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# THÈSE

En vue de l'obtention du

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Présentée et soutenue par

**Anastasiia SHCHEPETOVA**  
Le 17 Décembre 2014

Titre :

**Incomplete Information: the Role of Competition**

JURY

**Monsieur Zhijun CHEN, professeur, University of Auckland**  
**Monsieur Michele POLO, professeur, Università Bocconi**  
**Monsieur Andrew RHODES, maître de conférences, Université Toulouse 1**  
**Monsieur Thibaud VERGE, professeur, E.N.S.A.E.**

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**Ecole doctorale : Toulouse School of Economics**  
**Unité de recherche : GREMAQ - TSE**  
**Directeur de Thèse : Patrick REY**



L'université Toulouse I Capitole n'entend donner aucune approbation, ni improbation aux opinions émises dans cette thèse; elles doivent être considérées comme propres à leur auteur.



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# Abstract

This thesis analyzes the outcomes of competition in settings with incomplete and costly information.

The first two chapters explore in depth the incentives of firms to influence the amount of information available to consumers prior to their purchasing decisions. In many markets, firms have the potential to make their offers intentionally difficult for consumers to compare, for example, by adopting confusing presentation frames. We find that higher competition does not necessarily lead to better market outcomes for consumers.

The last chapter addresses a more general question as to whether competition leads to more or less informed decisions per unit of costs, in the setting where information is costly to obtain. I compare the adversarial and the inquisitorial systems of law enforcement and characterize the necessary and sufficient conditions for one scheme to dominate another and provide additional arguments in favor of the inquisitorial system.

The idea for the research questions studied in chapters one and two arose from observing the complexities of consumer credit offers which emerged from increasing competition in the banking sector in Ukraine. The presence of multiple clauses in credit contracts limited the extent to which credit offers could be compared and made the actual rate paid by consumers frequently far higher than the rate advertised on the initial publicity. Consider, for example, the following consumer loan conditions and try determine which consumer product carries the lower price: a 1.67% monthly interest rate with a 2.25% monthly commission on the total sum of credit, or a 12% annual interest rate, 11% upfront fee plus a 1.99% monthly commission on the total sum of credit including the upfront fee?

In addition the spread between publicized and actual effective interest rates varied from bank to bank. Despite there being a number of active banks competing in prices, the effective

interest rates far from reflected the actual costs, even if controlling for the associated country risks. The interest rates were quite dispersed for relatively homogeneous credit offers, which in turn suggests that the market outcome was unlikely to be competitive. Such examples are not limited to Ukraine and to the banking sector. Hardly comparable offers can be observed in various sectors such as electricity markets, mobile communications market, electronic goods, etc.

My research addresses the effects of this consumers, and the motivations of firms to undertake strategies that limit consumers' abilities to compare offers. When choosing the best deals, consumers look for the most advantageous combination of price and product match. Consumers differ in the amount of information they possess about prices and product features. In an oligopoly setting, we study the incentives of firms to make one or several dimensions of their offers intentionally confusing.

Firms can make their pricing structures and product features difficult to evaluate in order to limit the comparability of their own offers with competing ones. The mixed strategy equilibrium results in positive mark-ups. We analyze the strategic choices of firms and the relevant policy implications in three different settings: in markets with homogeneous, and markets with vertically and horizontally differentiated goods.

The first chapter of this thesis provides experimental evidence on the strategic limitation of price comparison by competing firms in a setting with homogenous products.

We develop a generalized theoretical framework following Carlin (2009) that accommodates both homogenous and vertically differentiated markets, and show that firms adapt to the competitiveness of the market by making price comparison costly for consumers, thereby departing from the concept of Bertrand competition. We then solve a simplified version of the game theoretical model and derive testable statistics on the effects of changes in market structure on equilibrium outcomes.

We test these predictions using a laboratory based experiment. The experimental data supports the theoretical predictions of the model. In particular, an increase in the number of competing firms leads to more costly price comparison for consumers. As a result, the share of uninformed consumers increases, which leads to higher average prices. Informed consumers still benefit from an increase in competition and pay lower prices. The firms that

charge higher prices tend to make it more costly for consumers to compare prices, while the firms that charge lower prices tend to make the comparison easier for consumers.

Furthermore, imposing an upper bound on the cost of comparison that firms can choose lowers market prices both for informed and uninformed consumers and reduces price dispersion. As an implication, a policy that limits the extent to which firms can make it costly for consumers to compare several offers would increase consumer welfare.

Chapter 2 extends the existing theoretical framework to accommodate horizontally differentiated products. Firms can choose to make their pricing structure and product features easier or harder to compare against competing offers. We find that price complexity increases with the price charged while product complexity decreases. For a high degree of product differentiation, there is a medium range of prices for which firms choose to make their offers fully transparent.

A higher number of firms shifts firms' choices towards complex prices and a transparent product match. The frequency of choosing fully transparent offers is non-monotonic in the number of firms; it increases with the number of firms when there are few firms in the market and decreases otherwise.

We further discuss relevant policy implications targeted at increasing market transparency. Increasing market transparency on the price dimension results in lower market prices but potentially worse product matches for consumers, while regulating transparency on the product dimension increases market prices but results in a better match. Welfare assessment therefore should take into account the importance of product matches for consumers.

The motivation for the research question that I explore in chapter three originates from observing the persistent differences in the way the US and European Union (EU) antitrust authorities operate. In the US, federal and state courts decide on cases prosecuted by national agencies (Federal Trade Commission in our example). In Europe, competition law enforcement is most often an administrative process in which agencies, such as the European Commission, also make decisions.

Chapter 3 of this thesis analyses the outcomes of strategic information disclosure under different institutional arrangements: the adversarial and the inquisitorial. A decision-maker



(DM) must take a binary decision faced with information provided by two persuaders: a firm that has an intrinsic interest in the final decision, and an expert who reacts only to monetary incentives designed by the DM. Under the adversarial arrangement, the expert is remunerated if the final decision is unfavorable to the firm. Under the inquisitorial arrangement, the remuneration is conditional only on the amount of disclosed information.

Common wisdom suggests that having two opposing parties competing to acquire information leads to higher efforts, and presumably to a higher level of precision for the final decision. At the same time, the possibility of remaining silent by withholding the unfavorable evidence, together with the associated higher costs of effort, undermines the superiority of the adversarial system.

I characterize the equilibrium of this disclosure game under the two arrangements, and provide the necessary and sufficient conditions for one system to dominate the other with respect to the level of precision of the final decision, net of the total cost of information acquisition. I find additional arguments in favor of the inquisitorial system.

# **Chapter 1**

## **Strategic Limitation of Price**

### **Comparison by Competing Firms. An Experimental Study**



## 1.1 Introduction

This paper provides experimental evidence on the strategic limitation of price comparison by competing firms. We develop a generalized theoretical framework that accommodates both homogenous and vertically differentiated markets and show that firms adapt to the competitiveness of the market by making price comparison costly for consumers. We find supportive evidence for the theoretical prediction that firms make their pricing structures hard to compare when charging high prices, and keep their prices easy to compare when charging low prices. The presence of more players on the market pushes firms to choose complex pricing structures more often, and as a result market prices increase.

In standard models of competition, consumers are assumed to be perfectly able to compare two different price offers and choose the best one. This leads to a Bertrand equilibrium outcome with prices equal to marginal costs, and the firm with the lowest price capturing the whole market. Empirical studies have shown, however, that consumers do not always fully compare the different price offers available, as there may be a cost to comparing the prices that they will pay. Subsequently, they may not choose the best offer available.

The challenges consumers experience in comparing different price offers are attributed in the literature to various psychological biases and to the fact that it is costly for consumers to compare several offers as pricing strategies can be quite complex. This leads to consumers having different information about various offers' attributes. Examples of hardly comparable offers can be seen in various sectors including credit offers in finance, mobile phone tariffs, the airline industry and electricity contracts. Prices could be presented in different measurement units, which are costly to convert into a single dimension. Offers could also include hidden charges, which create a cost for the consumer to find the complete price that they will pay. In addition, the mobile industry divides their tariffs into a large number of categories, making it costly for the individual consumer to calculate the total price they face.

Several recent works (Gabaix, Laibson (2005), Spiegel (2004), Pissone and Spiegel (2009), Carlin (2009)) show that firms respond to increased competition by employing more complex pricing schedules, rather than with more competitive pricing. Carlin (2009) in particular shows that high priced firms choose complex pricing structures, whereas low-priced firms choose transparent pricing structures. More players in the market results in

firms adopting complex pricing strategies more often.

The application of these theoretical models to policy design is, however, limited. Firstly the models rely on strong assumptions about information acquisition and result in a very complex equilibrium outcomes. Secondly it is difficult to empirically test these models, as it is hard to separate different possible reasons for pricing differentials, for example product differentiation or price discrimination may be misinterpreted as complex pricing strategies.

The main benefit of an experimental approach is that the laboratory setting allows us to control for these other factors, and limit our focus to the strategic choice of firms to make the comparison of their offers more difficult. The UK's Office of Fair Trading launched a high profile study in October 2009 in an attempt to understand what kind of pricing practices are more likely to harm consumers and the relevant policies that can minimize this harm. Kayalci (2010) finds experimental evidence that consumers tend to make more mistakes when faced with complex price structures. We contribute to the existing experimental literature by focusing our analysis solely on the behavior of firms, modeling consumers' behavior in accordance with the previous findings. This allows us to focus purely on the questions posed above, without the distortions of the real world. Therefore in our experiment we concentrate on the strategic behavior of firms and the resulting price distribution rather than on the behavior of buyers. We treat complexity of the price structure as the effort required for buyers to compare the price of a given firm with the prices of other firms in a market. We vary the number of firms between the sessions and the upper level of complexity choice possible within the sessions. We find supportive evidence for the theoretical prediction that the firms that choose higher prices make their pricing structures hard to compare, whereas the firms that choose lower prices make their prices transparent. Firms adopt to the competitiveness of the market by making their offers more difficult to compare. More players on the market makes firms choose complex pricing structures more often, and as a result the market prices increase.

We also contribute to the theoretical literature by generalizing the price-complexity game developed by Carlin (2009). He considers a set-up with homogeneous products, in which only a portion of all consumers are informed about all prices and the other portion remains uninformed. Firms affect these proportions by their individual choices of price complexity, thus exerting an externality on the demand that other firms face. The effect of the complexity

choice of one firm is independent of the complexity choices of other firms, i.e. the complexity choices of firms affect the proportion of informed consumers in an additive way. At equilibrium, price dispersion arises because firms compete strategically for a market share from both types of consumers. High price complexity is chosen by the firms that charge high prices with a fixed ex-ante probability that depends only on the number of firms.

We generalize the theoretical framework of Carlin by relaxing the additivity assumption about the way complexity choices of firms affect the proportion of informed consumers. We show that the qualitative results of Carlin (2009) still apply for a range of assumed functional forms. We then provide new quantitative predictions as to the frequency with which firms adopt complex or transparent pricing strategies. We further show how our framework can be used to accommodate markets with vertically differentiated products and thus contribute to the existing literature on obfuscation in vertically differentiated product markets. Alternatively, Armstrong and Chen (2008) study the market outcome in a setting with vertically differentiated products when some consumers are inattentive to product qualities. The equilibrium results in the provision of both low and high quality goods, even when the low quality goods are not socially desirable. In equilibrium firms take advantage of the existence of the inattentive consumers and ‘cheat’ from time to time by providing low-quality goods, which allows them to earn positive profits. We show that in the equilibrium both high and low quality firms can engage in obfuscation, depending on the shape of consumers’ preferences and the cost functions of firms.

The rest of the paper is organized as follows. Section 2 describes the theoretical set-up and the resulting equilibrium in the case of homogeneous products. We then demonstrate how our model can accommodate vertically differentiated products. In Section 3 we solve a simplified version of the theoretical model using the assumptions of Carlin (2009) and derive testable results on the effects of changes in market structure on equilibrium outcomes.

In Section 4 we introduce our experimental design, in which we test these predictions in the laboratory environment where human subjects acting as firms sell a homogeneous good to robot buyers. Section 5 analyses the results. Our preliminary experimental data supports the theoretical predictions of the model. However we find systematic discrepancies between the theoretical and observed price distributions. In all the treatments prices tend to be more dispersed than the theory predicts. Also the average market complexity in the treatment

with a lower upper bound on complexity choice is systematically lower than the predicted value. These last two results require further theoretical discussion and verification. Finally, Section 6 concludes.

## 1.2 Theoretical background

### 1.2.1 Model set-up

Suppose there is a large number of consumers who each desire to purchase, at most, one unit of a good. The maximum price any consumer will pay for the good - the consumer's valuation - is denoted by  $v$ . Consumers might be of two types: informed and uninformed. Firms can influence the proportion of the different types of consumers through their strategic choices.

#### Firms

$n$  firms have the same constant unit cost, normalized to zero. Each firm chooses the "level" of its prices,  $p_i \in [0, v]$ , as well as the complexity of its price structure,  $\rho_i \in \{\underline{\rho}, \bar{\rho}\}$ .

The complexity of the price structure represents how difficult it is to evaluate and compare a given price with competitors' prices; it can reflect the number of itemized fees included in the price structure, or the complexity of the frame being adopted - fees in small print, add-on prices, etc. The choice of complexity is costless and is firm's  $i$  private information. This assumption states that adopting a certain frame does not convey information about its individual complexity, but rather contributes to the overall market complexity in an unobservable way. So adopting a frame different to your competitor makes it harder to assess all the offers, but does not identify which frame is more complex.

The individual choices of complexity of price by firms determine the proportions of different types of consumers. That is, the proportion of *shoppers*,  $\lambda$ , and the proportion of *uninformed consumers*,  $(1 - \lambda)$ , are functions of the complexity index:

$$\lambda : [\underline{\rho}, \bar{\rho}]^n \rightarrow [0, 1]. \quad (1.1)$$

The proportion of *informed buyers* decreases with firm's individual choice of price complexity. This assumption reflects the idea that the harder it is to compare prices, the less consumers will choose to compare prices. As a result the proportion of *informed buyers* decreases while the proportion of *uninformed* consumers increases.

To summarize the effect of the firms' individual complexity choice on the proportion of informed buyers we adopt the following assumptions.

*Assumption 1*

$$1. \frac{d\lambda}{d\rho_i} < 0, \text{ for all } i. \quad (1.2)$$

Assumption one states that the proportion of the informed consumer decreases with firm's  $i$  individual price complexity. This assumption reflects the idea that the harder it is to compare prices, the less likely the consumers will do so.

## Timing

In period 1 each firm  $i$  simultaneously chooses its price level,  $p_i$ , and the complexity of its pricing schedule,  $\rho_i$ . Firms' individual complexity choices determine the resulting proportion of *uninformed* consumers and *informed buyers*. In period 2 the proportions of different types of consumers are determined and consumers make their purchase decisions. *Uninformed* consumers do not observe any information, therefore they choose a firm to buy from based on their rational expectations. *informed buyers* choose the product with the lowest price.

## Equilibrium

Define  $J^*$  to be the set of firms who quote the lowest price in equilibrium. Let  $n_j$  be the number of firms in  $J^*$ , so that the  $n_j$  firms in  $J^*$  split the demand from the informed consumers equally.

The profit of the firm  $i$  is given by:

$$\Pi_i = p_i \left[ \lambda \frac{1}{n_j} 1_{\{i \in J^*\}} + \frac{1}{n} (1 - \lambda) \right], \quad (1.3)$$



where the first term is the expected profit from *informed buyers*: firm  $i$  gets all *informed* buyers,  $\lambda$ , if it charges the lowest price out of  $n$  firms. In the case when more than one firm has the lowest price, each firm gets equal share of *informed buyers*.

The second term is the *uninformed* consumers. Firms equally share the demand from *uninformed* consumers.

Carlin (2009) shows that there is no pure strategy price equilibrium of this game. We thus concentrated on the mixed strategy equilibrium of this game, in which firms choose their prices according to equilibrium cdf  $F(p)$ . Before formally stating the result, we introduce more notations:

Let  $l$  be the number of firms out of  $n$ , that choose transparent price. Define  $\lambda(l, n)$ , to be the proportion of *informed* consumers when  $l$  firms out of  $n$  choose  $\underline{\rho}$ . Further denote by  $\widehat{\lambda}(l)$  the proportion of *informed* consumers for a given  $n$ . So  $\lambda(0)$  is therefore the proportion of *informed* consumers when all the firms choose complex prices,  $\bar{\rho}$ . We further denote by  $(1 - \underline{\Lambda})$  and  $(1 - \bar{\Lambda})$  the expected proportion of *uninformed* consumers when firm  $i$  chooses  $\bar{\rho}$  and  $\underline{\rho}$  respectively.

A symmetric mixed strategy equilibrium is characterized in the following proposition:

**Proposition 1** (*Equilibrium*) *There exists a unique symmetric mixed strategy equilibria, in which firms choose their prices according to equilibrium price distribution  $F^*(p)$  with the support  $[p_L, v]$  and there exists unique  $\widehat{p}, \widehat{F}$  such that:*

*i) Firms choose their complexity levels as follows:*

$$\rho^*(p) = \begin{cases} \underline{\rho} & p < \widehat{p} \\ \bar{\rho} & p > \widehat{p} \\ \rho \in \{\underline{\rho}, \bar{\rho}\} & p = \widehat{p}. \end{cases}$$

*ii) Firms choose  $\underline{\rho}$  with a unique probability,  $\widehat{F}$ , that is implicitly given by:*

$$[1 - \widehat{F}]^{n-1} = \frac{1}{n} \frac{\bar{\Lambda} - \underline{\Lambda}}{\widehat{\lambda}(1) - \widehat{\lambda}(0)}. \quad (1.4)$$

*such that  $F^*(\widehat{p}) = \widehat{F}$ .*

**Proof.** The detailed proof is provided in *Appendix A*. ■

Thus for a given  $\widehat{\lambda}(\cdot)$ , the firm's choice of price complexity is determined solely by the equilibrium price distribution and by the number of firms. Firms choose  $\rho$  so that to maximize the proportion of *informed* consumers (minimize the proportion of *uninformed* consumers) for the low range of prices, or to minimize it (maximize it) for the high range of prices. The choice to maximize the proportion of certain type of consumer depends on the expected profitability of attracting this type of consumer. On the one hand, when a firm charges low price, the probability of attracting all *informed* consumers is relatively high and this gives the firm incentives to maximize the share of *informed* consumers by choosing a transparent pricing structure and thus decreasing price complexity.

On the other hand, when a firm charges high price, the probability of attracting *informed* consumers becomes small and a firm's demand is coming mainly from the *uninformed* consumers. Hence, the firm would minimize the amount of the *informed* consumers by making its price complex in order to discourage price comparison and maximize the proportion of *uninformed* consumers.

Note that  $\bar{\Lambda} - \underline{\Lambda}$  can be rewritten as  $(1 - \underline{\Lambda} - (1 - \bar{\Lambda}))$ , the difference in the proportion of *uninformed* consumers when firm  $i$  decreases its price complexity from  $\bar{\rho}$  to  $\underline{\rho}$ . Firm  $i$  is indifferent between choosing transparent or complex price when the expected change in the demand from *informed* consumers is equal to the corresponding change in the expected demand from *uninformed* consumers.

Our model accommodates the result of Carlin (2009) as a particular case. Precisely, if the change in  $\lambda$  with respect to price complexity is independent of complexity choices of the other firms, i.e.  $\widehat{\lambda}(1) - \widehat{\lambda}(0) = \widehat{\lambda}(2) - \widehat{\lambda}(1) = \widehat{\lambda}(n) - \widehat{\lambda}(n-1)$ . In this case and due to the property of the binomial distribution,  $\bar{\Lambda} - \underline{\Lambda} = \widehat{\lambda}(1) - \widehat{\lambda}(0)$ . Thus, condition (1.4) becomes:

$$[1 - \widehat{F}]^{n-1} = \frac{1}{n}, \quad (1.5)$$

and firms choose complex prices with the ex-ante probability  $\frac{1}{n}^{\frac{1}{n-1}}$ . Thus, the firms choice of price and product complexity is determined solely by the equilibrium price distribution and by the number of firms as in Carlin (2009). However, for a broader class of  $\lambda$  function,

the probability of choosing different complexity levels, can be both smaller and greater than  $\frac{1}{n}$ , which we demonstrate later in numerical example for duopoly setting.

**Duopoly example:** Consider the case of two firms. Let us introduce the possible realization of  $\lambda$  in this case. There are three possible outcomes for the proportion of *informed* consumers:

$$\begin{aligned}\widehat{\lambda}(2) &\equiv \lambda(2, 2) \\ \widehat{\lambda}(1) &\equiv \lambda(1, 2) \\ \widehat{\lambda}(0) &\equiv \lambda(0, 2).\end{aligned}$$

For  $p < \widehat{p}$ , the expected profit of firm  $i$  is given by:

$$\Pi(p, \underline{p}, |\sigma_{-i}) = p \left\{ [1 - \widehat{F}] \widehat{\lambda}(1) + [\widehat{F} - F(p)] \widehat{\lambda}(2) + \frac{1}{2} (1 - \widehat{F} \widehat{\lambda}(2) - [1 - \widehat{F}] \widehat{\lambda}(1)) \right\}.$$

For  $p > \widehat{p}$ , the expected profit of firm  $i$  is given by:

$$\Pi(p, \bar{p}, |\sigma_{-i}) = p \left\{ [1 - \widehat{F}] \widehat{\lambda}(0) + \frac{1}{2} (1 - \widehat{F} \widehat{\lambda}(1) - [1 - \widehat{F}] \widehat{\lambda}(0)) \right\}$$

Equilibrium price distribution is given by:

$$F^*(p) = \begin{cases} \left[ \frac{(y-p)(\widehat{\lambda}(0) + (\widehat{\lambda}(1))^2 - \widehat{\lambda}(2) + \widehat{\lambda}(0)\widehat{\lambda}(2)) - 2y\widehat{\lambda}(0)\widehat{\lambda}(1) + 2p\widehat{\lambda}(1)\widehat{\lambda}(2)}{2p((\widehat{\lambda}(2))^2 - \widehat{\lambda}(0)\widehat{\lambda}(2))} \right] & \text{for } p < \widehat{p} \\ 1 - \left[ \frac{(y-p)(2\widehat{\lambda}(0)\widehat{\lambda}(1) + \widehat{\lambda}(2) - (\widehat{\lambda}(1))^2 - \widehat{\lambda}(0)\widehat{\lambda}(2))}{2p(\widehat{\lambda}(0)\widehat{\lambda}(2) - \widehat{\lambda}(0)^2)} \right] & \text{for } p > \widehat{p}. \end{cases}$$

In this case condition (1.4) becomes:

$$\widehat{F} = \frac{\widehat{\lambda}(1) - \widehat{\lambda}(0)}{\widehat{\lambda}(2) - \widehat{\lambda}(0)}.$$

In the case when  $\lambda$  is additive in complexity levels, Carlin (2009) predicts that for  $n = 2$ ,  $\widehat{F}$  is equal to one half. This result can be obtained as a particular case of our model, when  $\widehat{\lambda}(2) - \widehat{\lambda}(1) = \frac{\widehat{\lambda}(2) - \widehat{\lambda}(0)}{2}$ . However if  $\widehat{\lambda}(2) - \widehat{\lambda}(1) > \frac{\widehat{\lambda}(2) - \widehat{\lambda}(0)}{2}$ ,  $\widehat{F}$  is greater than one half and

vice versa.

## Entry effect

Given that the frequency with which firms choose complex prices depends on the number of firms, the important question is thus to investigate how the incentives of firms to choose complex prices change with the number of players in the market. Note that the number of firms affect both the probabilities of attracting different types of consumers and the marginal effect that  $\rho$  has on the proportion of *informed* consumers. Therefore the net effect of change in the number of firms depends on the interplay between these properties. In the following proposition we characterize a sufficient condition that leads to a decreases in  $\hat{F}$  as  $n$  increases.

**Proposition 2** :*ENTRY EFFECT: If  $\lambda(l, n)$  is logsupermodularity in differences:*

$$\frac{\lambda(l, n) - \lambda(l - 1, n)}{\lambda(1, n) - \lambda(0, n)} - \frac{\lambda(l, n') - \lambda(l - 1, n')}{\lambda(1, n') - \lambda(0, n')} \geq 0 \quad (1.6)$$

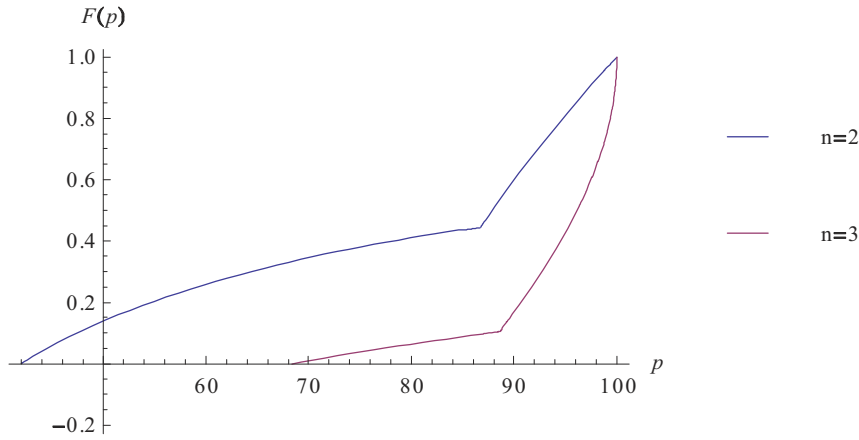
$$\text{for } l > 1 \text{ and } n > n', \quad (1.7)$$

*then when the number of firms increases, the frequency with which firms choose complex prices and transparent matching values increases.*

**Proof.** See Appendix B for a formal proof. ■

The property logsupermodularity in differences characterizes the curvature of  $\lambda$  function . For example if  $\lambda$  is convex in  $l$ , then (1.6) implies that  $\lambda(l, n')$  is more curved than  $\lambda(l, n)$ .

This condition implies that when the number of competitors increases, the expected demand from *informed* consumers decreases faster than the expected demand from *uninformed* consumers. Therefore, firms put more weight on the strategy that limits price comparison. If the number of firms is sufficiently large then all the firms choose complex prices and market price complexity increases as a result.

**Numerical example:****Figure 1: Effect of Number of Firms**

The graph above plots the equilibrium price distribution for  $n = 2$  with  $\lambda(0, 2) = 0.1$ ,  $\lambda(1, 2) = 0.05$ ,  $\lambda(2, 2) = 1$  and  $n = 3$  with  $\lambda(0, 3) = 0.05$ ,  $\lambda(1, 3) = 0.2$ ,  $\lambda(2, 3) = 0.6$  and  $\lambda(3, 3) = 1$ . For  $n = 3$  the frequency of choosing transparent prices/complex matching values ( $\hat{F}$ ) decreases with respect to  $n = 2$ . The equilibrium price distribution shifts downwards in comparison to  $n = 2$  and therefore prices increase with the number of firms. Thus this numerical example demonstrates that increasing the number of firms in this setting leads to higher and more complex prices and more transparent product characteristics.

Note that when  $\lambda$  is additive in the complexity choices of the firms, the property (1.6) is satisfied with the equality sign and the only affect that matter is the change in the probabilities of attracting different types of consumers. In this particular case Carlin (2009) shows that the probability that a firm chooses high complexity ( $\bar{\rho}$ ) is monotonically increasing in  $n$ . In the limit as  $n$  approaches infinity, all firms choose  $\bar{\rho}$ . The expected industry complexity is increasing in  $n$ .

As the number of firms increases firms choose complex prices more often. This result is quite intuitive as when the number of competitors increases, the probability of winning all the informed consumers decreases faster than the sure share of uninformed consumers. Firms therefore choose to put more weight on the strategy that ensures positive profits, i.e. they choose complex prices more often. As a result the overall market complexity increases, and therefore the equilibrium proportion of uninformed consumers increases as well.

The additivity assumption, makes permits to extend the above analysis to the setting when the complexity is chosen from the continuous interval. This setting is particularly amendable for the policy analysis, as the relevant policies to restore market transparency would be to introduce a cap on how complex the firms can go in their choices of pricing structures. We thus assume the  $\rho \in [\underline{\rho}, \bar{\rho}]$  and adopt the additivity assumption, i.e.  $\frac{d^2\lambda}{d\rho_i d\rho_j} = 0$  for all  $i \neq j$ . The effect of introducing the cap on complexity level on the equilibrium price distribution is summarized in the following proposition.

**Proposition 3** *The distribution of prices  $F(p)$  with the lower upper bound of complexity  $\bar{\rho}$  first order stochastically dominates that with a higher upper bound.*

**Proof.** The regulation of complexity affects the equilibrium price distribution in two ways: through the direct effect on  $\lambda$  and through the indirect effect on the frequency of choosing complex prices, which in turn depends on  $\lambda$ . The effect on  $\lambda$  is straight forward: a cap on price complexity increases  $\lambda$ , while a floor on price complexity reduces  $\lambda$ . The effect on  $F(\hat{p})$  is less obvious. Note that  $\hat{F}$  in case of linear  $\lambda$  is determined only by the number of firms and hence is unaffected by complexity regulation. Therefore the price distribution changes in the same direction as  $\lambda$ , thus increasing (lower prices) as a result of a price complexity cap and decreasing (higher prices) as a result of a price complexity floor. ■

Introducing a complexity cap in this setting decreases the equilibrium prices, and thus increases consumer welfare.

## Vertical differentiation

In this section we show that our results apply as well to the vertically differentiated market. Complexity in this case is interpreted as a tool that makes it more difficult to assess the net value of the product. For example, complexity can be viewed as a detailed and comprehensible disclosure of quality information, no-disclosure would correspond to a high complexity, whereas complete disclosure corresponds to a transparent offer.

Consider  $n$  firms competing in compete in price,  $p$ , and quality,  $q$ . The firms face marginal cost  $w(q)$  per unit of good of quality  $q$ , so that  $\partial w/\partial q > 0$  and have no capacity constraints.

In the market, there is a continuum of consumers of unit mass who each have a unit demand. The preferences of consumers over pair  $(p, q)$  are represented by utility function  $U(p, q)$ , which is strictly decreasing in  $p$ , strictly increasing in  $q$ , and continuous in  $(p, q)$ . Dubovik (2008) provides the necessary conditions that guarantee that for a given  $\lambda$ , there exists a mixed strategy equilibrium where firms randomize in margins, i.e. in the levels of utilities offered to consumers. In the equilibrium, higher margins,  $m$ , translate into lower consumer's utility. *Appendix C* provides in detail the assumptions.

This result implies that our above analysis applies directly to the vertically differentiated markets once prices are replaced by margins. Therefore proposition 1 reads as: for  $m > \hat{m}$ , firms choose high complexity of their offer and for  $m < \hat{m}$ , firms choose transparent offers. The equilibrium is characterized by both quality and price dispersion, with the exception of when preferences are quasi-linear in quality, i.e.  $U(p, q) = h(q) - p$ . In this case a unique quality and dispersed prices are chosen by all firms in the equilibrium, and there is a price dispersion in the equilibrium. Therefore the setting with homogeneous products can be viewed as a particular case of the setting with vertically differentiated products, i.e. the firms that choose higher prices choose high price complexity and vice versa.

