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"The dynamics of capital accumulation in the US: Simulations after Piketty"

by

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Abstract:

We calibrate a sequence of four nested models to study the dynamics of wealth accumulation. Individuals maximize a utility function whose arguments are consumption and investment. They desire to accumulate wealth for its own sake – this is not a life-cycle model. A competitive firm produces a single good from labor and capital; the rate of return to capital and the wage rate are market-clearing. The second model introduces political lobbying by the wealthy, whose purpose is to reduce the tax rate on capital income. The third model introduces differential rates of return to capitals of different sizes. The fourth model introduces inheritance and intergenerational mobility.

Key Words: Piketty, dynamics of wealth accumulation, intergenerational mobility, Kantian equilibrium

JEL codes: D31, D58, E37

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1. Introduction

We simulate the dynamics of wealth accumulation in a model calibrated to the US economy, with initial conditions taken from 2012 data. Although the model is inspired by Piketty (2014), we have added quasi-traditional micro-foundations: at each date there is an equilibrium where the return to capital and the real wage balance the markets for capital and labor. The equilibrium is only 'quasi'-traditional for the following reason. Agents do not maximize a discounted infinite sum of consumption over time, or even of consumption over time and a bequest, but rather a single-period utility function of consumption and savings, repeated in each period. That is, agents desire to consume and to *accumulate wealth for its own sake.* This assumption is inspired by Marx, who argued that, while pre-capitalist economies produced commodities, exchanging them for money in order to purchase a different commodity bundle ($C - M - C'$, à la Marx) , capitalist property relations produce agents whose aim is to transform money into more money, via the intermediary of commodities (labor power and capital), thus $M - C - M'$. Piketty (2014) argues – effectively, we believe – that the life cycle model, which is the $C - M - C'$ formulation – is not supported by the data. Moreover, our belief is that the very wealthy do not accumulate in order to leave bequests. They do so because 'money is life's report card,' as the caption of a *New Yorker* cartoon said. Warren Buffet (Andrew Carnegie) does not (did not) accumulate wealth in order to enrich his children; nor were his motives primarily philanthropic, although Buffet will (Carnegie did) use his winnings in the capitalist game for philanthropic ends. Accumulation for its own sake is the motivation for most members of the wealthy class, for success in the game of life is judged by one's wealth, often signaled by one's consumption.

We begin by postulating a distribution of wealth given by the 2012 US wealth distribution, from Piketty and Zucman (2013). We postulate a lognormal distribution of skills. We assume that the distribution of wealth is monotone increasing in an individual's skill. A firm, using a CES production function whose inputs are efficiency units of labor and capital, maximizes profits. Consumer-workers offer inelastically their entire endowment of skilled labor to the firm; they demand the consumption good and supply capital to the firm in order to maximize preferences described above. The interest rate and real wage equilibrate the markets for labor and capital. There are proportional taxes on capital and labor income, the revenues from which are returned as a demogrant to each worker. In the *basic model* these tax rates are the same, and an equilibrium is re-established at each period. The main fundamental changing over time is the distribution of capital/wealth; there is also an exogenously growing distribution of skills, and of the 'expected' standard of living.

We introduce sequentially three more models. In Model 2, we append to the basic model lobbying by the wealthy, who contribute to a fund that is spent to convince legislators to reduce the tax rate on capital income. We describe below how the wealthy overcome the free rider problem to generate a significant lobbying fund. Consequently, the tax rate on capital income now declines, while the tax rate on labor income is exogenous and fixed.

We next amend Model 2 by introducing differential rates of return to capital: those with large amounts of capital can attain a higher rate of return than those with small amounts. This is, again, an important fact discussed by Piketty (2014). In Model 3, this amendment accounts for a more dramatic concentration of capital than occurred in the first two models.

Finally, in Model 4, we add to Model 3 intergenerational mobility. We take a generation to last for 50 (or 25) years, and model this by assuming each individual has a probability of 2% (or 4%) of dying each year, upon which his capital passes down, without taxation, to his single offspring. The offspring's skill level – and hence her labor earnings – are not inherited, but are taken to be determined by the income intergenerational mobility matrix of Chetty et al (2014). The population dynamics are made explicit below. The dynamics of wealth again change.

Table 1 summarizes the main characteristics of the four nested models.

Model number	**Lobbying**	**Differential rates of return to capital**	**Intergenerational Mobility**
1	no	no	no
2	yes	no	no
3	yes	yes	no
4	yes	yes	yes

Table 1: Four nested models

We present the simplest model (Model 1) in section 2, with sections 3 to 5 devoted to Models 2 to 4. In each section, we first develop analytically the model, calibrate it and then provide and comment on results from numerical simulations. Section 6 concludes.

2. Model 1: Basic

We start by describing analytically the basic model developed, before calibrating it and presenting our numerical results.

A. Analytical description of the model

i. Endowments

There is a distribution of skill, s, denoted by a distribution function F on $[0,\infty)$ where $\overline{s} = \int sdF(s)$ denotes the average skill. There is a distribution of wealth at date 0 given by a function $S_0(s)$. It is postulated that wealth/capital is monotone increasing in s. We assume that skills increase (exogenously) by a factor of $(1+g)$ per annum.

ii. Production

There is a single good produced by a single firm, using a CES technology given by

$$y(K,L) = A(aK^{(\delta-1)/\delta} + (1-a)L^{(\delta-1)/\delta})^{\delta/(\delta-1)} , \tag{2.1}$$

where y , K and L are per capita income, capital, and labor in efficiency units. The only technical change in the model is induced by the exogenous increase in labor skills. The firm faces an interest rate r and a real wage per efficiency unit of labor w, and maximizes profits. Consequently, the demands for labor and capital by the firm at date t are determined by the first-order conditions from maximizing profits defined as $y(K_t,L_t) - w_tL_t - (r_t + d)K_t$:

$$r_t + d = ay_t^{1/\delta}K_t^{-1/\delta}A^{(\delta-1)/\delta} \text{ and } w_t = (1-a)y_t^{1/\delta}L_t^{-1/\delta}A^{(\delta-1)/\delta} , \tag{2.2}$$

where d is the annual rate of capital depreciation. Note that the firm replaces depreciated capital from income. Economic profits are zero.

iii. Preferences

Preferences are non-traditional. We assume there is a socially expected standard of living for people in this society, produced by a consumption level c_0 at date 0. This expected consumption level increases by a factor of $(1+g)$ per annum. We do not call this subsistence consumption – we set it at $100,000 in the simulations. It is the consumption level to which ordinary people aspire, which is generated by advertising and the media. (In the US, this level would define a successful middle-class life.) A sufficiently wealthy individual at date t chooses her consumption c and investment I to maximize a Stone-Geary utility function as follows:

$$\begin{aligned} &\max(c_t - c_0(1+g)^{t-1})^{\alpha} I_t^{1-\alpha} \\ &\text{s.t.}\quad c_t + I_t \le y_t(s) \\ &\qquad\; c_t \ge c_0(1+g)^{t-1} \end{aligned} \tag{2.3}$$

where $y_t(s)$ is the income of individual s at date t. If there is no solution to program (2.3), because income is insufficient to purchase the consumption level $c_0(1+g)^{t-1}$, then the individual consumes out of wealth. To be precise:

$$c_t(s) = \begin{cases} y_t(s) + S_{t-1}(s), \text{ if } y_t(s) + S_{t-1}(s) \le (1+g)^{t-1}c_0 \text{ (case 1)} \\ (1+g)^{t-1}c_0, \text{ if } y_t(s) \le (1+g)^{t-1}c_0 \le y_t(s) + S_{t-1}(s) \text{ (case 2)} \\ (1+g)^{t-1}c_0 + \alpha(y_t(s) - (1+g)^{t-1}c_0), \text{ if } y_t(s) > (1+g)^{t-1}c_0 \text{ (case 3)} \end{cases} \tag{2.4}$$

In case 1, the individual consumes his income plus his wealth $S_{t-1}(s)$, and those together do not suffice to generate the socially expected consumption of (2.4). In case 2, when her income does not suffice to allow socially expected consumption but her total asset position does, she consumes exactly socially acceptable consumption. In case 3, where her income alone suffices to allow socially acceptable consumption, she solves program (2.3) with α the marginal propensity to consume out of income. Thus, investment is given by:

$$I_t(s) = \begin{cases} -S_{t-1}(s) < 0, \text{ if case 1} \\ y_t(s) - c_0(1+g)^{t-1} < 0, \text{ if case 2} \\ y_t(s) - c_t(s) > 0, \text{ if case 3} \end{cases} . \tag{2.5}$$

The dynamics of wealth are given by:

$$S_t(s) = S_{t-1}(s) + I_t(s) \ . \tag{2.6}$$

Note that, because the firm replaces depreciated capital from its income, the investor can cash out his entire capital stock at the end of the period, and so the depreciation does not appear in equation (2.6).

(iv) Equilibrium

The market clearing equations are:

$$K_t = \int_0^\infty S_{t-1}(s)dF(s), \quad L_t = \int_0^\infty (1+g)^{t-1} s dF(s) = (1+g)^{t-1}\bar{s} \ . \tag{2.7}$$

(v) Income

We assume an exogenously given income tax rate τ , the revenues from which are returned to citizens as a demogrant. Thus income for an agent of type *s* in year *t* is given by:

$$y_t(s) = (1-\tau)(w_t s(1+g)^{t-1} + r_t S_{t-1}(s)) + \tau(w_t \bar{s}(1+g)^{t-1} + r_t K_t). \tag{2.8}$$

(vi) Summary of equations and solving of the model

The equations summarizing the model are :

1. FOC, profit maximization w.r.t. *K*:

$$K_t = \left(\frac{\left(\frac{r_t + d}{aA}\right)^{\delta-1} - a}{1-a} \right)^{\delta/(1-\delta)} (1+g)^{t-1}\bar{s} \tag{2.9}$$

2. FOC, profit maximization w.r.t. *L*:

$$w_t = (1-a)A\left(a\left(\frac{K_t}{(1+g)^{t-1}\bar{s}}\right)^{(\delta-1)/\delta} + 1 - a \right)^{1/(\delta-1)} \tag{2.10}$$

3. Definition of consumption: see equation (2.4)
4. Definition of investment: see equation (2.5)
5. Intergenerational transmission of wealth: see equation (2.6)
6. Market clearing of capital and labor markets: see equations (2.7)

7. Definition of income: see equation (2.8)

The full model is solved as follows, beginning with the wealth function $S_{t-1}(s)$ at the beginning of date t. K_t and L_t are determined by (2.7). Equations (2.9) and (2.10) determine r_t and w_t . $y_t(s)$ is determined by (2.8). Consumption and investment are determined by (2.4) and (2.5). $S_t(\cdot)$ is determined by (2.6). The next iteration begins.

B. Calibration of the model

We describe our calibration of preferences, the production function, and the initial distribution of capital.

(i) Preferences

One period is deemed to be one calendar year. We choose $c_0 = 100$ (thousands of dollars), which grows at a factor of $(1+g)$ each year, with g=0.01. We choose $\alpha = 0.6$, based on the fact that the propensity to consume for the wealthy is about 0.6 out of income.

(ii) Production function

We assume a CES production function given by (2.1). Piketty (2014) argues that a choice of the elasticity of substitution $\delta \in (1.3, 1.6)$ is implied by the historical distribution of capital's share in income. We choose $\delta = 1.5$.[1]

The capital income ratio K/y is 4.5 in the U.S. Depreciation is about 10% of GNP, which suggests a rate of depreciation $d = 0.02$.

Using (2.2), calculate that :

$$\frac{(r+d)K}{y} = a\left(\frac{y}{AK}\right)^{(1/\delta)-1}, \tag{2.11}$$

and hence

$$a = \left(\frac{rK}{y} + d\frac{K}{y}\right)\left(\frac{y}{AK}\right)^{1-1/\delta}. \tag{2.12}$$

[1] Karabarbounis and Neiman (2014) estimate an elasticity of substitution around 1.25 from cross-country variation in trends in rental rates and labor shares.

Taking $\frac{K}{y} = 4.5$ and capital's share in income $\frac{rK}{y} = 0.28$ (both from Piketty (2014)), (2.12) reduces to

$$a = (0.28 + 0.09)(4.5)^{-.333} A^{-.333} \; . \tag{2.13}$$

From the production function, we have:

$$1 = A(a(K/\text{y})^{.333} + (1-a)(L/\text{y})^{.333})^3 \; . \tag{2.14}$$

The distribution $F(s)$ of skills is taken to be lognormal. The unit of skill has no meaning: we take the median skill level to be 0.85 and the mean $\bar{s}$ to be 1. We then have that L=1 at period 1. Equation (2.14) then becomes

$$1 = A(a(K/\text{y})^{.333} + (1-a)(L/\text{y})^{.333})^3 = A(a(4.5)^{.333} + (1-a)(108.3)^{-.333})^3 \;, \tag{2.15}$$

where we calculate $L/\text{y} = (108.3)^{-1}$, using the facts that total income is 16.8×10^9 in thousands of dollars and the size of the labor force is 155×10^6. (Thus, income per worker is $108,300.) Solving (2.13) and (2.14) simultaneously for (a, A), we have $(a, A) = (0.070, 33.41)$.

(iii) The initial distribution of capital, $S_0(s)$

The initial distribution of wealth is taken from Saez and Zucman (2014), Appendix Table B1: Top wealth shares.[2] We assume that the wealth distribution is linear by parts over *s*, with 7 different brackets. The first bracket corresponds to no capital at all for the bottom half of the skill/wealth distributions. The remaining 6 brackets reproduce the top wealth shares reported by Saez and Zucman (2014) for 2012 (the most recent year for which data is available): 77.2% for the top 10%, 64.6% for the top 5%, 41.8% for the top 1%, 34.5% for the top 0.5%, 22% for the top 0.1% and 11.2% for the top 0.01%. Total capital at the beginning of period 1 is given by (ii) above (4.5 times $108,300).

