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Contracting Sequentially with Multiple Lenders: the Role of Menus

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Abstract

We study a capital market in which multiple lenders sequentially attempt at financing a single borrower under moral hazard. We show that restricting lenders to post take-it-or-leave-it offers involves a severe loss of generality: none of the equilibrium outcomes arising in this scenario survives if lenders offer menus of contracts. This result challenges the approach followed in standard models of multiple lending. From a theoretical perspective, we offer new insights on equilibrium robustness in sequential common agency games.

Keywords: Multiple Lending, Menus, Strategic Default, Common Agency, Bank Competition.

JEL Classification: D43, D82, G33.

1 Introduction

When firms negotiate with a bank, they commonly have access to a wide range of financial offers: loans of different size, maturity, interest rate and repayment schedule, with or without guarantees, or associated credit lines. Starting with the seminal work of Bester (1985), theoretical models of bank competition have rationalised the issuance of multiple financial contracts by a given bank as a device to screen different privately informed borrowers.[1] In this note, we argue that allowing banks to post menus of financial contracts, instead of take-it-or-leave-it offers, crucially affects equilibrium allocations even when there is complete information about the borrowers' characteristics.

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[1]See chapter 5 of Freixas and Rochet (1997) for an overview.

Our starting point is that firms do not always have fixed-size investment needs, that they fulfil by borrowing from a single bank at a single time. In practice, firm can choose variable investment amounts, at different periods, and borrow from several creditors.[2] This in turn can affect competition among banks. Indeed, when firms can borrow from different lenders who do not perfectly coordinate their offers, a creditor can be affected by his borrower's future debt issuances. If the borrower subsequently issues additional debt, a well known effect is that the default probability on the initial debt can increase, potentially inducing welfare losses. In this note, we explore how posting menus including several financial contracts can help creditors to protect themselves from this risk of debt dilution.

We consider a sequential version of the capital market described by Holmstrom and Tirole (1998), in which lenders sequentially make offers to a borrower, who then chooses an unobservable effort. Restricting attention to a two-lender case, we consider two scenarios: first, lenders can only make take-it-or-leave-it offers, then, they can post menus of financial contracts. We start by showing that, if the first lender is restricted to single offers, he cannot prevent being undercut by the second one. Thus, the first lender earns zero profit in any pure strategy equilibrium. We next show that the opportunity to choose menus of contracts crucially affects market equilibria. Indeed, by posting a menu of two non degenerate contracts, the first lender is able to prevent any profitable entry from the subsequent lender, therefore earning a monopolistic profit. As a consequence, none of the equilibrium outcomes arising when lenders are restricted to take-it-or-leave-it-offers survives the introduction of menus.

The literature on multiple lending has traditionally investigated two sets of issues. A first branch aims at understanding when it can be optimal for a firm to have several rather than a single lender. In these contexts, multiple lending can be an optimal response to the provision of monitoring activities (Winton (1995) and Park (2000)), firms' willingness to default (Bolton and Scharfstein (1996)), or informed lenders' ability to extract rents (Rajan (1992) and Berglof and von Thadden (1994)). Closer to our perspective, a second branch considers that multiple lending can arise from lenders' impossibility to fully control borrowers' trades, as in Bizer and DeMarzo (1992), Parlour and Rajan (2001), Bennardo et al. (2015), Kahn and Mookherjee (1998), Bisin and Guaitoli (2004), Brunnermeier and Oehmke (2013), Castiglionesi and Wagner (2012), Castiglionesi et al. (2015), and Donaldson et al. (2017). The above multiple lending models typically assume complete information over the borrower's characteristics, and restrict agents to post take-it-or-leave-it offers.

[2] Multiple banking relationships are pervasive in credit markets: Ongena and Smith (2000) document that the average number of bank relationships is between 5 and 6 in their sample of 1129 European firms. This is also true for small firms: Guiso and Minetti (2010) report that among small and medium sized US firms, half of those that borrow at all have more than one lender. See also Detragiache et al. (2000).