Depending on the shape of consumer preferences and the firms' cost functions, the equilibrium dispersion of margins translates into the equilibrium dispersion of price and quality pairs in the following way:

**Case 1** *In equilibrium* utility is strictly decreasing in  $p$ . This case is consistent with equilibrium level of quality being increasing or decreasing with the choice of price. so that *informed consumers* will buy the cheapest product. Decreasing utility is consistent with both increasing and decreasing  $g(p)$ :

a) the quality offered in the equilibrium is increasing in  $p$ .

In this case informed consumers choose the firm that offers the lowest price and the lowest quality as the marginal value of quality is low. Uninformed consumers get higher prices and higher qualities. Firms that offer higher quality and higher prices choose high complexity.

b) the quality offered in equilibrium is decreasing in  $p$ .

In this case in the equilibrium *informed* consumer still buy the cheapest good but now this good is of the highest quality. *Uninformed* consumers pay higher prices for a lower

quality compared to informed consumers. Under this assumption firms that offer higher prices and lower quality choose high complexity.

**Case 2:** Utility is strictly increasing in  $p$ .

In this case the *informed* consumers buy the most expensive good, as the marginal value of quality is high. Uninformed consumers on average buy goods of lower price and lower quality. Firms that offer low price and quality choose high complexity.

Note that in the The two cases described above show that at equilibrium several situation can emerge. When the preferences of consumers are such that they prefer lower price and lower quality, than it is the firms that produce high quality good that engage in obfuscation practices. On the other hand when consumers' preferences are such that fully informed consumers will purchase high quality good, it is low quality firms that choose engage in obfuscation. Therefore the quality level chosen by the firm is the poor indicator as to whether the firm engages in the obfuscation practice. Therefore policies that intend to regulate quality can backfire for the consumers in this case.

In the following section we test experimentally the incentive of firms to limit the comparison of their offers in the setting with homogenous goods, which in turn can be interpreted as the setting with vertically differentiated goods and quasi-linear preferences of consumer.

## 1.3 Experiment

### 1.3.1 Theoretical predictions and testable hypothesis

The model presented above provides theoretical predictions on the effects of changes in different market parameters. such as the number of firms and an upper bound on the complexity level. We test these predictions by varying the number of firms and the upper bound of price complexity. The additivity assumption is particularly amendable for the experimental setting as it allows us to explicitly solve the model for  $n = 2$  and  $n = 4$  and thus obtain the quantitative predictions.

For experimental purposes we adopt a particular specification for the proportion of in-



formed consumers:

$$\lambda = 1 - \frac{\sum_i^n \rho}{5n}$$

$$\rho \leq 1$$

The parametrization for complexity level is chosen for reasons of convenience, in order to maintain an integer number of resulting informed and uninformed buyers, and to obtain sufficient changes in prices. We further vary the number of firms from 2 to 4 and the upper bound of price complexity from 3 to 5.

We calculate the quantitative theoretical predictions for expected price, expected minimum price and expected complexity for our experimental parameters. Table 1 summarizes these predictions.

Table 1. Theoretical Predictions for Alternative Parameter Values						
$v = 100$						
$\rho = 0$						
	$Ep^{\min}(\text{informed})$		$Ep(\text{uninformed})$		$EK(\text{exp complexity})$	
	$\bar{\rho} = 3$	$\bar{\rho} = 5$	$\bar{\rho} = 3$	$\bar{\rho} = 5$	$\bar{\rho} = 3$	$\bar{\rho} = 5$
<b><math>n = 2</math></b>	<b>40</b>	<b>70</b>	<b>54</b>	<b>82</b>	<b>1.5</b>	<b>2.5</b>
<b><math>n = 4</math></b>	<b>31</b>	<b>57</b>	<b>64</b>	<b>84</b>	<b>1.9</b>	<b>3.16</b>

We take the treatment with  $n = 2$  and  $\bar{\rho} = 3$  as the *benchmark treatment*.

As Table 1 shows, compared to the benchmark treatment doubling the number of sellers increases the average price paid by uninformed buyers by almost 19% and decreases the price paid by informed buyers by 23%. The average market complexity increases as a result by 27%. Increasing  $\bar{\rho}$  from 3 to 5 increases the price paid by uninformed buyers by 52% and the price paid by informed buyers by 75%. The average market complexity increases as a result by 67%.

We compare our laboratory market outcomes to the quantitative predictions shown in this table, but based on previous experimental practices we do not expect such strong quantitative equilibrium predictions to hold very precisely. Our empirical analysis will therefore focus on the weaker, comparative static predictions summarized by the following hypotheses.

*Hypothesis 1. Complexity choice is positively correlated with price.*

The average market complexity is an externality in this model. By choosing individual complexity each firm affects the number of informed and uninformed buyers. So firms choosing lower prices would naturally want to increase the number of informed buyers by choosing a low level of complexity, thereby lowering the average market complexity. Firms that choose higher prices have opposite incentives.

The important prediction of the equilibrium described in the *Proposition 1* is that the firms charging a price above a certain threshold  $\hat{p}$  choose the highest complexity level. If the price is below this threshold, the firms choose the lowest level of complexity.

Instead we adopt a weaker hypothesis, that the firms that charge higher prices choose higher levels of complexity and the ones charging lower prices choose lower levels of complexity.

*Hypothesis 2. An increase in the upper bound of the available complexity level results in both higher average and minimum expected market prices.*

Increasing the upper bound on complexity allows firms to increase average market complexity and the resulting share of uninformed buyers for any given price. Therefore, as demand from uninformed buyers increases it is profitable for firms to increase the frequency of choosing high prices. This would result in a higher expected price as well as a higher expected minimum price compared to the benchmark treatment.

*Hypothesis 3. An increase in the number of firms results in higher average market complexity. Firms choose higher complexity more often when the number of competitors is higher.*

As the number of firms increases the probability that a given firm has the lowest price, and therefore the informed share of the market demand, decreases. Therefore, for a given price distribution, the range for which it is profitable for a firm to choose higher complexity would increase. This results in a higher probability of a firm choosing high levels of complexity, and hence a higher average market complexity.

*Hypothesis 4. An increase in the number of firms results in a higher average price and a lower minimum price in the market.*

Increasing the number of firms has two opposing effects: from one side competition for the informed buyers becomes more aggressive which results in a firm choosing a lower price when they choose low complexity. This in turn decreases the expected minimum price. From the other side increased competition for informed buyers makes firms choose higher prices more frequently and therefore maintain their guaranteed profits from the uninformed buyers. This results in a higher average price compared to the benchmark treatment with just two sellers.

### 1.3.2 Experimental design and procedures

The experiment modifies the standard posted-offer environment by introducing complexity levels and the number of firms as treatment variables. In this experiment we do not aim to test the behavior of buyers, but rather to concentrate on firms' strategic behavior. The subjects are the firms selling a homogeneous good to robot buyers. A computerized interface using the software z-Tree is used to conduct the experiment (Fischbacher, 1999).

The experiment consists of 8 sessions and was conducted at the Toulouse School of Economics in spring and autumn 2011. 12 graduate students participated in each session.

To disentangle the effects of changing the number of firms and the upper bound of complexity, we vary the number of firms across the sessions and the upper bound of complexity within the sessions. Six sessions constitute a Two-seller treatment and the other six will constitute a Four-Seller treatment. In each session subjects play a market game for real money rewards. Each session consists of three phases of 20 periods each (total 60 periods). In three sessions out of six, referred to as "Low-High-Low," the available upper bound of complexity ( $\bar{p}$ ) is low in the phase 1 (periods 1-20),  $\bar{p}$  is high in the phase 2 (periods 21-40) and  $\bar{p}$  is low in the phase 3 (periods 41-60) again. In the remaining three sessions the order is reversed "High-Low-High". Table 2 summarizes the experimental design.

Table 2 Experimental design

		Upper bound of complexity $\bar{\rho}$	
		High-Low-High	Low-High-Low
$n = 2$	2 sessions of 12 subjects	2 sessions of 12 subjects	2 sessions of 12 subjects
$n = 4$	2 sessions of 12 subjects	2 sessions of 12 subjects	2 sessions of 12 subjects

**Robot buyers.** Each market consists of 20 robot buyers who buy 4 units each. The computerized buyers are programmed to buy all 4 units from the cheapest firm if they are informed and split the units equally between all the firms if they are uninformed (each firm therefore gets 1 or 2 units of demand depending on the treatment). The amount of informed and uninformed buyers is determined by the resulting average market complexity. The higher the average complexity the higher the number of uninformed robots. This relationship is assumed to capture the fact that the more complex the market structure is the more difficult it is for buyers to calculate the actual price and more buyers optimally choose to remain uninformed. Several experimental studies provide evidence that human buyers make more mistakes when faced with complex prices (or products) and that the amount of mistakes increases with average complexity (Kayalci (2010), Sitzia (2008)), which justifies our modeling of the behavior of robot-buyers.

The use of computerized robot-buyers is motivated by several concerns. Firstly, the use of robot buyers allows us to abstract from the behavior of buyers and to concentrate on the behavior of firms. Secondly, employing human buyers may cause psychological factors that are common in laboratory environment but are not relevant in real markets to influence the market outcome. For example a high repetition of simple computational exercises can lead to a significant learning effect for human buyers. Also factors such as an endowment effect, lack of motivation to perform repeated calculations, or differences in cognitive abilities can appear in the laboratory and may not necessarily be relevant in the real markets. For example, in Abrams et al, (2000) human buyers were rejecting profitable transactions when faced with high prices, even when it was efficient to purchase.

**Firms.** At the beginning of each period subjects are randomly assigned to groups that form a market with two or four sellers depending on the session. In order to exclude the possibility of collusion, subjects do not know who is in their group and are randomly rematched

every period.

The complexity concept is introduced as an effort level that buyers have to exert in order to observe and compare prices. In each period sellers have to choose prices and the level of effort. The procedure is carefully explained to the subjects in instructions before the experiment starts. Instructions are available in *Appendix D*. The subjects are informed about the externality effect of their effort choice on other firms. The trade off between effort choice and the number of informed and uninformed buyers is also carefully explained. The proportion of uninformed buyers is proportional to average effort, so that the share of uninformed buyers is equal to the average resulting market complexity divided by 5. The motivation for the chosen parameters and functional forms is to maintain convenient integer numbers.

In each period subjects choose a price from the set  $\{0, 1, 2, \dots, 100\}$  and a level of effort, which corresponds to the complexity level. In "Low-High-Low," in phase 1 (periods 1-20) sellers choose the effort from the set  $\{0, 1, 2, 3\}$ , in phase 2 (periods 21-40) from the set  $\{0, 1, 2, 3, 4, 5\}$  and in phase 3 (periods 41-60) from the set  $\{0, 1, 2, 3\}$  again. In "High-Low-High" in phase 1 (periods 1-20) sellers choose effort from the set  $\{0, 1, 2, 3, 4, 5\}$ , in phase 2 (periods 21-40) from the set  $\{0, 1, 2, 3\}$  and in phase 3 (periods 41-60) from the set  $\{0, 1, 2, 3, 4, 5\}$  again.

*Example:* Consider phase 2 of the Four-seller treatment: if all the firms choose effort level 4 the amount of uninformed buyers would be 16 out of 20, or if seller 1 and 2 choose a level of effort equal to 0, seller 3 chooses level of effort 2, seller 4 chooses level of effort 4 the number of uninformed buyers would be 6 out of 20.

When all the subjects submit their price offers and effort levels, profits for each seller will be determined in experimental points, that are calculated as price times quantity sold. At the end of each period a 'Result Screen' is displayed on each computer with information on all the prices and levels of effort chosen by other sellers in the market, and quantities sold to informed and uninformed buyers in this period. Also each seller receives feedback on his individual performance: sales to informed buyers, sales to uninformed buyers, own point earnings in this period, as well as accumulated points over periods.

We adopt the argument of Morgan, Orzen and Sefton (2001) in choosing the nature of

feedback provided. In their paper they follow Cason and Friedman (2000), where at the end of each period a seller receives information about the entire price distribution. This set-up resulted in strong support for theoretical mixed strategies predictions. We believe that such a design helps subjects to develop an intuition regarding the best strategies, rather than expecting the subjects to correctly predict the equilibrium strategies, given the complexity of the equilibrium outcome.

At the end of a session each subject is paid a participation fee plus the accumulated points from all the 60 periods converted to euros at the certain rate.

## 1.4 Results

8 sessions of the experiment were conducted in the experimental laboratory of Toulouse School of Economics. Four sessions (Session 1, 2, 5, 6) employed a *Two-seller* treatment and the other four sessions (Session 3, 4, 7, 8) employed a *Four-seller* treatment. In Session 1, 2, 3 and 4,  $\bar{p}$  varied in the order 5 – 3 – 5. In sessions 5, 6, 7 and 8,  $\bar{p}$  varied in the order 3 – 5 – 3. Otherwise the procedures were the same for each session.

96 graduate students were recruited via E-mail announcement about the remunerated experiment where their earnings depend on their and other participants' decisions. 12 subjects participated in each session. Subjects accumulated earnings over the 60 periods. The average earnings are 12.75 euros in Two - seller treatments and 6 euros in Four-seller treatments for the session lasting between 50 and 70 minutes. At the end of each session, subjects completed a short post-experimental questionnaire and the question “*Would you be willing to take part in other experiments of this kind?*” received 100 percent affirmative responses.

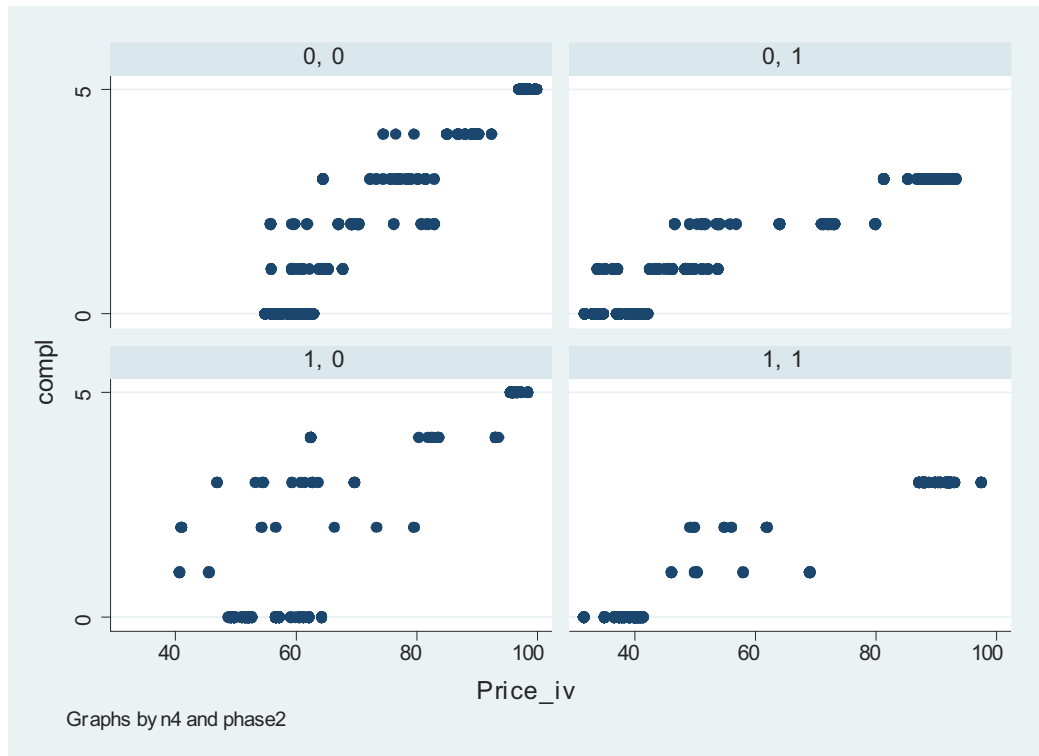
We consider phase 1 and 6 first periods of phase 2 and 3 as a learning period and base our analysis on the data from 14 last periods of phase 2 and 3 of each session. The results are summarized below.

*Result 1: Firms that choose higher prices, choose higher complexity.*

We estimate a relationship between the choice of complexity level and the price level using random effect GLS model.

In order to get an unbiased estimator of this effect we need to solve the possible endogeneity issue between the choice of complexity and the choice of price. We generate an instrumental variable for price to control for possible endogeneity issue. We construct the IV variable for each individual in the following way: we take the level of complexity chosen by a given individual and then compute an average of prices within other individuals with the same choice of the level of complexity. This produces an IV variable that is correlated with the complexity choice but uncorrelated with the error term.

The graph below represents a scatter plot of complexity and price by the number of firms (0 if  $n = 2$  and 1 if  $n = 4$ ) and by the upper bound of complexity level (1 if  $\bar{\rho} = 3$  and 0 if  $\bar{\rho} = 5$ ), so that  $\{0, 1\}$  represents a treatment with  $n = 2$  and  $\bar{\rho} = 3$  and  $\{1, 0\}$  represents a treatment with  $n = 4$  and  $\bar{\rho} = 4$  illustrates the positive correlation between IV variable and complexity.



The graph illustrates as well that when  $n = 4$  high complexity is chosen more often and for a larger price range that when  $n = 2$ .

The table below presents regression results for the "low" treatment ( $\bar{\rho} = 3$ ) and  $n = 4$ . The choice of price level has a positive significant effect on the choice of the complexity level. The effect is significant at five percent significance level.

Table 1.1: Dep = compl

Variable	Coefficient (Std. Err.)
Price_iv	0.057** (0.001)
Intercept	-2.138** (0.062)
N	
	471
Log-likelihood	
	.
$\chi^2_{(1)}$	
	4148.516

Table 3

The regressions for different treatments produce similar results and for this reason we omit their presentation

*Result 2. The average complexity level is higher in the Four-seller treatment compared with the Two-seller treatment.*

Theory predicts that the average market complexity increases with the number of firms. In the table below we can see that the average complexity level is systematically higher in the sessions employing 4 sellers than in the sessions employing 2 sellers as well as in the phases with the higher upper bound of complexity level ( $\bar{\rho} = 5$ ).

Average Complexity									
	$\bar{\rho}$	$n = 2$				$n = 4$			
Sessions		1	2	3	4	5	6	7	8
Experimental	5	2.0	2.3	2.4	2.5	2.4	3.1	3.2	3.2
	3	0.9	1.1	1.5	1.7	1.7	1.8	1.8	1.9
Theoretical	5	<b>2.5</b>				<i>3.2</i>			
	3	1.5				1.9			

Table 4

In fact for both treatments "high" and "low" the null hypothesis of no competitive static effect can be rejected at 1 percent significance level applying *Wilcoxon rank - sum* test. The *p-value* of 0.001 indicates the probability under the null hypothesis of obtaining a sum of



ranks of at least as large as that observed (with a rank of 1 assigned to the session yielding the lowest outcome and the sum is based on the ranks of the four-session treatment)

Figure 2 below displays five-period moving averages for the average complexity levels chosen by the subjects. The complexity levels tend to be higher in the Four - seller treatment compared to the Two - seller treatment. Therefore in the Four- seller treatment the average proportion of the uninformed buyers tend to be higher than in the treatment with just two sellers in the market.

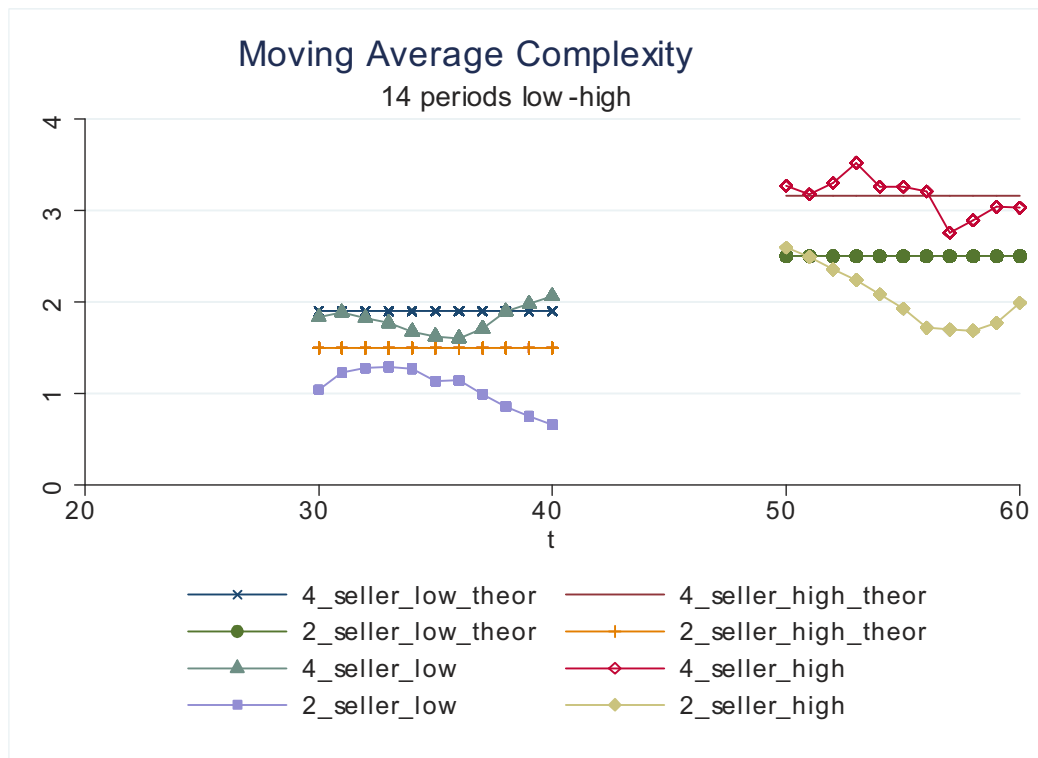


Figure 2

Note that in the treatment with  $n = 2$  the average complexity is systematically lower than a predicted value, while in the treatment with  $n = 4$  the average complexity fluctuates around the predicted value.

According to *Proposition 1* firms would choose either high or low (corner values) complexity levels. The frequency of choosing high complexity is predicted to be higher for  $n = 4$  than for  $n = 2$ . Figure 3 presents a histogram of complexity choices for  $n = 2$  and  $n = 4$ . We clearly observe the tendency to choose corner values of complexity interval as well as the supporting evidence for the fact that higher complexity levels are chosen more frequently in  $n = 4$  treatment.

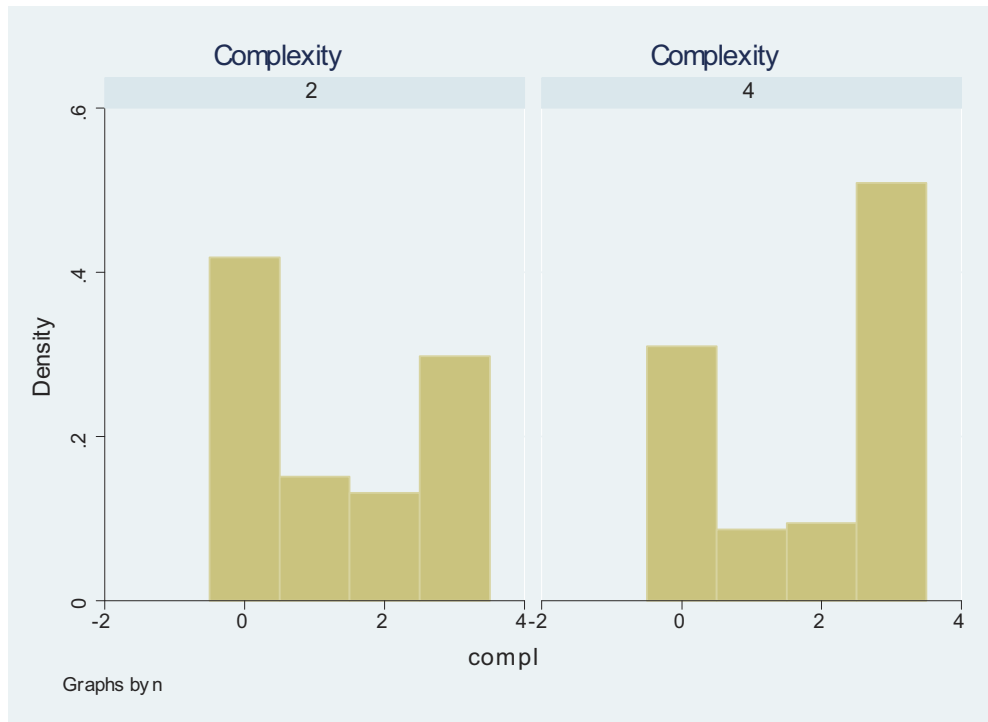


Figure 3

*Result 3 Average prices paid by uninformed consumers increase with number of firms and increase with the upper level of complexity.*

The theory predicts that the average price paid by uninformed buyers increases with the number of firms and with the upper level of complexity. The results of the *eight* sessions are summarized in the table below:

		Average Price								
		$\bar{p}$	$n = 2$				$n = 4$			
			1	2	3	4	5	6	7	8
Experimental	5	71	72	78	79	73	79	80	80	
	3	45	54	60	65	68	69	70	71	
Theoretical	5	<b>82</b>				<b>84</b>				
	3	<b>54</b>				<b>64</b>				

Table 5

The average price is systematically higher in the sessions employing 4 sellers than in the sessions employing 2 seller treatment as well as in the phases with upper complexity level is

5 than in the phases with upper complexity level being 3. This difference is significant by *Wilcoxon rank - sum test* at 1 percent significance level.

Figure 4 displays five-period moving averages of average prices in the Two- and Four-seller treatments, together with the theoretical predicted values.



Figure 4

Comparing the values of average prices across the two phases, we can observe a clear shifts in the predicted directions. The average prices tend to be higher when the upper level of complexity is high. So uninformed consumers are better off when the upper level of complexity is lower. Note that in the treatment with  $\bar{\rho} = 5$ , the prices are systematically lower than the predicted values. Risk-aversion of the players could be a candidate explanation for this result, however this result does not hold for  $\bar{\rho} = 3$  treatment.

We will come back later to the discussion of the effect of risk aversion on experimental result, when we analyze cumulative price distributions.

As to the effect of number of firms, average prices tend to be higher in a treatment with four seller compared to a treatment with two sellers. So in this case uninformed consumers are worse-off.

Result 5. Average minimum price paid by informed buyers decreases with the number of firms and increases with the upper level of complexity.

Minimum Price									
	$\bar{p}$	$n = 2$				$n = 4$			
Sessions		1	2	3	4	5	6	7	8
Experimental	5	60	60	67	67	30	48	51	51
	3	32	40	46	51	26	28	33	33
Theoretical	5	<b>70</b>				<b>57</b>			
	3	<b>40</b>				<b>31</b>			

Table 6

Experimental data clearly supports the theoretical prediction and again this result is significant by *Wilcoxon rank - sum test* at 1 percent significance level.

Figure 5 displays five-period moving averages for minimum prices in the Two- and Four-seller treatments.



Figure 5

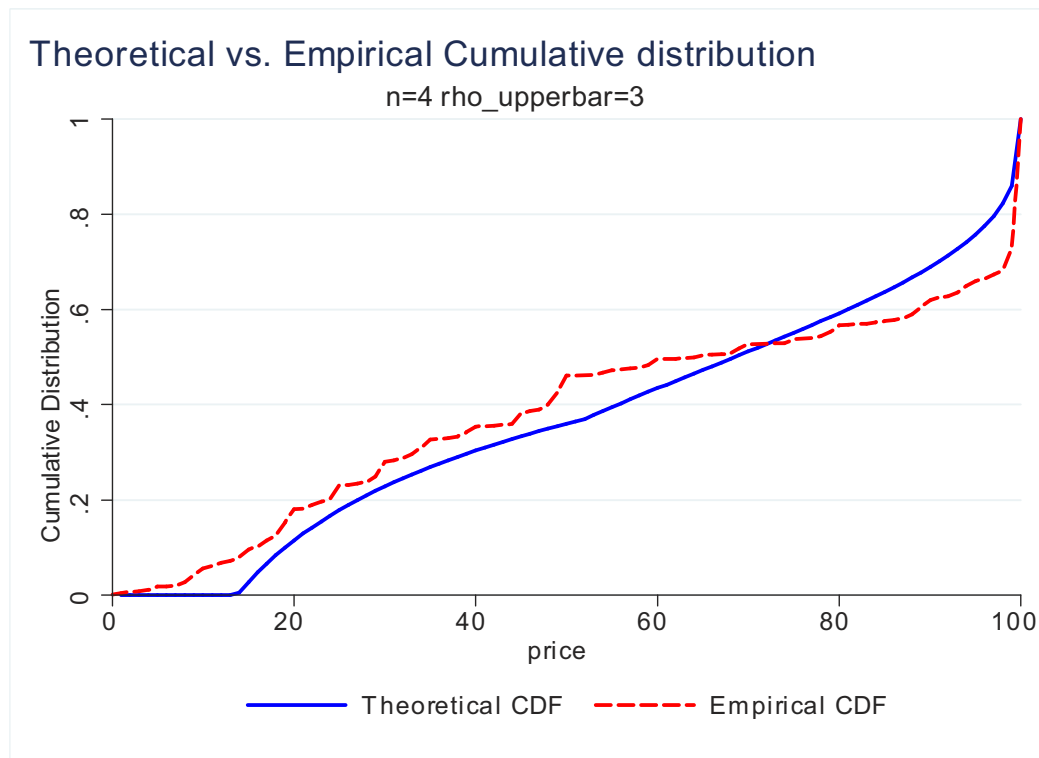


Figure 1.1: Figure 6

We can observe a clear shift in predicted by theory direction. So minimum prices are higher in the phases with high upper bound of the complexity level and lower in the treatment with 4 sellers compared to treatment with 2 sellers. We again observe the phenomenon of prices being systematically lower than predicted values in the treatment with higher upper bound of the complexity level.

Informed consumers are better off when the upper complexity level is lower and the competition is more intense in the sense of increased number of firms.

Our results presented so far clear provide support for the theoretical predictions.

However we observe some discrepancies between theoretical and empirical price distributions in each treatment. Figures 6-9 display the theoretical and empirical cumulative price distributions for all the treatments.

