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Refunding Emissions Taxes: The Case For A Three-Part Policy

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Abstract

This paper examines theoretically whether by combining both output based refunding and abatement expenditures based refunding it is possible to limit the negative consequences that a pollution tax imply for a polluting industry. We show that this is indeed the case by using a three-part policy where emissions are subject to a fee and where output and abatement expenditures are subsidized. When the industry is homogenous, it is possible to replicate the standard emission tax outcome by inducing a polluting firm to choose the production and emission levels obtained under any emission tax, without departing from budget balance. By construction, any polluter earns strictly more than under the standard tax alone without rebate, making this proposal more acceptable to the industry.

When firms are heterogenous, the refunding policy needed to replicate the standard emission tax outcome is personalized in the sense that at least the output subsidy should be type dependent and it is strictly prefered only from the industry's point of view to a standard environmental tax. We also explore the implications of uniform three-part refunding policies for a heterogenous industry.

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1 Introduction

Taxing sources of emissions is often difficult to implement in practice despite its theoretical appeal, since Pigou, to solve pollution problems in a cost-effective way. Traditional arguments against emission taxes encompass that they may harm industry's competitiveness, they may hit low income households hard and last but not least reducing domestic emissions may have zero effect on global pollution as increased foreign emissions may follow. As a consequence, it is not surprising that emissions taxes tends to be usually low and this typically undermines the efficiency of environmental policies (OECD, 2015).

It has long been recognized that earmarking the product of taxes to polluters is a way to diminish the economic burden to them and hence to potentially lessen political opposition to emissions taxes.[1] In particular, Aidt (2010) and Fredriksson and Sterner (2005) show that refunding increases the political feasibility of environmental taxation and that firms cleaner than average may even favor higher pollution taxes because they potentially benefit from the redistribution operated by the output based refunding policy. Refunding revenues to polluters in proportion to output is the way policy makers in Sweden have chosen with respect to the taxes collected on NOx emissions that cause ultimately acid rains (Sterner and Höglund-Isaksson, 2006).[2] Many authors have studied the properties of this refunding scheme with respect to the incentives to produce

[1] Although a breach in the principle of budgetary universality and even if the relative advantage in terms of political acceptability is difficult to measure, earmarking taxes is a popular strategy in many countries to secure funds towards e.g. governmental agencies or for specific purposes. In a country like France, it can represent 27% of total taxes collected by the government (Conseil des Prélèvements Obligatoires, 2018).

[2] There are other examples of refunding emission taxes through other channels. For instance, there is the tax on pesticides introduced in Denmark in 1996. There, two main channels were used to refund tax payments to farmers: first, there was a reduction on a tax on the value of land and second, to ensure that no specific sector of agriculture pays much more in tax revenue than the benefit it receives in terms of reduced land taxes, a compensation is decided on a yearly basis and transferred into specific funds which use is determined by farmers' organizations (e.g. to support marketing initiatives and research). Also the French water agencies system relies on emission fees paid by polluters that are refunded back to co-finance abatement expenditures (e.g. by installing some epuration systems).

and to abate pollution, compared to optimal policy (Fischer, 2001). While Sterner and Höglund-Isaksson (2006) study a perfectly competitive market structure, Gersbach and Requate (2004) concentrate on imperfectly competitive markets. Cato (2010) suggests that, when taking into consideration the endogeneity of market structure, there is a need to introduce an entry tax or license to avoid too much entry caused by the refunding policy. More recently, Bonilla et al. (2015) and Coria and Mohlin (2017) have shown how output based refunding of emission taxes can spur the adoption of new technologies when firms are not able to strategically influence the size of the refund.

Sterner and Turnheim (2009) have studied the Swedish refunded fee on NOx and concluded that the high tax allowed by the refunding did have significant impacts on innovation and on the observed decrease in NOx emission intensities. In Norway, the emissions payments for NOx are refunded in proportion to (observable) abatement expenditures. In a recent paper, Hagem et al. (2015) compare both output based and abatement based schemes and show that refunding necessarily implies a distortion on abatement (and production) decisions at the individual level compared to the situation where only emissions are taxed (at the same rate). More precisely, both output based and abatement expenditures based refunding induce a cost-ineffective provision of abatement as polluters put relatively too much effort into reducing emissions via abatement compared with reducing output. It follows that both schemes are welfare inferior to the optimal standard tax on emissions, because they lead to inferior output reductions.

This paper examines theoretically whether by combining both output based refunding and abatement expenditures based refunding it is possible to reduce these distortions. We actually show that this is indeed the case by using such a three-part policy where emissions are subject to a fee and where output and abatement expenditures are subject to a tax/subsidy. In particular, when the industry is homogenous, it is possible to replicate the standard emission tax outcome using such a policy: with the appropriate definition of the fee and of the output and abatement subsidies, the three-part refunding policy induces the polluting firm to choose the production and emission levels obtained under any emission tax. This is done without departing from budget balance which entails that (at least) a portion of taxes collected is returned back to polluters. By construction, the proposed regulation scheme entails that the polluter earns strictly more than under the standard tax alone without refunding, thereby making this proposal more acceptable by the industry.

More precisely, the design of the three-part policy follows simple rules. First, the emission fee necessarily differs from the standard emission tax, otherwise the only policy that gives the proper incentives with respect to production and pollution is the benchmark standard tax regulation without refunding. Second, if emissions are overtaxed compared to the standard emission tax situation, then to restore the proper incentives to produce and to pollute, it is necessary to tax abatement and to subsidize output. Conversely, for similar reasons, if emissions are undertaxed it is necessary to subsidize abatement and to tax production. Whether one or the other policy is adapted to a particular context actually depends on whether the budget constraint can hold. We show that this depends on the (local) properties of the abatement technology: if production strongly determines abatement expenditures, then the three-part refunding policy entails overtaxation of emissions while it is the reverse when either pollution mainly determines or production weakly determines abatement costs.[3]

When the industry is heterogenous, the refunding policy needed to replicate the standard emission tax outcome is now personalized in the sense that at least the output subsidy should be type dependent. Another result is that this three-part policy is strictly preferred only from the industry's point of view to a standard environmental

[3]More precisely, abatement expenditures are assumed to be increasing in production and decreasing in emissions. When abatement expenditures are raised by more than 1% when both production and pollution increase by 1%, we consider that production strongly determines abatement costs. Conversely, when abatement costs either decrease or is raised by less than 1%, we consider respectively that pollution mainly determines or that production only weakly determines abatement costs.

tax.

We then explore the implications of uniform three-part refunding policies in this context and find, by using an example, that it is possible for the refunding policy to replicate the standard tax outcome in the aggregate at the sector's level with respect to production and emissions without departing from the budget constraint. Abatement expenditures differ in general from those spent under a standard emission tax without refunding.

The policy implications of the analysis are clear. Because refunding emission taxes can help to ensure the political acceptability of strong environmental regulations (as in the Swedish NOx example), and because output based or abatement based refunding each generates some adverse incentives impacts, it could be interesting to go beyond by combining both refunding schemes. Under some conditions and a little more complexity for the regulation scheme (three instruments instead of two), a three-part policy allows to get rid of adverse incentives effects from the usual refunding channels and yet ensures that an exogenous refunding rate towards the polluting industry is met.

The paper is organized as follows. Section 2 lays out assumptions and notations as well as the two benchmarks of a standard emission tax and of output based versus abatement expenditures based refunding policies. In section 3, we consider the three-part refunding policy in a context with homogenous firms. Section 4 explores the case of heterogenous firms. Section 5 concludes.

2 The model

2.1 Assumptions and notations

Consider a competitive industry whose size is fixed to n firms indexed by $i = 1...n$. Product is homogenous and sold at (constant) price p. Production is also polluting and any firm has the possibility to control and reduce it by employing its optimal mix of output reduction and specific input use for abatement. We decompose the total cost of firm i as the sum of first a production cost $c_i(q_i)$, twice continuously differentiable, strictly increasing in production q_i and second an abatement cost function $a_i(q_i, e_i)$, twice continuously differentiable, strictly increasing in q_i, strictly decreasing in emissions e_i up to some level $e_i^\circ(q_i)$ and then increasing. Under *laissez-faire*, i.e. in the absence of pollution regulation, firm i would pollute up to the "selfish" level $e_i^\circ(q_i)$ that minimizes abatement costs.