The taxation rate τ is set at 0.35 throughout the paper.

[2] Available at http://gabriel-zucman.eu/files/uswealth/AppendixTables(Distributions).xlsx

C. Numerical results

We report in Table 1 the main results obtained from solving the model numerically from t=1 to t=25 (first column). The second column shows that the equilibrium interest rate r is fairly stable and decreases from 6.2% to 5.7% over 25 years. The equilibrium wage rate w increases from $68,240 to $71,014 over the same period. GDP per capita increases from $108,740 to $146,580. It is clear from Table 1 that both the growth rate of GDP and of average wealth is much lower than the equilibrium interest rate. So, in Piketty (2014)'s parlance, we obtain that $g<r$. At the same time, we obtain that the capital/output ratio increases from 4.5 to 5, but that the share of capital income (including depreciation) in total income remains fairly stable, increasing from 37.2% to 38.5%.

Insert Table 1 around here

The last 5 columns of Table 1 document the evolution of the wealth distribution over 25 years. The bottom half of the population does not accumulate any capital over that period. The patrimonial middle class (defined as agents in-between the fifth and ninth deciles in the skill/income/wealth distribution) sees its share of capital decrease from 22% to 17%, while the share of the 90th to 99th percentiles increases from 35.8% to 40%. The share of the top percentile then increases very slightly (from 42.1% to 42.8%), but the top 0.1% sees its share remaining fairly stable around 22%.

Figure 1 complements this information by comparing the distribution of capital as a function of skill in periods 1 (dashed line) and 25 (thick line). We first see that people immediately above the median skill ($s=0.85$) end up consuming all their capital, so that the share of people without any capital increases from 50% at time 1 (our calibration assumption) to 69.5% of the population at $t=25$ (corresponding to $s=1.13$). Individuals situated between the 69.5th and 72.7th percentiles have less capital (in absolute amount) at time 25 than at time 1, while the top 27% of the distribution sees its capital grow in absolute amounts. The wealth distribution remains linear by part following our calibration assumptions. The three vertical dashed lines correspond to the 50th, 90th and 99th percentiles of the skill distribution, respectively.

Insert Figure 1 around here

Figure 2 shows the density of the wealth distribution at time *t=25* as a function of skills *s* –i.e., it interacts the function *S(s, 25)* with the distribution of skills *F(s)*. The vertical dashed lines correspond to the 50th, 90th and 99th percentiles of the skill distribution. We see that the density is inversely U-shaped for the three groups spanned by these dashed lines: the 50-90, 90-99 and 99+. The density reaches its overall maximum for agents situated slightly above 90% in the distribution.

Insert Figure 2 around here

We postulate a distribution of skills that is lognormal. However, we know that the *income* distribution is well-approximated by a distribution that is lognormal for an interval of incomes up to a quite large income y^*, and a Pareto distribution on the income interval $y \geq y^*$. We wish to see how well the distribution of income generated by our simulations tracks this kind of distribution.

Jantzen and Volpert (2012) characterize the Lorenz curve of a hybrid income distribution, which exhibits left-sided self-similarity (where the degree of inequality repeats itself as we restrict ourselves to poorer and poorer fractions of the population) up to a threshold income level, and right-sided self-similarity (i.e., a Pareto distribution) above that threshold. Interestingly, the equation of the Lorenz curve is of the form:

$$L(x) = x^a(1-(1-x)^b) \ , \tag{2.16}$$

where $a > -1$ and $b > 0$; surprisingly, the Lorenz function has only two parameters. These authors fit the empirical income distributions of Atkinson, Piketty and Saez (2011), choosing parameters (a,b) by least-squares, and achieve remarkably good fits. We fit our simulated income distributions using least-squares and (2.16).

The result is depicted in Figure 3, where the equilibrium income distribution at period *t=25* is shown using dots while the fitted distribution (corresponding to a=-0.28 and b=0.37 in (2.16)) is represented by a continuous blue curve. We obtain what seems to be an excellent fit, with a R square of 0.932, although our numerical computations slightly underestimate the total income shares at both the bottom and the top of the distribution, compared to the fitted distribution. The Gini coefficient corresponding to the fitted Lorenz curve is 0.386.

Insert Figure 3 around here

To better judge how well our equilibrium income distribution fits this Lorenz curve, we compare our distribution with the Lorenz curve obtained from Jantzen and Volpert (2012) without the Pareto tail. They show that this curve is of the form:

$$L(x) = cx^p \,, \quad (2.17)$$

with two parameters c and p, which we choose by least-squares. Although we have two degrees of freedom when fitting our distribution to this family of Lorenz curves, as previously, we achieve a worse fit, with R2=0.9084. Figure 3 reports (in red) the best-fitted Lorenz curve without a Pareto tail. It is clear that, even though we assume a simple lognormal distribution of skills, our equilibrium income distribution after 25 periods is much closer to the hybrid distribution from Jantzen and Volpert (2012) (which they claim represents especially well the US income distribution) than to the one generated from their model without a Pareto tail.

We have also checked the robustness of our numerical results to the introduction of a Pareto tail in the skill distribution. We detail in Appendix how we have proceeded for the calibration of this hybrid skill distribution. The numerical results we obtain are very similar to the ones reported above – see Table xx in the Appendix, and compare it to Table 1 above. The figures corresponding to Figures 2 and 3 are also very similar and are omitted for the sake of brevity. Our results are then robust to the introduction of a Pareto tail in the skill distribution. We have also fitted the new equilibrium income distribution to the Jantzen and Volpert (2012) family of Lorenz curve, and we obtain a slightly better fit, as measure by the R^2 which increases from 0.932 to 0.955. The Gini coefficient associated with this fitted Lorenz curve increases slightly, from 0.386 to 0.391.

To summarize the results obtained with Model 1, we observe that the growth rate of income and of average wealth are significantly smaller than the interest rate (*g*<*r* using Piketty's terminology). We observe a capital deepening of the economy with time, with an increase in the capital/output ratio, but a stable share of capital in total income. This deepening goes hand in hand with a more unequal distribution of wealth, with the proportion of agents with no wealth increasing from 50% to 70% after 25 years. More generally, the 90th to 99th percentile of the wealth distribution gain shares at the expense of the bottom 90 percent, with the shares of the top 1% and the top 0.1% remaining roughly constant.

We now introduce lobbying by the wealthy into the picture, and assess its impact on the equilibrium dynamics of our model.

3. Model 2: Lobbying

A. Analytical model

Formally, the purpose of lobbying by the wealthy is to reduce the tax rate on capital income. The model will henceforth contain two tax rates, τ on labor income and τ^c on capital income. If the per capita (in the whole population) expenditure on lobbying at time *t* is $\bar{\sigma}_t$, then the tax rate τ_t^c is defined by:

$$\tau - \tau_t^c = k \frac{(\bar{\sigma}_t / (1+g)^{t-1})^\beta}{\beta} \qquad (3.1)$$

where *k* and β are parameters to be specified below. The effective cost of achieving a given outcome (tax rate τ_c) grows at rate *g*.

However, our conception of 'lobbying' is in reality much richer: it includes all those expenditures by owners of capital to protect their wealth from confiscation, which may occur not only through taxation but through other means. These expenditures include those needed to create an ideology among voters which is pro-capitalist and pro-laissez-faire, what has been called in recent years the neoliberal worldview. Thus, it includes expenditures on conservative think tanks, and on the transmission of this ideology through the media. It includes what Samuel Bowles has called 'guard labor,' labor needed to protect property from theft. Bowles and Jayadev (2014) argue that the amount of guard labor in the advanced capitalist countries is highly correlated with their Gini coefficients; they estimate that in the US in 2000, there where 200 workers employed in guard labor per 10,000 workers, the highest ratio of the advanced countries. (They include in guard labor, besides police and security guards, the military, prison officials, and weapons producers.) 'Lobbying' must also include those expenditures to prevent confiscation of profits by workers through labor organization; thus, the expenditures of the capitalist class in preventing the formation of unions, or destroying existing unions, must be included as well. Union-busting has been important in the United States during the last forty years, when the density of unionized workers has fallen from around 30% in 1960 to less than 10% today. (To see this is not simply an effect of structural changes in production, note that union density in Canada is roughly still 30% today as it was in 1960.) We would include in 'lobbying expenditures' corporate legal fees that are used to protect capital from confiscation by governments and workers, but not intra-capitalist transfers within a country, which do not change (to a first approximation) the ownership of capital by classes.

How is the budget for 'lobbying' raised in our model? The reduction in the tax rate on capital income is a public good for the wealthy, and so there is a free-rider problem in generating contributions to that budget under the usual assumptions about behavior. We solve the problem by assuming that those who contribute to lobbying optimize in the Kantian manner: the lobbying equilibrium is a multiplicative Kantian equilibrium of the game among contributors. (See Roemer (2014) for a discussion of Kantian equilibrium; see Roemer (2006) for a political model where the determination of citizen contributions to politics is modeled as a Kantian equilibrium.) In public-good games, the multiplicative Kantian equilibrium leads to a Pareto efficient solution of the public-good problem .

Let the lobbying expenditure of an agent of type s at date t be $\sigma_t(s)$. The individual's income at any contribution σ is:

$$y_t(\sigma;s) = (1-\tau)w_t s(1+g)^{t-1} + (1-\tau^c(\bar{\sigma}))r_t S_{t-1}(s) + \tau w_t \bar{s}_t (1+g)^{t-1} + \tau^c(\bar{\sigma})r_t K_t - \sigma \quad (3.2)$$

where $\bar{\sigma} = \int \sigma_t(s)dF(s)$. The equations (3.2) define payoff functions of a game for the citizens; a multiplicative Kantian equilibrium is a function $\sigma(\cdot)$ such that no player would advocate that all players change their contributions by any factor (the factor being the same for the entire society). The FOC for the solution of this problem is a schedule of contributions $\sigma(\cdot)$ satisfying :

$$\left.\frac{dy(\rho\sigma(s);s)}{d\rho}\right|_{\rho=1} = \left.\frac{d}{d\rho}\right|_{\rho=1} \Big((1-\tau^c(\rho\bar{\sigma}))r_t S_{t-1}(s) + \tau^c(\rho\bar{\sigma})r_t K_t - \rho\sigma(s)\Big) = 0 \ . \quad (3.3)$$

Expanding this expression gives:

$$\sigma(s) = \max[-(\tau^{c\prime}(\bar{\sigma}))r_t\,\bar{\sigma}(S_{t-1}(s) - K_t),0] \ , \quad (3.4)$$

where $\tau^{c\prime}$ is the derivative of the function $\tau^c(\bar{\sigma})$ implicitly defined by equation (3.1). It follows that positive contributions to lobbying are made by precisely those agents for whom $S_{t-1}(s) \geq K_t$ -- that is, those whose capital is at least equal to the average capital endowment in the society. Thus, those who contribute to the lobbying effort are exactly $\{s \geq s_t^*\}$ where s_t^* is defined by:

$$S_{t-1}(s_t^*) = K_t \ . \quad (3.5)$$

We have by definition:

$$\bar{\sigma}_t = \int_{s_t^*}^{\infty} \sigma_t(s)dF(s) \ . \quad (3.6)$$

Integrating equation (3.4) and dividing by $\bar{\sigma}$, we have:

$$1 = -\tau^{c\prime}(\bar{\sigma}_t)r_t \int_{s_t^*}^{\infty} (S_{t-1}(s) - K_t)dF(s) \quad (3.7)$$

from which we solve for $\bar{\sigma}_t$ and $\tau_t^c = \tau^c(\bar{\sigma}_t)$. Finally, we solve for the function $\sigma_t(\cdot)$ using (3.4).

A critic might well say that we should solve for the Kantian equilibrium of lobbying contributions not by having the investor maximize his income, but rather his utility. We have chosen not to do so, because when the investor maximizes income, we have a simple analytic solution for s_t^* (from (3.5)) and this simplifies the simulations considerably. This would not be the case were the investor to maximize her utility.

The equations summarizing the model are the same 7 equations given in subsection 2.A. (vi), where the definition of income is now given by (3.2). We add the following four equations to these seven:

8. Lobbying contribution of a type *s* investor

$$\sigma_t(s) = \begin{cases} 0, \text{ if } s \le s_t^* \\ -\tau^{c\prime}(\bar{\sigma}_t) r_t \bar{\sigma}_t (S_{t-1}(s) - K_t), \text{ if } s > s_t^* \end{cases} \tag{3.8}$$

9. Determination of per capita lobbying expenditures

$$(\bar{\sigma}_t)^{1-\beta} = k(1+g)^{(1-t)\beta} \left(\int_{s_t^*}^{\infty} r_t \left(S_{t-1}(s) - K_t \right) dF(s) \right) \tag{3.9}$$

10. Effect of lobbying on taxation of capital income

$$\tau^c(\bar{\sigma}) = \tau - k \frac{\left(\frac{\bar{\sigma}}{(1+g)^{t-1}} \right)^{\beta}}{\beta} \tag{3.10}$$

12. Definition of lowest type that contributes to lobbying: see equation (3.5)

As for solving the model, we proceed as previously to obtain K_t and L_t, and then r_t and w_t. $\bar{\sigma}_t$ is determined by equation (3.9), and $\sigma_t(s)$ is determined by equations (3.8) and (3.5). τ_t^c is determined by (3.10). $y_t(s)$ is determined by (3.2). Consumption, investment and saving at the end of period t are then obtained as previously, and we start the next iteration.

