldots, e^I\right) \in E = \bigtimes_{i=1}^{I} E^i$. Let Y_j be the finite set of actions available to principal j with $y_j \in Y_j$ a generic element of that set, and $Y = \bigtimes_{j=1}^{J} Y_j$. The functions $u^i : E \times Y \to \mathbb{R}$ and $v_j : E \times Y \to \mathbb{R}$ denote the payoff to agent i and to principal j, respectively.

Agents' efforts are observable, so each principal j can choose an action y_j contingent on the array of efforts e. We denote $\alpha_j : E^1 \times ... \times E^I \longrightarrow \Delta(Y_j)$, a pay-for-effort contract for principal j, with $\Delta(Y_j)$ being the set of probability distributions over Y_j. A pay-for-effort contract hence specifies a (possibly stochastic) action for every array of observed efforts. We let $\mathcal{A}_j$ be the set of pay-for-effort contracts for principal j with $\alpha_j \in \mathcal{A}_j$, and we denote $\mathcal{A} = \bigtimes_{j=1}^{J} \mathcal{A}_j$.

2.1 Competing mechanism games: equilibrium

We now describe a standard competing mechanism framework for complete information settings.[4] Communication occurs via the private messages sent by agents to principals, and the public mechanisms principals commit to. Specifically, we let $m_j^i \in M_j^i$ be a private message sent by agent i to principal j. A mechanism for principal j is the mapping $\gamma_j : M_j \to \mathcal{A}_j$, in which $M_j = \bigtimes_{i=1}^{I} M_j^i$ is the set of message profiles that principal j receives from

[4] We follow Epstein and Peters (1999) and Han (2007).

agents, with typical element $m_j = \left(m_j^1, \ldots, m_j^I\right)$. We denote $\Gamma_j^{M_j}$ the set of mechanisms available to principal j, and we let $\Gamma^M = \times_{j=1}^{J} \Gamma_j^{M_j}$.

The competing mechanism game unfolds as follows. First, principals simultaneously post mechanisms. Then, agents simultaneously take effort and communication decisions, and payoffs are determined. We let $\mu^i : \Gamma^M \to \Delta(M^i)$ be the message strategy of agent i, with $M^i = \times_{j=1}^{J} M_j^i$, and $\eta^i : \Gamma^M \times M^i \to \Delta(E^i)$ be her effort strategy. We denote $\beta^i = (\mu^i, \eta^i)$ a strategy for agent i, and $\beta = (\beta^1, \ldots, \beta^I)$ a profile of strategies. A pure strategy for principal j is a mechanism $\gamma_j \in \Gamma_j^{M_j}$. We let $U^i(\gamma_j, \gamma_{-j}, \beta)$ and $V_j(\gamma_j, \gamma_{-j}, \beta)$ be the corresponding expected utilities for agent i and principal j, respectively. We denote G^M the game in which agents send messages to principals through the sets $(M^1, ..., M^I)$ and principals post mechanisms $\gamma = (\gamma_j, \gamma_{-j}) \in \Gamma^M$.

We focus here on the subgame perfect Nash equilibria (SPNE) of G^M in which principals play pure strategies. We say that the agents' strategies (β^i, β^{-i}) constitute a continuation equilibrium relative to Γ^M if for every i and for every $\gamma \in \Gamma^M$, β^i maximizes $U^i(\gamma, \beta^i, \beta^{-i})$. The strategies (γ, β) constitute a SPNE in G^M if β is a continuation equilibrium and if, given γ_{-j} and β, for every $j = 1, \cdots J$: $\gamma_j \in \underset{\gamma_j' \in \Gamma_j^{M_j}}{argmax}\, V_j\left(\gamma_j', \gamma_{-j}, \beta\right)$.

As first documented by Epstein and Peters (1999), the set of equilibrium outcomes may crucially depend on the characteristics of the message spaces $(M^1, ..., M^I)$. In this complete information scenario, we focus on two situations.

1. Each message space M_j^i is a singleton. That is, principals are only allowed to post pay-for-effort contracts, specifying a (possibly stochastic) decision for every array of observed efforts. In this setting, a pay-for-effort contract coincides with a *direct* mechanism. We denote G^D the direct mechanisms game. As we discuss in Section 4, this game is relevant for most economic applications.