Our analysis suggests that lenders have instead an incentive to use menus of contracts to discipline their competitors.

From a theoretical standpoint, our findings can be interpreted in the light of the common agency literature, which analyses games in which principals compete through mechanisms in the presence of a single agent. Under complete information about the agent's characteristics, it is an established finding that restricting to take-it-or-leave-it offers involves a loss of generality, i.e. there may exist additional equilibria supported by more sophisticated mechanisms.[3] A second, and possibly more relevant, issue is to determine the *robustness* of equilibria in take-it-or-leave-it offers, that is, whether they survive to unilateral deviations towards arbitrary mechanisms. In our example, none of such equilibria survives if a principal deviates to a menu of contracts.[4] Indeed, considering the game in which menus are allowed, there exists a best response of the first investor to the equilibrium strategy of his opponent that cannot be characterized through simple take-it-or-leave-it offers. This suggests that further work is needed to identify robust equilibria in sequential common agency games.[5]

Overall, our results indicate that the financial instruments available to lenders are a key element to take into account when modelling competition in banking.

This note is organized as follows. Section 2 presents the model, section 3 analyzes the game in which banks are restricted to take-it-or-leave-it offers and Section 4 that in which banks post menus of contracts. Proofs are in the Appendix.

2 The model

We refer to the standard capital market model of Holmstrom and Tirole (1998), as reformulated by Attar et al. (2017). A risk-neutral entrepreneur has an endowment $A > 0$ and a variable size project: an investment of $I \in \mathbb{R}_+$ yields a verifiable cash flow GI if the project succeeds and 0 if it fails. The probability of success π_e depends on the entrepreneur's binary unobservable effort $e = \{L, H\}$, with $\pi_H > \pi_L$. If the entrepreneur

[3] The result, acknowledged as a failure of the revelation principle in common agency games, is documented in several examples (see, among others, Martimort and Stole (2002), and Peters (2001) for games of simultaneous offers, and Pavan and Calzolari (2009) for games of sequential offers).

[4] Restricting attention to the case in which principals make their offers simultaneously, Peters (2003) shows that every (pure strategy) equilibrium outcome of the take-it or leave-it offer game is also a (pure strategy) equilibrium outcome of the menus game. In this case, enlarging the strategy space of a single principal, while holding fixed the behavior of his rivals, does not create room for additional deviations. The result, established in Theorem 1 of Peters (2003), has typically been taken as a rationale for restricting attention to take-it or leave-it offers in economic applications. Our analysis shows that it does not extend to sequential settings.

[5] The role of menus in sequential common agency is also considered by Calzolari and Pavan (2008) who show that there exist equilibrium outcomes supported by indirect mechanisms that cannot be reproduced by menus. Here, we take a different perspective, and show that principals may find profitable to use menus when their opponents post take-it or leave-it offers.

selects $e = L$, she receives a private benefit $B \in \mathbb{R}_+$ per unit invested. The project has a positive net present value if and only if the entrepreneur selects $e = H$, that is:

$$\pi_H G > 1 > \pi_L G + B. \tag{1}$$

Asi in Holmstrom and Tirole (1998), to ensure that the optimal (second-best) investment if finite, we assume that

$$0 < \pi_H G - \frac{\pi_H B}{\Delta\pi} < 1, \tag{2}$$

where $\Delta\pi = \pi_H - \pi_L$.

The entrepreneur is protected by limited liability and can raise funds from two competing investors. If she raises I and invests $I + A$, pays back R in case of success, and 0 in case of failure,[6] her net payoff is $U(I, R, H) = \pi_H\big(G(I + A) - R\big) - A$ if $e = H$, and $U(I, R, L) = \pi_L\big(G(I + A) - R\big) + B(I + A) - A$ if $e = L$. Her reservation utility, $U(0) = (\pi_H G - 1)A$, is strictly positive given (1). The expected profit of investor i when he provides I_i and obtains R_i in case of success, is $V_i = \pi_e R_i - I_i$ with $e \in \{L, H\}$.