We also assume that total cost $c_i(.) + a_i(.,.)$ is strictly quasi-convex, which ensures that the profit function $\pi_i(q_i, e_i)$ is strictly quasi-concave. Denoting a_{ix} and a_{ixy} the corresponding first-order partial derivative and the second-order partial derivative respectively for $x, y \in \{e, q\}$, strict quasi-convexity of total cost means that $a_{iee} > 0$, $c_i'' + a_{iqq} > 0$ and $a_{iee}c_i'' + a_{iee}a_{iqq} - a_{iqe}^2 > 0$.

We will also make use of the two following assumptions that would prove useful in obtaining some comparative statics results.

Assumption 1 *The abatement cost function $a_i(.,.)$ is such that $a_{ieq} < 0$.*

Assumption 1 says that the marginal benefit of pollution for the firm, namely $-a_{ie} > 0$, is increasing in q_i. The more the firm produces the higher the cost savings from polluting are.

Assumption 2 *The abatement cost function $a_i(.,.)$ is such that the ratio $\frac{a_{ie}}{a_{iq}}$ is increasing in q_i and in e_i.*

It can be checked that assumption 2 holds whenever $|a_{ieq}|$ is sufficiently small and a_i convex in q.[4]

[4]Indeed, dropping the index i for clarity, we note that $\frac{\partial}{\partial q}(\frac{a_e}{a_q}) = \frac{a_q a_{qe} - a_e a_{qq}}{a_q^2}$ and $\frac{\partial}{\partial e}(\frac{a_e}{a_q}) = \frac{a_q a_{ee} - a_e a_{qe}}{a_q^2}$. Hence, it is sufficient that $|a_{qe}|$ is sufficiently close to 0 for assumption 2 to hold.

2.2 Standard emission tax as a benchmark

Let us first define the benchmark situation of a standard emission tax t without refunding (or when refunding corresponds to a lump sum transfer). When facing the emission tax t, firm i maximizes its (strictly quasi-concave) profit function with respect to production and emissions:

$$\max_{q_i, e_i} \pi_i = pq_i - c_i(q_i) - a_i(q_i, e_i) - te_i$$

and this leads to the following necessary and sufficient optimality conditions (assuming interior solutions):

$$\begin{aligned} p &= c_i'(q_i^*) + a_{iq}(q_i^*, e_i^*) \\ -a_{ie}(q_i^*, e_i^*) &= t. \end{aligned} \tag{1}$$

Firm i's optimal reaction to the emission tax t is governed by marginal cost pricing and equality between marginal abatement cost and marginal pollution price. For further reference, aggregate production is denoted $Q^*(t) = \sum_i q_i^*(t)$ and aggregate pollution is $E^*(t) = \sum_i e_i^*(t)$. Furthermore, aggregate abatement expenditures are denoted $A^*(t) = \sum_i a_i(q_i^*(t), e_i^*(t))$.

In terms of net profit, the regulated firm earns:

$$\pi_i^* = pq_i^* - c_i(q_i^*) - a_i(q_i^*, e_i^*) - te_i^*$$

which is assumed positive for any i and for the values of t considered.

In Appendix A, we show that comparative statics results with respect to price, emission tax and increase in abatement costs scale can be established as follows. First, quasi-concavity of profit implies that production is increasing in output price p whereas emissions are decreasing in the tax level t. Second, assumption 1 ensures that $\pi_{iqe} > 0$ or equivalently that the marginal profitability of q_i increases in emissions e_i. This in turn implies that production is decreasing in the tax level t while emissions are increasing in output price. Third, under assumption 2, an increase in the scale of abatement cost $a_i(q_i, e_i)$ leads to decreasing production and increasing emissions.

2.3 Output based versus abatement expenditures based refunding policies

Now consider the possibility for the government to refund emission payments collected back to the firms. We here revisit results obtained by Hagem et al. in a competitive setting with our notations (see also Gersbach and Requate (2004) for an analysis in markets with imperfect competition).

For a given emission tax t, it is interesting to compare a policy that entails refunding through a subsidy τ on output and an alternative policy refunding occurs through a subsidy s on abatement cost to a policy with no refunding of the fee. Whenever there is a subsidy, and whether it is output or abatement based, its level is given by the budget constraint that writes:

$$tE = \tau Q$$

in the output based refunding case and

$$tE = sA$$

in the abatement expenditures refunding case. The total emissions tax collected are returned back to polluters either proportionally to output or to abatement cost.

Given the above comparative statics results presented above, it is easy to recover the following results due to Hagem et al. (2015).

Proposition 1 (Hagem et al.) *Under assumptions 1 and 2, (i) for a given emission tax t and compared to a policy without refunding, an output based refunding policy (τ, t) implies too much pollution and an abatement expenditures based refunding policy (s, t) implies too few pollution, and (ii) for a given pollution target, the emission tax needed* *under output based refunding policy is larger than the tax without refunding whereas the tax needed under abatement expenditures based refunding policy is lower.*

Proof: Part (i) directly follows from Appendix A which states that, for a fixed price of pollution, subsidizing the abatement cost decreases pollution while subsidizing production will increase emissions. Similarly, part (ii) holds because Appendix A states that, in order to keep pollution constant, it is necessary to increase the pollution fee if one is going to subsidize production, while it is necessary to decrease the pollution fee if abatement expenditures are to be subsidized. ■

A two-part refunding policy generally involves a distortion on the pollution level for any firm and thus in aggregate, compared to what prevails under an emission tax without refunding. Intuitively, an output subsidy increases the marginal benefit of production which in turn increases the marginal incentives to pollute. Conversely, an abatement subsidy decreases the marginal benefit of pollution which in turn induces a downward distortion on pollution.

The opposite nature of the distortions brought by the two ways to refund emission taxes leads to study whether by combining both subsidies it is possible to reduce these distortions. We will first examine this possibility for a homogenous industry in the next section while the study of heterogenous industries is postponed to section 4.

3 Three-part refunding policies for a homogenous industry

3.1 Analysis

When the industry is composed of identical firms, the equilibrium outcome under a standard emission tax t is described by the system (1) that reduces to:

$$\begin{aligned} p &= c'(q^*) + a_q^* \\ -a_e^* &= t \end{aligned} \qquad (2)$$

where we denote, for the sake of exposition, $a^* \equiv a(q^*, e^*)$, $a_q^* \equiv a_q(q^*, e^*)$ and $a_e^* \equiv a_e(q^*, e^*)$.

The question we ask is whether it is possible to replicate this equilibrium outcome for a given tax t by using instead a three-part policy based on a per unit fee f on emissions, an output subsidy τ per unit and an abatement subsidy at rate s, without departing from budget balance. The natural advantage of the three-part policy would then be to leave a higher profit to firms and thus to make the regulation more acceptable to them. It turns out that the answer to this question is positive as we now show. We proceed by first defining the budget constraint of the government and then the optimal reaction of a representative polluter to a three-part refunding policy, before establishing our main result in this section.

We depart from Hagem et al. (2015) by assuming that there is also a revenue requirement R asked by the government from the regulation policy.[5] To avoid an arbitrary choice of R, we assume that

$$R = (1 - \delta)tE^*$$

with $\delta \in (0, 1)$. Hence, the government requires the refunding policy to collect at least a portion $1 - \delta$ of the emissions taxes $tE^*(t)$ that would be collected in the absence of refunding. When $\delta = 1$ there is complete refunding of emissions taxes towards polluters while for $\delta = 0$, the revenue requirement will actually impose the absence of refunding ($\tau = s = 0$) as it will be clear below. When $0 < \delta < 1$, there is only partial refunding of taxes to polluters and parameter δ represents the rate of refunding. This representation of the budget constraint allows for a richer modelling of possible refunding policies that can be only partial with a portion of taxes being spent purposedly elsewhere in the economy.

[5] Gersbach and Requate (2004) and Cato (2010) also consider the possibility of partial refunding.

To sum up, the budget constraint of the regulation schedule writes as:

$$fE = \tau Q + sA + R \tag{3}$$

where industry's emissions are E, total production is Q and total abatement expenditures A. Hence, all emissions taxes collected net of the revenue requirement R are given back to polluters under the form of output and abatement subsidies and this implicitly defines a relationship between emission tax, production and abatement subsidies.