B. Calibration

The tax rate on labor income is maintained at $\tau = 0.35$, as previously. To calibrate (3.11), we choose $(k, \beta) = (0.06, 0.40)$. We do not have a justification.

C. Numerical results

The evolution of our toy economy from *t=1* to *t=25* is very similar in Models 1 and 2, with the same trends in all variables. We then assess the impact of lobbying on the equilibrium by comparing the last row of Tables 1 and 2. The lobbying activity results in more than halving of the capital income tax rate (from 35% in Model 1 to 15%). This results in a slightly larger GDP after 25 periods ($148,640 per capita, compared to $146,580 without lobbying) and a slightly higher capital output ratio (5.08 against 4.97 at *t=25*). The equilibrium interest rate is slightly lower (5.62% against 5.74%) while the equilibrium wage rate is slightly higher ($71,680 compared to $71,015). The share of capital (including depreciation) in total income increases very slightly to 38.8%.

Insert Table 2 around here

At equilibrium, the top 17% of the income/wealth distribution contribute to lobbying (a proportion slightly decreasing with time) and their per capita (in the whole population) contribution increases from $1,489 at *t=1* to 2 604$ at *t=25*, representing 1.75% of GDP. This increase is faster than the exogenous growth rate g = 1%, resulting in an equilibrium capital income tax decreasing from 17.4% at t =1 to 15% after 25 periods.

In terms of wealth distribution, the opportunity to lobby benefits the top 10% (and especially the top 1%) at the expense of the rest of the distribution: the patrimonial middle class (50th to 90th percentile) sees its wealth share decrease to 15.6% at t=25 (compared to 16.9% without lobbying). The 90th to 99th percentile see their share increase by 0.4% thanks to lobbying (from 40.2% to 40.6%), the top 1% by 1% (from 42.8% to 43.8%) and the top 0.1% by 0.6% (from 21.8% to 22.4%). For the latter group, we now obtain that their wealth share increases with time, rather than decreasing without lobbying.[3]

The fitted Lorenz curve still looks quite close to the obtained distribution, with parameters a=-0.25 and b=0.33), although our numerical computations still slightly

[3] We obtain the same trends for all variables with the hybrid distribution of skills described in the Appendix, except that the top 0.1% wealth share remains decreasing with time.

underestimate the total income shares at both the bottom and the top of the distribution. The fit is actually better than in Model 1, with an R square increasing to 0.95. The fitted income distribution of Model 1 Lorenz dominates the one of Model 2, and the Gini coefficient increases from 0.386 to 0.441.[4]

The figures depicting wealth as a function of skill, the density of wealth distribution, and the Lorenz curves are sufficiently close to those presented in the previous section that we delay until the end of the paper the comparison of the curves for the four models.

We now summarize the impact of introducing lobbying. Lobbying results in a sizeable decrease of the capital income tax rate which generates a further capital deepening of the economy, measured by the capital output ratio. But the main impact of lobbying is distributional. The wealth distribution becomes even more unequal, with the shares of the top 10%, top 1% and top 0.1% increasing at the expense of the patrimonial middle class. Income inequality increases too, with the new income distribution being Lorenz dominated by the one generated by Model 1, increasing the Gini coefficient.

What is the magnitude of lobbying in the model? At $t = 25$, lobbying expenditures are 1.7% of GDP. In 2012, $6.5 billion was spent on elections, $3.3 billion on congressional lobbying, and about $1 billion by think tanks, most of which are conservative. This adds up to about 0.07% of GDP. Nevertheless, given our more inclusive conception of what the protection of capital requires, we do not believe that our figure of 1.7% is an overestimate. It may well be an underestimate considering what is at stake.

4. Model 3: Differential rates of return on capital

A. Analytical model and calibration

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[4] The fit is even better with the hybrid distribution and increases from 0.955 in Model 1 to 0.977 in Model 2. Introducing lobbying also increases the Gini coefficient with the hybrid skill distribution, from 0.391 to 0.448.

Large capitals earn significantly higher rates of return than small ones. Piketty (2014) estimates these rates of return using university endowments in the United States, which must publish their rates of return. He reports that Harvard, Princeton and Yale, with endowments much larger than \$1 billion, earned an average rate of return of 10.2% over the thirty year period 1980-2010. Endowments somewhat higher than \$1 billion but less than those of the top three earned 8.8%, endowments between \$500 million and \$1 billion, 7.8%, endowments between \$100 and \$500 million, 7.1%, and endowments less than \$100 million, 6.2%. All these numbers are much greater than capital per worker, which is $(\frac{K}{Y})Y/L = (4.5)(16.8\times 10^{12})/(155.\times 10^{6}) = \$490{,}000$. The average capital will earn much less than 6.1%, being invested by non-professionals in money-market funds, bank certificates of deposit, and mutual funds.

We model this as follows. We assume that the rate of return available for the capital owned by an agent of type s is given by:

$$\hat{r}_t(s) = \frac{r_t}{4}\left(1+\nu_t F(s)^2 + (5-\nu_t)F(s)^{40}\right) , \tag{4.1}$$

where r_t is the market-clearing interest rate and ν_t is a constant to be determined below. Thus the smallest capitals earn one-fourth the market-clearing rate, and the largest earn 1.5 times the market-clearing rate, about six times what the smallest capitals earn. The choice of functional form in (4.1) is ad hoc. It is chosen to guarantee:

- that the rates of return of $\frac{r_t}{4}$ and $1.5r_t$ are achieved at the endpoints of the interval, and
- to give a value ν_t in the interval $(0,5)$ as the solution of the equation

$$\int_0^\infty \hat{r}_t(s) S_{t-1}(s)\, dF(s) = r_t K_t . \tag{4.2}$$

From (4.2), capital income in the society aggregates to the market-clearing interest rate times the stock of capital. The interval in which ν_t lies according to the second bulleted property is necessary to ensure that $\hat{r}_t(\cdot)$ is an increasing function, which is obviously desirable[5]. As the reader will see below, the formulation of (4.1) gives the desired properties.

The equations summarizing the model are the same 11 equations given at the end of subsection 3.A., where *r(t)* is replaced by $\hat{r}_t(\cdot)$ where necessary in the equations determining income ((3.2)), individual ((3.8)) and per capita ((3.9)) lobbying contributions. We add the following two equations to these eleven:

12. Rate of return for investor of type *s*: see equation (4.1)
13. Consistency of rates of return with competitive rate of return: see equation (4.2)

As for solving the model, we proceed as previously to obtain K_t and L_t, and then r_t and w_t. Equations (4.1) and (4.2) determine ν_t and hence $\hat{r}_t(s)$. We then proceed as explained at the end of section 3.A to solve the other variables.

B. Numerical results

Introducing differentiated rates of return on capital leads to a smaller fraction of wealthy people lobbying (from 16.9% at *t=25* in Model 2 to 13.4% here) but with a much higher per capita lobbying contribution (from \$2,604 in Model 2 to \$3,491 here, increasing from 1.75% of GDP to 2.36%), leading to a smaller value of the capital income tax rate (which decreases from 15% in Model 2 to 12.5% here). Both GDP and capital at *t=25* decrease compared to Model 2 (GDP goes from \$148,640 per worker to \$147,960 while capital per worker decreases from \$755,810 to \$747,020). The capital/output ratio decreases slightly (from 5.08 to 5.05), resulting in a slightly larger interest rate (increasing from 5.62% to 5.66%) and a slightly smaller wage rate (decreasing from \$71,678 to \$71,461). The share of capital income (including depreciation) in total income barely changes, increasing from 38.77% to 38.68%.

[5] To accomplish both of the bulleted properties for the function $\hat{r}_t$, it is necessary to make the function increase very rapidly for $F(s)$ near one – hence the large exponent, 40.

Insert Table 5 around here

Figure 4 depicts the interest rate as a function of the location of the individual in the wealth distribution at date *t=25*. The horizontal line represents the equilibrium average rate of return of 5.66%. More than 90% of the population faces a rate of return of less than 2% (recall that more than 50% of the population has no capital income anyway), and 98.7% of the population faces a rate of return which is lower than the average. The concentration of wealth among the very top earners explains the skewness of the distribution of rates of return.

Insert Figure 4 around here

This skewness has a very large impact on the equilibrium distribution of wealth. The share of the patrimonial middle class drops consequently, from 15.6% in Model 2 at *t=25* to 9.9%. The share of the 90 to 99th percentiles also decreases (from 40.6% to 36.2%). The main beneficiaries of the introduction of differentiated rates of returns on capital are the top 1% (whose share increases from 43.8% to 53.9%) and the top 0.1% (whose share increases from 22.4% to 28.5%).[6]

The fitted Lorenz curve still looks quite close to the obtained distribution, with parameters a=-0.47 and b=0.27). The fit looks even better for the very top income, at the expense of a larger under-estimation (respectively, over-estimation) of the bottom (resp., middle) income levels – see Figure 5. The overall fit decreases, compared to Model 2, with an R square decreasing from 0.95 to 0.9. The income distribution with Model 3 is not Lorenz dominated by those generated by Model 1 nor 2, but the Gini coefficient increases from 0.441 to 0.468.[7]

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[6] We obtain the exact same trends when comparing numerical results obtained under Models 2 and 3 with the hybrid skill distribution described in the Appendix.

[7] Similarly, the value of R square decreases from 0.978 (Model 2) to 0.918 (Model 3) with the hybrid skill distribution, while the Gini index increases from 0.448 to 0.474.

Insert Figure 5 around here

To summarize, the introduction of differentiated rates of return to capital decreases the capital income tax rate, GDP and capital accumulation, but affects very little the equilibrium interest and wage rates. The main impact is indeed distributional, with the top 1% and top 0.1% increasing their wealth at the expense of the bottom 99%. Although there is no Lorenz domination relation between the income distributions generated by Models 2 and 3, we observe a further increase in the Gini coefficient.

5. Model 4: Intergenerational mobility

A. Analytical model and calibration

We now introduce death and inheritance. When an adult dies, we assume that his capital passes (untaxed) to his only child. However, the child will not in general have the skill/income capacity of the father. We use the 100×100 intergenerational income mobility matrix of Chetty et al (2014) to model this process.[8] An element p_{ij} of this matrix is the fraction of sons of fathers at the i^{th} centile of the income distribution who have incomes at the j^{th} centile of their cohort's income distribution. Indeed, we assume that the matrix $P = \{p_{ij}\}$ defines the mobility of *skill*, hence earned income.

To describe the dynamics, let us first suppose that all fathers die at once at the beginning of the year. If an *s* father dies at the beginning of year *t*, his son inherits $S_{t-1}(s)$. The son will be economically active beginning in year *t*. The sons are distributed on the skill distribution *F* according to *P*. Denote by Q^i the i^{th} centile of *F*, comprising a small interval of skills. Let $s \in Q^i$ and $s' \in Q^j$. Then the 'number' of sons who inherit from fathers of

[8] Online Data Table 1: National 100 by 100 transition matrix, available online at http://obs.rc.fas.harvard.edu/chetty/website/v2.1/online_data_tables.xls

skill s and end up at skill level s' will be equal to $100 p_{ij} f(s) f(s')$. Integrating this over $s' \in Q^j$ gives

$$\int_{Q^j} 100 p_{ij} f(s) f(s') ds' = p_{ij} f(s), \tag{5.1}$$

because $\int_{Q^j} f(s') dF(s') = \frac{1}{100}$, which is the correct number of sons of s fathers who end up at Q^j. If we add up these numbers over all j, we have:

$$\sum_j p_{ij} f(s) = f(s), \tag{5.2}$$

which is the total number of sons whose fathers were of skill s.

Now let's look at all fathers $s \in Q^i$. The number of sons who end up at some $s' \in Q^j$ will be $100 p_{ij} \int_{Q^i} f(s') f(s) ds = p_{ij} f(s')$. Summing over i, we have

$$\sum_i p_{ij} f(s') = f(s'), \tag{5.3}$$

which is correct number of sons at s'.

Now let's compute the total savings inherited by children when their fathers' estates pass to them. We are interested in the total savings at a generic skill level s' in the son's generation. These savings come from fathers' wealth at date $t-1$ -- so these are the savings at the beginning of date t for the sons, before they have augmented their savings with their own income (which will be with wages at skill level s'). The amount of inheritance at each value of father's s will be the same. The average inheritance for sons at s' in centile j will be:

$$\sum_{i=1}^{100} 100 p_{ij} \int_{Q^i} S_{t-1}(s) f(s) ds \equiv S^*_{t-1}(s'). \tag{5.4}$$

We now integrate this over all sons:

$$\begin{aligned} &\sum_{j=1}^{100}\sum_{i=1}^{100} 100 p_{ij} \int_{Q^j}\int_{Q^i} S_{t-1}(s) f(s) f(s') ds\, ds' = \sum_{j=1}^{100}\int_{Q^j} 100\left(\sum_{i=1}^{100} p_{ij}\left(\int_{Q^i} S_{t-1}(s) f(s) ds\right)\right) f(s') ds' = \\ &\sum_{j=1}^{100}\sum_{i=1}^{100}\left(\int_{Q^i} S_{t-1}(s) f(s) ds\right) p_{ij} = \sum_{i=1}^{100}\int_{Q^i} S_{t-1}(s) f(s) ds. = \int S_{t-1}(s) dF(s). \end{aligned} \tag{5.5}$$

which is the total inheritance of all sons – that is, total wealth.