2. Each message space M_j^i coincides with the set of pay-for-effort contracts $\mathcal{A}_j$. That is, the message spaces are sufficiently rich to allow every agent to communicate a direct mechanism to each principal j. We denote this game $G^{\mathcal{A}}$. Given these message spaces, Yamashita (2010) shows that a folk-theorem-like result can established.[5]

2.2 Robustness: principals' private communication

Our aim is to evaluate whether the equilibria of a given game G^M survive in a more general game in which principals can also privately communicate to agents. Private communication is

[5] Yamashita (2010)'s result is provided in a general incomplete information scenario. We revisit his construction in Section 3.

modeled following the standard approach developed in Myerson (1982). That is, we consider the following enlarged game. Given the array of messages $m_j \in M_j$ received from the agents, each principal j can now send a private signal $s_j^i \in S_j^i$ to each agent i. A mechanism for principal j is hence the mapping $\hat{\gamma}_j : M_j \to \Delta(\mathcal{A}_j \times S_j)$, in which $S_j = \times_{i=1}^{I} S_j^i$. Thus, given the messages m_j he receives, and the efforts e chosen by agents, the mechanism $\hat{\gamma}_j$ determines a joint probability distribution over his decisions in Y_j and his signals in S_j. As in Myerson (1982), we postulate that agent i privately observes the realization of each signal s_j^i and can therefore revise her prior information accordingly. Let $\Gamma_j^{M_j S_j}$ be the set of mechanisms available to principal j, and denote $\Gamma^{MS} = \times_{j=1}^{J} \Gamma_j^{M_j S_j}$. Throughout the paper, we refer to $\hat{\gamma}_j \in \Gamma_j^{M_j S_j}$ as to a mechanism with signals.

Mechanisms are publicly observed, but the message from agent i to principal j and the signal from principal j to agent i are only observed by i and j. Since signals are private, a principal can generate incomplete information among agents at the stage of choosing their efforts. There are two stages at which agent i moves in the game. First, having observed the mechanisms $\hat{\gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_J)$, she sends an array of messages $m^i = (m_1^i, \ldots, m_J^i)$ to the principals. Second, having observed her private signals $s^i = (s_1^i, \ldots, s_J^i)$, each agent i then chooses an effort $e^i \in E^i$. We take $\hat{\mu}^i : \Gamma^{MS} \to \Delta(M^i)$ to be the message strategy of agent i, and $\hat{\eta}^i : \Gamma^{MS} \times M^i \times S^i \to \Delta(E^i)$ to be her strategy in the effort game, with $S^i = \times_{j=1}^{J} S_j^i$. We correspondingly let $\hat{\beta}^i = (\hat{\mu}^i, \hat{\eta}^i)$ be a strategy for agent i. We denote G^{MS} the game in which agents send messages to principals through the sets $(M^1, ..., M^I)$, principals post mechanisms $\hat{\gamma} \in \Gamma^{MS}$ and send signals to agents through the sets $(S_1, ..., S_J)$.

Observe that, for each given $(M^1, ..., M^I)$, the game G^M can be interpreted as a degenerate G^{MS} game in which each S_j^i set is a singleton. In particular, we can write $\Gamma_j^{M_j} \subseteq \Gamma_j^{M_j S_j}$ for each j and S_j, and specify any mechanism γ_j as a degenerate $\hat{\gamma}_j$ in which, for every pair (m_j, e), the probability distribution over Y_j coincides with $\gamma_j(m_j, e)$ for each $s_j^i \in S_j^i$.[6]

Following Peters (2001), we say that an equilibrium (γ, β) of G^M is robust if, in every enlarged game, the original equilibrium survives to a unilateral deviation of a principal toward a more sophisticated mechanism. That is, if there exists *at least one* continuation equilibrium of the enlarged game which makes the deviation unprofitable.[7]

[6] A similar reasoning is used by Peters (2001) and Han (2007) to specify a direct mechanism as a degenerate indirect one.

[7] See Peters (2001), p. 1364, for a formal definition.

3 The role of private communication: an example

This section establishes a non-robustness result. We construct a set of equilibrium outcomes of a given game G^M and show that none of them survives if a principal can deviate to mechanisms with signals, that allow him to privately communicate with agents. More formally, we show that there exist equilibria in the game G^M that cannot be supported in an enlarged game with signals G^{MS}.

The result is important from the viewpoint of economic applications, which typically do not consider this channel of communication. In particular, we revisit the folk-theorem result established by Yamashita (2010), which is central to the recent developments of the competing mechanisms literature.

We therefore consider the game $G^{\mathcal{A}}$, as defined in Section 2, in which $M_j^i = \mathcal{A}_j$ for each $i = 1, \ldots, I$ and $j = 1, \ldots, J$. In $G^{\mathcal{A}}$, every agent i communicates a pay-for-effort contract, i.e. a direct mechanism, to each principal j. Following Yamashita (2010), we say that $\gamma_j^R : (\mathcal{A}_j)^I \times E \longrightarrow \Delta(Y_j)$ is a *recommendation* mechanisms for principal j if:

$$\gamma_j^R(m_j^i, \ldots, m_j^I) = \begin{cases} \alpha_j & \text{if } \left| \left\{ i : m_j^i = \alpha_j \right\} \right| \geq I - 1 \\ \text{any } \bar{\alpha}_j \in \mathcal{A}_j & \text{otherwise.} \end{cases}$$

A recommendation mechanism can be understood as having agents suggest to a principal the direct mechanism to be selected. The principal commits to follow any such recommendation if it is sent by at least $I - 1$ agents. Theorem 1 in Yamashita (2010) shows that, if principals post recommendation mechanisms, then every incentive compatible allocation yielding each principal a payoff above a given threshold can be supported at equilibrium.[8] For every unilateral deviation of a principal, the recommendation mechanisms of his opponents permit to the agents to implement a corresponding punishment to the deviator, which is key to compute the threshold payoff. Although Yamashita's analysis is developed in a general incomplete information setting, with agents taking no actions, the result naturally applies to the complete information framework we consider. Yet, a fundamental question is whether the corresponding equilibria survive the introduction of principals' private communication via mechanisms with signals.