Facing a three-part policy (τ, s, f), the optimal decisions for a representative firm are

$$(\hat{q}(\tau, s, f), \hat{e}(\tau, s, f)) \in \arg\max_{q,e} \pi(q, e) \equiv (p + \tau)q - c(q) - (1 - s)a(q, e) - fe \tag{4}$$

and the corresponding profit is

$$\hat{\pi}(\tau, s, f) = \pi(\hat{q}(\tau, s, f), \hat{e}(\tau, s, f)$$

Let us denote

$$\hat{a} \equiv a(\hat{q}, \hat{e}) \text{ with } \hat{a}_q = \left.\frac{\partial a}{\partial q}\right|_{q=\hat{q},\ e=\hat{e}} \text{ and } \hat{a}_e = \left.\frac{\partial a}{\partial e}\right|_{q=\hat{q},\ e=\hat{e}}$$

The FOCs are (for interior solutions):

$$\begin{aligned} p + \tau &= c'(\hat{q}) + (1 - s)\hat{a}_q \\ -(1 - s)\hat{a}_e &= f \end{aligned} \tag{5}$$

and second order conditions are (assuming $s < 1$):

$$\begin{aligned} \left.\frac{\partial^2 \pi}{\partial q^2}\right|_{q=\hat{q},\ e=\hat{e}} &= -c''(\hat{q}) - (1 - s)\hat{a}_{qq} < 0 \\ \left.\frac{\partial^2 \pi}{\partial e^2}\right|_{q=\hat{q},\ e=\hat{e}} &= -(1 - s)\hat{a}_{ee} < 0 \\ \left.\frac{\partial^2 \pi}{\partial q^2}\frac{\partial^2 \pi}{\partial e^2} - \left(\frac{\partial^2 \pi}{\partial q \partial e}\right)^2\right|_{q=\hat{q},\ e=\hat{e}} &= (1 - s)\left[\hat{a}_{ee}c''(\hat{q}) + (1 - s)(\hat{a}_{qq}\hat{a}_{ee} - \hat{a}_{qe}^2)\right] > 0 \end{aligned}$$

Let us denote

$$\sigma^* = 1 - \varepsilon^*_{a,e} - \varepsilon^*_{a,q}$$

where $\varepsilon^*_{a,e} = a^*_e e^*/a^* < 0$ is the elasticity of a w.r.t. e taken at the standard emission tax optimum q^*, e^* while $\varepsilon^*_{a,q} = a^*_q q^*/a^* > 0$ denotes the elasticity of a w.r.t. q and taken at the optimum q^*, e^*. The sum of elasticities $\varepsilon^*_{a,e} + \varepsilon^*_{a,q}$ represents the relative change of abatement expenditures when both production and pollution are raised by 1%.[6] It can be positive or negative as abatement expenditures are increasing in production but are decreasing in emissions. The value σ^* measures how much this sum of elasticities compares with 1. Hence, if $\sigma^* < 0$, this means that abatement expenditures raise by more than 1% when both production and emissions are raised by 1%. We describe such a situation as one where production strongly determines abatement expenditures. Conversely, when $\sigma^* > 0$, abatement expenditures either decrease or are raised by less than 1%, when both production and emissions increase by 1%. This means that respectively either pollution mainly determines or production weakly determines abatement expenditures.[7]

Overall, the sign of σ^* represents a local property (i.e. at the optimum q^*, e^*) of the abatement technology, depending on how production and pollution interact in expenditures directed towards pollution reduction.

Using the above definition of σ^* and comparing (2) and (5), we can establish the following result.

Proposition 2 *For a homogeneous industry of fixed size and for $\sigma^* < 0$ or $\sigma^* > \delta \frac{te^*}{a^*}$, there exists a unique three-part policy that entails any firm to choose $\hat{e} = e^*$ and $\hat{q} = q^*$*

[6]Indeed, from $a \equiv a(q, e)$ we obtain by differentiating $da = a_q dq + a_e de$. Dividing by a both members of the equality and introducing elasticies, we get $d \ln a = \varepsilon_{a,q} d \ln q + \varepsilon_{a,e} d \ln e$. Assuming that both q and e change by the same percentage, i.e. $d \ln q = d \ln e$, then $d \ln a = (\varepsilon_{a,q} + \varepsilon_{a,e}) d \ln e$.

[7]When $\sigma^* = 0$, then production and pollution are both determinants with equal strengh for abatement expenditures. As will be clear below, this limit case prevents the budget constraint from being met and should thus be excluded when looking for a three-part refunding policy.

while sustaining the budget constraint. It is characterized by

$$\begin{aligned} f &= (1-s)t \\ \tau &= -sa_q^* \\ s &= \frac{\delta te^*}{\sigma^* a^*} \end{aligned}$$

Moreover, any firm earns $\hat{\pi}(\tau, s, f) = \pi^* + \delta te^*$ *so that the three-part policy is strictly preferred by all firms to a standard emission tax as long as the refunding rate* δ *is strictly positive.*

Proof: See Appendix B. ■

The intuition is clear: with the three instruments contained in the three-part policy, one can mimic the standard emission tax outcome in terms of production and pollution without sacrificing the budget balance. Moreover, due to the (at least partial) refunding of taxes, the three-part policy (τ, s, f) is strictly preferred to the standard emission tax by all firms. When the government requires no refunding at all ($\delta = 0$), the three-part policy boils down to the no-refunding policy with $s = \tau = 0$ and $f = t$.

To interpret the policy exhibited, note first that τ and s necessarily have opposite signs (as $a_q > 0$). Second, f is necessarily different from t, otherwise we are back to the no-refunding policy: indeed, if $f = t$ this would set $1 - s = 1$ that is a zero abatement subsidy which then implies a zero subsidy on output ($\tau = 0$). Hence, it follows that, with a three-part refunding policy, emissions are either overtaxed or undertaxed, compared to the benchmark emission tax t.

If emissions are overtaxed, then the definition of f implies that abatement should be taxed ($s < 0$). Intuitively, overtaxing emissions induces the firm to potentially overinvest in abatement effort and hence a tax on abatement is needed to reduce these incentives. Also, output should be subsidized ($\tau > 0$) in order to counterbalance the marginal disincentives to produce because abatement expenditures are taxed.

Conversely, when emissions are undertaxed compared to t, then there is potentially underinvestment in abatement effort and a subsidy is introduced to stimulate abatement, along with a tax on output intended to counterbalance the increased marginal incentives to produce caused by the abatement subsidy.

Ultimately, whether the refunding policy is of the first or the second type depends on whether it is possible for the budget constraint to hold with one or the other policy. This in turn depends on the sign of σ^* as the budget constraint implies that necessarily s and σ^* should have the same sign. Indeed, by rewriting equation (18) and using the definition of R, we obtain:

$$(t-f)e^* + \tau q^* + sa^* = \delta te^*.$$

It follows that the amount $\delta te^* = \hat{\pi} - \pi^* > 0$, which is the money given back to the polluter thanks to the refunding policy, is spread into some abatement and output subsidies plus the difference between the standard tax and the emission fee payments. As shown in Appendix B, by using the definition of f and τ given in Proposition 2, we get that $(t-f)e^* + \tau q^* + sa^*$ can be written as being proportional to the abatement subsidy payment sa^*, the factor of proportionality being precisely σ^*. In total, we get as indicated in Proposition 2:

$$\sigma^* sa^* = \delta te^*. \tag{6}$$

Note that for (6) to hold, we need to exclude the case where $\sigma^* = 0$. Also, in Appendix B, we show that to preserve the quasi-concavity of profit, we exclude that $s > 1$ or equivalently that $\sigma^* < \delta \frac{te^*}{a^*}$ whenever it is positive. We deduce from Proposition 2 the following Corollary.

Corollary 1 *For $\delta \in (0,1)$ and t given, the three-part policy is characterized as follows:*

(i) *if σ^* is positive and greater than $\delta \frac{te^*}{a^*}$, then there is under-taxation of emissions ($f < t$), a subsidy on abatement ($s > 0$) and a tax on production ($\tau < 0$)*

(ii) *if σ^* is negative, then there is over-taxation of emissions ($f > t$), a tax on abatement ($s < 0$) and a subsidy on production ($\tau > 0$).*

3.2 Examples

Corollary 1 indicates that the refunding policy can take two different forms depending on the sign of σ^* which depends itself on the characteristics of the abatement technology. While the refunding policy in the case where σ^* is positive seems appealing, the policy when σ^* is negative seems unattractive at first sight because one has to tax abatement and to subsidize output. Nevertheless, both forms of the policy reach the same goal: the standard tax outcome is obtained in terms of production, emissions and abatement expenditures together with some money given back to polluters, whose size depends on the refunding rate δ chosen by the government and on the benchmark tax level t. To illustrate, we consider in this section two different specifications of the model.