Thus, in the sons' generation, there is, initially, heterogeneity of wealth at any skill level s'.

We now drop the assumption that all fathers die at the beginning of the year, and suppose, instead, that a fraction q of all fathers die at each skill level at the beginning of each year. Thus the capital at end of date t at skill level s will be an average of the capital of those who did not die and of the newly inheriting sons who arrive at skill level s. The average capital at skill level s at the end of the year is:

$$S_t(s) = (1-q)S_{t-1}(s) + qS^*_{t-1}(s) + I_t(s) \ . \tag{5.6}$$

The quantity in the first part of this convex combination is the average wealth of those who do not die at the beginning of date t -- call it 'survivors' capital'-- and $S^*_{t-1}(s)$ is the average wealth of sons who join type s at the beginning of date t.

However, we do not attempt to keep track of the heterogeneity of wealth at each skill level that occurs as a result of death and inheritance. We only track average wealth at each skill level. Thus, we aggregate at each skill level s at each date, and assign everyone of that skill level the average amount of capital from (5.6). If you are the son of a wealthy father, your inheritance will add wealth to the cohort at your skill level, but it will not benefit you especially.

The equations summarizing the model are the same 13 equations given at the end of subsection 4.A., except that we substitute equation (5.6) for (2.6) for the intergenerational transmission of wealth. We add the following equation to this set:

14. Wealth of an inheriting son, at the beginning of date t, whose own skill level is s' :

$$\sum_{i=1}^{100} 100 p_{ij} \int_{Q^i} S_{t-1}(s) f(s) ds \quad = S^*_{t-1}(s'), \text{ for } s' \in Q^j, \quad j = 1,...,100 \tag{5.7}$$

As for solving the model, we proceed as previously up to and including the solving of consumption and investment. $S^*_{t-1}(\cdot)$ is then determined by (5.7), and finally, $S_t(\cdot)$ is determined by (5.6).

B. Numerical results

We have performed two sets of computations, one where 2% of the population dies and is replaced at the end of each period (*q=0.02*) and one where 4% get replaced (*q=0.04*). The results we obtain are all monotone in *q*, so we only report those with *q=0.04*.

Insert Table 5 around here

Compared with Model 3, the introduction of social mobility increases the equilibrium interest rate (from 5.7% to 7% with *q=0.04*) and decreases the equilibrium wage rate (from $71,461 to $64,942). The share of wealthy agents lobbying increases from the top 13.4% of capital owners to 21%, but the per capita lobbying contribution decreases (from $3,491 or 2.36% of GDP to $2,153 or 1.68% of GDP), resulting in an increase in the capital income tax (from 12.5% to 16.5%). GDP decreases by 14% (from $147,965 per capita to $128,185), and the capital/output ratio decreases from 5.05 to 3.96, while the share of capital income decreases from 38.7% to 35.7%.

Social mobility is then detrimental to capital accumulation and to GDP, and results in less lobbying by wealthy agents. Its impact on the distribution of wealth is sizeable. First of all, the bottom half of the distribution starts accumulating a little bit of wealth (1.15% of total capital) for the first time in our simulations. Second, the patrimonial middle-class (with a wealth share increasing from 9.9% to 22.2%) and the 90th to 99th percentiles (with a share increasing from 36.2% to 40.3%) also benefit from social mobility. Finally, the losers are the top wealth owners, whose share decreases in a sizeable way (from 53.9% to 36.5% for the top 1%, and from 28.5% to 17% for the top 0.1%).

Social mobility has then a detrimental impact on production (a large decrease in GDP, and an even larger decrease in capital accumulation), but improves wealth inequality. The share of wealth owned by the top 1% and the top .1% decrease significantly, to the benefit of all others. The bottom half of the distribution accumulates some (modest) wealth, for the first time in our simulations, but the main winner is the patrimonial middle class, whose wealth share more than doubles. We obtain the exact same qualitative effects with the hybrid (lognormal-Pareto) skill distribution described in the Appendix.

To conclude, we provide a comparison of some results obtained with the four models, first in Table 6 and then in Figures 6 to 8.

Insert Table 6 around here

Table 6 reports the consumption levels of individuals at the median and the 99.5th percentile of the skill/income distribution, as well as the percentage of GDP dedicated to lobbying activities by the top wealth earners, the Gini coefficient of the fitted income distribution, and the value of the demogrant. The median individual attains his highest consumption level with Model 1. His consumption level decreases with the introduction of lobbying and of differentiated rates of return on capital, but increases with social mobility. The pattern is opposite at the 99.5th percentile, with consumption increasing with lobbying and with differentiated rates of return, but decreasing with mobility. A similar pattern holds for lobbying expenses and for the Gini coefficients, which are maximal in Model 3. We then obtain a very clear picture, with lobbying and differentiated rates of return on capital benefiting the very rich, at the expense of the median, while mobility works in the opposite direction.

The value of the demogrant decreases with the introduction of lobbying and of differentiated rates of return on capital, but also with the introduction of mobility. The latter effect may be counter-intuitive, because of the larger capital income tax rate in Model 4, and is due to the large decrease in capital accumulation (the *share* of total income devoted to the demogrant actually increases when we introduce social mobility). Social mobility then generates a trade-off between the consumption levels of the poorest (whose main source of income is the demogrant) and of the median agent.[9]

[9] Once more, we obtain similar qualitative results with the hybrid skill distribution introduced in the Appendix.

Insert Figures 6, 7 and 8 around here

Figure 6 depicts the absolute amount of wealth S as a function of skill *s* at *t=25* for the 4 models studied, while Figure 7 depicts the distribution of wealth (the vertical dashed lines correspond to the 50th, 90th and 99th percentiles of the skill distribution). Both figures tell a similar story. Figures 6 and 7 show that there is little difference between the first two models in terms of absolute individual wealth and thus also of wealth distribution. Figure 6 shows that the top 10% of the skill distribution accumulates more capital in absolute value in Model 2 (where lobbying is introduced), which results in a shift, in the capital distribution, from the patrimonial middle class to the top 10%. Figure 6 also shows that the introduction of differentiated returns to capital induces the top 1.5% to accumulate more capital in absolute value, while the others accumulate less. This move is confirmed by looking at Figure 7. Moreover, Figure 7 shows that the impact of moving to Model 3 on the wealth distribution is sizeable. Finally, allowing for social mobility reduces very significantly the capital accumulation (in absolute value) of the top 10% of the skill distribution. In terms of distribution, this benefits not only the bottom half of the distribution, but also the patrimonial middle class.[10]

Figure 8 shows the fitted Lorenz curves of the income distributions generated by the four models. The only Lorenz domination relationship is between Models 1 and 2, as stated above.

[10] The discontinuities associated with Model 4 on Figures 6 and 7 are due to the fact that we use a 100 by 100 mobility matrix (with each row/column representing a centile of the income distribution) rather than a smooth function.

6. Conclusions

In all models studied, the growth rates of income and of average wealth are significantly smaller than the interest rate ($g<r$ using Piketty's terminology). This goes in hand with a capital deepening of the economy with time, an increase in the capital/output ratio and a roughly stable share of capital in total income.

In the first model studied, we observe a more unequal distribution of wealth as time passes, with the proportion of agents with no wealth increasing, and more generally, the 90th to 99th percentile of the wealth distribution gain shares at the expense of the bottom 90 percent, with the shares of the top 1% and the top 0.1% remaining roughly constant.

The introduction of lobbying makes little difference with our calibration, although it decreases the capital income tax rate significantly and makes both the wealth and income distributions more unequal. Differentiated rates of return on capital and social mobility do affect the results much more, and in opposite directions. Differentiated returns benefit the top 1% and the top 0.1% of the wealth distribution, at the expense of all the others, and results in an increase of the Gini coefficient of the income distribution. Social mobility induces much less capital accumulation, but the shares of the top 1% and the top 0.1% decrease significantly to the benefit of all others, including those in the bottom half of the distribution, who have positive wealth uniquely in this scenario. The Gini coefficient of the income distribution decreases when social mobility is introduced. This improvement in wealth and income distributions is obtained at the price of a much lower GDP and overall capital accumulation, which in turn hurts the very poorest because of a decrease in the demogrant.

All these results continue to hold when we model a hybrid lognormal-Pareto skill distribution, rather than a purely lognormal one. The only differences are that we obtain a better fit to the Jantzen and Volpert (2012)'s Lorenz curve with the hybrid skill distribution, and that the Gini coefficients are larger than with the pure lognormal skill distribution, whatever the model considered.

[Table 7 about here]

Table 7 presents the Saez and Zucman (2014) top US wealth shares in 2012, which are virtually identical to the wealth shares in our four models at date $t = 1$, and then presents the top wealth shares at date $t = 25$ from our four models. It seems that the most significant difference between our simulation results and Piketty's history of recent capital accumulation is that in Model 4, when intergenerational mobility is introduced, the share of the top 1% and the top 0.1% of wealth-holders falls, albeit quite slowly. We think there are several possible explanations. The first is that ours is a competitive model, except for the introduction of variable rates of return in Models 3 and 4. We do note the rapid rate of increase of the wealth share of the top 1% and 0.1% in Model 3. In reality, the last thirty years in the United States may be best characterized as a period where capital has won significant gains through changing the bargaining conditions with labor, principally through the ascendency of neoliberal ideology and the concomitant attack on labor unions. One piece of evidence is that earnings have not increased with productivity. Another is the observed increase in the share of capital income. According to Piketty (2014, Figure 6.5), this share rose from 21% in 1975 to 28% in 2010; we do not see a comparable increase in our simulations, and this may be due to our competitive model. (In our first three models, capital's share is remarkably stable over the 25 year period, and in the fourth model it falls a little bit.) If indeed the increase of the wealth share at the very top of the distribution is due to the ascendency of neoliberalism it may have been a one-shot phenomenon, and we might expect the wealth shares of the top 1% and 0.1% to become more stable, as they are in our simulations. This view is not inconsistent with one held by Piketty, that the share of income (if not wealth) at the top in the US has been due to the setting of executive compensation in a non-competitive way. Indeed, the high salaries of top managers may simply be a transfer from workers facilitated by neoliberalism.

A second possible explanation has to do with the intergenerational transmission matrix of Chetty et al (2014) that we used. We employed this matrix to generate the intergenerational transmission of skills (which is to say labor income), while in reality it is a transmission matrix of all income of fathers to sons. There is also less persistence in wealth holding across generations in our model than there is in reality because we distribute all inherited wealth among the entire skill cohort that a son enters, rather than keeping track of dynasties. Thus, our model implements more mixing of wealth than in fact occurs. In the other direction, we have assumed less dispersion of wealth than may indeed occur, if wealthy families have more than one child who inherits. It seems that a more nuanced approach to intergenerational mobility than ours is required.

Appendix: Introducing a Pareto tail in the skill distribution

We assume that the skill distribution is lognormal (with cdf F and density *f*) up to skill *x**, and then follows a Pareto distribution for $x > x^*$. The CDF of the Pareto distribution is $P(x) = 1 - ax^{-1/\eta}$, with corresponding density $\frac{a}{\eta} x^{-(1+\eta)/\eta}$. Jones (2015) reports that $\eta = 0.6$ in the U.S. We then have two parameters to estimate: the threshold $x*$ and *a*.

In order to obtain a cut-off skill *x** that is not too large, we have to truncate the domain of the distribution function to [*0,B*], where we choose (arbitrary) that *B* = 6. (Recall that our lognormal distribution has a mean of one and a median of 0.85.)

We want the two density functions (lognormal and Pareto) to take the same value at *x**, so that the overall density function is continuous over the whole range. Let *b* equal the integral of the distribution function that we construct on [0,B] before normalizing it to be 1. The normalized density on the first part is $f(x)/b$. We then solve

$$f(x^*)/b = \frac{a}{\eta} x^{*-(1+\eta)/\eta},$$

from which we obtain

$$a = \frac{\eta f(x^*)}{b(x^*)^{-(1+\eta)/\eta}} .$$

We then solve the equation:

$$F(x^*)/b + \int_{x^*}^{B} \frac{a}{\eta} x^{-(1+\eta)/\eta} dx = 1$$

for *b*, using the formula for *a*. We obtain that *b*=1.0082 , *a*=0.193 and *x**=3.2, with $F(x^*)/b = 0.982$. In words, the distribution of skills is lognormal for the bottom 98.2% of the skill distribution, and Pareto for the top 1.8%.

Table 8 below reports the numerical results obtained under Model 1 with this hybrid distribution of skills.[11]

[11] Results obtained with the hybrid Lognormal-Pareto skill distribution for the four models are so similar to those obtained with the lognormal distribution that we do not report them here. They are available upon request from the authors.