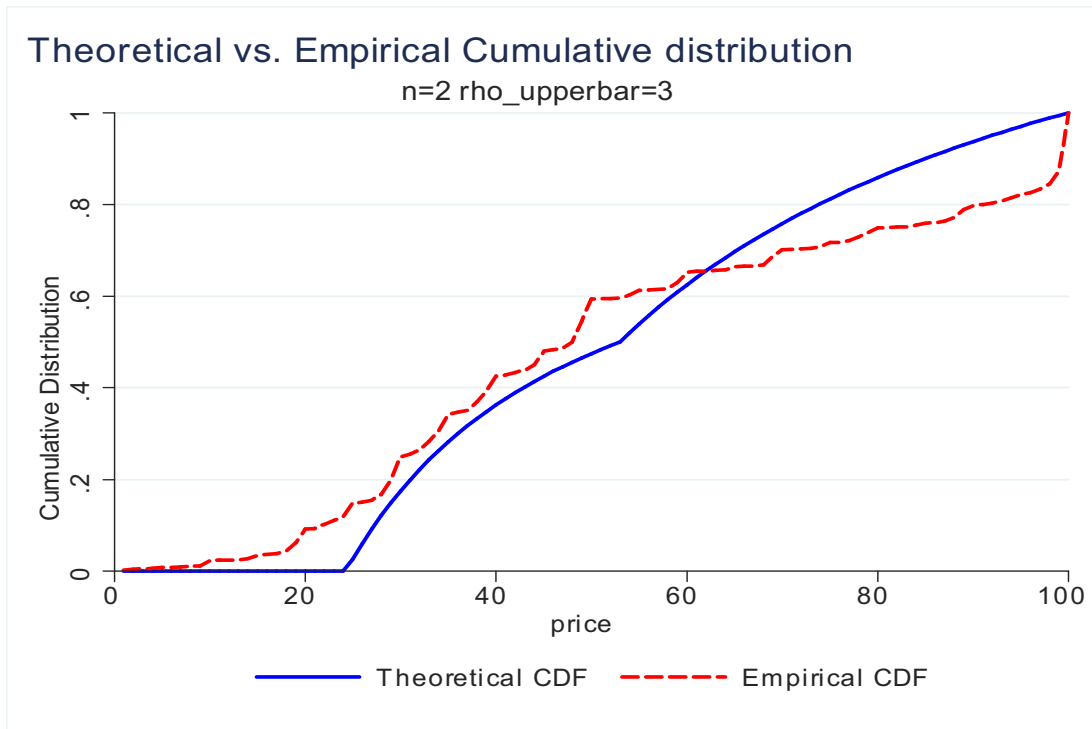


Figure 7

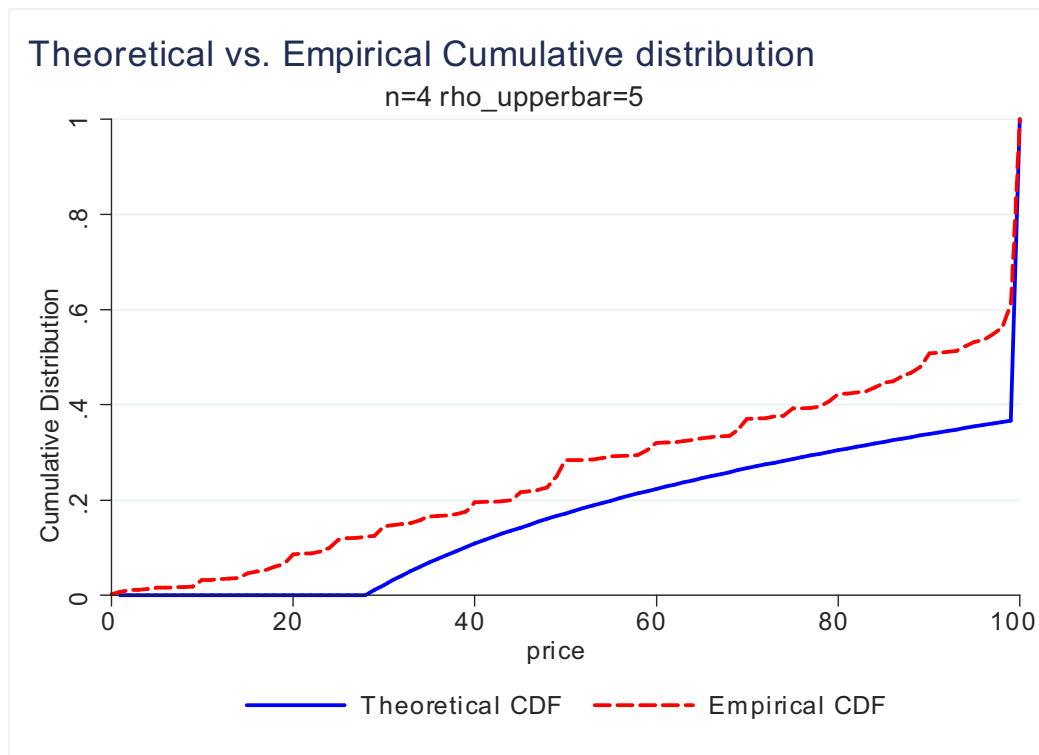


Figure 8

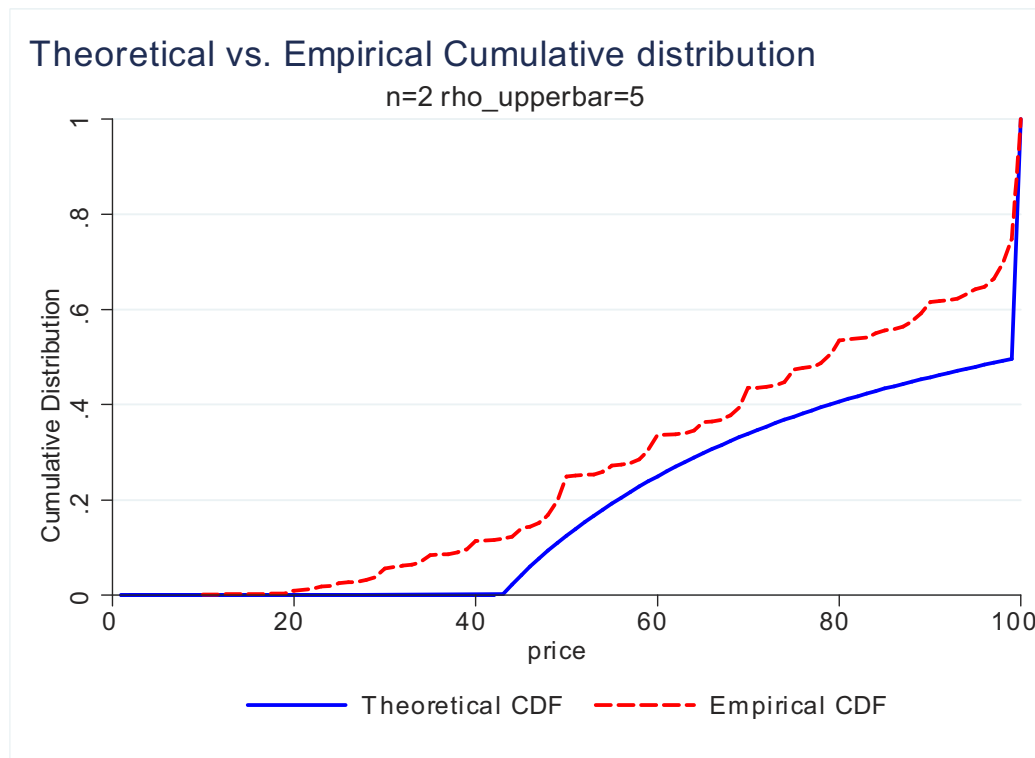


Figure 9

We can see that in "high" treatment the prices are systematically lower than theoretically predicted. However looking at the "low" treatment, we observe that theoretical and empirical price distributions cross, with prices being lower than theoretical predictions in a low range and higher in a high range. Morgan, Ozen and Sefton in their experimental study with exogenous proportion of uninformed consumers record similar discrepancies. However these discrepancies take place within different number of firms rather than within different proportions of uninformed consumers. One of the candidate explanations for observed gap between theoretical and empirical results is risk aversion of the subjects in the experiment. The theoretical results are derived for risk-neutral consumers, while in the reality the subjects are risk-averse. Morgan, Ozen and Sefton disregard such explanation, motivating this by the fact that if risk-aversion matters, than in the Four-seller treatment prices should be even higher than the theoretical predictions, which is not consistent with the observed result. This logic is indeed valid if firms cannot effect the proportion of informed and uninformed consumers.

In our setting, adding risk -aversion, might result in the following departure from the theoretical results derived under risk-neutrality: risk averse firms choose lower prices in the

range where they choose low price complexity and they choose higher prices in the range where they choose high price complexity than risk neutral firms. We do not formally derive this result for the moment, but just provide an intuition. Once firms choose high complexity they are exposed to higher risk of losing the uninformed consumers and therefore they do it only if return is sufficiently higher than under risk-neutrality, therefore they charge higher prices. However when firms choose low complexity they are ready to accept lower pay-off but to decrease the risk they face, therefore they charge lower prices, which results in higher and more sure probability to attract the informed consumers. In "high" treatment the threshold to choose high complexity is the maximum available price. That is why we might not observe the crossing phenomenon in this treatment.

## 1.5 Conclusions

Our experimental study contributes to the existing empirical literature on price dispersion providing additional evidence that in the presence of consumers that do not readily observe all the prices, the law of one price does not hold. Moreover, we find evidence that once having a possibility to ensure the presence of uninformed consumers, firms take an advantage of this opportunity and secure a positive proportion of uninformed consumers. Our experimental results confirm the theoretical predictions that increasing the number of firms results in more complex prices, higher proportion of uninformed consumers/ Moreover the average price that uninformed consumers pay increases with the number of firms, while the price that informed consumers pay decreases. The overall welfare is thus increases as a result. Though the experimental data is largely in line with all the theoretical predictions, we still observe as in previous experimental studies the discrepancies between theoretical and empirical cumulative distributions. However the argument of risk-aversion gains more support in our setting, though it stays only a tentative explanation for the moment. A further development for our work would be to introduce product heterogeneity into the theoretical model both vertical and horizontal ad to study the incentives of firms to introduce noise on the price dimension or on the product dimension, and therefore to affect either price or product comparison by consumers.



## 1.6 Appendix

### A: Equilibrium existence and uniqueness

#### Complexity choice

First let us consider profit at  $p = v$ . At this price firm  $i$  will not get any price shoppers and thus maximizes the proportion of uninformed consumers, by choosing  $\bar{\rho}$ . Let  $p_L$  be the lowest price of the equilibrium support. At  $p = p_L$  firm  $i$  gets all the price shoppers and shares the expected demand from uninformed consumers with other firms. Therefore it wants to maximize the proportion of informed consumers by choosing  $\underline{\rho}$ .

$$p \left\{ [1 - \widehat{F}]^{n-1} \widehat{\lambda}(1) + \frac{1}{n} (1 - \bar{\Lambda}) \right\} = \widehat{p} \left\{ [1 - \widehat{F}]^{n-1} \widehat{\lambda}(0) + \frac{1}{n} (1 - \underline{\Lambda}) \right\}$$

From where follows:

$$[1 - \widehat{F}]^{n-1} = \frac{1}{n} \frac{\bar{\Lambda} - \underline{\Lambda}}{\widehat{\lambda}(1) - \widehat{\lambda}(0)}, \quad (1.8)$$

where

$$\begin{aligned} \bar{\Lambda} &= \sum_{l=1}^n \binom{n-1}{l-1} [1 - \widehat{F}]^{n-l} [\widehat{F}]^{l-1} \widehat{\lambda}(l) \\ \underline{\Lambda} &= \sum_{l=0}^{n-1} \binom{n-1}{l} [1 - \widehat{F}]^{n-1-l} [\widehat{F}]^l \widehat{\lambda}(l). \end{aligned} \quad (1.9)$$

#### Uniqueness of $\widehat{F}$

Dividing (1.8) on  $[1 - \widehat{F}]^{n-1}$ , we obtain:

$$1 = \frac{1}{n} \frac{\sum_{l=1}^n \binom{n-1}{l-1} \left(\frac{\widehat{F}}{[1-\widehat{F}]}\right)^{l-1} (\widehat{\lambda}(l) - \widehat{\lambda}(l-1))}{\widehat{\lambda}(1) - \widehat{\lambda}(0)}.$$

The RHS of (1.8) is strictly increasing in  $\widehat{F}$  and when  $\widehat{F}$  equal to 0 it is equal to  $\frac{1}{n}$ , which is lower than 1. When  $\widehat{F}$  approaches 1 the RHS approaches infinity as  $\lim_{\widehat{F} \rightarrow 1} \frac{\widehat{F}}{[1-\widehat{F}]} = \infty$  for

$\widehat{F} < 1$ . Therefore as  $\widehat{F}$  increases the RHS crosses 1 from below, at a unique value if  $\widehat{F}$ .

## Price distribution

If all the competitors charge prices so that  $F(p) \leq \widehat{F}$ , then in equilibrium they also choose complexity pair  $(\bar{\rho})$ , thus the expected fraction of price shoppers is  $\widehat{\lambda}(0)$ . In equilibrium, the profits earned at each price charged (price such that  $F(p) \leq \widehat{F}$ ) should be equal. Therefore  $F^*(p)$  on  $(\widehat{F}, 1]$  should satisfy:

$$p_i \{ [1 - F(p)]^{n-1} \widehat{\lambda}(0) + \frac{1}{n} (1 - \underline{\Lambda}) \} = y(1 - \underline{\Lambda}) \frac{1}{n} \quad (1.10)$$

solving for symmetric equilibrium,  $F^*(p)$  is given by:

$$F^*(p) = 1 - \left[ \frac{y[(1 - \underline{\Lambda}) \frac{1}{n}] - p[(1 - \underline{\Lambda}) \frac{1}{n}]}{p \widehat{\lambda}(0)} \right]^{\frac{1}{n-1}} \quad (1.11)$$

Note that  $F^*(p)$  is uniquely defined on the interval  $(\widehat{F}, 1]$

Consider now prices such that  $F(p) > \widehat{F}$ . At this prices the level of complexity that maximize the expected profit is  $\rho = \underline{\rho}$ .

Let  $\bar{\Phi}(p)$  be the expected demand from price shoppers when firm  $i$  chooses transparent price/complex matching value. Then

$$\bar{\Phi}(p) = \sum_{l=1}^n \binom{n}{l} [1 - \widehat{F}]^{n-l} [\widehat{F} - F(p)]^{l-1} \widehat{\lambda}(l)$$

At any  $p$  the expected profit of firm  $i$  should be the same as when charging price  $\widehat{p}$  such that  $F(\widehat{p}) = \widehat{F}$ . So  $F^*(p)$  on  $[p_L, \widehat{p})$  solves:

$$p \{ \bar{\Phi}(p) + \frac{1}{n} (1 - \bar{\Lambda}) \} = \widehat{p} \{ [1 - F(\widehat{p})]^{n-1} \widehat{\lambda}(1) + \frac{1}{n} (1 - \bar{\Lambda}) \} \quad (1.12)$$

At the lowest price of the support,  $F(p_L) = 0$ , therefore  $p_L$  satisfies:

$$p_L \{ \bar{\Lambda} + \frac{1}{n} (1 - \bar{\Lambda}) \} = \widehat{p} \{ [1 - F(\widehat{p})]^{n-1} \widehat{\lambda}(1) + \frac{1}{n} (1 - \bar{\Lambda}) \}$$

The unique  $p_L$  is given by:

$$p_L = \frac{\widehat{p}\{[1 - F(\widehat{p})]^{n-1}\widehat{\lambda}(1) + \frac{1}{n}(1 - \overline{\Lambda})\}}{\overline{\Lambda} + \frac{1}{n}(1 - \overline{\Lambda})}$$

### Uniqueness of $F^*(p)$

We showed above that  $F^*(p)$  is uniquely determined on  $[\widehat{p}, v]$ . Note that  $p_L$  is uniquely determined. Therefore to show that  $F(p)$  is unique, it is sufficient to show that it is unique for any  $p$  of the support  $[p_L, v]$ . However on the interval  $[p_L, \widehat{p})$ , the uniqueness requires further examination. Consider (1.12), the right hand side (*RHS*) of this equation is given by  $\widehat{p}\{[1 - \widehat{F}]^{n-1}\widehat{\lambda}(1) + \frac{1}{n}(1 - \overline{\Lambda})\}$ , which is a singleton and does not vary with  $p$ . The left hand side (*LHS*) can be rewritten for a given  $p$  as following:

$$p\left\{\sum_{l=1}^n \binom{n}{l} [1 - \widehat{F}]^{n-l} [\widehat{F} - F]^{l-1} \widehat{\lambda}(l) + \frac{1}{n}(1 - \overline{\Lambda})\right\}. \quad (1.13)$$

In order to proof the uniqueness of  $F(p)$ , it is sufficient to show that there is a unique  $F$  that satisfies (1.12). Note that (1.13) is strictly decreasing with  $F$  and strictly increasing in  $p$ . Let  $F = 0$ , then at  $p = p_L$  *LHS* is equal to *RHS*, while at  $p > p_L$ , *LHS* is greater than the *RHS* for any  $p$  on  $[p_L, \widehat{p})$ . At the same time at  $F = \widehat{F}$ , the *LHS* is strictly smaller than *RHS* at  $p < \widehat{p}$ . Given that (1.13) is strictly decreasing with  $F$ , there exists a unique  $F$ , that satisfies (1.12).

## B: Increase in $n$ .

Expression (1.4) can be rewritten as following:

$$n - 1 = \frac{\sum_{l=1}^n \binom{n-1}{l-1} \left(\frac{\widehat{F}_n}{[1-\widehat{F}_n]}\right)^{l-1} (\lambda(l, n) - \lambda(l-1, n))}{\lambda(1, n) - \lambda(0, n)}. \quad (1.14)$$

We impose the following assumptions:

1. If all firms choose transparent prices and complex product features or equivalently all firms choose complex prices, the fraction of informed consumers is the same for any number

of competitors on the market, i.e.  $\lambda(0, n) = \lambda(0, n')$  and  $\lambda(n, n) = \lambda(n', n')$ ,  $\forall n, n'$ .

2. Let  $\widehat{F}_n$  denote  $\widehat{F}$  for  $n$  number of firms. Then from (1.14),  $\widehat{F}_n$  solves:

$$(n-1) = \frac{\sum_{l=1}^n \binom{n-1}{l-1} \left(\frac{\widehat{F}}{1-\widehat{F}}\right)^{l-1} (\lambda(l, n) - \lambda(l-1, n))}{(\lambda(1, n) - \lambda(0, n))}.$$

Then for  $n = 2$ ,  $\widehat{F}_2$  solves:

$$1 = \frac{\widehat{F}_2}{[1 - \widehat{F}_2]} \frac{\lambda(2, 2) - \lambda(1, 2)}{\lambda(1, 2) - \lambda(0, 2)}.$$

For  $n = 3$ ,  $\widehat{F}_3$  solves:

$$\begin{aligned} 2 &= 2 \frac{\widehat{F}_3}{[1 - \widehat{F}_3]} \frac{\lambda(2, 3) - \lambda(1, 3)}{\lambda(1, 3) - \lambda(0, 3)} + \left(\frac{\widehat{F}_3}{[1 - \widehat{F}_3]}\right)^2 \frac{\lambda(3, 3) - \lambda(2, 3)}{\lambda(1, 3) - \lambda(0, 3)}, \\ 1 &= \frac{\widehat{F}_3}{[1 - \widehat{F}_3]} \frac{\lambda(2, 3) - \lambda(1, 3)}{\lambda(1, 3) - \lambda(0, 3)} + \frac{1}{2} \left(\frac{\widehat{F}_3}{[1 - \widehat{F}_3]}\right)^2 \frac{\lambda(3, 3) - \lambda(2, 3)}{\lambda(1, 3) - \lambda(0, 3)}. \end{aligned}$$

Therefore it must be the case that:

$$\begin{aligned} \frac{\widehat{F}_2}{[1 - \widehat{F}_2]} \frac{\lambda(2, 2) - \lambda(1, 2)}{\lambda(1, 2) - \lambda(0, 2)} &= \frac{\widehat{F}_3}{[1 - \widehat{F}_3]} \frac{\lambda(2, 3) - \lambda(1, 3)}{\lambda(1, 3) - \lambda(0, 3)} + \frac{1}{2} \left(\frac{\widehat{F}_3}{[1 - \widehat{F}_3]}\right)^2 \frac{\lambda(3, 3) - \lambda(2, 3)}{\lambda(1, 3) - \lambda(0, 3)}, \\ \text{or } \frac{\widehat{F}_2}{[1 - \widehat{F}_2]} \frac{\lambda(2, 2) - \lambda(1, 2)}{\lambda(1, 2) - \lambda(0, 2)} &= \frac{\widehat{F}_3}{[1 - \widehat{F}_3]} \left(\frac{\lambda(2, 3) - \lambda(1, 3)}{\lambda(1, 3) - \lambda(0, 3)} + \frac{1}{2} \left(\frac{\widehat{F}_3}{[1 - \widehat{F}_3]}\right) \frac{\lambda(3, 3) - \lambda(2, 3)}{\lambda(1, 3) - \lambda(0, 3)}\right). \end{aligned}$$

Note that if

$$\frac{\lambda(2, 2) - \lambda(1, 2)}{\lambda(1, 2) - \lambda(0, 2)} < \frac{\lambda(2, 3) - \lambda(1, 3)}{\lambda(1, 3) - \lambda(0, 3)} + \frac{1}{2} \left(\frac{\widehat{F}_3}{[1 - \widehat{F}_3]}\right) \frac{\lambda(3, 3) - \lambda(2, 3)}{\lambda(1, 3) - \lambda(0, 3)},$$

then  $\widehat{F}_2$  must be larger than  $\widehat{F}_3$ .

Therefore  $\widehat{F}_2 > \widehat{F}_3$  if either  $\frac{\lambda(2,2)-\lambda(1,2)}{\lambda(1,2)-\lambda(0,2)} \leq \frac{\lambda(2,3)-\lambda(1,3)}{\lambda(1,3)-\lambda(0,3)}$  or  $\frac{\lambda(2,2)-\lambda(1,2)}{\lambda(1,2)-\lambda(0,2)} - \frac{\lambda(2,3)-\lambda(1,3)}{\lambda(1,3)-\lambda(0,3)} < \frac{1}{2} \left(\frac{\widehat{F}_3}{[1-\widehat{F}_3]}\right) \frac{\lambda(3,3)-\lambda(2,3)}{\lambda(1,3)-\lambda(0,3)}$ .

Consider now  $n = 4$ .

$\widehat{F}_4$  solves:

$$\begin{aligned} 3 &= 3 \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \frac{\lambda(2, 4) - \lambda(1, 4)}{\lambda(1, 4) - \lambda(0, 4)} + 3 \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right)^2 \frac{\lambda(3, 4) - \lambda(2, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right)^3 \frac{\lambda(4, 4) - \lambda(3, 4)}{\lambda(1, 4) - \lambda(0, 4)}, \\ 1 &= \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \frac{\lambda(2, 4) - \lambda(1, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right)^2 \frac{\lambda(3, 4) - \lambda(2, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \frac{1}{3} \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right)^3 \frac{\lambda(4, 4) - \lambda(3, 4)}{\lambda(1, 4) - \lambda(0, 4)} \\ \text{or } 1 &= \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \frac{\lambda(2, 4) - \lambda(1, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \left( \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right) \frac{\lambda(3, 4) - \lambda(2, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \frac{1}{3} \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right)^2 \frac{\lambda(4, 4) - \lambda(3, 4)}{\lambda(1, 4) - \lambda(0, 4)} \right). \end{aligned}$$

$\widehat{F}_3$  is larger than  $\widehat{F}_4$ , if

$$\frac{\lambda(2, 4) - \lambda(1, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right) \frac{\lambda(3, 4) - \lambda(2, 4)}{\lambda(1, 4) - \lambda(0, 4)} + \frac{1}{3} \left( \frac{\widehat{F}_4}{[1 - \widehat{F}_4]} \right)^2 \frac{\lambda(4, 4) - \lambda(3, 4)}{\lambda(1, 4) - \lambda(0, 4)} \quad (1.15)$$

$$\frac{\lambda(2, 3) - \lambda(1, 3)}{\lambda(1, 3) - \lambda(0, 3)} + \frac{1}{2} \left( \frac{\widehat{F}_3}{[1 - \widehat{F}_3]} \right) \frac{\lambda(3, 3) - \lambda(2, 3)}{\lambda(1, 3) - \lambda(0, 3)}. \quad (1.16)$$

When  $\frac{\lambda(2,4)-\lambda(1,4)}{\lambda(1,4)-\lambda(0,4)} \geq \frac{\lambda(2,3)-\lambda(1,3)}{\lambda(1,3)-\lambda(0,3)}$  and  $\frac{\lambda(3,4)-\lambda(2,4)}{\lambda(1,4)-\lambda(0,4)} \geq \frac{\lambda(3,3)-\lambda(2,3)}{\lambda(1,3)-\lambda(0,3)}$  then the condition (1.15) is satisfied.

The same logic can be applied for  $n$  larger than four.

## C: Vertical differentiation

We outline the assumptions necessary to establish the existence of the equilibrium. For formal proofs refer to Dubovik (2008).

*Assumption 1: The utility function is strictly decreasing in  $p$ , strictly increasing in  $q$ , and continuous in  $(p, q)$ .*

The outside option is given by  $U(p^r, q^r)$ . Consumers will purchase a good  $i$  if

$$U(p_i, q_i) \geq \max [U(p^r, q^r), U(p_j, q_j)].$$

i. The pairs  $(p^*, q^*)$  that are offered in equilibrium solve the problem

$$\begin{aligned} \max_{(p,q)} (p - w(q)) & \quad (1.17) \\ \text{s.t. } U(p, q) & \geq u, \end{aligned}$$

for every  $u$ , where  $u$  is a certain level of utility. Consumers and firms have opposing interests with respect to pair  $(p, q)$ , so it is reasonable to assume that this problem has a solution. For technical reasons Dubovik (2008) introduces a slightly stronger assumption to guarantee the uniqueness of the solution:

*Assumption 2: Problem (1.17) has a unique solution  $(q^*(u), p^*(u))$  for every  $u$ . There exists a continuous function  $g(p)$  such that  $q^*(u) = g(p^*(u))$  for given  $u$ .*

*Assumption 1* and *Assumption 2* allow to claim that the pairs  $(p^*, q^*)$  that would be offered in equilibrium lie on the contract curve, as choosing  $(p^*, g(p^*))$  strictly dominates all other choices. Also these assumptions imply that utility over a contract curve,  $U(p, g(p))$ , is strictly monotone in  $p$ .

Now let the equilibrium margin,  $m$ , be expressed as:

$$m(p) = p - w(g(p)).$$

Dubovik (2008) shows that the following is true (Lemma 1):

$$U(p_i, g(p_i)) > U(p_j, g(p_j)) \iff m(p_i) < m(p_j).$$

Therefore, choosing  $p$  and  $q$  translates for the firms into choosing  $m(p)$ .

The equilibrium is characterized by a price and a quality dispersion.<sup>1</sup> Firms make positive profits. In equilibrium only the lowest or the highest level of complexity is chosen. The firms that charge higher margins choose high complexity and the firms that choose low margins choose low complexity.

Depending on the shape of consumer preferences and the firms' cost functions, the equi-

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1. Except for the case when  $U(p, q) = h(q) - p$ . In this case there is a unique quality offered and dispersion exists only in prices.

librium dispersion of margins translates into the equilibrium dispersion of price and quality pairs in the following way:

**Case 1** Utility is strictly decreasing in  $p$ . In this case:

$$U(p_i, g(p_i)) > U(p_j, g(p_j)) \Leftrightarrow p_i < p_j$$

so that *informed consumers* will buy the cheapest product. Decreasing utility is consistent with both increasing and decreasing  $g(p)$ :

a)  $g(p)$  is increasing in  $p$ : this results in *informed* consumers choosing the lowest price and the lowest quality as the marginal value of quality is low. Uninformed consumers get higher prices and higher qualities.

b)  $g(p)$  is decreasing in  $p$ : in equilibrium *informed* consumers still buy the cheapest good but now the good is of the highest quality. *Uninformed* consumers pay higher prices for a lower quality compared to informed consumers.

**Case 2:** Utility is strictly increasing in  $p$ .

In this case the *informed* consumers buy the most expensive good, as the marginal value of quality is high. This case corresponds to  $g(p)$  being strictly increasing. Uninformed consumers on average buy goods of lower price and quality

## **D: Instructions**

### **GENERAL INSTRUCTIONS**

Welcome to the Experimental Laboratory of Toulouse School of Economics.

This is an experiment on individual decision making.

Please read carefully the instructions and raise your hand if you have any questions or doubts. It is important that you do not talk to any of the other participants in the room until the experiment is over.

You will earn an amount of money that depends on your own and other people's decisions.

The experiment consists of 3 phases, in each of which you will earn points. Your points will accumulate over all the 3 phases of the experiment. You will be paid for each of the 3 phases.

Points will be converted into cash using an exchange rate of 10,000 points = 1 Euro. Note that the more points an individual earns the more cash he will receive at the end of the experiment.

You will receive the instructions for each phase at the beginning of that phase.

We will now distribute the instructions for Phase 1.

### **INSTRUCTIONS for PHASE 1**

This phase consists of 20 periods.

In every period you will be a seller of a homogeneous good. There are 12 participants in today's experiment. You will be randomly matched every period into 6 markets with 2 sellers in each market, so that the participant who is the other seller in your market will change randomly in every period. The buyers are simulated by computerized "robots".



## Trading Instructions

- i. The cost of a unit of the good for the seller is zero. Therefore, every period you can sell as many units of good as the buyers demand.
- ii. In every period as a seller, you have to make two choices: the PRICE of the good and the level of EFFORT ( $E_i$ ) required for the buyers to observe the price and to compare it with the price offered by the other seller in your market:
  - The PRICE can be any integer from 0 to 100;
  - The level of EFFORT should be chosen from the set  $\{0;1;2;3;4;5\}$ . The choice of EFFORT is costless.

An example of the choice screen is given in the Figure below.

Period

1 out of 1

Remaining Time 25

Please choose your **PRICE** (integer) from 0 to 100

Choose the buyers' **EFFORT** level needed to observe the price

0  
 1  
 2  
 3  
 4  
 5

Continue

You have 30 seconds to submit your choice. Click on the Continue button to submit your PRICE and EFFORT decisions. The computer will wait until all the sellers have made their decisions before displaying the choices of the two sellers in each market.

- 3 The level of EFFORT chosen by the each of the two sellers determines how difficult it is for buyers to observe and compare the prices in the market. Together with the EFFORT choice of your competitor, your EFFORT choice contributes to the overall average EFFORT buyers need to exert to observe and compare prices in the market.

A level of average EFFORT of 0 corresponds to perfectly observable and comparable prices, while a level of average EFFORT, say, of 3 makes it harder for buyers to observe and compare the prices. Finally a level of average EFFORT of 5 makes it impossible for buyers to observe and compare the prices.

- iii. In every period, there are 20 identical robot buyers in each market. After the two sellers in the market have made their PRICE and EFFORT decisions, each buyer will purchase 4 units of the good. Buyers can be uninformed or informed.

Uninformed buyers do not observe the prices and buy an equal amount of 2 units from each seller, independently of their price.

Informed buyers perfectly observe all the prices and purchase all 4 units from the seller with the lowest price. If two sellers have the lowest price, the informed buyers purchase an equal amount of 2 units from each seller.

- iv. The number of uninformed and informed buyers in each period depends on yours and your competitor's EFFORT level choices. Buyers in a market become uninformed or informed after the market average EFFORT level,  $E_{av} = \frac{E_1 + E_2}{2}$ , is determined. More precisely,

- the number of uninformed buyers is proportional to the average EFFORT level and is  $4 \times E_{av} = \frac{E_1 + E_2}{2}$ .
- the remaining buyers are informed. Therefore their number is  $20 - (4 \times E_{av} = \frac{E_1 + E_2}{2})$ .

Notice that:

- the higher the overall average EFFORT level, the more buyers become uninformed.
- the lower the overall average EFFORT level, the more buyers become informed.

For the exact number of uninformed and informed buyers under different levels of EFFORT choices, refer to the table below.

Suppose that you are seller 1 in your market. The first column of the table indicates your possible choices of levels of EFFORT ( $E_1$ ) for each choice of level of EFFORT of

the other seller in your market. The second column indicates all the possible levels of EFFORT chosen by the other seller in your market ( $E_2$ ).