To analyze this issue, we construct an example of a complete information $G^{\mathcal{A}}$ game,

[8] In the incomplete information setting of Yamashita (2010), an incentive compatible allocation is an array of principals' decisions induced by a continuation equilibrium in which agents truthfully report their (exogenous) private information given the posted mechanisms.

in which recommendation mechanisms support multiple equilibrium allocations, and then show that none of these equilibria survives against a principal's unilateral deviations to mechanisms with signals.

Example. Consider a complete information setting with five players: two principals, $P1$ and $P2$, and three agents, $A1$, $A2$ and $A3$, who take contractible actions in the sets $E^1 = E^2 = \{a, b\}$ and $E^3 = \{a\}$. Let $P1$'s decision set be $Y_1 = \{y_{11}, y_{12}\}$, and $P2$'s one be $Y_2 = \{y_{21}, y_{22}\}$. The payoffs of the game are represented in the matrix below, in which the first two numbers in each cell denote the payoffs to $P1$ and $P2$, respectively, and the remaining three numbers denote the payoffs to $A1$, $A2$ and $A3$.

		y_{21}			y_{22}	
		a	b		a	b
y_{11}	a	$(2, 95, 10, 5, 1)$	$(2, \zeta, 3/2, 8, 1)$	a	$(2, \zeta, -1/10, 0, 1)$	$(2, \zeta, -1/10, 8, 1)$
	b	$(2, 0, 0, 0, 1)$	$(2, \zeta, 0, 10, 1)$	b	$(2, -1, 5, 5, 1)$	$(2, \zeta, 1, -1, 1)$
		a	b		a	b
y_{12}	a	$(2, 95, 10, 5, 1)$	$(2, \zeta, 3/2, 8, 1)$	a	$(2, \zeta, -1, 4, 1)$	$(2, \zeta, -1, 8, 1)$
	b	$(2, 5, 5, 5, 1)$	$(2, \zeta, -1, 4, 1)$	b	$(2, -1, 0, 0, 1)$	$(2, \zeta, 0, -1, 1)$

Table 1: The full payoff matrix of the game

In Table 1, the payoffs to $P1$ and $A3$ are constantly equal to 2 and to 1 respectively, we let $\zeta < -1$ be a loss for $P2$.[9] For the sake of simplicity, in the rest of the discussion we will use the reduced matrix below, in which we only report the payoffs of $P2$, $A1$ and $A2$, respectively.

		y_{21}			y_{22}	
		a	b		a	b
y_{11}	a	$(95, 10, 5)$	$(\zeta, 3/2, 8)$	a	$(\zeta, -1/10, 0)$	$(\zeta, -1/10, 8)$
	b	$(0, 0, 0)$	$(\zeta, 0, 10)$	b	$(-1, 5, 5)$	$(\zeta, 1, -1)$
		a	b		a	b
y_{12}	a	$(95, 10, 5)$	$(\zeta, 3/2, 8)$	a	$(\zeta, -1, 4)$	$(\zeta, -1, 8)$
	b	$(5, 5, 5)$	$(\zeta, -1, 4)$	b	$(-1, 0, 0)$	$(\zeta, 0, -1)$

Table 2: The reduced payoff matrix

In this example, a pay-for-effort contract can be conveniently represented by means of simple binary distributions over principals decisions. Specifically, we let $\delta_e = prob(y_{11}|e)$ be

[9]The value of ζ is used to identify the threshold value for $P2$ along the lines of Yamashita (2010). See Proposition 1 and, specifically, equation (1) for an explicit characterization of the values of ζ which sustain the result.

the probability with which $P1$ plays y_{11} if the effort array $e \in \{a,b\}^2$ is observed. That is, we write $\alpha_1 = (\delta_{aa}, \delta_{ab}, \delta_{ba}, \delta_{bb})$. Similarly, we let $\sigma_e = prob(y_{21}|e)$ be the probability with which $P2$ plays y_{21} if the effort array $e \in \{a,b\}^2$ is observed. That is, we write $\alpha_2 = (\sigma_{aa}, \sigma_{ab}, \sigma_{ba}, \sigma_{bb})$.

