Secret contracting in multilateral relations*

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Abstract

We develop a general, tractable framework of multilateral vertical contracting, which places no restriction on tariffs and fully accounts for their impact on downstream competition. Equilibrium tariffs are cost-based and replicate the outcome of a multi-brand oligopoly, a finding in line with the analysis of a recent merger.

We provide a micro-foundation for this framework, before analyzing the effect of RPM and price parity provisions, and of resale vs. agency business models. Finally, we extend the framework to endogenize the distribution network; we also consider mergers and show that their impact on the distribution network can dominate price effects.

JEL classification: L13, L42, D43, K21.

Keywords: bilateral contracting, vertical relationships, bargaining, vertical restraints, network formation, mergers.

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1 Introduction

We propose a flexible, tractable model of multilateral vertical contracting between upstream and downstream competitors, and extend it to endogenize the channel network.

Wholesale markets often involve interlocking multilateral relations. For instance, competing supermarkets carry the same rival brands, health insurers deal with the same care providers, and pay-TV operators offer the same channels. In intermediate goods markets, PC OEMs develop computers based on Intel and AMD chips, and Airbus and Boeing offer a choice of engines from General Electric, Rolls Royce and Pratt & Whitney. Yet, the vertical contracting literature mostly focuses on simpler market structures. For instance, much of the early literature focuses on an upstream or downstream monopolist,[1] or on competing vertical structures (e.g., franchise networks).[2]

Several papers consider multilateral relations, but with various restrictions. For instance, upstream competition comes from fringe suppliers[3] or perfect substitutes,[4] or contracts are restricted to linear or two-part tariffs.[5] Other papers, prompted by merger waves and policy debates in pay-TV[6] and healthcare[7] markets, either assume away the interplay between wholesale agreements and downstream outcomes (by restricting attention to lump-sum transfers), or account for it only partially (by assuming that upstream and downstream prices are set simultaneously).[8]

In the first part of this paper, we develop a model of multilateral interlocking relationships with upstream and downstream price competition,[9] allowing for any distribution of bargaining power. We do not restrict the tariffs that can be negotiated, and take into account their impact on downstream competition. As wholesale contracts are usually not publicly observable, we suppose that the negotiation outcomes are private information. Modelling secret contracting raises complex issues, even in simple bargaining settings with ultimatum offers. When receiving an out-of-equilibrium offer, a firm must conjec-

[1] See, e.g., Mathewson and Winter (1984) and Rey and Tirole (1986) on vertical coordination, Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994) on supplier's opportunism, and Bernheim and Whinston (1985, 1986, 1998), Marx and Shaffer (2007), Miklòs-Thal *et al.* (2011) and Rey and Whinston (2013) on exclusive dealing.

[2] See, e.g., Bonanno and Vickers (1988) and Rey and Stiglitz (1988, 1995) on strategic delegation, and Jullien and Rey (2007) and Piccolo and Miklòs-Thal (2012) on facilitating practices.

[3] This is a frequent assumption in the literature on private labels (see, e.g., Mills, 1995, and Gabrielsen and Sørgard, 2007). See also Hart and Tirole (1990) and Innes and Hamilton (2009).

[4] See, e.g., Salinger (1988), Ordover *et al.* (1990), de Fontenay and Gans (2005, 2014), and Nocke and White (2007, 2010).

[5] See, e.g., Dobson and Waterson (2007), Rey and Vergé (2010) and Allain and Chambolle (2011).

[6] See, e.g., Chipty and Snyder (1999) on the impact of horizontal mergers, Crawford and Yurukoglu (2012) on bundling, and Crawford *et al.* (2018) on vertical integration.

[7] See, e.g., Gowrisankaran *et al.* (2015) on hospital mergers, and Ho and Lee (2017) on competition among health insurance providers.

[8] Among the most recent papers, Gowrisankaran *et al.* (2015) and Ho and Lee (2017) focus on lump-sum transfers, whereas Crawford *et al.* (2018) assume that all (linear) prices are set simultaneously.

[9] Nocke and Rey (2018) study multilateral relations with Cournot downstream competition.

ture about the contracts signed by its rivals. As Bayesian updating does not restrict off-equilibrium beliefs, there are typically many (perfect Bayesian) equilibria. This has led the literature to rely on "reasonable" beliefs, such as passive or wary beliefs. Unfortunately, when downstream firms compete in prices, equilibria based on passive beliefs may not exist, and wary beliefs are rather intractable.[10] We define instead a bargaining equilibrium as follows. First, upstream negotiations are modelled using a "Nash-in-Nash" approach, which relies on the contract equilibrium concept developed by Crémer and Riordan (1987) and Horn and Wolinsky (1988): each contract is bilaterally efficient given the other equilibrium contracts, and the gains from trade are shared according to the parties' bilateral bargaining power.[11] Second, given their negotiated contracts, downstream firms compete in prices.

We first establish the existence of an equilibrium, and show that (as long as tariffs induce a "smooth behavior", in a sense made precise), equilibrium tariffs are *cost-based*, in that marginal input prices reflect marginal costs of production; as a result, downstream prices are the same as in a multi-brand oligopoly where downstream firms could produce all the inputs. The intuition is simple. To maximize their joint bilateral profit, which ignores the other firms' margins, firms adopt low input prices to enable the downstream firm to price aggressively. Conversely, if the other marginal input prices reflect marginal costs, then pricing at marginal cost makes the downstream firm a residual claimant for the joint bilateral profit and thus induces it to charge the bilateral optimal prices. Interestingly, this insight is in line with the results of Nilsen *et al.* (2016) who find that an upstream merger between Norwegian egg producers did not affect marginal input (and therefore retail) prices but only infra-marginal prices (e.g., franchise fees). Different tariffs generate different divisions of the equilibrium industry profit, however, more convex (resp., concave) tariffs giving a larger share to upstream (resp., downstream) firms.

We then provide a micro-foundation for these bargaining equilibria, which relies on a (non-cooperative, sequential) game of delegated negotiations: each firm has different agents negotiating with its different partners and, for each channel, one of the two agents is randomly selected to make a take-it-or-leave-it offer. Any bargaining equilibrium outcome can be sustained by a sequential equilibrium of this game of delegated negotiations; conversely, any "regular" equilibrium outcome of this game (in a sense made precise) corresponds to a bargaining equilibrium. Compared with the "direct negotiation game" in which firms would directly engage in bilateral negotiations (with the same random selection for the right to make a final offer), the candidate equilibria that survive

[10] Both issues arise even in the absence of upstream competition; see Rey and Vergé (2004).

[11] O'Brien and Shaffer (1992) apply this approach in an upstream monopoly setting. Since then, it has been used with various restrictions, both in the theoretical literature (e.g., Gans, 2007; Milliou and Petrakis, 2007; Allain and Chambolle, 2011) and the empirical literature (e.g., Crawford and Yurukoglu, 2012; Grennan, 2013; Gowrisankaran *et al.*, 2015). Because it combines the cooperative Nash-bargaining solution (for each vertical channel) with a non-cooperative Nash-equilibrium concept (across channels), Collard-Wrexler *et al.* (2019) have coined the terminology "Nash-in-Nash bargaining."

single-channel deviations are the same in the two games; assuming delegated negotiations however ensures existence, by ruling out multi-channel deviations. Adding a preliminary stage in which one side gets to make an offer, with the above lottery being used only if that offer is rejected, moreover generates deterministic outcomes and provides a similar micro-foundation for linear tariffs and/or publicly observable contracts.

To illustrate the flexibility of our approach, we study the impact of classic vertical restraints such as resale price maintenance (RPM) and price parity agreements (PPAs). Allowing RPM generates many equilibria: as retail prices are separately negotiated, firms can agree on any arbitrary marginal transfer prices (and share the profits as desired through, e.g., lump-sum fees), which however affect their negotiations with other partners. Furthermore, if price floors can sustain supra-competitive prices when brands are more substitutable than stores, price ceilings can do the same in the opposite case. This finding challenges the current antitrust approach towards RPM, which views inter-brand competition as likely to prevent anti-competitive effects, and treats maximum RPM more favorably than minimum RPM. By contrast PPAs, which require retailers to charge the same price for all brands, have no substantial impact on retail prices. They may limit the joint profit that a retailer can generate with a given supplier, but pricing at marginal cost still makes the retailer the residual claimant on this joint profit; as a result, equilibrium contracts are again cost-based. This contrasts with the view, common in policy circles, that retail PPAs are akin to RPM and should therefore be banned; it also suggests that the anti-competitive effects highlighted by the literature depends on the nature of the contracts that are considered (e.g., linear vs. non-linear tariffs).

We also use our approach to compare business models. Switching from the traditional resale model to the agency model often used by online marketplaces (where retail platforms obtain transaction-based commissions from suppliers) amounts to turning the model "upside-down". Platforms now play the role of upstream firms selling distribution services to suppliers who control the final prices and thus act as downstream firms. Equilibrium tariffs are again cost-based and the final outcome is the same as if suppliers were directly competing against each other at all retail locations. Whether equilibrium prices are higher under the wholesale model or the agency model is thus driven by whether competition is fiercer among suppliers or retailers.

In the second part of our paper, we endogenize the channel network. As the Nash-in-Nash approach implies that every channel is active in equilibrium,[12] we add a preliminary stage where each firm can choose which channels to activate. This determines the relevant network and the associated bargaining equilibrium. To avoid coordination issues, we focus on the coalition-proof Nash equilibria (CPNE) – see Bernheim *et al.* 1987.

In a setting with symmetric duopolies, we first show that, when downstream firms are

[12] See, e.g., de Fontenay and Gans (2014) and Collard-Wexler *et al.* (2019).

largely differentiated, each supplier deals with both to maximize demand. When instead they are close substitutes, each supplier deals with a single firm, to avoid profit dissipation through intrabrand competition. For the case of linear demands, there is always a unique CPNE, with the complete network if downstream firms are sufficiently differentiated, and exclusive dealing (each supplier dealing with a different downstream firm) otherwise.

Finally, we study the impact of mergers on prices and on the channel network. For any given network, and absent any efficiency gains, a downstream merger raises prices (by eliminating downstream competition between the merging parties), whereas an upstream merger has no impact on final prices (as marginal input prices remain cost-based). However, accounting for the possible impact on the distribution network can give rise to very different insights. Pre-merger, firms may limit the number of active channels to avoid profit dissipation through intrabrand competition. A *downstream merger* eliminates this concern and thus encourages firms to *expand the network*; as a result, such a merger may actually benefit consumers and increase social welfare. By contrast, an *upstream merger* enables the suppliers to coordinate their distribution decisions and can trigger *vertical foreclosure*, which harms consumers and decreases social welfare. Finally, a vertical merger induces the integrated supplier to charge higher (marginal) prices to rivals, which tends to raise retail prices and reduce consumer surplus and social welfare, but it may also expand or restrict the distribution network.

The paper is organized as follows. We first outline our setting (Section 2), characterize the bargaining equilibria (Section 3), and provide a micro-foundation (Section 4), before studying vertical restraints and alternative business models (Section 5). We then endogenize the channel network (Section 6), and examine the impact of mergers in this extended setting (Section 7). Finally, we apply our approach to publicly observable contracts (Section 8), before providing concluding remarks (Section 9).

2 The model

2.1 Setup

We consider a vertical chain in which $n \geq 2$ differentiated manufacturers, $M_1, ..., M_n$, distribute their goods through $m \geq 2$ differentiated retailers, $R_1, ..., R_m$.[13] For the sake of exposition, we assume constant returns to scale[14] and denote M_i's unit cost by c_i, for $i \in \mathcal{I} \equiv \{1, ..., n\}$, and R_j's unit cost by γ_j, for $j \in \mathcal{J} \equiv \{1, ..., m\}$.[15] The demand for brand i at store j (i.e., for "channel" $i - j$) is given by $D_{ij}(\mathbf{p})$ and is continuously differentiable in the price vector $\mathbf{p} = (p_{ij})_{(i,j) \in \mathcal{I} \times \mathcal{J}}$ whenever it is positive.[16]

[13] The analysis can be transposed to other vertically related industries.

[14] Allowing for non-linear cost functions is straightforward but notationally cumbersome.

[15] For ease of exposition, we use subscripts i and h for manufacturers, and j and k for retailers.

[16] This allows for "kinks" where demand becomes zero (e.g., when demand is linear).

We assume that wholesale contracts are purely "vertical": the contract between M_i and R_j specifies a transfer, t_{ij}, based solely on M_i's sales through R_j, q_{ij}. This excludes "horizontal" clauses such as exclusive dealing or market-share discounts,[17] but allows for any non-linear tariff $t_{ij}(q_{ij})$. We moreover focus on secret contracting: the terms negotiated between M_i and R_j (including whether they reached an agreement) are private information to the two parties. Finally, we assume that wholesale negotiations can influence retail pricing decisions. This leads us to consider the following timing:

Stage 1: Each M_i negotiates with each R_j a non-linear tariff $t_{ij}(q_{ij})$; these bilateral negotiations are simultaneous and secret.

Stage 2: Retailers simultaneously set retail prices for all the brands that they carry.

2.2 Bargaining equilibrium

As mentioned in the introduction, for tractability we follow the contract equilibrium approach pioneered by Crémer and Riordan (1987) and Horn and Wolinsky (1988), which requires contracts to be bilaterally efficient. We moreover allow for balanced bargaining, and denote by $\alpha_{ij} \in [0,1]$ the bargaining power of M_i in its bilateral negotiation with R_j. Specifically, in stage 2, each R_i chooses its prices, given the contracts it negotiated in the previous stage, and assuming that its rivals set the equilibrium retail prices. In stage 1, each M_i and each R_j negotiates a tariff that: (i) maximizes their joint profit, given the other equilibrium contracts and R_j's induced retail pricing behavior; and (ii) gives a share α_{ij} of the resulting gains from trade to M_i (and thus a share $1-\alpha_{ij}$ to R_i).

To state this formally, let us express the price vector as $\mathbf{p} = (\mathbf{p}_j, \mathbf{p}_{-j})$, where $\mathbf{p}_j = (p_{hj})_{h\in\mathcal{I}} = (p_{ij}, \mathbf{p}_{-i,j})$[18] is the vector of R_j's prices and $\mathbf{p}_{-j}$ the vector of all other retailers' prices. A "bargaining equilibrium" is then defined as follows:

Definition 1 (bargaining equilibrium) *A bargaining equilibrium is a vector of price responses $(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$, together with a vector of equilibrium tariffs $\mathbf{t^e} = (\mathbf{t}_j^{\mathbf{e}})_{j\in\mathcal{J}}$ and a vector of equilibrium prices $\mathbf{p^e} = (\mathbf{p}_j^{\mathbf{e}})_{j\in\mathcal{J}}$, such that:*

- *In stage 2, for every $j \in \mathcal{J}$, the price response $\mathbf{p}_j^R(\cdot)$:*
  - *maximizes R_j's profit for any $\mathbf{t}_j = (t_{ij})_{i\in\mathcal{I}}$ negotiated by R_j in stage 1,[19] given rivals' equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$:*
  $$\mathbf{p}_j^R(\mathbf{t}_j) \in \arg\max_{\mathbf{p}_j} \sum_{i\in\mathcal{I}} [(p_{ij} - \gamma_j) D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}) - t_{ij}(D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}))];$$
  - *satisfies $\mathbf{p}_j^R(\mathbf{t}_j^{\mathbf{e}}) = \mathbf{p}_j^{\mathbf{e}}$.*

[17] We consider price parity and other provisions in Section 5.

[18] With the convention that $p_{ij} = \infty$ when R_j does not carry M_i's brand.

[19] We assume that a tariff can be successfully negotiated only if it induces a well-behaved retail pricing problem. Alternatively, we could restrict attention to continuous tariffs and bounded demands (e.g., the monopoly demand for channel $i-j$ is finite for $p_{ij} = 0$ and becomes null for p_{ij} large enough).

- *In stage 1, for every $(i,j) \in \mathcal{I} \times \mathcal{J}$, the equilibrium tariff t_{ij}^e:*
  - *maximizes the joint profit of M_i and R_j, given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^{\mathbf{e}}$, its rivals' equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$, and R_j's price response, $\mathbf{p}_j^R(\mathbf{t}_j)$:*

$$t_{ij}^e \in \arg\max_{t_{ij}} \left\{ \begin{array}{c} [p_{ij}^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}) - c_i - \gamma_j] D_{ij}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}), \mathbf{p}_{-j}^{\mathbf{e}}) \\ + \sum_{k \in \mathcal{J} \setminus \{j\}} \left[\begin{array}{c} t_{ik}^e(D_{ik}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}), \mathbf{p}_{-j}^{\mathbf{e}})) \\ -c_i D_{ik}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}), \mathbf{p}_{-j}^{\mathbf{e}}) \end{array} \right] \\ + \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\begin{array}{c} [p_{hj}^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}) - \gamma_j] D_{hj}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}), \mathbf{p}_{-j}^{\mathbf{e}}) \\ -t_{hj}^e(D_{hj}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}), \mathbf{p}^{\mathbf{e}})) \end{array} \right] \end{array} \right\};$$

  - *gives M_i and R_j shares α_{ij} and $1-\alpha_{ij}$ respectively, of the additional profit generated by their relationship.*

2.3 Benchmark: multiproduct oligopoly

For future reference, it is useful to consider a hypothetical multiproduct oligopoly in which m differentiated firms $j \in \mathcal{J}$ could each produce at cost the n brands $i \in \mathcal{I}$. Let:

$$\pi_{ij}(\mathbf{p}) \equiv (p_{ij} - c_i - \gamma_j) D_{ij}(\mathbf{p}) \quad \text{and} \quad \pi_j(\mathbf{p}) \equiv \sum_{i \in \mathcal{I}} \pi_{ij}(\mathbf{p})$$

denote the profit that firm j then derives from brand i and in total, and let:

$$\mathbf{p}_j^r(\mathbf{p}_{-j}) \equiv \arg\max_{\mathbf{p}_j} \pi_j(\mathbf{p}_j, \mathbf{p}_{-j})$$

denote its best-response. We maintain the following regularity conditions:[20]

Assumption A (multiproduct oligopoly) There is a unique price vector $\mathbf{p}^*$ satisfying $\mathbf{p}_j^* \in \mathbf{p}_j^r(\mathbf{p}_{-j}^*)$ for $j \in \mathcal{J}$; this vector is moreover uniquely characterized by the first-order conditions, and such that $\mathbf{p}_j^* = \mathbf{p}_j^r(\mathbf{p}_{-j}^*)$ for $j \in \mathcal{J}$.[21] Furthermore, for every $(i,j) \in \mathcal{I} \times \mathcal{J}$:

(i) $D_{ij}(\mathbf{p}^*) > 0$; and,

(ii) $\sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}((\infty, \mathbf{p}_{-i,j}^*), \mathbf{p}_{-j}^*) > \sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}(\mathbf{p}^*)$; and

(iii) $\pi_j((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}^*)$ has a finite maximum in $\mathbf{p}_{-i,j}$.

Assumption A asserts that the hypothetical multiproduct oligopoly would have a unique equilibrium, with the usual features of product differentiation: each firm sells all

[20] See Vives (1999) for a discussion of the underlying assumptions on demand.

[21] That is, $\mathbf{p}^*$ is the unique solution to the set of first-order conditions $\{\partial \pi_j / \partial p_{ij} = 0\}_{i \in I, j \in J}$, and best-responses to equilibrium prices are also unique.

brands, but if it were to drop one brand, then it would earn more on the others. This implies that the contribution of any brand i to any firm j's profit, given by

$$\Delta_j^i \equiv \pi_j(\mathbf{p}^*) - \max_{\mathbf{p}_{-i,j}} \pi_j((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}^*),$$

is positive but lower than the equilibrium profit (see Lemma 1 in Appendix A):

$$0 < \Delta_j^i < \pi_{ij}(\mathbf{p}^*). \tag{1}$$

3 Equilibrium analysis

O'Brien and Shaffer (1992) show that, in the case of an upstream monopoly, secret contracting yields "cost-based" equilibrium tariffs: marginal wholesale prices reflect marginal costs. We now show that this insight carries over when there is upstream competition.

3.1 Two-part tariffs

We first establish the existence of a bargaining equilibrium in *cost-based two-part tariffs*, yielding the same retail outcome as the above hypothetical multiproduct oligopoly:

Proposition 1 (cost-based two-part tariffs) *There exists a unique equilibrium in which contracts are cost-based two-part tariffs; in this equilibrium:*

(i) $\mathbf{p^e} = \mathbf{p}^*$ *and, for every* $(i,j) \in \mathcal{I} \times \mathcal{J}$, $t_{ij}^e(q_{ij}) = t_{ij}^*(q_{ij}) \equiv \alpha_{ij}\Delta_j^i + c_i q_{ij}$*;*

(ii) for every $(i,j) \in \mathcal{I} \times \mathcal{J}$, M_i*'s and* R_j*'s equilibrium profits are respectively equal to:*

$$\Pi_{M_i}^e = \Pi_{M_i}^* \equiv \sum_{j \in \mathcal{J}} \alpha_{ij}\Delta_j^i \geq 0 \quad \textit{and} \quad \Pi_{R_j}^e = \Pi_{R_j}^* \equiv \pi_j(\mathbf{p}^*) - \sum_{i \in \mathcal{I}} \alpha_{ij}\Delta_j^i > 0.$$

Proof. See Appendix A. ∎

The intuition is simple. If the other channels adopt such tariffs, then the joint variable profit of M_i and R_j accounts for the full margins on R_j's sales of all brands, and only for those. To maximize this profit, it suffices to make R_j the residual claimant, which a cost-based tariff precisely achieves. All retailers then behave as if supplied at cost.

M_i obtains a share α_{ij} of the gains from trade, $\Delta_j^i > 0$, and its profit is thus positive whenever $\alpha_{ij} > 0$. However, as tariffs are cost-based, if R_j were to delist M_i, then R_j would benefit from the increase in the sales of rival brands, whereas M_i would not benefit from the increase in the sales of its brand through the other retailers. As a result, R_j obtains more than suggested by its intrinsic bargaining power: $\pi_{ij}(\mathbf{p}^*) - \alpha_{ij}\Delta_j^i > (1 - \alpha_{ij})\pi_{ij}(\mathbf{p}^*)$; in particular, it always obtains a positive profit, regardless of its bilateral bargaining power.

3.2 Equilibrium prices

Proposition 1 establishes the existence of a bargaining equilibrium in two-part tariffs, in which tariffs are cost-based. We now show that, conversely, as long as they induce a "smooth" retail behavior, equilibrium tariffs must be cost-based.

To introduce the notion of smooth retail behavior, fix a candidate bargaining equilibrium with tariffs $\mathbf{t}^{\mathbf{e}}$ and retail prices $\mathbf{p}^{\mathbf{e}}$, and suppose that the negotiation between M_i and R_j over the tariff t_{ij} induces instead R_j to sell a given quantity

$$q_{ij} \in Q_{ij} \equiv \{D_{ij}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}) \mid \mathbf{p}_j \in \mathbb{R}^n_+\}.$$

As the tariff t_{ij} affects R_j's profit only through $t_{ij}(q_{ij})$, R_j's price response, *conditional on selling* q_{ij} – and regardless of the tariff t_{ij} used to induce q_{ij} – is given by:[22]

$$\hat{\mathbf{p}}^{ij}_j(q_{ij}) \equiv \underset{\mathbf{p}_j \in \mathbf{P}^{ij}_j(q_{ij})}{\arg\max} \{(p_{ij} - \gamma_j)q_{ij} + \sum_{h \in \mathcal{I} \setminus \{i\}} \begin{array}{c} (p_{hj} - \gamma_j)D_{hj}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}) \\ -t^e_{hj}(D_{hj}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j})) \end{array} \}. \quad (2)$$

where $\mathbf{P}^{ij}_j(q_{ij}) \equiv \{\mathbf{p}_j | D_{ij}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}) = q_{ij}\}$ denotes the set of prices yielding q_{ij}. Let

$$\hat{q}^{ij}_{hk}(q_{ij}) \equiv D_{hk}(\hat{\mathbf{p}}^{ij}_j(q_{ij}), \mathbf{p}^{\mathbf{e}}_{-j})$$

denote the resulting quantities and

$$\hat{r}^{ij}_j(q_{ij}) \equiv \sum_{h \in \mathcal{I}} [\hat{p}^{ij}_{hj}(q_{ij}) - \gamma_j]\hat{q}^{ij}_{hj}(q_{ij})$$

denote the associated revenue for R_j, net of retail costs.

We say that the equilibrium tariffs $\mathbf{t}^{\mathbf{e}}_j$ induce R_j to adopt a smooth retail behavior if the conditional price responses $\hat{\mathbf{p}}^{ij}_j(q_{ij})$ satisfy the following conditions:

Definition 2 (smooth retail behavior) *For any $j \in \mathcal{J}$, the equilibrium tariffs $\mathbf{t}^{\mathbf{e}}_j$ induce a smooth retail behavior if they are differentiable and, for every $i \in \mathcal{I}$:*

(i) in the range $q_{ij} \in Q_{ij}$, $\hat{\mathbf{p}}^{ij}_j(q_{ij})$ is unique and differentiable;

(ii) $q^e_{ij} \in Int(Q_{ij})$ and the "diversion ratios" $\delta^{ij}_{jk} \equiv -(\hat{q}^{ij}_{ik})'(q^e_{ij})$, for $k \in \mathcal{J} \setminus \{j\}$, satisfy:

$$\delta^{ij}_{jk} \geq 0 \text{ for every } k \in \mathcal{J} \setminus \{j\} \text{ and } \sum_{k \in \mathcal{J} \setminus \{j\}} \delta^{ij}_{jk} < 1. \quad (3)$$

That is, when a retailer contemplates a marginal increase in the sales of a brand, it only marginally adjust its prices; condition (3) moreover asserts that total sales of the

[22] The superscript ij refers here to the bilateral negotiation between M_i and R_j.

brand would increase despite reduced sales through rival retailers.[23] The next Proposition shows that if the equilibrium tariffs induce a smooth retail behavior, then they must be cost-based, implying that the retail outcome remains the same as before:

Proposition 2 (cost-based tariffs) *Whenever the equilibrium tariffs induce all retailers to adopt a smooth retail behavior:*

(i) $(t_{ij}^e)'\left(q_{ij}^e\right) = c_i$ *for every* $(i,j) \in \mathcal{I} \times \mathcal{J}$*; and*

(ii) $\mathbf{p^e} = \mathbf{p}^*$.

Proof. See Appendix B. ■

The insight of Proposition 1 thus carries over to any equilibrium based on marginal considerations. The intuition is more involved and relies on an equilibrium argument: when negotiating with one retailer, a manufacturer has an incentive to undercut the margins charged to the other retailers; thus, in equilibrium, all upstream margins must be zero. To see this, consider a candidate equilibrium with arbitrary (smooth) upstream margins, $u_{ij}^e \equiv (t_{ij}^e)'\left(q_{ij}^e\right) - c_i$. In their negotiation, M_i and R_j seek to induce the quantity q_{ij} that maximizes their joint profit, taking as given that R_j will adjust its prices so as to maximize its own profit. However, q_{ij} must also maximize R_j's profit given the tariffs it faces; M_i's upstream margin must therefore neutralize the marginal impact of q_{ij} on its own profit. As decreasing q_{ij} by one unit would increase M_i's sales through every other R_k by δ_{jk}^{ij}, the negotiated margin must satisfy:

$$u_{ij}^e = \sum_{k \in \mathcal{J} \setminus \{j\}} \delta_{jk}^{ij} u_{ik}^e.$$

From (3), the sale lost by R_j would only partially be compensated by the other retailers' additional sales, and thus u_{ij}^e is a contraction of M_i's other margins – that is, the margin negotiated with R_j "undercuts" the margins that M_i charges to R_j's rivals. Hence, in equilibrium, all upstream margins are zero.

Remark: smooth retail behavior. In the case of an upstream monopoly, O'Brien and Shaffer (1992) show that equilibrium tariffs always induce a smooth retail behavior. Unfortunately, their reasoning does not carry over to the case of upstream competition, as R_j's response to M_i's tariff, say, now depends on M_i's rivals' tariffs; hence, R_j's response may no longer be "smooth" if, for instance, the other tariffs are discontinuous. Yet, we suspect that equilibrium tariffs are indeed likely to induce a smooth retail behavior.

Remark: two-part tariffs. Under standard regularity assumptions on demand, two-part tariffs induce a smooth retailer behavior; the equilibrium of Proposition 1 is then the unique equilibrium in two-part tariffs.

[23] Condition (3) is natural but not necessary; a weaker sufficient condition is the invertibility of the $m \times m$ matrix $\boldsymbol{\delta}^i$ with entries $\boldsymbol{\delta}^i(j,j) = 1$ and $\boldsymbol{\delta}^i(j,k) = -\delta_{ik}^{ij}$ for $k \neq j$.

3.3 Division of profits

Proposition 2 shows that, as long as tariffs induce all retailers to adopt a smooth retail behavior, there is a unique equilibrium outcome in terms of prices and quantities, and industry profit. Together with Proposition 1, it shows further that the division of this profit is also unique when two-part tariffs are used. However, other tariffs can sustain different profit allocations. For instance, under mild regularity assumptions[24], there exist bargaining equilibria that rely on the quadratic tariffs, $t_{ij}^{\sigma}(q_{ij}) = t_{ij}^{*}(q_{ij}) + \sigma(q_{ij} - q_{ij}^{*})^2$, where $(t_{ij}^{*})_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ are the cost-based two-part tariffs identified in Proposition 1 and $q_{ij}^{*} \equiv D_{ij}(\mathbf{p}^{*})$ denote the equilibrium quantities. Introducing the quadratic term does not affect the amount paid by R_j if it sticks to q_{ij}^{*}, but increases the amount that R_j would have to pay if it were to modify its prices and/or stop carrying another brand. It follows that introducing this convex term weakens R_j's bargaining position in its negotiations with the other suppliers. Conversely, manufacturers obtain a smaller share when the tariffs are concave (i.e., when $\sigma < 0$).

4 Micro-foundation

We now show that a bargaining equilibrium is an equilibrium outcome of a non-cooperative game in which each side makes a take-it-or-leave-it offer with a probability reflecting its bargaining power.[25] As already mentioned, with secret contracting these games have many equilibria; for example, any individually rational outcome can be sustained by "pessimistic" beliefs that interpret a deviant offer as a signal of aggressive offers to rivals, making the deviant offer likely to be rejected. This has led the literature to focus on specific beliefs. The above contract equilibrium approach is in line with "passive beliefs": a channel assumes that the others stick to the equilibrium tariffs when negotiating its own contract.[26] Unfortunately, when downstream firms compete in prices, multi-sided deviations may destroy all candidate equilibria with passive beliefs – even in the simpler case of an upstream monopoly.[27] To avoid this, we adopt a setting in which firms delegate the negotiations to partner-specific agents.[28] Specifically, each M_i has m agents, $M_i^1, ..., M_i^m$, each R_j has n agents, $R_j^1, ..., R_j^n$, and the negotiation between M_i and R_j is handled by M_i^j and R_j^i, each agent seeking to maximize the profit of its firm. The firms and their agents play the following "delegated negotiations" game Γ:

[24] For a complete analysis, see Online appendix A.

[25] See Collard-Wexler *et al.* (2019) for a micro-foundation of the Nash-in-Nash approach when the gains from trade are determined by the network of active channels. In our setting, however, the tariffs themselves also affect these gains, through their impact on downstream competition.

[26] See McAfee and Schwartz (1994); Hart and Tirole (1990) call it "market-by-market bargaining".

[27] See McAfee and Schwartz (1995) and Rey and Vergé (2004), as well as footnote 35.

[28] See the remark at the end of this section for further discussion of the role of delegated negotiations.

Stage 1. Two-step negotiations:

First step. For each $M_i - R_j$ pair, Nature randomly selects which side gets to make a take-it-or-leave it offer: M_i^j is selected with probability α_{ij}, and R_j^i is selected with complementary probability $1 - \alpha_{ij}$; the selection is only observed by the two agents, M_i^j and R_j^i, and selections are made independently across pairs.

Second step. For each $M_i - R_j$ pair, the selected agent, M_i^j or R_j^i, offers a tariff $t_{ij}(q_{ij})$ to its counterpart, who accepts or rejects it; all offers are simultaneous and secret, and all acceptance decisions are also simultaneous and secret.

Stage 2. Each R_j observes the tariffs negotiated by its agents, or the lack thereof; retailers then simultaneously set retail prices for the brand(s) that they carry.

We look for the sequential equilibria of this game Γ, which requires beliefs to be "consistent".[29] This implies that a deviation by one player conveys no information on other players' simultaneous moves.[30] Hence, in stage 1, the receiver of a deviant offer does not revise its beliefs about the tariffs negotiated by the other agents; and in stage 2, a retailer that faces a deviant contract believes that the other retailers still face the equilibrium tariffs, and will therefore stick to the equilibrium retail prices.

Let $\theta_{ij} \in \Theta_{ij} \equiv \{M_i^j, R_j^i\}$ denote the agent selected for making the offer in the negotiation between M_i and R_j, $\boldsymbol{\theta}_j \equiv (\theta_{ij})_{i\in\mathcal{I}} \in \Theta_j \equiv \Pi_{i\in\mathcal{I}}\Theta_{ij}$ denote the profile of selected agents in the negotiations between R_j and its suppliers, and $\boldsymbol{\theta} \equiv (\boldsymbol{\theta}_j)_{j\in\mathcal{J}} \in \Theta \equiv \Pi_{j\in\mathcal{J}}\Theta_j$ denote the profile of selected agents in all negotiations. Formally, a (sequential) equilibrium of game Γ is a vector of price responses, $(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$, together with a vector of equilibrium tariffs, $(\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ (where $\hat{\mathbf{t}}^{\boldsymbol{\theta}} = (\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j})_{j\in\mathcal{J}}$), a vector of equilibrium prices, $(\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ (where $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = (\hat{\mathbf{p}}_j^{\boldsymbol{\theta}_j})_{j\in\mathcal{J}}$), and beliefs $\mathbf{b} \equiv \{(b_{M_i^j}, b_{R_j^i})_{(i,j)\in\mathcal{I}\times\mathcal{J}}, (b_{R_j})_{j\in\mathcal{J}}\}$, such that:[31]

(i) In stage 2, for every $j \in \mathcal{J}$:

- R_j expects its rivals to face the equilibrium tariffs and charge the equilibrium prices;
- for any vector of tariffs $\mathbf{t}_j$ negotiated by R_j's agents in stage 1, the price response $\hat{\mathbf{p}}_j^R(\mathbf{t}_j)$ maximizes R_j's expected profit, given the other retailers' equilibrium prices;
- for every $\boldsymbol{\theta}_j \in \Theta_j$, $\hat{\mathbf{p}}_j^{\boldsymbol{\theta}_j} = \hat{\mathbf{p}}_j^R(\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j})$.

(ii) In stage 1, for every $i \in \mathcal{I}$, every $j \in \mathcal{J}$ and every selected agent $\theta_{ij} \in \Theta_{ij}$, letting $\tilde{\theta}_{ij} \in \Theta_{ij} \setminus \{\theta_{ij}\}$ denote the non-selected agent:

[29] See Kreps and Wilson (1982).

[30] Fudenberg and Tirole (1991) refer to this principle as "no-signaling-what-you-don't-know".

[31] Sequential equilibria have been defined for finite action spaces. We adapt here the definition by focusing on equilibrium tariffs and unilateral deviations from these tariffs.

- θ_{ij} and $\tilde{\theta}_{ij}$ (regardless of the tariff t_{ij} offered by θ_{ij}) believe that all other agents stick to their equilibrium behavior; they thus expect that R_j will charge $\hat{\mathbf{p}}_j^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}})$ whereas its rivals will stick to the equilibrium prices, $\hat{\mathbf{p}}_{-j}^{\boldsymbol{\theta}_{-j}} = (\hat{\mathbf{p}}_k^{\boldsymbol{\theta}_k})_{k \in \mathcal{J} \setminus \{j\}}$;

- $\tilde{\theta}_{ij}$ accepts any tariff t_{ij} that does not decrease the expected profit of its firm;

- θ_{ij} offers a tariff that maximizes the expected profit of its firm.[32]

A key feature of this game is that, as firms can share profit (e.g., through lump-sum transfers), their agents always seek to maximize their joint profit, regardless of which side makes the offer. That is, the tariff t_{ij} negotiated by M_i^j and R_j^i induces R_j to maximize M_i and R_j's joint profit, as in a bargaining equilibrium. Of course, which side makes the offer affects how the profit is shared: the offering side appropriates the bilateral gains from trade, leaving the receiving side indifferent between accepting or rejecting the offer. The probability α_{ij} therefore plays the same role as M_i's bargaining power in its bilateral relationship with R_j. Building on this, we show below that *any bargaining equilibrium can be replicated as an equilibrium of game* Γ; the converse moreover holds for any equilibrium of game Γ with "regular" tariffs and price responses, defined as follows:

Definition 3 (regular price responses and tariffs) *In game* Γ*:*

(i) the price responses $(\hat{\mathbf{p}}_j^R)_{j \in \mathcal{J}}$ *are said to be regular if they are invariant to lump-sum changes in tariffs: for any* $j \in \mathcal{J}$*, any tariffs* $\mathbf{t}_j$ *and any vector of fixed fees* $\mathbf{f} = (f_{ij})_{i \in \mathcal{I}} \in \mathbb{R}^n$*,* $\hat{\mathbf{p}}_j^R(\mathbf{t}_j + \mathbf{f}_j) = \hat{\mathbf{p}}_j^R(\mathbf{t}_j)$*; and*

(ii) the tariffs $\mathbf{t}^{\boldsymbol{\theta}}$ *are said to be regular if they depend on which side makes the offer only through a lump-sum transfer: for any* $(i,j) \in \mathcal{I} \times \mathcal{J}$*,* $t_{ij}^{M_i^j}(q_{ij}) - t_{ij}^{R_j^i}(q_{ij})$ *does not depend on* q_{ij}*.*

Price responses are trivially regular when best-responses are unique; when instead a retailer is indifferent between several optimal prices, imposing regularity amounts to making the actual price response independent of lump-sum changes in the retailer's expected profit function. Tariffs are regular when which side makes the offer does not affect firms' bargaining positions with other partners.[33] Together, these two requirements imply that bilateral bargaining power has no impact either on retail prices: $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$ for any $\boldsymbol{\theta} \in \Theta$.

The next Proposition establishes an equivalence between the above bargaining equilibria and the sequential equilibria of game Γ that have regular price responses and tariffs:

[32] Without loss of generality, attention can be restricted to acceptable tariffs, as the "null" tariff $t_{\emptyset}$, equal to 0 for $q_{ij} > 0$ and to $+\infty$ for $q_{ij} > 0$, is acceptable and mimics rejection.