EFFORT choice:		Average EFFORT ( $E_{av}$ ) = $(E_1 + E_2)/2$	Number of Uninformed buyers, ( $E_{av} \times 4$ )	Number of Informed buyers, ( $20 - (E_{av} \times 4)$ )
Your choice, $E_1$	Other's choice, $E_2$			
0	0	0	0	20
	1	0.5	2	18
	2	1	4	16
	3	1.5	6	14
	4	2	8	12
	5	2.5	10	10
1	0	0.5	2	18
	1	1	4	16
	2	1.5	6	14
	3	2	8	12
	4	2.5	10	10
	5	3	12	8
2	0	1	4	16
	1	1.5	6	14
	2	2	8	12
	3	2.5	10	10
	4	3	12	8
	5	3.5	14	6
3	0	1.5	6	14
	1	2	8	12
	2	2.5	10	10
	3	3	12	8
	4	3.5	14	6
	5	4	16	4
4	0	2	8	12
	1	2.5	10	10
	2	3	12	8
	3	3.5	14	6
	4	4	16	4
	5	4.5	18	2
5	0	2.5	10	10
	1	3	12	8
	2	3.5	14	6
	3	4	16	4
	4	4.5	18	2
	5	5	20	0

6 At the end of every period you will see a "Result Screen" as shown in the Figure below.

Period		1 out of 2	Remaining Time 35	
<b>Market Prices</b>	<b>Effort choice</b>	<b>Quantity sold to Uninformed Buyers</b>	<b>Quantity sold to Informed Buyers</b>	
50	2	8	64	
60	0	8	0	
<b>Your Price</b>		50		
<b>Your Effort Choice</b>		2		
<b>Your Quantity Sold</b>		72		
<b>Your Profit for this period:</b>		3600		
<b>Cumulative Profit for this Phase:</b>		3600		
<a href="#">Continue</a>				

The upper half of the "Result Screen" provides information about your market for the period just completed. The displayed information highlights the two sellers' PRICE

offers (including yours), the corresponding EFFORT choices and the corresponding good UNITS sold to uninformed and informed buyers respectively. The displayed information is ordered from the lowest price to the highest price. Recall that you will be randomly matched with a different seller in every period.

The lower half of the "Result Screen" contains information on your PROFIT earned in the period just completed and on your Cumulative Profit earned in this Phase until that period. To begin the next trading period, click on the Continue button on the lower right corner of your screen.

- v. Recall that the cost of each unit of the good is zero. Your PROFIT in each period consists of two parts: a profit earned from selling to uninformed buyers ( $Profit\_uninf$ ) and a profit from selling to informed buyers ( $Profit\_inf$ ). The two parts of PROFIT are given by:

$$Profit\_uninf = \_PRICE\_ \times \_Number\ of\ uninformed\ Buyers\_ \times \_2\ units$$

$$Profit\_inf = \_PRICE\_ \times \_Number\ of\ informed\ Buyers\_ \times \_4\ units$$

- vi. Your PROFIT at the end of each period is given by:

- if your price is not the lowest in your market,

$$PROFIT = Profit\_uninf$$

- if your price is the lowest in your market,

$$PROFIT = Profit\_uninf + Profit\_inf$$

- if your price is the same as the price of your competitor in your market,

$$PROFIT = Profit\_uninf + (Profit\_inf \div 2)$$

- vii. Recall that you will be paid the Cumulative Profit over all the 20 periods.

Example:

Suppose that in period  $t$  you choose  $PRICE = 50$  and  $EFFORT = 2$ . Suppose the other seller in your market chooses  $PRICE = 60$  and corresponding  $EFFORT = 0$ . The resulting market average EFFORT is  $(2+0)/2 = 1$ . The number of uninformed buyers is then  $4 \times 1 = 4$ . The number of informed buyers is therefore  $20 - (4 \times 1) = 16$ .

$$Profit\_uninf = 50 \times (4 \times 2 \text{ units}) = 400$$

$$\text{Profit\_inf} = 50 \times (16 \times 4 \text{ units}) = 3200$$

Your price is the lowest on the market. Therefore in period  $t$ ,

$$\text{Your PROFIT} = \textit{Profit\_uninf} + \textit{Profit\_inf} = 400 + 3200 = 3600 \textit{ points}$$

Are there any questions before we begin?

## **Chapter 2**

# **Confusopoly: Price versus Product Features**



## 2.1 Introduction

In standard models of competition, consumers are assumed to be perfectly able to compare several offers and to choose the one with the highest value to them. Empirically, however, studies have shown that consumers do not always choose the best offer. Consumers differ in their preferences and also in the information they possess about prices and different product attributes. In the literature, this phenomenon is attributed either to psychological biases or to the searching costs of consumers, i.e. prior to purchasing any product, consumers incur a cost of learning the relevant information. This in turn leads to positive mark-ups, price and quality dispersion and suboptimal choices by some consumers.<sup>1</sup> In these models, firms adopt their strategic choices in the presence of uninformed consumers without directly affecting the amount of information available to consumers. The introduction of free information sources such as internet comparison sites should have diminished if not alleviated the problem by fostering consumers' comparison-shopping.

However, product comparison does not seem to have become easier. Product differentiation, together with the increased variety of price structures and complex product features, play an important role in assessing the net product value and do not facilitate the task of cross comparison. In many markets, firms have the potential to make their offers intentionally confusing by adopting complex price structures (price complexity) and product attributes (product complexity).

The objective of this paper is to shed light on the incentives of firms to make their prices and product features difficult to evaluate and to compare across competitors. Examples of various price structures and product attributes can be seen in many sectors, including credit offers in finance, mobile phone tariffs, the airfare industry and electricity contracts as well as high tech products, computers, and mobile phones. However, consumers can face difficulties in choosing the best product even in relatively simple markets. Consider a simple situation of purchasing a shampoo from a large retailer. *Which offer carries the best value: a shampoo for everyday use for all types of hair for the price of 5 euros per bottle of 500 ml versus a shampoo for fine hair for the price of 6 euros for the volume of 400 ml with a free trial*

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1. The main examples of these works include the "search-theoretic" models of Stigler (1961), Rothschild (1973), Burdett and Judd (1983), and Stahl (1989, 1996) and the "information clearing house" models of Salop and Stiglitz (1977), Varian (1980), Rothenthal (1980), Morgan, Orzen and Sefton (2001).



*conditioner?*

Manufacturers and retailers invest a considerable amount of time and money into opportunities to differentiate their products, leverage their brands, set strategic prices, and reduce the effectiveness of consumer comparison across all offers. Costly comparison may affect what kind of information consumers acquire prior to making their purchasing decision. If market prices are too complex to understand, consumers may abandon the idea of price comparison and select a product with the highest matching value. On the other hand, if product features are difficult to understand, consumers might simply invest in price comparison and buy the cheapest product.

The question that we address is the following: if firms can influence consumers' shopping behavior by making price or product features (or both) easy or hard to compare with the offers of their competitors, which dimension (if any) will they choose to complicate consumers' shopping behavior, and under which conditions? Would the market outcome consist of more complex and barely comparable offers, or would competition correct for consumer biases and lead to more transparent market outcomes?

We distinguish between price and product complexity. Price complexity represents how difficult it is for consumers to evaluate and compare prices across all the firms. It can reflect the number of itemized fees included in the price structures of firms, or the complexity of the frame being adopted – fees in small print, add-on prices, etc. Product complexity represents how difficult it is to assess the matching values associated with product characteristic. It can be thought of as the number of features included in the product, and how easily these features and characteristics can be compared across products.

Scitovsky (1950) argued that sellers might have an incentive to confuse buyers by emphasizing the extent to which products differ and by stressing their technical, chemical or functional complexity, i.e. making their product features appear more complex.

Several recent works look at the incentive of firms to make their pricing structures complex. Carlin (2009) and Piccione and Spiegler (2009) find that firms adapt to the competitiveness of the market by making their price structures intentionally confusing and thus limiting the price comparison possible by consumers. A vast body of literature has analyzed the incentives of firms to adopt the practices of price and product complexity. However to

our knowledge, our paper is the first to consider the interaction between these practices in a competitive environment.

We find that price and product complexities are substitutes rather than complements. Firms that charge low prices tend to make their prices transparent but make product comparisons complicated for the consumer. Conversely, firms that charge high prices have incentives to make their product features transparent but tend to introduce complexity in the price dimension. When the products are sufficiently differentiated, there is a middle range of prices, such that firms have incentives to make their offers fully transparent and thus encourage informed purchasing decisions.

The closest to our work in terms of modelling is Carlin (2009). In this paper, only a proportion of all consumers are informed about the prices of homogeneous goods. Firms affect this proportion by their individual choices of price complexity, thus exerting an externality on the demand that other firms face. At equilibrium, price dispersion arises because firms compete strategically for market share from both types of consumers. High price complexity is chosen by the firms that charge high prices, with a fixed ex-ante probability that depends only on the number of firms. We extend this analysis to the differentiated products, and study the market outcomes when there are four types of consumers: fully informed, only price informed, only product match informed, and fully uninformed consumers. Firms affect each proportion of consumers by their individual choices of price and product complexity.

A number of works have considered a setting with heterogeneous goods, allowing firms to control the information on product features. Lewis and Sappington (1994) show that a monopolist might prefer not to provide any information about product characteristics, however Ivanov, (2013) shows that in a competitive environment with firms being price-takers, the market results in full disclosure of information about product characteristics by all suppliers. Gabaix and Laibson (2004), on the contrary, show that if consumers are subject to psychological biases, firms have incentives to make their products inefficiently complex. Similarly, Spiegler (2006) demonstrates that if goods have multiple dimensions and consumers cannot evaluate them all, firms have the incentive to make it difficult for consumers to compare the value of the goods.

The first paper that analyses the incentives for firms to provide various types of informa-

tion is that of Anderson and Renault (2000 b). The authors address similar questions in the context of a single seller. At equilibrium, price-only and price-and-characteristic advertising can arise depending on the relative strength of product differentiation and consumer search costs. When the search costs are large and the firm must advertise to bring in consumers, the firm may still prefer to keep consumers in the dark about how much they would like the product. Even when the firm finds it optimal to inform consumers on both their match values and the price level, the level of advertising is too small because the firm only accounts for its private benefit per consumer informed when determining how much to advertise, and not the extra benefit to consumers of making a valuable match.

Conflicting findings and the fact that product and price complexity have been analyzed on a one-at-a-time basis, calls for a broader analysis of firms' incentives to provide information on each dimension of their offers. Our work contributes to the existing literature by making a first step towards understanding the trade-off between price and product complexity choices of competing firms.

This paper is organized as follows. Section 2 introduces a model set-up. In Section 3 we characterize the equilibrium outcome, followed by the discussion in Section 4. Section 5 analysis relevant policy implications. Finally, Section 6 concludes.

## 2.2 Model set-up

We consider a market where  $n$  differentiated and risk neutral firms sell a substitute good to a mass market of ex-ante identical consumers. The marginal costs are normalized to zero. A unit mass of ex-ante identical and risk-neutral consumers has a unit demand and a "matching value",  $v_i$ , for the product produced by firm  $i$ . The matching value can be either high ( $v^H$ ) or low ( $v^L$ ) with equal probabilities and is ex-ante unknown to both sides. In this paper we consider highly differentiated products, such that  $v^H \geq 2v^L$ . We further provide some intuition as to the predictions of the model for the case when products are less differentiated.

Consumers are heterogeneous with respect to the information they hold on firms' price and matching value. There are four types: *uninformed*, *price shoppers*, *product shoppers* and

*experts*. *Experts* ( $\phi$ ) have full information on prices and corresponding matching values, *price shoppers* ( $\lambda$ ) have information on prices only, whereas *product shoppers* ( $\mu$ ) have information on matching values only, and finally, *uninformed* ( $1 - \phi - \lambda - \mu$ ) consumers have information on neither prices nor on matching values. Partitioning consumers into several groups based on the information available to them is standard in the search literature and has been referred to as an “all or nothing” search process or a “clearinghouse” search model. We follow Carlin (2009) in allowing the firms to directly influence the information available to consumers.

Each firm, as well as choosing the “level” of its price,  $p_i$ , chooses the “complexity” of its price structure (price complexity),  $\rho_i \in [\underline{\rho}, \bar{\rho}]$ , and of its product match (product complexity),  $\nu_i \in [\underline{\nu}, \bar{\nu}]$ . These choices are costless and remain the private information of firm  $i$ . This assumption implies that adopting a certain framing does not convey information about its individual complexity, but rather contributes to the overall market complexity in an unobservable way. So, adopting a framing different to the competitor makes it harder to assess all the offers, but does not identify which frame is more complex.

The proportion of each type of consumers is determined by price and product complexity choices of firms. The effect of complexity on the consumer population is captured mathematically as follows.

*Assumption 1* The proportion of experts, price shoppers and product shoppers is determined as following:

$$\phi = \alpha_1 - \beta_1 \sum_{i=1}^n \rho_i - \gamma_1 \sum_{i=1}^n \nu_i \quad (2.1)$$

$$\lambda = \alpha_2 - \beta_2 \sum_{i=1}^n \rho_i + \gamma_2 \sum_{i=1}^n \nu_i \quad (2.2)$$

$$\mu = \alpha_3 - \beta_3 \sum_{i=1}^n \nu_i + \gamma_3 \sum_{i=1}^n \rho_i \quad (2.3)$$

$$\lambda, \mu, \phi \in (0, 1).$$

This model can be viewed as a reduced form of the general set-up, where firms can directly influence the cost of obtaining information by consumers, and with consumers having

heterogeneous costs of evaluating prices and product features. The behavior of consumers can be explicitly modeled by introducing the cost of comparison among prices and product features for consumers. Consumers can be thought of as heterogeneous in their costs of obtaining price and product information. The costs of obtaining price and product information ( $c^p$  and  $c^e$ ) are identically and independently distributed across the population of consumers. The aggregate market complexity may alter the distribution of consumers' costs, in a way that affects the equilibrium proportion of different types of consumers in a similar manner to our assumption. Consumers observe their realized costs but do not observe the actual aggregate market complexity levels, and thus cannot use aggregate market complexity to infer information about the realized price distribution. Nevertheless, consumers form rational expectations with respect to equilibrium price distribution. However, due to the analytical complexity of the setting with respect to heterogeneous consumers, we present a simplified version of the model, where the proportion of different consumer types is solely determined by the firm's choice of how easy it is to compare their offer with the offers of competitors. From this, we abstract from the analysis of the individual consumer searching decision. Nevertheless, consumers form rational expectations with respect to equilibrium price distribution. However, due to the analytical complexity of the setting with respect to heterogeneous consumers, we present a simplified version of the model, where the proportion of different consumer types is determined solely by the firm's choice of how easy it is to compare their offer with the offers of competitors. From this, we abstract from the analysis of the individual consumer searching decision. Nevertheless, we believe that fully endogenizing the decision of consumers about information acquisition would result in similar qualitative predictions. Consumers evaluate a certain dimension if, and only if, the expected benefit of additional information exceeds the corresponding costs. In equilibrium, a population of consumers would consist of four different types that differ in the amount of information that they possess prior to their purchasing decision. A change in the cost of information acquisition affects the equilibrium proportion of each type of consumer. Having this interpretation in mind, we introduce the set of assumptions that characterizes the relationship between the proportions of different types of consumers, and the complexity choices of the firms.

The first part of the *Assumption 1* states that the proportion of experts and price shoppers decreases with the firm's individual choice of price complexity, while the proportion of

product shoppers and uninformed consumers increases with the price complexity. Similarly the proportion of experts and product shoppers decreases with product complexity, while the proportion of price shoppers and uninformed consumers increases respectively.

The map in (2.1) implies that complexity choices by individual firms make it not only difficult to understand the price and product match of that particular firm, but may also make it more difficult to compare other competing offers. This assumption captures the idea that each individual firm has an effect on the consumers' cost of information acquisition about other firms' offers. The relevance of this assumption is immediate in the duopoly setting. If firm 1 makes its price too complex to evaluate, less consumers will compare the prices of two firms, and thus the proportion of consumers that are fully aware of prices (price shoppers) would decrease. However, as the number of firms increases, the power of any one firm to influence the proportion of different types of consumers is not obvious. The following example demonstrates the relevance of this assumption in the oligopoly setting. Consider a consumer who, prior to leaving home, wants to create a spreadsheet file that contains information about prices and product features of all the firms in the industry. If all prices and product features were to be presented in a unique standardized manner, then, once all information is entered into the file, say from comparison sites, a special program would convert prices and product features to a single comparable dimension. However, if one firm adopts a different pricing frame to others, the program no longer works, and the consumer needs search how to modify a program or alternatively to manually enter all the information to the file. If this exercise is time and effort consuming, the consumer may abandon the idea to compare prices.

Another demonstration of the relevance of the assumption that each firm has a unilateral power to affect the number of different types of consumers is that the more complicated the price structure of an individual firm, the more likely consumers are to miscalculate the real price and thus to make uninformed choices (assuming that the firm cannot control the direction of these mistakes). Therefore, by increasing its price complexity, each firm decreases the likelihood of fully informed choices to be made by consumers, which in our setting is translated into an increased proportion of uninformed consumers. Kayalci (2010) finds experimental evidence that the rate of consumers' mistakes increases as more firms adopt complex pricing structures. The two examples demonstrate the relevance and applicability

of *Assumption 1* in practice.

The second part of *Assumption 1* implies that the complexity choice of one firm does not affect the inherent difficulty of evaluating a competing firm's offer.

We assume that all consumers observe the actual matching value and price after the purchase, and can return the product that they have purchased if the price exceeds the valuation, i.e. consumers are protected by limited liability. We assume that the highest price that firms can charge in the equilibrium cannot exceed  $v^L$ . In Section 3.2 we provide sufficient conditions on the proportion of experts and price shoppers such that charging the price above  $v^L$  is indeed not a profitable deviation and thus equilibrium prices are bounded by  $v^L$ .

## Timing

In period 1, each firm  $i$  simultaneously chooses its price level,  $p_i$ , the complexity of its pricing schedule,  $\rho_i$ , and the complexity of its product characteristics,  $\nu_i$ . Firms' individual complexity choices determine the resulting proportion of *uninformed* consumers, *price shoppers*, *product shoppers* and *experts*. In period 2, matching values are realized and consumers make their best purchase decisions based on the information available to them.

## 2.3 Equilibrium

We assume that all consumers observe the actual matching value and price after the purchase, and can return the product that they have purchased if the price exceeds its valuation, i.e. consumers are protected by limited liability.

Define  $J^*$  and  $K^*$  to be the set of firms who quote the lowest price and who have the highest net value respectively in equilibrium. Let  $n_j$  and  $n_k$  be the number of firms in  $J^*$  and  $K^*$ , who split the demand from the price shoppers and experts equally.

The profit of a firm  $i$  for given choices of complexity is:

$$\Pi_i = p_i \left[ \lambda \frac{1_{\{i \in J^*\}}}{n_j} + \mu \frac{1}{n} + \phi \frac{1_{\{i \in K^*\}}}{n_k} + (1 - \lambda - \mu - \phi) \frac{1}{n} \right], \quad (2.4)$$

where the first term is the expected profit from the *price shoppers*: firm  $i$  gets all the price shoppers,  $\lambda$ , if it charges the lowest price out of  $n$  firms. In the case when more than one firm has the lowest price, each firm gets an equal share of *price shoppers*.

The second term is the expected profit from expected profit from *product shoppers*. Firm  $i$  gets all *product shoppers* if it has the highest matching value and shares the proportion of product shoppers with other firms in the case of a tie. Firm  $i$  has a matching value  $v^H$  with probability  $\frac{1}{2}$ . The expected proportion of product shoppers is given by:

$$\left(\frac{1}{2}\right)^n \left[ \sum_{l=0}^{n-1} \binom{n-1}{l} \frac{\mu}{l+1} + \frac{\mu}{n} \right] = \frac{\mu}{n}. \quad (2.5)$$

The third term is the expected demand from experts: firm  $i$  gets all the experts if it offers the highest net value out of  $n$  firms and shares the demand in case of a tie.

Finally, the last term is the *uninformed* consumers. Firms equally share the demand from *uninformed* consumers.

Note that from (2.4), firms earn the same expected margin on both *product shoppers* and *uninformed* consumers.

Thus, firm  $i$ 's net expected profit can be rewritten as following:

$$\Pi_i = p_i \left[ \lambda \frac{1_{\{i \in J^*\}}}{n_j} + \phi \frac{1_{\{i \in K^*\}}}{n_k} + \frac{1}{n} (1 - \lambda - \phi) \right]. \quad (2.6)$$

We continue the analysis without a loss of generality as if with only three types of consumers. From herein, we refer to the total proportion of product shoppers and uninformed consumers as *non-price shoppers*.

A quick examination of the payoff of firm  $i$  suggests that there is a positive mark-up and price dispersion in the equilibrium, as some consumers do not observe prices and would therefore still buy at the positive prices. At the same time, a positive proportion of price shoppers ensure that a one-price equilibrium does not exist. The following *Lemma* formally states this result.

**Lemma 4** *There is no pure strategy equilibrium.*

**Proof.** Pure strategy equilibrium with zero profit cannot exist, as either firm can exploit



the positive proportion of non-price shoppers (which it can unilaterally maintain by corresponding complexity choice), and obtain a strictly positive profit. Thus in any equilibrium there must be positive profits. Symmetric equilibrium with positive profits cannot exist as either firm can slightly undercut its price and attract all the price shoppers (the positive proportion of which is guaranteed by its corresponding complexity choices), keeping the demand from product shoppers unchanged. The same arguments apply to asymmetric equilibria. In the equilibria where one firm offers the lowest price, it has incentives to slightly increase its price which results in a strictly higher profit. In the equilibria where several firms offer the lowest price, firms have an incentive to slightly undercut their prices in order to gain a share of price shoppers. Therefore, any equilibrium should be in mixed strategies and should be characterized by positive profits. ■

The equilibrium price dispersion is a typical feature of the search models when some consumers are uninformed about prices charged by individual firms. In this sense, our result is similar to that of Varian (1980), Rosenthal (1980), Stahl (1989) and Robert and Stahl (1993). Note that product differentiation does not affect this result as products are ex-ante homogeneous. We extend this argument to the setting with four types of consumers and analyze the corresponding equilibrium choices of complexities by the firms. In the following analysis, we concentrate on symmetric mixed strategy equilibrium, where firms randomize their prices and complexity choices. Intuitively, the asymmetric strategies are not consistent with price dispersion because they imply that the distribution of prices charged by some firms stochastically dominate those charged by other firms.<sup>2</sup> Uninformed consumers do not possess any information, therefore they buy a product from a random firm, as the expected net value of all products is the same for them. Price shoppers do not possess information on the realized matching values and buy the product with the lowest price, as in this case the expected matching value is the same across all firms. Product shoppers do not possess information on prices and buy the product that carries the highest matching value, as the expected price is the same across all firms. Experts know all of the information and make fully-informed purchasing decisions.

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2. Baye, M.R., Kovenock, D., & de Vries, C.G.(1992). It takes two to tango: Equilibria in a model of sales. *Games and Economic Behavior*, 493–510. doi:10.1016/0899-8256(92)90033-O

### 2.3.1 Profit

Denote by  $\Lambda(p_i, \rho_i, \nu_i, \sigma_{-i})$  and  $\Phi(p_i, \rho_i, \nu_i, \sigma_{-i})$  the conditional expectations of  $\lambda$  and  $\phi$  respectively, given a choice of  $\rho_i, \nu_i$  and  $p_i$  of firm  $i$  and the equilibrium strategies of other firms,  $\sigma_{-i}$ . We suppress the arguments of  $\Lambda$  and  $\Phi$ , unless it is necessary to make a specific point. Further denote by  $\Lambda_p$  the conditional expectations of  $\lambda$ , given a choice of  $\rho_i, \nu_i, p_i, \sigma_{-i}$  and that  $p_i = p_{\min}$  and by  $\Phi_p$  the conditional expectations of  $\phi$  given a choice of  $\rho_i, \nu_i, p_i, \sigma_{-i}$  and that  $v_i - p_i > \max(v_j - p_j)$  for  $j \neq i$  respectively.

Given the equilibrium price distribution,  $F(p)$ , the expected profit in (2.6) can be expressed as following:

$$\Pi_i = p_i[(1 - F(p))^{n-1}\Lambda_p + H(p)\Phi_p + \frac{1}{n}(1 - \Lambda - \Phi)], \quad (2.7)$$

where  $H(p)$  is the probability of firm  $i$  having the highest net value out of  $n$  products.

Whenever all of the firms have the same realization of  $v_i$  the probability of having the highest net value is equal to the probability of having the lowest price. Whenever firm  $i$  has a matching value  $v^L$  and at the same time any other firm has matching value  $v^H$ , the probability of having the highest net value is zero for this firm.

Whenever firm  $i$  has the highest valuation, then the probability of having the highest net value is equal to 1. Whenever other firms have valuation  $v^H$  together with firm  $i$  the probability of having the highest net value is equal to the probability of having the lowest price out of all firms that have valuation  $v^H$ .  $H(p)$  is therefore given by:

$$\left(\frac{1}{2}\right)^n \left[ \sum_{l=0}^{n-1} \binom{n-1}{l} (1-F)^l + (1-F)^{n-1} \right] = \left(\frac{1}{2}\right)^n [(1-F(p))^{n-1} + (2-F(p))^{n-1}].$$

$H(p)$  is strictly decreasing in  $p$ . Moreover, at the highest price of the equilibrium support, the probability of having the highest net value is positive and thus is higher than the probability of having the lowest price, but at the same time is lower than the probability of having the highest match, i.e.  $(1 - F(v^L))^{n-1} < H(v^L) < \frac{1}{n}$ . The opposite is true at the lowest price of the support: the probability of attracting experts is lower than the probability of having the lowest price but higher than the probability of having the highest match.

### 2.3.2 Covered market

Note that the expression of the profit in (2.7) relies on the assumption that the market is fully covered, i.e. the highest price that firms charge in the equilibrium cannot exceed  $v^L$ . We now characterize sufficient conditions on  $v^L$ , and on the proportions of *experts* and *price shoppers* such that charging any price above  $v^L$  is not a profitable deviation and thus equilibrium prices are bounded by  $v^L$ . Assume that  $v^L$  is the highest price charged in the mixed strategy equilibrium. Then if firm  $i$  deviates to any price  $p' > v^L$ , it will obtain the following profit:

$$p'[H(p')\Phi_p + \frac{1}{2n}(1 - \Lambda - \Phi)].$$

At a price higher than the highest price charged by other firms, firm  $i$  will attract experts if and only if it has the highest valuation out of  $n$  firms and other firms have high enough prices, i.e. with probability  $H(p') < (\frac{1}{2})^n$ , and it will keep all the uninformed consumers if and only if its valuation is  $v^H$ , i.e. with probability  $\frac{1}{2}$ .

This deviation will be unprofitable if:

$$p'[H(p')\Phi_p + \frac{1}{2n}(1 - \Lambda - \Phi)] < v^L[(\frac{1}{2})^n\Phi_p + \frac{1}{n}(1 - \Lambda - \Phi)].$$

Therefore if the proportion of price shoppers and experts and  $v^L$  is sufficiently low:

$$\frac{(1 - \Lambda - \Phi)}{\Phi_p} < \frac{n((\frac{1}{2})^n v^L - H(p')p')}{\frac{1}{2}p' - v^L}. \quad (2.8)$$

We further characterize the conditions necessary to guarantee that  $v^L$  is the highest price that firms charge in any equilibrium. Assume  $p^{\max} > v^L$  is the highest price of the equilibrium support. Then firm  $i$  would always want to deviate to  $v^L$  if the following is true:

$$\frac{(1 - \Lambda - \Phi)}{\Phi_p} < \frac{n(H(v^L)v^L - (\frac{1}{2})^n p^{\max})}{\frac{1}{2}p^{\max} - v^L}. \quad (2.9)$$

The less restrictive out of the two conditions described above guarantees that the highest charge priced in equilibrium is  $v^L$ .

The following proposition characterizes a unique symmetric mixed strategy equilibrium of the price/match complexity game.

**Proposition 5** (*equilibrium*) *There exists a unique symmetric mixed strategy equilibrium where firms choose their prices according to equilibrium price distribution  $F^*(p)$  with the support  $[p_L, v^L]$  and there exists unique  $\hat{p}, \hat{F}$  and  $\tilde{p}, \tilde{F}$  such that:*

*i) Firms choose their complexity levels as follows:*

$$\begin{aligned} \rho &= \underline{\rho} \text{ and } \nu = \bar{\nu} && \text{for } p < \tilde{p} \\ \rho &= \underline{\rho} \text{ and } \nu = \underline{\nu} && \text{for } \tilde{p} < p < \hat{p} \\ \rho &= \bar{\rho} \text{ and } \nu = \underline{\nu} && \text{for } p > \hat{p} \end{aligned}$$

*such that  $\tilde{p} = \hat{p}$  for  $n = 2$  and  $\tilde{p} < \hat{p}$  for  $n > 2$ .*

*ii) Firms choose a pair  $(\underline{\rho}, \bar{\nu})$  with a unique probability,  $\tilde{F}$ ; a pair  $(\underline{\rho}, \underline{\nu})$  with a unique probability  $\hat{F} - \tilde{F}$ , and pair  $(\bar{\rho}, \underline{\nu})$  with probability  $1 - \hat{F}$ , which are given by:*

$$\frac{d\Lambda}{d\rho_i} \left[ (1 - \hat{F})^{n-1} - \frac{1}{n} \right] = \frac{d\Phi}{d\rho_i} \left[ \frac{1}{n} - \left(\frac{1}{2}\right)^n \left( (1 - \hat{F})^{n-1} + (2 - \hat{F})^{n-1} \right) \right], \text{ and} \quad (2.10)$$

$$\frac{d\Lambda}{d\nu_i} \left[ (1 - \tilde{F})^{n-1} - \frac{1}{n} \right] = \frac{d\Phi}{d\nu_i} \left[ \frac{1}{n} - \left(\frac{1}{2}\right)^n \left( (1 - \tilde{F})^{n-1} + (2 - \tilde{F})^{n-1} \right) \right] \quad (2.11)$$

*such that  $F^*(\hat{p}) = \hat{F}$  and  $F^*(\tilde{p}) = \tilde{F}$ .*

*iii)  $F^*(p)$  is strictly decreasing in  $p$  and has no mass points.*

**Proof.** We outline below the key arguments used to prove the proposition. *Appendix A* contains a complete proof of the existence and full characterization of the mixed strategy equilibrium. ■

### 2.3.3 Existence

A simple examination of (2.6) reveals that the payoff of each firm is continuous except in the case when the price of firm  $i$  is the lowest on the market and is equal to the price of at least one of the competitors, and when the price of firm  $i$  is such that several firms have the highest net value. In this case, firm  $i$  can discontinuously increase (decrease) its profits by

lowering (raising) its price. Note that the assumption that  $v^H > v^L$  implies that the point of discontinuity arises only if several firms charge the same price which happens to be the lowest price. Dasgupta and Maskin (1986) show that *i) the continuity of the sum of the payoffs*, and *ii) the weak semi-continuity of each firm's payoff guarantees the existence of the mixed strategy equilibrium*.

*i)* The continuity of the sum of payoffs is guaranteed by the fact that at the points of the discontinuity the change in payoff serves as a transfer between firms that have the lowest price or the highest net value on the market. In this sense, the game at the points of discontinuity can be viewed as zero-sum game, since the sum of payoffs is continuous.

*ii)* Weak lower semi-continuity of  $\Pi_i$  at the points of discontinuity holds since a lower price than the one at which a tie occurs generates strictly positive profits.