Lenders offer menus, that is sets of financial contracts. Formally, a financial contract for lender i is $C_i = (I_i, R_i(.))$, where I_i is his investment, and $R_i(.) : \mathbb{R}_+ \to \mathbb{R}$ is the repayment he asks for when the project succeeds, as a function of the total cash flow.[7] The competition game unfolds as follows:

a) Investor 1 offers a menu of contracts M_1. Investor 2 observes M_1 and offers M_2.

b) Having observed M_1 and M_2, the entrepreneur chooses one contract in each menu and an effort level.[8]

c) The cash flow is realized and payments are made.

There is perfect information over investors' offers. A pure strategy for investor 1 is a menu M_1, and a pure strategy for investor 2 is a mapping that associates a menu M_2 to every M_1. A pure strategy for the entrepreneur associates to each array of menus the choice of one contract in each menu and an effort level. We focus on pure strategy subgame perfect equilibria (SPE).

Given limited liability, the entrepreneur can trade contracts which involve conflicting prescriptions, and induce *strategic default*. This is the case if $I = I_1 + I_2$ is such that $R = R_1(I) + R_2(I) > G(I + A)$. Under strategic default, the entrepreneur chooses $e = L$, and obtains $B(I_1 + I_2 + A) - A$. Investors

[6] Given the entrepreneur's limited liability, repayment is always zero if the project fails.
[7] Equivalently, the repayment can be contingent on the aggregate investment.
[8] Menus always include the null contract $C_0 = (0, 0)$ to incorporate the entrepreneur's participation decisions in a simple way.

cannot obtain R but receive a share of the final cash flow proportional to their investment. Because of (1), investors collectively make negative profit. We denote Ψ the set of aggregate investment-repayment pairs $(I, R) \in \mathbb{R}^2_+$ such that the entrepreneur is indifferent between $e = H$ and $e = L$, and call it the incentive frontier. On Ψ, we denote (I^m, R^m) the monopolistic allocation that maximizes the investors' profit subject to the entrepreneur's participation.[9] Last, in this example, we assume that

$$\pi_H < 2\pi_L, \tag{3a}$$

$$B > (\pi_H G - 1). \tag{3b}$$

3 The take-it or leave-it offers game

We first consider the scenario in which investors can at most post one non-degenerate contract. In this take-it-or-leave-it offers game, investor 2 successfully undercuts any profitable offer of investor 1, as illustrated by the following:

Proposition 1 *Investor 1 earns zero profit in any SPE. In addition, each profit level between zero and the monopolistic one for investor 2 can be supported at equilibrium.*

Intuitively, if investor 1 proposes a contract that grants him a strictly positive payoff, investor 2 can always select an investment I_2 and a repayment function $R_2(.)$ that undercut investor 1's offer. The repayment is designed to discourage the entrepreneur from over-investing by accepting both loans at a time. Investor 2 then appropriates all available rents providing exclusive financing to the entrepreneur, who optimally chooses $e = H$.

Investor 1 can therefore be active at equilibrium only by trading a zero-profit loan contract. We show that market equilibria can be constructed by letting the entrepreneur accept any zero-profit contract posted by investor 1, and complementing it with an additional offer of investor 2. The corresponding allocations are constrained (second-best) efficient and typically yield a strictly positive profit to investor 2.

4 Menus and sequential contracting

We then consider the general framework described in Section 2. In this menu game, the undercutting of investor 2 may be successfully prevented by the threats that investor 1 includes in his equilibrium menu. These

[9] Formally, $(I^m, R^m) \in \arg\max_{(I,R)} \pi_H R - I$ s.t. $U(I, R, H) \geq U(I, R, L)$ and $U(I, R, H) \geq U(0)$. In the solution, both constraints are binding and $(I^m, R^m) \in \Psi$.

threats take the form of additional, large investment contracts, designed to be traded by the entrepreneur together with any deviating offer of investor 2. The corresponding over-borrowing induces strategic default, and supports a positive profit for investor 1 at equilibrium. Specifically, we have the following:

Proposition 2 *In the unique SPE, investor 1 earns the monopolistic profit $V_1^* = \pi_H R^m - I^m$, and investor 2 earns zero profit.*

The proof of Proposition 2 shows, in addition, that the best response of investor 1 to a take-it-or-leave-it offer posted by his follower crucially involves the use of menus. Indeed, the opportunity to post additional offers is key to control the subsequent entrepreneur's behavior, and therefore to prevent any undercutting. Overall, allowing for menus dramatically modifies the market power of investor 1, and the distribution of equilibrium rents between investors.