A (stochastic) allocation induced by the direct mechanisms (α_1, α_2) and the agents continuation strategies (η^1, η^2, η^3) induced by (α_1, α_2) is the array

$$z = \left(\Sigma_e \delta_e \eta^1(e^1|\alpha_1,\alpha_2)\eta^2(e^2|\alpha_1,\alpha_2), \Sigma_e \sigma_e \eta^1(e^1|\alpha_1,\alpha_2)\eta^2(e^2|\alpha_1,\alpha_2), \eta^1(.|\alpha_1,\alpha_2), \eta^2(.|\alpha_1,\alpha_2), \{a\} \right),$$

in which $\eta^i(e^i|\alpha_1,\alpha_2)$ is the probability with which agent i plays e^i given (α_1,α_2). Thus, for a given $e = (e^1, e^2)$, $\delta_e \eta^1(e^1|\alpha_1,\alpha_2)\eta^2(e^2|\alpha_1,\alpha_2)$ is the probability of y_{11} given the mechanisms (α_1,α_2) and the continuation strategies (η^1,η^2,η^3), and $\sigma_e \eta^1(e^1|\alpha_1,\alpha_2)\eta^2(e^2|\alpha_1,\alpha_2)$ is the corresponding probability of y_{21}. Eventually, $\eta^i(.|\alpha_1,\alpha_2)$ is the probability distribution over e^i for agent $i = 1, 2$.

We hence say that a (stochastic) allocation z is *incentive feasible* if (η^1,η^2,η^3) is an equilibrium in the continuation game induced by (α_1,α_2).[10] We first point out two features of the set of incentive feasible allocations Z^{IF} that are key to our analysis.

Remark 1 *Any allocation supported in an equilibrium of $G^{\mathcal{A}}$ is incentive feasible.*

Remark 2 *Z^{IF} is non-empty. In particular, it includes the deterministic allocation $z_1 = (y_{12}, y_{21}, a, b, a)$. Indeed, if P1 commits to play $\{y_{12}\}$ regardless of the agents' efforts, and P2 commits to $\{y_{22}\}$, then it is a continuation equilibrium for A1 to play $\{a\}$, for A2 to play $\{b\}$ and for A3 to play $\{a\}$. This implements the payoffs $(2, \zeta, 3/2, 8, 1)$. A similar reasoning guarantees that Z^{IF} also includes the allocation $z_2 = (y_{11}, y_{22}, b, a, a)$, which implements the payoff profile $(2, -1, 5, 5, 1)$. Finally, it also includes the allocation $z_3 = (y_{12}, y_{21}, b, a, a)$, sustained by pay-for-effort contracts such that $P1$ who commits to play y_{12} when observing the efforts (b, a, a), and y_{11} otherwise; and $P2$ who commits to play y_{21} when observing the efforts (b, a, a), and y_{22} otherwise, so that (b, a, a) is a continuation equilibrium for the agents. This allocation implements the payoff profile $(2, 5, 5, 5, 1)$.*

Remark 1, which directly follows from the definition of incentive feasibility, parallels Lemma 1 in Yamashita (2010). The multiplicity of incentive feasible allocations documented

[10] Observe that Yamashita (2010) restricts attention to *deterministic* allocations. That is, agents play pure strategies in every continuation equilibrium, and principals cannot randomize over their decisions. Under this restriction existence of a continuation equilibrium is not guaranteed. We enlarge the analysis to random behaviors, therefore allowing for mixed strategy equilibria in each continuation game played by the agents.

in Remark 2 suggests the possibility to derive a folk-theorem-like result in the Example. This is established in the following analysis.

Proposition 1 *Every incentive feasible allocation yielding at least* -1 *to P2 can be sustained in an equilibrium of the game* $G^{\mathcal{A}}$ *in which principals play pure strategies.*

Proof. Let each principal $j = \{1, 2\}$ use the recommendation mechanism γ_j^R defined above. To show that any incentive feasible allocation yielding each principal a payoff above the threshold level -1 can be sustained at equilibrium, we first establish the following Lemma.

Lemma 1 *If P1 plays a given recommendation mechanism, then, for every pay-for-effort contract posted by P2, there exists an equilibrium in the agents' game yielding P2 a payoff lower than (or equal to)* -1.

Proof of Lemma 1. Let γ_1^R be the recommendation mechanism played by P1. For each pay-for effort contract $\alpha_2 = (\sigma_{aa}, \sigma_{ab}, \sigma_{ba}, \sigma_{bb})$ posted by $P2$, agents play a continuation game over the messages to send to P1 and over their efforts.

In this game, we construct the agents' equilibrium message strategy in such a way that they select a pay-for-effort contract (direct mechanism) $\alpha_1 \in \mathcal{A}_1$ such that $\delta_{ab} = \delta_{ba} = 1$ and $\delta_{bb} = \sigma_{bb}$ in the recommendation mechanism of P1. Given these messages, and recalling that $A3$ can only take one action, agents $A1$ and $A2$ play the effort game in Table 3.

	a	b
a	$11\sigma_{aa} + \frac{9}{10}\sigma_{ba}(1-\sigma_{aa}) - 1, \sigma_{aa} + 4(1 - \delta_{aa} + \delta_{aa}\sigma_{aa})$	$\frac{8}{5}\sigma_{ab} - \frac{1}{10}, 8$
b	$5(1-\sigma_{ba}), 5(1-\sigma_{ba})$	$0, \sigma_{bb}(6\sigma_{bb}+5) - 1$

Table 3: The effort game played by $A1$ and $A2$

Observe that in this game there is no pure strategy equilibrium in which $A1$ and $A2$ play (a, a). Indeed, if A1 plays a, A2 will choose b because $8 > \sigma_{aa} + 4(1 - \delta_{aa} + \delta_{aa}\sigma_{aa})$ for every $(\sigma_{aa}, \delta_{aa})$ pair.