[33] In equilibrium, each channel is typically indifferent between many cost-based tariffs (e.g., convex vs. concave), but the adopted shape drives the outcome of firms' other negotiations; see Section 3.3.

Proposition 3 (micro-foundation)

(*i*) *For any bargaining equilibrium $\mathcal{B}$, there exists a sequential equilibrium of game* Γ, *with regular price responses and tariffs, that yields the same retail outcome and gives all firms the same expected profits as $\mathcal{B}$.*

(*ii*) *Conversely, for any sequential equilibrium $\mathcal{E}$ of game* Γ *with regular price responses and tariffs, there exists a bargaining equilibrium yielding the same retail outcome and giving all firms the same expected profits as $\mathcal{E}$.*

Proof. See Online appendix B. ■

Any bargaining equilibrium $\{(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}, \mathbf{t}^\mathbf{e}, \mathbf{p}^\mathbf{e}\}$ can thus be replicated as a sequential equilibrium of game Γ with regular price responses and tariffs. The proof is constructive and relies on contingent tariffs, $\hat{t}_{ij}^{M_i^j} = t_{ij}^e + F_{ij}^{R_i}$ and $\hat{t}_{ij}^{R_j^i} = t_{ij}^e - F_{ij}^{M_i}$, where the fees $F_{ij}^{M_i}$ and $F_{ij}^{R_j}$ leave the receiving agent indifferent between accepting or rejecting the offer. The construction also relies on price responses that coincide with $(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$ when profits are single-peaked, and may otherwise slightly differ to ensure their regularity.

Conversely, any sequential equilibrium of game Γ with regular price responses $(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$ and regular tariffs $(\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ (implying that the equilibrium prices satisfy $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$ for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$) can be replicated as a bargaining equilibrium. There again, the proof is constructive and relies on the same price responses and on the expected tariffs $t_{ij}^e = E_{\theta_{ij}}[\hat{t}_{ij}^{\boldsymbol{\theta}_{ij}}]$.

Remark: on the role of delegated negotiations. To assess the role of delegation, consider the "direct negotiations" game Γ^D, derived from Γ by assuming that each firm assigns the same agent for all bilateral negotiations. The receiver of an unexpected offer may then wonder about what the deviating firm is offering to the others – the consistency requirement imposed on sequential equilibria has little bite in game Γ^D. As already noted, the literature often focuses on passive beliefs,[34] which is in line with the bargaining equilibrium approach and the spirit of the delegated negotiations game Γ: each channel takes as given the other equilibrium contracts when negotiating its own tariff. Indeed, any Perfect Bayesian Equilibrium with passive beliefs ("PBEPB" hereafter) of game Γ^D constitutes a sequential equilibrium of game Γ. Unfortunately, the converse does not hold: the a sequential equilibria of game Γ constitute the only candidate PBEPBs of game Γ^D but, with downstream Bertrand competition, these candidate PBEPBs may not survive multilateral deviations, where a firm deviates on its offers to multiple partners; as a result, PBEPBs may fail to exist in game Γ^D.[35] In other words, delegating negotiations

[34] See the papers mentioned in footnote 26 and Collard-Wexler *et al.* (2019) for a recent example.

[35] For the case of an upstream monopoly, the contract equilibrium characterized by O'Brien and Shaffer (1992) constitutes the only candidate PBEPB. Rey and Vergé (2004) show however that it does not survive multilateral deviations when downstream firms are insufficiently differentiated. By contrast, for downstream Cournot competition, existence of a PBEPB has been established by Hart and Tirole (1990) for an upstream monopoly, and extended by Nocke and Rey (2018) for an upstream duopoly. McAfee and Schwartz (1995) note however that existence problems arise again when negotiated tariffs become publicly observable before downstream decisions are made.

to distinct agents does not affect the set of candidate PBEPB outcomes surviving single-channel deviations, but ensures existence by preventing multi-channel deviations.

Remark: deterministic outcomes. In game Γ, the retail outcome is deterministic but, for each channel, the equilibrium tariff depends on which side makes the offer. It is however straightforward to extend the game so as to ensure that the equilibrium tariffs, too, are deterministic. To see this, consider the modified game $\hat{\Gamma}$, in which the following preliminary stage is added:

Stage 0**.** For each $M_i - R_j$ pair, M_i^j offers a tariff $t_{ij}(q_{ij})$ to R_j^i, who then accepts or rejects it;[36] all offers are simultaneous and secret, and all acceptance decisions are also simultaneous and secret. If the offer is accepted, the game directly proceeds to stage 2, otherwise it proceeds to stage 1.

Stage 1 now constitutes an outside option for stage 0. Consider a bargaining equilibrium $\mathcal{B} = \{(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}, \mathbf{t^e}, \mathbf{p^e}\}$ and the associated equilibrium of game Γ identified by Proposition 3, $\mathcal{E} = \{(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}, (\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\theta\in\Theta}, \mathbf{b}\}$; by construction, they satisfy $t_{ij}^e = E_{\theta_{ij}}[\hat{t}_{ij}^{\theta_{ij}}]$ (for $(i,j) \in \mathcal{I} \times \mathcal{J}$) and yield the same retail prices ($\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \mathbf{p^e}$ for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$) and the same expected profits, $\{\Pi_{M_i}^e\}_{i\in\mathcal{I}}$ and $\{\Pi_{R_j}^e\}_{j\in\mathcal{J}}$. Suppose now that, in the modified game $\hat{\Gamma}$, all players adopt the same strategies and beliefs as in $\mathcal{E}$ for stages 1 and 2, and consider the negotiation for channel $M_i - R_j$ in stage 0. As subsequent negotiations (in case of rejection at stage 0) are bilaterally efficient, and R_j^i can secure $\Pi_{R_j}^e$ by proceeding to stage 1, it is optimal for M_i^j to offer the expected tariff t_{ij}^e and for R_j^i to accept it. Hence, offering and accepting the tariffs $\mathbf{t^e} = E_{\boldsymbol{\theta}}[\hat{\mathbf{t}}^{\theta}]$ in stage 0, together with the strategies prescribed by $\mathcal{E}$ in the following stages, constitutes a sequential equilibrium of the modified game $\hat{\Gamma}$. It follows that there is an equivalence between the bargaining equilibria, the equilibria of game Γ with regular tariffs and price responses, and the deterministic equilibria of the modified game $\hat{\Gamma}$.

Remark: linear tariffs. When tariffs are restricted to be linear, bilateral negotiations are no longer efficient and Nash bargaining amounts to maximizing $\left(G_i^{ij}\right)^{\alpha_{ij}}(1-G_j^{ij})^{1-\alpha_{ij}}$, where G_l^{ij} denotes the gains from trade for firm $l = i, j$ when negotiating the tariff t_{ij}. We show in Online Appendix C that a bargaining equilibrium can still be replicated as an equilibrium of the modified game $\hat{\Gamma}$ for appropriate "bargaining parameters" $\boldsymbol{\beta} = \left(\beta_{ij}\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ (i.e., if the pair $M_i - R_j$ reaches stage 1, M_i^j gets to make a take-it-or-leave-it offer with probability β_{ij}) – the parameters $\boldsymbol{\beta}$ however depend on more variables than the weights $\boldsymbol{\alpha}$.

[36]Which agent (M_i^j or R_j^i) is selected to make the offer in this stage does not affect the analysis.

5 Vertical restraints and agency model

To illustrate the flexibility of our approach, we first consider the impact of vertical restraints, namely resale price maintenance (hereafter, RPM) and price parity agreements (hereafter, PPAs). These provisions are commonly observed in practice and both have triggered heated policy debates.[37] We then discuss how our results are affected when switching to the agency business model (in which the supplier remains the owner of goods and/or services and chooses the final prices), which is often adopted by online retail platforms. We only provide here a quick summary of the analysis and of the main results. The complete analysis is presented in Online appendix D.

5.1 Resale price maintenance

To allow for RPM, we suppose here that each $M_i - R_j$ pair can contract not only on a (non-linear) tariff $t_{ij}(q_{ij})$, but also – if it wishes to do so – on the retail price p_{ij}. The timing of wholesale negotiations and retail pricing decisions remains as before, with the caveat that in case of RPM, R_j sets the price p_{ij} that has been agreed upon.

Allowing for RPM does not destabilize the above cost-based tariff equilibria. Indeed, if the other channels sign cost-based tariffs, then a cost-based tariff t_{ij} induces R_j to maximize its joint profit with M_j, and there is no need for contracting on p_{ij}.[38]

However, RPM can sustain many other outcomes, even with simple two-part tariffs. As the wholesale price w_{ij} is no longer needed to "drive" the retail price p_{ij} (which can now be agreed upon through RPM), M_i and R_j can now set w_{ij} in any arbitrary way, adjusting the fixed fee F_{ij} so as to share the profit as desired. However, w_{ij} affects M_i's negotiation with the other retailers, as well as R_j's negotiation with the other manufacturers. For instance, when negotiating with M_h, R_j takes into account the impact of the price p_{hj} on its downstream margin on brand i, $p_{ij} - w_{ij}$. Likewise, when negotiating with R_k, M_i takes into account the impact of the price p_{ik} on its upstream margin on R_j's sales, $w_{ij} - c_i$. As there are $n \times m$ instruments (the wholesale prices) for $n \times m$ targets (the retail prices), it follows that, generically, any retail prices can be sustained in equilibrium.

Proposition 4 (RPM) *With RPM, any price vector* $\mathbf{p}$ *can generically be sustained.*

Proof. See Proposition D.1 in Online appendix D.1. ■

We have focused so far on "full RPM," where a retailer must charge the exact price negotiated with the manufacturer; our framework can also shed some light on the role of minimum RPM (i.e., price floors) and maximum RPM (i.e., price caps). For instance,

[37] PPAs have gained importance with the development of online platforms.

[38] Using RPM however reduces R_j's profit, as R_j can no longer adjust p_{ij} if another negotiation breaks down: this reduces R_j's disagreement payoff and, therefore, its equilibrium payoff.

restricting attention to symmetric equilibria, any price above the competitive price p^* can be sustained with minimum RPM (resp., maximum RPM) when there is more (resp., less) substitution among manufacturers' brands than among retailers' stores.

Proposition 5 (min vs max RPM) *Restricting attention to symmetric equilibria, any price $p > p^*$ can generically be sustained with minimum RPM (resp., maximum RPM) when there is more (resp., less) substitution among manufacturers than among retailers.*

Proof. See Proposition D.2 in Online appendix D.1. ■

To see the intuition, consider first retail pricing decisions. Absent RPM, when setting their prices retailers account for their downstream margins but ignore their partners' upstream margins. Hence, if upstream margins are positive, double marginalization problems arise: p_{ij} exceeds the level maximizing the joint profit of M_i and R_j, which calls for a price cap. Conversely, if upstream margins are negative, price floors are needed.

The next step is to determine the sign of upstream margins. With cost-based tariffs, each $M_i - R_j$ pair maximizes the profit generated by R_j's sales, which leads to a competitive outcome. When instead upstream margins are not zero, M_i and R_j moreover take into account the margins charged by M_i to R_j's rivals, but ignores the margins on R_j's sales of the other brands. It follows that, to sustain supra-competitive prices, negative upstream margins are required when there is more substitution upstream; price floors are then needed to counter retailers' excessive incentives to lower prices. When instead there is more substitution downstream, positive upstream margins are required, and price caps are then needed to counter retailers' excessive incentives to raise prices.[39]

5.2 Price parity agreements

We now turn to PPAs, which require the retailer to price the manufacturer's brand at the same level as (or no less/more than) competing brands. These provisions have triggered debates about their potential anti-competitive effects. For instance, in April 2010, the UK Office of Fair Trading (OFT) considered that bilateral agreements linking the retail price of a tobacco brand to the prices of competing brands (at the same stores) was anti-competitive and had the same adverse effects as RPM.[40]

To shed some light on this debate, we now consider a variant of our setting in which, in the second stage, retailers must charge the same price on all brands. We find that PPAs have little impact on the equilibrium outcome:

[39] Price floors thus have no effect in this case; by contrast, Allain and Chambolle (2011) find that industry-wide price floors are always anticompetitive.

[40] See Decision CA98/01/2010 of the Office of Fair Trading, Case CE/2596-03: Tobacco, 15 April 2010. This decision was later quashed by the Competition Appeals Tribunal (see the CAT Judgement [2011] CAT 41, 12 December 2011), who however did not discuss the possible anticompetitive effects of PPAs.

Proposition 6 (price parity agreements) *Under PPAs, in the class of bargaining equilibria based on differentiable tariffs and positive quantities:*

(i) equilibrium tariffs are all cost-based; and,

(ii) if $\mathbf{p}^*$ *is symmetric across brands, then prices remain equal to* $\mathbf{p}^*$*.*

Proof. See Proposition D.3 in Online appendix D.2. ∎

The insight of Proposition 2 thus carries over when retailers must set uniform prices across brands – requiring a smooth retail behavior moreover boils down to tariffs being differentiable and price responses being interior. PPAs thus have no impact on equilibrium tariffs, which remain cost-based. If in addition the equilibrium prices are already symmetric absent PPAs, then PPAs have no impact on retail prices either.

5.3 Agency model

We have focussed so far on the "resale" business model usually adopted by "brick-and-mortar" retailers: distributors buy goods or services from suppliers, and resell them to consumers. Online platforms often adopt instead an "agency" business model: suppliers sell directly to consumers, and platforms obtain commissions based on sales.

This amounts to turning the framework "upside-down". Manufacturers are now downstream and control retail prices and remunerate their upstream partners (i.e., the retailers/platforms) with (non-linear) commissions. The timing thus becomes:

Stage 1: Each pair negotiates a (possibly non-linear) commission schedule based on the volume of sales achieved by the manufacturer on the retailer's platform.

Stage 2: Manufacturers simultaneously set the retail prices for their products, for each platform that carries them.

It follows that, as long as manufacturers adopt a smooth pricing behavior, marginal commissions must reflect (upstream firms') distribution costs; the equilibrium outcome is therefore that of competition between "multi-store" firms. Whether this is more competitive than the previous multi-brand retail oligopoly depends on whether manufacturers or retailers are closer substitutes (see Online appendix D.3).

Price parity agreements (now requiring manufacturers to set the same prices on all platforms) have again no impact on the equilibrium outcome beyond imposing symmetry. That is, equilibrium tariffs remain cost-based and, when firms are symmetric at both stages of the vertical chains (and the equilibrium prices are symmetric in the absence of PPAs), then price parity agreements do not affect the equilibrium retail prices either. These insights are in sharp contrast with the recent literature on price parity agreements.

However, so far this literature has focused on either linear commissions[41] or constant revenue-sharing rules,[42] which generate contractual inefficiencies; instead, we allow here for general non-linear commissions and thus for efficient bilateral contracting.[43]

6 Endogenous network

Tariffs being cost-based, intrabrand competition dissipates profits when retailers are close substitutes; firms would then benefit from limiting the number of distribution channels. Yet, the above bargaining approach predicts that all channels are always active. To see why, suppose that firms negotiate cost-based two-part tariffs. In every bilateral negotiation, and regardless of which other channels are active, the manufacturer is then willing to supply for any non-negative fee, and the retailer is willing to add the manufacturer's brand to its portfolio if the fee is sufficiently low; they thus activate their channel.

To endogenize the channel network, the framework must therefore allow manufacturers and/or retailers to select explicitly their trading partners.[44] Prompted by the observation that many insurers limit the set of hospitals to which they offer access, Ho and Lee (2019) and Ghili (2018) allow insurers, in case of disagreement with a selected hospital, to replace it with an alternative hospital from outside the network. The outcome remains the same as Nash-in-Nash when networks are complete, but insurers can obtain more favorable terms by opting for selective networks, as hospitals must then compete to join the networks.[45] Lee and Fong (2013) adopt another, infinite horizon framework in which, at the beginning of every period, firms decide which new links to activate and/or to break (and incur a cost per added/withdrawn link); Nash-in-Nash bargaining then takes place within the resulting network.

We explore here an alternative approach, which consists in introducing a preliminary stage in which the distribution network is endogenously determined through a simultaneous "veto-game". This approach is similar in spirit to that of Lee and Fong (2013), but in a static setting. It turns out to remain reasonably tractable and yet predicts the emergence of selective distribution networks when retailers are close substitutes, as intuition suggests.

Formally we assume that, in a preliminary stage, manufacturers and retailers choose which channels to activate, each firm having veto power. That is, each retailer announces which manufacturer(s) it wishes to deal with (if any), and likewise each manufacturer

[41] See Boik and Corts (2016) and Johansen and Vergé (2017).

[42] See Johnson (2017) and Foros *et al.* (2017).

[43] Allowing for direct sales by suppliers would amount to adding a platform (the "direct sales" channel) offering intermediation services at cost, and would not affect the above insights.

[44] For an earlier analysis of buyer-seller network formation without downstream competition, see, e.g., Kranton and Minehart (2001).

[45] For instance, if hospitals are close substitutes, then dealing with a single hospital enables insurers to appropriate most of the profit, regardless of their bilateral bargaining power.

announces which retailer(s) it wishes to deal with (if any); these announcements are simultaneous and publicly observable. A channel then becomes active if and only if both partners wish to deal with each other. This preliminary stage determines the channel network, which gives rise a bargaining equilibrium defined along the same lines as before. It is well-known that veto games are subject to coordination problems that may generate a multiplicity of equilibria – in particular, there always exists a trivial equilibrium in which no channel becomes active. To avoid these coordination issues, we focus on Coalition-Proof Nash Equilibria (hereafter, CPNE).[46]

As the number of potential networks grows geometrically with the number of firms, in this section we focus on the simplest relevant case with two symmetric manufacturers, labelled M_A and M_B for convenience, and two symmetric retailers, R_1 and R_2. Manufacturers' and retailers' unit costs are respectively denoted by $c_A = c_B = c$ and $\gamma_1 = \gamma_2 = \gamma$, and, for any price vector $\mathbf{p} \equiv (p_{A1}, p_{B1}, p_{A2}, p_{B2})$, any $i \neq h \in \{A, B\}$ and any $j \neq k \in \{1, 2\}$, the demand for brand i at store j is given by:

$$D_{ij}(\mathbf{p}) \equiv D(p_{ij}, p_{hj}, p_{ik}, p_{hk}),$$

where the function $D(.)$ is continuously differentiable. Bargaining sharing rules, too, are symmetric: $\alpha_{ij} = \alpha$ for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$.

6.1 Bargaining equilibria

We provide in Online appendix E.1 a complete characterization of the bargaining equilibria for each distribution network, and summarize here their main features. As in the baseline model, tariffs are cost-based and there exists a unique equilibrium in two-part tariffs. When all firms activate at most one channel, the equilibrium outcome is also unique. Otherwise, there may exist multiple equilibrium outcomes, which differ in the division of profits. To ensure that firms' continuation payoffs are properly defined, throughout this section we focus on the equilibria based on two-part tariffs,[47] which are as follows.

• **Bilateral monopoly:** a single channel is active, say $i-j$. M_i and R_j obtain $\Pi_M^m \equiv \alpha\pi^m$ and $\Pi_R^m \equiv (1-\alpha)\pi^m$, respectively, where π^m denotes the monopoly profit obtained generated by the channel.

• **Exclusive dealing**: two unconnected channels are active, say $i-j$ and $h-k$. Manufacturers' and retailers' profits are $\Pi_M^{ED} \equiv \alpha\pi^{ED}$ and $\Pi_R^{ED} \equiv (1-\alpha)\pi^{ED}$, where π^{ED} denotes the per-channel profit in a duopoly where the two "products" are differentiated both upstream and downstream.

[46] See Bernheim *et al.* (1987). It follows that the analysis does not rely on a strong form of commitment, as no coalition of firms has an incentive to renegotiate the agreements.

[47] The analysis is thus valid when only two-part tariffs are feasible, or when firms favor two-part tariffs when they are indifferent between those and other non-linear tariffs.

• **Upstream foreclosure:** a single manufacturer deals with both retailers. Manufacturer's and retailers' profits are respectively $\Pi_M^{UF} \equiv 2\alpha\pi^{UF}$ and $\Pi_R^{UF} \equiv (1-\alpha)\,\pi^{UF}$, where π^{UF} denotes the per-channel profit in a duopoly where the two products are differentiated only downstream.

• **Downstream foreclosure:** a single retailer deals with both manufacturers. The manufacturers' and the retailer's profits are $\Pi_M^{DF} \equiv \alpha\left(2\pi^{DF} - \pi^m\right)$ and $\Pi_R^{DF} \equiv 2\,(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right)$, where π^{DF} denotes the per-channel profit when a downstream monopolist sells both brands.

• **Single exclusion:** a single channel, say $h-k$, is excluded. All firms are thus directly or indirectly connected, as retailers have a common manufacturer (namely, M_i), and one of them (R_j) also deals with the other manufacturer (M_k). We denote respectively by Π_{Mm}^{SE} and Π_{Rm}^{SE} the profits obtained by the multi-partner manufacturer and retailer (M_i and R_j) and by Π_{Ms}^{SE} and Π_{Rs}^{SE} the profits obtained by the single-partner manufacturer and retailer (M_h and R_k).

• **Interlocking relationships:** all channels are active; firms' profits are then:

$$\Pi_M^* = 2\alpha\left(2\pi^* - \hat{\pi}^*\right) \text{ and } \Pi_R^* = 2\left[(1-\alpha)\,\pi^* + \alpha\left(\hat{\pi}^* - \pi^*\right)\right],$$

where π^* denotes the equilibrium per-channel profit, whereas $\hat{\pi}^*$ denotes the profit that a retailer could achieve by dropping one brand.[48]

Two observations readily follow from manufacturers being imperfect substitutes:

• *Bilateral monopoly versus downstream foreclosure:* in both networks there is a single retailer, carrying only one brand in the first case, and both brands in the second case; brand differentiation thus yields: $2\pi^{DF} > \pi^m > \pi^{DF} > 0$.

• *Upstream foreclosure versus exclusive dealing:* in both networks there are two mono-brand retailers, carrying the same brand in the first case, and different brands in the second case; brand differentiation thus yields: $\pi^{ED} > \pi^{UF} > 0$.

6.2 Equilibrium network

We now study the CPNE of the network formation game. For expositional purposes, we restrict attention to cases where manufacturers do have bargaining power (i.e., $\alpha > 0$).[49]

We first note that at least two channels are active. Otherwise, a pair of vertically related inactive firms could generate a profit by activating their channel, and this deviating coalition would obviously be self-enforcing, as both firms would benefit from it. Furthermore, upstream foreclosure cannot arise: the excluded supplier (say, M_h) and either

[48] Using symmetry, π^* and $\hat{\pi}^*$ correspond to the profits π_j^* and π_j^{ij} defined in Proposition 1.

[49] When $\alpha = 0$, coalition-proofness has little bite, as manufacturers obtain no profit anyway. However, a unique equilibrium is selected as α tends to 0.

retailer (say, R_j) would gain from activating their channel (possibly in addition to the channel $i-j$): M_h would benefit from avoiding exclusion ($\Pi_M^{ED}, \Pi_{Ms}^{SE} > 0$), and R_j would benefit from dealing with a different supplier than R_k ($\max\{\Pi_R^{ED}, \Pi_{Rm}^{SE}\} \geq \Pi_R^{ED} > \Pi_R^{UF}$).

Intuitively, it is worth distributing a brand through both retailers only if they are substantially differentiated; otherwise, intrabrand competition dissipates profits without adding much demand. The following Proposition confirms this intuition by considering the two polar cases where retailers are either perfect substitutes or local monopolies:[50]

Proposition 7 (endogenous network)

(i) *When retailers do not compete against each other, the unique CPNE yields interlocking relationships.*

(ii) *When instead retailers are perfect substitutes:*

- *if* $\pi^{ED} > 2\pi^{DF} - \pi^m$, *the unique CPNE yields exclusive dealing;*
- *if* $\pi^{ED} < 2\pi^{DF} - \pi^m$, *the unique CPNE yields downstream foreclosure.*

Proof. See Online appendix E.2. ■

Interestingly, firms' relative bargaining power has no impact on the equilibrium network. When retailers are local monopolies, opening an additional channel always benefits both partners. When instead retailers are perfect substitutes, the network choice is driven by manufacturers, who want to deal with a single retailer; the relevant comparison is therefore between downstream foreclosure and exclusive dealing. As manufacturers obtain a share α of their contributions to their retailer's profit, the outcome follows from a comparison between these contributions – i.e., the channel profit π^{ED} under exclusive dealing, and the additional profit from expanding the brand portfolio, $2\pi^{DF} - \pi^m$, under downstream foreclosure.

To provide further insights, we study below the following linear demand specification, in which costs are normalized to zero ($c = \gamma = 0$) and, for $i \neq h \in \{A, B\}$ and $j \neq k \in \{1, 2\}$, the (inverse) demand for brand i at store j is given by, for some $\mu, \rho \in]0, 1[$:

$$P\left(q_{ij}, q_{hj}, q_{ik}, q_{hk}\right) = 1 - q_{ij} - \mu q_{hj} - \rho q_{ik} - \mu\rho q_{hk}.$$

The parameter μ (resp., ρ) reflects the degree of substitution between manufacturers (resp., retailers).[51] The next proposition confirms the previous insights:

[50] While we have so far ruled out these extreme cases for expositional purposes, it is straightforward to extend the previous analysis, as long as manufacturers remain imperfect substitutes.

[51] To limit the number of parameters, the price sensitivity across both manufacturers and retailers is supposed to be the product of those across manufacturers (μ) and across retailers (ρ). Similar insights obtain when making this assumption for the demand D rather than the inverse demand P, or when normalizing instead the demand so as to ensure that $P\left(q, q, q, q\right)$ remains constant as μ and ρ evolve.

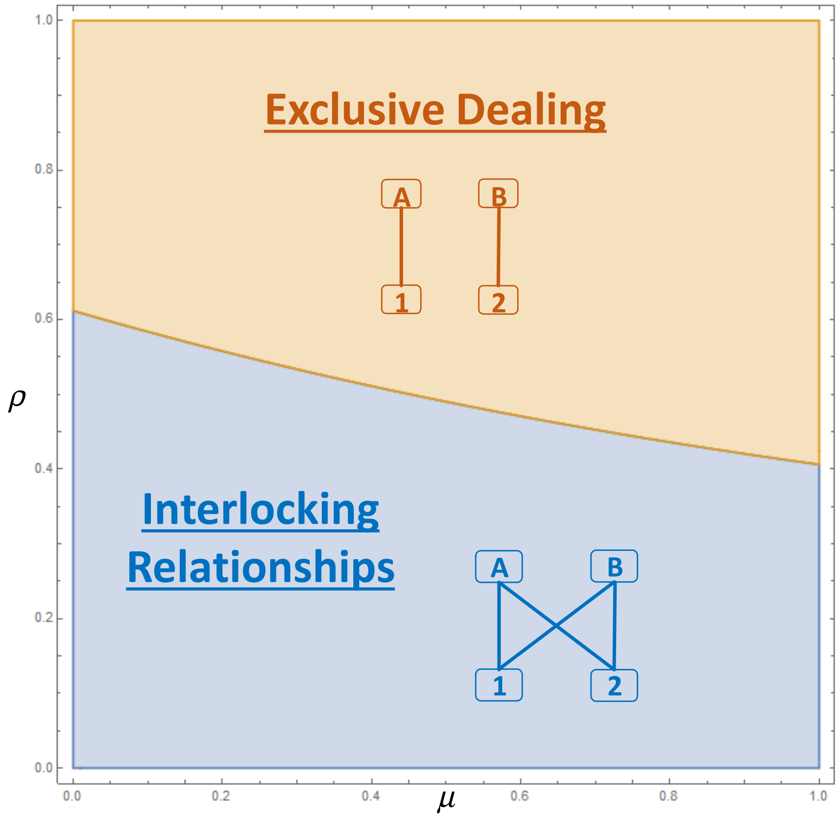

Figure 1: Equilibrium distribution network

Proposition 8 (endogenous network - linear demand) *For the above linear demand specification, there exists* $\rho^*(\mu) \in]0,1[$*, which is a decreasing function of* μ*, such that:*

- *if* $\rho < \rho^*(\mu)$*, then the unique CPNE yields interlocking relationships;*
- *if instead* $\rho \geq \rho^*(\mu)$*, then the unique CPNE yields exclusive dealing.*

Proof. See Online appendix E.3. ■

These insights are illustrated by Figure 1. There is again a unique CPNE, which does not depend on firms' relative bargaining powers (α): interlocking relationships arise when retailers are sufficiently differentiated, otherwise firms prefer avoiding intrabrand competition. For this linear demand example, we have that $\pi^{ED} > 2\pi^{DF} - \pi^m$; manufacturers thus favor exclusive dealing over downstream foreclosure ($\Pi_M^{ED} > \Pi_M^{DF}$), and retailers concur (to avoid exclusion).[52]

[52] This analysis thus provides a micro-foundation for exclusive dealing network structure, which has been the focus of many studies – see, e.g., Bonanno and Vickers (1988), Horn and Wolinski (1988), and Milliou and Petrakis (2007).

7 Mergers

We now consider the effect of horizontal (upstream or downstream) and vertical mergers. Whereas the literature on horizontal mergers often focuses on price effects,[53] our approach provides a natural framework for studying the impact on distribution networks as well. As we will see, taking this dimension into consideration can yield very different conclusions.[54]

For the sake of exposition, we stick to the above successive duopoly setting and maintain the focus on equilibria based on two-part tariffs. We provide a complete analysis in Online Appendix F and only highlight here the main findings.

7.1 Downstream merger

A merger between R_1 and R_2 creates a multi-location retail monopolist, R. For simplicity, we assume that manufacturers cannot discriminate according to the channel through which their products are sold (e.g., brick-and-mortar versus online sales); hence, R negotiates with each M_i a single two-part tariff, $t_i(q_i) = F_i + w_i q_i$. Equilibrium tariffs remain cost-based[55] but eliminating downstream competition raises prices to the monopoly level.

Beyond this classic horizontal effect, a downstream merger may also affect the distribution network: pre-merger, exclusivity can arise to avoid downstream competition; by creating a retail monopoly, the merger eliminates this motivation and makes interlocking relationships more likely. Indeed, for the linear demand specification, the unique CPNE always involves interlocking relationships. The merger may therefore benefit consumers by expanding product variety. This is for instance the case when retailers are good enough substitutes so that exclusive dealing arises pre-merger, and brand differentiation is so large that prices are then close to the monopoly level. The following proposition confirms this intuition for the linear demand specification:

Proposition 9 (downstream merger)

(*i*) *A downstream merger yields monopolistic retail prices for any given distribution network but makes interlocking relationships more likely. Hence, it reduces consumer surplus and total welfare when interlocking relationships already arise pre-merger, but otherwise expands the distribution network and can then increase consumer surplus and total welfare – all the more so if, pre-merger, the two channels are substantially differentiated, so that prices are already close to monopoly level.*

[53] Regarding horizontal mergers in vertically related markets see, e.g., von Ungern-Stenberg (1996) and Dobson and Waterson (1997) for downstream mergers and Horn and Wolinsky (1988) and Ziss (1995) for upstream mergers. More recently, Milliou and Sandonis (2018) consider the impact on product portfolio.

[54] There is a substantial literature on vertical integration and foreclosure; see, e.g., Salinger (1988), Ordover, Saloner and Salop (1990), Hart and Tirole (1990) and, more recently, Nocke and Rey (2018). We extend the insights of the last two papers to multiple upstream firms and price competition downstream.

[55] This is in line with Bernheim and Whinston (1985, 1986, 1998).

(*ii*) *For the linear demand specification considered above, the merger thus reduces consumer surplus and total welfare whenever* $\rho < \rho^*(\mu)$*; when instead* $\rho \geq \rho^*(\mu)$*, there exist* $\hat{\mu}_S(\rho)$ *and* $\hat{\mu}_W(\rho)$*, which are decreasing functions of* ρ *satisfying* $\hat{\mu}_S(1) = \hat{\mu}_W(1) = 0$*, such that the merger reduces consumer surplus (resp., total welfare) if* $\mu < \hat{\mu}_S(\rho)$ *(resp.,* $\mu < \hat{\mu}_W(\rho)$*) and increases it if* $\mu > \hat{\mu}_S(\rho)$ *(resp.* $\mu > \hat{\mu}_W(\rho)$*).*

Proof. See Online appendix F.1. ■

Pre-merger, the condition $\rho \geq \rho^*(\mu)$ ensures that exclusive dealing arises and the conditions $\mu < \hat{\mu}_s(\rho)$ (for $s = S, W$) ensure that brand differentiation induces high prices; as a result, the network-expansion effect of the merger more than compensates the price increase to the monopoly level: taking into consideration this network effect thus reverses the standard conclusion based on prices.

7.2 Upstream merger

A merger between M_A and M_B creates a multi-brand upstream monopolist, M. For simplicity, we assume that M then bundles the two brands and thus negotiates with each R_j a unique fixed fee F_j, besides the wholesale prices w_{Aj} and w_{Bj}.[56] Equilibrium tariffs still remain cost-based; hence, for any given distribution network, the merger affects neither wholesale nor retail prices, but only the division of profit.

The merger may however alter the equilibrium network, and affect consumers in this way. For example, when retailers are close substitutes, competing manufacturers would rather distribute their products through different retailers, so as to improve their bargaining position; M may instead decide to sell both brands through the same retailer, so as to avoid downstream competition and increase industry profit. Likewise, where competing manufacturers would opt for interlocking relationships, M may restrict the distribution of one brand to improve the profitability of its other brand. We have:

Proposition 10 (upstream merger)

(*i*) *An upstream merger does not affect retail prices for in any given distribution network but may generate (complete or partial) vertical foreclosure, in which case it reduces consumer surplus and total welfare.*

(*ii*) *For the above linear demand specification, there is still a unique CPNE post-merger, and the merger either has no impact on the network (and, thus, on consumers and total welfare), or alters it in a way that reduces both consumer surplus and total*

[56] The bargaining equilibrium is then similar to a game of delegated negotiations, in which M has two agents, each negotiating a bundled tariff with a retailer. Likewise, in the previous case of an downstream merger, the bargaining equilibrium is similar to a game in which R has two agents, each negotiating a single tariff with a manufacturer.

welfare; specifically, there exist $\tilde{\rho}(\mu)$ and $\overline{\rho}(\mu)$, which are decreasing functions of μ satisfying $0 < \tilde{\rho}(\mu) < \rho^*(\mu) < \overline{\rho}(\mu) < 1$ for $\mu > 0$, such that:

- *if $\rho \geq \overline{\rho}(\mu)$, then the merger alters the network from exclusive dealing to downstream foreclosure;*
- *if instead $\tilde{\rho}(\mu) < \rho < \rho^*(\mu)$, then the merger alters the network from interlocking relationships to exclusive dealing;*
- *otherwise, the merger has no impact on the network.*

Proof. See Online appendix F.2. ■

Hence, despite the absence of direct price effects, taking into consideration the effect of an upstream merger on the distribution network can give rise to competition concerns: an *horizontal merger* between suppliers may trigger *vertical foreclosure*, as the merged entity may stop supplying some products to some retailers.

7.3 Vertical merger

A merger between M_i and R_j creates a vertically integrated firm, I, that interacts with the independent M_h and R_k. M_h's tariffs remain cost-based but, as R_k now competes with I's downstream subsidiary, either I stop supplying R_k, or it increases its wholesale price ($w_{ik} > c$).[57] In addition, in the latter case, I's downstream subsidiary now competes less aggressively (despite facing cost-based tariffs), as it takes into account the upstream margin earned on R_k's sales. As a result, for any given network in which R_k carries M_i's brand, the merger raises retail prices, which reduces consumer surplus and total welfare.

The merger may also alter the distribution network. Where an independent M_i can limit intrabrand competition only through exclusivity, I can now achieve this by raising w_{ik}; hence, the merger may induce R_k to carry both brands rather than M_h's brand only. However, I also internalizes the impact of carrying M_h's brand on the profitability of its own brand, which makes interlocking relationships less likely. Indeed, we have:

Proposition 11 (vertical merger) *For the above linear demand specification:*

(i) When $\rho < \rho^*(\mu)$ (interlocking relationships pre-merger), a vertical merger raises the wholesale price charged to the independent retailer, and either does not affect the network or induces the integrated firm to drop the rival brand; it thus increases retail prices and reduces both consumer surplus and total welfare.

(ii) When instead $\rho \geq \rho^*(\mu)$ (exclusive dealing pre-merger), there exists $\rho^{IR}(\mu, \alpha)$ and $\rho^{ED}(\mu, \alpha) > \max\left\{\rho^*(\mu), \rho^{IR}(\mu, \alpha)\right\}$ such that:

[57] In the formal analysis, for the sake of exposition we assume that the wholesale price w_{ik} is observed by the downstream subsidiary R_j, and thus becomes "public". However, even if w_{ik} were not observed by R_j, I would still have an incentive to raise it so as to limit the competition faced by R_j (see, e.g., Hart and Tirole 1990).

– *If $\rho > \rho^{ED}(\mu, \alpha)$, then the merger has no network or price effect; it thus has no impact on consumer surplus and total welfare.*
– *If instead $\rho \leq \rho^{IR}(\mu, \alpha)$, then the merger fully expands the distribution network, which increases consumer surplus and total welfare.*
– *Otherwise, the merged firm supplies the rival retailer, but charges a positive margin; as a result, the merger reduces consumer surplus (it also reduces total welfare if retailers are close enough substitutes).*

Proof. See Online appendix F.3. ■

The simulations performed for the case of a linear demand show that the parameter regions in which a vertical merger is either neutral or pro-competitive are rather small (see, e.g., Figure 4 in Online appendix F.3), suggesting that the upward price pressure that it creates is likely to dominate any network expansion benefit.

8 Observable contracts

While our assumption of secret contracting is natural for many industries, it is worth noting that the same framework, as well as its micro-foundation, can be used as well when wholesale tariffs are common knowledge among downstream firms. To see this, modify the previous two-stage game by replacing its stage 2 with:

Stage 2 (observable contracts): Retailers, having observed all wholesale tariffs, simultaneously set retail prices for the brand(s) that they carry.