In our example, equilibrium menus have a natural interpretation. The menu posted by the first investor includes a contract, not traded at equilibrium, which specifies a high amount of investment. This contract serves the role of a threat. The borrower finds optimal to trade it, and default on her aggregate loans, whenever the second lender tries to enter the market. This additional offer, therefore, prevents the dilution of the first debt contract. In practice, the option to obtain a large loan is embedded in many loan contracts. For instance, banks do not set a priori a loan amount that they are willing to lend. Instead, their credit committees define an internal credit limit, e.g. a maximum credit exposure, with each client.[10] Banks also grant lines of credit to their clients, that allow firms to obtain additional funds from their bank without negotiating new loans.[11] Our result suggests that lenders can strategically exploit these contractual features to increase their profits.

5 Conclusion

In this note, we highlight that whether lenders can offer multiple or single contracts affects their ability to control firms' borrowing policy, and the resulting competition outcome. In addition, the equilibrium menus offered by lenders feature similarities with actual lending practices, which sheds light on the strategic role of credit lines: lenders offer loans with credit lines to discourage future debt issuance and protect lenders' rents. This provides new insights on the role of credit lines in capital markets.

[10]See Degryse et al. (2016) for an empirical analysis of the impact of multiple lending on banks' internal credit limit.
[11]See Sufi (2009) for an empirical analysis of lines of credit.

Appendix

PROOF OF PROPOSITION 1

The proof establishes the following two Lemmas.

Lemma 1 *Investor 1 earns zero profit in any SPE.*

Proof. Suppose, by contradiction, that there is a SPE in which investor 1 earns a positive profit. Denote $(I_i^*, R_i^*(.))$ the contract that the entrepreneur trades with investor $i = 1, 2$ at equilibrium, with $I_1^* > 0$, $I_2^* \geq 0$, and $I^* = I_1^* + I_2^*$. The entrepreneur's equilibrium payoff $U^* = U\big(I_1^* + I_2^*, R_1^*(I^*) + R_2^*(I^*), H\big)$ must be such that

$$U^* \geq \underline{U}(I_1^*, R_1^*(I_1^*), H), \tag{4}$$

where $\underline{U}(I_1^*, R_1^*(I_1^*), H) = \max\{U(0), U\big(I_1^*, R_1^*(I_1^*), H\big), U(I_1^*, (G - \frac{B}{\Delta\pi})(I_1^* + A), H)\big)\}$. Indeed, U^* is necessarily larger than the reservation payoff $U(0)$, and than $U\big(I_1^*, R_1^*(I_1^*), H\big)$, the payoff corresponding to investing $I_1^* + A$ and choosing $e = H$. Also, given that $I_1^* \leq I^*$, and since $e = H$ is chosen at equilibrium, we have

$$U(I_1^*, (G - \frac{B}{\Delta\pi})(I_1^* + A), H) \leq U(I^*, (G - \frac{B}{\Delta\pi})(I^* + A), H) \leq U^*, \tag{5}$$

which implies (4).[12] For a given $(I_1^*, R_1^*(I_1^*))$, denote $(\bar{I}_2, \bar{R}_2) \in \Psi$ the investment-repayment such that $U\big(\bar{I}_2, \bar{R}_2, H\big) = \underline{U}(I_1^*, R_1^*(I_1^*), H)$. Existence and the uniqueness of $(\bar{I}_2, \bar{R}_2)$ are guaranteed by the continuity and the linearity of $U(I, R, H)$. It also follows from the definition of $\underline{U}(I_1^*, R_1^*(I_1^*), H)$ that $\bar{I}_2 \geq I_1^*$.