Insert Table 8 here

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Model 1 : benchmark

Table 1 : Equilibrium results as a function of t for Model 1

t	r	w	GDP	K[t]	$\frac{K}{GDP}$	$\frac{(r+d)K}{GDP}$	$\frac{wL}{GDP}$	shareK, 0−50	shareK, 50−90	shareK,90−99	shareK top 1%	share top 0.1%
1	0.0626065	68.2403	108.742	490.3	4.50883	0.372459	0.627541	0.	0.221455	0.357878	0.420668	0.221095
2	0.0625394	68.2732	109.909	496.166	4.51433	0.37261	0.62739	0.	0.215739	0.361377	0.422884	0.221951
3	0.0624353	68.3244	111.133	502.643	4.52289	0.372845	0.627155	0.	0.210715	0.364561	0.424725	0.222611
4	0.0623014	68.3905	112.408	509.647	4.53393	0.373148	0.626852	0.	0.206269	0.36748	0.426251	0.223105
5	0.0621435	68.4688	113.727	517.116	4.54701	0.373507	0.626493	0.	0.202315	0.370173	0.427512	0.223461
6	0.061966	68.5572	115.086	524.999	4.56178	0.373911	0.626089	0.	0.198798	0.372705	0.428581	0.223717
7	0.0617755	68.6527	116.48	533.215	4.57774	0.374347	0.625653	0.	0.195608	0.375008	0.429385	0.223836
8	0.0615663	68.7581	117.916	541.866	4.59536	0.374826	0.625174	0.	0.192749	0.377197	0.430055	0.223886
9	0.0613493	68.8681	119.381	550.795	4.61375	0.375326	0.624674	0.	0.190162	0.37926	0.430578	0.22386
10	0.0611237	68.9832	120.877	560.026	4.63301	0.375847	0.624153	0.	0.187817	0.381211	0.430973	0.223769
11	0.0608911	69.1028	122.404	569.545	4.65301	0.376387	0.623613	0.	0.185684	0.383063	0.431254	0.22362
12	0.0606528	69.2261	123.959	579.34	4.67365	0.376943	0.623057	0.	0.183739	0.384826	0.431435	0.223421
13	0.0604099	69.3526	125.542	589.399	4.69484	0.377512	0.622488	0.	0.181963	0.386511	0.431526	0.223177
14	0.0601634	69.482	127.152	599.715	4.71651	0.378092	0.621908	0.	0.180338	0.388125	0.431538	0.222892
15	0.0599141	69.6139	128.79	610.282	4.7386	0.378681	0.621319	0.	0.178849	0.389673	0.431478	0.222573
16	0.0596627	69.7479	130.453	621.094	4.76105	0.379278	0.620722	0.	0.177484	0.391164	0.431352	0.222221
17	0.0594098	69.8837	132.143	632.147	4.78382	0.379882	0.620118	0.	0.176231	0.392601	0.431169	0.221841
18	0.0591557	70.0212	133.858	643.438	4.80686	0.380491	0.619509	0.	0.17508	0.393989	0.430932	0.221436
19	0.0589011	70.1601	135.599	654.965	4.83015	0.381104	0.618896	0.	0.174021	0.395328	0.430643	0.221005
20	0.058646	70.3004	137.366	666.731	4.85367	0.381722	0.618278	0.	0.173051	0.396632	0.430317	0.220557
21	0.0583914	70.4414	139.158	678.719	4.87734	0.382341	0.617659	0.	0.172159	0.397895	0.429947	0.220089
22	0.0581368	70.5836	140.975	690.944	4.90119	0.382963	0.617037	0.	0.17134	0.399121	0.42954	0.219603
23	0.0578829	70.7265	142.817	703.401	4.92518	0.383587	0.616413	0.	0.170588	0.400314	0.429098	0.219101
24	0.0576297	70.8702	144.685	716.09	4.9493	0.384212	0.615788	0.	0.1699	0.401476	0.428625	0.218585
25	0.0573775	71.0144	146.578	729.01	4.97352	0.384838	0.615162	0.	0.16927	0.402608	0.428123	0.218056

Figure 1 : Wealth as a function of skill s for the model 1 at t = 25

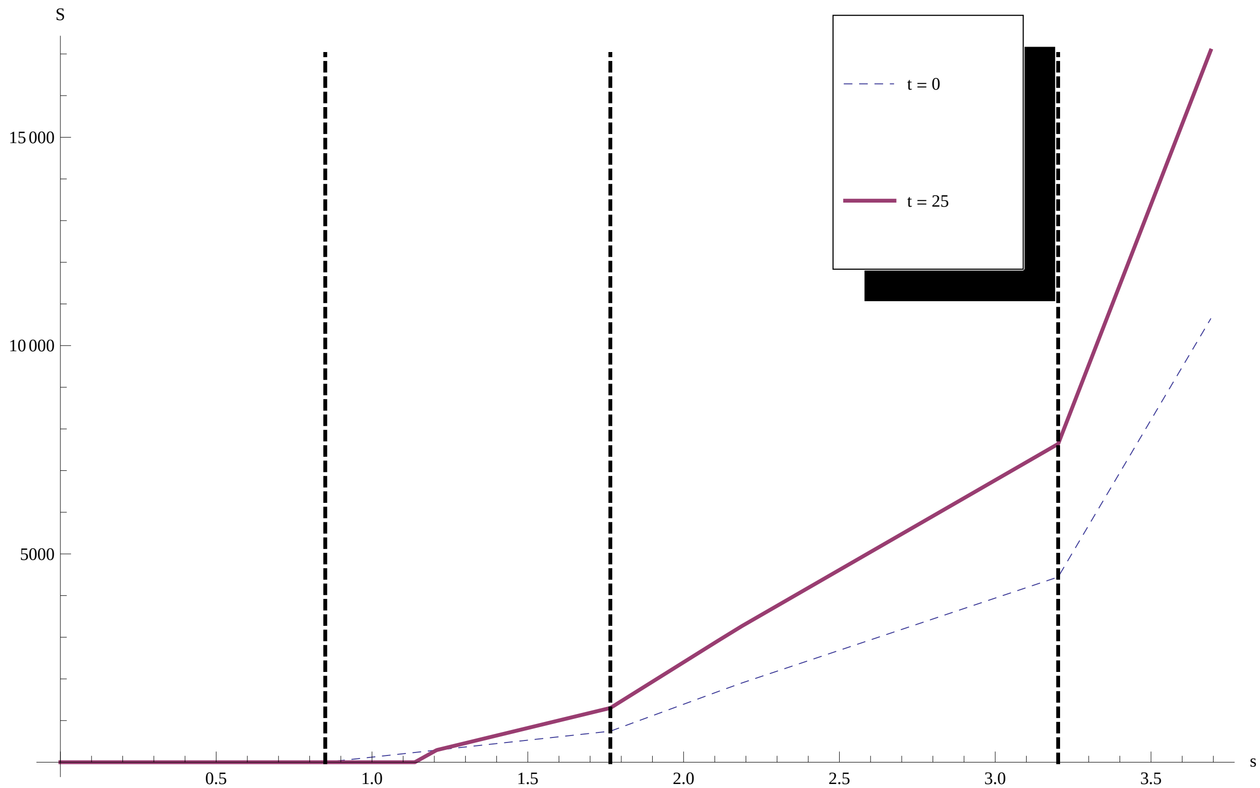


Figure 2 : Wealth distribution as a function of skill for model 1 at t = 25

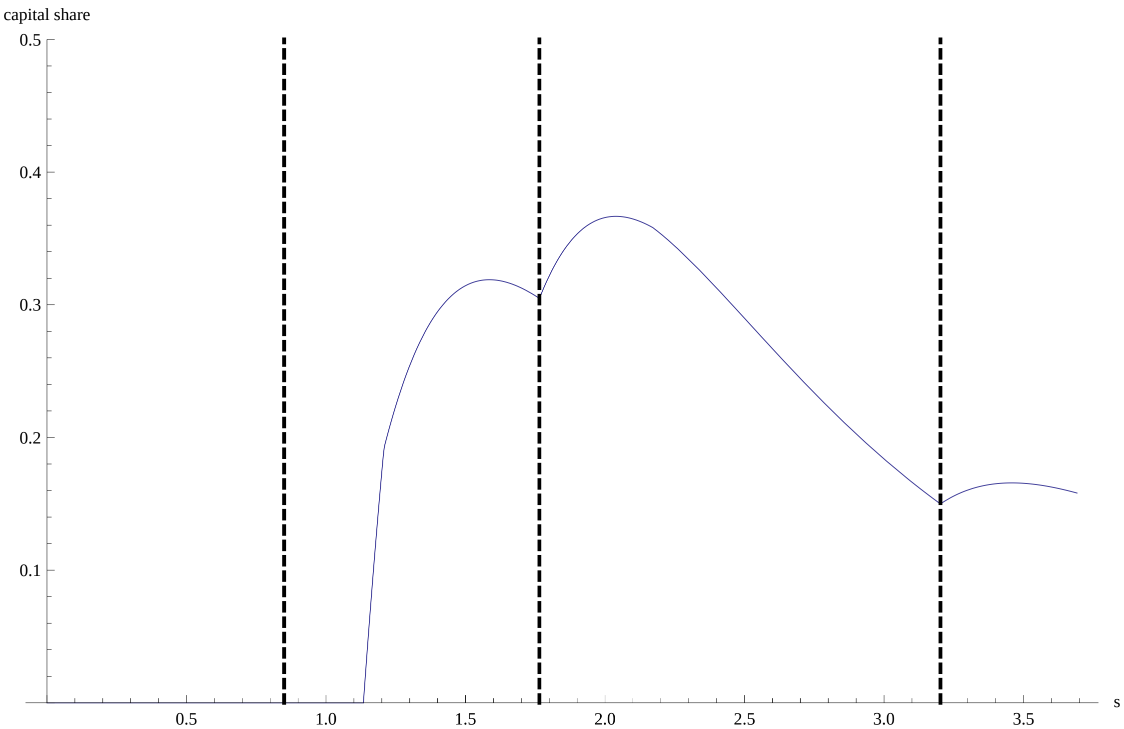
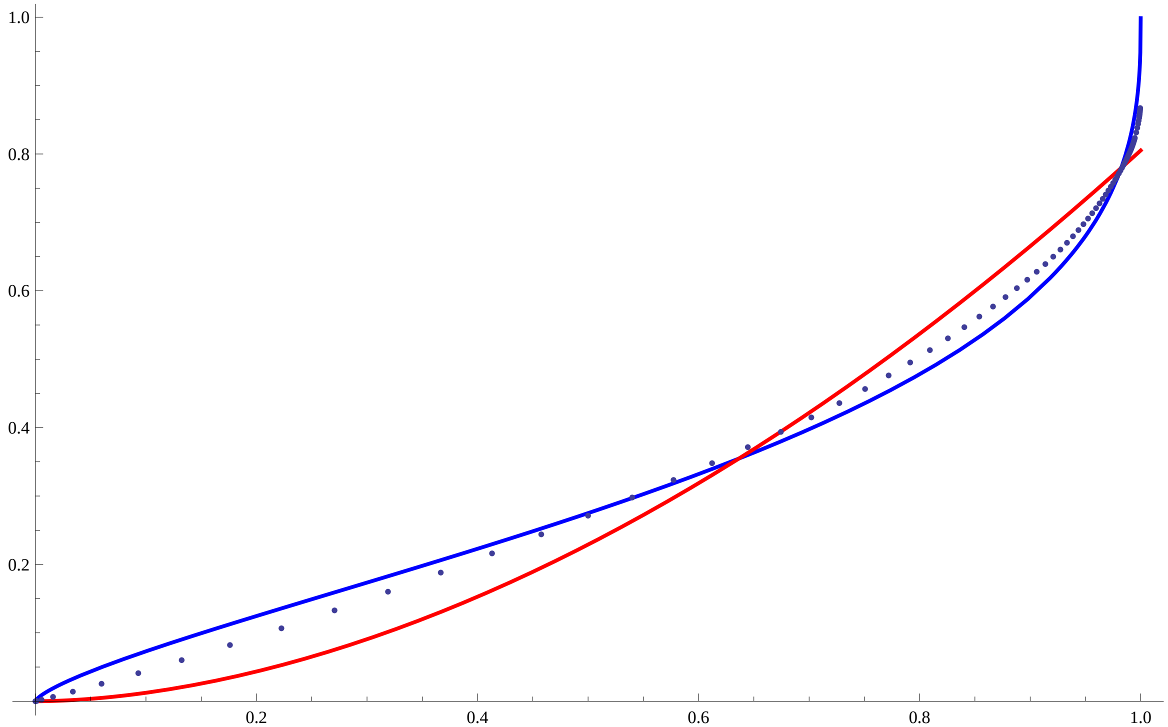


Figure 3 : Lorenz curves at t = 25, obtained (dots), fitted with lognormal (blue curve) and hybrid lognormal - Pareto (red curve), for Model 1



Model 2 (with lobbying)