A "bargaining equilibrium" of this modified game can then defined as follows. In stage 2, retail prices constitute a Nash equilibrium given the negotiated tariffs. In stage 1, each $M_i - R_j$ pair negotiates a tariff $t_{ij}(q_{ij})$ that: (i) maximizes the joint profit of M_i and R_j, given the other equilibrium contracts and the resulting retail price equilibrium; and (ii) gives a share α_{ij} of the additional profit generated by a successful negotiation to M_i (and thus a share $1 - \alpha_{ij}$ to R_i). Formally:

Definition 4 (observable contracts) *A bargaining equilibrium with observable contracts is a vector of price responses $\mathbf{p}^R(\mathbf{t}) = (\mathbf{p}_j^R(\mathbf{t}))_{j \in \mathcal{J}}$, together with a vector of equilibrium tariffs $\mathbf{t}^\mathbf{e} = (\mathbf{t}_j^\mathbf{e})_{j \in \mathcal{J}}$ and a vector of equilibrium prices $\mathbf{p}^\mathbf{e} = (\mathbf{p}_j^\mathbf{e})_{j \in \mathcal{J}}$, such that:*

- *In stage 2, the price responses $\mathbf{p}^R(\mathbf{t})$:*
  – *constitute a Nash equilibrium, for any $\mathbf{t}$ negotiated in stage 1:*
$$\forall j \in \mathcal{J}, \mathbf{p}_j^R(\mathbf{t}) \in \arg\max_{\mathbf{p}_j} \sum_{i \in \mathcal{I}} \left[\begin{array}{c} (p_{ij} - \gamma_j) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^R(\mathbf{t})\right) \\ -t_{ij}\left(D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^R(\mathbf{t})\right)\right) \end{array} \right];$$
  – *satisfy $\mathbf{p}^\mathbf{e} = \mathbf{p}^R(\mathbf{t}^\mathbf{e})$.*

- *In stage 1, for every $(i,j) \in \mathcal{I} \times \mathcal{J}$, the equilibrium tariff t^e_{ij}:*
  - *maximizes the joint profit of M_i and R_j, given R_j's other equilibrium tariffs, $\mathbf{t}^{\mathbf{e}}_{-i,j}$, and the retail price responses, $\mathbf{p}^R(\mathbf{t})$; that is, $t_{ij} = t^e_{ij}$ maximizes:*

$$\begin{aligned}
&\left(p^R_{ij}\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right) - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right)\right) \\
&+ \sum_{k \in \mathcal{J} \setminus \{j\}} \left[t^e_{ik}\left(D_{ik}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right)\right)\right) - c_i D_{ik}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right)\right)\right] \\
&+ \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\begin{array}{c} \left(p^R_{hj}\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right) - \gamma_j\right) D_{hj}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right)\right) \\ -t^e_{hj}\left(D_{hj}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}, \mathbf{t}^{\mathbf{e}}_{-j}\right)\right)\right) \end{array}\right];
\end{aligned}$$

  - *gives M_i and R_j shares α_{ij} and $1 - \alpha_{ij}$ respectively, of the additional profit generated by their relationship.*

These bargaining equilibria can be generated by the game of delegated negotiations Γ^O, derived from game Γ by replacing its stage 2 with the above "Stage 2 (observable contracts)". As tariffs are publicly observed at the beginning of stage 2, each continuation game constitutes a proper subgame, and it is thus natural to look for the subgame perfect equilibria (SPEs hereafter) of game Γ^O. Ensuring the existence of Nash equilibria for any set of wholesale tariffs is however problematic – for example, sufficiently concave tariffs would generate convex profit functions and discontinuous price responses. A solution consists in focusing on two-part tariffs, which allow for bilateral efficiency without raising convexity issues: the existence of continuation equilibria is then guaranteed if, in the downstream market where m multiproduct firms compete against each other, there exists a Nash equilibrium for any profile of constant unit costs.

In what follows, we therefore focus on two-part tariffs of the form $t_{ij}(q_{ij}) = F_{ij} + w_{ij} q_{ij}$, which we denote by $t_{ij} = \{w_{ij}, F_{ij}\}$. For the sake of exposition, we further assume that, in case of multiple equilibria, the selection of the continuation equilibrium depends on unit costs (and thus on wholesale prices), and not on fixed costs (franchise fees); that is, the price responses can be expressed as $\mathbf{p}^R(\mathbf{w})$, where $\mathbf{w} = (\mathbf{w}_j)_{j \in \mathcal{J}}$ denotes the vector of wholesale prices. With this restriction, the next proposition establishes a perfect correspondence between the bargaining equilibria and the SPEs of game Γ^O:

Proposition 12 (micro-foundation: observable two-part tariffs)

(i) For any bargaining equilibrium of the form $\mathcal{B} = \{\mathbf{p}^R(\mathbf{w}), \mathbf{t}^{\mathbf{e}} = \{\mathbf{w}^{\mathbf{e}}, \mathbf{F}^{\mathbf{e}}\}, \mathbf{p}^{\mathbf{e}}\}$, there exist $(\mathbf{F}^{\boldsymbol{\theta}})_{\boldsymbol{\theta} \in \Theta}$ such that $\mathcal{E} = \{(\hat{\mathbf{p}}^R_j(\mathbf{t}_j))_{j \in \mathcal{J}}, (\hat{\mathbf{t}}^{\boldsymbol{\theta}} = \{\mathbf{w}^{\mathbf{e}}, \mathbf{F}^{\boldsymbol{\theta}}\})_{\boldsymbol{\theta} \in \Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \mathbf{p}^{\mathbf{e}})_{\boldsymbol{\theta} \in \Theta}\}$ constitutes a SPE of game Γ^O, giving all firms the same expected profits as $\mathcal{B}$.

(ii) Conversely, for any SPE of game Γ^O of the form $\mathcal{E} = \{\hat{\mathbf{p}}^R(\mathbf{w}), (\hat{\mathbf{t}}^{\boldsymbol{\theta}} = \{\hat{\mathbf{w}}, \hat{\mathbf{F}}^{\boldsymbol{\theta}}\})_{\boldsymbol{\theta} \in \Theta}, \hat{\mathbf{p}}\}$, $\mathcal{B} = \{(\hat{\mathbf{p}}^R_j(\mathbf{t}_j))_{j \in \mathcal{J}}, \mathbf{t}^{\mathbf{e}} = \{\hat{\mathbf{w}}, \mathbf{F}^{\mathbf{e}} = E_{\boldsymbol{\theta}}[\hat{\mathbf{F}}^{\boldsymbol{\theta}}]\}, \hat{\mathbf{p}}\}$ constitutes a bargaining equilibrium, giving all firms the same expected profits as $\mathcal{E}$.

Proof. See Online appendix G.1. ■

The intuition is the same as for secret contracts. Consider the bilateral negotiation between M_i and R_j, say, in game Γ^O. Given the retail price response $\mathbf{p}^R\left(w_{ij}, \mathbf{w}^{\mathbf{e}}_{-i,j}, \mathbf{w}^{\mathbf{e}}_{-j}\right)$, the selected agent chooses the wholesale price w_{ij} that maximizes the joint profit of the two firms. Hence, which side makes the offer has no impact on the wholesale prices, which coincide with bargaining equilibrium ones. Which side makes the offer however affects the fixed fees, as the selected agent appropriates the bilateral gains from trade; as a result, expected fixed fees coincide with those negotiated in a bargaining equilibrium.

Intuitively, the equilibrium wholesale prices are now above cost. Indeed, starting from cost-based tariffs, a slight increase in w_{ij}, say, generates only a second-order loss of efficiency in the bilateral relationship between M_i and R_j (as $w_{ij} = c_i$ would then maximize the joint profit of M_i and R_j), but generates a first-order strategic benefit, by inducing the *other* retailers to raise their prices (assuming, as is often the case, that retail prices are strategic complements). This, in turn, implies that retail prices and industry profit are higher under public contracting than under secret contracting. Yet, we would expect the outcome to be somewhat competitive.

To explore this further, consider a market structure in which: (i) costs are symmetric ($c_i = c$ and $\gamma_j = \gamma$); (ii) demand is symmetric and such that a uniform increase in all the prices of a retailer decreases its demand; (iii) total industry profit is concave in prices and maximal for symmetric monopoly prices ($p^M_{ij} = p^M$); (iv) all products are (imperfect) substitutes and retail prices are strategic complements; and: (v) symmetric wholesale prices $w_{ij} = w$ generate a symmetric retail price equilibrium $p_{ij} = p^R(w)$, where $p^R(w)$ increases with w and is such that $p^R\left(w^M\right) = p^M$ for some $w^M > c$. Suppose further that:

Assumption A^O(observable contracts) Starting from a symmetric outcome where all wholesale prices are equal to w, increasing w_{ij}, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$:

(*i*) decreases the total quantity sold by M_i;

(*ii*) increases the total quantity sold by any other M_h, for $h \neq i$, as well as the total quantity sold by any other R_k, for $k \neq j$.

Under Assumption A, secret contracting in two-part tariffs yields a unique equilibrium, where wholesale prices are at cost: $w_{ij} = c$; Assumption A^O ensures that retail prices are thus also symmetric: $p^*_{ij} = p^*$. The next proposition confirms the intuition that public contracting generates in that case higher prices and profits:

Proposition 13 (public contracting raises prices and profits) *Under Assumptions A and A^O, any bargaining equilibrium with symmetric observable two-part tariffs $t^O_{ij} = \left\{w^O, F^O\right\}$ generates positive upstream margins (i.e., $w^O > c$) and symmetric retail prices $p_{ij} = p^O$ that lie between the competitive and monopoly levels: $p^* < p^O < p^M$.*

Proof. See Online appendix G.2. ■

Remark: on the role of delegated negotiations. Proposition 12 shows that bargaining equilibria can be again interpreted as (here, subgame perfect) equilibria of a delegated negotiations game. Assuming that firms delegate the bilateral negotiations with their partners to distinct agents again ensures existence. This is achieved not only by limiting the scope for multilateral negotiations, as for secret contracting, but also by limiting the scope for "multilateral responses" to a deviation, which would otherwise arise with public contracting. For example, in the direct negotiation game considered by Rey and Vergé (2010), in which manufacturers have all the bargaining power in the bilateral negotiations, a small reduction in one wholesale price charged by M_i to R_j, say, may induce *any* of R_j's rivals to reject the offer made by *any* of M_i's rivals. As a result, even for a simple successive linear duopoly model such as the one considered in Proposition 13, multilateral deviations and responses prevent the existence of a SPE in most of the parameter range. Assuming delegated negotiations also affect pricing incentives, however. For example, when negotiating with R_j, M_i^j takes as given the fixed fee that M_i^k is negotiating with R_k. By contrast, in the case of direct negotiations, M_i would take into account the fact that a reduction in w_{ij}, which is likely to induce R_j to price more aggressively and reduces R_k's profit, would induce a reduction in the fixed fee that could be charged to R_k. Ignoring this effect is thus likely to induce manufacturers to price more aggressively.

Remark: deterministic outcomes and linear tariffs. As for game Γ, the retail equilibrium outcome of game Γ^O is deterministic but the negotiated tariffs depend on which side makes the offer. It is however straightforward to extend again the game so as to ensure that the equilibrium tariffs, too, are deterministic. Consider the extended game $\hat{\Gamma}^O$, in which in a preliminary stage 0, one side can make an offer; the game proceeds as in Γ^O if the offer is rejected, otherwise it proceeds directly to the retail pricing stage. The same reasoning as for secret tariffs (with the caveat that any change in the tariffs accepted at stage 0 or 1 is now observed by all retailers before stage 2) applies; as a result, there is an equivalence between the bargaining equilibria, the equilibria of game Γ^O, and the deterministic equilibria of the modified game $\hat{\Gamma}^O$.

The same reasoning carries over to the case of observable linear tariffs: as before, any bargaining equilibrium with observable linear tariffs can be replicated as an equilibrium of the extended game $\hat{\Gamma}^O$ for appropriate bargaining parameters $\boldsymbol{\beta}$.

9 Conclusion

In the first part of this paper, we develop a framework for the analysis of multilateral vertical relations. The key features are (secret, bilateral) upstream negotiations, followed by downstream price competition. The setting allows for any number of firms, any degree of product differentiation, and any cost or demand asymmetry, at each stage of the vertical chain; it also allows for any bargaining power within each vertical channel, and places no restriction on the tariffs that can be negotiated. To fix ideas, we cast the exposition in a manufacturer – retailer setting, but the framework can be applied as well to other contexts: media content and TV channels, hospitals and health insurance providers, manufacturers and part suppliers, and so forth.

An appealing feature of this framework lies in its tractability. We show that equilibrium tariffs are cost-based, whenever they induce a smooth downstream behavior (i.e., a small change in the quantity sold for one brand by a retailer triggers only small changes in the quantities sold for the other brands by that same retailer). The equilibrium downstream outcome thus replicates that of a multiproduct oligopoly. The division of the profits however depends on the shape of the equilibrium tariffs: downstream firms get a higher (resp., lower) share of the industry profit when tariffs are convex (resp., concave).

We provide a micro-foundation that relies on a non-cooperative game of delegated negotiations. The bargaining equilibria correspond to the sequential equilibria of this game, and correspond to the candidate perfect equilibria with passive beliefs of a game of direct negotiations, as characterized by single-sided deviations; focussing on delegated negotiations ensures existence by discarding the possibility of multi-sided deviations.

To illustrate the versatility of this framework, we consider several extensions. We first consider resale price maintenance (RPM) provisions, where the retail price of a product is contractually set by its manufacturer. We show that even purely vertical, bilateral RPM agreements drastically affect competition; in particular, they can sustain industry-wide monopoly prices, thus eliminating inter-brand as well as intrabrand competitive pressures. We also find that both maximum and minimum RPM can be used to raise prices above their competitive levels, an insight at odds with the current legal treatment of RPM, which treats minimum RPM substantially more harshly than maximum RPM.

We then turn to price parity agreements that restrict a retailer's pricing policy across competing brands. While antitrust agencies have sometimes viewed these price parity agreements as a restriction of competition, similar to minimum RPM, in our setting these contractual clauses are instead rather ineffective – they do not substantially affect the equilibrium outcome, beyond imposing symmetry.

We also use our framework to study the agency business model widely adopted by online retailers and intermediation platforms. This amounts to turning the initial resale setting upside-down: manufacturers are now downstream and set final price, whereas retailers (or intermediation platforms) are upstream. The above insight carries over: as long as firms can negotiate non-linear commissions, these must be cost-based. The equilibrium outcome then replicates that of direct competition between multi-platform firms. Likewise, price parity agreements (linking prices across distribution platforms) do not substantially affect the equilibrium outcome, beyond imposing symmetry.

In the second part of this paper, we endogenize the channel network by introducing a preliminary stage in which firms choose which channels to activate. To obtain a complete characterization, we restrict attention to successive symmetric duopolies. In the polar case where downstream firms are local monopolies, the unique (coalition-proof) equilibrium has all channels being active. When downstream firms are instead perfect substitutes, the unique equilibrium involves either exclusive dealing (each upstream firm dealing with a different downstream firm) or downstream foreclosure (both upstream firms dealing with a common downstream firm). When demand is linear, there is always a unique (coalition-proof) equilibrium, with all channels being active if retailers are sufficiently differentiated, and exclusive dealing otherwise.

Finally, we use our extended framework to study the impact of mergers on the network as well as on prices. Interestingly, this may lead to rather different conclusions than when focusing on price effects. In particular, a downstream merger may expand the distribution network and benefits consumers and society, despite the elimination of downstream competition. Conversely, an upstream merger can trigger vertical foreclosure and be anti-competitive despite the absence of direct price effects.

That upstream contract terms are private and not observable by rival suppliers or customers appears plausible in many markets. The implication that tariffs are then cost-based is moreover in line with the empirical analysis of Nilsen *et al.* (2016), who find that an upstream merger between Norwegian egg producers did not have any impact on consumer prices, but only on the division of profits between producers and retailers. Yet, other markets may be more transparent. It can therefore be useful to consider the case of observable contracts. This appears difficult in the absence of any restriction on admissible tariffs, but we show how to apply the above framework to the case of observable two-part tariffs. The case of secret or observable linear contracts is also considered as well. Which assumption about the informational context or the relevant type of tariffs provides the best fit could be empirically tested.

It would also be interesting to compare the predictions of our network formation framework (which is static and uses coalition-proofness as an equilibrium selection device) with those of alternative approaches, such as the dynamic approach developed by Lee and Fong (2013) (using Markov-perfection as an equilibrium selection device). Finally, the flexibility and tractability of the approach studied in this paper makes it a good instrument to study firms' decisions over other dimensions, such as product portfolio or investment in production capacity or innovation.

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Appendix

A Proof of Proposition 1

Let

$$\pi_{ij}^* \equiv \pi_{ij}(\mathbf{p}^*),\ \pi_j^* \equiv \pi_j(\mathbf{p}^*) = \max_{\mathbf{p}_j} \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^*) \text{ and } \pi_j^{ij} \equiv \max_{\mathbf{p}_{-i,j}} \pi_j((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}^*)$$

denote the equilibrium profits achieved by firm j on brand i and in total, and in case of a negotiation break-down. We first establish the following lemma:

Lemma 1 *Under Assumption A, in equilibrium:*

$$0 < \Delta_j^i \equiv \pi_j^* - \pi_j^{ij} < \pi_{ij}^*.$$

Proof. We first establish the first inequality; using $\pi_j^* = \max_{\mathbf{p}_j} \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$, we have:

$$\Delta_j^i = \max_{\mathbf{p}_j} \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \max_{\mathbf{p}_{-i,j}} \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right).$$

That the last expression is positive then follows from the fact that: (i) in the determination of π_j^{ij}, R_j is constrained to set $q_{ij} = 0$; and (ii) from Assumption A, maximizing $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$ with respect to $\mathbf{p}_j$ leads to $D_{ij}\left(\mathbf{p}^*\right) > 0$.

To establish the second inequality, note that:

$$\begin{aligned}
\pi_j^{ij} &= \max_{\mathbf{p}_{-i,j}} \sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) \\
&\geq \sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) \\
&> \sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}\left(\mathbf{p}^*\right) \\
&= \pi_j^* - \pi_{ij}^*,
\end{aligned}$$

where the strict inequality stems from Assumption A. ■

We now prove Proposition 1. To establish existence, fix a candidate equilibrium in which each $M_i - R_j$ pair, for $i \in \mathcal{I}$ and $j \in \mathcal{J}$, signs the cost-based two-part tariff $t_{ij}^*\left(q_{ij}\right) = F_{ij}^* + c_i q_{ij}$, where:

$$F_{ij}^* = \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right),$$

and retail prices are equal to $\mathbf{p}^*$. Consider the negotiation between M_i and R_j. Given their other equilibrium tariffs, $\left(t_{ik}^*\right)_{k \in \mathcal{J} \setminus \{j\}}$ and $\left(t_{hj}^*\right)_{h \in \mathcal{I} \setminus \{i\}}$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^*$, they seek to maximize their joint profit, equal to:

$$\begin{aligned}&\left(p_j - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) + \sum_{k \in \mathcal{J} \setminus \{j\}} F_{ik}^* \\ &+ \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\left(p_{hj} - c_h - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - F_{hj}^*\right].\end{aligned}$$

Assumption A ensures that this joint profit is maximal for $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$. Furthermore, given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^*$, adopting a tariff t_{ij} leads R_j to maximize its own profit, equal to:

$$\begin{aligned}&\left(p_{ij} - \gamma_j\right) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - t_{ij}\left(D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)\right) \\ &+ \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\left(p_{hj} - c_i - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - F_{hj}^*\right].\end{aligned}$$

A cost-based two-part tariff in the form $t_{ij}\left(q_{ij}\right) = F_{ij} + c_i q_{ij}$ is then optimal, as it makes R_j's variable profit equal to the joint variable profit of M_i and R_j.

To complete the proof of existence, it suffices to show that the fixed fees satisfy the Nash bargaining assumption. Let Δ_{M_i} denote the impact of a successful negotiation on M_i's profit and Δ_{R_j} denote the impact on R_j's profit. Under Nash bargaining, M_i must obtain a fraction α_{ij} of the bilateral joint surplus generated by the negotiation, $\Delta_{M_i}+\Delta_{R_j}$:

$$\Delta_{M_i} = \alpha_{ij}\left(\Delta_{M_i} + \Delta_{R_j}\right). \tag{4}$$

In the candidate equilibrium, manufacturers derive their profits from fixed fees, whereas retailers are residual claimants; hence, M_i and R_j respectively obtain:

$$\Pi_{M_i}^* = \sum_{k \in \mathcal{J}} F_{ik}^* \text{ and } \Pi_{R_j}^* = \pi_j^* - \sum_{h \in \mathcal{I}} F_{hj}^*.$$

If the negotiation between M_i and R_j were to break down, M_i would simply collect the other retailers' fixed fees and R_j would sell the other brands, adjusting its prices so as to maximize its own profit. They would therefore respectively obtain:

$$\Pi_{M_i}^{ij} = \sum_{k \in \mathcal{J} \setminus \{j\}} F_{ik}^* \text{ and } \Pi_{R_j}^{ij} = \pi_j^{ij} - \sum_{h \in \mathcal{I} \setminus \{i\}} F_{hj}^*.$$

Hence, $\Delta_{M_i} = F_{ij}^*$ and $\Delta_{R_j} = \Delta_j^i - F_{ij}^*$; the Nash bargaining rule (4) thus yields:

$$F_{ij}^* = \alpha_{ij} \Delta_j^i.$$

The candidate equilibrium thus indeed constitutes an equilibrium. Conversely, whenever the equilibrium contracts are cost-based tariffs:

- Given its rivals' equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$, R_j's profit (gross of fixed fees) coincides with $\pi_j\left(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}\right)$, and thus its equilibrium prices must satisfy $\mathbf{p}^{\mathbf{e}}_j \in \mathbf{p}^r_j\left(\mathbf{p}^{\mathbf{e}}_{-j}\right)$; Assumption A therefore ensures that retail prices are equal to $\mathbf{p}^{\mathbf{e}} = \mathbf{p}^*$.

- The Nash bargaining rule then uniquely pins down the equilibrium fixed fees.

We now turn to the last part of the Proposition. Manufacturers obtain:

$$\Pi^*_{M_i} = \sum_{j \in \mathcal{J}} \alpha_{ij} \Delta^i_j,$$

From Lemma 1, their profit is thus positive as long as $\alpha_{ij} > 0$ (as $\Delta^i_j > 0$), but they obtain less than a share α_{ij} of the equilibrium channel profit (as $\Delta^i_j < \pi^*_{ij}$). It follows that retailers get more than a share $1 - \alpha_{ij}$ of the channel profits they generate. In particular, they obtain a positive profit regardless of their bargaining power (namely, even when $\alpha_{ij} = 1$):

$$\Pi^*_{R_j} = \pi^*_j - \sum_{i \in \mathcal{I}} F^*_{ij} = \pi^*_j - \sum_{i \in \mathcal{I}} \alpha_{ij} \Delta^i_j \geq \pi^*_j - \sum_{i \in \mathcal{I}} \Delta^i_j > \pi^*_j - \sum_{i \in \mathcal{I}} \pi^*_{ij} = 0,$$

where the strict inequality derives from Lemma 1.

B Proof of Proposition 2

Part (i). Consider the negotiation between M_i and R_j, given the equilibrium tariffs negotiated for the other brands, $\mathbf{t}^{\mathbf{e}}_{-i,j}$, and the other retailers' equilibrium prices, $\mathbf{p}^{\mathbf{e}}_{-j}$. Choosing the tariff t_{ij} that maximizes the joint profit of M_i and R_j amounts to choosing the quantity q_{ij} sold by R_j at the retail competition stage, taking into account that R_j will price so as to maximize its own profit. The equilibrium quantity q^e_{ij} therefore maximizes:

$$\hat{r}^{ij}_j\left(q_{ij}\right) - c_i q_{ij} - \sum_{h \in \mathcal{I} \setminus \{i\}} t^e_{hj}\left(\hat{q}^{ij}_{hj}\left(q_{ij}\right)\right) + \sum_{k \in \mathcal{J} \setminus \{j\}} \left[t^e_{ik}\left(\hat{q}^{ij}_{ik}\left(q_{ij}\right)\right) - c_i \hat{q}^{ij}_{ik}\left(q_{ij}\right)\right],$$

where $\hat{q}^{ij}_{hk}(q_{ij}) \equiv D_{hk}(\hat{\mathbf{p}}^{ij}_j(q_{ij}), \mathbf{p}^{\mathbf{e}}_{-j})$ and $\hat{r}^{ij}_j(q_{ij}) \equiv \sum_{h \in \mathcal{I}}[(\hat{p}^{ij}_{hj}\left(q_{ij}\right) - \gamma_j)\hat{q}^{ij}_{hj}(q_{ij})]$. As $q^e_{ij} \in Int\left(Q_{ij}\right)$, it thus satisfies the following first-order condition:

$$(\hat{r}^{ij}_j)'\left(q^e_{ij}\right) - c_i - \sum_{h \in \mathcal{I} \setminus \{i\}} (t^e_{hj})'(q^e_{hj}) \times (\hat{q}^{ij}_{hj})'(q^e_{ij}) + \sum_{k \in \mathcal{J} \setminus \{j\}} [(t^e_{ik})'(q^e_{ik}) - c_i](\hat{q}^{ij}_{ik})'(q^e_{ij}) = 0. \quad (5)$$

However, R_j chooses q_{ij} so as to maximize its own profit, equal to:

$$\hat{r}^{ij}_j(q_{ij}) - t^e_{ij}(q_{ij}) - \sum_{h \in \mathcal{I} \setminus \{i\}} t^e_{hj}(\hat{q}^{ij}_{hj}(q_{ij})).$$

The equilibrium quantity q_{ij}^e must therefore also satisfy R_j's first-order condition:

$$(\hat{r}_j^{ij})'(q_{ij}^e) = (t_{ij}^e)'(q_{ij}^e) + \sum_{h \in \mathcal{I} \setminus \{i\}} (t_{hj}^e)'(q_{hj}^e) \times (\hat{q}_{hj}^{ij})'(q_{ij}^e). \tag{6}$$

Combining (5) and (6) yields:

$$u_{ij}^e - \sum_{k \in \mathcal{J} \setminus \{j\}} \delta_{jk}^{ij} u_{ik}^e = 0, \tag{7}$$

where $\delta_{jk}^{ij} \equiv -(\hat{q}_{ik}^{ij})'(q_{ij}^e)$ denotes the equilibrium diversion ratio of brand i's sales from R_j to R_k, and $u_{ik}^e \equiv (t_{ik}^e)'(q_{ik}^e) - c_i$ denote M_i's equilibrium margin on the sales through R_k. That is, these margins satisfy:

$$\boldsymbol{\delta}^i \cdot \begin{bmatrix} u_{i1}^e \\ \vdots \\ u_{im}^e \end{bmatrix} = 0,$$

where the $m \times m$ matrix δ^i is such that

$$\boldsymbol{\delta}^i(j,k) = \begin{cases} 1 & \text{if} \quad k = j, \\ -\delta_{jk}^{ij} & \text{if} \quad k \neq j. \end{cases}$$

This matrix is diagonal dominant, as $\left|\boldsymbol{\delta}^i(j,j)\right| = 1 > \sum_{k \in \mathcal{J} \setminus \{j\}} \delta_{jk}^{ij} = \sum_{k \in \mathcal{J} \setminus \{j\}} \left|\boldsymbol{\delta}^i(j,k)\right|$; it it is therefore non-singular, implying that M_i's equilibrium tariffs must be cost-based: the only solution is

$$u_{ij}^e = 0 \iff (t_{ij}^e)'(q_{ij}^e) = c_i$$

for every $j \in \mathcal{J}$.

Part (ii). When all tariffs are cost-based and induce smooth retail behaviors, the equilibrium prices satisfy the first-order conditions of each retailer's profit maximization program, that is, for $i \in \mathcal{I}$ and $j \in \mathcal{J}$:

$$\begin{aligned} 0 &= D_{ij}(\mathbf{p^e}) + \sum_{h \in \mathcal{I}} [p_{hj} - (t_{hj}^e)'(q_{hj}^e) - \gamma_j] \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p^e}) \\ &= D_{ij}(\mathbf{p^e}) + \sum_{h \in \mathcal{I}} (p_{hj} - c_h - \gamma_j) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p^e}) \\ &= \frac{\partial \pi_j}{\partial p_{ij}}(\mathbf{p^e}). \end{aligned}$$

These conditions thus coincide with those characterizing $\mathbf{p}^*$ and Assumption A then ensures that retail prices are $\mathbf{p^e} = \mathbf{p}^*$.

Online appendix for
Secret contracting with multilateral relations

A Division of profits

Proposition 2 shows that, as long as tariffs induce all retailers to adopt a smooth retail behavior, equilibrium prices and quantities, and thus total industry profit, are the same as in a multiproduct oligopoly. Together with Proposition 1, it shows further that the division of this profit is also uniquely defined when two-part tariffs are used. However, other tariffs can sustain different profit allocations. To see this, our next proposition considers quadratic tariffs of the form:

$$t_{ij}^{\sigma}\left(q_{ij}\right)=F_{ij}\left(\sigma\right)+c_i q_{ij}+\sigma\left(q_{ij}-q_{ij}^{*}\right)^{2},$$

where $q_{ij}^{*}=D_{ij}\left(\mathbf{p}^{*}\right)$ and $F_{ij}\left(\sigma\right)$ remains to be determined. For the sake of exposition, we assume that these tariffs generate a smooth retail response, even if a negotiation breaks down; that is:

Assumption A'. For σ not too negative and any $j \in \mathcal{J}$:

(i) Maximizing

$$\pi_j\left(\mathbf{p}_j,\mathbf{p}_{-j}^{*}\right)-\sum_{i\in\mathcal{I}}\sigma\left[D_{ij}\left(\mathbf{p}_j,\mathbf{p}_{-j}^{*}\right)-q_{ij}^{*}\right]^{2}$$

with respect to $\mathbf{p}_j$ yields a unique price response, which is uniquely characterized by first-order conditions.

(ii) For any $i \in \mathcal{I}$:

(a) Maximizing

$$\pi_j\left(\left(\infty,\mathbf{p}_{-i,j}\right),\mathbf{p}_{-j}^{*}\right)-\sum_{h\in\mathcal{I}\setminus\{i\}}\sigma\left[D_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}\right),\mathbf{p}_{-j}^{*}\right)-q_{hj}^{*}\right]^{2}$$

with respect to $\mathbf{p}_{-i,j}$ yields a unique price reaction, denoted $\mathbf{p}_{-i,j}^{ij}\left(\sigma\right)$, which is a continuous function of σ;

(b) This price reaction is such that $D_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}^{ij}\left(0\right)\right),\mathbf{p}_{-j}^{*}\right)\neq q_{hj}^{*}$ for some $h\in\mathcal{I}\setminus\{i\}$ and $D_{ik}\left(\left(\infty,\mathbf{p}_{-i,j}^{ij}\left(0\right)\right),\mathbf{p}_{-j}^{*}\right)\neq q_{ik}^{*}$ for some $k\in\mathcal{J}\setminus\{j\}$.

Assumptions A'(i) and A'$(ii.a)$ are satisfied, for instance, when the revenue function (letting $\bar{\mathbf{p}}_j\left(\mathbf{q}_j\right)$ denote the vector of inverse residual demands, satisfying $\left(D_{ij}\left(\bar{\mathbf{p}}_j,\mathbf{p}_{-j}^{\mathbf{e}}\right)\right)_{i\in I}=\mathbf{q}_j$)

$$r_j\left(\mathbf{q}_j\right)\equiv\sum_{i\in I}\left[\bar{p}_{ij}\left(\mathbf{q}_j\right)-\gamma_j\right]q_{ij}$$

is strictly concave. Assumption A'($ii.b$) simply asserts that breaking down a negotiation between a manufacturer and a retailer affects the manufacturer's sales in at least one other retailer's stores, as well as the retailer's sales of at least one other brand. We have:

Proposition A.1 (division of profit) *There exists $\overline{\sigma} > 0$ such that:*

(i) for any σ satisfying $|\sigma| < \overline{\sigma}$, there exists an equilibrium in which each pair $M_i - R_j$ signs a cost-based tariff of the form $t_{ij}^{\sigma}(q_{ij})$, for some $F_{ij}(\sigma)$, and all retail prices are equal to $\mathbf{p}^*$; and,

(ii) within this class of equilibria, each M_i obtains a profit $\Pi_{M_i}(\sigma)$, which is such that $\Pi_{M_i}(\sigma) > \Pi_{M_i}^*$ (resp., $\Pi_{M_i}(\sigma) < \Pi_{M_i}^*$) for $\sigma > 0$ (resp., $\sigma < 0$).

Proof. Consider a candidate equilibrium where retail prices are equal to $\mathbf{p}^*$ and each $M_i - R_j$ pair signs a contract:

$$t_{ij}^{\sigma}(q_{ij}) = F_{ij}(\sigma) + c_i q_{ij} + \sigma\left(q_{ij} - q_{ij}^*\right)^2,$$

for an appropriately chosen $F_{ij}(\sigma)$.

We first check that $\mathbf{p}^*$ constitutes a retail price equilibrium when these contracts are in place. In response to $\mathbf{p}_{-j}^*$, R_j chooses its prices $\mathbf{p}_j$ so as to maximize:

$$\pi_j^{\sigma}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) \equiv \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \sum_{i \in \mathcal{I}} \sigma\left[D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{ij}^*\right]^2.$$

It follows that $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$, which maximizes $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$ and leads to $D_{ij}(\mathbf{p}^*) = q_{ij}^*$, satisfies the first-order conditions, that is, for every $h \in \mathcal{I}$:

$$\left.\frac{\partial \pi_j^{\sigma}}{\partial p_{hj}}\right|_{\mathbf{p}_j = \mathbf{p}_j^*} = \frac{\partial \pi_j}{\partial p_{hj}}(\mathbf{p}^*) - 2\sigma \sum_{i \in \mathcal{I}} \left(q_{ij}^* - q_{ij}^*\right) \frac{\partial D_{ij}}{\partial p_{hj}}(\mathbf{p}^*) = \frac{\partial \pi_j}{\partial p_{hj}}(\mathbf{p}^*) = 0.$$

Assumption A'(i) then ensures that $\mathbf{p}_j = \mathbf{p}_j^*$ constitutes R_j's unique price response to the tariffs $\mathbf{t}_j^{\sigma}$.

In the negotiation between M_i and R_j, given their other equilibrium tariffs, $(t_{ik}^{\sigma})_{k \in \mathcal{J} \setminus \{j\}}$ and $\left(t_{hj}^{\sigma}\right)_{h \in \mathcal{I} \setminus \{i\}}$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^*$, the two firms seek to maximize their joint profit, which is now equal to:

$$\begin{aligned}&\left(p_j - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) + \sum_{k \in \mathcal{J} \setminus \{j\}} \left\{F_{ik}(\sigma) + \sigma\left[D_{ik}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{ik}^*\right]^2\right\} \\ &+ \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\left(p_{hj} - c_h - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - \left\{F_{hj}(\sigma) + \sigma\left[D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2\right\}\right].\end{aligned}$$

By construction, $\mathbf{p}_j^* = \mathbf{p}_j^r\left(\mathbf{p}_{-j}^*\right)$ satisfies the associated first-order conditions for $\sigma = 0$. As the additional terms in σ are of the form $2\sigma(D_{hk}(\mathbf{p}_j, \mathbf{p}_{-j}^*) - q_{hk}^*)$, charging $\mathbf{p}_j = \mathbf{p}_j^*$ (which leads to $D_{hk}(\mathbf{p}^*) = q_{hk}^*$ for every $(h,k) \in \mathcal{I} \times \mathcal{J}$), still satisfies these first-order conditions for $\sigma \neq 0$, . Furthermore, for $\sigma = 0$, the joint profit is uniquely maximal for $\mathbf{p}_j = \mathbf{p}_j^*$. It follows that it remains maximal at $\mathbf{p}_j^*$ for $|\sigma|$ low enough.

Likewise, from Lemma 1 (see Appendix A), M_i and R_j have an incentive to deal with each other when $|\sigma|$ is low enough. The tariffs $\mathbf{t}_j^\sigma$ then sustain an equilibrium in which retail prices are set to $\mathbf{p}^*$ and each channel $i - j$ generates a profit π_{ij}^*, to be shared according to the Nash bargaining rule.

Let us now evaluate the impact of σ on the division of profit. In equilibrium, each M_i derives all of its profit through the fixed fees:

$$\Pi_{M_i}(\sigma) = \sum_{j \in \mathcal{J}} F_{ij}(\sigma),$$

whereas each R_j obtains $\Pi_{R_j}(\sigma) = \pi_j^* - \sum_{i \in \mathcal{I}} F_{ij}(\sigma)$. If the negotiation with M_i were to break down, R_j would adjust its prices $\mathbf{p}_{-i,j}$ so as to maximize:

$$\pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) - \sum_{h \in \mathcal{I} \setminus \{i\}} \sigma \left[D_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2.$$

From Assumption A'$(ii.a)$, this yields a unique price response, $\mathbf{p}_{-i,j}^{ij}(\sigma)$, which is a continuous function of σ. Letting:

$$\pi_j^{ij}(\sigma) \equiv \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}(\sigma)\right), \mathbf{p}_{-j}^*\right) - \sum_{h \in \mathcal{I} \setminus \{i\}} \sigma \left[D_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}(\sigma)\right), \mathbf{p}_{-j}^*\right) - q_{hj}^*\right]^2$$

denote the associated value, and $q_{ik}^{ij}(\sigma) \equiv D_{ik}\left(\left(\infty, \mathbf{p}_{-i,j}^{ij}(\sigma)\right), \mathbf{p}_{-j}^*\right)$ denote M_i's sales through every other retailer R_k, M_i's and R_j's disagreement payoffs are respectively equal to:

$$\Pi_{M_i}^{ij}(\sigma) = \sum_{k \in \mathcal{J} \setminus \{j\}} \left\{F_{ik}(\sigma) + \sigma \left[q_{ik}^{ij}(\sigma) - q_{ik}^*\right]^2\right\} \text{ and } \Pi_{R_j}^{ij}(\sigma) = \pi_j^{ij}(\sigma) - \sum_{h \in \mathcal{I} \setminus \{i\}} F_{hj}(\sigma).$$

Hence, the impact of a successful negotiation on profits are respectively given by:

$$\Delta_{M_i} = F_{ij} - \sigma \sum_{k \in \mathcal{J} \setminus \{j\}} \left[q_{ik}^{ij}(\sigma) - q_{ik}^*\right]^2 \quad \text{and} \quad \Delta_{R_j} = \pi_j^* - F_{ij} - \pi_j^{ij}(\sigma),$$

and the Nash bargaining rule (4) yields:

$$F_{ij}(\sigma) = \alpha_{ij}\left[\pi_j^* - \pi_j^{ij}(\sigma)\right] + (1 - \alpha_{ij})\,\sigma \sum_{k \in \mathcal{J} \setminus \{j\}} \left[q_{ik}^{ij}(\sigma) - q_{ik}^*\right]^2.$$

Therefore:

$$\Pi_{M_i}(\sigma) = \sum_{j \in \mathcal{J}} \left\{\alpha_{ij}\left[\pi_j^* - \pi_j^{ij}(\sigma)\right] + (1 - \alpha_{ij})\,\sigma \sum_{k \in \mathcal{J} \setminus \{j\}} \left[q_{ik}^{ij}(\sigma) - q_{ik}^*\right]^2\right\},$$

and Assumption A'($ii.a$) ensures that this expression is a continuously differentiable function of σ. Furthermore, using the envelope theorem yields:

$$\frac{d\pi_j^{ij}}{d\sigma}(0) = -\sum_{h\in\mathcal{I}\setminus\{i\}} \left[q_{hj}^{ij}(0) - q_{hj}^{*}\right]^2.$$

We thus have:

$$\Pi'_{M_i}(0) = \sum_{j\in\mathcal{J}} \left\{ \alpha_{ij} \sum_{h\in\mathcal{I}\setminus\{i\}} \left[q_{hj}^{ij}(0) - q_{hj}^{*}\right]^2 + (1-\alpha_{ij}) \sum_{k\in\mathcal{J}\setminus\{j\}} \left[q_{ik}^{ij}(0) - q_{ik}^{*}\right]^2 \right\} > 0,$$

where the strict inequality follows from Assumption A'($ii.b$): for $\alpha_{ij} > 0$, $q_{hj}^{ij}(0) \neq q_{hj}^{*}$ for some $h \neq j$, and for $\alpha_{ij} = 0$, $q_{ik}^{ij}(0) \neq q_{ik}^{*}$ for some $k \neq j$.