### 2.3.4 Complexity choices

The profit of firm  $i$  increases with the proportion of price shoppers and experts if the probability of having the lowest price, and the highest net value is high enough. Therefore, for a high probability of having the lowest price firms have incentives to maximize the expected demand from price sensitive consumers. For a low probability of having the lowest price, firms have incentives to minimize the proportion of price-insensitive consumers. The cut-off probabilities are such that the change in the expected demand of price shopper and experts resulting from an increase in the complexity levels is equal to the corresponding change in the proportion of non-price shoppers. We show that the probability with which firms choose different complexity levels depends on the number of firms in the market, and for  $n > 2$  on the relative magnitude of the derivative of  $\lambda$  and  $\phi$  with respect to complexity levels. The maximum and minimum boundaries of the probability with which firms choose different complexity levels depend only on the number of firms, and constitute a testable theoretical prediction.

Low price complexity maximizes both the expected proportion of price shoppers and experts at the same time and minimizes the proportion of non-price shoppers. High match complexity maximizes the proportion of price shoppers but at the same time minimizes the proportion of experts. Therefore, firms have incentives to minimize the proportion of price

shoppers when the probability of having the highest net value exceeds the probability of having the lowest price. Thus the two thresholds for price and match complexity choices may not be the same. In the duopoly setting, the probability of having the lowest price is equal to the probability of having the highest net value, therefore the two thresholds are the same and equal to one half. For the highly differentiated product ( $v^H > 2v^L$ ) and  $n > 2$ , firms choose complex matching values less often than transparent prices. The corresponding prices at which the probability thresholds are obtained characterize the price range for different complexity choices of the firms. The uniqueness of equilibrium price distribution ensures the uniqueness of the respective price thresholds.

In summary, for prices low enough firms have incentives to choose transparent prices and complex product matches, and for prices high enough the incentives change towards complex prices and transparent matches. For the middle range of prices, the resulting complexity choices depend on whether  $\hat{p}$  is greater or smaller than  $\tilde{p}$ . When the products are highly differentiated for  $p \in [\tilde{p}, \hat{p}]$ , firms choose fully transparent offers, i.e. a pair of complexity levels  $(\rho, \nu)$ . In Section 4 we discuss the robustness of these results for a low degree of product differentiation.

The choice of the extreme values of complexity levels in equilibrium is driven by the linearity property of  $\lambda$ ,  $\mu$ , and  $\phi$ . However, the monotonic relationship between prices and complexity levels described in *Proposition 1* holds even when relaxing the linearity assumption.

Restricting attention to either duopoly setting, or to the setting with only partially informed consumers (i.e. the proportion of experts is zero), results in a unique threshold for complexity choices,  $\tilde{F} = \hat{F}$ . In this case, there is a one to one trade-off between price and match complexity: firms choose transparent prices and complex match for  $p < \hat{p}$  and complex prices with transparent match for  $p > \hat{p}$ . The ex-ante frequency with which firms choose the latter combination depends only on the number of firms and is equal to  $\frac{1}{n^{\frac{1}{n-1}}}$ . Note that our setting generates, as a particular case, a result that is identical to Carlin (2009) in the price-complexity game with homogeneous products and two types of consumers: price shoppers and uninformed. This result arises due to the additivity property of  $\lambda$ . Firms face a trade off between price-sensitive and price-insensitive consumers. When prices are low more weight is put on price-sensitive consumers, therefore the optimal complexity choice is such

that maximizes the proportion of price shoppers, i.e.  $(\underline{\rho}, \underline{\nu})$  in our setting. Instead, when prices are higher than  $\hat{p}$ , firms minimize the proportion of price shoppers and choose  $(\bar{\rho}, \underline{\nu})$  respectively.

## 2.4 Discussion

When the proportion of experts is positive and the number of firms is greater than two, the frequency with which firms choose transparent prices and complex matching values are no longer equal, and firms choose to make their offers fully transparent with positive probability. The numerical example below compares the frequencies with which firms choose different complexity levels for  $n = 2$  and  $n = 3$  cases.

### 2.4.1 Complexity choices: example with $n = 2$ and $n = 3$ .

For  $n = 2$ , the threshold for choosing transparent versus complex prices, and the threshold for choosing complex versus transparent matching values, are given by:

$$\hat{F} = \tilde{F} = \frac{1}{2}.$$

In the duopoly setting, firms never choose to be fully transparent. This is due to the fact that the probability of attracting experts is always between the probability of attracting price shoppers and attracting non-price shoppers. Therefore, firms only need to choose between maximizing or minimizing the proportion of price shoppers and the expected proportion of experts is irrelevant for the complexity choices.

Let us now consider the case of three firms,  $n = 3$ . For simplicity we consider a case when the proportions of price shoppers and experts are symmetric with respect to the complexity levels, i.e.  $\frac{d\Lambda}{d\rho_i} = \frac{d\Phi}{d\rho_i}$  and  $\frac{d\Lambda}{d\nu_i} = -\frac{d\Phi}{d\nu_i}$ .

In this case  $\hat{F}$  is determined as follows:

$$\begin{aligned} \frac{5}{4}(1 - \hat{F})^2 + \frac{1}{4}(1 - \hat{F}) + \frac{1}{8} - \frac{2}{3} &= 0 \\ 1 - \hat{F} &\approx 0.57. \end{aligned}$$

The expression for  $\tilde{F}$  is:

$$\begin{aligned} \frac{3}{4}(1 - \tilde{F})^2 - \frac{1}{4}(1 - \tilde{F}) - \frac{1}{8} &= 0 \\ 1 - \tilde{F} &= \frac{1}{6}(1 + \sqrt{7}) \approx 0.61. \end{aligned}$$

In this case, firms choose a pair  $(\underline{\rho}, \bar{\nu})$  with probability  $\tilde{F} = 0.39$ , fully transparent offers with probability  $\hat{F} - \tilde{F} = 0.04$ , and a pair  $(\bar{\rho}, \underline{\nu})$  with probability  $1 - \hat{F} = 0.57$ . Compared with a duopoly case, there is now a region where firms choose to be fully transparent, and the frequency of choosing transparent prices decreases, whereas the frequency of choosing transparent match values increases as a result. Note that as  $n$  increases from 2 to 3, the frequency with which firms choose complex matches decreases, while the frequency with which firms choose complex prices increases. This suggests that stronger competition, in the sense of there being more firms on the market, leads to more transparent matching values and to more complex pricing structures at the same time. However, in our example, the frequency with which firms choose fully transparent offers increases. Therefore, the overall impact of more intense competition on market transparency requires further investigation. The following proposition formally states this result for any number of firms. For simplicity, we keep the symmetry assumption, however we believe that the result is robust without imposing a symmetry property, but this would require a more detailed investigation.

### 2.4.2 Competition and complexity

**Proposition 6** *ENTRY EFFECT: When the number of firms increases,  $F(\tilde{p})$  and  $F(\hat{p})$  decreases. The difference ( $F(\tilde{p})$  and  $F(\hat{p})$ ) if  $\tilde{p} > p_L$ , and decreases otherwise. In the limit, when  $n$  goes to infinity both price thresholds approach  $p_L$ , and the frequency with which firms choose complex price/transparent match approaches to one.*

**Proof.** Appendix B ■

Increasing  $n$  decreases the probability of having the lowest price faster than the probability of having the highest net value, or the probability of attracting product shoppers and uninformed consumers. Therefore, as  $n$  increases, firms decrease the frequency of choosing the complexity pair that maximizes the proportion of price shoppers. Therefore as  $n$



increases firms decrease the frequency of choosing the complexity pair that maximizes the proportion of price shoppers. As  $n$  increases further, the probability of attracting experts becomes smaller than  $(1/n)$  and thus firms more often choose a pairing of complexity that maximizes the joint proportion of product shoppers and uninformed consumers.

*Proposition 5* implies that a higher proportion of firms will add complexity to their prices and make their products more transparent when there is greater competition. Thus, we can draw a novel empirical prediction from the analysis: industry concentration should be negatively correlated with price complexity, and positively correlated with product complexity, *ceteris paribus*. In our experimental study of price complexity game in the homogeneous goods setting, we find evidence of negative correlation between industry concentration and price complexity. However, for intermediate values of  $n$ , increasing  $n$  leads to a higher frequency with which firms choose fully transparent offers, and therefore can in fact result in a more transparent market. As  $n$  approaches infinity, this frequency approaches zero, and therefore the market outcome is characterized by complex prices and transparent matching values.

This result should be interpreted with a certain degree of caution as the relationship between the proportions of different types of consumers and complexity levels is given exogenously in our setting, and, thus, does not capture the effect of a change in price dispersion when the number of firms increases.

## 2.5 Policy implications

The following section explores how regulation in the form of a complexity cap (lower  $\bar{\rho}$  and  $\bar{\nu}$ ) or a complexity floor (higher  $\underline{\rho}$  and  $\underline{\nu}$ ) would affect the price distribution and the aggregate market complexity. Complexity levels can be regulated by introducing a respective cap on how complex each offer can be, i.e. standardizing the framework for representing prices and product features, or alternatively, by inducing the disclosure of full information by firms for the purposes of creating a comparison technology that allows the reduction of comparison to fewer dimensions. Ease of comparison can also be potentially controlled by imposing a floor on how complex the offers can be. For example, forbidding comparative advertising can impose a limit on how transparent the market is. We characterize the conditions under

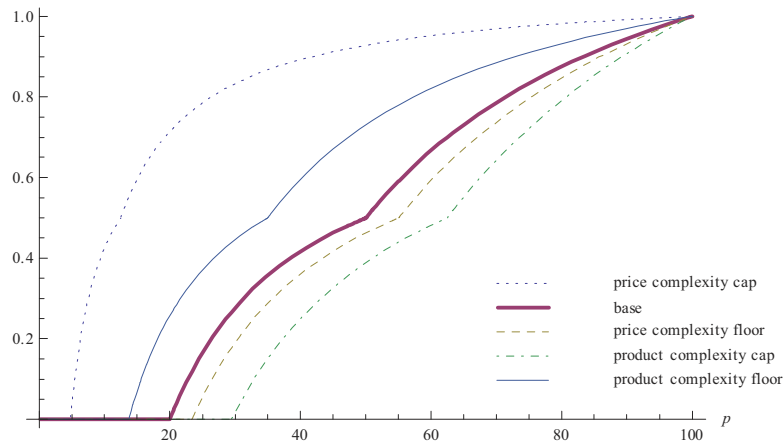
which these policies lead to an increase or decrease in prices.

**Proposition 7** (*Complexity regulation*) *Imposing a cap on price complexity or a floor on product complexity decreases equilibrium prices. Imposing a cap on product complexity or a floor on price complexity increases equilibrium prices.*

**Proof.** The respective thresholds for complexity choices are independent of the complexity choices, and depend only on the number of firms and on the degree of product differentiation. Therefore, the regulation of complexity levels affects price distribution via the effect on the proportions of different types of consumers and not via the change in the frequency of choosing different complexity levels. The higher the proportion of price insensitive consumers (product shoppers and uninformed consumers), the more firms put weight on higher prices. The higher the proportion of price shoppers, the more weight firms put on lower prices. The net effect on price distribution due to the change in the proportion of experts is less clear cut. On the one hand, the lower proportion of experts relaxes price competition. On the other hand, a lower proportion of experts may lead to a higher proportion of price experts, and therefore more intense price competition. Therefore, the net effect depends on the relative change of the proportion of other consumers when the proportion of experts changes. A decrease in price complexity increases the amount of price shoppers and experts, and decreases the amount of product shoppers and uninformed consumers. Therefore, the policy that decreases the upper bound or increases the lower bound of the price complexity leads to higher and lower prices respectively. As the product complexity increases, a proportion of price shoppers increases, while the proportion of experts decreases. In this case, the joint proportion of uninformed consumers and product shoppers is unchanged (*Assumption 1*). A reduction in the proportion of experts is fully absorbed by an increase in the proportion of price shoppers. Given that the latter are more price sensitive, the firms have more incentive to put more weight on lower prices. As a consequence, the decrease in the upper bound of product complexity leads to higher prices, while the increase in the lower bound of product complexity leads to lower prices respectively. ■

Below is the plot of  $F^*(p)$  for  $n = 2$ ,  $v^L = 100$  and different values of  $\bar{\rho}, \bar{\nu}$  and  $\underline{\rho}, \underline{\nu}$ . We take as a baseline  $\bar{\rho} = \bar{\nu} = 1$  and  $\underline{\rho} = \underline{\nu} = 0$ .

It can be seen from Figure 1 that the price distribution shifts up (prices decrease) as a cap on price complexity or a floor on product complexity is imposed. The threshold price for complexity choice shifts left. When a cap is imposed on product complexity instead or a floor on price complexity, the price distribution shifts down (prices increase), and the threshold price shifts to the right.



A regulator that wants to decrease market prices, should set a ceiling on price complexity, i.e. how complex firms can go in their price structures, or a product complexity floor, i.e. to limit the extent to which firms can make their product transparent. Instead, setting a cap on product complexity can result in an adverse effect on price distribution and market prices consequently increase.

### 2.5.1 Welfare implications

The welfare effect depends crucially on whether or not the reduction in prices is more important for consumers than the disutility of the ‘bad’ match. When price complexity decreases, all consumers benefit from price reduction. However, the proportion of product shoppers decreases, therefore there are fewer consumers that have information about the best match value. Thus, the proportion of consumers that switch from product shoppers to price shoppers benefit from lower prices, but at the same time choose less often the best match value. If the disutility of consumers from a mismatch is lower than the additional utility from lower prices, then policies that lower prices are socially desirable. However, if the mismatch results in a higher disutility than the relative gain in prices, consumers can be better off from the

policies that encourage match comparison, even if the latter triggers higher prices. For example, if the utility of consumers is given by  $v - p$ , as implicitly assumed in our setting, then the loss of a consumer from a mismatch is greater than any possible price reduction. Therefore, the net effect on consumer welfare depends on the relative change in the corresponding consumer proportions times the net benefit for each type of consumer.

### 2.5.2 Lower product differentiation

We have provided the analysis for the market that is characterized by a high degree of product differentiation. The remaining question is what happens in the middle range of the equilibrium price support, when products are moderately differentiated, or more precisely when  $v^H - v^L < v^L$ .

A lower dispersion of valuations affects the probability of attracting experts,  $H(p)$ . If this probability increases with respect to the probability of attracting price shoppers, then the above results are reinforced. However, if this probability decreases sufficiently as a result of lower valuation dispersion, then the two probabilities would cross at a higher price, so that the case that  $\hat{p}$  becomes smaller than  $\tilde{p}$  cannot be ruled out. For  $n = 2$  case  $H(p)$  takes the following form:

$$H(p) = \frac{1}{2}(1 - F(p)) + \frac{1}{4}(1 - F(p + v^H - v^L)) + \frac{1}{4}(1 - F(p - v^H - v^L))$$

Note that it is greater than in the case when  $v^H - v^L > v^L$  if  $[2 - F(p + v^H - v^L) - F(p - v^H - v^L)] > 1$ , which in turn depends on the equilibrium properties of  $F(p)$ . For a sufficiently low price range the probability of attracting price shoppers remains high so firms choose the complexity levels that maximize a proportion of price shoppers. As price increases this probability decreases, however  $H(p)$  decreases sufficiently fast so that the probability of attracting price shoppers becomes lower than the probability of attracting non-price shoppers,  $\frac{2}{n}$ , before crossing  $H(p)$ . In this case, it becomes more profitable for firms to minimize the proportion of experts in the middle range of prices, and at once switch their complexity choices to those that maximize the proportion of non-price shoppers.

As  $n$  increases,  $H(p)$  presumably would decrease slower than  $(1 - F(p))^{n-1}$ , and the gap

between the two decreases. In this case, there can be a shift to a full transparency range for some intermediate values of  $n$ . Thus, as  $n$  increases, the two price thresholds approach each other until  $\tilde{p}$  again becomes lower than  $\hat{p}$ . However, more detailed analysis is needed before drawing any conclusions. Due to the technical complexity of the task, we leave this analysis beyond the scope of this paper.

## 2.6 Conclusions

The adopted framework allows us to analyze the incentives of firms to obfuscate price or product features and to predict the change in the market structure as a result of new entries or regulatory interventions. The reduced form analysis sheds light on the trade-off that firms choose when deciding which information to make available for consumers.

The existing literature on obfuscation shows that firms choose to limit the product net-value comparison by consumers. However, the dimension that firms might obfuscate is usually exogenously chosen: either price or product features. In this paper, we provide an analysis of firms' incentives to limit comparison by consumers of the net value of differentiated products. The firms choose which dimension, if any, they obfuscate. We find that price and product complexities are substitutes rather than complements. Firms either choose to limit price comparison or product comparison. The equilibrium results in a mixed pricing strategy by firms, and the choice of price and product complexity is determined by the prices of the individual firms, the equilibrium distribution of the prices, and the number of competing firms. Firms that charge low prices choose transparent prices, however much they try to limit comparison of their product features with those of competing firms. In other words, when choosing low prices, firms tend to encourage price shopping and limit the extent to which their products are viewed as niche products by consumers, in order to maximize the share of consumers that make their purchasing decisions based on price comparison. Firms that charge prices in the middle range of the equilibrium support, choose fully transparent products in the case of highly differentiated products, and possibly choose fully complex offers in the case of low product heterogeneity. Whereas firms that charge higher prices tend to encourage consumer shopping based on product features comparison, by making their price structures complex and their products transparent. Having a transparent product guaran-

tees some captive consumers for the firm on the one hand, and limiting price comparison relaxes price competition on the other hand. The frequency with which the firms choose different complexity levels depend only on the number of firms, and the relative change in the proportion of price shoppers and experts from the change in the complexity levels. Information on the number of firms is sufficient to derive testable theoretical predictions as boundaries of the frequency with which firms choose different complexity levels. When the number of firms increases in the market, the choice of complex prices and transparent products becomes more frequent, as the probability of winning the price competition decreases for each firm when the total number of firms increases. Therefore, this results in consumers making suboptimal choices on a price dimension.

The policies targeted at regulating price and product complexity have different predictions as to the effect on equilibrium price distribution. We consider two relevant policies: a cap on the upper limit of price or product complexity; or a floor on the lower bound of price or product complexity. Our analysis shows that imposing a cap on price complexity or introducing a floor on product complexity results in lower market prices. In the case of the imposition of a cap on product complexity and a floor on price complexity, the effect on aggregate market complexity levels is the same, however market prices increase as a result. When product complexity is regulated, consumers shift from price comparison to product comparison, which in turn relaxes price competition for firms and therefore results in higher market prices. Even though both policies lead to more transparent markets, regulating price complexity appears to be more efficient for lowering market prices than regulating product complexities. Together, with the result that increasing the number of firms leads to more complex and higher prices, the efficiency advantage of price complexity regulation becomes more pronounced. However, the impact on consumer welfare remains ambiguous, in the case of a high disutility from the ‘bad’ product match. Our analysis constitutes a first attempt to analyze market outcomes when firms directly influence the information available to consumers prior to purchasing decisions. A richer model is needed in order to assess in detail the net impact on consumer behavior and total welfare.

## 2.7 Appendix

### A: Mixed Strategy Equilibrium

#### A.1 Existence and uniqueness

**Proof.**

i. The expected profit of firm  $i$  is given by:

$$\Pi_i(p_i, \rho_i, \nu_i | \sigma_{-i}) = p([1 - F(p)]^{n-1} \Lambda_p + H(p) \Phi_p + \frac{1}{n}(1 - \Lambda - \Phi)). \quad (2.12)$$

A simple examination of (2.12) reveals that the payoff of each firm is continuous except in the instances when the price of firm  $i$  price is the lowest on the market and equals the price of at least one of the competitors. In this case, firm  $i$  can discontinuously increase (decrease) its profits by lowering (raising) its price. Dasgupta and Maskin (1986) provide necessary and sufficient conditions for the existence of the mixed strategy equilibrium in the games with multi-dimensional strategy set in the case of such discontinuities (Theorem 6\*):

*i) the continuity of the sum of payoffs;*

*ii) the continuity of each firm's payoff and weak semi continuity at points of discontinuity (when several firms charge the lowest price or happen to have the highest net-value).*

We now outline the intuition as to why these conditions are satisfied.

i) The continuity of the sum of payoffs is guaranteed by the fact that at the points of the discontinuity the change in payoff serves as a transfer between firms that have the lowest price or the highest net value on the market. In this way, the game at the points of discontinuity can be viewed as zero-sum game, as, the sum of payoffs is continuous.

ii) Note that the payoff is continuous in complexity levels. Weak lower-semi continuity of  $\Pi_i$  in price demands that the deviation to the lower prices at the points of discontinuity to be profitable. Consider that the point of discontinuity is such that at this price firm  $i$  shares the proportion of price experts with  $n_j$  firms, but keeps its proportion of experts. Deviating to a lower price than the point of discontinuity results in a strictly positive profit, as it is at this

price that firm  $i$  keeps the demand from non-price shoppers and experts unchanged and gains all the price shoppers. For prices close enough to the point of discontinuity, the deviation is strictly profitable. Now consider that a tie occurs at the price such that firm  $i$  shares a proportion of it price shoppers with  $n_j$  firms, and a proportion of experts with  $n_i$  firms. Then, the deviation to the lower price in the neighborhood of the point of discontinuity is even more profitable than in the case when firm  $i$  shares only the demand of price shoppers. Therefore, weak lower semi-continuity of  $\Pi_i$  holds. ■

### *Strictly decreasing in $p$*

**Proof.** Suppose there exists an interval  $[p_a, p_b]$  within  $[0, v^L]$  such that  $F(p_b) - F(p_a) = 0$ . Then for any  $\tilde{p}$  such that  $p_a < \tilde{p} < p_b$ ,  $[1 - F(\tilde{p})]^{n-1} = [1 - F(p_a)]^{n-1}$ . Since  $\tilde{p}[1 - F(\tilde{p})]^{n-1} > p_a [1 - F(p_a)]^{n-1}$  and  $\tilde{p}[1 - (1 - F(\tilde{p}))^{n-1}] > p_a [1 - (1 - F(p_a))^{n-1}]$ , then there exists a profitable deviation. Thus,  $F(p_b) - F(p_a) \neq 0$  for any interval  $[p_a, p_b]$  within  $[0, v^L]$ . ■

### *No mass points*

**Proof.** If some  $p$  is charged with a positive probability, then there is a positive probability of a tie at  $p$ . Note that there cannot be a positive mass at a price that results in zero profits, as any positive mass on the price slightly higher will generate positive profits. Consider a point mass at any  $p \in (0, v^L]$ . It must be the case that the number of mass points is countable. Then there exists a price slightly smaller (by  $\varepsilon$ ) than  $p$  that is charged with the probability zero. Consider a deviating firm charging  $p$  with probability zero and  $p - \varepsilon$  with a positive probability. Such deviation increases the demand from price shoppers while keeping the demand from product shoppers unchanged, and thus profitable. Therefore, in equilibrium, no mass points can exist. ■

### *Characterization and uniqueness of equilibrium price distribution*

**Proof.** Denote by  $\Lambda(p_i, \rho_i, \nu_i, \sigma_{-i})$  and  $\Phi(p_i, \rho_i, \nu_i, \sigma_{-i})$  the conditional expectations of  $\lambda$  and  $\phi$  respectively, given a choice of  $\rho_i, \nu_i$  and  $p_i$  of firm  $i$  and the equilibrium strategies of other firms,  $\sigma_{-i}$ . Further denote by  $\Lambda_p$  the conditional expectations of  $\lambda$ , given a choice of  $\rho_i, \nu_i, p_i, \sigma_{-i}$  and that  $p_i = p_{\min}$  and by  $\Phi_p$  the conditional expectations of  $\phi$  given a choice



of  $\rho_i, \nu_i, p_i, \sigma_{-i}$  and that  $v_i - p_i > \max(v_j - p_j)$  for  $j \neq i$  respectively. Let  $\underline{\Phi}$  and  $\bar{\Phi}$  denote  $\Phi$  when firm  $i$  chooses  $(\bar{\rho}, \underline{\nu})$  and  $(\underline{\rho}, \bar{\nu})$  respectively. Then equilibrium price distribution should be such that firms are indifferent between any price charged.  $F(p)$  is determined by:

$$\begin{aligned} & p[(1 - F(p))^{n-1}\Lambda_p + (\frac{1}{2})^n[(1 - F(p))^{n-1} + (2 - F(p))^{n-1}]\Phi_p + \frac{1}{n}(1 - \Lambda - \Phi)] \\ &= v^L[(\frac{1}{2})^n\underline{\Phi} + \frac{1}{n}(1 - \underline{\Lambda} - \underline{\Phi})]. \end{aligned} \quad (2.13)$$

■

From where we can determine  $p_L$ :

$$p_L[\Lambda_p(\underline{\rho}, \bar{\nu}) + (\frac{1}{2})\Phi_p(\underline{\rho}, \bar{\nu}) + \frac{1}{n}(1 - \Lambda(\underline{\rho}, \bar{\nu}) - \Phi(\underline{\rho}, \bar{\nu}))] = v^L[(\frac{1}{2})^n\Phi_p(\bar{\rho}, \underline{\nu}) + \frac{1}{n}(1 - \Lambda(\bar{\rho}, \underline{\nu}) - \Phi(\bar{\rho}, \underline{\nu}))]$$

$$p_L = \frac{v^L[(\frac{1}{2})^n\Phi_p(\bar{\rho}, \underline{\nu}) + \frac{1}{n}(1 - \Lambda(\bar{\rho}, \underline{\nu}) - \Phi(\bar{\rho}, \underline{\nu}))]}{\Lambda_p(\underline{\rho}, \bar{\nu}) + (\frac{1}{2})\Phi_p(\underline{\rho}, \bar{\nu}) + \frac{1}{n}(1 - \Lambda(\underline{\rho}, \bar{\nu}) - \Phi(\underline{\rho}, \bar{\nu}))}.$$

Note that  $p_L$  is a singleton and is independent of  $F(p)$ .

Therefore to show that  $F(p)$  is unique, it is sufficient to show that it is unique for any  $p$  of the support  $[p_L, v^L]$ . Consider (2.13), the right hand side (*RHS*) of this equation is given by  $v^L[(\frac{1}{2})^n\underline{\Phi} + \frac{1}{n}(1 - \underline{\Lambda} - \underline{\Phi})]$ , which does not vary with  $p$ . The left hand side (*LHS*) of (2.13) for a given  $p$  is strictly decreasing with  $F$ , as both  $\Lambda_p(\underline{\rho}, \bar{\nu})$  and  $\Phi_p(\underline{\rho}, \bar{\nu})$  are strictly decreasing with  $F$ .

In order to prove the uniqueness of  $F(p)$ , it is sufficient to show that there is a unique  $F$  that satisfies (2.13). Let  $F = 0$ , then at  $p = p_L$  *LHS* should be equal to *RHS* by definition of  $F(p)$ , while at  $p > p_L$ , *LHS* is strictly greater than the *RHS* when  $F = 0$  for any  $p$  on  $[p_L, v^L]$ . At the same time at  $F = 1$ , the *LHS* is strictly smaller than *RHS* at  $p < v^L$ . Given that the *LHS* is strictly decreasing with  $F$ , there exists a unique  $F$ , that satisfies (2.13).

## A.2 Complexity choices

**Proof.** Below we provide a detailed proof for the choice of complexity levels. The proof

consists of three parts. We first analyze the firms' choice of price complexity, followed by the analysis of match complexity choice. We finalize the proof by showing that  $\widehat{F} \geq \widetilde{F}$ . ■

### *Price complexity*

**Proof.** The first order conditions with respect to  $\rho_i$  for a given  $p_i$  is:

$$(1 - F(p))^{n-1} \frac{d\Lambda_p}{d\rho_i} + H(p) \frac{d\Phi_p}{d\rho_i} - \frac{1}{n} \left( \frac{d\Lambda}{d\rho_i} + \frac{d\Phi}{d\rho_i} \right). \quad (2.14)$$

Given that  $\frac{d^2\lambda}{d\rho_i d\rho_j} = \frac{d^2\phi}{d\rho_i d\rho_j} = 0$ , implies that  $\frac{d\Lambda_p}{d\rho_i} = \frac{d\Lambda}{d\rho_i}$  and  $\frac{d\Phi_p}{d\rho_i} = \frac{d\Phi}{d\rho_i}$ . Therefore, (2.14) can be simplified to:

$$\frac{d\Lambda}{d\rho_i} \left[ (1 - F(p))^{n-1} - \frac{1}{n} \right] + \frac{d\Phi}{d\rho_i} \left[ H(p) - \frac{1}{n} \right]. \quad (2.15)$$

As  $\frac{d\Lambda}{d\rho_i} < 0$ , (2.15) equal to zero when  $((1 - F(p))^{n-1} + H(p) - \frac{2}{n})$  is equal to zero. Several results follow immediately from the properties of  $H(p)$  and  $((1 - F(p))^{n-1})$  discussed above. Both  $((1 - F(p))^{n-1})$  and  $H(p)$  are strictly decreasing in price, therefore (2.15) is strictly increasing in price. At the lowest price of the equilibrium support,  $p_L$ ,  $F(p_L) = 0$  and therefore  $[(1 - F(p_L))^{n-1} - \frac{1}{n}] > 0$  and  $H(p_L)$  is greater than  $\frac{1}{n}$ , therefore (2.15) is negative. At  $p = v^L$ ,  $F(v^L)$  is equal to one, and  $H(v^L)$  is lower than  $\frac{1}{n}$ , therefore (2.15) is positive. Combining this result with the fact that (2.15) is strictly increasing in price, concludes the proof that there exists a unique  $\widehat{p}$  such that at  $p = \widehat{p}$  (2.15) is equal to zero. Therefore,  $\widehat{p}$  is characterized by:

$$\frac{d\Lambda}{d\rho_i} \left[ (1 - F(\widehat{p}))^{n-1} - \frac{1}{n} \right] + \frac{d\Phi}{d\rho_i} \left[ H(\widehat{p}) - \frac{1}{n} \right] = 0. \quad (2.16)$$

For  $p > \widehat{p}$ , (2.15) is positive, and firms choose  $\rho = \bar{\rho}$ . For  $p < \widehat{p}$ , firms choose  $\rho = \underline{\rho}$ , while at  $p = \widehat{p}$  firms are indifferent between all price complexity levels. ■

### *Match complexity*

**Proof.** The FOC with respect to  $\nu_i$  for a given price are given by:

$$(1 - F(p))^{n-1} \frac{d\Lambda_p}{d\nu_i} + H(p) \frac{d\Phi_p}{d\nu_i} - \frac{1}{n} \left( \frac{d\Lambda}{d\nu_i} + \frac{d\Phi}{d\nu_i} \right). \quad (2.17)$$

Given that  $\frac{d^2\lambda}{d\rho_i d\rho_j} = \frac{d^2\phi}{d\rho_i d\rho_j} = 0$ , implies that  $\frac{d\Lambda_p}{d\rho_i} = \frac{d\Lambda}{d\rho_i}$  and  $\frac{d\Phi_p}{d\rho_i} = \frac{d\Phi}{d\rho_i}$ . Therefore (2.17) can be simplified to:

$$\frac{d\Lambda}{d\nu_i} \left[ (1 - F(p))^{n-1} - \frac{1}{n} \right] + \frac{d\Phi}{d\nu_i} \left[ H(p) - \frac{1}{n} \right]. \quad (2.18)$$

As  $\frac{d\Lambda}{d\nu_i} > 0$  while  $\frac{d\Phi}{d\nu_i} < 0$ , the overall sign of (2.18) depends on the relative magnitude of  $\frac{d\Lambda}{d\nu_i}$  and  $\frac{d\Phi}{d\nu_i}$ . It is reasonable to assume that an increase in product complexity generates higher absolute change in the proportion of price shoppers than in the proportion of experts. When match complexity increases, *experts* shift towards price comparison, while *product shoppers* become either *price shoppers* or remain *uninformed*. The change in the proportion of *price shoppers*, therefore, accommodates both the change in the fraction of *experts* and the change in the proportion of *product shoppers* that have switched towards price comparison. Therefore,  $\frac{d\Lambda}{d\nu_i}$  should be greater or equal to  $\frac{d\Phi}{d\nu_i}$ . Keeping this assumption in mind, we can examine a sign of (2.18) at  $p_L$  and  $v^L$ . Thus, applying the properties of  $H(p)$  discussed above, at  $p_L$ , (2.18) is positive, while at  $v^L$  it is negative.