Thus, when A2 chooses b, A1's payoff by choosing a is $8/5\sigma_{ab} - 1/10$ which is positive if $\sigma_{ab} \geq 1/16$. In this case, (a, b, a) is an equilibrium in the agents' game and $P2$'s payoff is equal to $\zeta < -1$.

Alternatively, if $\sigma_{ab} < 1/16$, $A1$'s payoff from choosing a when $A2$ chooses b is strictly lower than zero. In this case, for every $(\sigma_{ba}, \sigma_{bb})$ such that $\sigma_{bb}(6\sigma_{bb} + 5) - 1 \geq 5(1 - \sigma_{ba})$, (b, b, a) is an equilibrium of the agents' game, and $P2$'s payoff is again equal to $\zeta < -1$.

Consider now any $(\sigma_{ba}, \sigma_{bb})$ such that $\sigma_{bb}(6\sigma_{bb} + 5) - 1 < 5(1 - \sigma_{ba})$. If $A1$ plays b, it is then optimal for $A2$ respond with a. Now, for each $(\sigma_{aa}, \sigma_{ba})$ such that $11\sigma_{aa} + 9/10(1 -$

$\sigma_{aa}) - 1 \leq 5(1-\sigma_{ba})$, (b,a,a) is an equilibrium in the agents' game for each $\delta_{aa} \in [0,1]$. The corresponding payoff for $P2$ is $-(1-\sigma_{ba}) \leq -1$.

Finally, for each $(\sigma_{aa}, \sigma_{ba})$ such that $11\sigma_{aa} + 9/10\delta_{aa}(1-\sigma_{aa}) - 1 > 5$, we set $\delta_{aa} = 1$. It follows that the agents' game has a mixed strategy equilibrium in which $A1$ and $A2$ randomize over (a,b), while $A3$ plays $\{a\}$. For any such $(\sigma_{aa}, ..., \sigma_{bb})$, and given the corresponding $(\delta_{aa}, ..., \delta_{bb})$, we let ϕ be the probability with which A1 plays a, and τ that with which A2 plays a. The payoff to P2 in this completely mixed equilibrium is $\phi\tau(10\sigma_{aa} + \zeta(1-\sigma_{aa})) - (1-\phi)\tau(1-\sigma_{ba}) + (1-\tau)\zeta$. We choose ζ to guarantee that this payoff is smaller than -1. This is the case if

$$\zeta < \bar{\zeta} \equiv \min_{\sigma_{aa},\sigma_{ba}} \frac{(1-\phi)\tau(1-\sigma_{ba}) - 1 - \phi\tau 10\sigma_{aa}}{\phi\tau(1-\sigma_{aa}) + (1-\tau)}. \tag{1}$$

Observe that the fraction that defines $\bar{\zeta}$ takes finite values for every $(\sigma_{aa}, \sigma_{ba})$, and given the induced mixed strategy equilibrium (ϕ, τ). Indeed, if $\sigma_{aa} = 1$, the mixed strategy equilibrium degenerates to $(\phi = 1, \tau = 0)$. It follows that $\bar{\zeta}$ is well defined.

To complete of the proof of the lemma, it is enough to remark that, for each profile of mechanisms (γ_1^R, α_2), it is an equilibrium in the agents' game to send messages to P1 so to select $\delta_{ab} = \delta_{ba} = 1, \delta_{bb} = \sigma_{bb}$, and δ_{aa} as determined above, and to play the corresponding equilibrium efforts. This is a direct implication of the fact that P1 uses a recommendation mechanisms in the presence of three agents. As a consequence, following any deviation of P2 to a pay-for-effort contract α_2, agents coordinate on a continuation equilibrium that yields him at most -1. ■

This reasoning reproduces that of Lemma 2 of Yamashita (2010) and shows that -1 is the threshold payoff for $P2$. Hence, as in Theorem 1 of Yamashita (2010), every incentive feasible allocation yielding $P2$ at least a payoff of -1 can be sustained at equilibrium. ■

It then follows from Remark 2 that all (incentive feasible) *deterministic* allocations can be supported at equilibrium except those yielding $P2$ the minimal payoff ζ. The following lemma characterizes, in addition, the maximal payoff available to $P2$ among *all* incentive feasible allocations.

Lemma 2 *There is no allocation $z \in Z^{IF}$ yielding P2 a payoff strictly greater than* 5.