It follows that $\Pi'_{M_i}(\sigma) > 0$ for σ close to 0; hence, in that range, $\Pi_{M_i}(\sigma) > \Pi^{*}_{M_i} = \Pi_{M_i}(0)$ (resp., $\Pi_{M_i}(\sigma) < \Pi^{*}_{M_i}$) for $\sigma > 0$ (resp., $\sigma < 0$). ■

Hence, while there is a unique retail equilibrium outcome, replicating that of a multiproduct oligopoly, manufacturers and retailers can share the resulting profit in various ways. With the above quadratic tariffs, manufacturers obtain a bigger share when marginal wholesale prices increase with the quantity being traded, as this degrades the retailers' outside option in case a negotiation breaks down. To see why, start with the equilibrium two-part tariffs $t_{ij}^{*}(q_{ij}) = F_{ij}^{*} + c_i q_{ij}$ used in Proposition 1, and introduce a convex term, $\sigma\left(q_{ij} - q_{ij}^{*}\right)^2$ with $\sigma > 0$, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. Modifying the tariff in this way does not affect the amount paid by R_j if it sticks to the equilibrium quantity q_{ij}^{*}, but increases the amount that R_j would have to pay if it were to modify its prices and/or stop carrying another brand. It follows that introducing this convex term weakens R_j's bargaining position in its negotiations with the other suppliers. Conversely, manufacturers obtain a smaller share when the tariffs are concave (i.e., when $\sigma < 0$).

B Micro-foundation: Proof of Proposition 3

Part (i). Fix a bargaining equilibrium $\mathcal{B} = \left\{ \left(\mathbf{p}_j^R(\mathbf{t}_j)\right)_{j\in\mathcal{J}}, \mathbf{t}^{\mathbf{e}}, \mathbf{p}^{\mathbf{e}} \right\}$ and, for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$:

- let

$$\begin{aligned} \Pi^e_{M_i} &\equiv \sum_{k\in\mathcal{J}} \left[t_{ik}\left(D_{ik}(\mathbf{p}^{\mathbf{e}})\right) - c_i D_{ik}(\mathbf{p}^{\mathbf{e}})\right], \\ \Pi^e_{R_j} &\equiv \sum_{h\in\mathcal{I}} \left[\left(p^e_{hj} - \gamma_j\right) D_{hj}(\mathbf{p}^{\mathbf{e}}) - t_{hj}\left(D_{hj}(\mathbf{p}^{\mathbf{e}})\right)\right], \end{aligned}$$

  respectively denote the equilibrium profits of M_i and R_j;

- let
$$\mathbf{p}_{-i,j}^{ij} \in \underset{\mathbf{p}_{-i,j}=\left(p_{hj}\right)_{h\in\mathcal{I}\setminus\{i\}}}{\arg\max} \sum_{h\in\mathcal{I}\setminus\{i\}} \left[\begin{array}{c} \left(p_{hj}-\gamma_j\right) D_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \\ -t_{hj}^{e}\left(D_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right) \end{array} \right]$$
denote R_j's price response if the negotiation with M_i breaks down, and
$$\begin{aligned} \Pi_{M_i}^{ij} &\equiv \sum_{k\in\mathcal{J}\setminus\{j\}} \left[t_{ik}^{e}\left(D_{ik}\left(\left(\infty,\mathbf{p}_{-i,j}^{ij}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right) - c_i D_{ik}\left(\left(\infty,\mathbf{p}_{-i,j}^{ij}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right], \\ \Pi_{R_j}^{ij} &\equiv \sum_{h\in\mathcal{I}\setminus\{i\}} \left[\left(p_{hj}^{ij}-\gamma_j\right) D_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}^{ij}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) - t_{hj}^{e}\left(D_{hj}\left(\left(\infty,\mathbf{p}_{-i,j}^{ij}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right)\right], \end{aligned}$$
respectively denote the resulting profits for M_i and R_j; and

- for every $\theta_{ij} \in \Theta_{ij}$, let
$$\hat{F}_{ij}^{\theta_{ij}} \equiv \begin{cases} \Pi_{R_j}^{e} - \Pi_{R_j}^{ij} & \text{if } \theta_{ij} = M_i^j, \\ -\left(\Pi_{M_i}^{e} - \Pi_{M_i}^{ij}\right) & \text{if } \theta_{ij} = R_j^i, \end{cases}$$
reflect the benefit of the bilateral relationship for θ_{ij}'s firm.

These fees balance each other in expectation:

Lemma B.1 (bargaining fees) *For every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$:*
$$E_{\theta_{ij}}\left[\hat{F}_{ij}^{\theta_{ij}}\right] = 0.$$

Proof. We have:
$$\begin{aligned} E_{\theta_{ij}}\left[\hat{F}_{ij}^{\theta_{ij}}\right] &= \alpha_{ij}\hat{F}_{ij}^{M_i^j} + \left(1-\alpha_{ij}\right)\hat{F}_{ij}^{R_j^i} \\ &= \alpha_{ij}\left(\Pi_{R_j}^{e} - \Pi_{R_j}^{ij}\right) - \left(1-\alpha_{ij}\right)\left(\Pi_{M_i}^{e} - \Pi_{M_i}^{ij}\right) \\ &= 0, \end{aligned}$$
where the last equality follows from Nash bargaining (equation (4)). ■

Let $\left(\hat{\mathbf{p}}^{\boldsymbol{\theta}}\right)_{\boldsymbol{\theta}\in\Theta}$ denote the price vector such that $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \mathbf{p}^{\mathbf{e}}$ for every $\boldsymbol{\theta} \in \Theta$ and, for every $j \in \mathcal{J}$, let $\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j} = \left(\hat{t}_{ij}^{\boldsymbol{\theta}_{ij}}\right)_{i\in\mathcal{I}}$ denote the tariffs $\hat{t}_{ij}^{\theta_{ij}} = t_{ij}^{e} + \hat{F}_{ij}^{\theta_{ij}}$ and $\hat{\mathbf{p}}_j^{R}\left(\mathbf{t}_j\right)$ denote the following candidate price response:

- $\hat{\mathbf{p}}_j^{R}(\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j}) = \mathbf{p}_j^{\mathbf{e}}$ for every $\boldsymbol{\theta}_j \in \Theta_j$;
- for every $i \in \mathcal{I}$ and every $\boldsymbol{\theta}_{-i,j} = (\theta_{hj})_{h\in\mathcal{I}\setminus\{i\}}$, $\hat{\mathbf{p}}_j^{R}(t_{i,j}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}) = \mathbf{p}_j^{R}(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}})$ (where $\hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}} = (\hat{t}_{hj}^{\theta_{hj}})_{h\in\mathcal{I}\setminus\{i\}}$ and $\mathbf{t}_{-i,j}^{\mathbf{e}} = (t_{hj}^{e})_{h\in\mathcal{I}\setminus\{i\}}$) whenever $t_{ij} \notin \{\hat{t}_{ij}^{M_i^j}, \hat{t}_{ij}^{R_j^i}\}$;
- if there is no agreement between R_j and $\{M_i\}_{i\in\overline{\mathcal{I}}}$, where $\overline{\mathcal{I}} \subset \mathcal{I}$ and $\overline{\mathcal{I}} \neq \varnothing$, then (with the convention that $t_{ij}(\cdot) = 0$ and $\hat{p}_{ij}^{R}\left(\mathbf{t}_j\right) = +\infty$ for $i \in \overline{\mathcal{I}}$, and expressing R_j's

tariffs as $\mathbf{t}_j = (\mathbf{0}, \bar{\mathbf{t}}_j)$, where $\bar{\mathbf{t}}_j = (t_{hj})_{h\in\mathcal{I}\setminus\overline{\mathcal{I}}}$, and its price vector as $\mathbf{p}_j = (\infty, \bar{\mathbf{p}}_j)$, where $\bar{\mathbf{p}}_j = (\bar{p}_{hj})_{h\in\mathcal{I}\setminus\overline{\mathcal{I}}}$):

$$\left(\hat{p}^R_{hj}\left(\mathbf{0}, \bar{\mathbf{t}}_j\right)\right)_{h\in\mathcal{I}\setminus\overline{\mathcal{I}}} \in \underset{\bar{\mathbf{p}}_j=\left(\bar{p}_{hj}\right)_{h\in\mathcal{I}\setminus\overline{\mathcal{I}}}}{\arg\max} \sum_{h\in\mathcal{I}\setminus\overline{\mathcal{I}}} \left[\begin{array}{c} \left(\bar{p}_{hj}-\gamma_j\right) D_{hj}\left(\left(\infty, \bar{\mathbf{p}}_j\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \\ -t_{hj}\left(D_{hj}\left(\left(\infty, \bar{\mathbf{p}}_j\right), \mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \end{array}\right],$$

with the restriction that, in case of multiple optima:

– the selected best-response is unaffected by lump-sum changes in tariffs; that is, for any $\mathbf{f}_j = (f_{hj})_{h\in\mathcal{I}\setminus\overline{\mathcal{I}}} \in \mathbb{R}_+^{\left|\mathcal{I}\setminus\overline{\mathcal{I}}\right|}$:

$$\hat{p}^R_{hj}\left(\mathbf{0}, \bar{\mathbf{t}}_j + \mathbf{f}_j\right) = \hat{p}^R_{hj}\left(\mathbf{0}, \bar{\mathbf{t}}_j\right) \text{ for every } h \in \mathcal{I}\setminus\overline{\mathcal{I}};$$

– in the particular case where there is no agreement with a single M_i, and all other tariffs correspond to the bargaining equilibrium tariffs (that is, $\mathbf{t}_{-i,j} = \mathbf{t}^{\mathbf{e}}_{-i,j}$), the selected best-response corresponds to the prices that would arise in the bargaining equilibrium:

$$\hat{\mathbf{p}}^R_{-i,j}\left(0, \mathbf{t}^{\mathbf{e}}_{-i,j}\right) = \mathbf{p}^{ij}_{-i,j};$$

- in all other cases, $\hat{\mathbf{p}}^R_j(\mathbf{t}_j) = \mathbf{p}^R_j(\mathbf{t}_j)$.

The price response $\hat{\mathbf{p}}^R_j(\cdot)$ coincides with the price response of the bargaining equilibrium, $\mathbf{p}^R_j(\cdot)$, whenever R_j's profit has a unique maximum. In case of multiple maxima, $\hat{\mathbf{p}}^R_j(\mathbf{t}_j)$ differs from $\mathbf{p}^R_j(\mathbf{t}_j)$ only if $\mathbf{p}^R_j(\cdot)$ were to pick different maxima when altering the tariffs $\mathbf{t}_j$ by some constants. We now show that $((\hat{\mathbf{p}}^R_j(\mathbf{t}_j))_{j\in\mathcal{J}}, (\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta})$, together with consistent beliefs, constitutes an equilibrium of game Γ.

We first note that this candidate equilibrium gives all firms the same expected profits as the bargaining equilibrium $\mathcal{B}$: the price responses to the equilibrium tariffs are designed to generate the same retail prices as in $\mathcal{B}$ (regardless of which side gets to make the offers), and the expected equilibrium tariffs (where the expectation refers to which side makes the offers) coincide with the bargaining equilibrium tariffs. It can further be noted that the tariffs $\hat{t}^{\theta_{ij}}_{ij}$ are such that $\hat{t}^{M^j_i}_{ij}$ gives R_j its disagreement profit in the bargaining equilibrium $\mathcal{B}$, $\Pi^{ij}_{R_j}$, and conversely $\hat{t}^{R^i_j}_{ij}$ gives M_i its disagreement profit in the bargaining equilibrium $\mathcal{B}$, $\Pi^{ij}_{M_i}$.

We now study stage 2, and consider a given retailer R_j, for some $j \in \mathcal{J}$. With consistent beliefs, following any deviation on $\mathbf{t}_j$ (including the absence of an agreement), R_j expects the other retailers' agents to have negotiated the equilibrium tariffs, and thus it expects its rivals to charge the equilibrium prices, which are the same as in the bargaining equilibrium $\mathcal{B}$: $\hat{\mathbf{p}}^{\boldsymbol{\theta}_{-j}}_{-j} = \mathbf{p}^{\mathbf{e}}_{-j}$ for every $\boldsymbol{\theta}_{-j} \in \Theta_{-j}$. Therefore:

• Following disagreement with one or several manufacturers, $\hat{\mathbf{p}}^R_j(\cdot)$ constitutes an appropriate price response, as it maximizes R_j's expected profit, given its beliefs that all other retailers charge $\mathbf{p}^{\mathbf{e}}_{-j}$.

• If instead R_j agreed on a tariff t_{ij} with every M_i, for $i \in \mathcal{I}$, then by construction $\mathbf{p}^R_j(\mathbf{t}_j)$ constitutes an appropriate price response to $\mathbf{t}_j = (t_{ij})_{i\in\mathcal{I}}$. Furthermore, if $\mathbf{t}_j = \left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right)$ for some $i \in \mathcal{I}$ and some $\boldsymbol{\theta}_{-i,j} \equiv (\theta_{hj})_{h\in\mathcal{I}\setminus\{i\}}$, then $\mathbf{p}^R_j\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right)$ also constitutes an appropriate price response to $\mathbf{t}_j$, as the tariffs $\hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}$ coincide with the bargaining equilibrium tariffs $\mathbf{t}^{\mathbf{e}}_{-i,j}$ up to some constants. The same applies to the particular case where $t_{ij} = \hat{t}^{\theta_{ij}}_{ij}$ for some $\theta_{ij} \in \Theta_{ij}$, in which case $\mathbf{t}_j = (\hat{t}^{\theta_{ij}}_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}) = (\hat{\mathbf{t}}^{\boldsymbol{\theta}_{\cdot,j}}_j)$ and $\mathbf{p}^{\mathbf{e}}_j = \mathbf{p}^R_j(\mathbf{t}^{\mathbf{e}}_j)$, which maximizes R_j's profit when it faces $\mathbf{t}^{\mathbf{e}}_j$, also maximizes R_j's profit when it faces $\hat{\mathbf{t}}^{\boldsymbol{\theta}_{\cdot,j}}_j$, as the tariffs $\hat{\mathbf{t}}^{\boldsymbol{\theta}_{\cdot,j}}_j$ coincide with the tariffs $\mathbf{t}^{\mathbf{e}}_j$ up to some constants.

It follows that $\hat{\mathbf{p}}^R_j(\cdot)$ indeed constitutes an appropriate price response for R_j.

We now turn to stage 1, and study the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. In the bargaining equilibrium $\mathcal{B}$, the tariff t^e_{ij} maximizes their joint profit, given the other retailers' prices, the tariffs $\mathbf{t}^{\mathbf{e}}_{i,-j} = (t^e_{ik})_{k\in\mathcal{J}\setminus\{j\}}$ and $\mathbf{t}^{\mathbf{e}}_{-i,j} = (t^e_{hj})_{h\in\mathcal{I}\setminus\{i\}}$ negotiated with the other partners, and R_j's price response $\mathbf{p}^R_j(\cdot)$; that is, $t_{ij} = t^e_{ij}$ maximizes:

$$\begin{aligned}
\Pi^R_{M_i-R_j}(t_{ij}) \equiv\ & \left[p^R_{ij}\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right) - c_i - \gamma_j\right] D_{ij}\left(\mathbf{p}^R_j\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \\
& + \sum_{k\in\mathcal{J}\setminus\{j\}} \left[\begin{array}{c} t^e_{ik}\left(D_{ik}\left(\mathbf{p}^R_j\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \\ -c_i D_{ik}\left(\mathbf{p}^R_j\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \end{array}\right] \\
& + \sum_{h\in\mathcal{I}\setminus\{i\}} \left\{\begin{array}{c} \left[p^R_{hj}\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right) - \gamma_j\right] D_{hj}\left(\mathbf{p}^R_j\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \\ -t^e_{hj}\left(D_{hj}\left(\mathbf{p}^R_j\left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \end{array}\right\}.
\end{aligned}$$

In the above candidate equilibrium of game Γ, the agents M^j_i and R^i_j expect all other agents to negotiate the equilibrium tariffs and the other retailers to charge the equilibrium prices. Hence, when signing a tariff t_{ij} they expect R_j to charge $\hat{\mathbf{p}}^R_j\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right)$ for any realization $\boldsymbol{\theta}_{-i,j} = (\theta_{hj})_{h\in\mathcal{I}\setminus\{i\}} \in \boldsymbol{\Theta}_{-i,j} = \Pi_{h\in\mathcal{I}\setminus\{i\}}\Theta_{hj}$. Therefore, they expect the joint profit of their two firms to be given by:

$$\hat{\Pi}^R_{M_i-R_j}(t_{ij}) \equiv E_{\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}}\left[\tilde{\Pi}^R_{M_i-R_j}(t_{ij}; \boldsymbol{\theta}_{i,-j}, \boldsymbol{\theta}_{-i,j})\right],$$

where $\boldsymbol{\theta}_{i,-j} \equiv (\theta_{ik})_{k\in\mathcal{J}\setminus\{j\}}$ and:

$$\begin{aligned}
\tilde{\Pi}^R_{M_i-R_j}(t_{ij}; \boldsymbol{\theta}_{i,-j}, \boldsymbol{\theta}_{-i,j}) \equiv\ & \left[\hat{p}^R_{ij}\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right) - c_i - \gamma_j\right] D_{ij}\left(\hat{\mathbf{p}}^R_j\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \\
& + \sum_{k\in\mathcal{J}\setminus\{j\}} \left[\begin{array}{c} \hat{t}^{\theta_{ik}}_{ik}\left(D_{ik}\left(\hat{\mathbf{p}}^R_j\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \\ -c_i D_{ik}\left(\hat{\mathbf{p}}^R_j\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \end{array}\right] \\
& + \sum_{h\in\mathcal{I}\setminus\{i\}} \left\{\begin{array}{c} \left[\hat{p}^R_{hj}\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right) - \gamma_j\right] D_{hj}\left(\hat{\mathbf{p}}^R_j\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \\ -\hat{t}^{\theta_{hj}}_{hj}\left(D_{hj}\left(\hat{\mathbf{p}}^R_j\left(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \end{array}\right\}
\end{aligned} \tag{8}$$

If feasible, it would be optimal for the selected agent, θ_{ij}, to offer a tariff that maximizes the expected joint profit $\hat{\Pi}^R_{M_i-R_j}(t_{ij})$, and leaves the other agent indifferent between accepting or rejecting the offer. To conclude the argument, we now show that the tariff $\hat{t}^{\theta_{ij}}_{ij}$ achieves precisely this.

• *For any $\theta_{ij} \in \Theta_{ij}$, the tariff $\hat{t}^{\theta_{ij}}_{ij}$ maximizes the expected joint profit of M_i and R_j.* Picking $t_{ij} \notin \{\hat{t}^{\theta_{ij}}_{ij}\}_{\theta_{ij}\in\Theta_{ij}}$ would yield (using $\hat{\mathbf{p}}^R_j(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}) = \mathbf{p}^R_j(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j})$):

$$\begin{aligned}
\tilde{\Pi}^R_{M_i-R_j}(t_{ij};\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}) \equiv\ & \left[p^R_{ij}\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right) - c_i - \gamma_j\right] D_{ij}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right) \\
&+ \sum_{k\in\mathcal{J}\backslash\{j\}} \left[\begin{array}{c} \hat{t}^{\theta_{ik}}_{ik}\left(D_{ik}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \\ -c_i D_{ik}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right) \end{array}\right] \\
&+ \sum_{h\in\mathcal{I}\backslash\{i\}} \left\{\begin{array}{c} \left[p^R_{hj}\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right) - \gamma_j\right] D_{hj}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right) \\ -\hat{t}^{\theta_{hj}}_{hj}\left(D_{hj}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \end{array}\right\}.
\end{aligned}$$

From Lemma B.1, we have:

$$E_{\theta_{hk}}\left[\hat{t}^{\theta_{hk}}_{hk}(q_{hk})\right] = t^e_{hk}(q_{hk}) + E_{\theta_{hk}}\left[\hat{F}^{\theta_{hk}}_{hk}\right] = t^e_{hk}(q_{hk}). \tag{9}$$

Therefore:

$$\begin{aligned}
\hat{\Pi}^R_{M_i-R_j}(t_{ij}) =\ & \left[p^R_{ij}\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right) - c_i - \gamma_j\right] D_{ij}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right) \\
&+ \sum_{k\in\mathcal{J}\backslash\{j\}} \left[\begin{array}{c} t^e_{ik}\left(D_{ik}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \\ -c_i D_{ik}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right) \end{array}\right] \\
&+ \sum_{h\in\mathcal{I}\backslash\{i\}} \left\{\begin{array}{c} \left[p^R_{hj}\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right) - \gamma_j\right] D_{hj}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right) \\ -t^e_{hj}\left(D_{hj}\left(\mathbf{p}^R_j\left(t_{ij},\mathbf{t}^{\mathbf{e}}_{-i,j}\right),\mathbf{p}^{\mathbf{e}}_{-j}\right)\right) \end{array}\right\} \\
=\ & \Pi^R_{M_i-R_j}(t_{ij}).
\end{aligned} \tag{10}$$

Picking instead $t_{ij} = \hat{t}^{\theta_{ij}}_{ij}$, for any $\theta_{ij} \in \Theta_{ij}$, yields (using $\hat{\mathbf{p}}^R_j\left(\hat{t}^{\theta_{ij}}_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right) = \mathbf{p}^{\mathbf{e}}_j$):

$$\begin{aligned}
\tilde{\Pi}^R_{M_i-R_j}\left(\hat{t}^{\theta_{ij}}_{ij};\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}\right) \equiv\ & \left(p^e_{ij} - c_i - \gamma_j\right) D_{ij}(\mathbf{p}^{\mathbf{e}}) \\
&+ \sum_{k\in\mathcal{J}\backslash\{j\}} \left[\hat{t}^{\theta_{ik}}_{ik}\left(D_{ik}(\mathbf{p}^{\mathbf{e}})\right) - c_i D_{ik}(\mathbf{p}^{\mathbf{e}})\right] \\
&+ \sum_{h\in\mathcal{I}\backslash\{i\}} \left[\left(p^e_{hj} - \gamma_j\right) D_{hj}(\mathbf{p}^{\mathbf{e}}) - \hat{t}^{\theta_{hj}}_{hj}\left(D_{hj}(\mathbf{p}^{\mathbf{e}})\right)\right],
\end{aligned}$$

and thus, using (9):

$$\begin{aligned}
\hat{\Pi}^R_{M_i-R_j}\left(\hat{t}^{\theta_{ij}}_{ij}\right) &= \left(p^e_{ij}-c_i-\gamma_j\right)D_{ij}\left(\mathbf{p^e}\right)+\sum_{k\in\mathcal{J}\setminus\{j\}}\left[t^e_{ik}\left(D_{ik}\left(\mathbf{p^e}\right)\right)-c_iD_{ik}\left(\mathbf{p^e}\right)\right] \\
&\quad+\sum_{h\in\mathcal{I}\setminus\{i\}}\left[\left(p^e_{hj}-\gamma_j\right)D_{hj}\left(\mathbf{p^e}\right)-t^e_{hj}\left(D_{hj}\left(\mathbf{p^e}\right)\right)\right] \\
&= \Pi^R_{M_i-R_j}\left(t^e_{ij}\right). \qquad (11)
\end{aligned}$$

As $\Pi^R_{M_i-R_j}(t_{ij})$ is maximal for $t_{ij}=t^e_{ij}$, it follows from (10) and (11) that $\hat{\Pi}^R_{M_i-R_j}(t_{ij})$ is indeed maximal for $t_{ij}=\hat{t}^{\theta_{ij}}_{ij}$, for any $\theta_{ij}\in\Theta_{ij}$.

• *The agent R^i_j is indifferent between accepting or rejecting the tariff $\hat{t}^{M^j_i}_{ij}$.* If the agents M^j_i and R^i_j do not reach an agreement, R_j's expected profit is given by (with the convention that $\mathbf{t}_j=(0,\mathbf{t}_{-i,j})$):

$$\hat{\Pi}^{ij}_{R_j}\equiv E_{\boldsymbol{\theta}_{-i,j}}\left[\sum_{h\in\mathcal{I}\setminus\{i\}}\left\{\begin{array}{c}\left[\hat{p}^R_{hj}\left(0,\mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right)-\gamma_j\right]D_{hj}\left(\infty,\hat{\mathbf{p}}^R_{-i,j}\left(0,\mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right),\mathbf{p^e}_{-j}\right)\\ -t^{\theta_{hj}}_{hj}\left(D_{hj}\left(\infty,\hat{\mathbf{p}}^R_{-i,j}\left(0,\mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right),\mathbf{p^e}_{-j}\right)\right)\end{array}\right\}\right].$$

The price response $\mathbf{p}^R_{-i,j}(0,\cdot)$ is invariant to lump-sum changes in tariffs, and the tariffs $\mathbf{t}^{M^j_i}_{-i,j}$ and $\mathbf{t}^{R^i_j}_{-i,j}$ only differ from $\mathbf{t^e}_{-i,j}$ by some fixed fees. We thus have:

$$\hat{\mathbf{p}}^R_{-i,j}\left(0,\mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right)=\hat{\mathbf{p}}^R_{-i,j}\left(0,\mathbf{t^e}_{-i,j}\right)=\mathbf{p}^{ij}_{-i,j}.$$

Hence, R_j's expected disagreement profit can be expressed as:

$$\begin{aligned}
\hat{\Pi}^{ij}_{R_j} &= \sum_{h\in\mathcal{I}\setminus\{i\}}\left\{\begin{array}{c}\left(p^{ij}_{hj}-\gamma_j\right)D_{hj}\left(\infty,\mathbf{p}^{ij}_{-i,j},\mathbf{p^e}_{-j}\right)\\ -E_{\theta_{hj}}\left[t^{\theta_{hj}}_{hj}\left(D_{hj}\left(\infty,\mathbf{p}^{ij}_{-i,j},\mathbf{p^e}_{-j}\right)\right)\right]\end{array}\right\} \\
&= \sum_{h\in\mathcal{I}\setminus\{i\}}\left[\begin{array}{c}\left(p^{ij}_{hj}-\gamma_j\right)D_{hj}\left(\infty,\mathbf{p}^{ij}_{-i,j},\mathbf{p^e}_{-j}\right)\\ -t^e_{hj}\left(D_{hj}\left(\infty,\mathbf{p}^{ij}_{-i,j},\mathbf{p^e}_{-j}\right)\right)\end{array}\right] \\
&= \Pi^{ij}_{R_j},
\end{aligned}$$

where the second equality stems from (9), and the last one from the definition of $\mathbf{p}^{ij}_{-i,j}$. But as already noted, accepting the tariff $\hat{t}^{M^j_i}_{ij}$ gives R_j precisely this profit, $\Pi^{ij}_{R_j}$.

• *The agent M^j_i is indifferent between accepting or rejecting the tariff $\hat{t}^{R^i_j}_{ij}$.* In the absence of an agreement, M_i's expected profit is given by:

$$\hat{\Pi}^{ij}_{M_i}\equiv E_{\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}}\left[\sum_{k\in\mathcal{J}\setminus\{j\}}\left\{\begin{array}{c}t^{\theta_{ik}}_{ik}\left(D_{ik}\left(\infty,\hat{\mathbf{p}}^R_{-i,j}\left(0,\mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right),\mathbf{p^e}_{-j}\right)\right)\\ -c_iD_{ik}\left(\infty,\hat{\mathbf{p}}^R_{-i,j}\left(0,\mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right),\mathbf{p^e}_{-j}\right)\end{array}\right\}\right].$$

Using again $\hat{\mathbf{p}}^R_{-i,j}\left(0, \mathbf{t}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}\right) = \hat{\mathbf{p}}^R_{-i,j}\left(0, \mathbf{t}^{\mathbf{e}}_{-i,j}\right) = \mathbf{p}^{ij}_{-i,j}$, this expected profit can be expressed as:

$$\begin{aligned}
\hat{\Pi}^{ij}_{M_i} &= \sum_{k \in \mathcal{J}\setminus\{j\}} \left\{ E_{\boldsymbol{\theta}_{ik}} \left[t^{\theta_{ik}}_{ik} \left(D_{ik}\left(\infty, \mathbf{p}^{ij}_{-i,j}, \mathbf{p}^{\mathbf{e}}_{-j}\right)\right)\right] - c_i D_{ik}\left(\infty, \mathbf{p}^{ij}_{-i,j}, \mathbf{p}^{\mathbf{e}}_{-j}\right) \right\} \\
&= \sum_{k \in \mathcal{J}\setminus\{j\}} \left[t^{e}_{ik} \left(D_{ik}\left(\infty, \mathbf{p}^{ij}_{-i,j}, \mathbf{p}^{\mathbf{e}}_{-j}\right)\right) - c_i D_{ik}\left(\infty, \mathbf{p}^{ij}_{-i,j}, \mathbf{p}^{\mathbf{e}}_{-j}\right) \right] \\
&= \Pi^{ij}_{M_i},
\end{aligned}$$

which, as already noted, is precisely the profit that M^j_i obtains when accepting the tariff $\hat{t}^{R^i_j}_{ij}$.

Part (ii). Fix an equilibrium of game Γ, $\mathcal{E} = \left\{ \left(\hat{\mathbf{p}}^R_j(\mathbf{t}_j)\right)_{j\in\mathcal{J}}, \left(\hat{\mathbf{t}}^{\boldsymbol{\theta}}\right)_{\boldsymbol{\theta}\in\Theta}, \left(\hat{\mathbf{p}}^{\boldsymbol{\theta}}\right)_{\boldsymbol{\theta}\in\Theta}, \mathbf{b} \right\}$, such that price responses and tariffs are regular, and so $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$ for any $\boldsymbol{\theta} \in \Theta$, and the beliefs $\mathbf{b}$ are consistent. Consider now the tariffs $\mathbf{t}^{\mathbf{e}} = \left(t^e_{ij}\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ where, for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$:

$$t^e_{ij}(q_{ij}) = E_{\theta_{ij}}\left[\hat{t}^{\theta_{ij}}_{ij}(q_{ij})\right] = \alpha_{ij}\hat{t}^{M^j_i}_{ij}(q_{ij}) + (1-\alpha_{ij})\,\hat{t}^{R^i_j}_{ij}(q_{ij}). \tag{12}$$

We now show that $\left\{ \left(\hat{\mathbf{p}}^R_j(\mathbf{t}_j)\right)_{j\in\mathcal{J}}, \mathbf{t}^{\mathbf{e}}, \mathbf{p}^{\mathbf{e}} = \hat{\mathbf{p}} \right\}$ constitutes a bargaining equilibrium.

We start again with stage 2, and consider a given R_j, for some $j \in \mathcal{J}$. As $\mathbf{p}^{\mathbf{e}} = \hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$, R_j expects the other retailers to charge the same prices as in the equilibrium $\mathcal{E}$. Hence, $\hat{\mathbf{p}}^R_j(\mathbf{t}_j)$ constitutes an appropriate price response to any tariffs $\mathbf{t}_j$ negotiated by R_j's agents in stage 1:

$$\begin{aligned}
\hat{\mathbf{p}}^R_j(\mathbf{t}_j) &\in \arg\max_{\mathbf{p}_j} E_{\boldsymbol{\theta}_{-j}} \left[\sum_{i\in\mathcal{I}} \left\{ \begin{array}{c} (p_{ij} - \gamma_j)\, D_{ij}\left(\mathbf{p}_j, \hat{\mathbf{p}}^{\boldsymbol{\theta}_{-j}}_{-j}\right) \\ -t_{ij}\left(D_{ij}\left(\mathbf{p}_j, \hat{\mathbf{p}}^{\boldsymbol{\theta}_{-j}}_{-j}\right)\right) \end{array} \right\} \right] \\
&= \arg\max_{\mathbf{p}_j} \left\{ \sum_{i\in\mathcal{I}} \left[(p_{ij} - \gamma_j)\, D_{ij}(\mathbf{p}_j, \hat{\mathbf{p}}) - t_{ij}\left(D_{ij}(\mathbf{p}_j, \hat{\mathbf{p}})\right) \right] \right\}.
\end{aligned}$$

The regularity of the equilibrium tariffs implies that, for any $h \in \mathcal{I}$, $\hat{t}^{M^j_h}_{hj}(\cdot)$ and $\hat{t}^{R^h_j}_{hj}(\cdot)$ coincide, up to a constant; it follows from the definition of the tariff t^e_{hj} that it also differs from $\hat{t}^{M^j_h}_{hj}(\cdot)$ or $\hat{t}^{R^h_j}_{hj}(\cdot)$ only by a constant. The regularity of price responses then ensures that, along the equilibrium path, prices are indeed equal to $\mathbf{p}^{\mathbf{e}}_j$:

$$\hat{\mathbf{p}}^R_j\left(\mathbf{t}^{\mathbf{e}}_j\right) = \hat{\mathbf{p}}^R_j\left(\hat{\mathbf{t}}^{\boldsymbol{\theta}_j}_j\right) = \hat{\mathbf{p}}^{\boldsymbol{\theta}_j}_j = \mathbf{p}^{\mathbf{e}}_j.$$

We now turn to stage 1, and study the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. In the equilibrium $\mathcal{E}$, the tariff offered by the selected agent, θ_{ij}, maximizes the expected profit of its firm, among those that are acceptable by the other agent. As profits can easily be shared through lump-sum fees (which do not affect retailers' pricing decisions, as price responses are regular), it follows that the tariff $\hat{t}_{ij}^{\theta_{ij}}$ maximizes the expected joint profit of M_i and R_j, given the other retailers' prices and the tariffs negotiated with the other partners; that is, $t_{ij} = \hat{t}_{ij}^{\theta_{ij}}$ maximizes:

$$\hat{\Pi}_{M_i-R_j}^{R}\left(t_{ij}\right) \equiv E_{\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}}\left[\tilde{\Pi}_{M_i-R_j}^{R}\left(t_{ij};\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}\right)\right],$$

where $\tilde{\Pi}_{M_i-R_j}^{R}\left(t_{ij};\boldsymbol{\theta}_{i,-j},\boldsymbol{\theta}_{-i,j}\right)$ is given by (8). As the price responses are regular and, as noted above, the tariffs only differ by some constants, we have:

$$\hat{p}_{ij}^{R}\left(t_{ij},\hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}\right) = \hat{p}_{ij}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right).$$

The expression of $\hat{\Pi}_{M_i-R_j}^{R}\left(t_{ij}\right)$ can therefore be simplified to:

$$\begin{aligned}
\hat{\Pi}_{M_i-R_j}^{R}\left(t_{ij}\right) &= \left[\hat{p}_{ij}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right) - c_i - \gamma_j\right] D_{ij}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \\
&\quad + \sum_{k\in\mathcal{J}\setminus\{j\}}\left\{\begin{array}{c} E_{\boldsymbol{\theta}_{ik}}\left[\hat{t}_{ik}^{\theta_{ik}}\left(D_{ik}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right)\right] \\ -c_i D_{ik}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \end{array}\right\} \\
&\quad + \sum_{h\in\mathcal{I}\setminus\{i\}}\left\{\begin{array}{c} \left[\hat{p}_{hj}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right) - \gamma_j\right] D_{hj}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \\ -E_{\boldsymbol{\theta}_{hj}}\left[\hat{t}_{hj}^{\theta_{hj}}\left(D_{hj}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right)\right] \end{array}\right\}
\end{aligned}$$

Using $t_{hk}^{e}\left(\cdot\right) = E_{\theta_{hk}}\left[\hat{t}_{hk}^{\theta_{hk}}\left(\cdot\right)\right]$ for any $h\in\mathcal{I}$ and $k\in\mathcal{J}$, we then have:

$$\begin{aligned}
\hat{\Pi}_{M_i-R_j}^{R}\left(t_{ij}\right) &= \left[\hat{p}_{ij}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right) - c_i - \gamma_j\right] D_{ij}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \\
&\quad + \sum_{k\in\mathcal{J}\setminus\{j\}}\left[\begin{array}{c} t_{ik}^{e}\left(D_{ik}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right) \\ -c_i D_{ik}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \end{array}\right] \\
&\quad + \sum_{h\in\mathcal{I}\setminus\{i\}}\left\{\begin{array}{c} \left[\hat{p}_{hj}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right) - \gamma_j\right] D_{hj}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right) \\ -t_{hj}^{e}\left(D_{hj}\left(\hat{\mathbf{p}}_{j}^{R}\left(t_{ij},\mathbf{t}_{-i,j}^{\mathbf{e}}\right),\mathbf{p}_{-j}^{\mathbf{e}}\right)\right) \end{array}\right\},
\end{aligned}$$

where the right-hand side corresponds to $\Pi_{M_i-R_j}^{R}\left(t_{ij}\right)$, the joint profit of M_i and R_j in their bilateral negotiation of the bargaining game, taking as given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^{\mathbf{e}}$, as well as rivals' equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$, and R_j's price response, $\mathbf{p}_{j}^{R}\left(\mathbf{t}_j\right)$. As by construction $t_{ij} = \hat{t}_{ij}^{\theta_{ij}}$ maximizes $\hat{\Pi}_{M_i-R_j}^{R}\left(t_{ij}\right)$, it thus also maximizes $\Pi_{M_i-R_j}^{R}\left(t_{ij}\right)$. Furthermore, as R_j's price response is regular and t_{hk}^{e} differs from $\hat{t}_{hk}^{\theta_{hk}}$ only by some constant, it follows that $t_{ij} = t_{ij}^{e}$, too, maximizes $\Pi_{M_i-R_j}^{R}\left(t_{ij}\right) = \hat{\Pi}_{M_i-R_j}^{R}\left(t_{ij}\right)$. Finally, it is straightforward to check that t_{ij}^{e} gives M_i and R_j shares α_{ij} and $1-\alpha_{ij}$, respectively, of the additional profit generated by their relationship, and that the resulting expected profits are the same as in the equilibrium $\mathcal{E}$.