3.2.1 A quadratic specification

Let us assume that all firms share the following cost and abatement functions, $c(q) = cq^2/2$ and $a(q,e) = k(q - e/\gamma)^2/2$. The interpretation is that if x denotes the abatement effort then the emission level is proportional to production minus the abatement effort, i.e. $e = \gamma(q - x)$ and the abatement effort entails a quadratic cost $kx^2/2$. Straightforward computations lead to:

$$\begin{aligned} q^* &= \frac{p - t\gamma}{c} > 0 \text{ if } t < p/\gamma \\ e^* &= \frac{p\gamma}{c} - t\gamma^2\left(\frac{k+c}{kc}\right) > 0 \text{ if } t < \frac{k}{k+c}\frac{p}{\gamma} \\ e^*/q^* &= \gamma\frac{p - \left(\frac{k+c}{k}\right)t\gamma}{p - t\gamma} \end{aligned}$$

Note that the production level is strictly decreasing in γ. Moreover, both the emission level e^* and the emission intensity ratio e^*/q^* are first increasing and then decreasing in γ (or equivalently in q). Hence, this formulation generates a hump shaped emission curve as positive parameter γ decreases and consequently when production grows. Emissions are low when the size of firms in terms of their production is small or large while they are higher for firms with intermediary sizes. The same is true for the emission intensity ratio. This feature of the specification is consistent with at least the fact that the lower pollution intensity of large firms is an observation widely documented in the empirical literature (see e.g. Andreoni and Levinson, 1998, or Wang and Wheeler, 2005).

The maximum of pollution is in $1/\gamma = 2t(k+c)/(pk)$. Abatement effort $q^* - e^*/\gamma = t\gamma/k$ is increasing in γ as long as $e^* > 0$. We thus assume that the condition $t < \frac{k}{k+c}\frac{p}{\gamma}$ holds so that pollution remains strictly positive. We also find that $\sigma^* = -1$, i.e. a negative constant, and this implies that emissions are overtaxed in the refunding policy.

Applying straightforwardly Proposition 2, we obtain the following result.

Proposition 3 *Assume that all firms share the following cost and abatement functions, $c(q) = cq^2/2$ and $a(q,e) = k(q - e/\gamma)^2/2$. Then the unique three-part policy is such that:*

$$\begin{aligned} s &= \frac{2\delta}{c}\left(k + c - k\frac{p}{t\gamma}\right) < 0 \\ \tau &= 2\delta\gamma\frac{k+c}{c}\left(\frac{k}{k+c}\frac{p}{\gamma} - t\right) > 0 \\ f &= t(1 - 2\delta\frac{k+c}{c}) + 2\delta\frac{p}{\gamma} > t \end{aligned}$$

where one has to overtax emissions, to tax abatement and to refund taxes through a production subsidy.

Recall that small firms (in terms of their production level) are characterized by high values of γ. The refunding policy is such that the tax on abatement in absolute value $|s|$ is decreasing in γ as well as the emission fee f and the output subsidy τ.

Hence, when the industry is composed of small firms, the refunding policy entails an emission fee closer to the standard tax t and lower output subsidy and abatement tax in absolute value, compared to an industry with larger firms ceteris paribus.

3.2.2 A Cobb-Douglas based specification

Let us still assume that $c(q)$ is still quadratic, $c(q) \equiv cq^2/2$, but that now the abatement technology associates emissions and a specific input l with unit price to final production according to a Cobb-Douglas form ($q = l^\beta e^\alpha$), so that the abatement cost function writes:

$$a(q, e) = q^{1/\beta} e^{-\alpha/\beta}$$

with $\alpha, \beta > 0$. Note that this function is increasing in q, convex in q if and only if $\beta < 1$ and decreasing convex in e.[8] Also, assumption 1 holds as $a_{qe} < 0$. The abatement function is also quasi convex if and only if $\alpha + \beta < 1$, i.e. when there are decreasing returns to scale for the underlying Cobb-Douglas production function.

The two elasticities $\varepsilon^*_{a,e}$ and $\varepsilon^*_{a,q}$ (and so σ^*) do not actually depend on t and are constants given by:

$$\begin{aligned} \varepsilon^*_{a,e} &= \frac{a^*_e e^*}{a^*} = -\frac{\alpha}{\beta} \\ \varepsilon^*_{a,q} &= \frac{a^*_q q^*}{a^*} = \frac{1}{\beta} \end{aligned}$$

and we obtain

$$\sigma^* = 1 - \varepsilon^*_{a,e} - \varepsilon^*_{a,q} = \frac{\alpha + \beta - 1}{\beta}.$$

The abatement subsidy is also a constant equal to:

$$s = -\frac{\delta \varepsilon^*_{a,e}}{\sigma^*} = \frac{\delta \alpha}{\alpha + \beta - 1}.$$

To avoid the possibility of having $s > 1$, we assume that $\delta < \max(1, (\alpha + \beta - 1)/\alpha)$. Note also that the case of constant returns to scale ($\alpha + \beta = 1$) is excluded as this leads

[8] We assume that emissions can be at most equal to some exogenous level $\bar{e} > 0$ which represents the selfish level that would occur in the absence of emission taxation (”laissez faire” policy).

to $\sigma^* = 0$ which makes it impossible to satisfy the budget constraint with a three-part refunding policy.

We sum up our main result in the following Proposition.

Proposition 4 *Assume that all firms share the following cost and abatement functions,* $c(q) = cq^2/2$ *and* $a(q,e) = q^{1/\beta}e^{-\alpha/\beta}$ *with* $\alpha, \beta > 0$. *Then, the unique three-part policy entails an abatement subsidy rate given by*

$$s = \frac{\delta\alpha}{\alpha + \beta - 1}$$

and $s < (>)0$ *if and only if there are decreasing (increasing) returns to scale in the abatement technology* $(\alpha + \beta < (>)1)$.

To illustrate, Figure 1 depicts the refunding policy for the case of IRS (panel (a)) and DRS (panel (b)) as a function of the benchmark tax t, and for specific values of the parameters chosen so that the total cost $c(q) + a(q,e)$ is quasi convex in (q,e), which ensures quasi concavity of the profit function.[9]