Proof. For a given profile of pay-for-effort contracts, $\alpha_1 = (\delta_{aa}, \delta_{ab}, \delta_{ba}, \delta_{bb})$ and $\alpha_2 = (\sigma_{aa}, \sigma_{ab}, \sigma_{ba}, \sigma_{bb})$, since $A3$ can only play $\{a\}$, we analyze the effort game played by $A1$ and $A2$, that is

	a	b
a	$10\sigma_{aa} - (1 - \delta_{aa} + \frac{1}{10}\delta_{aa})(1 - \sigma_{aa}),$ $5\sigma_{aa} + 4(1 - \delta_{aa})(1 - \sigma_{aa})$	$\frac{3}{2}\sigma_{ab} - (1 - \delta_{ab} + \frac{1}{10}\delta_{ab})(1 - \sigma_{ab}), 8$
b	$5(1 - \sigma_{ba})\delta_{ba} + 5\sigma_{ba}(1 - \delta_{ba}),$ $5(1 - \sigma_{ba})\delta_{ba} + 5\sigma_{ba}(1 - \delta_{ba})$	$\delta_{bb}(1 - \sigma_{bb}) - \sigma_{bb}(1 - \delta_{bb}),$ $10\delta_{bb}\sigma_{bb} + 4\sigma_{bb}(1 - \delta_{bb}) - (1 - \sigma_{bb})$

which coincides with the game in Table 3, without assuming $\delta_{ab} = \delta_{ba} = 1$ and $\delta_{bb} = \sigma_{bb}$. We therefore have the following

	a	b
a	$11\sigma_{aa} + \frac{9}{10}\delta_{aa}(1 - \sigma_{aa}) - 1,$ $\sigma_{aa} + 4(1 - \delta_{aa} + \delta_{aa}\sigma_{aa})$	$\frac{5}{2}\sigma_{ab} + \frac{9}{10}\delta_{ab}(1 - \sigma_{ab}) - 1, 8$
b	$5(\sigma_{ba} + \delta_{ba}) - 10\sigma_{ba}\delta_{ba},$ $5(\sigma_{ba} + \delta_{ba}) - 10\sigma_{ba}\delta_{ba}$	$\delta_{bb} - \sigma_{bb},$ $\sigma_{bb}(6\delta_{bb} + 5) - 1$

Table 4: The efforts' game played by $A1$ and $A2$, induced by (α_1, α_2)

The inequality $\zeta < \bar{\zeta}$ postulated in (1) guarantees that $P2$'s payoff is strictly below 5 in any fully mixed equilibrium of the agents' game. For P2 to get a payoff strictly above 5, principals' mechanisms should be designed so to guarantee that there exists an equilibrium in which (a, a) is chosen with positive probability and at most one agent randomizes.

First, observe that A2 strictly prefers to choose b rather than a whenever $\sigma_{aa} + 4(1 - \delta_{aa} + \delta_{aa}\sigma_{aa}) < 8$, which can be rewritten as $\sigma_{aa}(1 + 4\delta_{aa}) < 4(1 + \delta_{aa})$. It is immediate to check that the inequality is satisfied for every $\delta_{aa} \geq 0$, and $\sigma_{aa} \geq 0$. Thus, there is no equilibrium in which $A1$ plays a and $A2$ randomizes between a and b.

Next, consider the case in which $A2$ plays a. Then, it must be $\sigma_{aa} = 1$, otherwise $P2$ will get ζ with positive probability. In this case, $A1$ will never randomize between a and b. In fact, by playing a she gets $10 > 5(\sigma_{ba} + \delta_{ba}) - 10\sigma_{ba}\delta_{ba} \in [0, 5]$ for every $\sigma_{ba} \geq 0$ and $\delta_{ba} \geq 0$. Hence, there is no incentive feasible allocation yielding P2 a payoff strictly above 5. ■

Taken together, Proposition 1 and Lemma 2 imply that recommendation mechanisms can sustain as equilibria of the game $G^{\mathcal{A}}$ all incentive feasible allocations which yield a payoff above -1 and at most equal to 5 to $P2$. This provides a generalized version of the Yamashita (2010) main theorem in a complete information setting in which random behaviors are allowed.[11]

[11] See Xiong (2013) for a version of the folk theorem of Yamashita (2010) that does not rely on the restriction to deterministic behaviors.

We now show that none of these equilibria survives if we allow for private communication from principals to agents. That is, we show that, if $P1$ plays a recommendation mechanism, then $P2$ can get a payoff strictly above 5 by using a mechanism with signals. As a consequence the (pure strategy) equilibria of $G^{\mathcal{A}}$ are not robust, since there exists a specific G^{MS} game, and a corresponding profitable deviation for $P2$. This is formally shown in the following proposition.