C Micro-foundation for linear tariffs

Consider a bargaining equilibrium for some vector of bargaining weights $\boldsymbol{\alpha} = (\alpha_{ij})_{(i,j)\in\mathcal{I}\times\mathcal{J}} \in [0,1]^{nm}$, and let $\mathbf{w^e}(\boldsymbol{\alpha})$ and $\mathbf{p^e}(\boldsymbol{\alpha})$ denote the equilibrium wholesale and retail prices, $\Pi^e_{M_i}(\boldsymbol{\alpha})$ and $\Pi^e_{R_j}(\boldsymbol{\alpha})$ respectively denote M_i's and R_j's profits, and $\underline{\Pi}^j_{M_i}(\boldsymbol{\alpha}) \leq \Pi^e_{M_i}(\boldsymbol{\alpha})$ and $\underline{\Pi}^i_{R_j}(\boldsymbol{\alpha}) \leq \Pi^e_{R_j}(\boldsymbol{\alpha})$ respectively denote their disagreements payoffs – i.e., the profits that they would obtain if the other equilibrium tariffs remained unchanged but they decided not to deal with each other. As the outcomes of the bilateral negotiations are given by the Nash bargaining rule, we have:

$$w^e_{ij}(\boldsymbol{\alpha}) = \arg\max_{w_{ij}} \left\{ \left[\Pi^R_{M_i}\left(w_{ij}, \mathbf{w^e}_{-\{ij\}}(\boldsymbol{\alpha})\right) - \underline{\Pi}^j_{M_i}\right]^{\alpha} \left[\Pi^R_{R_j}\left(w_{ij}, \mathbf{w^e}_{-\{ij\}}(\boldsymbol{\alpha})\right) - \underline{\Pi}^i_{R_j}\right]^{1-\alpha} \right\}$$

where $\Pi^R_{M_i}\left(\mathbf{w}_j, \mathbf{w^e}_{-j}(\boldsymbol{\alpha})\right)$ and $\Pi^R_{M_i}\left(\mathbf{w}_j, \mathbf{w^e}_{-j}(\boldsymbol{\alpha})\right)$ denote M_i's and R_j's profits in the continuation equilibrium where R_j faces wholesale prices w_j and expects all other retailers to face wholesale prices $\mathbf{w}^e_{-j}(\boldsymbol{\alpha})$ and charge retail prices $\mathbf{p^e}_{-j}(\boldsymbol{\alpha})$; the resulting equilibrium profits, $\Pi^e_{M_i}(\boldsymbol{\alpha})$ and $\Pi^e_{R_j}(\boldsymbol{\alpha})$, lie on the Pareto frontier generated by varying w_{ij}.

We now show that this bargaining equilibrium can be replicated as an equilibrium of the game $\hat{\Gamma}$ for some vector of "bargaining parameters" $\boldsymbol{\beta} = \left(\beta_{ij}\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}} \in [0,1]^{nm}$ (i.e., if the pair $M_i - R_j$ reaches stage 1, M^j_i gets to make a take-it-or-leave-it offer with probability β_{ij}). For the sake of exposition, we assume here that, in stage 0, the agent of the upstream firm is always selected to make an offer to its counterpart. The analysis is qualitatively the same if the downstream agent were instead selected, although the specific choice of the bargaining parameters $\boldsymbol{\beta}$ would be affected.

Specifically, we construct an equilibrium in which the wholesale tariffs offered at stage 0 are equal to $\mathbf{w^e}(\boldsymbol{\alpha})$ and these tariffs are accepted by retailers' agents at that stage. If this is indeed the case, all firms obtain the same profits as in the bargaining equilibrium with weights $\boldsymbol{\alpha}$. Fix $i \in \mathcal{I}$ and $j \in \mathcal{J}$, and consider what happens if R^i_j rejects M^j_i's offer but all other tariffs are accepted at stage 0. With probability β_{ij}, M^j_i gets to make a (final) counter-offer and thus leaves R^i_j a profit just equal to the disagreement payoff $\underline{\Pi}^e_{R_j}(\boldsymbol{\alpha})$. With probability $1 - \beta_{ij}$, R^i_j gets to make the final offer and it thus achieves at least $\Pi^e_{R_j}(\boldsymbol{\alpha})$: it can secure this profit by offering a wholesale price $w_{ij} = w^e_{ij}(\boldsymbol{\alpha})$, which M^j_i is willing to accept (as it is the outcome of the Nash bargaining rule, which never harms the negotiating parties). Therefore, R^i_j's outside option in game $\hat{\Gamma}(\boldsymbol{\beta})$ is of the form $\hat{\Pi}_j\left(\beta_{ij}\right)$, which is non-decreasing in β_{ij} and such that $\hat{\Pi}_j(0) = \underline{\Pi}^e_{R_j}(\boldsymbol{\alpha}) \leq \Pi^e_{R_j}(\boldsymbol{\alpha}) \leq \hat{\Pi}_j(1)$. There thus exists β_{ij} such that $\hat{\Pi}_j\left(\beta_{ij}\right) = \Pi^e_{R_j}(\boldsymbol{\alpha})$. This value of β_{ij} induces M^j_i, in stage 0, to choose a wholesale price w_{ij} that lies on the Pareto frontier and gives R_j at least $\Pi^e_{R_j}(\boldsymbol{\alpha})$; this leads M^j_i to choose $w_{ij} = w^e_{ij}(\boldsymbol{\alpha})$, and profits thus correspond to the equilibrium profits of the bargaining equilibrium. As a consequence, any bargaining equilibrium outcome can be replicated as the equilibrium outcome of some game $\hat{\Gamma}(\boldsymbol{\beta})$.

The reverse statement also holds for the equilibria of the game $\hat{\Gamma}(\boldsymbol{\beta})$ that are Pareto efficient and such that, in stage 0, each downstream agent (i) is indifferent between accepting the initial offer or waiting for the stochastic game Γ, and (ii) accepts the offer. Let $\hat{\mathbf{w}}^{\mathbf{e}}(\boldsymbol{\beta})$ and $\hat{\mathbf{p}}^{\mathbf{e}}(\boldsymbol{\beta})$ denote the equilibrium wholesale and retail prices, $\hat{\Pi}^e_{M_i}(\boldsymbol{\beta})$ and $\hat{\Pi}^e_{R_j}(\boldsymbol{\beta})$ respectively denote M_i's and R_j's profits, and $\hat{\underline{\Pi}}^j_{M_i}(\boldsymbol{\beta}) \leq \hat{\Pi}^e_{M_i}(\boldsymbol{\beta})$ and $\hat{\underline{\Pi}}^i_{R_j}(\boldsymbol{\beta}) \leq \hat{\Pi}^e_{R_j}(\boldsymbol{\beta})$ respectively denote their disagreements payoffs – i.e., the profits that they would obtain if the other equilibrium tariffs remained unchanged but they decided not to deal with each other (at all). These profits also constitute the profits that M_i^j and R_j^i can respectively secure even if they are not selected to make the final offer at stage 1. Finally, denote by $\bar{\Pi}^i_{R_j}(\boldsymbol{\beta})$ the maximal profit that R_j can achieve if it has to guarantee to M_i a profit at least equal to $\hat{\Pi}^e_{M_i}$. Remember that, in equilibrium, R_j is indifferent between accepting and rejecting the initial offer made by M_i^j at stage 0, implying that its equilibrium profit $\hat{\Pi}^e_{R_j}(\boldsymbol{\beta})$ satisfies:

$$\hat{\Pi}^e_{R_j}(\boldsymbol{\beta}) = \beta_{ij}\hat{\underline{\Pi}}^i_{R_j}(\boldsymbol{\beta}) + \left(1 - \beta_{ij}\right)\bar{\Pi}^i_{R_j}(\boldsymbol{\beta}).$$

Consider now a bargaining equilibrium, and suppose that the equilibrium wholesale prices are $w_{hk} = \hat{w}^e_{hk}(\boldsymbol{\beta})$ for every $hk \neq ij$, and focus on the bargaining between M_i and R_j for different values of the bargaining parameter α_{ij}. For $\alpha_{ij} = 0$, the outcome is the same as if R_j^i were to make a take-it-or-leave-it offer, and R_j's profit is therefore equal to $\bar{\Pi}^i_{R_j}(\boldsymbol{\beta})$. When $\alpha_{ij} = 1$, the bargaining solution is the same as if M_i^j were to make a take-it-or-leave-it offer and R_j's profit is therefore equal to $\hat{\underline{\Pi}}^i_{R_j}(\boldsymbol{\beta})$. As the bargaining weight α_{ij} continuously increases from 0 to 1, R_j's profit decreases from $\bar{\Pi}^i_{R_j}(\boldsymbol{\beta})$ to $\hat{\underline{\Pi}}^i_{R_j}(\boldsymbol{\beta})$; hence, there exists a value for α_{ij} for which the bargaining solution yields a profit equal to $\hat{\Pi}^i_{R_j}(\boldsymbol{\beta})$ to R_j. As the outcome moreover lies on the Pareto frontier, it must be the case that M_j obtains exactly $\hat{\Pi}^e_{M_i}(\boldsymbol{\beta})$, and that $w_{ij} = \hat{w}^e_{ij}(\boldsymbol{\beta})$. Therefore, for any Pareto efficient equilibrium of the game $\hat{\Gamma}(\boldsymbol{\beta})$ such each retailer is indifferent between accepting the initial offer or waiting for the stochastic game Γ but accepts at the initial stage, there exists a vector of bargaining weights $\boldsymbol{\alpha}$ for which the bargaining equilibrium would yield exactly the same outcome.

D Vertical restraints and agency model

D.1 Resale price maintenance

We suppose here that manufacturers and retailers can adopt RPM provisions; that is, each $M_i - R_j$ pair can contract not only on a (non-linear) tariff $t_{ij}(q_{ij})$, but also – if the two firms wish to do so – on the retail price p_{ij}. The timing of wholesale negotiations and retail pricing decisions remains as before, with the caveat that in case of an RPM agreement between M_i and R_j, the retailer simply sets the agreed retail price p_{ij} in stage 2.

We first note that allowing firms to adopt RPM provisions does not destabilize the cost-based tariff equilibria identified above. Specifically, when the other channels sign cost-based tariffs, a cost-based tariff precisely induces the retail price that maximizes the joint profit of the manufacturer and the retailer,[1] and therefore they do not need to contract on the retail price. Retailers, however, get a lower share of the industry profit when RPM is used in equilibrium. This is because they can no longer adjust the prices they charge for the rival brands if their negotiations with a manufacturer were to fail. For instance, when dealing with M_i, R_j's disagreement payoff – and therefore, the equilibrium payoff – is lower when R_j has a RPM contract with other manufacturers.

RPM can, however, be used to sustain many other outcomes. To see this, suppose that bilateral profits are well-behaved when firms rely on two-part tariffs of the form $t_{ij}(q_{ij}) = F_{ij} + w_{ij}q_{ij}$. Namely:

Assumption B. For any $i \in \mathcal{I}$ and $j \in \mathcal{J}$, any wholesale prices $(w_{hk})_{(h,k)\neq(i,j)\in\mathcal{I}\times\mathcal{J}}$ and any prices $(p_{hk})_{(h,k)\neq(i,j)\in\mathcal{I}\times\mathcal{J}}$, the gross joint profit of M_i and R_j, given by:

$$\left(p_{ij} - c_i - \gamma_j\right) D_{ij}(\mathbf{p}) + \sum_{k\in\mathcal{J}\setminus\{j\}} \left(w_{ik} - c_i\right) D_{ik}(\mathbf{p}) + \sum_{h\in\mathcal{I}\setminus\{i\}} \left(p_{hj} - w_{hj} - \gamma_j\right) D_{hj}(\mathbf{p}),$$

is strictly quasi-concave[2] in p_{ij} and maximal for a finite price level.

Let $\mathbf{\Lambda}(\mathbf{p})$ denote the $nm \times nm$ matrix such that the term in row $l(i,j) \equiv (i-1)m + j$ and column $l(h,k)$, for $i, h \in \mathcal{I}$ and $j, k \in \mathcal{J}$, is given by:

$$\Lambda_{l(i,j),l(h,k)}(\mathbf{p}) = \begin{cases} \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) & \text{if } h \neq i \text{ and } k = j, \\ -\frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) & \text{if } h = i \text{ and } k \neq j, \\ 0 & \text{otherwise.} \end{cases}$$

We have:

Proposition D.1 (RPM) *When RPM is allowed:*

(i) there exists an equilibrium based on cost-based two-part tariffs and RPM, which replicates the multiproduct oligopoly prices and quantities, but gives retailers a lower share of profit than in the absence of RPM; and,

(ii) any price vector $\mathbf{p}$ *such that* $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ *can be sustained in equilibrium.*

Proof. *Part (i).* Assuming that all other channels, $h - k$ (i.e., for every $(h,k) \neq (i,j)$) sign cost-based two-part tariffs $\bar{t}^*_{hk}(q) = \bar{F}^*_{hk} + c_h q_{hk}$ and agree, through RPM, to set retail

[1] For instance, in the equilibrium based on two-part tariffs characterized in Proposition 1, the equilibrium contract $t^*_{ij}(q_{ij}) = F^*_{ij} + c_i q_{ij}$ induces R_j to maximize the joint profit of the pair $M_i - R_j$.

[2] If the demand for the channel $i - j$ drops to zero when the price p_{ij} is high enough, then the strict quasi-concavity should hold in the price range where $D_{ij}(\cdot) > 0$. A similar comment applies to Assumptions C and D.

prices $p_{hk} = p_{hk}^*$, the joint profit of M_i and R_j is given by:

$$\begin{aligned}\Pi_{M_i - R_j} &= \left(p_{ij} - c_i - \gamma_j\right) D_{ij}\left(\left(p_{ij}, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) + \sum_{k \in \mathcal{J} \setminus \{j\}} \bar{F}_{ik}^* \\ &\quad + \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\left(p_{hj}^* - c_h - \gamma_j\right) D_{hj}\left(\left(p_{ij}, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) - \bar{F}_{hj}^*\right].\end{aligned}$$

As the variable part of this profit coincides with $\pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right)$, Assumption A ensures that it is maximized for $p_{ij} = p_{ij}^*$. Therefore, M_i and R_j can maximize their joint profit by agreeing to charge p_{ij}^*. Furthermore, as this joint profit does not depend on their own tariff (in particular, the tariff t_{ij} no longer affects R_j's prices, which are here set through RPM), they can also sign a cost-based two-part tariff.

As firms negotiate cost-based two-part tariffs, $\Delta_{M_i} = \bar{F}_{ij}^*$ for every $i \in \mathcal{I}$; hence, from Nash bargaining, for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$ the fixed fee $\bar{F}_{ij}^*$ is given by:

$$\bar{F}_{ij}^* = \Delta_{M_i} = \alpha_{ij}\left(\Delta_{M_i} + \Delta_{R_j}\right) = \alpha_{ij}\left(\pi_j^* - \bar{\pi}_j^{ij}\right), \text{ where } \bar{\pi}_j^{ij} = \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right).$$

As retailers cannot adjust their prices in case of disagreement, the additional profit generated by a successful negotiation is (weakly) larger than in the absence of RPM:

$$\begin{aligned}\pi_j^* - \bar{\pi}_j^{ij} &= \pi_j^* - \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) \\ &\geq \pi_j^* - \max_{\mathbf{p}_{-i,j}} \pi_j\left(\left(\infty, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}^*\right) = \pi_j^* - \pi_j^{ij} > 0,\end{aligned}$$

where the strict inequality stems from Lemma 1 (see Appendix A).

There thus exists an equilibrium where firms negotiate cost-based two-part tariffs and RPM is used (and retail prices are equal to $\mathbf{p}^*$). M_i's and R_j's equilibrium profits are then given by:

$$\bar{\Pi}_{M_i}^* = \sum_{j \in \mathcal{J}} \alpha_{ij}\left(\pi_j^* - \bar{\pi}_j^{ij}\right) \text{ and } \bar{\Pi}_{R_j}^* = \pi_j^* - \sum_{i \in \mathcal{I}} \alpha_{ij}\left(\pi_j^* - \bar{\pi}_j^{ij}\right).$$

It follows that, as long as $\alpha_{ij} > 0$, manufacturers obtain a positive profit, which is moreover (weakly) greater than what they would obtain in the absence of RPM (namely, $\Pi_{M_i}^* = \sum_{j \in \mathcal{J}} \alpha_{ij}\left(\pi_j^* - \pi_j^{ij}\right)$). However, they still obtain less than a share α_{ij} of the equilibrium channel profit:

$$\pi_j^* - \bar{\pi}_j^{ij} < \pi_{ij}^*.$$

To see this, it suffices to note that, from Assumption A(ii):

$$\bar{\pi}_j^{ij} = \sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}\left(\left(\infty, \mathbf{p}_{-i,j}^*\right), \mathbf{p}_{-j}^*\right) > \sum_{h \in \mathcal{I} \setminus \{i\}} \pi_{hj}\left(\mathbf{p}^*\right) = \pi_j^* - \pi_{ij}^*.$$

It follows that retailers still get more than a share $1 - \alpha_{ij}$ of the profits they generate, and thus obtain a positive profit.

Part (ii). Fix a price vector $\mathbf{p}$ satisfying $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ and consider a candidate equilibrium in which each pair $M_i - R_j$ agrees on setting the retail price to p_{ij}, and on a two-part tariff based on some wholesale price w_{ij}. Note that the condition $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ implies that all quantities are positive. Indeed, if we had $D_{ij}(\mathbf{p}) = 0$ for some $(i,j) \in \mathcal{I} \times \mathcal{J}$, then an increase in p_{ij} could not affect the demand for any other channel (that is, we would have $\partial D_{hj}/\partial p_{ij}(\mathbf{p}) = \partial D_{ik}/\partial p_{ij}(\mathbf{p}) = 0$ for any $h \neq i$ and any $k \neq j$); hence, the row $l(i,j) \equiv (i-1)m + j$ would only have zeros, implying $|\mathbf{\Lambda}(\mathbf{p})| = 0$.

Given the agreements signed by the other channels, M_i and R_j are willing to reach an agreement, as they can replicate the no-agreement outcome by agreeing on a prohibitively high price for their channel (together with a tariff satisfying $t_{ij}(0) = 0$). Furthermore, if M_i and R_j were to deviate to some $\check{t}_{ij}$ and to a different retail price $\check{p}_{ij} \neq p_{ij}$, their joint profit (gross of fixed fees) would be given by:

$$\begin{aligned}
\Pi_{M_i - R_j}(\check{p}_{ij}) &= \left(\check{p}_{ij} - c_i - \gamma_j\right) D_{ij}\left(\left(\check{p}_{ij}, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}\right) \\
&\quad + \sum_{k \in \mathcal{J} \setminus \{j\}} \left(w_{ik} - c_i\right) D_{ik}\left(\left(\check{p}_{ij}, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}\right) \\
&\quad + \sum_{h \in \mathcal{I} \setminus \{i\}} \left(p_{hj} - w_{hj} - \gamma_j\right) D_{hj}\left(\left(\check{p}_{ij}, \mathbf{p}_{-i,j}\right), \mathbf{p}_{-j}\right),
\end{aligned}$$

which depends only on the deviating retail price $\check{p}_{ij}$, and not on the deviating wholesale tariff $\check{t}_{ij}(q_{ij})$. Hence, M_i and R_j have no incentive to deviate from the specified wholesale tariff. Furthermore, under Assumption B, this joint profit has a unique maximum, characterized by the first-order condition. Hence, M_i and R_j have no incentive to deviate from the specified retail price, p_{ij}, whenever $\Pi'_{M_i - R_j}(p_{ij}) = 0$, that is:

$$\begin{aligned}
&D_{ij}(\mathbf{p}) + \left(p_{ij} - c_i - \gamma_j\right) \frac{\partial D_{ij}}{\partial p_{ij}}(\mathbf{p}) \\
&+ \sum_{k \in \mathcal{J} \setminus \{j\}} \left(w_{ik} - c_i\right) \frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) + \sum_{h \in \mathcal{I} \setminus \{i\}} \left(p_{hj} - w_{hj} - \gamma_j\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) = 0,
\end{aligned}$$

which can be rewritten as:

$$\sum_{h \in \mathcal{I} \setminus \{i\}} \left(w_{hj} - c_h\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) - \sum_{k \in \mathcal{J} \setminus \{j\}} \left(w_{ik} - c_i\right) \frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) = \mu_{ij}(\mathbf{p}),$$

where:

$$\mu_{ij}(\mathbf{p}) \equiv D_{ij}(\mathbf{p}) + \sum_{h \in \mathcal{I}} \left(p_{hj} - c_h - \gamma_j\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}).$$

It follows that, if $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$, there exists a unique vector of wholesale prices, $\mathbf{w}(\mathbf{p})$ satisfying the above equations for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$.

Finally, the equilibrium fixed fees $F_{ij}(\mathbf{p})$ (for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$) can be uniquely identified using $F_{ij}(\mathbf{p}) = \Delta_{M_i}(\mathbf{p})$ and the Nash bargaining rule:

$$\begin{aligned} F_{ij}(\mathbf{p}) &= \alpha_{ij}\left[\Delta_{R_j}(\mathbf{p}) + \Delta_{M_i}(\mathbf{p})\right] \\ &= \alpha_{ij}\left\{ \begin{array}{c} \sum_{h\in\mathcal{I}}\left[p_{hj} - c_i - \gamma_j\right] D_{hj}(\mathbf{p}) \\ + \sum_{k\in\mathcal{J}\setminus\{j\}}\left[w_{ik}(\mathbf{p}) - c_i\right]\left[D_{ik}(\mathbf{p}) - D_{hk}\left((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}\right)\right] \\ + \sum_{h\in\mathcal{I}}\left[p_{hj} - w_{hj}(\mathbf{p}) - \gamma_j\right]\left[D_{hj}(\mathbf{p}) - D_{hk}\left((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}\right)\right] \end{array} \right\}. \end{aligned}$$

Conversely, starting from an equilibrium in which each channel $i - j$ agrees on a wholesale unit price equal to $w_{ij}(\mathbf{p})$ (associated with the corresponding fixed fee $F_{ij}(\mathbf{p})$) and a retail price equal to p_{ij}, no manufacturer-retailer pair has an incentive to deviate to another wholesale and/or retail price. ■

Proposition D.1 shows that with RPM, many prices can arise in equilibrium. In particular, whenever $\left|\mathbf{\Lambda}\left(\mathbf{p}^M\right)\right| \neq 0$, RPM enables the firms to sustain monopoly prices. The proof is constructive, and consists of exhibiting two-part tariffs which, together with RPM, sustain the targeted prices.

The intuition is simple. By construction, the joint profit of M_i and R_j does not depend on the "internal" wholesale price w_{ij}. As it is no longer needed to "drive" the retail price p_{ij} (which can now be directly agreed upon through RPM), this internal wholesale price w_{ij} can thus be fixed in any arbitrary way, adjusting the fixed fee F_{ij} so as to share the profit as desired. However, this internal price affects M_i's negotiation with every other retailer R_k, as well as R_j's negotiation with every other manufacturer M_h, and can thus be set so as to sustain the targeted retail prices. As there are nm "instruments"(the wholesale prices) for nm "targets"(the retail prices), it follows that, generically, an equilibrium can be constructed around any profile of retail prices.

More precisely, in the absence of RPM and with cost-based tariffs, R_j takes into consideration the full margin on its sales; it thus chooses p_{ij} so as to maximize:

$$\pi_j(\mathbf{p}) = \sum_{h\in\mathcal{I}}\left(p_{hj} - c_h - \gamma_j\right) D_{hj}(\mathbf{p}).$$

Let:

$$\mu_{ij}(\mathbf{p}) \equiv \frac{\partial \pi_j(\mathbf{p})}{\partial p_{ij}} = D_{ij}(\mathbf{p}) + \sum_{h\in\mathcal{I}}\left(p_{hj} - c_h - \gamma_j\right)\frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) \tag{13}$$

denote the impact of a marginal increase in p_{ij} on the above profit. In the absence of RPM, Assumption A implies that equilibrium retail prices $\mathbf{p}^*$ are such that $\mu_{ij}(\mathbf{p}^*) = 0$ for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$.

With RPM, p_{ij} is instead chosen by M_i and R_j, who, for given vectors of wholesale prices $\mathbf{w}_{i,-j} = (w_{ik})_{k\in\mathcal{J}\setminus\{j\}}$ and $\mathbf{w}_{-i,j} = (w_{hj})_{h\in\mathcal{I}\setminus\{i\}}$, now ignore the upstream margin on R_j's sales of any rival brand h, $w_{hj} - c_h$, but do account for the upstream margin on M_i's sales through any rival store k, $w_{ik} - c_i$. Hence, under Assumption B, and for any given retail price profile $\mathbf{p}$, to ensure that M_i and R_j stick to p_{ij} it suffices to pick $\mathbf{w}_{i,-j}$ and $\mathbf{w}_{-i,j}$ so as to satisfy their first-order condition. This amounts to satisfy:

$$\sum_{h \in \mathcal{I} \setminus \{i\}} (w_{hj} - c_h) \frac{\partial D_{hj}}{\partial p_{ij}} (\mathbf{p}) - \sum_{k \in \mathcal{J} \setminus \{j\}} (w_{ik} - c_i) \frac{\partial D_{ik}}{\partial p_{ij}} (\mathbf{p}) = \mu_{ij} (\mathbf{p}). \tag{14}$$

That is, the differential impact of a marginal increase of p_{ij} on the upstream margins of the channels $M_i - R_k$, for $k \in \mathcal{J} \setminus \{j\}$, and $M_h - R_j$, for $h \in \mathcal{I} \setminus \{i\}$, should off-set $\mu_{ij}(\mathbf{p})$. The condition $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ ensures the existence of a wholesale price vector $\mathbf{w}$ satisfying the above equations for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$, in which case these wholesale prices are moreover uniquely defined. Fixed fees are then uniquely determined through the bargaining sharing rule.

So far we have considered "full RPM," where a retailer is required to charge the exact price negotiated with the manufacturer. The analysis can also shed some light on the role of minimum RPM (i.e., price floors) and maximum RPM (i.e., price caps). For the sake of exposition, we focus here on symmetric manufacturers and retailers,[3] and on symmetric equilibria, where $w_{ij} = w$ and $p_{ij} = p$ for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$. The condition $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$ then amounts to $\lambda_M(p) \neq \lambda_R(p)$, where:

$$\lambda_M(p) \equiv \sum_{h \in \mathcal{I} \setminus \{i\}} \frac{\partial D_{hj}}{\partial p_{ij}} (p, ..., p) \text{ and } \lambda_R(p) \equiv \sum_{k \in \mathcal{J} \setminus \{j\}} \frac{\partial D_{ik}}{\partial p_{ij}} (p, ..., p)$$

denote the impact of a change in the price of brand i in store j on, respectively, the sales of the others brands at store j (interbrand price sensitivity of demand) and on the sales of brand i in the other stores (intrabrand price sensitivity of demand).[4] Thus, whenever $\lambda_M(p) \neq \lambda_R(p)$, there exists an equilibrium based on two-part tariffs and RPM, in which all retail prices are equal to p and all wholesale prices are equal to $w = \bar{w}(p)$, where (using (14)):

$$\bar{w}(p) \equiv c + \frac{\mu(p)}{\lambda_M(p) - \lambda_R(p)}, \tag{15}$$

where $\mu(p) = \mu_{ij}(p, ..., p)$ denotes, as before, the marginal impact given by (13), of an increase in one retailer's price on the profit generated by that retailer when it faces cost-based tariffs.

To ensure that price caps or price floors induce the expected outcomes, we introduce the following regularity conditions:

Assumption C. For any $p > p^*$ such that $\lambda_M(p) \neq \lambda_R(p)$:

[3] Symmetry among manufacturers means $c_i = c$ and $D_{ij}(\mathbf{p}) = D_{hj}(\sigma_{ih}^M(\mathbf{p}))$ for any $j \in J$ and any $i, h \in I$, where $\sigma_{ih}^M(\mathbf{p})$ is derived from $\mathbf{p}$ by swapping the prices of brands i and h in each retailer's stores. Likewise, symmetry among retailers means $\gamma_j = \gamma$ and $D_{ij}(\mathbf{p}) = D_{ik}\left(\sigma_{jk}^R(\mathbf{p})\right)$ for any $i \in I$ and any $j, k \in J$, where $\sigma_{jk}^R(\mathbf{p})$ is derived from $\mathbf{p}$ by swapping R_j's and R_k's prices for each brand.

[4] The symmetry assumptions ensure that these parameters are also symmetric.

(i) for any $j \in \mathcal{J}$, R_j's gross profit $\sum_{i\in\mathcal{I}} (p_{ij} - \bar{w}(p) - \gamma) D_{ij} (\mathbf{p}_j, p, ..., p)$ is strictly quasi-concave in $\mathbf{p}_j = (p_{ij})_{i\in\mathcal{I}}$; and,

(ii) the function $\mu(p)$ satisfies $\mu'(p) < 0$.

Finally, to rule out large deviations in the bilateral negotiations, we introduce another technical assumption. For any $p > p^*$ and $w \neq c$, for any $i \in \mathcal{I}$ and any $j \in \mathcal{J}$, let denote by $\hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p) = \left(\hat{p}_{hj}^{ij}(p_{ij}; w, p)\right)_{h\in\mathcal{I}\setminus\{i\}}$ the prices that R_j would like to charge on the other brands, conditional on charging p_{ij} for brand i and on facing price caps (if $w > c$) or price floors (if $w < c$) set to p on the other brands; that is:

- if $w > c$,

$$\hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p) \equiv \begin{array}{l} \underset{\mathbf{p}_{-i,j}}{\arg\max} \displaystyle\sum_{h\in\mathcal{I}} (p_{hj} - w - \gamma) D_{hj} ((p_{ij}, \mathbf{p}_{-i,j}), p, ..., p) \\ \qquad\qquad \text{s.t. } p_{hj} \leq p \text{ for any } h \in \mathcal{I} \setminus \{i\}; \end{array}$$

- if $w < c$,

$$\hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p) \equiv \begin{array}{l} \underset{\mathbf{p}_{-i,j}}{\arg\max} \displaystyle\sum_{h\in\mathcal{I}} (p_{hj} - w - \gamma) D_{hj} ((p_{ij}, \mathbf{p}_{-i,j}), p, ..., p) \quad . \\ \qquad\qquad \text{s.t. } p_{hj} \geq p \text{ for any } h \in \mathcal{I} \setminus \{i\}. \end{array}$$

We can now state our last assumption, namely, that the joint profit of M_i and R_j remains well-behaved when R_j faces price caps or price floors for the other brands (whether or not these constraints are binding). Specifically:

Assumption D. For any $i \in \mathcal{I}$ and $j \in \mathcal{J}$, any wholesale price w and any retail price p, the gross joint profit of M_i and R_j, given by:

$$\begin{aligned} &(p_{ij} - c - \gamma) D_{ij} \left((p_{ij}, \hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p)), p, ..., p\right) \\ &+ \sum_{k\in\mathcal{J}\setminus\{j\}} (w - c) D_{ik} \left((p_{ij}, \hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p)), p, ..., p\right) \\ &+ \sum_{h\in\mathcal{I}\setminus\{i\}} \left(\hat{p}_{hj}^{ij}(p_{ij}; w, p) - w - \gamma\right) D_{hj} \left((p_{ij}, \hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p)), p, ..., p\right), \end{aligned}$$

is strictly quasi-concave in p_{ij} and maximal for a finite price level.

We then have:

Proposition D.2 (min vs. max RPM) *Restricting attention to symmetric equilibria, any price $p > p^*$ can be sustained with minimum RPM (resp., maximum RPM) when there is more (resp., less) substitution among manufacturers' brands than among retailers' stores, that is, when $\lambda_M(p) > \lambda_R(p)$ (resp., $\lambda_M(p) < \lambda_R(p)$).*

Proof. The proof consists in showing that the equilibria characterized in the proof of Proposition D.1 for the case of fixed RPM can also be sustained with price caps or price floors. We first study in which direction the retailers would like to adjust their prices, were they free to do so, starting from the two-part tariff cum RPM equilibrium identified by Proposition D.1, in which retail prices are set to p and all wholesale prices are set to $\bar{w}(p)$. This determines whether price floors or price caps are needed to sustain this equilibrium. Second, we show that following a small deviation in one of its prices, a retailer finds it optimal to stick to the equilibrium price p for the other brands. This validates the first order conditions characterized in the proof of Proposition D.1, and thus the relationship between p and $\bar{w}(p)$. The strict quasi-concavity of the bilateral joint profit then concludes the argument.

Consider a situation in which all retail prices are set to p and all wholesale prices are set to $\bar{w}(p)$, characterized by (15).[5] Starting from this situation, by adjusting the price p_{ij}, R_j could obtain a profit, gross of fixed fees, equal to:

$$[p_{ij} - \bar{w}(p) - \gamma] D_{ij}(p_{ij}, p, ..., p) + \sum_{h \in \mathcal{I} \setminus \{i\}} [p - \bar{w}(p) - \gamma] D_{hj}(p_{ij}, p, ..., p).$$

Thus, the impact of a marginal increase in one retailer's price on that retailer's profit is given by (using (15)):

$$\begin{aligned} D(p, ..., p) - [p - \bar{w}(p) - \gamma][\lambda(p) - \lambda_M(p)] &= \mu(p) + [\bar{w}(p) - c][\lambda(p) - \lambda_M(p)] \\ &= [\bar{w}(p) - c][\lambda(p) - \lambda_R(p)], \end{aligned}$$

where:

$$\lambda(p) \equiv -\frac{\partial D_{ij}}{\partial p_{ij}}(p, ..., p) > 0$$

denotes the own-price sensitivity of demand. As retailers are differentiated, and thus imperfect substitutes, $\lambda_R(p) < \lambda(p)$ (that is, when the price of a particular brand increases in one store, and thus consumers buy less of that brand in that store, consumers only partially report the lost demand for the brand to different stores). Furthermore, under Assumption C(i) R_j's profit is strictly quasi-concave in its prices ($\mathbf{p}_j$); hence, retailers have an incentive to lower their prices if $\bar{w}(p) < c$, and to raise them if $\bar{w}(p) > c$. In other words, price floors are needed to sustain p if $\bar{w}(p) < c$, and price caps are instead needed to sustain p if $\bar{w}(p) > c$.

The price constraints (price caps or price floors) are by construction binding whenever $\bar{w}(p) \neq c$. By continuity, this remains true when, say, M_i and R_j adopt a price p_{ij} that slightly departs from the symmetric price p. Hence, in the event of such a (marginal) deviation, the constraints imposed by the other manufacturers continue to be binding, and R_j thus continues to charge prices equal to p for the other brands. It follows that,

[5] The equilibrium fixed fee $F(p)$ is also uniquely defined and determined by the surplus-sharing rule.

as in the case of fixed RPM, (15) ensures that such a marginal deviation is not profitable for M_i and R_j. That is, (15) still constitutes the relevant first-order condition when fixed RPM is replaced with minimum RPM (when $\bar{w}(p) < c$) or maximum RPM (when $\bar{w}(p) > c$). The strict quasi-concavity assumption E ensures that global deviations in p_{ij} are not profitable either. ∎

To understand the underlying intuition, consider first the retail pricing decisions. If retailers were free to set their prices, they would do so taking into consideration their downstream margins but ignoring their partners' upstream margins. Hence, if upstream margins are positive, classic double marginalization problems arise: the price of any brand at any store would be higher than what would maximize the joint profit of the manufacturer and the retailer, and price caps are therefore needed. Conversely, if upstream margins are negative, retailers would be tempted to adopt too low prices, and price floors are thus needed.

The next step is to determine whether positive or negative upstream margins are needed to sustain supra-competitive retail prices. If tariffs were cost-based, each negotiating pair would aim at maximizing the profit generated by the retailer's sales (on all brands); but then, each pair would have an incentive to undercut the others' prices.[6]

When relying instead on a wholesale price $w \neq c$, each pair moreover takes into account the impact of their joint decision on the manufacturer's margins earned on the sales of its brand at the other stores, which, in a symmetric situation, is given by

$$(w - c) \sum_{k \in \mathcal{J} \setminus \{j\}} \frac{\partial D_{ik}}{\partial p_{ij}} (p, ..., p) = (w - c) \lambda_R (p),$$

but however ignores the impact of their decision on the upstream margins earned on the retailer's sales of the other brands, which is given by

$$(w - c) \sum_{h \in \mathcal{I} \setminus \{i\}} \frac{\partial D_{hj}}{\partial p_{ij}} (p, ..., p) = (w - c) \lambda_M (p).$$

Therefore, in order to sustain the equilibrium price (i.e., discourage undercutting it), the net balance of these two effects should be positive, which amounts to

$$(w - c) [\lambda_R (p) - \lambda_M (p)].$$

It follows that in order to raise prices above p^*, negative upstream margins are required when $\lambda_M (p) > \lambda_R (p)$, in which case price floors are needed to counter retailers' excessive incentives to lower prices; when instead $\lambda_M (p) < \lambda_R (p)$, positive upstream margins are required, and price caps are then needed to counter retailers' excessive incentives to raise prices.[7]

[6] To see this formally, consider a situation where all retail prices are equal to $p > p^*$. By construction, $\mu(p^*) = 0$, and thus, from Assumption C(ii), $\mu(p) < 0$ for $p > p^*$.

[7] Price floors thus have no effect in this case; by contrast, Allain and Chambolle (2011) find that

Remark: Price caps and price floors. Moving from full RPM to price floors or price caps may also affect the division of profit, as R_j's disagreement payoffs may be affected. If the negotiation between M_i and R_j were to fail, R_j would be tempted to react by optimally revising the retail prices $\mathbf{p}_{-i,j}$ it charges the other brands. Such adjustment is impossible under full RPM, but may become feasible under a price floor or price ceiling. When such a change is indeed feasible, R_j's disagreement payoffs – and thus the equilibrium division of profit – are affected.

D.2 Price parity agreements

We now turn to the role of price parity agreements (PPAs). A PPA is a contractual provision requiring the retailer to price the manufacturer's brand at the same level as competing brands. Variants of such PPAs may be slightly less restrictive and simply prevent the retailer from charging less for competing brands, or more for competing brands.