Both  $((1 - F(p))^{n-1}$  and  $H(p)$  are strictly decreasing with  $p$ , with  $(1 - F(p))^{n-1}$  being greater than  $H(p)$  at  $p = p_L$  and  $(1 - F(p))^{n-1}$  being smaller than  $H(p)$  at  $p = v^L$ . Plugging the expression for  $H(p)$  and rearranging the terms results in following condition that characterizes  $\tilde{p}$ :

$$\underbrace{(1 - F(\tilde{p}))^{n-1} \left( \frac{d\Lambda}{d\nu_i} + \left(\frac{1}{2}\right)^n \frac{d\Phi}{d\nu_i} \right)}_1 + \underbrace{(2 - F(\tilde{p}))^{n-1} \left(\frac{1}{2}\right)^n \frac{d\Phi}{d\nu_i}}_2 = \frac{1}{n} \left( \frac{d\Lambda}{d\nu_i} + \frac{d\Phi}{d\nu_i} \right). \quad (2.19)$$

The first term of the LHS of (2.19) is positive and strictly decreasing in  $p$ , while the second term is negative and also strictly decreasing in  $p$  but at a lower rate than the first term, as  $\frac{d\Lambda}{d\nu_i} > 0$ . There exists a unique  $\bar{p}$  such that for  $p > \bar{p}$  the first term is greater than the second term, and for  $p < \bar{p}$  the first term is smaller than the second term. Therefore, for  $p < \bar{p}$  the LHS of (2.19) is decreasing in  $p$ , while for  $p > \bar{p}$ , the LHS is increasing in  $p$ . At  $p = \bar{p}$  the LHS=0, and is smaller than the RHS, therefore  $\tilde{p}$  should be smaller than  $\bar{p}$ . Therefore,  $\tilde{p} \in [p_L, \bar{p})$ . At this range of prices, the LHS is strictly decreasing, therefore on this interval the LHS crosses the RHS only once and at  $\tilde{p}$ . We now have to show that for  $p \in [\bar{p}, v^L)$  the LHS does not cross the RHS. Note that on this interval the LHS is strictly increasing in  $p$ , and at  $v^L$  the FOC (2.18) is negative. Thus, the LHS and RHS do not cross.

Therefore, for  $p > \tilde{p}$ , (2.18) is negative, and firms choose  $\nu = \underline{\nu}$ . For  $p < \tilde{p}$ , firms choose  $\nu = \bar{\nu}$ , while at  $p = \tilde{p}$  firms are indifferent between all the match complexity levels ■

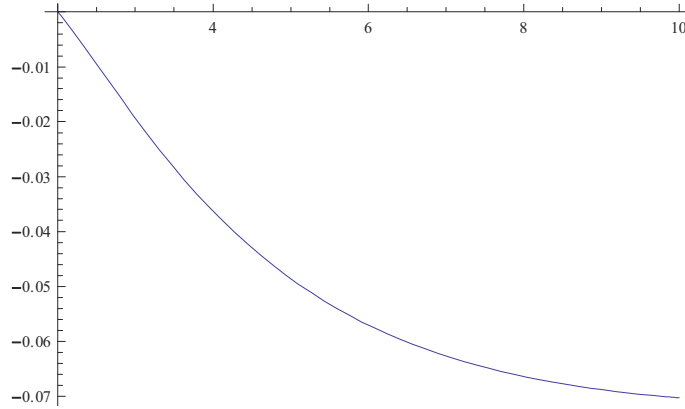
### Comparison of thresholds

The FOC for price complexity (2.15) evaluated at  $\tilde{p}$  is given by:

$$\frac{d\Lambda}{d\rho_i} / \frac{d\Phi}{d\rho_i} \left[ (1 - \tilde{F})^{n-1} - \frac{1}{n} \right] - \frac{d\Lambda}{d\nu_i} / \frac{d\Phi}{d\nu_i} \left[ (1 - \tilde{F})^{n-1} - \frac{1}{n} \right]. \quad (2.20)$$

Note that the sign of (2.20) depends on the sign of  $(1 - \tilde{F})^{n-1} - \frac{1}{n}$ .

A picture below plots the value of (2.18) at  $F = 1 - (\frac{1}{n})^{n-1}$ .



FOC wrt to  $\nu$  evaluated at  $F = 1 - (\frac{1}{n})^{n-1}$

The sign of (2.18) at this value is negative, therefore  $\tilde{F}$  must be lower than  $1 - (\frac{1}{n})^{n-1}$  and therefore (2.20) is positive, which implies that  $\hat{F}$  must be greater than  $\tilde{F}$ , and therefore  $\tilde{p} < \hat{p}$ .

## B: Proof of Proposition 2

Using the symmetry assumption ( $\frac{d\Lambda}{d\rho} = \frac{d\Phi}{d\rho}$  and  $\frac{d\Lambda}{d\nu} = \frac{d\Phi}{d\nu}$ ),  $\tilde{F}$  is given by:

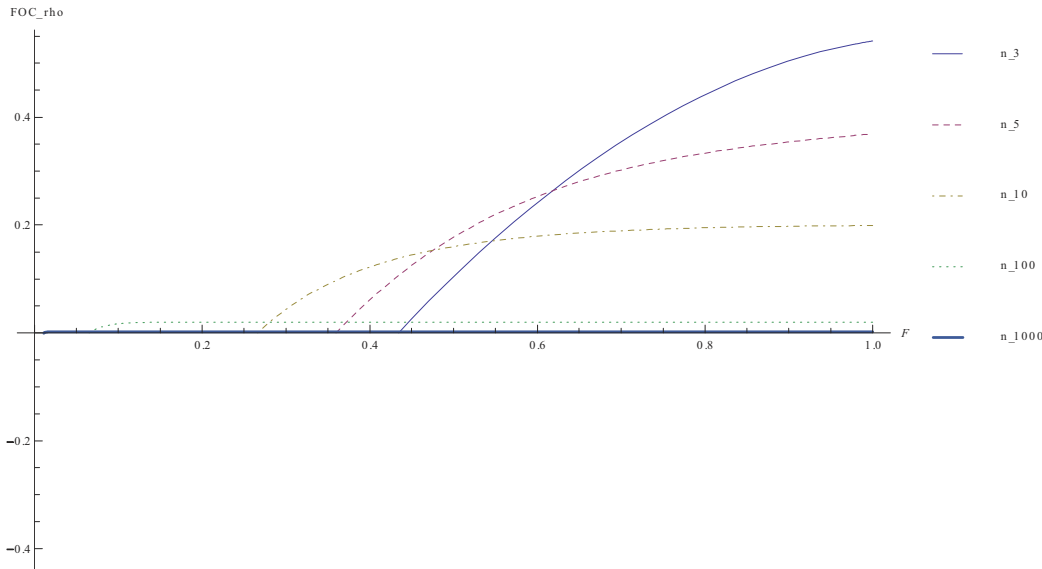
$$\frac{(2^n - 1)^{\frac{1}{n-1}} - 2}{(2^n - 1)^{\frac{1}{n-1}} - 1} < 1.$$

The derivative of  $\tilde{F}$  with respect to  $n$  is given by:

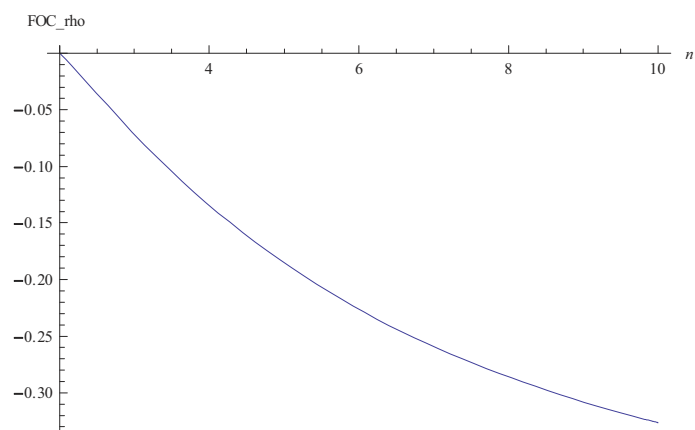
$$\frac{(2^n - 1)^{\frac{1}{n-1}} \left( \frac{2n \text{Log}[2]}{(2^n - 1)(n-1)} - \frac{\text{Log}[2n-1]}{(n-1)^2} \right)}{(2^n - 1)^{\frac{1}{n-1}} - 1)^2} < 0.$$

In the limit when  $n$  goes to infinity  $F(\hat{p})$  goes to 0.

As both  $(\frac{1}{2})^{n-1}(1 - F(\hat{p}))^{n-1}$  and  $(\frac{1}{2})^{n-1}(1 - F(\tilde{p}))^{n-1}$  decrease with  $n$  faster than  $\frac{1}{n}$ ,  $F(\hat{p})$  decreases with  $n$  as well. Figure (2.7) plots FOC for  $\rho$  for different values of  $n$  and  $F$ . Note that the value of  $F$  such that this FOC are equal to zero ( $\hat{F}$ ) decreases with  $n$  and approaches zero for  $n$  large.



Note from the figure (2.7): the difference  $(F(\hat{p}) - F(\tilde{p}))$  increases with  $n$ , until  $F(\hat{p})$  reaches zero. This implies that the frequency with which firms choose transparent offers increases with  $n$  until  $F(\hat{p})$  reaches zero, and decreases thereafter. At the same time, the frequency with which firms choose transparent price/complex match decreases, while the frequency with which firms choose complex price/transparent math increases, and in the limit when  $n$  goes to infinity, reaches one.



FOC wrt  $\rho$  evaluated at  $\tilde{F}$  as a function of  $n$



## Chapter 3

# Inquisitorial versus Adversarial System and the Right to Remain Silent





### 3.1 Introduction

In this paper we compare the *adversarial* and the inquisitorial systems of law enforcement. The differences in the way the US and European Union (EU) antitrust authorities operate is our primary application. The motivating example for our analysis is based on merger control. In the US, federal and state courts decide on cases prosecuted by national agencies (Federal Trade Commission in our example). In Europe, competition law enforcement is most often an administrative process in which agencies, such as the European Commission, also make decisions.<sup>1</sup>

In the *adversarial* system of law, the *DM*—typically a court—does not collect the evidence itself, and rules based on the information provided by the opposing parties (e.g. a US administration body versus a firm). In contrast, in the inquisitorial (or the administrative) system of law, the *DM* relies on its administration to gather information both favorable and unfavorable to the firm.

However, the EU law enforcement system is not truly inquisitorial since the European Commission as a *DM* compensates for the relative absence of well organized plaintiffs, and acts to some extent as a prosecutor.<sup>2</sup>

The European Commission's dual role as a prosecutor and judge is heavily criticized by practitioners and legal scholars alike. For example, Venit (2009) considers that the dual role of the Commission together with the 'all-too-human tendency of any regulatory agency to be biased by its *raison d'être*' are the main factors that negatively affect the quality of evidence collection in the administrative system.

In this paper, we investigate whether or not a *DM* prefers its agent (i.e. its administration) to be biased when gathering and submitting evidence (and thus act as an adversary to the firm), or instead to be neutral (and thus complement the firm's (biased) information).

In the existing literature, the two systems are usually considered in their extreme interpretation. The *adversarial* system is represented by two polarized parties who gather information and engage in the strategic disclosure of evidence, and a *DM* who passively

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1. Also, final decisions can be appealed in both jurisdictions. In this paper we abstract from the appeal process.

2. Neven (2006) refers to this as the inquisitorial model with prosecutorial bias.

rules based on the provided information. The *inquisitorial* system of law is instead represented by a single agent whose objective is to seek the "truth" by collecting evidence, with no room for information disclosure by the parties with a stake in the matter. Relying on such a representation of the two systems of law, Shavell (1989), Shin (1994, 1998), Dewatripont and Tirole (1999), and Gromb and Martimort (2007) find that the *adversarial* system performs better in terms of the quantity of information revealed, and thus the level of precision of the final decision.

However, experimental evidence available to us contradicts the superiority of the *adversarial* approach, and finds only weak empirical evidence in favor of its superiority. For example, Lind, Thibaut and Walker (1973) run a series of experiments to study the relative performance of the two systems. They find that on average, the *adversarial* system is more robust to the *DM*'s bias, but that it does not lead to more information. The *adversarial* system does not seem to instigate more searches for information, even though it does lead to advocates who are initially confronted with unfavorable evidence to exert greater effort.

The key novelty of our framework consists of allowing the interested party to submit information in both systems of law enforcement. In practice, antitrust enforcement proceedings in either system contain elements in common. One such common element is the right for interested parties to submit any information relevant to the case. The right to remain silent, in the sense of withholding the incriminating evidence, also applies to both systems of law enforcement. Finally, in both systems, the law enforcement agencies are meant to represent the public interest, and arguably respond exclusively to the incentives provided by their respective institutional arrangements.

We adopt the informational framework of Angelucci (2012) to compare the performance of the two systems. There are two pieces of evidence available to the parties, which, if they are gathered and revealed, serve as an informative signal about the underlying state of nature. The two parties, the administration and the firm, exert costly effort to (possibly) come into possession of no, one, or two pieces evidence.

The first party is an independent agent that has an intrinsic interest to convince the *DM* to rule in her favor (e.g. a firm). The second party is an expert (e.g. an administration), to whom the *DM* delegates information acquisition responsibilities. The expert is *a priori*

indifferent to the final decision, and reacts only to the monetary incentives provided by the *DM*. In our motivating example, based primarily on merger control, the first party represents a firm that notifies the merger and the second party represents the case team of the enforcing authority.

By choosing the judicial system, the *DM* chooses a remuneration scheme for the expert. Under the *adversarial* system the agent is remunerated if the final decision is unfavorable to the independent agent. In contrast, under the *inquisitorial* system, the agent is remunerated based on the amount of information delivered. In addition, we allow for parties to choose whether to look only for favorable information, only for unfavorable information, or for any type of information, through their choice of information acquisition technology (either general or specialized). Acquiring information of a certain type (either favorable or unfavorable) is on average more likely when using specialized technology than when using general technology.

We characterize the equilibrium of the game, and provide conditions under which one system performs better than the other with respect to the level of precision of the final decision, net of the total cost of information acquisition.

Both systems have a number of advantages and disadvantages. The main advantage of the *adversarial* approach is that sound competition between the two opposing parties forces both parties to exert a higher level of effort, and to develop special skills necessary in order to acquire evidence in support of their best interests. This leads to a higher amount of information produced, and therefore a higher level of precision for the final decision.

Milgrom and Roberts (1986) show that when parties have opposing interests, where evidence necessarily favors one side over the other, all relevant information ends up being disclosed.

We find a similar result in the sense that the effort levels of both parties are strategic complements under both systems. The effort of the firm, for a given level of effort by the expert, is higher under the *adversarial* system. However, the full disclosure result found in Milgrom and Roberts (1986) rests on the assumptions that evidence is non-manipulable, and is costless to obtain.

We show that the higher level of effort exerted by the firm in the *adversarial* system of law is not sufficient to offset the loss of information arising from the strategic withdrawal of

useful information by the expert. The amount of information revealed under the *adversarial* system is higher than under the *inquisitorial* system if, and only if, the specializing search technology sufficiently increases the likelihood of finding evidence of the desired type. A higher level of precision comes at the cost of effort, which is the main disadvantage of the *adversarial* approach.

The main strength of the *inquisitorial* system of law is the unbiased acquisition of evidence by the expert, and also presumably, the more accurate decisions that follow. We show that less information of higher accuracy can lead to a superiority of the *inquisitorial* approach over the the *adversarial* approach.

We find that for a low relative efficiency of specialized technologies, the *inquisitorial* system is always superior to the *adversarial* one, resulting in both a higher level of precision of the final decision and lower total costs. In contrast, if specialized technologies are highly efficient, the difference in the two systems is non-monotonic in the levels of equilibrium efforts of both parties. The *adversarial* system performs better for sufficiently low and high levels of equilibrium efforts, whereas the *inquisitorial* system remains superior for intermediate levels of effort. The intuition for this result can be summarized as follows: for highly efficient specialized technologies, the *adversarial* system of law always leads to a final decision of higher accuracy, but with higher costs. For low values of equilibrium effort of the firm, the increase in the level of precision of the final decision always offsets the associated increase in costs. This result is reversed as the effort of the firm increases. Finally, when the effort levels of both parties are sufficiently high, the two systems have the same equilibrium levels of effort (and therefore costs), but the level of precision of the final decision is higher under the *adversarial* system. The main determinants of the equilibrium efforts are the stakes of the parties, i.e. the profit of the firm and the compensation of the expert that in turn depends on the loss of a  $DM$  from an erroneous decision.

Further, we show that for a range of parameters, the  $DM$  can increase its total payoff by committing not to accept the evidence of the independent party. In these instances, the *inquisitorial* system naturally dominates the *adversarial* one.

In our setting, we restrict attention to the allocation of the burden of proof by rule. We assume that if no evidence is submitted by two parties, the  $DM$  rules in the favor of firm,

i.e. the burden of proof is born by the expert. We further relax the exogenous allocation of the burden of proof and characterize the conditions under which such an allocation can arise as an equilibrium outcome. We find that under the *inquisitorial* system, such allocation of the burden of proof will never arise in equilibrium, while it constitutes an equilibrium under the *adversarial* system for low enough values of the equilibrium effort of the independent agent. Indeed in practice, such allocations of the burden of proof are more typical for the *adversarial* system, but less so for the *inquisitorial* system.

Our analysis provides supportive arguments and additional justification for the non-*adversarial* system.

The rest of the paper is organized as follows: Section 2 introduces the set-up. Section 3 analyses the equilibrium of interest and the level of precision of the final decision under the two systems. Section 4 discusses the endogenous allocation of the burden of proof. Section 5 concludes.

## 3.2 Set-up of the model

### 3.2.1 The parties

We consider a set-up with three risk neutral players: persuader  $F$ , persuader  $E$  and decision-maker  $DM$ . The motivating example of our analysis is based on merger assessment. Thus, it is convenient to interpret persuader  $F$  as a firm that notifies a merger and persuader  $E$  as an expert team of antitrust authority. The  $DM$  in our case is either a judge or an antitrust agency that takes a formal decision either to clear or to block a merger,  $m = 1$  or  $m = 0$  respectively.  $F$  receives a positive profit  $\Pi$  if the merger is cleared ( $m = 1$ ) and 0 otherwise. Based on merger control example, our model stays fairly general and applies to a number of different settings.

The state of nature  $\omega$  can be either ‘good’ or ‘bad’,  $\omega \in \Omega = \{g, b\}$ . The  $DM$  incurs a loss  $L$  from blocking the merger if the state is ‘good’ and clearing the merger if the state ‘bad’. In case of a correct decision, the  $DM$ ’s loss is zero.  $E$  is *a priori* indifferent about whether the merger is cleared or not. She responds only to monetary incentives from the  $DM$ .

### 3.2.2 Information structure

There are two pieces of decision-relevant information that pertain to the case:  $x$  and  $y$ . The evidence in our setting represents a verifiable piece of hard information. The first piece of evidence,  $x$ , is drawn from a countable set  $X$ . The second piece of evidence,  $y$ , is drawn from a countable set  $Y$ . The whole set of evidence is denoted by  $z = (x, y)$ , and is drawn according to the joint pdf  $f_\omega(z)$ , with marginal probabilities  $g_\omega(x)$  and  $h_\omega(y)$ .

Further, a set  $X$  can be divided into two subsets:  $X^g$  and  $X^b$ , where  $X \equiv X^g \cup X^b$  so that  $x \in X^g$  (respectively,  $x \in X^b$ ) if  $g_g(x) > g_b(x)$  (respectively,  $g_g(x) < g_b(x)$ ) and in this case  $x$  is referred to as ‘good’ (respectively, ‘bad’). Similar dichotomies apply to  $Y$  and  $Z$ .

#### Information acquisition

To acquire information the parties exert unobservable and unverifiable effort,  $e_i$ , at a cost  $C(e_i)$ .<sup>3</sup> The *DM* cannot obtain information on his own but can delegates the evidence collection to his expert  $E$  in exchange for monetary remuneration. Both  $F$  and  $E$  potentially have access to both pieces of evidence. The evidence gathered by party  $F$  or  $E$  is that party’s private information. In addition,  $F$  and  $E$  can choose to disclose (or not) any evidence at their disposal. The submission involves some arbitrary small costs  $c$  per piece of evidence submitted.<sup>4</sup> The *DM* rules based on the evidence submitted by persuaders. In the case when no evidence is submitted, the *DM* rules in the favor of the firm by a rule, which is public information. In this case we say that party  $E$  carries the burden of proof. In this paper, we do not provide the analysis under alternative allocation of the burden of proof. However, we do provide some intuitive predictions for this case, which is discussed in more detail in Section 4. No evidence submission is the only instance when the *DM* can commit ex-ante to a decision rule. We assume that the *DM* cannot commit to any particular decision rule ex-ante if one of the parties submits any evidence.

There are two technologies available for information collection: general (*g-technology*) and specialized (*s-technology*). The choice of technologies is unobservable to the *DM*. The two technologies differ in the probability of obtaining ‘good’ and ‘bad’ evidence.

3. We assume  $C(e_i)$  to be increasing and convex.

4. The positive costs serve to break the indifference between the decision to submit the evidence that does not affect the final decision or to not submit.

Upon exerting effort  $e_i$  and using *g-technology*, persuader  $i$  comes into possession of only  $x$  with probability  $\alpha e_i$ , only  $y$  with probability  $\alpha e_i$ , both  $x$  and  $y$  with probability  $\alpha e_i$  and acquires no evidence with probability  $1 - 3\alpha e_i$ , where  $\alpha \in (0, \frac{1}{3}]$ . Parameter  $\alpha$  represents the efficiency of *g-technology*. The ex-ante probabilities of finding ‘good’ or ‘bad’ evidence when using *g-technology* are thus equal.

*S-technology* in turn allows the parties to specialize their search on either ‘good’ or ‘bad’ evidence. The probability of finding a desired type of evidence using *s-technology*, conditional on the fact that this evidence exists, is higher than using general technology, while the probability of finding an undesired type of evidence is lower. Conditional on the fact that there exists a desired piece of evidence, persuader  $i$  comes into possession of only this piece of evidence with probability  $\alpha(1+s)e_i$ . Similarly conditional on the existence of an undesirable piece of evidence, persuader  $i$  finds only this piece of evidence with probability  $\alpha(1-s)e_i$ . If the realized evidence is such that both pieces of evidence are favorable (or unfavorable) to persuader  $i$ , she obtains both pieces with probability  $\alpha(1+s)e_i$  (or  $\alpha(1-s)e_i$ ) and finds nothing with probability  $[1 - 3\alpha(1+s)e_i]$  (or  $[1 - 3\alpha(1-s)e_i]$ ) respectively. If two pieces of evidence happen to be conflicting, then persuader  $i$  comes into possession of both  $x$  and  $y$  with probability  $\alpha e_i$  and finds nothing with probability  $1 - 3\alpha e_i$ . Parameter  $s$  determines a relative degree of specialization of *s-technology* with respect to *g-technology*, such that  $\alpha(1+s) \in (0, \frac{1}{3}]$ . If  $s = 0$ , both technologies are identical.

It is important to stress that the ex-ante probability of obtaining a specific type of evidence is independent of the state of the nature. Conditional on the fact that this evidence exists, the probability of obtaining it is the same in both states. Therefore, the equilibrium strategies of the evidence collection and submission should be independent of the state of nature.

Having two different technologies reflects the idea that one can choose a certain direction of research. Consider testing a drug, where one can run a number of trials in order to test a concrete effect without looking for all the possible side effects. In the merger example, the analysis conducted by the firm can be oriented towards quantifying the efficiency gain and less so towards the coordinated effects. A lawyer who has developed an expertise in a narrow domain is more likely to be hired in this domain than a lawyer with a more general training.



### Remuneration schemes

There are two observable outcomes for which contracting is possible: a final decision and submitted evidence. We restrict our attention to two remuneration schemes: a decision-based scheme and an information-based scheme. Only these two schemes appear to be relevant, as contracting on the type of evidence is not effort enhancing, and the choice of the latter is not conditional on the prevailing state.

The judicial system is *adversarial* when  $E$ 's remuneration is conditional on the final decision and is *inquisitorial* when  $E$ 's remuneration is conditional on the amount of evidence submitted.

### 3.2.3 Timing of the game

The game proceeds as follows:

1. Nature draws  $\omega$  and  $z$ ; their realizations are unobserved by all players.
2. The  $DM$  chooses a remuneration scheme (thus, judicial system).
3.  $F$  and  $E$  choose a search technology and invest in evidence acquisition.
4. The parties decide whether to submit (all or part of) their evidence.
5. The  $DM$  rules.

### 3.2.4 Payoffs

Let  $\nu_\omega(e_F, e_E)$  denote the probability that the merger is cleared given the state of nature  $\omega$ . From now on we will suppress the argument of  $\nu_\omega$  and specify it if needed.

*E's payoff*

Let  $\{\mathbf{w}_E^A, \mathbf{w}_E^I\}$  be the vectors of monetary transfers from the  $DM$  to  $E$  under the *adversarial* and *inquisitorial* systems accordingly. Note that under the *adversarial* system, the  $DM$  will pay a positive wage to the agent  $E$  only if the final outcome is different from the one in the case when no evidence was submitted, i.e. if, and only if, the merger is blocked. Under the *inquisitorial* system, the  $DM$  will pay a positive wage if, and only if, the agent

submits two pieces of evidence. In order to ensure the submission of acquired evidence, the wage for two pieces of evidence should be higher than the wage for one piece of evidence. Otherwise,  $E$  would always withhold the second piece of evidence. It is sufficient to remunerate a small submission cost in order to ensure the submission of one piece of evidence. Therefore, in order to ensure a positive effort, it is sufficient to remunerate the agent (with the wage greater than the submission costs) only if both pieces of evidence are submitted.

Under the *inquisitorial* system,  $E$ 's will receive  $w_E^I$  with the probability that she acquires two pieces of evidence and 0 otherwise, and that she would incur the corresponding costs of the effort.

Under the *adversarial* system,  $E$ 's payoff is given by:

$$U_E = \frac{1}{2}(2 - \nu_g - \nu_b)w_E^A - C(e_E). \quad (3.1)$$

$F$ 's payoff under the two systems is given by:

$$U_F = \frac{1}{2}(\nu_g + \nu_b)\Pi - C(e_F). \quad (3.2)$$

$DM$ 's payoff under the two systems is given by:

$$U_{DM} = -\frac{1}{2}(1 - \nu_g + \nu_b)L - C(e_F) - C(e_E). \quad (3.3)$$

### 3.3 Equilibrium

Throughout the analysis, we assume that the burden of proof is allocated on  $E$  by rule, i.e. in the absence of any evidence submitted the merger is cleared. Later, we relax the assumption that the allocation burden of proof is exogenous and will examine whether or not such a decision can constitute an equilibrium.

#### 3.3.1 Equilibrium concept and putative equilibrium

We focus on pure-strategy subgame perfect equilibrium, in which:

1.  $F$  submits  $x$  only if  $x \in X^g$ , and  $y$  only if  $y \in Y^g$ .
2.  $E$  submits  $x$  only if  $x \in X^b$ , and  $y$  only if  $y \in Y^b$ .

If the  $DM$  observes only  $x$  (or only  $y$ ), he allows the merger if, and only if,  $x \in X^g$  (or  $y \in Y^g$ ), and blocks the merger otherwise. If the  $DM$  observes  $z$ , he allows the merger if, and only if,  $z \in Z^g$ , and blocks the merger otherwise.

### 3.3.2 Analysis

Given the putative equilibrium behavior of the  $DM$ , we first characterize the persuaders' submission strategies and choice of searching technology in *Lemma 1* and *Lemma 2* respectively. *Lemma 3* summarizes strategic complementarities that arise during the information acquisition stage. *Proposition 1* then establishes the existence of the conjectured equilibrium. We further provide the conditions on off-the-equilibrium beliefs such that this equilibrium survives the trembling hand refinement, i.e. the equilibrium still exists even if the parties make submission mistakes with a negligible probability.

#### Further assumptions and notations

The possible realizations of evidence can be classified into four events. Evidence is said to be *consistent* and favorable to  $F$  (**event 1**, with probability  $p_{1,\omega}$ ), whenever the pair  $(x, y)$  is such that  $x \in X^g$  and  $y \in Y^g$ . Evidence is said to be consistent and unfavorable to  $F$  (favorable to  $E$ ) (**event 4**, with probability  $p_{4,\omega}$ ), whenever the pair  $(x, y)$  is such that  $x \in X^b$  and  $y \in Y^b$ . Evidence is instead said to be *conflicting* whenever  $(x, y)$  is such that either  $x \in X^g$  and  $y \in Y^b$  or  $x \in X^b$  and  $y \in Y^g$ . In this case, evidence is nevertheless overall favorable to  $F$  (**event 2**, with probability  $p_{2,\omega}$ ) if  $z \in Z^g$ , and overall unfavorable to  $F$  (**event 3**, with probability  $p_{3,\omega}$ ) if  $z \in Z^b$ .

Finally, we assume that (i) distributions are symmetric in the sense that  $p_{1,g} = p_{4,b}$ ,  $p_{4,g} = p_{1,b}$ ,  $p_{2,g} = p_{3,b}$ , and  $p_{3,g} = p_{2,b}$ ; and (ii) such that  $p_{1,g} > p_{4,g}$  and  $p_{2,g} > p_{3,g}$ . *Appendix B* provides an example of the distribution of  $x$  and  $y$  that results in the joint distributions that satisfies our assumptions.

**Lemma 8** *i. Persuader  $E$  submits all the evidence that she acquires under the inquisi-*

*torial system.*

- ii. Persuader  $F$  and persuader  $E$  submit the minimum amount of evidence favorable to them under the adversarial system.*

**Proof.**

- i. The first part of the lemma is trivial and follows immediately from the definition of the *inquisitorial* system.
- ii. When the payoff of a persuader is conditional on a final decision, it is worth submitting any acquired evidence if, and only if, it increases the chances to revert an otherwise unfavorable final decision. An unfavorable piece of evidence either does not affect or reverts an otherwise favorable decision, thus persuaders never submit unfavorable evidence to them. Submitting a favorable piece of evidence that does not affect the chances of reverting an unfavorable outcome only triggers a positive cost of submission, and therefore only a favorable piece evidence that increases the chances of ‘winning’ would be submitted. Therefore, if two pieces of favorable evidence are acquired, only one piece of evidence would be submitted.