Proposition 2 *Consider a game G^{MS}, in which $M_j^i = \mathcal{A}_j$ for every (i,j), $S_2^i = \{a,b\}$ and S_1^i is a singleton for every i. Suppose that, in this game, P1 posts the recommendation mechanism γ_1^R. Then, there is a $\hat{\gamma}_2 \in \Gamma_2^{\mathcal{A}_2 S_2}$ which yields P2 a payoff strictly greater than 5 in every continuation equilibrium.*

Proof. In this game G^{MS}, P2 can send private signals to agents, from the set $S_2^i = \{a,b\}$ for every i. Consider the following probability distribution over such signals: with probability $k > 0$ he privately communicates a to all agents, with probability $(1-k)$ he privately communicates b to $A1$ and a to $A2$ and $A3$. All the other signals are sent with probability zero. Let $P2$ associate to such signals a simple decision rule, which selects y_{21} when (a,a,a) is sent, and and y_{22} when (b,a,a) is sent, for every combination of agents' efforts and messages. We denote this deviating mechanism $\hat{\gamma}_2$.

The mechanisms $\left(\gamma_1^R, \hat{\gamma}_2\right)$ generate a game of incomplete information between the agents. Given the private signals they receive, they are differently informed over P2's decisions, which crucially affects their effort choices. The corresponding effort game played by $A1$ and $A2$ ($A3$ can only take the effort a) can be described as follows.

While both mechanisms are public, neither the signal received by $A1$ nor the realization of the lottery associated to $\hat{\gamma}_2$ are known to $A2$ and $P1$. Specifically, from the perspective of $A1$, when she receives the signal a she knows that with probability one P2 has chosen y_{21}. By choosing a she gets 10 if A2 plays a and 3/2 if A2 plays b. By choosing b, instead, she gets $5(1-\delta_{ba})$ if A2 plays a and $-(1-\delta_{bb})$ if A2 plays b. Playing a is hence a strictly dominant effort strategy for A1 irrespective of any pay-for-effort contract selected in $\mathcal{A}_1$, i.e. for every δ_{ba} and δ_{bb}. Alternatively, if she receives the signal b she knows that with probability one P2 chooses y_{22}. By choosing a she gets $-(1/10)\delta_{aa} - (1-\delta_{aa})$ if A2 plays a and $-(1/10)\delta_{ab} - (1-\delta_{ab})$ if A2 plays b. By choosing b, instead, she gets $5\delta_{ba}$ if A2 plays a and δ_{bb} if A2 plays b. Playing b is a strictly dominant effort strategy for A1 for every $(\delta_{aa}, \delta_{ab}, \delta_{ba}, \delta_{bb})$.

The above remarks allow to pin down A2's beliefs on A1's equilibrium behavior. From the perspective of A2, signals are uninformative, hence she chooses a if:

$$k[5\delta_{aa} + 5(1 - \delta_{aa})] + (1 - k)[5\delta_{ba}] \geq 8k - (1 - k) \quad (2)$$

which boils down to a condition on k, that is

$$k \leq \frac{5\delta_{ba} + 1}{5\delta_{ba} + 4},$$

which is satisfied for every δ_{ba} if $k \in (0, 1/4)$.

Thus, for every array of messages that $A1$, $A2$, and $A3$ may send to P1, i.e. for every $\delta = (\delta_{aa}, \delta_{ab}, \delta_{ba}, \delta_{bb})$, there exists a mechanism $\hat{\gamma}_2$ with $k \in (0, 1/4)$, that induces a unique equilibrium in the agents' effort game, in which $A1$ plays according to the signal she receives, $A2$ and $A3$ play a. This yields P2 a payoff of $95k + (1 - k)(-1)$ which is strictly greater than 5 for $k > 1/16$. Therefore, given $P1$'s mechanism γ_1^R, any mechanism $\hat{\gamma}_2$ in which $k \in (1/16, 1/4)$ guarantees $P2$ a payoff strictly above 5. ■

The logic of the proof of Proposition 2 can be resumed as follows. When posting the mechanism $\hat{\gamma}_2$, P2 effectively induces an incomplete information game between the agents. In particular, given their private signals, A1 and A2 have different posterior probability distributions over the decisions implemented by P2. Specifically, P2 correlates his decisions with the signals he sends to agents so that, when receiving his signal, A1 becomes perfectly informed, while the signal received by A2 is uninformative. This difference crucially affects the unique continuation equilibrium of the agents' effort game, generating a joint probability distribution over E, Y_1 and Y_2 that cannot be reproduced by any pay-for-effort contract chosen by P2.

Our analysis shows that the standard construction used to derive folk theorem results in competing mechanism games heavily relies on restricting principals' communication to be public.

4 Discussion

1. The G^M game in which each M_j^i space is a singleton plays a central role in economic applications. In this game, which we denoted G^D in Section 2, competition between principals takes place in the absence of any private communication, and principals post pay-for-effort contracts. The game G^D provides, in particular, a generalized version of the traditional models of lobbying of Bernheim and Whinston (1986), Dixit et al. (1997), and Prat and Rustichini (2003).