Table 2 : Equilibrium results as a function of t for Model 2

t	r	w	1−F[s∗]	σbar	τc	GDP	K[t]	$\frac{K}{GDP}$	((r + d) K)/ GDP	$\frac{wL}{GDP}$	shareK, 0−50	shareK, 50−90	shareK,90−99	shareK top 1%	ShareK, top 0.1%
1	0.0626065	68.2403	0.173762	1.48856	0.174127	108.742	490.3	4.50883	0.372459	0.627541	0.	0.219816	0.358513	0.42167	0.221634
2	0.0625343	68.2757	0.174368	1.53842	0.172503	109.915	496.24	4.51475	0.372622	0.627378	0.	0.212726	0.36253	0.424748	0.222954
3	0.062412	68.3359	0.174694	1.58667	0.17101	111.161	502.983	4.5248	0.372898	0.627102	0.	0.206533	0.366133	0.427334	0.224017
4	0.0622498	68.4161	0.174811	1.63368	0.169627	112.47	510.412	4.53819	0.373265	0.626735	0.	0.201087	0.369396	0.429517	0.224867
5	0.0620562	68.5123	0.174777	1.67979	0.168332	113.835	518.434	4.55427	0.373706	0.626294	0.	0.196272	0.372386	0.431375	0.225548
6	0.0618384	68.6211	0.174638	1.72531	0.167108	115.247	526.963	4.57246	0.374203	0.625797	0.	0.191972	0.375108	0.432919	0.226064
7	0.0615981	68.742	0.174405	1.77021	0.165953	116.708	535.999	4.59267	0.374753	0.625247	0.	0.18813	0.377637	0.434233	0.226462
8	0.0613428	68.8714	0.174121	1.81488	0.164847	118.208	545.446	4.6143	0.375341	0.624659	0.	0.184674	0.379988	0.435338	0.226751
9	0.0610745	69.0084	0.173798	1.85938	0.163787	119.746	555.291	4.63723	0.375961	0.624039	0.	0.181554	0.382185	0.436261	0.226947
10	0.0607958	69.152	0.173447	1.90384	0.162765	121.321	565.508	4.66125	0.376609	0.623391	0.	0.178725	0.384248	0.437027	0.227062
11	0.0605088	69.301	0.173082	1.94831	0.161779	122.931	576.077	4.68619	0.37728	0.62272	0.	0.176152	0.386193	0.437655	0.227106
12	0.0602154	69.4547	0.172709	1.99289	0.160822	124.573	586.981	4.71193	0.377969	0.622031	0.	0.173804	0.388034	0.438162	0.227089
13	0.0599169	69.6124	0.172335	2.03766	0.159892	126.248	598.206	4.73835	0.378674	0.621326	0.	0.171657	0.389782	0.438562	0.227018
14	0.0596147	69.7736	0.171965	2.08267	0.158985	127.953	609.744	4.76536	0.379392	0.620608	0.	0.169687	0.391448	0.438865	0.226898
15	0.0593097	69.9378	0.171602	2.12796	0.158099	129.689	621.585	4.79288	0.380121	0.619879	0.	0.167877	0.39304	0.439084	0.226736
16	0.0590027	70.1046	0.17125	2.17359	0.157232	131.455	633.723	4.82084	0.380859	0.619141	0.	0.16621	0.394565	0.439225	0.226536
17	0.0586945	70.2736	0.170909	2.21959	0.156383	133.25	646.154	4.84918	0.381604	0.618396	0.	0.164673	0.39603	0.439297	0.226302
18	0.0583858	70.4445	0.170583	2.26601	0.155549	135.074	658.872	4.87786	0.382355	0.617645	0.	0.163253	0.397438	0.439305	0.226037
19	0.0580768	70.6172	0.17027	2.3128	0.154731	136.927	671.878	4.90684	0.383111	0.616889	0.	0.161941	0.3988	0.439261	0.225746
20	0.0577686	70.7912	0.169975	2.3601	0.153925	138.807	685.159	4.93604	0.383869	0.616131	0.	0.160726	0.400114	0.439162	0.225429
21	0.0574608	70.9666	0.169695	2.40788	0.153132	140.717	698.727	4.96549	0.384631	0.615369	0.	0.159601	0.401385	0.439015	0.225089
22	0.0571541	71.1431	0.169431	2.45691	0.152327	142.654	712.577	4.99513	0.385395	0.614605	0.	0.158556	0.402614	0.438823	0.224728
23	0.0568486	71.3206	0.169182	2.50502	0.151579	144.621	726.712	5.02495	0.38616	0.61384	0.	0.15759	0.403813	0.438597	0.224351
24	0.056545	71.4987	0.168952	2.55433	0.15082	146.614	741.116	5.05487	0.386925	0.613075	0.	0.156693	0.404976	0.438331	0.223955
25	0.056243	71.6776	0.168737	2.60426	0.150069	148.637	755.807	5.08493	0.387691	0.612309	0.	0.155861	0.406107	0.438032	0.223545

Model 3 (with lobbying and differential rates of return)

Table 3 : Equilibrium results as a function of t for Model 3

t	r	ν	w	1−F[s∗]	σbar	τc	GDP	K[t]	$\frac{K}{\text{GDP}}$	$((r+d)K)/\text{GDP}$	$\frac{\text{wL}}{\text{GDP}}$	shareK, 0−50	shareK, 50−90	shareK,90−99	shareK top 1%	ShareK, top 0.1%
1	0.0626065	1.51974	68.2403	0.173762	1.98082	0.152835	108.742	490.3	4.50883	0.372459	0.627541	0.	0.216021	0.357999	0.42598	0.224161
2	0.0625959	1.41893	68.2455	0.172823	2.04037	0.151278	109.842	495.356	4.5097	0.372483	0.627517	0.	0.205307	0.361317	0.433377	0.228027
3	0.0625283	1.32704	68.2787	0.171574	2.09914	0.149806	111.022	501.29	4.51524	0.372635	0.627365	0.	0.195674	0.36405	0.440277	0.231643
4	0.0624141	1.24239	68.3349	0.170101	2.15743	0.148405	112.27	507.981	4.52463	0.372893	0.627107	0.	0.186966	0.366281	0.44675	0.235046
5	0.0622618	1.16369	68.4101	0.168468	2.21554	0.14706	113.58	515.338	4.5372	0.373238	0.626762	0.	0.179059	0.368081	0.45286	0.238267
6	0.0620784	1.08986	68.5012	0.166726	2.2735	0.145767	114.945	523.279	4.55241	0.373655	0.626345	0.	0.171843	0.369505	0.458652	0.241331
7	0.0618693	1.02016	68.6056	0.16491	2.33178	0.144508	116.36	531.751	4.56986	0.374132	0.625868	0.	0.16523	0.370603	0.464168	0.244258
8	0.0616393	0.953905	68.7212	0.163051	2.39006	0.143293	117.821	540.705	4.5892	0.374659	0.625341	0.	0.159146	0.371413	0.469441	0.247068
9	0.0613919	0.890605	68.8464	0.16117	2.44886	0.142103	119.325	550.102	4.61013	0.375227	0.624773	0.	0.153527	0.371972	0.474501	0.249773
10	0.0611305	0.829811	68.9797	0.159282	2.50815	0.140938	120.868	559.913	4.63243	0.375831	0.624169	0.	0.14832	0.372308	0.479372	0.252388
11	0.0608576	0.771176	69.12	0.157401	2.56786	0.139799	122.449	570.112	4.6559	0.376465	0.623535	0.	0.14348	0.372446	0.484074	0.254922
12	0.0605754	0.714417	69.2663	0.155537	2.62827	0.138678	124.067	580.68	4.68038	0.377124	0.622876	0.	0.138967	0.372409	0.488625	0.257384
13	0.0602857	0.65928	69.4177	0.153695	2.68937	0.137573	125.719	591.598	4.70574	0.377804	0.622196	0.	0.134748	0.372214	0.493038	0.259781
14	0.0599901	0.605582	69.5736	0.151881	2.75122	0.136484	127.404	602.856	4.73185	0.378501	0.621499	0.	0.130794	0.371877	0.497328	0.262121
15	0.0596897	0.553131	69.7335	0.150098	2.81382	0.13541	129.122	614.442	4.75863	0.379214	0.620786	0.	0.127081	0.371415	0.501505	0.264409
16	0.0593858	0.501774	69.8967	0.14835	2.87725	0.134348	130.871	626.345	4.78598	0.379939	0.620061	0.	0.123584	0.370838	0.50558	0.26665
17	0.0590793	0.451393	70.0628	0.146638	2.94161	0.133295	132.651	638.559	4.81383	0.380675	0.619325	0.	0.120286	0.370156	0.509559	0.268847
18	0.058771	0.401888	70.2315	0.144962	3.00683	0.132254	134.462	651.08	4.84212	0.381419	0.618581	0.	0.11717	0.369381	0.51345	0.271006
19	0.0584615	0.353155	70.4025	0.143324	3.07298	0.131223	136.303	663.903	4.8708	0.38217	0.61783	0.	0.11422	0.36852	0.517261	0.273128
20	0.0581514	0.305101	70.5754	0.141723	3.1401	0.130201	138.173	677.023	4.89982	0.382928	0.617072	0.	0.111423	0.367581	0.520996	0.275217
21	0.0578413	0.257676	70.75	0.140159	3.2082	0.129187	140.073	690.438	4.92913	0.38369	0.61631	0.	0.108766	0.366571	0.524662	0.277276
22	0.0575316	0.210786	70.9261	0.138632	3.27729	0.128182	142.002	704.146	4.95869	0.384455	0.615545	0.	0.106241	0.365497	0.528262	0.279307
23	0.0572226	0.164383	71.1035	0.137141	3.34744	0.127183	143.961	718.146	4.98848	0.385224	0.614776	0.	0.103836	0.364363	0.5318	0.281312
24	0.0569147	0.118422	71.282	0.135685	3.41863	0.126192	145.948	732.436	5.01846	0.385994	0.614006	0.	0.101544	0.363175	0.535281	0.283292
25	0.0566082	0.0728587	71.4615	0.134263	3.49095	0.125207	147.965	747.018	5.04862	0.386765	0.613235	0.	0.0993559	0.361937	0.538707	0.285249

Figure 4 : Rate of return on capital as a function of skill s in Model 3

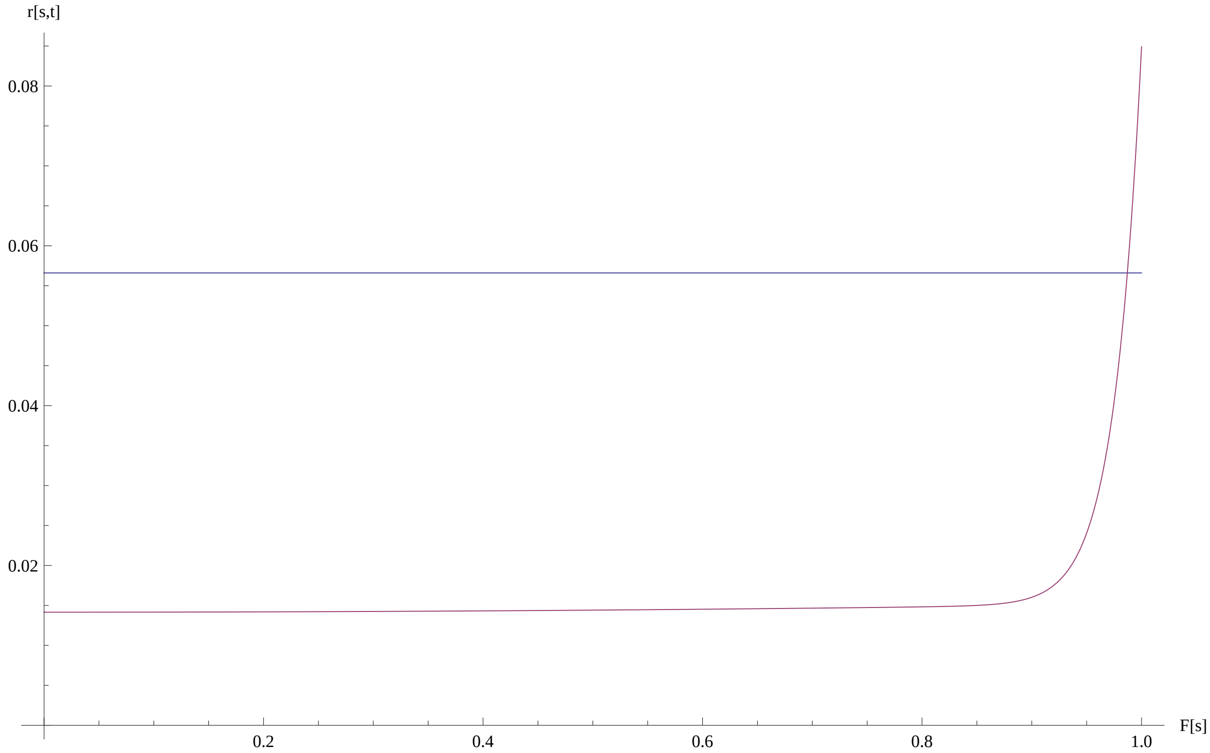
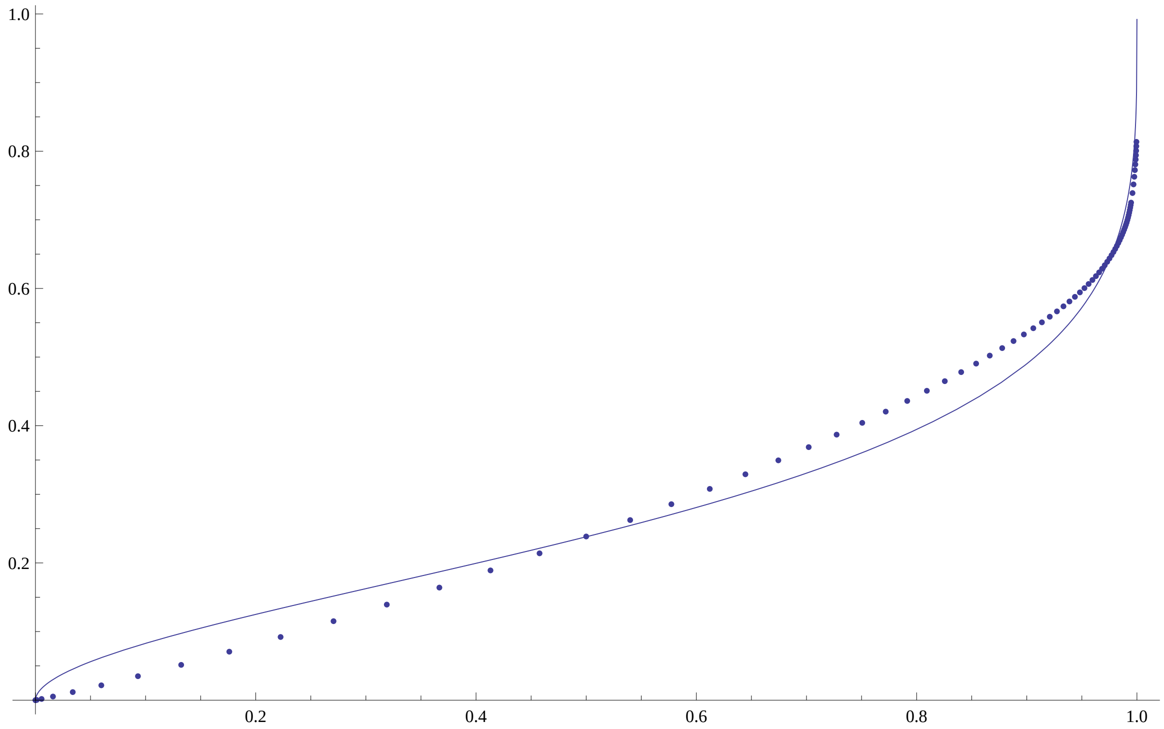