These provisions have recently triggered debates about their potential anti-competitive effects. In April 2010, the UK Office of Fair Trading (OFT) imposed £225 million fines against tobacco manufacturers and retailers over retail pricing strategies. The OFT considered that manufacturers and retailers had entered into bilateral agreements linking the retail price of a tobacco brand to the prices of competing brands (at the same stores). Those retail price parity agreements were deemed to be anti-competitive by the OFT, who judged that they had the same adverse effects as RPM.[8]

We now show that in our framework, a PPA is not a substitute for RPM. To see this, we adapt the previous two-stage game of wholesale negotiations and retail pricing decisions as follows:

- in the first stage, each $M_i - R_j$ pair can also adopt a PPA (in addition to agreeing on a tariff $t_{ij}(q_{ij})$); and,
- in the second stage, a retailer that has accepted a PPA must set the same retail price for all the brands it carries.

Obviously, imposing uniform prices across brands can affect retailers' pricing behavior when they would otherwise wish to charge asymmetric prices. In particular, the "conditional price responses" introduced in Section 3.2 are now $\bar{\mathbf{p}}_j^{ij}(q_{ij}) = (\bar{p}_j^{ij}(q_{ij}), ..., \bar{p}_j^{ij}(q_{ij}))$ satisfying:

$$D_{ij}\left(\bar{\mathbf{p}}_j^{ij}(q_{ij}), \mathbf{p}_{-j}^{\mathbf{e}}\right) = q_{ij},$$

where $\mathbf{p}^{\mathbf{e}} = \left(p_{ij}^{e}\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ is the vector of equilibrium prices.

industry-wide price floors are always anticompetitive.

[8] See Decision CA98/01/2010 of the Office of Fair Trading, Case CE/2596-03: Tobacco, 15 April 2010. This decision was later quashed by the Competition Appeals Tribunal (see the CAT Judgement [2011] CAT 41, 12 December 2011), who however did not discuss the possible anticompetitive effects of PPAs.

Assumption E. For every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$, whenever it is positive, the demand function $D_{ij}(\mathbf{p})$ satisfies:

(i) $\sum_{h\in\mathcal{I}} \partial D_{ij}(\mathbf{p}) / \partial p_{hj} < 0$;

(ii) $\sum_{h\in\mathcal{I}} \partial D_{ij}(\mathbf{p}) / \partial p_{hk} > 0$ for any $k \in \mathcal{J} \setminus \{j\}$; and,

(iii) In addition, $\sum_{h\in\mathcal{I}} \sum_{k\in\mathcal{J}} \partial D_{ik}(\mathbf{p}) / \partial p_{hj} < 0$.

Assumption E is rather innocuous and simply relies on products being differentiated. Part (i) requires that R_j's sales of M_i's brand decrease when R_j uniformly increases the price of all brands, whereas part (ii) assumes that the same sales increase when a rival retailer uniformly increases its prices. Finally, part (iii) ensures that when R_j uniformly increases all of its prices, the total sales of M_i's brand through all retailers decreases (i.e., the direct effects on the sales through R_j dominates).

The following proposition shows that firms cannot strategically use PPAs to depart from cost-based tariffs, and thus cannot affect the equilibrium outcome beyond imposing symmetry:

Proposition D.3 (price parity agreements) *In the class of equilibria based on differentiable tariffs and price parity agreements where all equilibrium quantities are positive:*

(i) equilibrium tariffs are all cost-based, that is, marginal wholesale prices reflect marginal costs of production; and,

(ii) if firms are symmetric at both stages of the vertical chain,[9] *then all prices are the same as if in the absence of any price parity agreement.*

Proof. *Part (i).* Consider a candidate equilibrium where the equilibrium tariffs are t_{ij}^e for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$, and all equilibrium quantities are positive and the equilibrium retail prices are given by the price vector $\mathbf{p^e}$ such that, for every $j \in \mathcal{J}$, $p_{ij}^e = p_j^e$ for all $i \in \mathcal{I}$.

If such an equilibrium exists, the price p_j^e must maximize R_j's profit when it faces the tariffs $\mathbf{t}_j^\mathbf{e}$ and anticipates rival retail prices $\mathbf{p}_{-j}^\mathbf{e}$:

$$p_j^e \in \arg\max_{p_j} \left\{ \sum_{h\in\mathcal{I}} \left[\left(p_j - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^\mathbf{e}\right) - t_{hj}^e\left(D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^\mathbf{e}\right)\right) \right] \right\}.$$

Alternatively, one can write R_j's maximizing program as choosing a quantity q_{ij} for M_i's brand. Under the price parity requirement, choosing a quantity q_{ij} amounts to choosing the price $\bar{p}_j^{ij}(q_{ij})$ such that, for $\bar{\mathbf{p}}_j^{ij}(q_{ij}) = \left(\bar{p}_j^{ij}(q_{ij}), \ldots, \bar{p}_j^{ij}(q_{ij})\right)$:

$$D_{ij}\left(\bar{\mathbf{p}}_j^{ij}(q_{ij}), \mathbf{p}_{-j}^\mathbf{e}\right) = q_{ij}. \tag{16}$$

Assumption E ensures that such a price $\bar{p}_j^{ij}(q_{ij})$ exists and is continuously differentiable as long as $q_{ij} \leq q^{\max}\left(\mathbf{p}_{-j}^\mathbf{e}\right) \equiv D_{ij}\left((0,\ldots,0), \mathbf{p}_{-j}^\mathbf{e}\right)$. Therefore, when it faces the

[9] See Footnote 3 for a precise expression of this symmetry assumption.

tariffs $\mathbf{t}_j = \left(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j}\right)$ and anticipates that its rivals set their equilibrium prices, $\mathbf{p}^{\mathbf{e}}_{-j}$, R_j chooses the quantity q_{ij} that maximizes:

$$\overline{r}^{ij}_j\left(q_{ij}\right) - t_{ij}\left(q_{ij}\right) - \sum_{h \in \mathcal{I} \setminus \{i\}} t^e_{hj}\left(\overline{q}^{ij}_{hj}\left(q_{ij}\right)\right)$$

where $\overline{q}^{ij}_{hk}\left(q_{ij}\right)$ denotes R_k's sales of the M_h's brand, for $h \neq i \in \mathcal{I}$ and $k \in \mathcal{J}$, and is given by:

$$\overline{q}^{ij}_{hk}\left(q_{ij}\right) \equiv D_{hk}\left(\overline{\mathbf{p}}^{ij}_j\left(q_{ij}\right), \mathbf{p}^{\mathbf{e}}_{-j}\right) \tag{17}$$

and $\overline{r}^{ij}_j\left(q_{ij}\right)$ denotes R_j's retail revenues (net of the retailing costs)

$$\overline{r}^{ij}_j\left(q_{ij}\right) \equiv \left[\overline{p}^{ij}_j\left(q_{ij}\right) - \gamma_j\right] q_{ij} + \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\overline{p}^{ij}_j\left(q_{ij}\right) - \gamma_j\right] \overline{q}^{ij}_{hj}\left(q_{ij}\right).$$

To maximize their joint profit, subject to the PPA, M_i and R_j should adopt a tariff t_{ij} inducing the quantity q_{ij} that maximizes:

$$\overline{r}^{ij}_j\left(q_{ij}\right) - c_i q_{ij} - \sum_{h \in \mathcal{I} \setminus \{i\}} t^e_{hj}\left(\overline{q}^{ij}_{hj}\left(q_{ij}\right)\right) + \sum_{k \in \mathcal{J} \setminus \{j\}} \left[t^e_{ik}\left(\overline{q}^{ij}_{ik}\left(q_{ij}\right)\right) - c_i \overline{q}_{ik}\left(q_{ij}\right)\right].$$

Therefore, to induce the quantity $q^e_{ij} > 0$ that maximizes their joint profit, M_i and R_j need to agree on an equilibrium tariff t^e_{ij} that satisfies (using $\overline{q}^{ij}_{ik}\left(q^e_{ij}\right) = q^e_{ik}$):

$$\left(t^e_{ij}\right)'\left(q^e_{ij}\right) - c_i + \sum_{k \in \mathcal{J} \setminus \{j\}} \left[\left(t^e_{ik}\right)'\left(q^e_{ik}\right) - c_i\right] \left(\overline{q}^{ij}_{ik}\right)'\left(q^e_{ij}\right) = 0.$$

For any $i \in \mathcal{I}$, the equilibrium upstream margins $u^e_{ij} = \left(t^e_{ij}\right)'\left(q^e_{ij}\right) - c_i$, for $j \in \mathcal{J}$, thus satisfy:

$$\overline{\boldsymbol{\delta}}^i \cdot \begin{bmatrix} u^e_{i1} \\ \vdots \\ u^e_{im} \end{bmatrix} = 0, \tag{18}$$

where $\overline{\boldsymbol{\delta}}^i$ denotes the $m \times m$ matrix such that the term in row $j \in \mathcal{J}$ and column $k \in \mathcal{J}$ is given by:

$$\overline{\boldsymbol{\delta}}^i\left(j,k\right) = \begin{cases} 1 & \text{if } k = j, \\ -\overline{\delta}^{ij}_{ik} & \text{otherwise.} \end{cases}, \text{ where } \overline{\delta}^{ij}_{ik} = -\left(\overline{q}^{ij}_{ik}\right)'\left(q^e_{ij}\right)$$

Conversely, to induce R_j to sell a given quantity q_{ij}, it suffices to adopt a continuously differentiable tariff $t_{ij}\left(\cdot\right)$ that is sufficiently convex and q_{ij} is characterized by the first-order condition.

We now conclude the proof by showing that, under Assumption E, the matrix $\overline{\boldsymbol{\delta}}^i$ is invertible. Differentiating (17) (for $h = i$), yields:

$$\bar{\delta}_{ik}^{ij} = -\left[\sum_{h\in\mathcal{I}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)\right]\left(\bar{p}_j^{ij}\right)'\left(q_{ij}^{e}\right). \tag{19}$$

Differentiating (16), we get:

$$\left(\bar{p}_j^{ij}\right)'\left(q_{ij}\right) = \frac{1}{\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\bar{\mathbf{p}}_j^{ij}\left(q_{ij}\right), \mathbf{p}_{-j}^{\mathbf{e}}\right)} < 0. \tag{20}$$

Using (20), equation (19) can be rewritten as:

$$\bar{\delta}_{ik}^{ij} = -\frac{\sum_{h\in\mathcal{I}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)}{\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)} > 0,$$

where the inequality stems from Assumption E. Indeed, parts (i) and (ii) of that assumption respectively imply that $\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right) < 0$ and $\sum_{h\in\mathcal{I}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right) > 0$. It follows that the matrix $\bar{\boldsymbol{\delta}}^i$ is diagonally dominant, as for every $j \in \mathcal{J}$ we have:

$$\begin{aligned}
\left|\bar{\boldsymbol{\delta}}^i(j,j)\right| - \sum_{k\in\mathcal{J}\setminus\{j\}}\left|\bar{\boldsymbol{\delta}}^i(j,k)\right| &= 1 - \sum_{k\in\mathcal{J}\setminus\{j\}} \frac{\sum_{h\in\mathcal{I}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)}{-\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)} \\
&= \frac{-\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right) - \sum_{k\in\mathcal{J}\setminus\{j\}}\sum_{h\in\mathcal{I}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)}{-\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)} \\
&= \frac{-\sum_{h\in\mathcal{I}}\left[\frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right) + \sum_{k\in\mathcal{J}\setminus\{j\}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)\right]}{-\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)} \\
&= -\frac{\sum_{h\in\mathcal{I}}\left[\sum_{k\in\mathcal{J}} \frac{\partial D_{ik}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)\right]}{-\sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^{\mathbf{e}}\right)} \\
&> 0,
\end{aligned}$$

where the inequality stems from Assumption E (parts (i) and (iii)). It follows that the matrix $\bar{\boldsymbol{\delta}}^i$ is invertible, and thus (18) yields $\left(t_{ij}^e\right)'\left(q_j^e\right) = c_i$ for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$.

Part (ii). Given the equilibrium tariffs $\mathbf{t}^{\mathbf{e}}$, the equilibrium prices must be such that for any $j \in \mathcal{J}$, p_j^e maximizes R_j profit, that is:

$$p_j^e \in \arg\max_{p_j}\left\{\sum_{h\in\mathcal{I}}\left[\left(p_j - \gamma_j\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}\right) - t_{hj}^e\left(D_{hj}\left(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}\right)\right)\right]\right\}.$$

This maximization program also writes as:

$$p_j^e \in \arg\max_{p_j} \left\{ \pi_j\left(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}\right) + \sum_{i\in\mathcal{I}} \left[t_{ij}^e\left(D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}\right)\right) - c_i D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}\right)\right] \right\}.$$

Given that we focus here on interior symmetric equilibria, the equilibrium retail price p_j^e must satisfy the first-order condition:

$$\sum_{i\in\mathcal{I}} \left\{ \frac{\partial \pi_j}{\partial p_{ij}}\left(\mathbf{p}^{\mathbf{e}}\right) + \left[\left(t_{ij}^e\right)'\left(q_{ij}^e\right) - c_i\right] \frac{\partial D_{ij}}{\partial p_{ij}}\left(\mathbf{p}^{\mathbf{e}}\right) \right\} = 0 \iff \sum_{i\in\mathcal{I}} \frac{\partial \pi_j}{\partial p_{ij}}\left(\mathbf{p}^{\mathbf{e}}\right) = 0. \quad (21)$$

By definition the prices $\mathbf{p}^*$ satisfy this last condition, as $\partial\pi_j\left(\mathbf{p}^*\right)/\partial p_{ij} = 0$ for every $i \in \mathcal{I}$. Moreover, when firms are symmetric at both stages of the vertical chain, the equilibrium price vector $\mathbf{p}^*$ is symmetric, in the sense that for every $j \in \mathcal{J}$, $p_{ij}^* = p_j^*$. Therefore, $\mathbf{p}^*$ is a solution to the set of first-order conditions given by equation (21) for every $j \in \mathcal{J}$.

Finally, using symmetry, equation (21) simplifies to $\partial\pi_j\left(\mathbf{p}^{\mathbf{e}}\right)/\partial p_{ij} = 0$. Under Assumption A, this system of first-order conditions has a unique solution, which ensures that we must have $p^e = p^*$. ■

The adoption of PPAs thus does not affect the previous analysis. Pricing at marginal cost again makes a retailer the residual claimant for the profit it can generate together with a given manufacturer – even if this profit is limited due to the imposition of uniform prices – and thus induces the retailer to maximize this joint profit (possibly subject to the uniform price restriction). It follows that in equilibrium, all contracts are cost-based.

Remark: smooth tariffs. Proposition D.3 is more general than Proposition 2 as it applies to all equilibria based on differentiable tariffs, regardless of whether or not they would induce a smooth retail behavior in the absence of PPAs. The reason is that by imposing uniform prices across brands, PPAs *de facto* ensure that retail behavior will be smooth. In particular, the equivalent of the assumption $\left|\boldsymbol{\delta}^i\right| \neq 0$ (namely, $\left|\bar{\boldsymbol{\delta}}^i\right| \neq 0$, replacing the impact of q_{ij} on the "adjusted demands" $\hat{q}_{ik}^{ij}\left(q_{ij}\right)$, obtained with R_j's conditional price response, with the mechanical impact that a change in the quantity q_{ij} will have on the other quantities $\bar{q}_{ik}^{ij}\left(q_{ij}\right)$ of brand i sold by the other retailers R_k, given that R_j has to charge the same price $\bar{p}_j^{ij}\left(q_{ij}\right)$ for all brands) always holds when retailers are subject to PPAs.

Remark: price caps and price floors. The above analysis focuses on "pure" PPAs, which require retailers to charge the same price for all brands; any manufacturer can thus unilaterally impose this price uniformity. As mentioned above, in practice a variant consists of preventing retailers from charging prices that exceed those of rival brands. Obviously, the outcome is the same as with pure PPAs when all manufacturers adopt this variant, as retailers are then *de facto* constrained to charge the same price for all brands. While this paper does not formally study the case where a limited number of manufacturers adopt this variant, it should be clear that the proof of Proposition D.3 readily extends to this case. A similar comment applies when retailers are instead required to charge no less than for rival brands, or when a limited number of retailers are subject to a PPA or one of its variants.

D.3 Agency model

We have been focussing so far on the "resale" business model, where the distributor buys the goods and/or services from the suppliers, and then resells them to consumers (hence, absent RPM, it is the distributor who sets consumer prices). If such a model is standard for "brick-and-mortar" retailers, online retail platforms often adopt instead an "agency" business model in which the supplier remains the owner of its goods and/or services, and chooses the prices at which it offers them on the platforms; each distributor then obtains commissions on the sales made through its platform.

To study this agency business model within our framework, in this section we adapt the timing of negotiations and pricing decisions as follows:

1. Each $M_i - R_j$ pair negotiates a (possibly non-linear) commission schedule $\tilde{t}_{ij}(q_{ij})$, based on the volume of sales q_{ij} achieved by M_i through R_j's platform. As before, these bilateral negotiations are simultaneous and secret; and,

2. Each M_i sets the retail prices for its product on each platform that carries it; in this section we will refer to M_i's prices as $\tilde{\mathbf{p}}_i = (\tilde{p}_{ij})_{j\in\mathcal{J}}$.

The bargaining equilibria of this game are defined accordingly. In the second stage (retail pricing decisions), each manufacturer chooses its prices assuming that its rivals set the equilibrium retail prices, $\tilde{\mathbf{p}}^{\mathbf{e}}_{-i} = \left(\tilde{p}^e_{hj}\right)_{h\in\mathcal{I}\setminus\{i\},j\in\mathcal{J}}$. In the first stage, each $M_i - R_j$ pair negotiates a schedule $\tilde{t}_{ij}(q_{ij})$ that: (i) maximizes its joint profit, given the other equilibrium contracts and the resulting retail pricing behavior; and (ii) gives a share $\alpha_{ij} \in [0,1]$ of the additional profit generated by a successful negotiation to the manufacturer (and thus a share $1-\alpha_{ij}$ to the retailer).

Formally, a bargaining equilibrium is a vector of price responses $\left(\tilde{\mathbf{p}}^R_i\left(\tilde{\mathbf{t}}_i\right)\right)_{i\in\mathcal{I}}$, together with a vector of equilibrium commission schedules $\tilde{\mathbf{t}}^{\mathbf{e}} = \left(\tilde{t}^e_{ij}\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ and a vector of equilibrium prices $\tilde{\mathbf{p}}^{\mathbf{e}} = (\tilde{\mathbf{p}}^{\mathbf{e}}_i)_{i\in\mathcal{I}}$ such that:

- In the second stage:
  - for every $i\in\mathcal{I}$ and any vector of schedules $\tilde{\mathbf{t}}_i = \left(\tilde{t}_{ij}\right)_{j\in\mathcal{J}}$ negotiated by M_i in the first stage, M_i's pricing strategy is given by:

$$\tilde{\mathbf{p}}^R_i\left(\tilde{\mathbf{t}}_i\right) \in \arg\max_{\tilde{\mathbf{p}}_i}\left\{\sum_{j\in\mathcal{J}}\left[\left(\tilde{p}_{ij}-c_i\right)D_{ij}\left(\tilde{\mathbf{p}}_i,\tilde{\mathbf{p}}^{\mathbf{e}}_{-i}\right) - \tilde{t}_{ij}\left(D_{ij}\left(\tilde{\mathbf{p}}_i,\tilde{\mathbf{p}}^{\mathbf{e}}_{-i}\right)\right)\right]\right\};$$

– the equilibrium prices and commission schedules satisfy $\tilde{\mathbf{p}}_i^{\mathbf{e}} = \tilde{\mathbf{p}}_i^R\left(\tilde{\mathbf{t}}_i^{\mathbf{e}}\right)$.

- In the first stage, each schedule $\tilde{t}_{ij}^e$:
  – maximizes the joint profit of M_i and R_j, taking as given M_i's other equilibrium schedules, $\tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}$, rivals' equilibrium prices, $\tilde{\mathbf{p}}_{-i}^{\mathbf{e}}$, and M_i's pricing strategy in the second stage, $\tilde{\mathbf{p}}_i^R\left(\tilde{\mathbf{t}}_i\right)$:

$$
\begin{aligned}
\tilde{t}_{ij}^e \in \arg\max_{\tilde{t}_{ij}} \quad & \Bigg\{ \left(\tilde{p}_{ij}^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right) - c_i - \gamma_j\right) D_{ij}\left(\tilde{\mathbf{p}}_i^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right), \tilde{\mathbf{p}}_{-i}^{\mathbf{e}}\right) \\
& + \sum_{h \in \mathcal{I} \setminus \{i\}} \left[\begin{array}{c} \tilde{t}_{hj}^e\left(D_{hj}\left(\tilde{\mathbf{p}}_i^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right), \tilde{\mathbf{p}}_{-i}^{\mathbf{e}}\right)\right) \\ -\gamma_j D_{hj}\left(\tilde{\mathbf{p}}_i^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right), \tilde{\mathbf{p}}_{-i}^{\mathbf{e}}\right) \end{array} \right] \\
& + \sum_{k \in \mathcal{J} \setminus \{j\}} \left[\begin{array}{c} \left(\tilde{p}_{ik}^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right) - c_i\right) D_{ik}\left(\tilde{\mathbf{p}}_i^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right), \tilde{\mathbf{p}}_{-i}^{\mathbf{e}}\right) \\ -\tilde{t}_{ik}^e\left(D_{ik}\left(\tilde{\mathbf{p}}_i^R\left(\tilde{t}_{ij}, \tilde{\mathbf{t}}_{i,-j}^{\mathbf{e}}\right), \tilde{\mathbf{p}}_{-i}^{\mathbf{e}}\right)\right) \end{array} \right] \Bigg\};
\end{aligned}
$$

  – gives M_i and R_j shares α_{ij} and $1 - \alpha_{ij}$, respectively, of the additional profit generated by their relationship.

It is straightforward to see that this definition of a bargaining equilibrium amounts to turning the previous framework "upside-down": manufacturers are now downstream (they control retail prices), whereas retailers/platforms are upstream. As before, however, commissions are non-linear payment schedules paid by downstream firms (here, the manufacturers) to their upstream partners (the retailers).

We thus simply need to adapt our initial assumptions to conclude that as long as commissions induce a smooth retail pricing behavior by manufacturers, equilibrium commissions are cost-based and the outcome is similar to that of a multi-store oligopoly in which n firms directly compete against each other at m retail locations. Formally, the modified assumption is:

Assumption Ã: multi-store oligopoly. There is a unique price vector $\tilde{\mathbf{p}}^*$ satisfying $\tilde{\mathbf{p}}_i \in \tilde{\mathbf{p}}_i^r\left(\tilde{\mathbf{p}}_{-i}\right) \equiv \arg\max_{\tilde{\mathbf{p}}_i}\left\{\sum_{j \in \mathcal{J}}\left(\tilde{p}_{ij} - c_i - \gamma_j\right) D_{ij}\left(\tilde{\mathbf{p}}\right)\right\}$ for every $i \in \mathcal{I}$; it is characterized by first-order conditions and such that $\tilde{\mathbf{p}}_i^* = \tilde{\mathbf{p}}_i^r\left(\tilde{\mathbf{p}}_{-i}^*\right)$ for every $j \in \mathcal{J}$, and $D_{ij}\left(\tilde{\mathbf{p}}^*\right) > 0$ for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$.

Under Assumption Ã, and in the class of contracts inducing the manufacturers to adopt a smooth pricing behavior, all commission schedules must be cost-based, in the sense that marginal commission rates must reflect marginal costs of distribution; hence, the equilibrium outcome replicates that of direct competition between multi-store firms (that is, $\tilde{\mathbf{p}}^{\mathbf{e}} = \tilde{\mathbf{p}}^*$). Moreover in this framework, price parity agreements (i.e., agreements between manufacturers and retailers requiring that manufacturers set the same prices on all platforms) have no impact on the equilibrium outcome beyond imposing symmetry. More precisely, equilibrium tariffs are once again cost-based in the sense that marginal commissions reflect marginal costs of distribution (i.e., the intermediaries' costs). In addition, when firms are symmetric at both stages of the vertical chains (and the equilibrium prices are symmetric in the absence of PPAs), then price parity agreements do not affect the equilibrium retail prices either.

The result that Price Parity Agreements (PPAs) have no impact on prices in the agency model contrasts with the recent literature on these agreements. However, so far this literature has focused on either linear commissions[10] or constant revenue-sharing rules,[11] which generate contractual inefficiencies; instead, we allow for general non-linear commissions and thus for efficient bilateral contracting. Foros *et al.* (2017) also consider constant revenue-sharing rules but study the platforms' choice between setting final prices (traditional wholesale model) or delegating these pricing decisions to suppliers (agency model). They show that a coordination failure may arise, whereby the agency model may fail to be adopted (even though it would increase all firms' profits); PPAs can then be used to facilitate the adoption of the agency model, thus leading to higher prices for consumers.

E Endogenous network

E.1 Bargaining equilibria

In this subsection, we study the bargaining equilibria for the distribution networks considered in Section 6.1. For the sake of exposition, we assume that demand satisfies standard regularity assumption ensuring that two-part tariffs induce retailers to adopt a smooth retail behavior. Using similar arguments as in Propositions 1 and 2, we show that there exists a unique equilibrium in two-part tariffs, in which these tariffs are cost-based, and characterize the equilibrium profits.

E.1.1 Bilateral monopoly

Suppose first that a single channel is active, say $i - j$. In this case, firms maximize their joint profit by negotiating a cost-based tariff and generate in this way a profit

$$\pi^m \equiv \max_p (p - c - \gamma) D(p, \infty, \infty, \infty).$$

As both firms would obtain zero profit in case of a negotiation break-down, M_i's and R_j's equilibrium profits are respectively equal to:

$$\Pi_{M_i} = \Pi_M^m \equiv \alpha \pi^m \quad \text{and} \quad \Pi_{R_j} = \Pi_R^m \equiv (1 - \alpha) \pi^m.$$

[10]See Boik and Corts (2016) and Johansen and Vergé (2017).
[11]See Johnson (2017).

These equilibrium profits can, for instance, be sustained with the following two-part tariff:

$$t_{ij}(q) = \alpha \pi^m + cq.$$

E.1.2 Exclusive dealing

Suppose now that two unconnected channels are active, say $i-j$ and $h-k$. Given the equilibrium retail price p^e_{hk} set by R_k and the tariff t_{ij} that it faces, R_j chooses the price $p^R_{ij}(t_{ij})$ that maximizes its retail profit, that is:

$$p^R_{ij}(t_{ij}) \equiv \arg\max_p \left[(p-\gamma) D\left(p, \infty, \infty, p^e_{hk}\right) - t_{ij}\left(D\left(p, \infty, \infty, p^e_{hk}\right)\right)\right].$$

The joint profit of M_i and R_j, equal to

$$\left(p^R_{ij}(t_{ij}) - c - \gamma\right) D\left(p^R_{ij}(t_{ij}), \infty, \infty, p^e_{hk}\right),$$

is thus maximized when the tariff t_{ij} is cost-based. Therefore, in any equilibrium, each tariff is cost-based and each channel generates a profit

$$\pi^{ED} \equiv \left(p^{ED} - c - \gamma\right) D\left(p^{ED}, \infty, \infty, p^{ED}\right),$$

where the price p^{ED} is such that:

$$p^{ED} = \arg\max_p (p - c - \gamma) D\left(p, \infty, \infty, p^{ED}\right).$$

As both firms would again obtain zero profit in case of a negotiation break-down, M_i's and R_j's equilibrium profits are respectively equal to:

$$\Pi_{M_i} = \Pi^{ED}_M \equiv \alpha \pi^{ED} \quad \text{and} \quad \Pi_{R_j} = \Pi^{ED}_R \equiv (1-\alpha)\pi^{ED}.$$

These equilibrium profits can be sustained with the following two-part tariffs:

$$t_{ij}(q) = t_{hk}(q) = \alpha \pi^{ED} + cq.$$

E.1.3 Upstream foreclosure

In the case where a single manufacturer, say M_i, deals with both retailers, O'Brien and Shaffer (1992) have shown that equilibrium tariffs are cost-based (see Proposition 3, p. 305). In equilibrium, each channel thus generates a profit

$$\pi^{UF} \equiv \left(p^{UF} - c - \gamma\right) D\left(p^{UF}, \infty, p^{UF}, \infty\right),$$

where the price p^{UF} is such that:

$$p^{UF} = \arg\max_{p} (p - c - \gamma) D\left(p, \infty, p^{UF}, \infty\right).$$

If the negotiation with R_j were to break down, R_j would be excluded and obtain zero profit. In equilibrium, it thus obtains a share $1 - \alpha$ of the bilateral gains from trade. As retailers are the residual claimants for the profit generated by their channel (the manufacturer obtaining its profits through the fixed fees), these bilateral gains from trade coincide with the profit generated by the channel.[12] It follows that the manufacturer and the two retailers' equilibrium profits are respectively equal to:

$$\Pi_{M_i} = \Pi_M^{UF} \equiv 2\alpha\pi^{UF} \text{ and } \Pi_{R_1} = \Pi_{R_2} = \Pi_R^{UF} \equiv (1-\alpha)\pi^{UF}.$$

These equilibrium profits can be sustained with the following two-part tariffs:

$$t_{i1}(q) = t_{i2}(q) = \alpha\pi^{UF} + cq.$$

E.1.4 Downstream foreclosure

In the case where a single retailer, say R_j, deals with both manufacturers, Bernheim and Whinston (1985, 1998) have shown that equilibrium tariffs are then cost-based. In equilibrium, each channel thus generates a profit

$$\pi^{DF} \equiv \max_{p} (p - c - \gamma) D(p, p, \infty, \infty).$$

If the negotiation with M_i were to break down, M_i would be excluded and obtain zero profit. In equilibrium, it thus obtains a share α of the bilateral gains from trade, which are here equal to $2\pi^{DF} - \pi^m$, as M_i and R_j jointly earn $2\pi^{DF} - F_{hj}$ when reaching an agreement, and $\pi^m - F_{hj}$ otherwise. In equilibrium, manufacturers' profits are therefore given by:

$$\Pi_{M_A} = \Pi_{M_B} = \Pi_M^{DF} \equiv \alpha\left(2\pi^{DF} - \pi^m\right)$$

whereas the retailer's profit is equal to:

$$\Pi_{R_j} = \Pi_R^{DF} \equiv 2(1-\alpha)\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right).$$

These equilibrium profits can be sustained with the following two-part tariffs:

$$t_{Aj}(q) = t_{Bj}(q) = \alpha\left(2\pi^{DF} - \pi^m\right) + cq.$$

[12] That is, the bilateral joint profit of M_i and R_j is given by $\pi^{UF} + F_k$ if they reach an agreement, and by F_k otherwise; hence, the bilateral gains from trade are equal to π^{UF}.

E.1.5 Single exclusion

Suppose finally that a single channel, say $h-k$, remains inactive. All firms are thus directly or indirectly connected, as M_i deals with both retailers, and R_j deals with both manufacturers.

It is straightforward to check that the same reasoning as in the baseline model (with the convention here that $p_{hk}=+\infty$ and $t_{hk}(\cdot)=q_{hk}=0$) implies that the equilibrium upstream margin u_{hj}^{SE} satisfies (noting that $q_{hk}=0$ implies $\delta_{jk}^{hj}=0$):

$$u_{hj}^{SE}=0,$$

whereas the equilibrium margins u_{ij}^{SE} and u_{ik}^{SE} satisfy:

$$\begin{aligned} u_{ij}^{SE}-\delta_{jk}^{ij}u_{ik}^{SE} &= 0, \\ u_{ik}^{SE}-\delta_{kj}^{ik}u_{ij}^{SE} &= 0. \end{aligned}$$

The condition 3 then implies $\delta_{jk}^{ij}\delta_{kj}^{ik}\neq 1$, which in turn yields $u_{ij}^{SE}=u_{ik}^{SE}=0$.

We now turn to the equilibrium retail prices. For the sake of exposition, we use the subscripts J, M and R to refer respectively to the *joint* channel of the two multi-channel firms (here, $i-j$), the other channel of the multi-channel *manufacturer* (here, $i-k$), and the other channel of the multi-channel retailer (here, $h-j$). Given that equilibrium tariffs are cost-based, the equilibrium retail prices p_J^{SE} and p_R^{SE} must satisfy:

$$\left(p_J^{SE},p_R^{SE}\right)=\arg\max_{(p_J,p_R)}\left\{\begin{array}{c}(p_J-c-\gamma)\,D\left(p_J,p_R,p_M^{SE},\infty\right)\\ +(p_R-c-\gamma)\,D\left(p_R,p_J,\infty,p_M^{SE}\right)\end{array}\right\},$$

whereas the equilibrium price p_M^{SE} satisfies:

$$p_M^{SE}=\arg\max_{p_M}(p_M-c-\gamma)\,D\left(p_M,\infty,p_J^{SE},p_R^{SE}\right).$$

In what follows, we assume that these prices are unique. We denote by

$$\pi_m^{SE}\equiv\left(p_J^{SE}-c-\gamma\right)D\left(p_J^{SE},p_R^{SE},p_M^{SE},\infty\right)+\left(p_R^{SE}-c-\gamma\right)D\left(p_R^{SE},p_J^{SE},\infty,p_M^{SE}\right)$$

the profit generated by the multi-channel retailer (R_j), and by

$$\pi_s^{SE}\equiv\left(p_M^{SE}-c-\gamma\right)D\left(p_M^{SE},\infty,p_J^{SE},p_R^{SE}\right)$$

the profit generated by the single-channel retailer (R_k). Finally, let

$$\hat{\pi}_J\equiv\max_p(p-c-\gamma)\,D\left(p,\infty,p_M^{SE},\infty\right)\text{ and }\hat{\pi}_R=\max_p(p-c-\gamma)\,D\left(p,\infty,\infty,p_M^{SE}\right)$$

denote the profit that the multi-channel retailer (R_j) could generate by focusing instead, respectively, on the joint channel $(M_i - R_j)$, and on the other channel $(M_h - R_j)$.

We now focus on two-part tariffs and derive the individual equilibrium profits. We respectively denote M_i's and M_h's profits by $\Pi_{M_i} = \Pi^{SE}_{Mm}$ and $\Pi_{M_h} = \Pi^{SE}_{Ms}$, where the subscripts Mm and Ms respectively refer to the multi-channel and single-channel manufacturers. With a similar convention, we respectively denote R_j's and R_k's profits by $\Pi_{R_j} = \Pi^{SE}_{Rm}$ and $\Pi_{R_k} = \Pi^{SE}_{Rs}$. As firms negotiate cost-based two-part tariffs, from Nash bargaining (equation (35) the fixed fees are of the form $F = \alpha\Delta$, where Δ is the extra joint profit generated by a successful negotiation. We thus have:

$$F_{ij} = \alpha\left(\pi^{SE}_m - \hat{\pi}_R\right) > 0,\ F_{hj} = \alpha\left(\pi^{SE}_m - \hat{\pi}_J\right) > 0 \text{ and } F_{ik} = \alpha\pi^{SE}_s > 0.$$

Manufacturers' profits are therefore respectively given by

$$\Pi_{M_i} = \Pi^{SE}_{Mm} \equiv \alpha\left(\pi^{SE}_m + \pi^{SE}_s - \hat{\pi}_R\right) \quad \text{and} \quad \Pi_{M_h} = \Pi^{SE}_{Ms} \equiv \alpha\left(\pi^{SE}_m - \hat{\pi}_J\right),$$

and retailers' profits are respectively given by

$$\Pi_{R_j} = \Pi^{SE}_{Rm} \equiv (1-\alpha)\,\pi^{SE}_m + \alpha\left(\hat{\pi}_J + \hat{\pi}_R - \pi^{SE}_m\right) \quad \text{and} \quad \Pi_{R_k} = \Pi^{SE}_{Rs} \equiv (1-\alpha)\,\pi^{SE}_s.$$

E.2 Proof of Proposition 7

We consider the two polar cases in turn.

E.2.1 No retail competition

Consider first the case where retailers are active in independent geographic markets. Each geographic market can then be analyzed separately and, building on the analysis already presented in the text, in any CPNE both brands must be carried in each market. Finally, it is straightforward to check that this indeed constitutes a CPNE.

Consider the geographic market of R_j, say. In the candidate CPNE, R_j carries both brands, each channel generates π^M, and firms' profits are respectively given by $\Pi_A = \Pi_B = \alpha\left(2\pi^M - \pi^m\right)(>0)$ and $\Pi_{R_j} = 2(1-\alpha)\,\pi^M + 2\alpha\left(\pi^m - \pi^M\right)(>0)$. Obviously, in the preliminary stage manufacturers have no incentive to deviate (either unilaterally, or as a coalition), as they can only change the distribution network by exiting the market. Likewise, the retailer has no incentive to reduce the number of brands it carries as:

$$2(1-\alpha)\,\pi^M + 2\alpha\left(\pi^m - \pi^M\right) > (1-\alpha)\,2\pi^M > (1-\alpha)\,\pi^m,$$

where the first inequality stems from brand differentiation and the second from the fact that the retailer generates more profit when it carries both brands. Moreover, a deviation involving the "grand coalition" (i.e., R_j together with both manufacturers) would either have no effect (if all firms remain active) or require the exit of one firm, which the firm would reject. Finally, suppose that R_j deviates with one manufacturer. To make the deviation profitable for the manufacturer, it must exclude the other brand. In the continuation bargaining game, the remaining active channel generates π^m and R_j obtains $(1-\alpha)\,\pi^m < \Pi_{R_j}$, making the deviation unprofitable for R_j. It follows that "interlocking relationships" (i.e. here, both brands being carried in each retailer's territory) indeed constitutes a CPNE.

E.2.2 Perfect retail substitutes

Consider now the case where retailers are perfect substitutes.

We first note that each brand will be carried by a single retailer. To see this, consider a candidate equilibrium in which M_i, say, deals with both retailers. As tariffs are cost-based, retailers face the same marginal cost, and intrabrand competition leads them to simply pass on this cost to consumers. As a result, retailers derive zero profit from the sales of M_i's product, and thus M_i obtains zero profit as well. But then, M_i would profitably deviate by refusing to deal with one retailer: the other retailer would then generate a profit from selling M_i's product, and M_i would obtain a share of that profit.

As both brands must be sold (from the reasoning at the beginning of Section 6.2), it follows that the only candidate CPNE networks are "exclusive dealing" and "downstream foreclosure".