Consider first persuader  $F$  in the case where  $F$  acquires two pieces of evidence. Submitting a favorable piece of evidence reverts an unfavorable to  $F$  outcome if, and only if: *i*) the evidence is conflicting but overall favorable to  $F$ ; and *ii*)  $E$  submits an unfavorable piece of evidence. In this case,  $F$  would submit a favorable piece of evidence only if the evidence acquired is conflicting and overall favorable to  $F$  (*event 2*). Consider the case when  $F$  acquires only one piece of evidence and it turns out to be favorable to  $F$ . Then, submitting it either does not affect or increases the chances of reverting an otherwise unfavorable decision if *event 2* is realized. Thus, when  $F$  acquires one favorable piece of evidence it always submits it. Consider now the incentives of persuader  $E$ . Submitting ‘*bad*’ evidence reverts an otherwise unfavorable final outcome to  $E$  in more instances than for persuader  $F$ , as  $E$  carries the burden of proof. A ‘*bad*’ piece of evidence, if submitted, always reverts an otherwise unfavorable decision to  $E$  in the following events: evidence is conflicting and overall unfavorable to  $F$  (*event 3*); evidence is consistent and unfavorable to  $F$  (*event 4*); and if evidence is overall

unfavorable to  $E$  but  $F$  fails to submit a ‘good’ evidence. Therefore, if  $E$  acquires at least one piece of ‘bad’ evidence, she always submits it.

■

Prior to evidence acquisition persuaders choose a searching technology. The next *Lemma* characterizes persuaders’ choices. We assume that once indifferent between the two technologies, persuaders choose *g-technology*. This assumption can be supported by introducing an arbitrary small fixed cost  $k$  for adopting *s-technology*.

**Lemma 9 (choice of search technology)** *Persuader  $E$  chooses  $s$ -technology under the adversarial system and  $g$ -technology under the inquisitorial system, while persuader  $F$  always chooses  $s$ -technology.*

**Proof.** The ex-ante probability of acquiring two pieces of evidence when using *g-technology* is equal to that when using *s-technology*. Therefore under the *inquisitorial* system, persuader  $E$  is indifferent between the two technologies, and thus chooses *g-technology*. Under the *adversarial* system, the ex-ante probability of acquiring a desirable piece of evidence is greater when using *s-technology* than when using *g-technology*. Therefore,  $F$  always uses *s-technology*. Similarly, for persuader  $F$ , the probability of acquiring a ‘good’ piece of evidence is higher when using *s-technology*. ■

Building on *Lemma 1* and *Lemma 2*, we can now write the corresponding payoffs of the parties under two systems. Note that the *DM* has no monetary constraints, and the wage serves as a pure transfer from the *DM* to  $E$ . Therefore in each system, the *DM* can always set such  $w_E^{*A}$  and  $w_E^{*I}$  that implement the first best level of effort, given the choice of remuneration scheme and the effort of persuader  $F$ . From now on, we will therefore refer to the *DM*’s first best level of effort as  $E$ ’s effort evaluated at  $w_E^{*A}$  and  $w_E^{*I}$ . We will abstract from  $E$ ’s payoff in the rest of the analysis and refer to  $e_E$  as if chosen directly by the *DM* under each system, given  $E$ ’s submission strategy.

### 1. Inquisitorial system

$$U_{DM}^I = \frac{1}{2} [1 - (p_{4,b} - p_{4,g})(3\alpha e_E) - (p_{2,g} - p_{2,b})\alpha e_E(1 + 2\alpha e_F)](-L) - C(e_F) - C(e_E) \quad (3.4)$$

$$U_F^I = \sum_{\Omega} \frac{1}{2} [1 - p_{2,\omega}(\alpha e_E(1 - (2\alpha)e_F) - p_{3,\omega}2\alpha e_E - p_{4,\omega}3\alpha e_E)\Pi - C(e_F)]. \quad (3.5)$$

The payoff of the *DM* can be written as the loss from an erroneous decision in the absence of evidence net of the difference between the correction of existing errors and production of new errors due to submitted evidence. When no evidence is available, the *DM* always takes the decision in favour of *F*, therefore failing to block the merger when the state is bad (*type I* error), but avoiding to block the merger in the good state (avoiding *type II* errors). By taking into consideration available evidence, the *DM* reduces *type I* errors and increases *type II* errors, so that the level of precision of the final decision increases as a result. In the event where both pieces of evidence are favorable to *F* (*event 1*), the acquisition of evidence does not bring any new information, and does not affect the decision of the *DM*. However in the event where both pieces of evidence are unfavorable to *F*, acquiring at least one piece of evidence prevents an erroneous decision if the state is ‘*bad*’, but increases the error if the state is ‘*good*’. Due to the fact that this event is more likely in the ‘*bad*’ state, submitted evidence decreases the expected error. In the event when evidence is conflicting but overall favorable to *F*, the *DM* increases *type II* error if *F* fails to find a positive piece of evidence (with probability  $(1 - 2\alpha e_F)$ ) and *E* finds only a negative piece of evidence (with probability  $\alpha e_E$ ) in the ‘*good*’ state and decreases *type I* error otherwise. Finally in the event when evidence is conflicting but overall unfavorable to *F* (*event 3*), finding a negative piece of evidence (with probability  $2\alpha e_E$ ) reduces *type II* error in the ‘*bad*’ state and increases *type I* error in the ‘*good*’ state, with the latter being less likely. Note that the reduction of errors in the event 3 is greater than the increase in the errors in the *event 2*, therefore the level of precision of the final decision increases with information available.

With probability  $\sum_{\Omega} p_{1,\omega}$  the evidence is consistent and favorable to persuader *F* (*event 1*). In these instances, *F* wins independently as to what type of information the other party obtains. With probability  $\sum_{\Omega} p_{2,\omega}$ , evidence is conflicting but overall favorable to *F* (*event 2*). In this case, *F* loses only if she does not obtain a favorable piece of evidence (this happens with probability  $(1 - 2\alpha e_F)$ ) and at the same time, *E* obtains the only unfavorable piece of evidence (this happens with probability  $\alpha e_E$ ). With probability  $\sum_{\Omega} p_{3,\omega}$ , evidence is conflicting but overall unfavorable to *F* (*event 3*). Under such a scenario, *F* wins if *E* does not obtain the unfavorable piece of evidence (this happens with probability  $2\alpha e_E$ ). Finally,

with probability  $\sum_{\Omega} p_{4,\omega}$ , both pieces of evidence are unfavorable to  $F$ . In this case,  $F$  wins only if  $E$  fails to obtain any evidence (this happens with probability  $(1 - 3\alpha e_E)$ ).

## 2. Adversarial system

$$U_{DM}^A = \frac{1}{2} [1 - (p_{4,b} - p_{4,g})3\alpha(1+s)e_E - (p_{2,g} - p_{2,b})(2+s)^2\alpha^2 e_E e_F] (-L) - C(e_F) - C(e_E) \quad (3.6)$$

$$U_F^A = \sum_{\Omega} \frac{1}{2} [(1 - (2+s)\alpha e_E)(p_{2,\omega}(1 - (2+s)\alpha e_F) + p_{3,\omega}) - p_{4,\omega}3\alpha(1+s)e_E] \Pi - C(e_F). \quad (3.7)$$

With probability  $\sum_{\Omega} \frac{1}{2} p_{1,\omega}$  evidence is consistent and favorable to persuader  $F$  (*event 1*). In these instances,  $F$  wins independently as to what type of information  $E$  obtains. With probability  $\sum_{\Omega} p_{2,\omega}$ , evidence is conflicting but overall favorable to  $F$  (*event 2*).  $F$  always wins, except in the event when she fails to obtain a favorable piece of evidence and  $E$  succeeds in obtaining the unfavorable one (this happens with probability  $(1 - (2+s)\alpha e_F)(2+s)\alpha e_E$ ). With probability  $\sum_{\Omega} p_{3,\omega}$ , evidence is conflicting but overall unfavorable to  $F$  (*event 3*). Under such a scenario,  $F$  wins if  $E$  does not obtain the unfavorable piece of evidence (this happens with probability  $(2+s)\alpha e_E$ ). Finally, with probability  $\sum_{\Omega} p_{4,\omega}$ , both pieces of evidence are unfavorable to  $F$ . In this case,  $F$  wins only if  $E$  fails to obtain any evidence (this happens with probability  $(1 - 3\alpha(1+s)e_E)$ ).

Note that for equal levels of efforts of parties under the two systems and for  $s = 0$ , the two systems differ only in the level of precision of the final decision in the *event 2*. The *inquisitorial* system protects  $F$  more often from being mistakenly accused and fails to accuse more often when  $F$  is guilty. Anticipating this,  $F$  has incentives to exert more effort for evidence collection under the *adversarial* system, all else being equal.

The following *Lemma* formally summarizes the nature of strategic complementarities prevailing in this game under the two systems. Throughout the analysis, we adopt a quadratic specification for the cost function such that  $C(e_i) = \frac{1}{2}e_i^2$ . However, our results apply to a more general convex cost function.

**Lemma 10** (*reaction functions*) *The efforts of the parties are strategic complements.*

**Proof.** Differentiating the payoffs of the parties we obtain:

1. *Adversarial* system:

$$e_F^{*A} = \frac{1}{2} \Pi \sum_{\Omega} [(2+s)^2 \alpha^2 e_E p_{2,\omega}], \quad (3.8)$$

$$e_E^{*A} = \frac{1}{2} \alpha L [3(1+s)(p_{1,g} - p_{1,b}) + \alpha(2+s)^2 (p_{2,g} - p_{2,b}) e_F]. \quad (3.9)$$

2. *Inquisitorial* system:

$$e_F^{*I} = \frac{1}{2} \Pi \sum_{\Omega} [2\alpha^2 e_E p_{2,\omega}], \quad (3.10)$$

$$e_E^{*I} = \frac{1}{2} \alpha L [3(p_{1,g} - p_{1,b}) + (p_{2,g} - p_{2,b})(1 + 2\alpha e_F)]. \quad (3.11)$$

■

The first thing to note is that it is the existence of two pieces of evidence that drives the non-zero effort level of both persuaders. If only one piece of evidence is available, only the party that carries the burden of proof would exert a positive level of effort under both systems. The fact that an otherwise unfavorable decision can be reverted if extra piece of information is provided drives the incentive of both persuaders to invest in information collection.

#### *F's reaction function*

Persuader  $F$  has a chance to revert an otherwise unfavorable decision by providing additional evidence if  $E$  obtained an unfavorable piece of evidence, and if evidence is overall favorable to  $F$ . Thus, the higher the chances that  $E$  will acquire and thus submit only a negative piece of evidence in the event that such evidence is overall favorable to  $F$  ( $p_{2,\omega}$ ), the higher is  $F$ 's effort. Note that the probability that  $E$  will submit only a negative piece of evidence is greater under the *adversarial* system than under the *inquisitorial* system. Thus for a given  $e_E$ , the effort of  $F$  is greater under the *adversarial* system than under the *inquisitorial* system.



*E's reaction function*

$E$ 's reaction function evaluated at  $w_E^{*A}$  and  $w_E^{*I}$ , represents the  $DM$ 's constrained first best choice of  $E$ 's effort, which increases with  $F$ 's effort under both systems. As  $F$  increases her effort, the probability of the  $DM$  blocking the 'good' merger decreases, and each extra unit of  $E$ 's effort contributes to a further reduction of type II errors, i.e. allowing 'bad' merger. Under the *adversarial* system, *type II* errors are more likely than under the *inquisitorial* system. Therefore, an increase in  $F$ 's effort triggers sharper a reaction, and thus a larger increase in  $E$ 's effort. So, the increase in  $F$ 's effort triggers a larger increase in  $E$ 's effort under the *adversarial* system.

For a given level of  $F$ 's effort, first best  $DM$ 's effort is higher under the *inquisitorial* system for  $s$  small and under the *adversarial* system for  $s$  large enough. The efficiency of an additional unit of  $E$ 's effort under the *adversarial* system in bringing up useful information increases with  $s$ . When the degree of specialization is relatively low, an extra unit of  $E$ 's effort is more efficient under the *inquisitorial* system than under the *adversarial* system. Under the *inquisitorial* system  $E$  is not being strategic enough, and reveals all of the information that she acquires. Therefore, in this case, the  $DM$  prefers  $E$  to incur a higher effort under the *inquisitorial* system than under the *adversarial* system. As the degree of specialization increases, the probability of acquiring and thus submitting favorable evidence increases under the *adversarial* system. An extra unit of  $E$ 's effort becomes more efficient in producing useful information, and thus the optimal level of the  $DM$ 's effort increases. For  $s$  sufficiently large, the  $DM$  chooses a higher effort of  $E$  under the *adversarial* system than under the *inquisitorial* system.

The following proposition shows that the above conjectured equilibrium exists, and that the  $DM$  reaches its decision as if being a fully Bayesian player.

**Proposition 11 (Existence of Equilibrium)** *i) There exists a pure strategy equilibrium, in which along the equilibrium path the  $DM$  reaches its conjectured decision as if based solely on the informativeness of submitted evidence, and this equilibrium is unique of its kind. In the case of no evidence submitted the  $DM$  rules in favor of  $F$  by rule. This equilibrium can be supported by the off-equilibrium beliefs that support the conjectured decision.*

*ii) This equilibrium survives a trembling hand criterion as long as  $e_E^{*A}$  is low enough.*

**Proof.** See *Appendix A*. ■

A simple decision rule leads to the same final decision as if a *DM* would have updated his rational beliefs based on the information at his disposal, and based on the anticipated equilibrium behavior of both parties. Two factors drive this result: *i*) both states are equally likely and all the players share common priors; and *ii*) the information acquisition efforts of both parties, together with the choices of searching technology, are not conditional on the state of nature, and therefore the informativeness of the evidence submitted outweighs the strategic considerations. *Appendix A* contains a detailed proof of this result and specifies the off-equilibrium beliefs that support this equilibrium. We show that this equilibrium is robust, even when allowing for a small probability of submission mistakes by the parties, i.e. it survives a trembling hand refinement. The *DM*, when faced with an unexpected deviation, believes that a persuader mistakenly discloses evidence with a certain probability. We show that such off-equilibrium beliefs are sufficient to sustain the equilibrium under the *inquisitorial* system. Under the *adversarial* system, such beliefs sustain the equilibrium only if the equilibrium effort of *E* under the *adversarial* system ( $e_E^{*A}$ ) is low enough, i.e. if the degree of the specialization of *s-technology* or  $\Pi$ , or  $L$  are high enough. For  $e_E^{*A}$  sufficiently high, additional assumptions are needed on the off-equilibrium beliefs in order to sustain the equilibrium. For example, when *E* fails to submit any evidence, the *DM* (if fully Bayesian), would always rule in the favor of *F*. If *E* equilibrium is high under *adversarial* system, then the probability of her finding a desirable piece of evidence, conditional on it existing, is sufficiently high. Therefore the fact that *E* did not submit any evidence under the *adversarial* system is more likely to suggest that *E* found a ‘good’ piece of evidence, and therefore withholds it. *E* is more likely to find a ‘good’ piece of evidence in a ‘good’ state than in a ‘bad’ state, therefore the *DM* should clear the merger independently on what type of evidence *F* submitted in this case. However, the behaviour of the *DM* off-equilibrium path does not affect the equilibrium outcome, as *F* does not have incentives to deviate and submit unfavorable to her evidence. If *DM*’s beliefs are such that when faced with the unexpected deviation by the firm and no evidence submitted by the expert, the *DM* believes that the mistake is more likely in the ‘bad’ state than in the ‘good’ state, this would sustain the equilibrium for the *adversarial* system, even for high equilibrium efforts of the expert.

### 3.3.3 Comparison of two systems

*Proposition 1* states that under both systems, a simple decision rule that the *DM* applies constitutes a sub-game perfect Nash equilibrium. However, which system generates a higher *DM*'s payoff requires further examination. *Lemma 3* shows that for a given level of *E*'s effort, persuader *F* exerts higher effort under the *adversarial* system than under the *inquisitorial* system. More effort triggers a higher level of precision of the final decision, *ceteris paribus*. However, more effort also affects the effort decision of the other party and in addition triggers higher costs. From *Lemma 3*, *E*'s equilibrium effort (for a given  $e_F$ ) is lower under the *adversarial* system for  $s$  low enough. Therefore, a necessary condition for the *adversarial* system to generate a higher *DM*'s payoff is that a higher *F*'s effort should result in a higher level of precision of the final decision, i.e. it should result in fewer errors. If this condition is violated, the *inquisitorial* system would always result in a higher *DM*'s payoff, as it involves lower total costs. However, a higher level of precision is only a necessary condition. A sufficient condition would require that a marginal increase in the level of precision of a final decision due to a higher effort outweighs the corresponding increase in the total costs. The following proposition characterizes the level of precision of a final decision under the two systems.

**Proposition 12 (payoff of DM)** *1. For  $s = 0$  the inquisitorial system always generates a higher level of precision of the final decision, and therefore a higher *DM*'s payoff than the adversarial system.*

*2. There exists  $\hat{s} < 1$ , such that for  $s > \hat{s}$  the level of precision of a final decision under the adversarial system is higher than under the inquisitorial system.*

**Proof.** Appendix C.1. ■

When both technologies are equally efficient in obtaining different types of evidence, more evidence is expected to be submitted under the *inquisitorial* system. A higher effort of persuader *F* is not sufficient to outweigh a lower effort of persuader *E* in this case. Intuitively, this occurs due to the fact that a unit change in *E*'s effort generates a higher change in the expected quantity of the evidence submitted than a unit change in *F*'s effort. In the *adversarial* system, both persuaders are strategic, and submit only favorable evidence

to them, while under the *inquisitorial* system, expert  $E$  submits all the information acquired. This result is reversed when the degree of the specialization of  $s$ -technology increases. In this case, the efficiency of the unit of effort with respect to the information submitted increases for the *adversarial* system. Therefore, the *adversarial* system results in a higher than expected amount of evidence revealed by the persuaders.

The next proposition characterizes the conditions under which the increase in the total cost associated with higher effort is lower than the increase in a precision of a final decision. *Proposition 3* therefore defines the sufficient conditions under which the *adversarial* system results in a higher  $DM$ 's payoff.

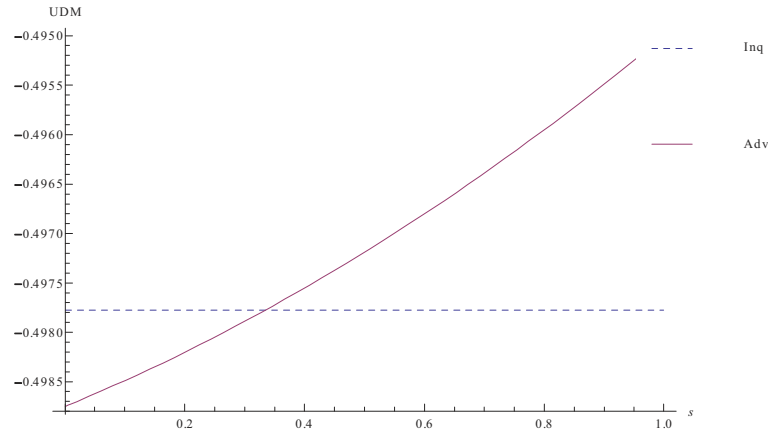
**Proposition 13** 1. For  $s > \hat{s}$  and  $e_F^{*I}$  sufficiently small the *adversarial* system results in a higher  $DM$ 's payoff.

2. For  $e_F^{*I} = e_F^{*A} = 1$  there exists  $\tilde{s} < 1$ , such that for  $s > \tilde{s}$  the *adversarial* system results in a higher  $DM$ 's payoff.

**Proof.** Appendix C.2 ■

The proof is based on several arguments. Using the fact that the  $DM$ 's payoff under the *adversarial* system increases with  $s$ , we show that for a given level of  $e_E^*$  (if  $F$  exerts no effort, i.e.  $\Pi = 0$ ), there exists  $\hat{s}$  such that the *adversarial* system results in a strictly higher  $DM$ 's payoff for  $s > \hat{s}$ . The difference in the level of precision of the final decision is strictly positive and linear in  $e_F^*$  while the difference in costs is zero. Applying the continuity property of the  $DM$ 's loss and cost function in  $e_F^*$  and  $e_E^*$  we argue that for  $\Pi$  slightly higher than zero, this result still holds. As  $\Pi$  increases, the *adversarial* system results in higher costs and the *inquisitorial* system generates a higher  $DM$ 's payoff.

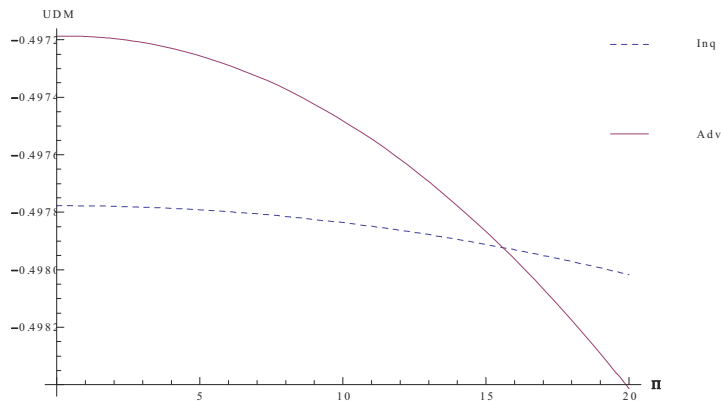
Figures 1 and 2 below provide a numerical example of this result for a quadratic cost function:



$$\Pi = L = 1$$

Figure 1

The *DM's* payoff under the *adversarial* system increases in the degree of specialization of *s-technology*, and crosses the *DM's* payoff under the *inquisitorial* system for  $\Pi$  low enough.



$$s = 0.7$$

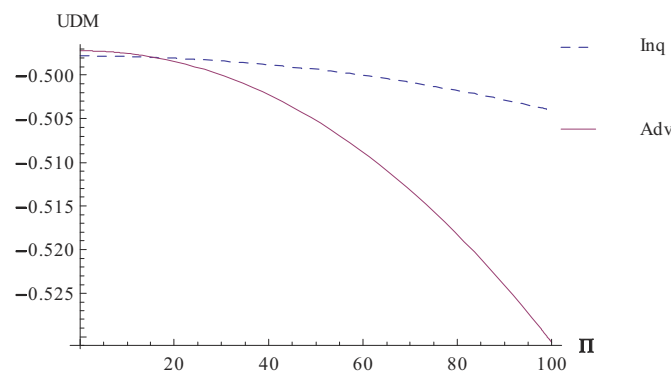
Figure 2

However, as  $\Pi$  increases,  $F$ 's probability of obtaining no evidence reaches zero and therefore the effort is at maximum under the two systems. We show that for the same level of costs, the *adversarial* system generates a higher level of precision of the decision for  $s$  high enough, and therefore a higher *DM's* payoff.

Propositions 2 and 3 state that the necessary condition for the *adversarial* system to dominate the *inquisitorial* system is the high degree of specialization of *s-technology*, i.e.

research specialization should substantially increase the probability of obtaining a desired type of evidence, given that it exists. For a low degree of specialization, a higher effort of the firm for a given effort of the agent does not offset the loss of information from the strategic ‘silence’ of the latter under the *adversarial* system. Given that the degree of specialization is sufficiently high, the *adversarial* system will perform better than the *inquisitorial* system if, and only if, the stake of the firm is either sufficiently low or sufficiently high. Thus, the increase in the level of precision of the final decision offsets the corresponding increase in the costs of effort. For the intermediate values of  $F$ 's stake, the increase in the level of precision of the final decision is not sufficient to offset the respective increase in the costs, and therefore the *inquisitorial* system dominates the *adversarial* system, even for a high degree of specialization.

Note that for  $\Pi$  high enough, the  $DM$ 's payoff under both systems is lower than if the decision was taken randomly, due to the higher costs of a joint effort. If the  $DM$  could commit to not take into consideration evidence submitted by  $F$ , he would increase his payoff. Figure 3 demonstrates this point.



If the  $DM$  commits to not take into consideration  $F$ 's evidence and if the burden of proof is allocated to  $E$ , the *inquisitorial* system results in a higher  $DM$ 's payoff for low degree of specialization of *s-technology*, and vice versa.

### 3.4 Burden of proof

We now relax the assumption that the burden of proof is exogenously imposed on the players, and investigate whether or not such an equilibrium spontaneously arises.

**Proposition 14 (No evidentiary rules)** *1. Under the adversarial system, for  $\Pi$  sufficiently small, there exists a pure strategy equilibrium in which the decision maker reaches its decision ‘as if’ based solely on the informativeness of the submitted evidence*

*2. Under the inquisitorial system, the allocation of the burden of proof on  $E$  can never arise as an equilibrium outcome.*

**Proof.** Appendix D. ■

The intuition for this result is similar to the one described above for the trembling hand criterion. The allocation of the burden of proof on  $E$  leads to the latter extracting more effort than  $F$ , for  $\Pi$  sufficiently small. In this case, under the *adversarial* system, the *DM* is rationally inferring from the fact that  $E$  is not disclosing evidence (despite her being the better informed persuader), that she is in fact withholding unfavorable information, and thus rules in favor of  $F$ . In practice, the *adversarial* system adopts such an allocation of burden of proof quite frequently. However, under the *inquisitorial* system,  $E$  would not withhold any information, and thus  $E$  does not submit any information only if she does not succeed in obtaining it. On the other hand,  $F$  does not submit any information either if she did not find any information or she found negative information, or if she found two pieces of evidence and the evidence is unfavorable to her. This situation is more likely to occur in the bad state, and thus ruling in favor of  $F$  is not consistent with a Bayesian update of information. Therefore, the *DM* should be able to commit to a rule of allocation of burden of proof. A natural extension would be to conduct a similar analysis in the case when the burden of proof is carried by  $F$ , i.e. if no evidence is submitted, the *DM* blocks the merger. It appears that such an allocation of the burden of proof can arise as an equilibrium outcome under the *inquisitorial* system, which justifies the frequent use of such an allocation of the burden of proof in the *inquisitorial* system. The efforts of persuaders are strategic substitutes in this case. However, the existence of an equilibrium described in *Proposition 1* requires more detailed examination, which we intend to complete as the next step of our

work. We further aim to investigate the performance of both systems under the alternative allocation of the burden of proof. The intuition suggests that the *inquisitorial* system would further increase its performance in this case. This calls for a comparison of the performance of the *inquisitorial* system, under the alternative allocation of the burden of proof, with the *adversarial* system, under the actual allocation of the burden of proof.

### 3.5 Conclusions

In this paper we compare two judicial systems: the *adversarial* and the *inquisitorial*. By choosing between two remuneration systems, a decision-maker encourages an agent to either be biased or neutral with respect to a defendant. We depart from the standard literature on this topic by allowing the defendant to submit the acquired evidence under both systems, and by allowing for a broader informational structure. We motivate our set-up by a merger control example, where the defendant is the firm that wants to merge with another firm, and the agent is the competition agency in charge of the investigation. Both the agency and the firm invest in the acquisition of hard evidence. An additional piece of information, once acquired, can revert the *DM's* decision to block or to clear the merger. This triggers strategic complementarities in the effort choices of both parties. The *adversarial* arrangement results in the firm exerting a higher level of effort, all other things being equal. On the one hand, a higher level of effort means more information on the firm's side; while on the other hand, it triggers higher costs. At the same time, the agent withholds the acquired information in the case where it is unfavorable to her, and therefore some information is wasted. Under the *inquisitorial* arrangement, the *DM's* agent is not strategic and submits all of the information that she acquires. We have characterized the equilibrium of this game under both systems. The *DM* applies a simple rule for a final decision that appears to be fully consistent with the Bayesian updating of the beliefs based on the available information. We characterize the necessary and sufficient conditions for one system to be superior to another in terms of the level of precision of the final decision, net of the associated costs from the acquisition of the information. We find additional supportive arguments for the *inquisitorial* system, which always dominates the *adversarial* system if the specialized searching technology available to the parties is not efficient to obtain a desired type of evidence. In this case, the *inquisitorial*



system results in more precise final decision-making, as well as in a lower cost of the information acquisition. If the degree of the specialization of the *s-technology* is high enough, then the *adversarial* system generates a higher level of precision of the final decision, and at the same time, higher costs. The difference in the net *DM*'s payoff under both systems is non-monotonic in the equilibrium effort of the firm. We show that either for sufficiently low values or for sufficiently high values of the equilibrium effort of the firm, the *adversarial* system dominates the *inquisitorial* system. For the intermediate values of *F*'s equilibrium effort the conclusion is reversed, and the *inquisitorial* system results in much lower costs of the level of precision. We also show that for high values of the equilibrium efforts of the parties, the *DM* might be better served by committing to not consider the information provided by the firm. These results are generated under a certain assumptions about the *DM*'s final decision in the absence of any evidence. In particular, we have assumed throughout the analysis that if no evidence is submitted, the *DM* rules in the favor of the firm and clears the merger. Our results are further reinforced under the alternative allocation of the burden of proof, but the detailed comparison of the performance of the two systems in this case is left for future research. We show that such an allocation of the burden of proof can arise as the equilibrium outcome under the *adversarial* system, but can never arise under the *inquisitorial* system, and thus should be maintained as a commitment. This result provides additional intuition as to why the *inquisitorial* system seldom allocates the burden of proof to the prosecuting party. A more detailed exploration of these only partially-addressed questions shapes our future research opportunities.

## 3.6 Appendix

### A: Proof of Proposition 1

The evidence submission strategies stated in *Lemma 1* were characterized under the assumption that the *DM* takes his decision as following:

a) Under the *inquisitorial* system and under the *adversarial* system for the values of  $L$  and  $\Pi$  such that the equilibrium effort of  $e_E$  is low enough.

1. decides to allow the merger, i.e. sets  $m = 1$ , if in a possession of either nothing or  $x \in X^g, y \in Y^g$ , or  $z \in Z^g$ , and

2. decides to disallow the merger, i.e. sets  $m = 0$  if in possession of either  $x \in X^b, y \in Y^b$ , or  $z \in Z^b$ .

b) Under the *adversarial* system for the values of  $L$  and  $\Pi$  such that the equilibrium effort of  $e_E$  is sufficiently high.

1. decides to allow the merger, i.e. sets  $m = 1$  if expert  $E$  fails to obtain any evidence or if in a possession of either nothing or  $x \in X^g, y \in Y^g$ , or  $z \in Z^g$ .

2. decides to disallow the merger, i.e. sets  $m = 0$  if in possession of  $x \in X^b, y \in Y^b$ , or  $z \in Z^b$  and expert  $E$  submits at least one piece of evidence.

We now must verify that such behavior indeed constitutes an equilibrium under the two systems. We first analyze behavior and corresponding beliefs on the equilibrium path, taking evidence submission strategies as stated in *Lemma 1*. We then show that this behavior remains optimal off-equilibrium path, assuming that, when faced with an unexpected deviation, the *DM* believes that a submission mistake was made with probability  $\epsilon$  (per piece of evidence mistakenly submitted).

### Equilibrium path

On the equilibrium path under both systems, the *DM* may either receive the whole set of evidence  $z$ , one piece of evidence ( $x$  or  $y$ ), or nothing. The decision  $m = 1$  constitutes an equilibrium if  $EU_{DM}(m = 1) > EU_{DM}(m = 0)$ , which implies that conditional on the evidence submitted, the ‘good’ state is more likely to prevail than the ‘bad’ state.