Figure 5 : Lorenz curve at t = 25, obtained (dots) and fitted (curve), in Model 3



Model 4 (with lobbying, differential rates of return and mobility)

Table (Not in paper) : Equilibrium results as a function of t for Model 4 with 2 % death rate / mobility

t	r	ν	w	1−F[s*]	σbar	τc	GDP	K[t]	$\frac{K}{GDP}$	$((r+d)K)/GDP$	$\frac{wL}{GDP}$	shareK, 0−50	shareK, 50−90	shareK,90−99	shareK top 1%	ShareK, top 0.1%
1	0.0626065	1.51974	68.2403	0.173762	1.98082	0.152835	108.742	490.3	4.50883	0.372459	0.627541	0.00580602	0.220607	0.355472	0.418645	0.219796
2	0.0625772	1.51543	68.2546	0.173993	2.00878	0.152514	109.864	495.623	4.51123	0.372525	0.627475	0.00578709	0.215157	0.358473	0.421111	0.220548
3	0.062717	1.47575	68.1862	0.175	2.04074	0.152053	110.796	498.561	4.4998	0.37221	0.62779	0.00577091	0.210289	0.361173	0.423301	0.221157
4	0.0628315	1.44067	68.1303	0.175668	2.07221	0.151629	111.767	501.885	4.49047	0.371953	0.628047	0.00575685	0.20591	0.363604	0.425252	0.221642
5	0.0629244	1.40956	68.0851	0.176296	2.10324	0.151238	112.772	505.549	4.48293	0.371744	0.628256	0.00574466	0.201943	0.365824	0.427018	0.222034
6	0.0630014	1.38119	68.0478	0.176796	2.13396	0.150876	113.806	509.475	4.47669	0.371572	0.628428	0.00573397	0.198321	0.367864	0.428632	0.222348
7	0.0630643	1.35542	68.0173	0.177183	2.16462	0.150532	114.867	513.64	4.47161	0.371431	0.628569	0.00572428	0.194997	0.369725	0.43009	0.222583
8	0.0631144	1.33208	67.9931	0.177471	2.19513	0.150209	115.954	518.03	4.46757	0.371319	0.628681	0.00573009	0.191935	0.371456	0.431444	0.222767
9	0.0631542	1.31055	67.9738	0.177677	2.22568	0.1499	117.063	522.613	4.46436	0.37123	0.62877	0.00572226	0.189103	0.373051	0.432683	0.222893
10	0.0631851	1.29064	67.959	0.177814	2.25609	0.149609	118.195	527.372	4.46187	0.371161	0.628839	0.00571525	0.186475	0.374535	0.433833	0.222976
11	0.0631966	1.27547	67.9534	0.17783	2.28652	0.149333	119.363	532.47	4.46095	0.371136	0.628864	0.0056943	0.184018	0.375928	0.434907	0.223022
12	0.0632248	1.25491	67.9398	0.17792	2.31711	0.149065	120.52	537.359	4.45868	0.371073	0.628927	0.00568651	0.181666	0.377102	0.435765	0.22296
13	0.0632238	1.24212	67.9403	0.177842	2.34765	0.148812	121.726	542.749	4.45876	0.371075	0.628925	0.00568288	0.179589	0.378435	0.436833	0.223007
14	0.0632403	1.22381	67.9324	0.177846	2.37834	0.148567	122.922	547.918	4.45743	0.371038	0.628962	0.00567797	0.177594	0.379571	0.437705	0.222956
15	0.0632411	1.20952	67.932	0.177756	2.40922	0.14833	124.15	553.384	4.45737	0.371036	0.628964	0.00567348	0.175729	0.380632	0.438516	0.222878
16	0.0632365	1.19638	67.9342	0.177632	2.44007	0.148106	125.398	558.991	4.45774	0.371047	0.628953	0.00566913	0.173955	0.381635	0.439283	0.222778
17	0.0632289	1.18383	67.9378	0.177487	2.471	0.147892	126.662	564.703	4.45835	0.371063	0.628937	0.00566521	0.172294	0.382583	0.440008	0.22266
18	0.0632184	1.17193	67.9429	0.177321	2.50246	0.147673	127.943	570.524	4.4592	0.371087	0.628913	0.00566151	0.17072	0.383478	0.44069	0.222523
19	0.0632049	1.16061	67.9494	0.177138	2.53396	0.147466	129.241	576.451	4.46028	0.371117	0.628883	0.00565811	0.169236	0.384327	0.441338	0.22237
20	0.0631887	1.14992	67.9572	0.176939	2.56557	0.147268	130.556	582.486	4.46158	0.371153	0.628847	0.00565471	0.167828	0.385115	0.441932	0.222193
21	0.0631686	1.14019	67.9669	0.176718	2.59741	0.147075	131.89	588.65	4.4632	0.371198	0.628802	0.00565177	0.166491	0.385888	0.442522	0.222016
22	0.0631468	1.13075	67.9774	0.176489	2.62938	0.146889	133.24	594.908	4.46495	0.371247	0.628753	0.00564892	0.165223	0.386613	0.443071	0.221821
23	0.0631268	1.12084	67.9871	0.17627	2.66178	0.146702	134.601	601.203	4.46657	0.371291	0.628709	0.00564694	0.164043	0.38735	0.443645	0.22164
24	0.0631079	1.11052	67.9962	0.176058	2.69414	0.146529	135.974	607.544	4.46809	0.371334	0.628666	0.00564436	0.162896	0.387997	0.444132	0.221419
25	0.0630815	1.10229	68.009	0.175807	2.72713	0.146348	137.372	614.084	4.47022	0.371392	0.628608	0.00564224	0.161826	0.388634	0.444617	0.221198

Table 5 : Equilibrium results as a function of t for Model 4 with 4 % death rate / mobility

t	r	ν	w	1−F[s∗]	σbar	τc	GDP	K[t]	$\frac{K}{GDP}$	$\frac{(r+d)K}{GDP}$	$\frac{wL}{GDP}$	shareK, 0−50	shareK, 50−90	shareK,90−99	shareK top 1%	ShareK, top 0.1%
1	0.0626065	1.51974	68.2403	0.173762	1.98082	0.152835	108.742	490.3	4.50883	0.372459	0.627541	0.011612	0.225194	0.352945	0.411311	0.215432
2	0.062559	1.61157	68.2636	0.175	1.97703	0.153769	109.886	495.884	4.51272	0.372566	0.627434	0.0116006	0.22496	0.35557	0.408928	0.213135
3	0.0629044	1.62188	68.0949	0.178383	1.98277	0.154321	110.574	495.873	4.48455	0.371789	0.628211	0.0115908	0.224853	0.358081	0.406527	0.210844
4	0.0632437	1.63273	67.9307	0.181723	1.98849	0.154874	111.276	495.975	4.45716	0.371031	0.628969	0.0115827	0.224836	0.360505	0.404134	0.208571
5	0.0635768	1.64431	67.7711	0.184992	1.99445	0.155417	111.993	496.188	4.43054	0.37029	0.62971	0.0115758	0.224889	0.362846	0.401753	0.206319
6	0.0639057	1.65608	67.615	0.187846	2.00064	0.155949	112.722	496.486	4.40451	0.369564	0.630436	0.0115697	0.224973	0.365114	0.399393	0.204093
7	0.0642316	1.66792	67.4618	0.190914	2.00566	0.156526	113.462	496.849	4.37898	0.368848	0.631152	0.0115642	0.22507	0.367337	0.397082	0.201905
8	0.0645549	1.67982	67.3111	0.193914	2.01337	0.156998	114.213	497.273	4.35389	0.368142	0.631858	0.0115591	0.225156	0.369513	0.394816	0.199755
9	0.0648773	1.69136	67.1623	0.195	2.01985	0.157518	114.973	497.731	4.3291	0.367443	0.632557	0.0115556	0.225251	0.37168	0.392636	0.197662
10	0.0651994	1.70236	67.0149	0.195535	2.0268	0.158019	115.741	498.215	4.30458	0.366747	0.633253	0.0115506	0.225306	0.373753	0.390447	0.195578
11	0.0655208	1.71306	66.8691	0.19787	2.03379	0.158518	116.517	498.732	4.28033	0.366058	0.633942	0.0115464	0.225342	0.375813	0.388335	0.193546
12	0.065841	1.72362	66.7252	0.20013	2.04098	0.159009	117.302	499.287	4.2564	0.365374	0.634626	0.0115425	0.225336	0.37786	0.386298	0.191565
13	0.0661625	1.73351	66.5819	0.202327	2.04855	0.159486	118.094	499.846	4.2326	0.364692	0.635308	0.0115392	0.225313	0.379895	0.384333	0.189633
14	0.0664854	1.74262	66.4393	0.204248	2.05876	0.159865	118.892	500.408	4.20892	0.36401	0.63599	0.0115353	0.22524	0.381879	0.382403	0.18773
15	0.066806	1.75192	66.299	0.205501	2.06428	0.160417	119.701	501.023	4.18563	0.363338	0.636662	0.0115325	0.225135	0.383849	0.380541	0.185874
16	0.067128	1.76049	66.1593	0.206488	2.07205	0.160886	120.516	501.641	4.16245	0.362666	0.637334	0.0115289	0.224983	0.385804	0.378744	0.184062
17	0.0674509	1.76856	66.0204	0.20735	2.08043	0.161333	121.338	502.268	4.13942	0.361996	0.638004	0.0115183	0.22466	0.387494	0.376769	0.182176
18	0.0677504	1.782	65.8926	0.208016	2.08858	0.161788	122.196	503.23	4.11824	0.361377	0.638623	0.0115206	0.224524	0.389623	0.37529	0.180542
19	0.0680977	1.78352	65.7457	0.208747	2.09749	0.162216	123.005	503.572	4.09391	0.360664	0.639336	0.0115179	0.224247	0.391562	0.373711	0.178872
20	0.0684244	1.78983	65.6087	0.209295	2.10588	0.162663	123.847	504.212	4.07124	0.359997	0.640003	0.0115134	0.223898	0.393424	0.372135	0.177214
21	0.0687483	1.79655	65.4741	0.209738	2.11469	0.163096	124.701	504.911	4.04898	0.35934	0.64066	0.0115104	0.223534	0.395322	0.370666	0.175618
22	0.0690775	1.80172	65.3385	0.210109	2.1242	0.163504	125.557	505.561	4.02655	0.358675	0.641325	0.0115067	0.223124	0.397172	0.369226	0.174047
23	0.0694045	1.80712	65.2049	0.210387	2.13335	0.163925	126.424	506.261	4.00448	0.358019	0.641981	0.0115035	0.222691	0.399017	0.367852	0.172519
24	0.0697336	1.81173	65.0716	0.210592	2.1428	0.164336	127.297	506.955	3.98247	0.357361	0.642639	0.0114988	0.222197	0.400788	0.366481	0.171002
25	0.0700572	1.81732	64.9417	0.210706	2.15257	0.164737	128.185	507.743	3.96103	0.356719	0.643281	0.0114964	0.221711	0.402621	0.365235	0.169554

Comparison of the 4 models

Table 6 : Consumption and lobbying for the 4 models at t = 25

Model	c[median,25]	c[0.995,25]	Lobby/GDP	Gini	Demogrant
1	96.018	583.739	0	0.38631	46.1994
2	88.5172	668.046	1.75 %	0.441436	38.2333
3	87.1849	899.987	2.36 %	0.468321	37.0527
4-q=4%	95.5686	660.111	1.679 %	0.417872	34.7204