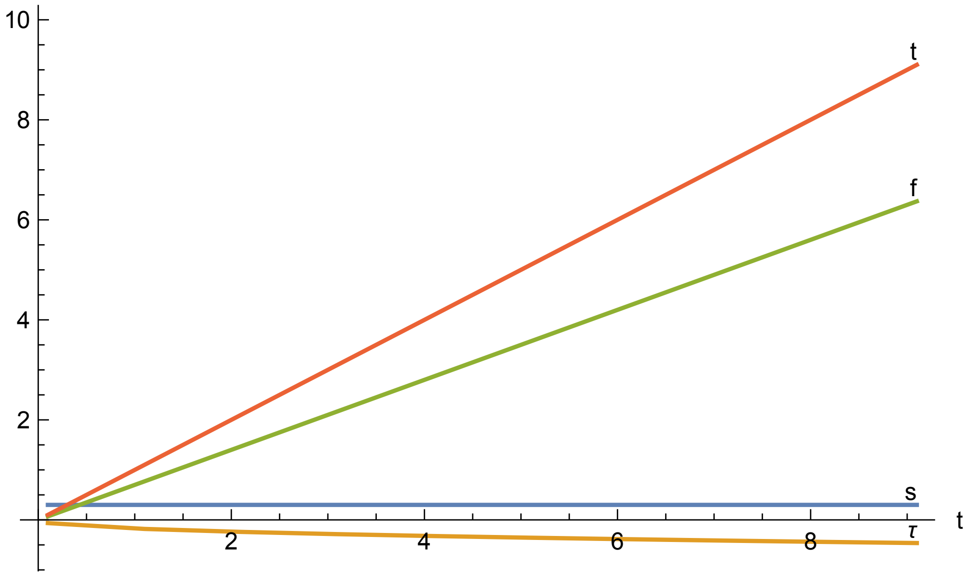
4 Extension to heterogenous industries

4.1 Personalized three-part refunding policies

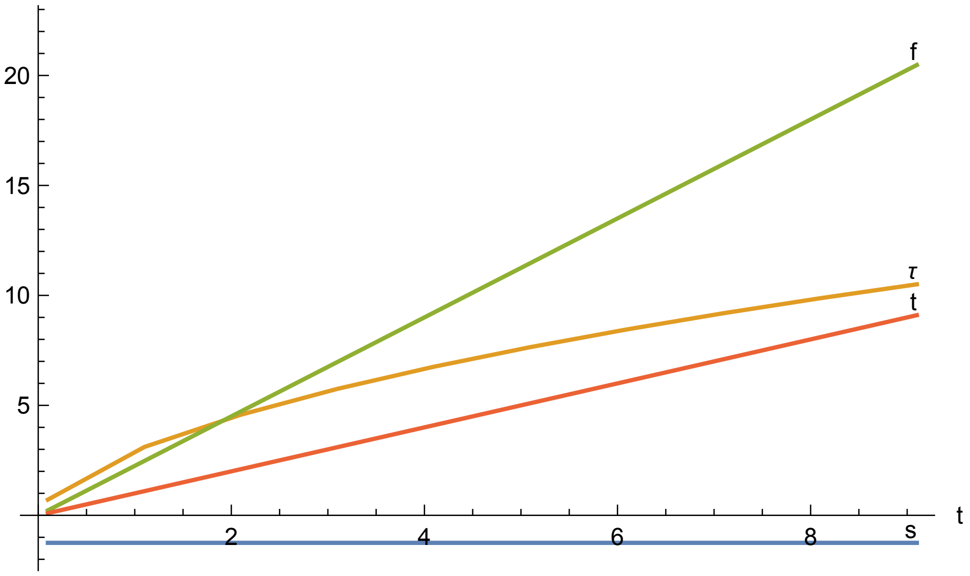
In this section, we examine the case of heterogenous industries with respect to cost conditions. The three-part policy able to replicate the standard emission tax outcome while sustaining the budget constraint is a priori *personalized* and should thus be indexed by i : f_i, τ_i and s_i. Faced with this three-part policy, firm i chooses $\hat{q}_i(\tau_i, s_i, f_i)$ and $\hat{e}_i(\tau_i, s_i, f_i)$ such that:

$$\begin{aligned} p + \tau_i &= c_i'(\hat{q}_i) + (1 - s_i)\hat{a}_{iq} \\ -(1 - s_i)\hat{a}_{ie} &= f_i \end{aligned} \qquad (7)$$

[9]We use $p = 20$, $c = 20$, and a refunding rate of $\delta = 50\%$. For the case of IRS, we set $\alpha = 1.5$ and $\beta = 2$. For the case of DRS, we set $\alpha = 0.5$ and $\beta = 0.3$. We also normalize the mass of firms to 1.



(a): Increasing returns to scale, $s > 0$, $f < t$ and $\tau < 0$.



(b): Decreasing returns to scale, $s < 0$, $f > t$ and $\tau > 0$.

Figure 1: Three-part refunding policies for the Cobb-Douglas based specification.

As above, comparing (1) with (7) allows to derive the following result.

Proposition 5 *For a heterogenous industry of fixed size, there exists at least a three-part policy that entails any firm i to choose $\hat{e}_i = e_i^*$ and $\hat{q}_i = q_i^*$ while sustaining the budget constraint (provided firm's problem concavity is preserved). They are characterized by:*

$$\begin{aligned} f_i &= (1 - s_i)t \\ \tau_i &= -s_i a_{iq}^* \\ \sum_i \sigma_i^* a_i^* s_i &= \delta t E^* \end{aligned}$$

where

$$\sigma_i^* = 1 - \varepsilon_{a_i,e}^* - \varepsilon_{a_i,q}^*.$$

Moreover, aggregate net profit is larger than the one without rebate ($\hat{\Pi} - \Pi^* = \delta t E^* > 0$) so that the three-part policy is strictly preferred from the industry's point of view to a standard environmental tax.

Proof: See Appendix C. ■

One particular policy of interest is where one takes $s_i = s \equiv tE^* / \sum_i \sigma_i^* a_i^*$ and $f_i = f = (1 - s)t$ for any i and where $\tau_i = -s a_{iq}^*$. For this policy, the sign of $\sum_i \sigma_i^* a_i^*$, weighted sum of abatement expenditures, is the key element. Assume that $\sum_i \sigma_i^* a_i^* > 0$. In that case, the policy prescribes a uniform subsidy $s > 0$ on abatement, a uniform tax on emissions that under-taxes emissions ($f < t$) and a personalized tax $\tau_i < 0$ on production. Conversely, if $\sum_i \sigma_i^* a_i^* < 0$, then the policy prescribes a tax on abatement ($s < 0$), a tax on emissions that over-taxes emissions ($f > t$) and a personalized subsidy on production $\tau_i > 0$.

Two remarks are to be made. First, at the individual level, refunding is profitable if only if $s\sigma_i^* > 0$, that is when s and σ_i^* have the same sign. While it is always profitable for the industry as a whole, it may not be profitable for all members of the industry.

Second, incomplete information will in general impede the implementation of such personalized output subsidies. However, suppose that asymmetric information occurs with respect to some abatement function parameter, then as production, pollution and abatement expenditures are assumed observable, asymmetric information poses no problem as the regulator can always deter any non truth-telling strategy by observing all these variables. But if abatement expenditures can be manipulated by the firm then truth telling must be induced by an appropriate choice of revelation mechanisms. While the search for incentive compatible refunding policies is beyond the scope of this paper, we instead investigate what can be done with a uniform policy (τ, s, f). This is the purpose of the next section.

4.2 Uniform three-part refunding policies

We now consider a uniform policy of the type (τ, s, f) implemented in a sector composed of n heterogenous polluting firms as described in section 4. Non-personalized policies will not make it possible to have firms choosing q_i^* and e_i^* in general. Nevertheless, it is still possible to find a three-part refunding policy that satisfies the budget constraint and that leads to the same aggregate pollution and production levels that prevail under any emission tax t without refunding. For this, a solution (τ, s, f) to the following system for given t must exist:

$$\hat{E}(\tau, s, f) - E^*(t) = 0 \tag{8}$$

$$\hat{Q}(\tau, s, f) - Q^*(t) = 0 \tag{9}$$

$$f\hat{E}(\tau, s, f) - \tau\hat{Q}(\tau, s, f) - s\hat{A}(\tau, s, f) - (1-\delta)tE^*(t) = 0 \tag{10}$$

Existence of a solution can be deduced from using the Implicit Function Theorem and so by checking that the Jacobian matrix of the system is non singular. However, studying the properties of this system of equations with this level of generality is not very informative. Hence, we instead prove below that such a solution exists for a particular example relying on the specification used in section 3.2.1.

More importantly, a uniform three-part refunding policy solution of the system above is unable in general to always ensure that the industry benefits from the refund, because the uniform policy makes it impossible to have firms choosing individually q_i^* and e_i^*. As a consequence, the aggregate costs of production and abatement under the three-part refunding policy and under the standard tax policy generally differ. To see this, recall that the aggregate profit of the industry under the three-part refunding policy can be written as follows:

$$\hat{\Pi} = (p + \tau)\hat{Q} - \hat{C} - (1 - s)\hat{A} - f\hat{E}$$

where $\hat{C} = \sum_i c_i(\hat{q}_i)$, $\hat{A} = \sum_i a_i(\hat{q}_i, \hat{e}_i)$ with $\hat{q}_i(\tau, s, f)$ and $\hat{e}_i(\tau, s, f)$ being the optimal decisions for polluter i facing the policy (τ, s, f). Recall also that the aggregate profit under the standard emission tax t is:

$$\Pi^* = pQ^* - C^* - A^* - tE^*$$

where $C^* = \sum_i c_i(q_i^*)$. Computing the difference and using (8), (9), (10) and simplifying, we obtain finally:

$$\hat{\Pi} - \Pi^* = \delta t E^* + C^* + A^* - \hat{C} - \hat{A}$$

It follows that $\hat{\Pi} - \Pi^* > 0$ only if the uniform refunding policy does not increase too much the aggregate cost of production (gross of emissions payments) compared to the benchmark case of a standard emission tax t. Although the refunding policy ensures stability of aggregate production and emission by definition, it will reallocate these quantities among the different polluters compared to the benchmark case. If this reallocation of production and emissions ends up by increasing too much the sum of production and abatement costs for the industry, the refunding policy may not be beneficial to the firms as a whole.

To illustrate these results, we now specify the model and assume that there are n firms indexed by $i = 1...n$. The cost of production is $c_i(q_i) = cq_i^2/2$ and abatement cost is $a_i(q_i, e_i) = k(q_i - e_i/\gamma_i)^2/2$ as in section 3.2.1. Firms are heterogeneous according to parameter γ_i. The number of firms of type γ_i is n_i and $\sum_i n_i = n$. Let us denote the mean $\bar{\gamma}$ and the variance σ_γ^2 of parameter γ.