Let  $\tilde{z}$  be the pair of evidence submitted by the parties.

The conditional probability that the state of nature is ‘good’ given the submitted evidence is:

$$P(\omega = g|\tilde{z}) = \frac{P(z = \tilde{z}|\omega = g)P(\omega = g)}{P(z = \tilde{z})}. \quad (3.12)$$

The conditional probability that the state of nature is ‘bad’ given the submitted evidence

is thus:

$$P(\omega = b|\tilde{z}) = \frac{P(z = \tilde{z}|\omega = b)P(\omega = b)}{P(z = \tilde{z})}. \quad (3.13)$$

A ‘good’ state is more likely to prevail given the submitted evidence if  $P(z = \tilde{z}|\omega = g) > P(z = \tilde{z}|\omega = b)$ .

Under the *adversarial* system, only four outcomes are possible on the equilibrium path: no evidence is submitted, party  $F$  only submits a ‘good’ piece of evidence, party  $E$  only submits a ‘bad’ piece of evidence, or both party  $F$  and party  $E$  submit ‘good’ and ‘bad’ pieces respectively.

Under the *inquisitorial* system more outcomes are possible on the equilibrium path: no evidence submitted, party  $F$  does not submit any evidence and at the same time party  $E$  submits either one or two pieces of evidence, party  $F$  submits a ‘good’ piece of evidence and at the same time party  $E$  does not submit any evidence or submits one piece of evidence (either ‘good’ or ‘bad’) or two pieces of evidence.

We now check that in all the instances the decision of the  $DM$  described above constitutes an equilibrium.

**Two pieces of evidence are submitted** Note that once both pieces of evidence are submitted, independently of who submitted which piece of evidence, the probability of the good state is more likely if  $f_g > f_b$ , i.e. if, and only if,  $z \in Z^g$ . To gain further intuition of this result, consider the case when  $E$  and  $F$  each submit one piece of evidence.

Suppose that  $F$  submits only  $x$  and  $E$  submits only  $y$ , i.e.  $\tilde{z}_F = (x, \emptyset)$  while  $\tilde{z}_E = (\emptyset, y)$ , where  $x \in X^g$ ,  $y \in Y^b$  and  $z \in Z^g$ .

1. Consider first the *inquisitorial* system.

The evidence submitted suggests that *event 2* (conflicting but overall favorable evidence) is realized. The probability of such realization in ‘good’ and ‘bad’ states is given by  $f_g$  and  $f_b$  respectively.  $E$  submits a ‘bad’ piece of evidence only in the case she acquired only this evidence (this happens with probability  $\alpha e_E^*$ ).  $F$  submits a ‘good’ piece of evidence either in the case she found only this piece of evidence (with probability  $\alpha e_F^*(1 + s)$ ) or in the case

she found two pieces of evidence (with probability  $\alpha e_F^*$ ).

Therefore the conditional probabilities of ‘good’ and ‘bad’ states are:

$$P(z = \tilde{z} | \omega = g) = f_g(z)(2 + s)\alpha^2 e_F^* e_E^* \quad (3.14)$$

$$P(z = \tilde{z} | \omega = b) = f_b(z)(2 + s)2\alpha^2 e_F^* e_E^*. \quad (3.15)$$

The *DM* rules in favor of *F* if and only if:

$$f_g(z)2\alpha^2 e_F^* e_E^* > f_b(z)(2 + s)2\alpha^2 e_F^* e_E^*. \quad (3.16)$$

2. Consider now the *adversarial system*. The behavior of *F* is the same under the *adversarial* system as under the *inquisitorial* system. However, *E* submits a ‘bad’ piece of evidence either if she acquired only this piece of evidence (with probability  $(1 + s)\alpha e_E^*$ ) or if she acquired two pieces of evidence (with probability  $\alpha e_E^*$ ). The corresponding conditional probabilities are thus given by:

$$P(z = \tilde{z} | \omega = g) = f_g(z)(2 + s)^2 \alpha^2 e_F^* e_E^* \quad (3.17)$$

$$P(z = \tilde{z} | \omega = b) = f_b(z)(2 + s)^2 \alpha^2 e_F^* e_E^*. \quad (3.18)$$

*DM* rules in favor of *F* if, and only if:

$$f_g(z)(2 + s)^2 \alpha^2 e_F^* e_E^* > f_b(z)(2 + s)^2 \alpha^2 e_F^* e_E^*. \quad (3.19)$$

**Proof.** As  $f_g(z \in Z^g) > f_b(z \in Z^g)$  by definition, the inequalities (3.16) and (3.19) hold.

■

Due to the fact that the probabilities of submitting  $z$  in both states are equal, it is the joint density of  $z$  that determines which state is more likely.

The proof is identical for all the other cases when the submission strategies by parties lead to two pieces of evidence being revealed.

Therefore, we would further consider only partial submission cases.

**One piece of evidence submitted** a) We first consider the case when  $F$  submits a ‘good’ evidence and  $E$  submits nothing, i.e.  $\tilde{z}_F = (x, \emptyset)$  while  $\tilde{z}_E = (\emptyset, \emptyset)$ , where  $x \in X^g$ .

Let us denote by  $\bar{p}_{j,\omega,x}$  and  $\bar{p}_{j,\omega,y}$  the probabilities of event  $j = 1, \dots, 4$ , conditional on being in state of nature  $\omega$  and for a given realization of, respectively,  $x$  or  $y$ .

1. The conditional probabilities of each state under *inquisitorial system* are given by:

$$P(x|\omega = g) = g_g(x)(1 - 3\alpha e_E^*)\alpha e_F^*[(1 + s) + \bar{p}_{2,g,x}] \quad (3.20)$$

$$P(x|\omega = b) = g_b(x)(1 - 3\alpha e_E^*)\alpha e_F^*[(1 + s) + \bar{p}_{2,b,x}]. \quad (3.21)$$

$F$  submits one ‘good’ piece of evidence if:

i) she acquired only 1 piece of evidence and this evidence was ‘good’ (with probability  $(1 + s)\alpha e_F^*$ )

ii) she acquired two pieces of evidence and the second piece was ‘bad’ but the evidence is overall favorable to  $F$  (with probability  $\alpha e_F^*\bar{p}_{2,g,x}$ ). Recall that  $F$  does not submit any evidence if she acquires two pieces of evidence that are ‘good’ and if she acquires two pieces of evidence that are overall unfavorable to her.

$E$  at the same time does not submit any evidence, only if she does not indeed acquire any evidence (with probability  $(1 - 3\alpha e_E^*)$ )

$DM$  rules of favour of  $F$  if following inequality is satisfied:

$$g_g(x)(1 - 3\alpha e_E^*)\alpha e_F^*[(1 + s) + \bar{p}_{2,g,x}] > g_b(x)(1 - 3\alpha e_E^*)\alpha e_F^*[(1 + s) + \bar{p}_{2,b,x}]. \quad (3.22)$$

2. The corresponding probabilities under the *adversarial system* are given by:

$$P(x|\omega = g) = \quad (3.23)$$

$$g_g(x)[\alpha e_F^*((1 + s) + \bar{p}_{2,g,x})(1 - (\bar{p}_{2,g,x} + \bar{p}_{3,g,x})(2 + s)\alpha e_E^*)]$$

$$P(x|\omega = b) = \quad (3.24)$$

$$g_b(x)[\alpha e_F^*((1 + s) + \bar{p}_{2,b,x})(1 - (\bar{p}_{2,b,x} + \bar{p}_{3,b,x})(2 + s)\alpha e_E^*)].$$

Under the *adversarial* system,  $F$  submission strategy is unchanged. However,  $E$  does not submit any evidence in more instances: when she obtained either nothing or a ‘good’ piece of evidence (with probability  $(1 - (\bar{p}_{2,b,x} + \bar{p}_{3,b,x}))(2 + s)\alpha e_E^*$ ). Note that  $\bar{p}_{2,\omega,x} + \bar{p}_{3,\omega,x} = 1 - \bar{p}_{1,\omega,x}$

$DM$  rules of favour of  $F$  if the following inequality is satisfied:

$$\begin{aligned} & g_g(x)[\alpha e_F^*((1 + s) + \bar{p}_{2,g,x})(1 - (1 - \bar{p}_{1,g,x})(2 + s)\alpha e_E^*)] \\ & > g_b(x)[\alpha e_F^*((1 + s) + \bar{p}_{2,b,x})(1 - (1 - \bar{p}_{1,g,x})(2 + s)\alpha e_E^*)]. \end{aligned} \quad (3.25)$$

**Proof.** given that  $x \in X^g$  implies  $g_g(x) > g_b(x)$ , together with  $\bar{p}_{1,g,x} < \bar{p}_{1,b,x}$  and  $\bar{p}_{2,g,x} > \bar{p}_{2,b,x}$ , inequalities (3.22) and (3.25) are satisfied. ■

b) We now analyze the case when  $E$  submits one ‘bad’ piece of evidence and  $F$  submits nothing. Using the same reasoning as for the case above the  $DM$  rules in favour of  $E$  if following inequalities hold:

1. Inquisitorial system

$$\begin{aligned} & g_g(x)\alpha e_E^*[1 - (2 + s)\alpha e_F^* + \bar{p}_{3,g,x}\alpha e_F^* + \bar{p}_{4,g,x}(1 + (2 + s)\alpha e_F^*)] \\ & < g_b(x)\alpha e_E^*[1 - (2 + s)\alpha e_F^* + \bar{p}_{3,b,x}\alpha e_F^* + \bar{p}_{4,b,x}(1 + (2 + s)\alpha e_F^*)]. \end{aligned} \quad (3.26)$$

2. Adversarial system

$$\begin{aligned} & g_g(x)[(2 + s)\alpha e_E^*((1 - \bar{p}_{1,g,x})(1 - (2 + s)\alpha e_F^*) + \bar{p}_{3,g,x}\alpha e_F^*) + 3(1 + s)\alpha e_E^*\bar{p}_{4,g,x}] \\ & < g_b(x)[(2 + s)\alpha e_E^*((1 - \bar{p}_{1,g,x})(1 - (2 + s)\alpha e_F^*) + \bar{p}_{3,b,x}\alpha e_F^*) + 3(1 + s)\alpha e_E^*\bar{p}_{4,b,x}]. \end{aligned} \quad (3.27)$$

**Proof.** Inequalities (3.26) and (3.27) are satisfied, as  $\bar{p}_{4,g,x} < \bar{p}_{4,b,x}$ , and  $\bar{p}_{3,g,x} < \bar{p}_{3,b,x}$ . ■

c) Under the *inquisitorial system* we are still left with two outcomes to analyze:

1.  $E$  submits ‘good’ evidence and  $F$  submits nothing, i.e.  $\tilde{z}_F = (\emptyset, \emptyset)$  while  $\tilde{z}_E = (x, \emptyset)$ , where  $x \in X^g$ .

2. Both  $F$  and  $E$  submit ‘good’ evidence.

However, we can observe that if  $m = 1$  constitutes equilibrium in the first case, then it should constitute equilibrium in the second case as well, as it is more likely in the second case that the true state is ‘good’.

Therefore, we are only left with the first case to analyze.

$DM$  indeed rules in favour of  $F$  if, and only if:

$$\begin{aligned} & g_g(x)\alpha e_E^*[1 - (2 + s)\alpha e_F^* + \bar{p}_{1,g,x}(1 - (2 + 3s)\alpha e_F^*) + \bar{p}_{3,g,x}\alpha e_F^*] \\ & > g_b(x)\alpha e_E^*[(1 - (2 + s)\alpha e_F^*) + \bar{p}_{1,b,x}(1 - (2 + 3s)\alpha e_F^*) + \bar{p}_{3,b,x}\alpha e_F^*]. \end{aligned} \quad (3.28)$$

**Proof.** After several algebraic manipulations, one can show that the above inequality is satisfied as:

- i)  $(\bar{p}_{1,g,x} - \bar{p}_{1,b,x})(1 - (2 + 3s)\alpha e_F^*) > (\bar{p}_{3,b,x} - \bar{p}_{3,g,x})\alpha e_F^*$  as  $(\bar{p}_{1,g,x} - \bar{p}_{1,b,x}) > (\bar{p}_{3,b,x} - \bar{p}_{3,g,x})$
- ii)  $(1 - (2 + 3s)\alpha e_F^*) > \alpha e_F^*$  as  $(1 + s)\alpha \leq \frac{1}{3}$  by the assumption. ■

### Off-equilibrium path and trembling hand criterion

We now consider out-of-equilibrium behavior.

When faced with an unexpected event, the  $DM$  believes that a party made a mistake (per piece of evidence mistakenly submitted) with probability  $\epsilon$ .

Consider the outcome in which  $F$  submits only a ‘bad’ piece of evidence while  $E$  submits nothing, i.e.  $\tilde{z}_F = (x, \emptyset)$  while  $z_E = (\emptyset, \emptyset)$ , where  $x \in X^b$ . The  $DM$  then disallows the merger iff:

1. Under the *inquisitorial system* the following should be true:

$$\begin{aligned} & g_b(x)(1 - 3\alpha e_E^*)\alpha(1 - s)e_F^*\epsilon(1 + \bar{p}_{4,b,x}) \\ & > g_g(x)(1 - 3\alpha e_E^*)\alpha(1 - s)e_F^*\epsilon(1 + \bar{p}_{4,g,x}). \end{aligned} \quad (3.29)$$

Since  $x \in X^b$ , events 2, 3 and 4 are the only relevant ones. In case  $y \in Y^g$ , it cannot be

the case that  $F$  observed it, for otherwise it would have submitted it. Thus, since  $\tilde{z}_F = (x, \emptyset)$  it must be the case that persuader  $F$  acquired only  $x$  (with probability  $(1-s)\alpha e_F^*$ ), or both  $x$  and  $y$  in the event 4 (with probability  $2(1-s)\alpha e_F^*$ ) and made a submission mistake with probability  $\epsilon$ . Since we also have that  $z_E = (\emptyset, \emptyset)$ , then  $E$  did not succeed in acquiring information, which happens with probability  $(1-3\alpha e_E^*)$ .

Inequality (3.29) holds since  $\bar{p}_{4,b,x} > \bar{p}_{4,g,x}$  when  $x \in X^b$ . It can be easily shown that all the other scenarios in which a submission mistake could occur work in a similar manner under the *inquisitorial* system.

2. Under the *adversarial* system, the submission of such evidence can occur only in the event 2, 3 and 4. In the event 1, only ‘good’ evidence is realized.

$F$  submits a bad piece of evidence in the events 2 and 3 if she has found only this piece of evidence and made a mistake in submitting it (with probability  $(1-s)\alpha e_F^*\epsilon$ ), and in the event 4, if she has found either only one or both pieces of evidence (probability  $3(1-s)\alpha e_F^*\epsilon$ ).  $E$ , on the other hand, does not submit any evidence under the *adversarial* system either if she acquired nothing or if she acquired ‘good’ evidence. In the events 2 and 3,  $E$  would acquire no evidence or a ‘good’ evidence with the probability  $(1-\alpha e_E^*(2+s))$ . In the event 4,  $E$  submits nothing only if she indeed acquires nothing which happens with a relatively low probability under the *adversarial* system,  $1-3(1-s)\alpha e_E^*$ .

Replacing  $(\bar{p}_{2,\omega,x} + \bar{p}_{3,\omega,x})$  by  $(1-\bar{p}_{4,\omega,x})$  one can express the corresponding conditional probabilities for each state as:

$$P(x|\omega = b) = \tag{3.30}$$

$$g_b(x)(1-s)\alpha e_F^*\epsilon[(1-\alpha e_E^*(2+s)) + \bar{p}_{4,b,x}((2-(7+8s)\alpha e_E^*))]$$

$$P(x|\omega = g) = \tag{3.31}$$

$$g_g(x)(1-s)\alpha e_F^*\epsilon[(1-\alpha e_E^*(2+s)) + \bar{p}_{4,g,x}((2-(7+8s)\alpha e_E^*))].$$

Thus,  $P(x|\omega = b)$  is greater than  $P(x|\omega = g)$  only  $(2-(7+8s)\alpha e_E^*) \geq 0$ , i.e. for  $e_E^* \leq \frac{2}{\alpha(7+8s)}$ . Otherwise,  $P(x|\omega = g) > P(x|\omega = b)$ .

This result appears to be quite intuitive, as for a high equilibrium effort of  $E$ , the probability of finding nothing in the event 4 is very small, and thus event 4 is highly unlikely to



be realized, conditional on the fact that  $E$  did not acquire any evidence. Therefore, in this case, events 2 and 3 are more likely. Given that  $(\bar{p}_{2,g,x} + \bar{p}_{3,g,x}) > (\bar{p}_{2,b,x} + \bar{p}_{3,b,x})$ , the ‘good’ state is more likely than the ‘bad’ state.

For the low enough  $E$ ’s equilibrium level of effort, the probability of  $E$  not obtaining any evidence is higher and the conditional probability of the event 4 is higher than of the events 2 and 3. In this case, the ‘bad’ state is more likely to prevail than the ‘good’ state.

## B: Example of joint distribution

The first piece of evidence,  $x$ , is drawn from a countable set  $X \in \{-\infty, \dots, -n, \dots, -2, -1, 1, \dots, n, \dots, \infty\}$  with marginal probability  $g_\omega(x) = p + 2\varepsilon(x, \omega)$ . The second piece of evidence,  $y$ , is drawn from a countable set  $Y \in \{-\infty, \infty\}$  with marginal probability  $h_\omega(y) = p + 2\varepsilon(y, \omega)$ ,

where:

$$\varepsilon(x, \omega) = \begin{cases} \varepsilon^x & \text{for } x > 0 \text{ and for } \omega = g \\ -\varepsilon^{|x|} & \text{for } x < 0 \text{ and for } \omega = g \\ -\varepsilon^x & \text{for } x > 0 \text{ and for } \omega = b \\ \varepsilon^{|x|} & \text{for } x < 0 \text{ and for } \omega = b, \end{cases} \quad (3.32)$$

where  $\sum_{x=-\infty}^{\infty} (p_x + 2\varepsilon(x, \omega)) = 1$ . Similar properties apply to  $\varepsilon(y, \omega)$ .

According to our assumptions  $x \in X^g$  (respectively,  $x \in X^b$ ) if  $g_g(x) > g_b(x)$  (respectively,  $g_g(x) < g_b(x)$ ) and in this case  $x$  is referred to as ‘good’ (respectively, ‘bad’). Similar dichotomies apply to  $Y$  and  $Z$ . In our example  $x \in X^g$ , if  $x > 0$  and  $y \in Y^g$  if  $y > 0$ . Otherwise,  $x \in X^b$  and  $y \in Y^g$ .

$z \in Z^g$  if both  $x$  and  $y$  are positive or if  $x > 0$  and  $y < 0$  and  $x > |y|$  or if  $x < 0$  and  $y > 0$  and  $y > |x|$ .

$z \in Z^b$  if both  $x$  and  $y$  are negative or if  $x > 0$  and  $y < 0$  and  $x < |y|$  or if  $x < 0$  and  $y > 0$  and  $y < |x|$ .

The joint pdf is given by  $f_\omega(x, y) =_\omega (x) * h_\omega(y)$ .

The probability that both  $x$  and  $y$  are ‘good’ (event 1) in each state is given by:

$$p_{1,\omega} = \sum_{x=1}^{\infty} \sum_{y=1}^{\infty} f_{\omega}(x, y). \quad (3.33)$$

The probability that both  $x$  and  $y$  are ‘bad’ (*event 4*) in each state is given by:

$$p_{4,\omega} = \sum_{x=-\infty}^{-1} \sum_{y=-\infty}^{-1} f_{\omega}(x, y). \quad (3.34)$$

The probability that evidence are conflicting but overall ‘good’ (*event 2*) in each state is given by:

$$p_{2,\omega} = \sum_{x=1}^{\infty} \sum_{y=-\infty}^{-1} f_{\omega}(x, y) 1_{\{x>|y|\}} + \sum_{x=-\infty}^{-1} \sum_{y=1g}^{\infty} f_{\omega}(x, y) 1_{\{y>|x|\}} + 2 \sum_{x=-\infty}^{-1} \sum_{y=1g}^{\infty} f_{\omega}(x, y) \frac{1_{\{y=|x|\}}}{2}. \quad (3.35)$$

Finally, the probability that evidence is conflicting but overall ‘bad’ (*event 3*) in each state is given by:

$$p_{3,\omega} = \sum_{x=1}^{\infty} \sum_{y=-\infty}^{-1} f_{\omega}(x, y) 1_{\{x<|y|\}} + \sum_{x=-\infty}^{-1} \sum_{y=1g}^{\infty} f_{\omega}(x, y) 1_{\{y<|x|\}} + 2 \sum_{x=-\infty}^{-1} \sum_{y=1g}^{\infty} f_{\omega}(x, y) \frac{1_{\{y=|x|\}}}{2}. \quad (3.36)$$

Consider a following numerical example:

let  $p = (\frac{1}{2})^{x+1}$  and  $\varepsilon = \frac{1}{10}$ , then

$$g_g(x) = \begin{cases} (\frac{1}{2})^{x+1} + 2(\frac{1}{10})^x & \text{for } x > 0 \\ (\frac{1}{2})^{x+1} - 2(\frac{1}{10})^{|x|} & \text{for } x < 0 \end{cases} \quad (3.37)$$

$$g_b(x) = \begin{cases} (\frac{1}{2})^{x+1} - 2(\frac{1}{10})^x & \text{for } x > 0 \\ (\frac{1}{2})^{x+1} + 2(\frac{1}{10})^{|x|} & \text{for } x < 0 \end{cases} \quad (3.38)$$

This distribution generates the joint distribution that is consistent with our assumptions,

i.e:

$$\begin{aligned}
 p_{1,g} &= p_{4,b} = \frac{169}{324} \\
 p_{4,g} &= p_{1,b} = \frac{25}{324} \\
 p_{2,g} &= p_{3,b} = \frac{95}{324} \\
 p_{3,g} &= p_{2,b} = \frac{35}{324}.
 \end{aligned} \tag{3.39}$$

## C: Proof of Propositions 3 and 4

### The level of precision of the *DM*'s decision

1.  $s = 0$

From *Lemma 3*, for any convex and continuous cost function the following inequality is satisfied:

$$1 < \frac{e_F^A}{e_F^I} \leq 2. \tag{3.40}$$

Thus, the total costs are higher under the *adversarial* system for a given level of  $e_E$ . For  $s = 0$ , the difference in the level of precision of the final decision, or in the expected loss from an erroneous decision between the *adversarial* and the *inquisitorial* systems, can be written as:

$$-\frac{1}{2}L(1 + 2\alpha(e_F^I - 2e_F^A))(p_{2,g} - p_{2,b}). \tag{3.41}$$

This difference is positive for  $e_F^A > \frac{1}{4\alpha} + \frac{e_F^I}{2}$ . As  $e_F^{*I}$  is always higher than  $e_F^{*A}$ , the smallest value it can take is equal to  $e_F^{*A}$ . If condition (3.41) is not satisfied for the smallest value of  $e_F^{*I}$ , then it will not be satisfied for any other values of  $e_F^{*I}$ . Thus, the above inequality can be rewritten as the following:

$$e_F^A > \frac{1}{2\alpha}. \tag{3.42}$$

As the smallest value that  $\alpha$  can take is  $\frac{1}{3}$ , this inequality is satisfied only for the values of  $e_F^{*A}$  greater than 1. Therefore, the *adversarial* system generates lower expected loss for  $e_F^{*A} > 1$ . Note that  $e_F^{*A}$  represents the probability in our set-up, and therefore cannot exceed

one by definition. This concludes the proof that for  $s = 0$ , the *adversarial* system generates strictly lower *DM*'s payoff than the *inquisitorial* system.

2. Using an Envelope Theorem and differentiating (3.6) with respect to  $s$  establishes that the level of precision of final decision under the *adversarial* system is increasing in  $s$ , and for  $s = 1$  is higher under the *adversarial* system than under the *inquisitorial*.

### Comparison of *DM*'s payoff

The proof is divided into two parts. We first show that at  $s \geq \hat{s}$  and  $\Pi$  sufficiently low the *adversarial* system is superior to the *inquisitorial* system. We then show that for values of  $\Pi$  such that  $e_F^I = 1$  the *adversarial* system is superior to the *inquisitorial* system, followed by a demonstration that there exists intermediate values of  $\Pi$ , for which this result is reversed.

Consider the difference in *DM*'s payoff at  $e_F^A = e_F^{*A}$ ,  $e_F^I = e_F^{*I}$  and  $e_E = e_E^{*I}$  for both systems. After several algebraic manipulations the difference in the *DM*'s payoff under two systems is given by:

$$\frac{1}{2}\alpha L e_E^{*I} [3(p_{1,g} - p_{1,b})s - (p_{2,g} - p_{2,b})(1 - \alpha e_F^{*I} (\frac{2+s}{2} - 2))] - (\alpha e_F^{*I})^2 \frac{1}{2} [(\frac{2+s}{4} - 1)]. \quad (3.43)$$

Note that for the *adversarial* system  $e_E$  is not the optimal level of effort. Therefore, if the payoff under the *adversarial* system is higher than under the *inquisitorial* system at the suboptimal level of  $E$ 's effort, this result will be reinforced at the optimal  $E$ 's level for the *adversarial* system.

1. At  $s \geq \hat{s}$  and  $\Pi = 0$ , this difference is strictly positive as  $\alpha e_F^{*I} = 0$  while  $e_E^{*I} > 0$ . At the same time, the difference in the *DM*'s expected loss and the difference in associated costs increase in  $\Pi$  as both  $e_E^{*I}$  and  $e_F^{*I}$  increase in  $\Pi$ . Given the continuity of the *DM*'s loss function, it must be the case that at least for the small enough values of  $\Pi$  the *adversarial* system generates higher *DM*'s payoff.

Consider now  $\Pi$  large enough that both  $e_F^A = e_F^{*A} = 1$ . Then the difference in the expected payoff of the *DM* under the two systems is given by:

$$\frac{1}{2}\alpha L e_E^{*I} [3(p_{1,g} - p_{1,b})s - (p_{2,g} - p_{2,b})(1 - \alpha((2+s)^2 - 2))], \quad (3.44)$$

which is always greater than zero for  $s \geq \hat{s}$ . Therefore, for  $\Pi$  large enough the *adversarial* system dominates the *inquisitorial* system.

2. Consider now the difference in the *DM*'s payoff at  $e_F^A = e_F^{*A}$ ,  $e_F^I = e_F^{*I}$  and  $e_E = e_E^{*A}$  for both systems. For a given  $e_E$ ,  $e_F^{*A}$  increases faster with  $\Pi$  than  $e_F^{*I}$ , and therefore  $e_F^{*A}$  reaches the value of one for lower values of  $\Pi$  than  $e_F^{*I}$ .

From *Lemma 2*, it follows that for  $e_F^{*A} = 1$ ,  $e_E^{*A}$  does not depend on  $e_F^{*A}$  and therefore on  $\Pi$ .

From what follows, that for  $\Pi$  such that  $e_F^{*A} = 1$  and  $e_F^{*I} < 1$  the difference between the *DM*'s payoff under the *adversarial* and the *inquisitorial* systems can be expressed as follows:

$$\frac{1}{2}\alpha L e_E^{*A} [3(p_{1,g} - p_{1,b})s - (p_{2,g} - p_{2,b})(1 - \alpha((2+s)^2 - 2\alpha e_F^{*I}))] - \frac{1}{2}[1 - (\alpha e_F^{*I})^2]. \quad (3.45)$$

Note that the difference in expected loss is increasing and linear in  $e_F^{*I}$ , while the difference in costs is increasing and convex in  $e_F^{*I}$ . Given that for  $e_F^{*I} = 0$  and  $s \geq \hat{s}$  the difference in the expected cost is greater than the difference in costs, then there exists  $\Pi$  such that  $e_F^{*A} = 1$  and  $e_F^{*I} < 1$  where the two curves cross.

## D: Proof of Proposition 4 on endogenous allocation of the burden of proof

In *Appendix A* we have considered the cases of the full or partial submission of the evidence, maintaining the exogenous decision rule in the favor of  $F$  in the case when no evidence are submitted. We now check whether or not such a decision can arise as an equilibrium outcome.

Using the same reasoning as described above, the corresponding conditional probability of each state is given by:

1. *Inquisitorial system*:

$$P(z = \emptyset | \omega = g) = \quad (3.46)$$

$$(1 - 3\alpha e_E^*)[1 - p_{1,g} 2\alpha e_F^*(1 + s) - p_{2,g}\alpha e_F^*(1 + s) - p_{3,g}\alpha e_F^*(2 + s)]$$

$$P(z = \emptyset | \omega = b) = \quad (3.47)$$

$$(1 - 3\alpha e_E^*)[1 - p_{1,b} 2\alpha e_F^*(1 + s) - p_{2,b}\alpha e_F^*(1 + s) - p_{3,b}\alpha e_F^*(2 + s)]$$

$$\begin{aligned} & p_{1,g} 2\alpha e_F^*(1 + s) + p_{2,g}\alpha e_F^*(1 + s) + p_{3,g}\alpha e_F^*(2 + s) \quad (3.48) \\ < & p_{1,b} 2\alpha e_F^*(1 + s) + p_{2,b}\alpha e_F^*(1 + s) + p_{3,b}\alpha e_F^*(2 + s) \end{aligned}$$

$$\alpha e_F^*(1 + s) + p_{1,g} \alpha e_F^*(1 + s) + p_{3,g}\alpha e_F^* > \alpha e_F^*(1 + s) + p_{1,b} \alpha e_F^*(1 + s) + p_{3,b}\alpha e_F^*. \quad (3.49)$$

2. *Adversarial system:*

$$P(z = \emptyset | \omega = g) = \quad (3.50)$$

$$\begin{aligned} & 1 - p_{1,g}2\alpha(1 + s)e_F^* - p_{4,g}3\alpha(1 + s)e_E^* - p_{2,g}\alpha(2 + s)(e_E^* + e_F^* - \alpha(2 + s)e_F^*e_E^*) \\ & - p_{3,g}\alpha(e_F(1 + s) + e_E^*(2 + s) - \alpha e_F^*e_E^*(1 + s)(2 + s)) \end{aligned}$$

$$P(z = \emptyset | \omega = b) = \quad (3.51)$$

$$\begin{aligned} & 1 - p_{1,b}2\alpha(1 + s)e_F^* - p_{4,b}3\alpha(1 + s)e_E^* - p_{2,b}\alpha(2 + s)(e_E^* + e_F^* - \alpha(2 + s)e_F^*e_E^*) \\ & - p_{3,b}\alpha(e_F^*(1 + s) + e_E^*(2 + s) - \alpha e_F^*e_E^*(1 + s)(2 + s)) \end{aligned}$$

$$P(z = \emptyset | \omega = g) > P(z = \emptyset | \omega = b) \text{ if}$$

$$(p_{1,g} - p_{4,g})(1 + s)(3e_E^* - 2e_F^*) > (p_{2,g} - p_{3,g})((e_F^* - \alpha e_F^*e_E^*(2 + s))). \quad (3.52)$$

Inequality (3.52) is satisfied if  $e_E^* > e_F^*$ , or more precisely if

$$e_E^* > \frac{p_{2,g} - p_{3,g}}{3(p_{1,g} - p_{4,g})(1 + s)} + \frac{1}{3}e_F^*,$$

which occurs for  $\Pi$  sufficiently low and/or  $L$  sufficiently large.

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