It is therefore a relevant question from the viewpoint of applications whether the equilibria of G^D survive when principals deviate to more complex mechanisms involving some communication. Theorem 1 in Han (2007) provides a positive answer, identifying a set of equilibria that are robust against unilateral deviations to mechanisms *with no signals*. These are the pure strategy *strongly robust* equilibria of G^D, that is, the SPNE in which no principal j can profitably deviate to a mechanism $\gamma'_j \in \Gamma_j$, regardless of the continuation equilibrium selected by agents.[12] Thus, a strongly robust equilibrium of G^D is also an (strongly robust) equilibrium of any G^M game.

Going back to our example, recall that, as pointed out in Remark 2, there exists an incentive feasible allocation yielding P2 his *maximal* payoff of 5. Then, as an implication of Lemma 2, this allocation can be supported in a strongly robust equilibrium of the game G^D. At equilibrium, P1 plays y_{12} when observing the efforts (b, a), and y_{11} otherwise; P2 plays y_{21} when observing the efforts (b, a), and y_{22} otherwise; A1 and A2 play b and a, respectively. It hence follows by Theorem 1 in Han (2007) that these behaviors constitute an equilibrium in any G^M game. At the same time, however, the proof of Proposition 2 shows that, if P1 plays the mechanism above, then P2 can profitably deviate to the mechanism with signals $\hat{\gamma}_2$. Thus, posting these mechanisms does not constitute an equilibrium in a game with signals G^{MS}. Overall, our analysis suggests that pure strategy equilibria of complete information games in which principals post pay-for-effort contracts may not be robust against unilateral deviations towards sophisticated mechanisms.

2. Our example shares with Yamashita (2010) the focus on recommendations mechanisms. In contrast with Yamashita (2010), however, we do not consider any (exogenous) incomplete information, and we let agents take (contractible) actions, so to cope with economic applications. One may therefore ask whether mechanisms with signals keep playing a key role in pure incomplete information settings. To answer this question, observe that, when information is incomplete and principals play recommendation mechanisms, agents take two relevant decisions. First, they "recommend" to each principal the direct mechanism he should post; second, they simultaneously report a type to each principal. From the viewpoint of a given principal j, the messages (types) that agents send to his opponents can be seen as hidden actions. Indeed, by selecting a profile of decisions in all posted direct mechanisms, such messages may *indirectly* affect principal j's payoff. He may therefore gain by generating uncertainty amongst agents when they play their message game, using the same logic we developed when considering the effort game in our example. That is, principal j may design a mechanism with signals, so to privately communicate with each type of each agent *before* messages (types) are sent. The corresponding continuation equilibrium over messages may

[12]See Han (2007), p. 613, for a formal definition of strongly robust equilibria. The result of his Theorem 1 does not naturally extend to mixed strategy equilibria, as he shows in Example 1.

induce a correlation between principals' decisions that cannot be reproduced without private signals, similarly to the correlation generated between e, y_1 and y_2 in our example by the mechanism $\hat{\gamma}_2$.

3. More generally, we suggest that equilibria supported by standard mechanisms, in which principals' private communication is disregarded, do not survive when this restriction is removed. Our example makes this point by simply enlarging the communication opportunities of P2, who is allowed to choose signals in the set $\{a, b\}$. In the corresponding game with signals, following the deviation to $\hat{\gamma}_2$, and upon receiving their private signals, agents play an incomplete information game given the pay-for-effort contract α_1 selected in γ_1^R. It is therefore natural to ask whether P1 can effectively "block" the deviation to $\hat{\gamma}_2$ by trying to extract the agents' private information it generates. That is, P1 may further ask agents to communicate this information and modify his decision accordingly. However, this additional possibility turns out not to be effective since the unique continuation equilibrium of the agents' effort game induced by $\hat{\gamma}_2$ is not affected by any further change in α_1. Indeed, playing in accordance with the received signal is a dominant strategy for A1 regardless of α_1, and the inequality (2) characterizing A2's best reply holds for every α_1 if $k \in (0, 1/4)$.

5 Conclusion

Since principals cannot in general be prevented from privately communicating with agents, our result suggest that further work is needed to identify a "safe" class of mechanisms supporting robust equilibria. To be relevant for applications, the corresponding messages and signals must be sufficiently simple and tractable. In this respect, a natural candidate is the class of direct mechanisms introduced by Myerson (1982) for generalized principal-agent problems. In complete information settings, they require that the set of signals available to each principal coincides with the set of agents' actions. However, were his opponents posting such mechanisms, principal j may gain by eliciting the agents' private information embedded in the signals. To do so, he would need to make signals contingent on each state of the world, i.e. on each array of his opponents' signals. Using such a larger signals' space allows the deviating principal to support different correlated equilibria in each agents' continuation game induced by the different array of signals they may receive from other principals.[13] In these circumstances, identifying a robust equilibrium may be very demanding. Indeed, each $-j$ principal could attempt at sending signals contingent on the (contingent) signals of

[13]This effect is highlighted in the competing mechanism game with moral hazard presented in Example 1 of Attar et al. (2010).

principal j, potentially leading to an infinite regress problem. This suggests that there could be scope for extending the Epstein and Peters (1999) construction to cope with principals' private communication.

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