We then turn to investors' equilibrium profit. By assumption, $V_1^* = \pi_H R_1^*(I^*) - I_1^* > 0$, and $V_2^* = \pi_H R_2^*(I^*) - I_2^* \geq 0$. Thus, we have $V_2^* < V_1^* + V_2^* \leq \min\{(\pi_H G - 1)I^m, \pi_H \bar{R}_2 - \bar{I}_2\}$, where the first inequality follows from $V_1^* > 0$, and the second one from $U^* \geq \underline{U}(I_1^*, R_1^*(I_1^*)) = U(\overline{I}_2, \overline{R}_2, H)$.

We next show that there exists a pure strategy for investor 2 yielding him a profit $V_2' > V_2^*$. For each offer $(I_1, R_1(.))$ of investor 1, let investor 2 post the take-it or leave-it offer $(I_2, R_2^\varepsilon(.))$ where I_2 is such that $\frac{\pi_H}{\Delta\pi} B(I_2 + A) - A = \underline{U}(I_1, R_1(I_1), H)$, $R_2^\varepsilon(I) = G(I + A)$ for $I \neq I_2$, and $R_2^\varepsilon(I_2) = (G - \frac{B}{\Delta\pi})(I_2 + A) - \varepsilon$,

[12] If a investment-repayment pair (I, R) belongs to the incentive frontier Ψ, it must be that $R = (G - \frac{B}{\Delta\pi})(I + A)$, that is, $U(I, (G - \frac{B}{\Delta\pi})(I + A), H) = U(I, (G - \frac{B}{\Delta\pi})(I + A), L)$. It follows that, since $e = H$ in any SPE, the equilibrium repayment $R_1^*(I^*) + R_2^*(I^*) = R^*(I^*)$ must be smaller than $(G - \frac{B}{\Delta\pi})(I^* + A)$, otherwise the borrower would choose $e = L$. This guarantees that the second inequality in (5) is satisfied.

with $\varepsilon > 0$. Observe that, if investor 1 posts $(I_1^*, R_1^*(.))$, the strategy above prescribes investor 2 to set $I_2 = \bar{I}_2$. We now show that, following this strategy, investor 2 successfully undercuts $(I_1^*, R_1^*(.))$ by inducing the entrepreneur to invest only $I_2 + A$ selecting $e = H$, which yields him a profit strictly above the equilibrium one.

Indeed, if the entrepreneur selects $e = H$, her (unique) optimal choice is to raise I_2 only: $R_2^{\varepsilon}(.)$ is such that if the entrepreneur raises $I_2 + I_1$, she optimally chooses $e = L$. Furthermore, since $U(\bar{I}_2, \bar{R}_2 - \varepsilon, H) = U(\bar{I}_2, \bar{R}_2, H) + \pi_H \varepsilon > \underline{U}(I_1^*, R_1^*(I_1^*), H)$, the entrepreneur strictly prefers to raise I_2 only, rather than raising I_1^* only, or raising nothing.[13] It remains to show that the entrepreneur optimally chooses $e = H$ at the deviation stage. Since $U(\bar{I}_2, \bar{R}_2 - \varepsilon, H) > U(\bar{I}_2, \bar{R}_2 - \varepsilon, L)$ by construction, we only have to consider the alternative situation in which she chooses $e = L$ and raises $I_1^* + \bar{I}_2$. In this case, given $R_2^{\varepsilon}(.)$, the entrepreneur strategically defaults and gets

$$B(I_1^* + \bar{I}_2 + A) - A < B\left(\frac{\pi_L}{\Delta\pi} + 1\right)(\bar{I}_2 + A) - A = \underline{U}(I_1^*, R_1^*(I_1^*)) < U(I_2, R_2^{\varepsilon}(I_2), H), \tag{6}$$

where the inequality follows from $(3a)$ and from $\bar{I}_2 + A \geq \bar{I}_2 \geq I_1^*$. Thus, given $(I_1^*, R_1^*(.))$, there is a strategy for investor 2 inducing a unique continuation equilibrium in which the entrepreneur chooses $e = H$ and only raises funds from him. The corresponding profit to investor 2 is $V_2' = \min\{(\pi_H G - 1)I^m, \pi_H \bar{R}_2 - \bar{I}_2\} - \pi_H \varepsilon > V_2^*$. This constitutes a contradiction.