Using (7), we immediately obtain that:

$$\begin{aligned}\hat{q}_i &= \frac{p + \tau - f\gamma_i}{c} \\ \hat{e}_i &= \gamma_i(\hat{q}_i - \frac{f\gamma_i}{k(1-s)})\end{aligned}$$

This in turn allows to compute aggregate values for production, emissions and abatement expenditures as follows:

$$\begin{aligned}\hat{Q} &= \frac{n\,(p + \tau - f\bar{\gamma})}{c} \\ \hat{E} &= n\bar{\gamma}\frac{p+\tau}{c} - nf(\frac{1}{c} + \frac{1}{k(1-s)})(\sigma_\gamma^2 + \bar{\gamma}^2) \\ \hat{A} &= \frac{nf^2(\sigma_\gamma^2 + \bar{\gamma}^2)}{2k(1-s)^2}.\end{aligned}$$

We look for τ, s and f such that the system (8),(9),(10) hold for the above values of $\hat{Q}$, $\hat{E}$ and $\hat{A}$. We show in Appendix D that there are two solutions (τ_1, s_1, f_1) and (τ_2, s_2, f_2), say respectively solution 1 and 2. Following the remark above, we also impose the additional constraint that a refunding policy should benefit the industry as a whole to our search and it appears that this holds only for solution 1 as explained below.

We depict the tax t and the two solutions 1 and 2 in Figure 2, as a function of the initial tax t for a specific set of parameters.[10] Figure 2 panel (a) indicates that solution 1 entails a tax on abatement and tax refunding through output subsidy along with emissions being overtaxed compared to the initial tax t. On the contrary, Figure

[10] We use $p = 15$, $c = 1$, $k = 1/2$, $\bar{\gamma} = 0.7$, $\sigma_\gamma = 0.3$, $\delta = 1$ and $n = 1$. Tax t is allowed to vary between 1 and 6. A tax $t > 1$ ensures a corresponding abatement subsidy $s < 1$. At $t = 6$, aggregate pollution cancels out.

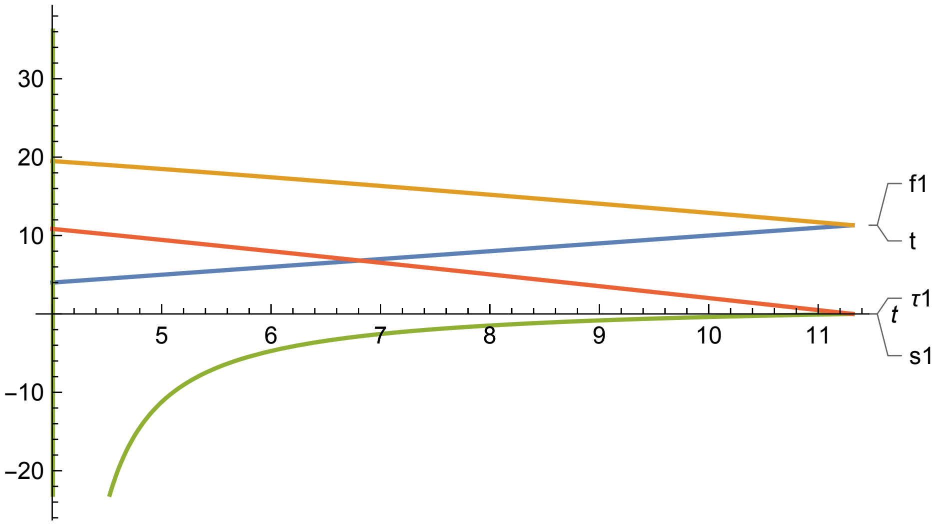
2 panel (b) shows that solution 2 entails a tax on output and tax refunding through abatement subsidy along with emissions being undertaxed.

By construction, both solutions lead to same aggregate production Q^* and emissions E^* depicted on Figure 3 panel (a), but they differ in terms of aggregate abatement expenditures and cost of production: While solution 1 entails lower abatement expenditures but higher production cost ($\hat{C}_1 \geq C^*$, $\hat{A}_1 \leq A^*$), solution 2 entails higher abatement expenditures but lower production costs ($\hat{C}_2 \leq C^*$, $\hat{A}_2 \geq A^*$) at the aggregate level. This is because the solution 1 implies a reallocation of production (and pollution) towards firms with low values of γ while it is the reverse for the solution 2. As shown by Figure 3 panel (b), it turns out that only the three-part refunding policy (τ_1, s_1, f_1) is beneficial for the industry, compared to the standard tax benchmark.

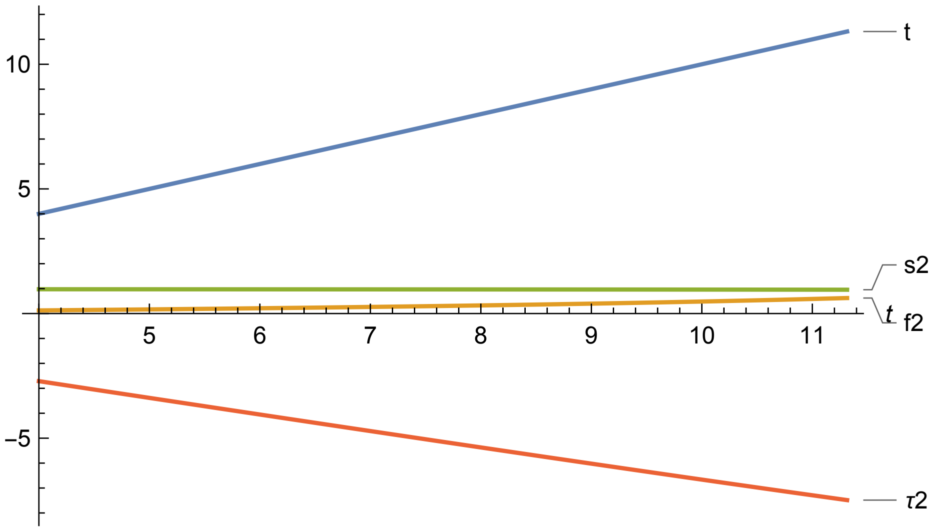
More precisely, for the solution 1, we prove in Appendix E that $\hat{q}_i > q_i^*$ iff $\gamma_i < \bar{\gamma}$ and that $\hat{e}_i > e_i^*$ iff $\gamma_i < \bar{\gamma} + \sigma_\gamma^2/\bar{\gamma}$. The refunding policy (τ_1, s_1, f_1) thus implies that large firms (with low γ) are producing more and are polluting more at the expense of smaller firms, compared to the standard tax benchmark. As aggregate production and pollution remain unchanged, the redistribution operated by this uniform three-part refunding policy is clearly in favor of the larger firms.

5 Conclusion

In this paper, we have shown that a three-part refunding policy can help to alleviate the drawbacks of either pure output based refunding or pure abatement expenditure based refunding. In particular, when the regulated industry is homogenous, it is possible to replicate the standard emission tax outcome using such a policy: with the appropriate definition of the fee and of the output and abatement subsidies, the three-part refunding policy induces the polluting firm to choose the production and emission levels obtained under any emission tax, without departing from budget balance. By construction, the proposed regulation scheme entails that the polluter earns strictly more than under the

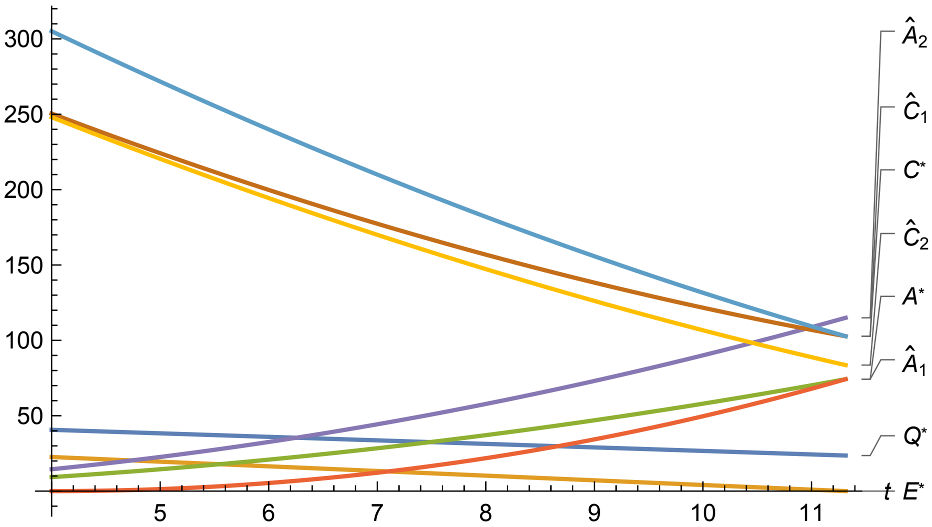


(a): Tax on abatement ($s < 0$), emissions are overtaxed ($f_1 > t$) and refunding through output subsidy ($\tau_1 > 0$).