Figure 6 : S (s, t) for the 4 models at t = 25

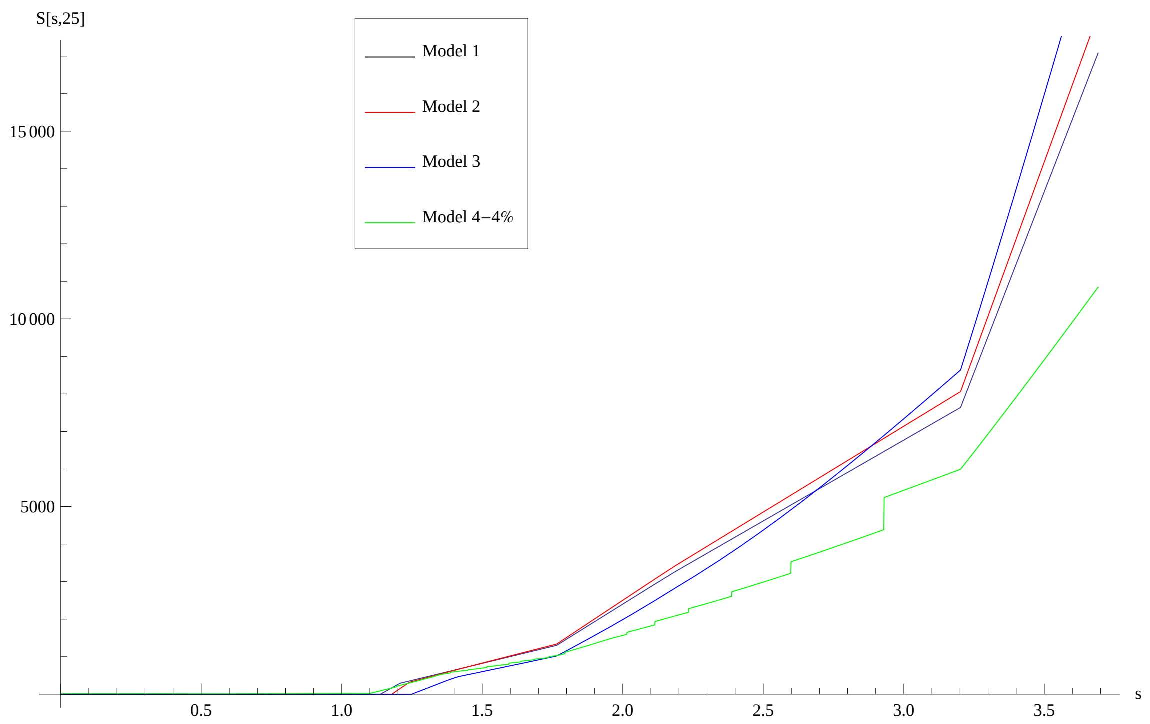


Figure 7 : Wealth distribution as a function of skill for the 4 models at t = 25

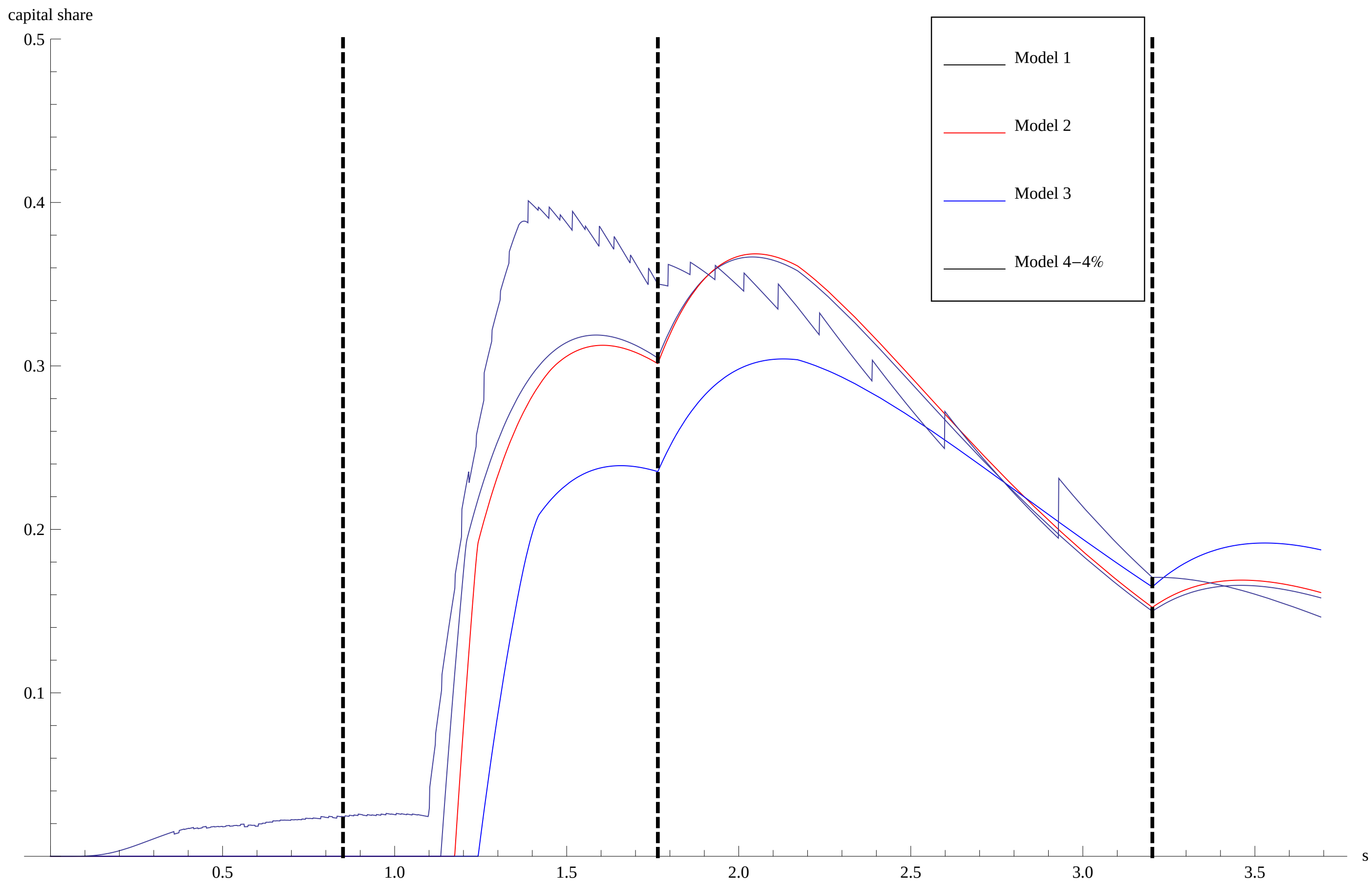


Figure 8 : Lorenz curves for the 4 models at t = 25

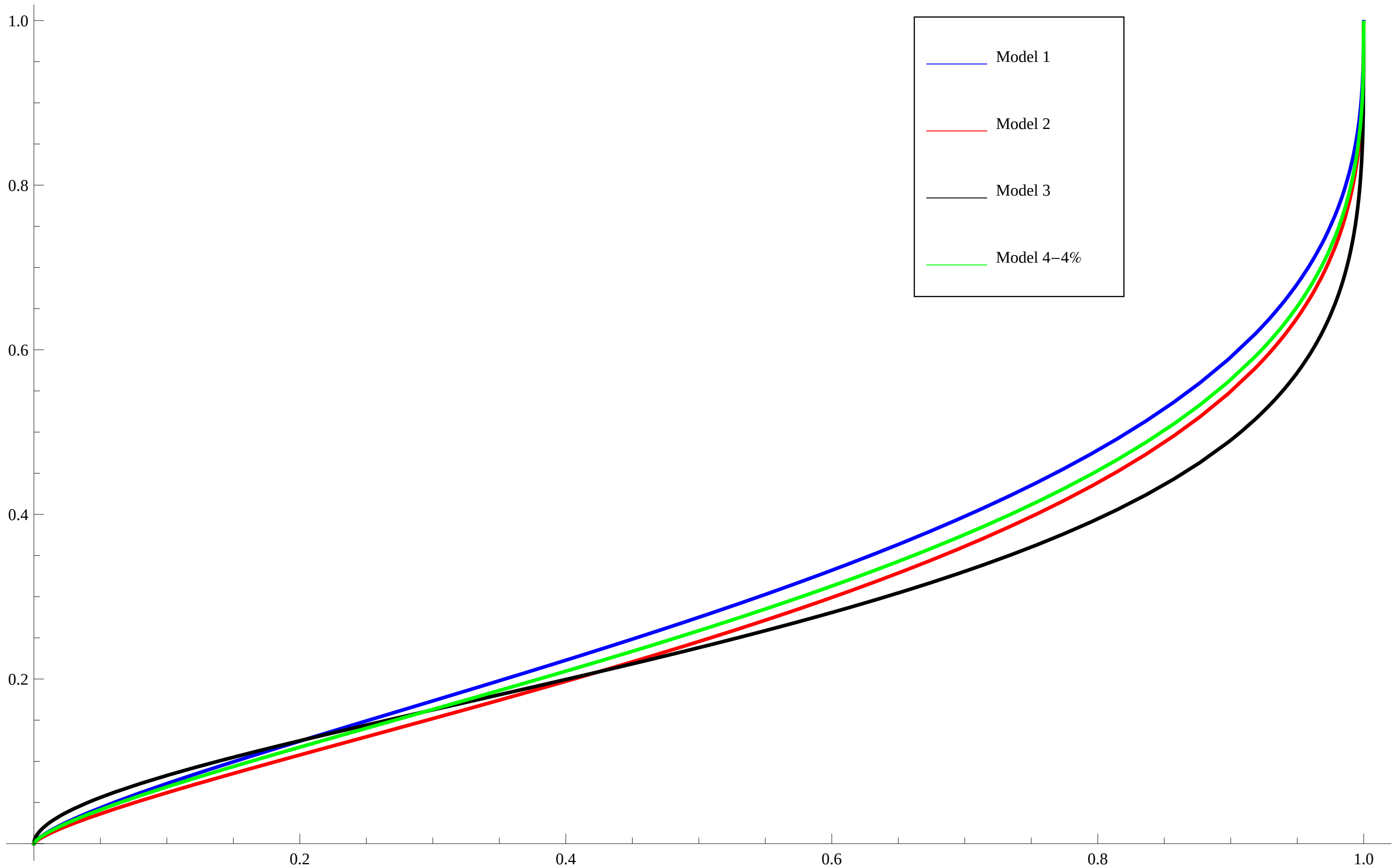


Table 7 : Top wealth shares from Saez – Zucman (S – Z) in 2012 and in our four models (2037)

Model	Top decile	Top centile	Top 0.1%
S–Z 2012	77.2	41.8	22.
1–2037	82.8	42.8	21.8
2–2037	84.4	43.8	22.3
3–2037	90.	53.8	28.5
4–2037	76.7	36.3	17.

Table 8 : Equilibrium results as a function of t for Model 1 with the Lognorma – Pareto skill distribution

t	r	w	GDP	K[t]	$\frac{K}{GDP}$	$\frac{(r+d) K}{GDP}$	$\frac{wL}{GDP}$	shareK, 0–50	shareK, 50–90	shareK,90–99	shareK top 1%	share top 0.1%
1	0.0635969	67.7615	110.704	490.3	4.42894	0.370246	0.629754	0.	0.221665	0.357912	0.420455	0.220813
2	0.0634755	67.8195	111.954	496.92	4.43861	0.370515	0.629485	0.	0.216157	0.361447	0.422429	0.22139
3	0.0633179	67.895	113.263	504.155	4.4512	0.370865	0.629135	0.	0.211332	0.364669	0.424032	0.221774
4	0.0631316	67.9847	114.622	511.922	4.46618	0.371281	0.628719	0.	0.207079	0.367627	0.425326	0.221997
5	0.0629224	68.0861	116.027	520.16	4.48309	0.371749	0.628251	0.	0.20331	0.370362	0.426361	0.222086
6	0.062695	68.197	117.474	528.82	4.5016	0.37226	0.62774	0.	0.199952	0.372907	0.427174	0.222062
7	0.0624528	68.3158	118.959	537.865	4.52144	0.372806	0.627194	0.	0.196948	0.375286	0.427798	0.221942
8	0.062199	68.4413	120.48	547.267	4.5424	0.373381	0.626619	0.	0.19425	0.377522	0.42823	0.221741
9	0.0619359	68.5723	122.034	557.	4.5643	0.37398	0.62602	0.	0.191818	0.379634	0.428551	0.221468
10	0.0616654	68.7081	123.621	567.048	4.587	0.374599	0.625401	0.	0.189621	0.381634	0.428748	0.221135
11	0.061389	68.8479	125.238	577.395	4.61038	0.375234	0.624766	0.	0.18763	0.383536	0.428837	0.220749
12	0.0611082	68.9912	126.886	588.032	4.63434	0.375883	0.624117	0.	0.185822	0.38535	0.42883	0.220317
13	0.060824	69.1374	128.562	598.946	4.65881	0.376544	0.623456	0.	0.184178	0.387085	0.428739	0.219844
14	0.0605373	69.2861	130.267	610.133	4.68371	0.377213	0.622787	0.	0.182681	0.38875	0.428572	0.219336
15	0.0602489	69.4371	132.	621.584	4.70898	0.37789	0.62211	0.	0.181316	0.39035	0.428337	0.218796
16	0.0599594	69.5899	133.76	633.297	4.73457	0.378574	0.621426	0.	0.18007	0.39189	0.428042	0.218229
17	0.0596695	69.7443	135.548	645.267	4.76044	0.379262	0.620738	0.	0.178934	0.393377	0.427692	0.217637
18	0.0593794	69.9001	137.362	657.492	4.78656	0.379954	0.620046	0.	0.177896	0.394814	0.427292	0.217024
19	0.0590897	70.0571	139.203	669.97	4.81288	0.380649	0.619351	0.	0.176949	0.396206	0.426847	0.216391
20	0.0588007	70.2152	141.072	682.699	4.83938	0.381347	0.618653	0.	0.176086	0.397555	0.426362	0.215741
21	0.0585127	70.3741	142.966	695.68	4.86604	0.382046	0.617954	0.	0.175299	0.398864	0.425839	0.215075
22	0.0582258	70.5338	144.888	708.911	4.89283	0.382746	0.617254	0.	0.174584	0.400136	0.425282	0.214396
23	0.0579403	70.6941	146.836	722.393	4.91974	0.383446	0.616554	0.	0.173933	0.401372	0.424691	0.213703
24	0.0576563	70.855	148.811	736.131	4.94676	0.384147	0.615853	0.	0.173346	0.402579	0.424077	0.213001
25	0.0573743	71.0162	150.812	750.112	4.97382	0.384846	0.615154	0.	0.172815	0.403753	0.423433	0.212289