We next show that any profit between zero and the monopolistic one for investor 2 can be supported at equilibrium.

Lemma 2 *For any $V_2^* \in [0, \pi_H I^m - R^m]$, there exists a SPE in which investor 2 's profit is exactly V_2^*.*

Proof. Take any $V_2^* \in [0, \pi_H I^m - R^m]$. Let $(I^*, R^*) \in \Psi$ be the investment-repayment pair such that $\pi_H R^* - I^* = V_2^*$. Let also I_1^* be the investment level such that $U(I_1^*, I_1^*/\pi_H) = (\pi_H G - 1)(I_1^* + A) = U(I^*, R^*, H) \geq U(0)$. Consider the following strategies for investors:

1. Investor 1 posts $\{(0, 0), (I_1^*, R_1^*)\}$, where the non degenerate contract involves the constant repayment $R_1^* = I_1^*/\pi_H$.

2(i). If investor 1 posts $\{(0, 0), (I_1^*, R_1^*)\}$, investor 2 posts $\{(0, 0), (I_2^*, R_2^*(\cdot))\}$ with

$$I_2^* = I^* - I_1^* \qquad R_2^*(I_1^* + I_2^*) = R^* - R_1^* \equiv R_2^* \qquad R_2^*(I) = G(I + A)\ \forall I \neq I_1^* + I_2^*. \tag{7}$$

[13] The result obtains since, by construction, $\underline{U}(I_1^*, R_1^*(I_1^*), H) = \max\big(U(I_1, R_1(\cdot), H), U(0)\big)$.

2(ii). If investor 1 does not post $\{(0,0),(I_1^*, R_1^* = I_1^*/\pi_H)\}$,then investor 2 posts $\{(0,0),(I^*, R^*)\}$ where, once again, the non degenerate contract involves a constant repayment.

Observe that $U(I^*, R^*, H) = U(I_1^*, R_1^*(I_1^*), H) \geq U(I_1^*, (G - \frac{B}{\Delta\pi})(I_1^* + A))$, where the last inequality follows from (2), which guarantees that (I^*, R^*) coincides with the pair $(\bar{I}_2, \bar{R}_2)$ defined in the proof of Lemma 1.

We show that these strategies are part of a SPE, in which the entrepreneur invests $I^* = I_1^* + I_2^*$ earning earns $U^* = U(I^*, R^*, H)$, with $R^* = R_1^* + R_2^*(I^*)$. Investor 2 earns V_2^*, which is the maximal profit available to investors when the entrepreneur's payoff is fixed to be U^*, and given that the borrower chooses $e = H$.

See first that, given the offers above, if the entrepreneur selects $e = H$, then she optimally invests $I_1^* + I_2^*$. Indeed, given $R_2^*(.)$, she optimally chooses $e = L$ whenever she only trades with investor 1. In addition, since $U^* = U(I_1^*, R_1^*(I_1^*), H)$, the entrepreneur does not strictly prefer to trade with investor 1 only. We next show that $e = H$ is an optimal effort choice on the equilibrium path. Three cases must be considered. If the entrepreneur raises I_1^* only, choosing $e = L$ yields $U(I_1^*, I_1^*/\pi_H, L) < U(I_1^*, I_1^*/\pi_H, H) = U^*$. If the entrepreneur raises $I_1^* + I_2^*$, choosing $e = L$ she gets $U(I^*, R^*, L) = U(I^*, R^*, H)$. If she raises I_2^* only, she necessarily defaults obtaining $B(I^* - I_1^* + A) - A$. One therefore has

$$B(I^* - I_1^* + A) - A < B(I^* + A) - A < \frac{\pi_H}{\Delta\pi} B(I^* + A) - A = U(I^*, R^*, H),$$

where the second inequality follows from $\pi_L < \pi_H$, and the equality is implied by $(I^*, R^*) \in \Psi$. This guarantees the optimality of $e = H$ on the equilibrium path.