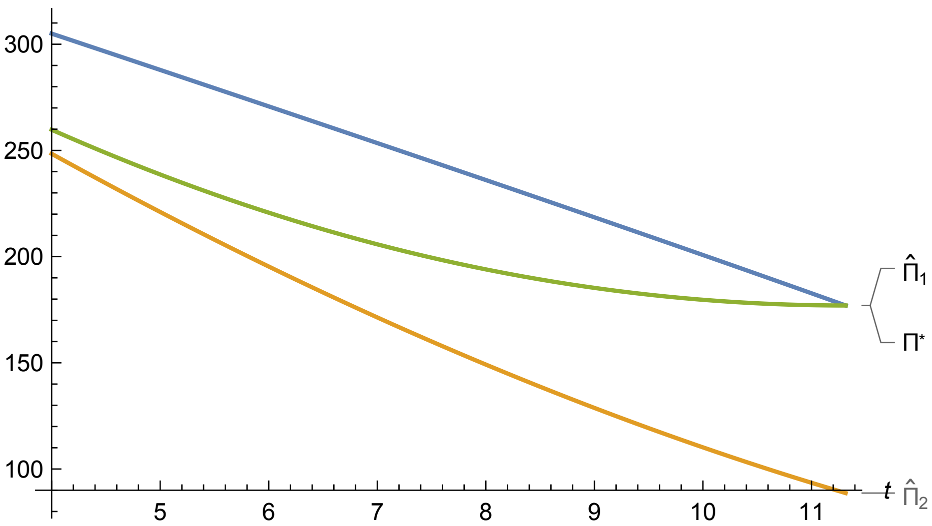


(b): Tax on output ($\tau_2 < 0$), emissions are undertaxed ($f_2 < t$) and refunding through abatement subsidy ($s > 0$).

Figure 2: Solutions 1 and 2 for a uniform three-part refunding policy.



(a): Production (Q^*), emissions (E^*), costs (C^*, $\hat{C}_1$ and $\hat{C}_2$) and abatement expenditures (A^*, $\hat{A}_1$ and $\hat{A}_2$).



(b): Industry's profit (Π^*, $\hat{\Pi}_1$ and $\hat{\Pi}_2$).

Figure 3: Production, emissions, costs and profits under uniform three-part refunding policies.

standard tax alone without rebate, thereby making this proposal highly acceptable.

When the industry is heterogenous, the refunding policy needed to replicate the standard emission tax outcome is now personalized in the sense that at least the output subsidy should be type dependent. Another result is that this three-part policy is strictly preferred from the industry's point of view to a standard environmental tax. By specifying the model, we also show that a uniform three-part refunding policies can also replicate standard tax outcome in the aggregate at the sector's level with respect to production and emissions without departing from the budget constraint.

Overall, in this paper, we have shown that there are ways to design refunded pollution taxes so that negative impacts on the industry's profit are attenuated while still inducing firms to limit emissions. Obviously, a refunding policy lessens the industry's reluctance to the pollution tax, but this comes at the drawback of making entry easier in the market which may impair pollution reduction in the long run (see Cato, 2010, for a possible remedy based on an entry license). Also, such policies however require information on emissions, outputs and abatement expenditures. It would thus be interesting to investigate the design of such refunding policies when these informations are private and/or can be observed at a cost by the regulator (see e.g. Bontems and Bourgeon, 2005, and Macho-Stadler and Perez-Castrillo,2006, for analysis of pollution taxes under costly observability of pollution). Finally, another interesting extension would be to analyze the design of three part refunding policies for imperfectly competitive market structures. We leave this to future research.

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Appendix

A Comparative statics

Dropping the firm's index for the sake of clarity and introducing a positive scale parameter θ for abatement cost function, the system (1) rewrites as:

$$\begin{aligned} p &= c'(q^*) + \theta a_q(q^*, e^*) \\ -\theta a_e(q^*, e^*) &= t. \end{aligned}$$

Differentiating totally this system and dropping arguments, we obtain:

$$\begin{aligned} dq &= \frac{1}{\Delta}\left[a_{qe}dt + a_{ee}dp - (a_q a_{ee} - a_e a_{qe})d\theta\right] \\ de &= \frac{1}{\Delta}\left[(a_e c'' - \theta(a_e a_{qq} - a_q a_{qe}))d\theta - (c'' + \theta a_{qq})\, dt - \theta a_{qe} dp\right] \end{aligned}$$

where $\Delta = a_{ee}c'' + \theta(a_{ee}a_{qq} - a_{qe}^2) > 0$ under quasi-convexity of total cost $c(.) + a(.,.)$. We obtain that:

$$\frac{\partial q}{\partial p} = \frac{1}{\Delta}a_{ee} > 0 \text{ and } \frac{\partial e}{\partial t} = -\frac{c'' + \theta a_{qq}}{\Delta} < 0.$$

Also, under assumption 1, we obtain that:

$$\frac{\partial q}{\partial t} = \frac{1}{\Delta}a_{qe} < 0 \text{ and } \frac{\partial e}{\partial p} = -\frac{a_{qe}}{\Delta} > 0.$$

Finally, assumption 2 allows to state that:

$$\frac{\partial q}{\partial \theta} = -\frac{(a_q a_{ee} - a_e a_{qe})}{\Delta} < 0 \text{ and } \frac{\partial e}{\partial \theta} = -\frac{(a_e a_{qq} - a_q a_{qe}))}{\Delta} > 0.$$

B Proof of Proposition 2

To get $\hat{q}(\tau, s, f) = q^*$ and $\hat{e}(\tau, s, f) = e^*$, in view of the FOCs (2) and (5), it is sufficient to take

$$\begin{aligned} f &= t + s a_e^* = (1 - s)t \\ \tau &= -s a_q^*. \end{aligned}$$

Also, the subsidy s is given by the budget constraint (3) that now writes:

$$nfe^* = n\tau q^* + nsa^* + R \tag{11}$$

or equivalently with $R = (1-\delta)tne^*$

$$\begin{aligned} (t + sa_e^*)\, e^* &= -sa_q^* q^* + sa^* + (1-\delta)te^* \\ s &= \frac{\delta te^*}{a^* - a_e^* e^* - a_q^* q^*} = \frac{\delta te^*}{\sigma^* a^*} \end{aligned}$$

where $\sigma^* = 1 - \varepsilon_{a,e}^* - \varepsilon_{a,q}^*$ with $\varepsilon_{a,e}^* = a_e^* e^*/a^* < 0$ and $\varepsilon_{a,q}^* = a_q^* q^*/a^* > 0$ and recalling that $t = -a_e^*$ from (1). Observe that $\sigma^* = 0$ needs to be ruled out otherwise the budget equality constraint cannot hold.

Also, to preserve quasi concavity of the polluter's program, we need $s < 1$ which amounts to assume that

$$\sigma^* > \frac{\delta te^*}{a^*}$$

whenever is is positive.

Finally, by construction, the difference in terms of net profit is

$$\begin{aligned} \hat{\pi}(\tau, s, f) - \pi^* &= (p+\tau)q^* - c(q^*) - (1-s)a^* - fe^* - pq^* + c(q^*) + a^* + te^* \\ &= \tau q^* + sa^* + (t-f)e^* \\ &= fe^* - (1-\delta)te^* + (t-f)e^* = \delta te^* \end{aligned}$$

by using the budget constraint.

C Proof of Proposition 5

To get $\hat{q}_i = q^*$ and $\hat{e}_i = e^*$, in view of the FOCs (1) and (7), it is sufficient to take

$$\begin{aligned} f_i &= (1-s_i)t \\ \tau_i &= -s_i a_{iq}^*. \end{aligned} \tag{12}$$

Moreover, the budget constraint writes:

$$\sum_i f_i e_i^* = \sum_i \tau_i q_i^* + \sum_i s_i a_i^* + (1-\delta)tE^* \tag{13}$$

Using (12) and replacing in (13), we get

$$\sum_i (1-s_i)te_i^* = -\sum_i s_i a_{iq}^* q_i^* + \sum_i s_i a_i^* + (1-\delta)tE^*$$

which implies

$$\sum_i \sigma_i^* a_i^* s_i = \delta t E^* \tag{14}$$

with $\sigma_i^* = 1 - \varepsilon_{a_i,e}^* - \varepsilon_{a_i,q}^*$.