In the case of exclusive dealing, each firm has a single trading partner, and thus its outside option in case of disagreement yields zero profit. The channel profit π^{ED} is thus simply shared in proportion $(\alpha, 1-\alpha)$. Each manufacturer obtains $\Pi_M^{ED} \equiv \alpha\pi^{ED}$ and each retailer obtains $\Pi_R^{ED} \equiv (1-\alpha)\,\pi^{ED}$. In case of downstream foreclosure, each manufacturer again has a single trading partner, but now one retailer carries both brands.[13] As a result, in case of disagreement with one manufacturer, the retailer would still obtain a share of the bilateral monopoly profit π^m. As a result, manufacturers now obtain $\Pi_M^{DF} \equiv \alpha\left(2\pi^{DF} - \pi^m\right)$, whereas the selected retailer obtains $\Pi_R^{DF} \equiv 2\,(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right)$.

Note that when starting from a candidate CPNE involving either exclusive dealing or downstream foreclosure:

- deviations by a coalition activating more than two channels are irrelevant: at least one manufacturer (who has to be part of the deviating coalition) would be dealing with both retailers, and this manufacturer would have an incentive to (unilaterally) deviate from the coalition so as to deal instead with a single retailer;

- all active firms obtain a positive profit, and thus none of them has an incentive to deviate by simply refusing to deal; in the same vein, in case of downstream foreclosure, the active retailer has no incentive to close down any channel: with only one active channel (that is, under bilateral monopoly) the retailer would only obtain

[13] As retailers are perfect substitutes here, the active retailer generates the industry-wide monopoly profit (that is, $2\pi^{DF} = \Pi^M$).

$\Pi_R^m = (1-\alpha)\,\pi^m$, whereas with both active channels (downstream foreclosure) the retailer obtains:

$$\Pi_R^{DF} = 2(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right) > (1-\alpha)\,2\pi^{DF} > (1-\alpha)\,\pi^m = \Pi_R^m,$$

where the last inequality stems from the fact that the retailer generates more profit when it carries both brands.

We now consider the other potential deviations for each of the two candidate equilibrium networks.

• *Exclusive dealing.* Consider a candidate CPNE in which, say, M_i deals with R_j whereas M_h deals with R_k. In the light of the above remarks, deviations leading to fewer, or to more active channels are irrelevant. Likewise, a coalition deviating to upstream foreclosure is irrelevant (as intrabrand competition would then dissipate all profits). Therefore, the only relevant deviation is for a coalition to move to downstream foreclosure. Suppose, for instance, that M_i and R_k agree to open their channel (in addition to the $h-k$ channel) and foreclose R_j (that is, M_i and R_k now deal with each other, whereas M_i stops dealing with R_j but R_k keeps dealing with M_h):

- this deviation is always profitable for R_k, whose profit increases from $\Pi_R^{ED} = (1-\alpha)\,\pi^{ED}$ to:

  $$\Pi_R^{DF} = 2(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right) > (1-\alpha)\,2\pi^{DF} > (1-\alpha)\,\pi^{ED} = \Pi_R^{ED},$$

  where the first inequality stems from the fact that a channel profit is maximal when all other channels are inactive (and thus $\pi^m > \pi^{DF}$), whereas the second inequality stems from the fact that industry-wide profit is larger when the two brands are carried by the same retailer (so that $2\pi^{DF} > \pi^{ED}$);

- by contrast, this deviation is profitable for M_i if and only if:

  $$\Pi_M^{DF} = \alpha\left(2\pi^{DF} - \pi^m\right) > \Pi_M^{ED} = \alpha\pi^{ED}.$$

It follows that exclusive dealing is a CPNE network if and only if $\pi^{ED} \geq 2\pi^{DF} - \pi^m$.

• *Downstream foreclosure.* Consider now a candidate CPNE in which the two manufacturers deal with a single common retailer, say, R_j. Using the same reasoning as above, the only relevant deviation is now for a coalition to move to exclusive dealing. Suppose, for instance, that M_h stops dealing with R_i and forms a coalition with R_k to open their channel (that is, M_h and R_k now deal with each other, whereas R_j keeps dealing with M_i but no longer deals with M_h):

- this deviation is always profitable for R_k, whose profit is now positive whereas it would otherwise be excluded;

- by contrast, this deviation is profitable for M_h if and only if:

$$\Pi_M^{ED} = \alpha \pi^{ED} > \Pi_M^{DF} = \alpha \left(2\pi^{DF} - \pi^m\right).$$

It follows that downstream foreclosure is a CPNE network if and only if $\pi^{ED} \leq 2\pi^{DF} - \pi^m$.

Summing-up, exclusive dealing constitutes the unique CPNE network if $\pi^{ED} > 2\pi^{DF} - \pi^m$, whereas downstream foreclosure constitutes the unique CPNE network if instead $\pi^{ED} < 2\pi^{DF} - \pi^m$. In the limit case where $\pi^{ED} = 2\pi^{DF} - \pi^m$, both network configurations can arise in a CPNE.

E.3 Proof of Proposition 8

We already know that no firm can be fully excluded in equilibrium, which leaves us with only three candidate networks for a CPNE: exclusive dealing, single exclusion (i.e., three active channels) and interlocking relationships. We consider them in turn.

E.3.1 Exclusive dealing

Consider a candidate CPNE yielding exclusive dealing. Without loss of generality, we can restrict attention to candidate strategies where firms are willing to deal with a single partner, as this minimizes the number of alternative networks that a coalition could achieve. Thus, consider a candidate equilibrium in which M_i and R_j, on the one hand, and M_h and R_k, on the other hand, only want to deal with each other.

We first note that these strategies constitute indeed a Nash-equilibrium of the network formation game as, by unilaterally deviating, a firm can affect the network only by excluding itself from the market. Furthermore, given these equilibrium strategies, the coalition of manufacturers, the coalition of retailers and the coalition consisting of M_i and R_j (resp., M_h and R_k) cannot profitably deviate: indeed, any deviation affecting the network could have been achieved through a unilateral deviation.

Finally, given these strategies, any network that can be achieved by a deviating coalition of three firms can also be achieved by a two-firm coalition. Therefore, we only need to consider deviations by the coalition consisting of M_i and R_k (by symmetry, the same analysis applies to the coalition consisting of M_h and R_j) or by the "grand coalition" (all four players).

- *Deviations by the coalition $M_i - R_k$.* When considering deviations by the coalition consisting of M_i and R_k, looking for self-enforcing deviations amounts to looking for Pareto-undominated Nash-equilibria of the two-player game between M_i and R_k, keeping fixed the strategies of M_h and R_j – i.e., taking as given that M_h only wants to deal with R_k, and R_j only wants to deal with M_i.

As noted above, M_i and M_h dealing exclusively with R_j and R_k respectively, constitutes a Nash equilibrium of this two-player game. And as M_i and R_k obtain a positive profit in this exclusive dealing network, we can restrict attention to alternative Nash equilibria in which they both have at least one trading partner. Furthermore, we have:

(i) if M_i is willing to deal only with R_k, then R_k's best-response is to deal with both manufacturers (as downstream foreclosure gives R_k a greater profit than bilateral monopoly);

(ii) if M_i is willing to deal with both retailers, then R_k prefers dealing exclusively with M_h to dealing exclusively with M_i (as competition is softer when the retailers carry different brands);

(iii) if R_k is willing to deal with both suppliers, then M_i prefers dealing exclusively with R_j to dealing exclusively with R_k, as the condition $\pi^{ED} > 2\pi^{DF} - \pi^m$ implies $\Pi_M^{ED} > \Pi_M^{DF}$.

The first two observations imply that there is no Nash equilibrium in which R_k deals exclusively with M_i. The third one implies that there is no Nash equilibrium in which R_k deals with both suppliers and R_j is excluded from the market. Therefore, besides exclusive dealing (with channels $i - j$ and $h - k$ being active), the only other network that may arise in a Nash-equilibrium of the two-player game is one where only channel $h - j$ remains inactive (i.e., single exclusion).

In addition, the above observations imply that, starting from a candidate Nash equilibrium yielding a connected network (i.e., single exclusion), for each partner the only relevant deviation consists of switching to exclusive dealing, by refusing to deal with its other trading partner. Therefore, the connected network constitutes a Nash equilibrium if and only if M_i and R_k both (weakly) prefer it to exclusive dealing, that is, if and only if:

$$\pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R \geq \pi^{ED} \text{ and } (1-\alpha)\,\pi_m^{SE} + \alpha\left(\hat{\pi}_J + \hat{\pi}_R - \pi_m^{SE}\right) \geq (1-\alpha)\,\pi^{ED}. \tag{22}$$

For the linear demand specified above: (i) the first condition in (22) amounts to $\rho \leq \rho^*(\mu)$, where the threshold $\rho^*(\mu)$ is the unique solution to $\pi^{ED} = \pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R$, and is such that $\rho^*(\mu) \in (0,1)$ and it strictly decreases as μ increases[14]; and (ii) when this first condition holds, then $\pi_m^{SE} > \pi^{ED}$, and thus the second condition in (22) holds strictly for any $\alpha \in [0,1]$.

Therefore:

[14] The threshold $\rho^*(\mu)$ is the unique solution in $[0,1]$ of:

$$\begin{aligned}\left(4+\mu^4\rho^4\right)\left(4-8\rho+3\rho^2-\rho^3\right)+4\mu^2\rho^2\left(1-\rho+2\rho^2\right)+\mu^4\rho^4\left(1+\rho\right)&\\ -4\mu\rho\left(2-\mu\rho+\mu^2\rho^2\right)\left(6-11\rho+6\rho^2-\rho^3\right)&=0.\end{aligned}$$

- when $\rho < \rho^*(\mu)$, exclusive dealing and connected networks can both be supported as a Nash-equilibrium of the two-player game, and the connected network in which M_i is the multi-partner supplier is strictly preferred by both M_i and R_k;

- when $\rho = \rho^*(\mu)$, both types of networks can be supported as a Nash-equilibrium of the two-player game, but M_i is indifferent between exclusive dealing and being the multi-partner supplier in a connected network;

- finally, when $\rho > \rho^*(\mu)$, exclusive dealing is the unique network that can be supported as a Nash-equilibrium of the two-player game.

It follows from these observations that, when $\rho < \rho^*(\mu)$, starting from the candidate Nash-equilibrium with exclusive dealing, there exists a self-enforcing profitable deviation for the coalition made of M_i and R_k. When instead $\rho \geq \rho^*(\mu)$, there is no self-enforcing profitable deviation for this coalition (as at least one firm – namely, M_i – would not strictly benefit from such a deviation).

• *Deviations by the grand coalition.* Consider now a deviation by the grand coalition. To be profitable, the grand coalition needs to increase the number of active channels: this can be done by switching either to a connected network or to interlocking relationships. However, switching to a connected network can already be achieved by the coalition $M_i - R_k$ and including more players in the coalition only makes the deviation less likely to be self-enforcing. Moreover, whenever $\rho \geq \rho^*(\mu)$, we have $\pi^{ED} > 2(2\pi^* - \hat{\pi})$ implying that manufacturers cannot benefit from such a deviation.

Summing-up, there exists a CPNE leading to exclusive dealing if and only if $\rho \geq \rho^*(\mu)$.

E.3.2 Interlocking relationships

Consider now a candidate CPNE leading to interlocking relationships (i.e., where all channels are active). By construction, in such an equilibrium all firms must be willing to deal with both of their trading partners. It follows that any deviating network that could be achieved by a coalition made of the manufacturers and at least one retailer (resp., the retailers and at least one manufacturer) could also be achieved by the coalition of manufacturers (resp., retailers). Hence, there is no need to consider deviations by coalitions of three or more firms, and we can instead restrict attention to unilateral deviations and deviations by two-firm coalitions.

As exiting the market is not profitable (as all firms are profitable in the equilibrium generated by interlocking relationships), to rule out unilateral deviations, it suffices to check that no firm prefers dealing with a single partner, which amounts to:

$$2(2\pi^* - \hat{\pi}^*) \geq \pi_m^{SE} - \hat{\pi}_J \quad \text{and} \quad 2(1-\alpha)\pi^* + 2\alpha(\hat{\pi}^* - \pi^*) \geq (1-\alpha)\pi_s^{SE}. \tag{23}$$

For the linear demand specification:

- $2\pi^* > \pi_s^{SE}$, and thus the second condition in (23) holds strictly for any $\alpha \in [0,1]$;
- the first condition in (23) holds instead if and only if $\rho \leq \rho^*(0)$.

Therefore, there exists a Nash-equilibrium leading to interlocking relationships if and only if $\rho \leq \rho^*(0)$. Next, we consider (self-enforcing) deviations by two-firm coalitions.

• *Deviations by the coalition of manufacturers.* Deviations by the coalition of manufacturers are self-enforcing if they constitute Pareto-undominated Nash-equilibria of the two-player game between M_A and M_B, taking R_1 and R_2' strategies as given. As retailers are willing to deal with both suppliers, in this two-player game each manufacturer freely determines which of its two distribution channels will be active.

Exiting the market is again never a best-response. Furthermore, from the above observation, in response to M_h dealing with both retailers, M_i is also willing to deal with both retailers when $\rho \leq \rho^*(0)$, and strictly prefers doing so (rather than dealing exclusively with one retailer) if $\rho < \rho^*(0)$. If instead M_h chooses to deal with one retailer only (say, R_k):

- M_i prefers to deal exclusively with R_j (so as to induce the exclusive dealing network) to dealing exclusively with R_k (as this would lead to the foreclosure of R_j, which is less profitable for M_i, as $\pi^{ED} > 2\pi^{DF} - \pi^m$ for the linear demand specification);
- And M_i strictly prefers dealing with both retailers rather than dealing exclusively with R_j whenever $\pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R > \pi^{ED}$, that is, whenever $\rho < \rho^*(\mu)$.

As $\rho^*(\mu)$ is a decreasing function of μ, it follows from the above observations that, when $\rho < \rho^*(\mu)$, there exists a unique Nash-equilibrium of the above two-player manufacturer game, and this equilibrium induces interlocking relationships.

When instead $\rho \geq \rho^*(\mu)$, there also exists a Nash-equilibrium of the two-player game leading to exclusive dealing. It can furthermore be checked that, for the linear demand specification, manufacturers then prefer the outcome generated by exclusive dealing to the outcome generated by interlocking relationships; that is, $\rho \geq \rho^*(\mu)$ implies $\pi^{ED} > 2[2\pi^* - \hat{\pi}^*]$. Hence, even when interlocking relationships can be supported as a Nash-equilibrium (which is the case when $\rho \leq \rho^*(0)$), there exists a self-enforcing deviation (to exclusive dealing) for the coalition of manufacturers. In what follows, we thus focus on the case $\rho < \rho^*(\mu)$.

• *Deviations by the coalition of retailers.* Deviations by the coalition of retailers are self-enforcing if they constitute Pareto-undominated equilibria of the two-player game between R_1 and R_2, taking M_A and M_B's strategies as given. As manufacturers are willing to deal with both distributors, in this two-player game each retailer freely determines which of the two brands it will carry. Building on the previous observations, exiting the market is never a best-response and, if a retailer chooses to carry both brands, then the other retailer strictly prefers carrying both brands as well. In addition, $\rho < \rho^*(\mu)$ implies

$\Pi_{Rm}^{SE} > \Pi_R^{ED}$ (that is, the second part of in (22) holds); hence, if a retailer chooses to carry a single brand, the other retailer strictly prefers carrying both brands. Carrying both brands thus constitutes a strictly dominant strategy for each retailer, implying that, starting from the Nash-equilibrium with interlocking relationships, there is no self-enforcing profitable deviation by the coalition of retailers.

• *Deviations by the coalition* $M_i - R_j$. Finally, consider a coalition made of a supplier (say, M_i) and a retailer (say, R_j). When $\rho < \rho^*(\mu)$:

- when M_i (resp., R_j) deals with both retailers (resp., manufacturers), R_j's (resp., M_i's) best-response is to deal with both manufacturers (resp., retailers);
- when R_j is willing to deal exclusively with M_i, M_i's (unique) best-response is to deal with both retailers.

Moreover, when M_i deals exclusively with R_k, R_j has two best-responses (dealing with M_h exclusively, or accepting to deal with both manufacturers) that yield the same connected network, where channel $i - j$ remains inactive. Likewise, when R_j deals exclusively with M_h, M_i has two best-responses (dealing with R_k exclusively, or accepting to deal with both retailers) leading to the same network.

This implies that this two-player game has two Nash-equilibria, one leading to interlocking relationships and one leading to a connected network (with channel $i-j$ remaining inactive). But in this last case, M_i and R_j would strictly prefer to activate channel $i - j$. Hence, the equilibrium with single exclusion is strictly Pareto-dominated, implying that there is no self-enforcing deviation for the coalition $M_i - R_j$.

Summing-up, there exists a CPNE with interlocking relationships if and only if $\rho < \rho^*(\mu)$.

E.3.3 Single exclusion

We finally show that there never exists a CPNE with three active channels (i.e., single exclusion). To see this, consider a candidate CPNE with only one channel, say $h - k$, is inactive.

When $\rho < \rho^*(0)$, we have seen that both conditions in (23) strictly hold. It follows that there exists a self-enforcing deviation for the coalition $M_h - R_k$, which consists of activating the fourth channel (in addition to the other ones).

Furthermore, when $\rho > \rho^*(\mu)$, we have seen that condition in (22) is violated. Therefore, M_i would find it profitable to unilaterally deviate and deal exclusively with R_k.

As $\rho^*(\mu)$ is a decreasing function of μ, the above analysis implies that there always exists either a profitable unilateral deviation (when $\rho > \rho^*(\mu)$), or a self-enforcing deviation by a two-firm coalition (when $\rho < \rho^*(0)$). Hence, there never exists a CPNE with three active channels.

F Mergers

We now apply our bargaining equilibrium approach to evaluate the effect of mergers in the above successive duopoly setting. To ensure that firms' payoffs are properly defined, we maintain the focus on equilibria based on two-part tariffs.

F.1 Downstream merger

A merger between the two retailers creates a new entity, R, which is a multi-location monopolist. As mentioned in the text, for the sake of exposition we assume that manufacturers can no longer discriminate according to the channel through which their products are sold, and so R negotiates with each M_i a single two-part tariff, $t_i(q_i) = F_i + w_i q_i$. A bargaining equilibrium is then similar to a delegated negotiation game in which R uses two different agents simultaneously negotiating with the two manufacturers.[15]

We first study the bargaining equilibria for each distribution network, and show that equilibrium tariffs are still always cost-based. We then characterize the equilibrium fixed fees, and show that they are unique. Finally, we look for the CPNE of the network formation game, which we compare with the pre-merger outcome.

Throughout this section, we assume that R's monopolistic price response is uniquely defined and that the resulting quantities are "well-behaved". Specifically, let (with the convention that $w_{ij} = p_{ij} = \infty$ and $D_{ij} = 0$ if brand i is not present at location j)

$$\mathbf{p^m}(\mathbf{w}) \equiv \arg\max_{\mathbf{p}} \sum_{i\in\{A,B\},j\in\{1,2\}} \left(p_{ij} - w_{ij} - \gamma_j\right) D_{ij}(\mathbf{p})$$

denote R's price response to wholesale prices $\mathbf{w}$, and $\mathbf{q^m}(\mathbf{w})$ denote the corresponding quantities (i.e., $q_{ij}^m(\mathbf{w}) = D_{ij}(\mathbf{p^m}(\mathbf{w}))$ if brand i is present at location j, and $q_{ij}^m(\mathbf{w}) = 0$ otherwise). Likewise, let $\pi_R(\mathbf{w})$, $\pi_A(\mathbf{w})$ and $\pi_B(\mathbf{w})$ denote the resulting profits (gross of fixed fees) for R, M_A and M_B respectively. We also maintain the following adaptation of Assumption A (specifically, the existence and uniqueness of a retail response) and condition (3):

Assumption F1 (multi-brand multi-location monopoly) For any vector of wholesale prices $\mathbf{w}$, there is a unique vector $\mathbf{p}^m(\mathbf{w})$; furthermore, the retail quantity for each (active) brand responds to its wholesale price:

$$\text{for every } i \in \{A,B\}, \sum_{j\in\{1,2\}} \frac{\partial q_{ij}^m}{\partial w_{ij}}(\mathbf{w}) \neq 0.$$

[15] If manufacturers could charge different prices for each distribution channel, then the relevant delegated negotiation game would have R using four different agents, one for each channel.

F.1.1 Interlocking relationships

We start with interlocking relationships, where the retailer carries both brands at both locations. In the last stage, R obtains a profit (gross of the fixed fees F_A and F_B) given by:

$$\pi_R(\mathbf{w}) \equiv \max_{\mathbf{p}} \left\{ \sum_{j\in\{1,2\}} \left[(p_{ij} - w_i - \gamma) D_{ik}(\mathbf{p}) + (p_{hj} - w_h - \gamma) D_{hj}(\mathbf{p}) \right] \right\}$$

.

Using the envelope theorem, we thus have:

$$\frac{\partial \pi_R}{\partial w_{hk}}(\mathbf{w}) = -q_{hk}^m(\mathbf{w}). \tag{24}$$

Consider now the negotiation between R and M_i, for $i \in \{A, B\}$. Given the wholesale price negotiated with the competing manufacturer, w_h^{IR}, R and M_i choose $w_{i1} = w_{i2} = w_i$ so as to maximize their joint profit, given by:

$$\begin{aligned} \pi_i + \pi_R\left(w_i, w_h^{IR}, w_i, w_h^{IR}\right) - F_h &= \sum_{j\in\{1,2\}} (w_i - c)\, q_{ij}^m\left(w_i, w_h^{IR}, w_i, w_h^{IR}\right) \\ &\quad + \pi_R\left(w_i, w_h^{IR}, w_i, w_h^{IR}\right) + F_k - F_h. \end{aligned}$$

Using (24), the first-order condition with respect to w_i yields:

$$\left(w_i^{IR} - c\right) \sum_{j\in\{1,2\}} \frac{\partial q_{ij}^m}{\partial w_i}\left(\mathbf{w}^{IR}\right) = 0.$$

Assumption F1 then ensures that, in any equilibrium in two-part tariffs, tariffs are cost-based, that is, $w_i^{IR} = c$ for any $i \in \{A, B\}$.[16]

If the negotiation between M_i and R were to break down, M_i would be excluded whereas R would keep selling M_h's product at both locations; the industry profit would thus be $2\hat{\pi}^{UF}$ rather than $4\pi^M$ where:

$$\hat{\pi}^{UF} \equiv \max_p (p - c - \gamma) D(p, \infty, p, \infty) \quad \text{and} \quad \pi^M \equiv \max_p (p - c - \gamma) D(p, p, p, p).$$

As manufacturers obtain zero profit in case of a negotiation breakdown, in equilibrium they obtain a share α of the bilateral gains from trade; as manufacturers obtain their profits through the fixed fees, and retailers are thus the residual claimants, these bilateral gains coincide with the increase in industry profit; hence, the equilibrium profits are:

$$\Pi_{M_A} = \Pi_{M_B} = \Pi_M^M \equiv 2\alpha\left(2\pi^M - \hat{\pi}^{UF}\right) \text{ and } \Pi_R = \Pi_R^M \equiv 4(1-\alpha)\pi^M + 4\alpha\left(\hat{\pi}^{UF} - \pi^M\right).$$

[16] More generally, under relatively mild assumptions, any post-merger bargaining equilibrium involves cost-based tariffs, but additional equilibria (based on non-linear tariffs other than two-part tariffs) can sustain different profit-sharing between the monopolistic retailer and the manufacturers.

F.1.2 Bilateral monopoly

If a single channel is active, say $i - j$, the merger does not affect the bargaining equilibrium. M_i's and R's profits are thus respectively given by:

$$\Pi_{M_i} = \Pi_M^m \equiv \alpha \pi^m \quad \text{and} \quad \Pi_R = \Pi_R^m \equiv (1-\alpha)\pi^m.$$

F.1.3 Downstream foreclosure

The merger does not affect the bargaining outcome either when R carries both brands but operates only one location. The equilibrium profits are then:

$$\Pi_{M_A} = \Pi_{M_B} = \Pi_M^{DF} \equiv \alpha \left(2\pi^{DF} - \pi^m\right) \text{ and } \Pi_R = \Pi_R^{DF} \equiv 2\left(1-\alpha\right)\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right).$$

F.1.4 Exclusive dealing

When two unconnected channels are active, say $i - j$ and $h - k$, the situation is similar to downstream foreclosure, except that the two active products are more differentiated. The equilibrium tariffs are therefore cost-based, and the industry profit is equal to $2\hat{\pi}^{ED}$, where:

$$\hat{\pi}^{ED} \equiv \max_p (p - c - \gamma) D(p, \infty, \infty, p),$$

and the equilibrium profits are:

$$\Pi_{M_i} = \Pi_{M_h} = \Pi_M^{ED} \equiv \alpha\left(2\hat{\pi}^{ED} - \pi^m\right) \text{ and } \Pi_R = \Pi_R^{ED} \equiv 2\left(1-\alpha\right)\hat{\pi}^{ED} + 2\alpha(\pi^m - \hat{\pi}^{ED}).$$

F.1.5 Upstream foreclosure

When a single manufacturer, say M_i, negotiates with R a tariff applying to both locations, the situation is the same as under bilateral monopoly, except that (total) demand for brand i is here equal to $\hat{D}_i(p) = 2D(p, \infty, p, \infty)$. The equilibrium tariffs are therefore cost-based, and the industry profit is equal to $2\hat{\pi}^{UF}$, and the equilibrium profits are:

$$\Pi_{M_i} = \Pi_M^{UF} \equiv 2\alpha\hat{\pi}^{UF} \quad \text{and} \quad \Pi_R = \Pi_R^{UF} \equiv 2\left(1-\alpha\right)\hat{\pi}^{UF}.$$

F.1.6 Single exclusion

Finally, suppose that a single channel, say $h - k$, remains inactive (and so the active channels are $i - 1$, $i - 2$ and $h - j$, for $i \neq h \in \{A, B\}$ and $j \neq k \in \{1, 2\}$). When negotiating over w_i, R and M_i seek to maximize their joint profit, given by:

$$\pi_i + \pi_R\left(w_i, w_h^{SE}, w_i, \infty\right) - F_h = \sum_{l \in \{1,2\}} \left(w_i - c\right) q_{il}^m \left(w_i, w_h^{SE}, w_i, \infty\right) + \pi_R\left(w_i, w_h^{SE}, w_i, \infty\right) - F_h.$$

Using (24), the first-order condition with respect to w_i yields again:

$$\left(w_i^{SE} - c\right) \sum_{l \in \{1,2\}} \frac{\partial q_{il}^m}{\partial w_i} \left(\mathbf{w}^{SE}\right) = 0.$$

Likewise, when negotiating over w_h, M_h and R seek to:

$$\pi_h + \pi_R \left(w_i^{SE}, w_h, w_i^{SE}, \infty\right) - F_i = \left(w_h - c\right) q_{hj}^m \left(w_i^{SE}, w_h, w_i^{SE}, \infty\right) + \pi_R \left(w_i^{SE}, w_h, w_i^{SE}, \infty\right) - F_i,$$

which, using again (24), leads to the first-order condition:

$$\left(w_h^{SE} - c\right) \frac{\partial q_{hj}^m}{\partial w_h} \left(\mathbf{w}^{SE}\right) = 0.$$

Assumption F1 then ensures that both tariffs are cost-based: $w_i^{SE} = w_h^{SE} = c$. The resulting total profit is then

$$\Pi^{SE} \equiv \pi_R \left(c, c, c, \infty\right).$$

As manufacturers obtain zero profit in case of a negotiation breakdown, in equilibrium they obtain a share α of the bilateral gains from trade, which coincide with their contribution to industry profit; hence their equilibrium profits are given by:

$$\Pi_{M_i} = \Pi_m^{SE} \equiv \alpha \left(\Pi^{SE} - \pi^m\right), \; \Pi_{M_h} = \Pi_s^{SE} \equiv \alpha \left(\Pi^{SE} - 2\pi^{UF}\right),$$

and R thus obtains:

$$\Pi_R = \Pi_R^{SE} \equiv (1-\alpha)\Pi^{SE} + \alpha \left(2\pi^{UF} + \pi^m - \Pi^{SE}\right).$$

F.1.7 Post-merger distribution network and impact on consumer surplus

• *Equilibrium distribution network.* As products are imperfect substitutes, we have:

$$4\pi^M > \Pi^{SE} > 2\hat{\pi}^{ED} > 2\max\{\pi^{DF}, \hat{\pi}^{UF}\} > 2\min\{\pi^{DF}, \hat{\pi}^{UF}\} > \pi^m.$$

Hence, a manufacturer is always strictly better off when R sells its brand at both locations rather than at a single location. For manufacturers, it is thus a (weakly) dominant strategy to activate both channels with R.

In addition, in our linear demand example, it can be checked that " interlocking relationships" is always R's preferred channel configuration. Combining these two observations, the unique coalition-proof distribution network involves interlocking relationships.

• *Impact on prices and consumer surplus.* Following a merger between the two retailers, the equilibrium distribution network always involves interlocking relationships and prices are at the industry-profit maximizing level. This implies that the merger harms consumers whenever the pre-merger equilibrium distribution network already involves interlocking relationships. The merger may however benefit consumers by expanding distribution network. This is for instance the case when retailers are close substitutes, so that exclusive dealing arises pre-merger, and brands do not compete (maximal differentiation between brands), as the two brands are then already sold at monopoly prices pre-merger; hence, in that case consumers are not affected by any price increase, but benefit post-merger from increased variety as they can find both brands at both locations.

For the linear demand specification, interlocking relationships arise pre-merger whenever $\rho < \rho^* (\mu)$, in which case the merger harms consumers and society, as it raises prices without any off-setting benefit in variety. When instead $\rho \geq \rho^* (\mu)$, the merger expands the distribution network, from exclusive dealing to interlocking relationships. In that case, there exist two thresholds $\hat{\mu}_S (\rho)$ and $\hat{\mu}_W (\rho)$ (where $\hat{\mu}_S (\rho)$ and $\hat{\mu}_W (\rho)$ are decreasing function of ρ such that $\hat{\mu}_S (1) = \hat{\mu}_W (1) = 0$ and $\hat{\mu}_S (\rho) < \hat{\mu}_W (\rho)$ for $\rho < 1$)[17] such that, despite price increases, by expanding the distribution network the downstream merger increases consumer surplus if and only if $\mu < \hat{\mu}_S (\rho)$, and increases total welfare if and only if $\mu < \hat{\mu}_W (\rho)$. Hence, the downstream merger benefits consumer if and only if: (i) retailers are close enough substitutes, namely, $\rho > \rho^* (\mu)$, so that the pre-merger distribution network involves exclusive dealing; and yet the combination of brand and retail differentiation yields prices that are so high that increasing them further to the monopoly level does not offset the benefit from expanding the network. These insights are illustrated by Figure 2.

F.2 Upstream Merger

A merger between the two manufacturers creates a new entity, M, which is a multi-brand monopolist dealing with competing distributors. For simplicity, we assume that M bundles the two brands and thus negotiates with each R_j, in addition to the wholesale prices w_{Aj} and w_{Bj}, a single fixed fee F_j. A bargaining equilibrium is then similar to a game with delegated negotiations where M uses two different agents simultaneously negotiating with the two retailers.[18] This analysis is also similar to that of upstream foreclosure (i.e., one manufacturer dealing with two retailers) except that the manufacturer sells here two differentiated products.

[17] The threshold $\hat{\mu}_S (\rho)$ is the unique solution in $[0,1]$ to:

$$2(1-\rho) - 2(1+\rho)\mu - 3\rho^2\mu^2 + \rho^3\mu^3 = 0;$$

whereas the threshold $\hat{\mu}_W (\rho)$ is the unique solution in $[0,1]$ to:

$$6(1-\rho) - 2\left(3+\rho-2\rho^2\right)\mu + (4-5\rho)\mu^2 + 3\rho^3\mu^3 = 0.$$

[18] In the absence of bundling, the relevant delegated negotiation game would have M using four different agents, one for each distribution channel.

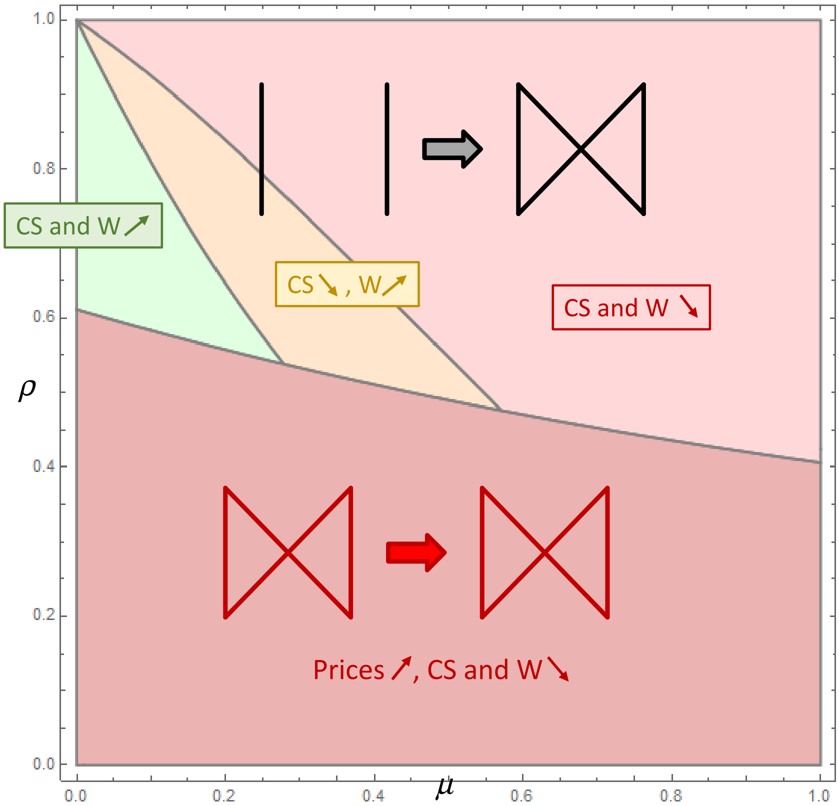

Figure 2: Impact of a downstream merger

We first study the bargaining equilibria for each possible distribution network, and show that equilibrium tariffs remain again cost-based. We then characterize the equilibrium fixed fees, and show that they are unique. Finally, we solve for the CPNE of the distribution network formation game, which we compare with the pre-merger outcome.

Throughout this section, we assume that the retail equilibrium outcome is uniquely defined and "well-behaved." Specifically, for any $j \in \{1,2\}$, any wholesale prices $\mathbf{w}_j = (w_{Aj}, w_{Bj})$ and any retail prices $\mathbf{p} = (p_{ij})_{i\in\{A,B\},j\in\{1,2\}}$, let

$$\tilde{\pi}_j(\mathbf{p}; \mathbf{w}_j) \equiv \sum_{i\in\{A,B\}} (p_{ij} - w_{ij} - \gamma) D_{ij}(\mathbf{p})$$

denote R_j's retail profit (gross of fixed fees), with the convention that $w_{ij} = p_{ij} = \infty$ and $D_{ij} = 0$ if R_j does not carry brand i; and, for any $j \neq k \in \{1,2\}$, any wholesale prices $\mathbf{w}_j = (w_{Aj}, w_{Bj})$ and any rival's retail price $\mathbf{p}_k = (p_{Ak}, p_{Bk})$, let

$$\tilde{p}^r_j(\mathbf{p}_k, \mathbf{w}_j) \equiv \arg\max_{\mathbf{p}_j} \tilde{\pi}_j(\mathbf{p}_j, \mathbf{p}_k; \mathbf{w}_j)$$

denote R_j's best-response to its rivals' prices $\mathbf{p}_{-j}$. We also maintain the following adaptation of Assumption A and condition (3):

Assumption F2. For any vector of wholesale prices $\mathbf{w}$ and any vector of retail prices

$\mathbf{p}_k$ (with $k \neq j \in \{1,2\}$), the best-response function $\widetilde{\mathbf{p}}_j^r(\mathbf{p}_k, \mathbf{w}_j)$ is uniquely characterized by the first-order conditions for any $j \in \{1,2\}$. In addition, the best-response functions satisfy:

(*i*) $\widetilde{\mathbf{p}}_j^r(\mathbf{p}_k; c, c) = \mathbf{p}_j^r(\mathbf{p}_k)$, defined in Section 2.3;

(*ii*) for any vector of wholesale prices $\mathbf{w}_j$ and vector of retail prices $\mathbf{p}_k$, and for any $i \neq h \in \{A, B\}$:

$$\frac{\partial \tilde{p}_{ij}^r}{\partial w_{ij}}(\mathbf{p}_k, \mathbf{w}_j) > \frac{\partial \tilde{p}_{hj}^r}{\partial w_{ij}}(\mathbf{p}_k, \mathbf{w}_j) \geq 0.$$

F.2.1 Interlocking relationships

We start with interlocking relationships, where each retailer carries both brands. Consider the negotiation between M and R_j, for $j \in \{1,2\}$. If they agree on wholesale prices $\mathbf{w}_j = (w_{Aj}, w_{Bj})$, in the last stage R_j expects to obtain

$$\tilde{\pi}_j\left(\tilde{\mathbf{p}}_j^r\left(\mathbf{p}_k^{IR}, \mathbf{w}_j\right), \mathbf{p}_k^{IR}; \mathbf{w}_j\right) = \max_{\mathbf{p}_j} \tilde{\pi}_j\left(\mathbf{p}_j, \mathbf{p}_k^{IR}; \mathbf{w}_j\right),$$

which, using the envelope theorem, satisfies:

$$\frac{\partial}{\partial w_{ij}}\left\{\tilde{\pi}_j\left(\tilde{\mathbf{p}}_j^r\left(\mathbf{p}_k^{IR}, \mathbf{w}_j\right), \mathbf{p}_k^{IR}; \mathbf{w}_j\right)\right\} = -D_{ij}\left(\tilde{\mathbf{p}}_j^r\left(\mathbf{p}_k^{IR}, \mathbf{w}_j\right), \mathbf{p}_k^{IR}\right). \tag{25}$$

Given the wholesale prices negotiated by M with R_k, $\mathbf{w}_k^{IR} = \left(w_{Ak}^{IR}, w_{Bk}^{IR}\right)$, M and R_j choose $\mathbf{w}_j = (w_{Aj}, w_{Bj})$ so as to maximize their joint profit, given by:

$$\mathbf{w}_j^{IR} = \arg\max_{\mathbf{w}_j} \left\{\tilde{\pi}_j\left(\tilde{\mathbf{p}}_j^r\left(\mathbf{p}_k^{IR}, \mathbf{w}_j\right), \mathbf{p}_k^{IR}; \mathbf{w}_j\right) + \sum_{h\in\{A,B\}} (w_{hj} - c) D_{hj}\left(\tilde{\mathbf{p}}_j^r\left(\mathbf{p}_k^{IR}, \mathbf{w}_j\right), \mathbf{p}_k^{IR}\right) \right.$$
$$\left. + \sum_{h\in\{A,B\}} \left(w_{hk}^{IR} - c\right) D_{hk}\left(\tilde{\mathbf{p}}_j^r\left(\mathbf{p}_k^{IR}, \mathbf{w}_j\right), \mathbf{p}_k^{IR}\right)\right\}$$

Using (25), the first-order condition with respect to w_{ij} yields:

$$\sum_{h\in\{A,B\}} \left(w_{hj}^{IR} - c\right) \sum_{g\in\{A,B\}} \frac{\partial D_{hj}}{\partial p_{gj}} \frac{\partial \tilde{p}_{gj}^r}{\partial w_{ij}} + \sum_{h\in\{A,B\}} \left(w_{hk}^{IR} - c\right) \sum_{g\in\{A,B\}} \frac{\partial D_{hk}}{\partial p_{gj}} \frac{\partial \tilde{p}_{gj}^r}{\partial w_{ij}} = 0.$$

Assumption F2 then ensures that wholesale prices are at cost: $w_{ij}^{IR} = c$ for every $i \in \{A, B\}$ and $j \in \{1,2\}$.