We next show that none of the investors can profitably deviate. Consider first investor 2. Given $(I_1^*, R_1^*(\cdot))$, he can profitably deviate only by granting the entrepreneur a payoff strictly above $U(I_1^*, R_1^*(I_1^*), H)$. However, given that (I^*, R^*) belongs to the incentive frontier Ψ, any such deviation necessarily yields a profit smaller than V_2^* to investor 2.

Finally, suppose that investor 1 posts a contract $(I_1, R_1(\cdot)) \neq (I_1^*, R_1^*(\cdot))$; then, recalling that $(I^*, R^*) = (\bar{I}_2, \bar{R}_2)$, the arguments developed in Lemma 1 can be used to show that investor 2 undercuts investor 1 granting the entrepreneur a payoff of $U(I_1, R_1(I_1), H)$. The equilibrium strategy of the entrepreneur can be constructed by letting her trading with investor 2 only, whenever she is indifferent between several options. This in turn blocks any profitable deviation of investor 1. ■

PROOF OF PROPOSITION 2

Consider the candidate equilibrium menus (M_1^*, M_2^*) with $M_2^* = \{(0,0)\}$, and $M_1^* = \{(0,0), (I^m, R_1(.)); (\hat{I}^m, \hat{R}_1(.))\}$, with $\hat{I}^m$ such that $U(I^m, R^m, H) = B(\hat{I}^m + A) - A$. The repayment functions $R_1(.)$ and $\hat{R}_1(.)$ are defined as

$$R_1(I) = \begin{cases} R^m & \text{if } I = I^m, \\ G(I+A) & \text{if } I \neq I^m, \end{cases}$$

and $\hat{R}_1(I) = G(I+A)$.

Given these menus, it is a best reply for the entrepreneur to select the contract $(I^m, R_1(.))$ in M_1^* and to choose $e = H$. That is, she trades the monopolistic allocation for lenders (I^m, R^m). Since investor 1 earns a monopolistic profit, he has no incentive to deviate. It is then sufficient to show that there is no profitable deviation for investor 2 either. Any such deviation can, without loss of generality, be represented by a simple menu $M_2' = \{(I_2', R_2'(.)), (0,0)\}$, with $I_2' > 0$, which incorporates only one non degenerate contract. Following the deviation, two situations must be considered:

1. The borrower chooses $(I_2', R_2'(.))$ in M_2', and $(I^m, R_1(.))$ in M_1^*. In this case, given $R_1(.)$, she optimally chooses $e = L$ which makes unprofitable the original deviation.

2. The borrower chooses $(I_2', R_2'(.))$ in M_2', and and $(0,0)$ in M_1^*. In this case, she earns $U(I_2', R_2'(I_2'), H)$ by choosing $e = H$. Yet, by choosing $e = L$, she can obtain $B(\hat{I}^m + I_2' + A) - A$. We can hence write

$$\begin{aligned} U(I_2', R_2'(I_2'), H) - (B(\hat{I}^m + I_2' + A) - A) &< U(0) + (\pi_H G - 1)I_2' - (B(\hat{I}^m + I^* + A) - A) \\ &= (\pi_H G - 1 - B)I_2' < 0. \end{aligned}$$

The first inequality follows from the fact that, for the deviation to be profitable, it must be that $\pi_H R_2'(I_2') - I_2' > 0$, which implies that $U(I_2', R_2'(I_2'), H) < U(I_2', \frac{1}{\pi_H} I_2', H) = U(0) + (\pi_H G - 1)I_2'$. The last inequality follows from (3b) and $I_2' > 0$, showing that $e = H$ is not an optimal effort choice. This establishes that the monopolistic allocation for investor 1 is supported at equilibrium, and concludes the proof. ■

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