Moreover, the difference between net profits is:

$$\begin{aligned} \hat{\pi}_i - \pi_i^* &= (p+\tau_i)\, q_i^* - c_i(q_i^*) - (1-s_i)a_i(q_i^*, e_i^*) - f_i e_i^* - (pq_i^* - c_i(q_i^*) - a_i(q_i^*, e_i^*) - te_i^*) \\ &= \tau_i q_i^* + s_i a_i^* + (t - f_i)e_i^* \end{aligned}$$

Replacing with the values obtained for the instruments τ_i and f_i and recalling that $t = -a_{ie}^*$, we obtain:

$$\hat{\pi}_i - \pi_i^* = s_i \left(a_i^* - a_{iq}^* q_i^* - a_{ie}^* e_i^* \right) = s_i a_i^* \sigma_i^*$$

Summing over i and using (14), we thus get $\hat{\Pi} - \Pi^* = \sum_i \sigma_i^* a_i^* s_i = \delta t E^* > 0$.

D Example

From the expression for individual production,

$$\hat{q}_i = \frac{p + \tau - f\gamma_i}{c}$$

we get by summing over i :

$$\hat{Q} = \frac{n\,(p + \tau - f\bar{\gamma})}{c}$$

Also from

$$\hat{e}_i = \gamma_i(\hat{q}_i - \frac{f\gamma_i}{k(1-s)})$$

we obtain similarly

$$\begin{aligned}\hat{E} &= \sum_i n_i\gamma_i(\hat{q}_i - \frac{f\gamma_i}{k(1-s)}) \\ &= \sum_i n_i\gamma_i(\frac{p+\tau-f\gamma_i}{c} - \frac{f\gamma_i}{k(1-s)}) \\ \hat{E} &= n\bar{\gamma}\frac{p+\tau}{c} - nf(\frac{1}{c}+\frac{1}{k(1-s)})(\sigma_\gamma^2+\bar{\gamma}^2)\end{aligned}$$

Finally, for abatement expenditures we obtain:

$$\hat{A} = \sum_i n_i \frac{k(\hat{q}_i - \hat{e}_i/\gamma_i)^2}{2} = \frac{f^2}{2k(1-s)^2}\sum_i n_i\gamma_i^2 = \frac{nf^2(\sigma_\gamma^2+\bar{\gamma}^2)}{2k(1-s)^2} \quad (15)$$

The system to be solved in (τ, s, f) rewrites as follows:

$$\hat{Q} = Q^* \quad (16)$$

$$\hat{E} = E^* \quad (17)$$

$$f\hat{E} = \tau\hat{Q} + s\hat{A} + (1-\delta)tE^* \quad (18)$$

Equation (16) can be expressed as

$$\frac{n\,(p+\tau-f\bar{\gamma})}{c} = \frac{n\,(p-t\bar{\gamma})}{c}$$

from which we deduce that

$$\tau = (f-t)\bar{\gamma} \quad (19)$$

Also, equation (17) writes as

$$n\bar{\gamma}\frac{p+\tau}{c} - nf(\frac{1}{c}+\frac{1}{k(1-s)})(\sigma_\gamma^2+\bar{\gamma}^2) = n\bar{\gamma}\frac{p}{c} - nt(\frac{1}{c}+\frac{1}{k})(\sigma_\gamma^2+\bar{\gamma}^2)$$

and replacing τ using (19) allows to obtain

$$\bar{\gamma}^2\frac{(f-t)}{c} - f(\frac{1}{c}+\frac{1}{k(1-s)})(\sigma_\gamma^2+\bar{\gamma}^2) = -t(\frac{1}{c}+\frac{1}{k})(\sigma_\gamma^2+\bar{\gamma}^2)$$

which simplifies into

$$\frac{f}{1-s} = t + \lambda(t-f) \quad (20)$$

where $\lambda \equiv \frac{k}{c}\frac{\sigma_\gamma^2}{\sigma_\gamma^2+\bar{\gamma}^2} \geq 0$. We can deduce f as a function of s :

$$f = t\frac{(1+\lambda)(1-s)}{1+\lambda(1-s)}$$

When there is homogeneity, $\sigma_\gamma^2 = 0$ and thus $\lambda = 0$ and we recover $f = (1-s)t$ as in Proposition 2.

Last, using (16) and (17), the budget constraint (18) rewrites simply as

$$\begin{aligned} \left[f - t(1-\delta)\right] E^* &= \tau Q^* + s\hat{A} \\ \left[f - t(1-\delta)\right] E^* &= (f-t)\bar{\gamma}Q^* + s\hat{A} \end{aligned}$$

or using (15) and the definition of E^* and Q^*:

$$\left[f - t(1-\delta)\right] E^* = (f-t)\bar{\gamma}Q^* + s\frac{nf^2(\sigma_\gamma^2+\bar{\gamma}^2)}{2k(1-s)^2}$$

This allows to compute $s/(1-s)$ as a function of f :

$$\frac{s}{1-s} = \frac{2k}{\sigma_\gamma^2+\bar{\gamma}^2}\frac{f(E^*-\bar{\gamma}Q^*)+t(\bar{\gamma}Q^*-(1-\delta)E^*)}{t+\lambda(t-f)}$$

Hence, it follows that:

$$\frac{1}{1-s} = 1 + \frac{2k}{\sigma_\gamma^2+\bar{\gamma}^2}\frac{f(E^*-\bar{\gamma}Q^*)+t(\bar{\gamma}Q^*-(1-\delta)E^*)}{t+\lambda(t-f)}$$

and replacing in (20):

$$f\left(1 + \frac{2k}{\sigma_\gamma^2+\bar{\gamma}^2}\frac{f(E^*-\bar{\gamma}Q^*)+t(\bar{\gamma}Q^*-(1-\delta)E^*)}{t+\lambda(t-f)}\right) = t+\lambda(t-f)$$

which amounts to solve a polynomial equation of degree 2 in f. We denote the two solutions f_1 and f_2. And the corresponding values of τ and s by respectively τ_1, τ_2, s_1 and s_2.

E Reallocation of production and pollution between firms

We compare $\hat{q}_i$ and $\hat{e}_i$ taken at the solution 1 (f_1, τ_1, s_1) with their counterparts q_i^* and e_i^* in the standard tax benchmark. Let us start with quantities:

$$\begin{aligned}\hat{q}_i - q_i^* &= \frac{p + \tau_1 - f_1\gamma_i}{c} - \frac{p - t\gamma_i}{c} \\ &= \frac{\tau_1 + (t - f_1)\gamma_i}{c}\end{aligned} \tag{21}$$

From (19), we know that $\tau_1 = (f_1 - t)\bar{\gamma}$ and replacing in (21), we get:

$$\hat{q}_i - q_i^* = \frac{(f_1 - t)(\bar{\gamma} - \gamma_i)}{c}$$

As solution 1 entails over-taxation of emissions $(f_1 > t)$, we obtain that $\hat{q}_i > q_i^* \Leftrightarrow \gamma_i < \bar{\gamma}$.

Now we compare the emission levels:

$$\begin{aligned}\hat{e}_i - e_i^* &= \gamma_i(\hat{q}_i - \frac{f_1\gamma_i}{k(1-s_1)}) - \gamma_i(q_i^* - \frac{t\gamma_i}{k}) \\ &= \gamma_i\left(\frac{(f_1 - t)(\bar{\gamma} - \gamma_i)}{c} + \frac{\gamma_i}{k}\left(t - \frac{f_1}{1 - s_1}\right)\right)\end{aligned} \tag{22}$$

Using (20) and replacing in (22), we obtain:

$$\begin{aligned}\hat{e}_i - e_i^* &= \gamma_i\left(\frac{(f_1 - t)(\bar{\gamma} - \gamma_i)}{c} - \frac{\gamma_i}{k}\lambda(t - f_1)\right) \\ &= \gamma_i(f_1 - t)\left(\frac{\bar{\gamma} - \gamma_i}{c} + \frac{\gamma_i}{k}\lambda\right)\end{aligned}$$

Recall that $\lambda \equiv \frac{k}{c}\frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \bar{\gamma}^2}$ so that by rearranging,

$$\hat{e}_i - e_i^* = \frac{\gamma_i(f_1 - t)\bar{\gamma}^2}{c(\sigma_\gamma^2 + \bar{\gamma}^2)}\left(\frac{\sigma_\gamma^2}{\bar{\gamma}} + \bar{\gamma} - \gamma_i\right)$$

Once again, using that $f_1 > t$, it follows that $\hat{e}_i > e_i^* \Leftrightarrow \gamma_i < \bar{\gamma} + \frac{\sigma_\gamma^2}{\bar{\gamma}}$.