If the negotiation between M and R_j were to break down, R_j would be excluded whereas M would still obtain F_k from R_k; in equilibrium, R_j thus obtains a share $1 - \alpha$ of its contribution to the bilateral joint profit, which is equal to $2\pi^*$.[19] It follows that the

[19] The joint bilateral profit of M and R_j is equal to $2\pi^* + F_k$ if they reach an agreement and to F_k otherwise.

equilibrium profits are:

$$\Pi_M = \widetilde{\Pi}_M^{IR} \equiv 4\alpha\pi^* \quad \text{and} \quad \Pi_{R_1} = \Pi_{R_2} = \widetilde{\Pi}_R^{IR} \equiv 2(1-\alpha)\pi^*.$$

F.2.2 Bilateral monopoly

If a single channel is active, say $i{-}j$, the merger does not affect the bargaining equilibrium. M's and R_j's profits are thus, respectively:

$$\Pi_M = \widetilde{\Pi}_M^{m} \equiv \alpha\pi^m \quad \text{and} \quad \Pi_{R_j} = \widetilde{\Pi}_R^{m} \equiv (1-\alpha)\pi^m.$$

F.2.3 Upstream foreclosure

When the manufacturer sells only one brand, say brand i, the merger does not affect the bargaining equilibrium. The manufacturer and the retailers' profits are thus, respectively:

$$\Pi_M = \widetilde{\Pi}_M^{UF} \equiv 2\alpha\pi^{UF} \quad \text{and} \quad \Pi_{R_1} = \Pi_{R_2} = \widetilde{\Pi}_R^{UF} \equiv (1-\alpha)\pi^{UF}.$$

F.2.4 Downstream foreclosure

When M negotiates with a single retailer, say R_j, over both brands, the parties aim to maximize their joint profit and this is again achieved by negotiating a cost-based tariff, i.e., $w_{ij}^e = w_{hj}^e = c$. Industry profit is then equal to $2\pi^{DF}$ and this profit is divided between M and R_j according to the $(\alpha, 1-\alpha)$ bargaining rule as profits would be equal to 0 were the negotiation to fail. The individual profits are therefore:

$$\Pi_M = \widetilde{\Pi}_M^{DF} \equiv 2\alpha\pi^{DF} \quad \text{and} \quad \Pi_{R_j} = \widetilde{\Pi}_R^{DF} \equiv 2(1-\alpha)\pi^{DF}.$$

F.2.5 Exclusive dealing

When two unconnected channels are active, say $i-j$ and $h-k$, the situation is similar to upstream foreclosure, except that the two active products are now more differentiated. Equilibrium tariffs are therefore cost-based and the industry profit is equal to $2\pi^{ED}$. Individual profits are then:

$$\Pi_M = \widetilde{\Pi}_M^{ED} \equiv 2\alpha\pi^{ED} \quad \text{and} \quad \Pi_{R_1} = \Pi_{R_2} = \widetilde{\Pi}_R^{ED} \equiv (1-\alpha)\pi^{ED}.$$

F.2.6 Single exclusion

Finally, suppose that only one channel, say $h-k$, remains inactive. In the negotiation between M and R_j, the parties take the wholesale price negotiated by M with R_k, w_{ik}^{SE}, as given. They thus choose:

$$\mathbf{w}_j^{SE} = \arg\max_{\mathbf{w}_j} \left\{ \begin{array}{c} \tilde{\pi}_j\left(\widetilde{\mathbf{p}}_j^r\left(p_{ik}^{SE}, \infty; \mathbf{w}_j\right), p_{ik}^{SE}, \infty; \mathbf{w}_j\right) + \left(w_{ik}^{SE} - c\right) D_{ik}\left(\widetilde{\mathbf{p}}_j^r\left(p_{ik}^{SE}, \infty; \mathbf{w}_j\right), p_{ik}^{SE}, \infty\right) \\ + \sum_{g \in \{A,B\}} \left(w_{gj} - c\right) D_{gj}\left(\widetilde{\mathbf{p}}_j^r\left(p_{ik}^{SE}, \infty; \mathbf{w}_j\right), p_{ik}^{SE}, \infty\right) \end{array} \right\}$$

Applying the envelope theorem to R_j's retail pricing program yields:

$$\frac{\partial}{\partial w_{ij}}\left\{\tilde{\pi}_j\left(\tilde{\mathbf{p}}_j^r\left(p_{ik}^{SE},\infty;\mathbf{w}_j\right),p_{ik}^{SE},\infty;\mathbf{w}_j\right)\right\}=-D_{ij}\left(\tilde{\mathbf{p}}_j^r\left(p_{ik}^{SE},\infty;\mathbf{w}_j\right),p_{ik}^{SE},\infty\right).$$

Using this, the first-order condition of the above joint profit maximization program with respect to $w_{\hat{\imath}j}$, for $\hat{g}=A,B$, yields:

$$\sum_{g\in\{A,B\}}\left(w_{gj}^{SE}-c\right)\sum_{\tilde{g}\in\{A,B\}}\frac{\partial D_{gj}}{\partial p_{\tilde{g}j}}\frac{\partial\tilde{p}_{\tilde{g}j}^r}{\partial w_{\hat{g}j}}+\left(w_{ik}^{SE}-c\right)\sum_{\tilde{g}\in\{A,B\}}\frac{\partial D_{ik}}{\partial p_{\tilde{g}j}}\frac{\partial\tilde{p}_{\tilde{g}j}^r}{\partial w_{\hat{g}j}}=0. \tag{26}$$

Similarly, in the negotiation between M and R_k, the parties that the wholesale prices negotiated by M with R_j, $\mathbf{w}_j^{SE}=(w_{ij}^{SE},w_{hj}^{SE})$, as given. They thus choose:

$$w_{ik}^{SE}=\arg\max_{w_{ik}}\left\{\begin{array}{c}\tilde{\pi}_k\left(\tilde{p}_{ik}^r\left(\mathbf{p}_j^{SE},w_{ik}\right),\infty,\mathbf{p}_j^{SE};w_{ik}\right)+\left(w_{ik}-c\right)D_{ik}\left(\tilde{p}_{ik}^r\left(\mathbf{p}_j^{SE},w_{ik}\right),\infty,\mathbf{p}_j^{SE}\right)\\+\sum_{g\in\{A,B\}}\left(w_{gj}^{SE}-c\right)D_{gj}\left(\tilde{p}_{ik}^r\left(\mathbf{p}_j^{SE},w_{ik}\right),\infty,\mathbf{p}_j^{SE}\right)\end{array}\right\}$$

Using the envelope theorem for R_k's retail pricing program, the first-order condition of this joint profit maximization program yields:

$$\left[\sum_{g\in\{A,B\}}\left(w_{gj}^{SE}-c\right)\frac{\partial D_{gj}}{\partial p_{ik}}+\left(w_{ik}^{SE}-c\right)\frac{\partial D_{ik}}{\partial p_{ik}}\right]\frac{\partial\tilde{p}_{ik}^r}{\partial w_{ik}}=0. \tag{27}$$

Conditions (26) and (27) are satisfied by $w_{ij}^{SE}=w_{hj}^{SE}=w_{ik}^{SE}=c$. Assumption F2($ii$), combined with imperfect substitution between brands and retailers, ensures that this is the only equilibrium.

Equilibrium tariffs being cost-based, the equilibrium fixed fee negotiated between M and R_l, for any $l\in\{1,2\}$, is equal to $F_l=\alpha\Delta_{R_l}$. It follows that

$$F_j^{SE}=\alpha\pi_m^{SE}\quad\text{and}\quad F_k^{SE}=\alpha\pi_s^{SE}.$$

Hence, M obtains a share α of the industry profit

$$\Pi_M=\widetilde{\Pi}_M^{SE}\equiv\alpha\left(\pi_m^{SE}+\pi_s^{SE}\right)$$

and retailers obtain:

$$\Pi_{R_j}=\widetilde{\Pi}_{R_m}^{SE}\equiv(1-\alpha)\,\pi_m^{SE}\quad\text{and}\quad\Pi_{R_k}=\widetilde{\Pi}_{R_s}^{SE}\equiv(1-\alpha)\,\pi_s^{SE}.$$

F.2.7 Post-merger distribution network and impact on consumer surplus

• *Equilibrium distribution network.* For all distribution networks, M's profits are proportional to α and retailers' profits are always proportional to $1-\alpha$; this is because M generates its profits exclusively through fixed fees and a retailer is excluded if its negotiation with M fails. Hence, the decisions about activating or not a channel are independent of α.

Furthermore, imperfect substitutability implies that $2\pi^{DF} > \pi^{m}$ and $\pi^{ED} > \pi^{UF}$. Therefore, bilateral monopoly and upstream foreclosure cannot arise in a CPNE: a coalition formed by M and the active retailer would find it profitable to activate their second channel in the first case, and the grand coalition would rather switch to exclusive dealing in the second. In addition, for any strategy chosen by its rival, a retailer always prefers activating both channels (whenever possible) rather than a single one, and activating any channel rather than being excluded. Therefore, in a CPNE the distribution network must be M's preferred one: otherwise, M could profitably deviate, either by unilaterally closing some of the channels, or by forming a coalition with one or both retailer(s) to induce a "switch" to its preferred distribution network.

M always gets a share α of the industry profit; hence, bilateral monopoly and upstream foreclosure can never be its preferred distribution network, and we only need to compare the industry profit with interlocking relationships ($4\pi^{*}$), exclusive dealing $\left(2\pi^{ED}\right)$, downstream foreclosure $\left(2\pi^{DF}\right)$ and single exclusion $\left(\pi_{m}^{SE}+\pi_{s}^{SE}\right)$. In our linear setting, single exclusion always yields a lower profit than either interlocking relationships (if $\rho < \rho^{*}(0)$) or downstream foreclosure (if $\rho > \rho^{*}(0)$). Comparing the remaining profits, there exist two thresholds[20] $\tilde{\rho}(\mu)$ and $\bar{\rho}(\mu)$ that are decreasing functions of μ such that $\rho^{*}(\mu) < \tilde{\rho}(\mu) < \bar{\rho}(\mu)$ for $\mu \in]0,1]$, $\tilde{\rho}(0) = \rho^{*}(0)$ and $\bar{\rho}(0) = 1$, such that the CPNE of the network formation game involves:[21]

- interlocking relationships if and only if $\rho \leq \tilde{\rho}(\mu)$;
- exclusive dealing if and only if $\tilde{\rho}(\mu) \leq \rho < \bar{\rho}(\mu)$;
- downstream foreclosure if and only if $\rho \geq \bar{\rho}(\mu)$.

• *Impact on prices and consumer surplus and total welfare.* Keeping the distribution network constant, an upstream merger has no impact on wholesale and retail prices, and thus does not affect consumers or total welfare. However, the merger may well alter the equilibrium distribution network and therefore have an impact on variety and prices, and thus on consumer surplus and welfare.

- When $\rho \geq \bar{\rho}(\mu)$ (i.e., retailers are close substitutes), M prefers selling both brands to a single common retailer (downstream foreclosure) rather than selling each one

[20] The threshold $\tilde{\rho}(\mu)$ is the unique solution in $[0,1]$ to

$$4(1-\mu)-4\left(2-\mu-\mu^{2}\right)\rho+3\left(1+\mu-2\mu^{2}\right)\rho^{2}-\left(1+4\mu-3\mu^{2}-2\mu^{3}\right)\rho^{3}+\mu\left(1+\mu-2\mu^{2}\right)\rho^{4}=0,$$

whereas $\bar{\rho}(\mu)$ is the unique solution in $[0,1]$ to: $4-4(1+\mu)\rho+3\mu\rho^{2}-\mu^{2}\rho^{3}=0$.

[21] For $\rho = \tilde{\rho}(\mu)$, two CPNE co-exist: the manufacturers and the retailers are all indifferent between interlocking relationships and exclusive dealing. But for $\rho = \bar{\rho}(\mu)$, if M is indifferent between exclusive dealing and downstream foreclosure, the coalition formed by M and R_j can profitably (in the sense of Pareto-dominance) deviate to downstream foreclosure as this network is strictly preferred by R_j.

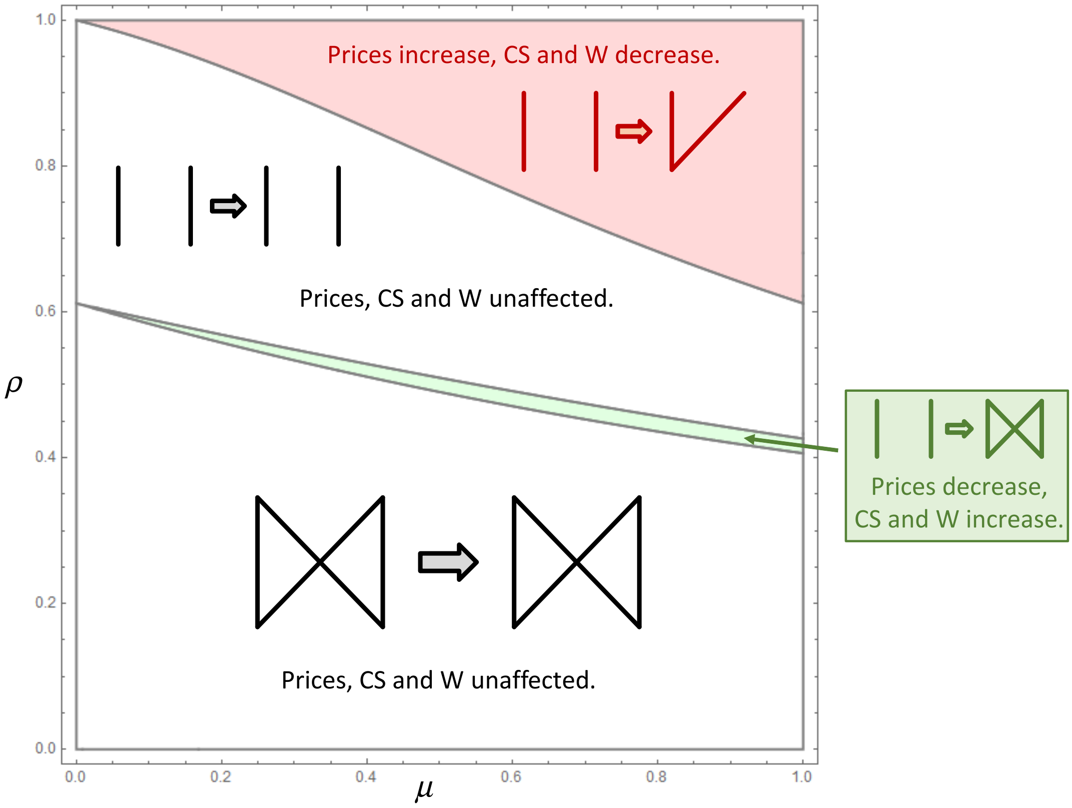

Figure 3: Impact of an upstream merger

of them to a different retailer (exclusive dealing); in that case, the merger does not really affect variety (two channels are available pre- as well as post-merger, which involve distinct brands and either the same retailer or two different retailers; because of symmetry, keeping prices constant this would have no impact on consumers or profits), but raises prices by avoiding downstream competition. Hence, it reduces consumer surplus and total welfare.

- When $\rho^*(\mu) \leq \rho < \tilde{\rho}(\mu)$, M extends the distribution network and opts for interlocking relationships instead of exclusive dealing, which increases consumer surplus and total welfare by both increasing variety and decreasing prices.

Hence, for the linear demand specification, the merger may either be consumer surplus and welfare-neutral (network is unaffected), consumer surplus and welfare increasing (for a small set of parameter with intermediate degree of substitution between retailers), or consumer surplus and welfare-decreasing (when retailers are good substitutes). These insights are illustrated by Figure 3.

F.3 Vertical Merger

A vertical merger between one manufacturer, say M_i, and one retailer, say R_j, creates a vertically integrated firm, I, dealing with an independent (i.e., non-integrated) manufacturer, M_h, and an independent retailer, R_k. We focus in what follows on bargaining equilibria based on two-part tariffs to ensure that firms' profits continue to be properly defined. Moreover, when considering the possible distribution networks, we assume that, by default, the vertically integrated channel $i-j$ is always active. We thus need to consider eight possible distribution networks:

- Bilateral monopoly: only channel $i-j$ is active (m).
- Exclusive dealing: only channels $i-j$ and $h-k$ are active (ED).
- Downstream foreclosure: only channels $i-j$ and $h-j$ are active (DF).
- Upstream foreclosure: only channels $i-j$ and $i-k$ are active (UF).
- Single exclusion: all channels but $h-k$ are active (hk).
- Single exclusion: all channels but $i-k$ are active (ik).
- Single exclusion: all channels but $h-j$ are active (hj).
- Interlocking relationships: all channels are active (IR).

By construction, the integrated manufacturer supplies its downstream subsidiary at cost: $w_{ij}=c$. We moreover show below that the independent manufacturer, M_h, always offers cost-based tariffs to every available partner. Hence, at most one wholesale price, w_{ik}, may vary. For the sake of exposition, in what follows we assume that the continuation retail equilibrium has always a unique outcome and is "well-behaved". Specifically, for any wholesale prices $\mathbf{w}=(w_{ik},w_{hj},w_{hk})$, with the convention that $w_{ik}=\infty$ if the channel $i-k$ is inactive, and that

$$w_{hl}=\begin{cases} c & \text{if the channel } h-l \text{ is active,} \\ \infty & \text{otherwise,} \end{cases}$$

and for any retail prices $\mathbf{p}=(\mathbf{p}_j,\mathbf{p}_k)$, with the convention that $p_{gl}=\infty$ if the channel $g-l$ is inactive, for $g\in\{A,B\}$ and $l\in\{1,2\}$, let

$$\pi_I(\mathbf{p};\mathbf{w})\equiv\sum_{g\in\{A,B\}}(p_{gj}-c-\gamma)\,D_{gj}(\mathbf{p})+(w_{ik}-c)\,D_{ik}(\mathbf{p}),$$

denote the vertically integrated firm's profit (gross of fixed fees), and

$$\pi_R(\mathbf{p};\mathbf{w})\equiv(p_{ik}-w_{ik}-\gamma)\,D_{ik}(\mathbf{p})+(p_{hk}-c-\gamma)\,D_{hk}(\mathbf{p})$$

denote the independent retailer's profit (again gross of fixed fees), with the convention that the term corresponding to any given channel $g-l$ is zero if that channel is inactive.

For any $g \in \{A, B\}$, the independent R_k's conditional price response $\hat{\mathbf{p}}_k^{gk}(q_{gk})$ and revenue function $\hat{r}_k^{gk}(q_{gk})$ are defined as before (see section 3.2); that is, given the equilibrium tariffs $\mathbf{w}^e$ and retail prices $\mathbf{p}^{\mathbf{e}}$:

$$\hat{\mathbf{p}}_k^{gk}(q_{gk}) \equiv \underset{\{\mathbf{p}_k | D_{gk}(\mathbf{p}_k, \mathbf{p}_j^{\mathbf{e}}) = q_{gk}\}}{\arg\max} \sum_{m \in \{A,B\}} (p_{mk} - w_{mk}^e - \gamma) D_{mk}(\mathbf{p}_k, \mathbf{p}_j^{\mathbf{e}})$$

and

$$\hat{r}_k^{gk}(q_{gk}) = \sum_{m \in \{A,B\}} (\hat{p}_{mk}^{gk}(q_{gk}) - \gamma) \hat{q}_{mk}^{gk}(q_{gk}),$$

where

$$\hat{q}_{mk}^{gk}(q_{gk}) \equiv D_{mk}\left(\hat{\mathbf{p}}_k^{gk}(q_{gk}), \mathbf{p}_j^{\mathbf{e}}\right).$$

By contrast, when negotiating with the independent M_h, the integrated R_j now takes into account the upstream margin on the sales of brand i through R_k. As it is supplied at cost by M_i, its conditional price response $\hat{\mathbf{p}}_j^{hj}(q_{hj})$ and the revenue function $\hat{r}_j^{hj}(q_{hj})$ are therefore modified and given by:

$$\hat{\mathbf{p}}_j^{hj}(q_{hj}) \equiv \underset{\{\mathbf{p}_j | D_{hj}(\mathbf{p}_j, \mathbf{p}_k^{\mathbf{e}}) = q_{hj}\}}{\arg\max} \left\{ \begin{array}{c} (p_{ij} - c - \gamma) D_{ij}(\mathbf{p}_j, \mathbf{p}_k^{\mathbf{e}}) + \left(p_{hj} - w_{hj}^e - \gamma\right) D_{hj}(\mathbf{p}_j, \mathbf{p}_k^{\mathbf{e}}) \\ + (w_{ik}^e - c) D_{ik}(\mathbf{p}_j, \mathbf{p}_k^{\mathbf{e}}) \end{array} \right\}$$

and

$$\hat{r}_j^{hj}(q_{hj}) \equiv \sum_{m \in \{A,B\}} \left[(\hat{p}_{mj}^{hj}(q_{hj}) - \gamma) \hat{q}_{mj}^{hj}(q_{hj}) \right] + (w_{ik}^e - c) \hat{q}_{ik}^{hj}(q_{hj}),$$

where, for $m \in \{A, B\}$:

$$\hat{q}_{mj}^{hj}(q_{hj}) \equiv D_{mj}\left(\hat{\mathbf{p}}_j^{hj}(q_{hj}), \mathbf{p}_k^{\mathbf{e}}\right).$$

To ensure the existence of a smooth retail behavior, we adapt the assumptions of the baseline model as follows:

Assumption F3.

(i) For any wholesale prices $\mathbf{w} = (w_{ik}, w_{hj}, w_{hk}) \in \mathbb{R} \times \{c, \infty\}^2$, there exists a unique retail price equilibrium $\mathbf{p}^R(\mathbf{w}) = (\mathbf{p}_j^R(\mathbf{w}), \mathbf{p}_k^R(\mathbf{w}))$, with the convention that $p_{gl}^R = \infty$ if channel $g - l$ is inactive, which:

(a) is uniquely characterized by the first-order conditions of the programs

$$\mathbf{p}_j^R(\mathbf{w}) = \arg\max_{\mathbf{p}_j} \pi_I\left(\mathbf{p}_j, \mathbf{p}_k^R(\mathbf{w}); \mathbf{w}\right) \text{ and } \mathbf{p}_k^R(\mathbf{w}) = \arg\max_{\mathbf{p}_k} \pi_R\left(\mathbf{p}_j^R(\mathbf{w}), \mathbf{p}_k; \mathbf{w}\right);$$

(b) increases with the wholesale price w_{ik}, that is, for any $g \in \{A, B\}$ and any $l \in \{1, 2\}$:

$$\frac{\partial p_{gl}^R}{\partial w_{ik}} > 0;$$

(ii) For the channels involving M_h, the conditional price responses, $\hat{\mathbf{p}}_1^{h1}(q_{h1})$ and $\hat{\mathbf{p}}_2^{h2}(q_{h2})$ are both unique and differentiable, the diversion ratios, δ_{12}^{h1} and δ_{21}^{h2} satisfy

$$0 \leq \delta_{12}^{h1}, \delta_{21}^{h2} < 1,$$

and the equilibrium quantities satisfy $q_{h1}^e \in Int\,(Q_{h1})$ and $q_{h2}^e \in Int\,(Q_{h2})$.

Finally, we respectively denote by

$$\pi_I^R(\mathbf{w}) \equiv \pi_I\left(\mathbf{p}^R(\mathbf{w});\mathbf{w}\right) \text{ and } \pi_R^R(\mathbf{w}) \equiv \pi_R\left(\mathbf{p}^R(\mathbf{w});\mathbf{w}\right)$$

the profits (gross of fixed fees) of the vertically integrated firm and of the independent retailer in the retail price equilibrium $\mathbf{p}^R(\mathbf{w})$, for $\mathbf{w} = (w_{ik}, w_{hj}, w_{hk}) \in \mathbb{R} \times \{c, \infty\}^2$.

F.3.1 Bargaining equilibria

No relationship between I and R_k When the vertically integrated firm does not supply its rival retailer, the merger has no impact, neither on the equilibrium outcome nor on the off-equilibrium outcomes in case of a negotiation break-down: the integrated firm always supplies its own subsidiary at cost, and the previous reasoning can be used to check that the independent manufacturer always offers cost-based tariffs to any available partner. The profits of the independent firms are therefore the same as pre-merger, and the profit of the integrated firm is simply the sum of the profits that its subsidiaries would obtain pre-merger. Hence, we have:

- Bilateral monopoly:

$$\hat{\Pi}_I^m = \pi^m,\ \hat{\Pi}_M^m = \alpha\pi^m \text{ and } \hat{\Pi}_R^m = (1-\alpha)\,\pi^m.$$

- Exclusive dealing:

$$\hat{\Pi}_I^{ED} = \pi^{ED},\ \hat{\Pi}_M^{ED} = \alpha\pi^{ED} \text{ and } \hat{\Pi}_R^{ED} = (1-\alpha)\,\pi^{ED}.$$

- Downstream foreclosure:

$$\hat{\Pi}_I^{DF} = 2\,(1-\alpha)\,\pi^{DF} + \alpha\pi^m,\ \hat{\Pi}_M^{DF} = \alpha\left(2\pi^{DF} - \pi^m\right) \text{ and } \hat{\Pi}_R^{DF} = 0.$$

- Single exclusion of channel $i-k$:

$$\hat{\Pi}_I^{ik} = (1-\alpha)\,\pi_m^{SE} + \alpha\hat{\pi}_R,\ \hat{\Pi}_M^{ik} = \alpha\left(\pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R\right) \text{ and } \hat{\Pi}_R^{ik} = (1-\alpha)\,\pi_s^{SE}.$$

Interlocking relationships

Consider now the case where all channels are active.

• *The independent manufacturer's tariffs are cost-based:* $w_{hj}^{IR} = w_{hk}^{IR} = c$.

Consider first the negotiation between the two independent firms, M_h and R_k. In this negotiation, the two firms take the other three equilibrium contracts and the equilibrium prices set by the vertically integrated retailer R_j as given. Choosing a wholesale price w_{hk} is thus equivalent, for the pair $M_h - R_k$, to choosing a quantity q_{hk} sold by R_k at the retail competition stage, taking into account that R_k will choose the prices $\hat{\mathbf{p}}_k^{hk}(q_{hk})$ maximizing its profit. The equilibrium quantity q_{hk}^{IR} thus maximizes:

$$\hat{r}_k^{hk}\left(q_{hk}\right) - cq_{hk} - w_{ik}^{IR}\hat{q}_{ik}^{hk}\left(q_{hk}\right) + \left(w_{hj}^{IR} - c\right)\hat{q}_{hj}^{hk}\left(q_{hk}\right).$$

In addition, when choosing its retail prices (i.e., choosing q_{hk} given $\hat{\mathbf{p}}_k^{hk}\left(q_{hk}\right)$), R_k maximizes its own profit, that is:

$$\hat{r}_k^{hk}\left(q_{hk}\right) - w_{hk}q_{hk} - w_{ik}^{IR}\hat{q}_{ik}^{hk}\left(q_{hk}\right).$$

From Assumption F3(ii), the equilibrium quantity q_{hk}^{IR} must satisfy the first-order conditions associated with these two optimization programmes; we thus have:

$$\begin{aligned} \left(\hat{r}_k^{hk}\right)'\left(q_{hk}\right) &= c + w_{ik}^{IR}\left(\hat{q}_{ik}^{hk}\right)'\left(q_{hk}\right) + \left(w_{hj}^{IR} - c\right)\delta_{kj}^{hk} \\ \left(\hat{r}_k^{hk}\right)'\left(q_{hk}\right) &= w_{hk} + w_{ik}^{IR}\left(\hat{q}_{ik}^{hk}\right)'\left(q_{hk}\right). \end{aligned}$$

Combining these conditions yields, using as before the notation $u_{gl}^{IR} = w_{gl}^{IR} - c$ for the upstream margin on channel $g - l$, for $g \in \{A, B\}$ and $l \in \{1, 2\}$:

$$u_{hk}^{IR} = \delta_{kj}^{hk} u_{hj}^{IR}. \tag{28}$$

Consider now the negotiation between M_h and I (for the sales of brand h through R_j). When choosing its retail prices or quantities, I now takes into account the margin u_{ik}^{IR} earned on its sales to R_k. Therefore, choosing a wholesale price w_{hj} is thus equivalent, for the pair $M_h - I$, to choosing a quantity q_{hj} sold through R_j at the retail competition stage, taking into account that I will choose the prices $\hat{\mathbf{p}}_j^{hj}(q_{hj})$ maximizing its profit. The equilibrium quantity q_{hj}^{IR} thus maximizes:[22]

$$\hat{r}_j^{hj}\left(q_{hj}\right) - c\hat{q}_{ij}^{hj}\left(q_{hj}\right) - cq_{hj} + \left(w_{hk}^{IR} - c\right)\hat{q}_{hk}^{hj}\left(q_{hj}\right).$$

In addition, when choosing its retail prices (i.e., choosing the quantity q_{hj} given $\hat{\mathbf{p}}_j^{hj}\left(q_{hj}\right)$),

[22] Recall that the revenue $\hat{r}_j^{hj}\left(q_{hj}\right)$ takes into account the margin u_{ik}^{e} earned by I on M_i's sales to R_k, given by $\hat{q}_{ik}^{hj}(q_{hj})$.

I maximizes:

$$\hat{r}_j^{hj}\left(q_{hj}\right)-c\hat{q}_{ij}^{hj}\left(q_{hj}\right)-w_{hj}q_{hj}.$$

From Assumption F3(ii), the equilibrium quantity q_{hj}^{IR} satisfies both associated first-order conditions, and combining them yields:

$$u_{hj}^{IR}=\delta_{jk}^{hj}u_{hk}^{IR} \tag{29}$$

From Assumption F3(ii), the diversion ratios are positive and lower than 1; hence, conditions (28) and (29) yield a unique solution: $u_{hj}^{IR}=u_{hk}^{IR}=0$; that is, the tariffs negotiated between M_h and I and between M_h and R_k continue to be cost-based in equilibrium.

- *The tariff negotiated between the integrated firm and the independent retailer exhibits a positive margin:* $u_{ik}^{IR}>0$. Consider now the negotiation between the integrated firm I and the independent retailer R_k over the tariff t_{ij}. Although only one wholesale price, w_{ik}, varies in that negotiation (I and R_j taking all other equilibrium tariffs as given), any change in that wholesale price affects all retail prices in the continuation game, as the integrated R_j now "observes" the tariff before setting its prices and thus reacts to it. The profits of the integrated firm I and of the independent retailer R_k (gross of fixed fees) are thus respectively given by $\pi_I^R\left(w_{ik},c,c\right)$ and $\pi_R^R\left(w_{ik},c,c\right)$. Using the envelope theorem, we have:

$$\begin{aligned}
\left.\frac{\partial\pi_I^R}{\partial w_{ik}}\left(\mathbf{w}\right)\right|_{\mathbf{w}=\left(w_{ik},c,c\right)} &= D_{ik}\left(\mathbf{p}^R\left(\mathbf{w}\right)\right)\\
&\quad+\sum_{g\in\{A,B\}}\left(p_{gj}-c-\gamma\right)\sum_{m\in\{A,B\}}\frac{\partial D_{gj}}{\partial p_{mk}}\left(\mathbf{p}^R\left(\mathbf{w}\right)\right)\frac{\partial p_{mk}^R}{\partial w_{ik}}\left(\mathbf{w}\right)\\
&\quad+\left(w_{ik}-c\right)\sum_{m\in\{A,B\}}\frac{\partial D_{ik}}{\partial p_{mk}}\left(\mathbf{p}^R\left(\mathbf{w}\right)\right)\frac{\partial p_{mk}^R}{\partial w_{ik}}\left(\mathbf{w}\right),\\
\left.\frac{\partial\pi_R^R}{\partial w_{ik}}\left(\mathbf{w}\right)\right|_{\mathbf{w}=\left(w_{ik},c,c\right)} &= -D_{ik}\left(\mathbf{p}^R\left(\mathbf{w}\right)\right)\\
&\quad+\left(p_{ik}-w_{ik}-\gamma\right)\sum_{m\in\{A,B\}}\frac{\partial D_{ik}}{\partial p_{mj}}\left(\mathbf{p}^R\left(\mathbf{w}\right)\right)\frac{\partial p_{mj}^R}{\partial w_{ik}}\left(\mathbf{w}\right)\\
&\quad+\left(p_{hk}-c-\gamma\right)\sum_{m\in\{A,B\}}\frac{\partial D_{hk}}{\partial p_{mj}}\left(\mathbf{p}^R\left(\mathbf{w}\right)\right)\frac{\partial p_{mj}^R}{\partial w_{ik}}\left(\mathbf{w}\right).
\end{aligned}$$

When negotiating over the wholesale price w_{ik}, I and R_k maximize their joint profit, which amounts to maximizing the total industry profit (as M_h obtains its own profit only through fixed fees):

$$\Pi^R\left(\mathbf{w}\right)\equiv\pi_I^R\left(\mathbf{w}\right)+\pi_R^R\left(\mathbf{w}\right),$$

taking into account the impact that any change in w_{ik} has on retail prices, $\mathbf{p}^R(w_{ik}, c, c)$, in the continuation equilibrium. The equilibrium wholesale price thus solves:

$$w_{ik}^{IR} = \arg\max_{w_{ik}} \Pi^R(w_{ik}, c, c).$$

For the sake of exposition, we assume here that this optimization program is well-behaved:

Assumption G. For any wholesale prices $\mathbf{w} = (w_{ik}, w_{hj}, w_{hk}) \in \mathbb{R} \times \{c, \infty\}^2$, the joint bilateral profit of the integrated firm and the independent retailer in the continuation equilibrium, $\Pi^R(\mathbf{w})$, is quasi-concave in w_{ik}.

It follows that, in equilibrium, I charges a positive margin on its sales to R_k. To see this, it suffices to note that, for $w_{ik} = c$, we have:

$$\begin{aligned}
\left.\frac{\partial \Pi^R}{\partial w_{ik}}(\mathbf{w})\right|_{\mathbf{w}=(c,c,c)} &= \sum_{g\in\{A,B\}} \left[p_{gj}^R(\mathbf{w}) - c - \gamma\right] \sum_{m\in\{A,B\}} \frac{\partial D_{gj}}{\partial p_{mk}}\left(\mathbf{p}^R(\mathbf{w})\right) \frac{\partial p_{mk}^R}{\partial w_{ik}}(\mathbf{w}) \\
&\quad + \sum_{g\in\{A,B\}} \left[p_{gk}^R(\mathbf{w}) - c - \gamma\right] \sum_{m\in\{A,B\}} \frac{\partial D_{gk}}{\partial p_{mj}}\left(\mathbf{p}^R(\mathbf{w})\right) \frac{\partial p_{mj}^R}{\partial w_{ik}}(\mathbf{w}) \\
&> 0,
\end{aligned}$$

where the inequality stems from product substitution ($\partial D_{gk}/\partial p_{mj} > 0$ and $\partial D_{gj}/\partial p_{mk} > 0$ for any $g, m \in \{A, B\}$), Assumption F3($i.b$) and the fact that each retailer charges at least one positive downstream margin:

- This is obvious for R_k, as (i) charging non-positive margins on both brands cannot constitute R_k's retail price response, as this would yield a non-positive profit, whereas deviating to slightly positive margins would generate instead a positive profit; and (ii) starting from a situation in which R_k would charge a positive margin on one brand, say brand g, and a negative one on the other brand, say brand m, R_k would profitable deviate by eliminating the negative margin (i.e., increasing p_{mk} to $c + \gamma$), keeping the positive margin unchanged (i.e., maintaining p_{gk} to the same level): this would avoid the loss on brand m, and moreover boosts the sales of brand g, on which R_k earns a positive margin.

- The same observation applies to I when $w_{ik} = c$, as I's entire profit then comes from its retail activities.

Hence, starting from $w_{ik} = c$, the firms wish to increase w_{ik}. The quasi-concavity of the joint bilateral profit Π^R then implies that the negotiated price is strictly above cost: $w_{ik}^{IR} > c$. As the integrated R_j internalizes the impact of its pricing decisions on the margin u_{ik}^{IR} earned by M_i, it follows that it prices less aggressively than pre-merger.

• *Equilibrium fixed fees.* Consider first the negotiation between M_h and I over F_{hj}. As before M_h derives all of its profit from the fixed fees, $F_{hj} + F_{hk}^{IR}$. If the negotiation succeeds, then I gets $\pi_I^{IR} - F_{hj} + F_{ik}^{IR}$, where

$$\pi_I^{IR} \equiv \pi_I^R \left(w_{ik}^{IR}, c, c \right).$$

If instead the negotiation fails, then it gets $\pi_I^{hj} + F_{ik}^{IR}$, where (as R_k, being unaware of the negotiation break-down, sticks to $\mathbf{p}_k^R \left(w_{ik}^{IR}, c, c \right)$):

$$\pi_I^{hj} \equiv \max_{p_{ij}} \left\{ \begin{array}{c} \left(p_{ij} - c - \gamma \right) D \left(p_{ij}, \infty, \mathbf{p}_k^R \left(w_{ik}^{IR}, c, c \right) \right) \\ + \left(w_{ik}^{IR} - c \right) D \left(\mathbf{p}_k^R \left(w_{ik}^{IR}, c, c \right), p_{ij}, \infty \right) \end{array} \right\}.$$

The fixed fee is therefore $F_{hj}^{IR} = \alpha \Delta_I^{hj}$, where $\Delta_I^{hj} \equiv \pi_I^{IR} - \pi_I^{hj}$.

Consider now the negotiation between M_h and R_k over F_{hk}. Once again, M_h derives all of its profit from the fixed fees, $F_{hj}^{IR} + F_{hk}$. If the negotiation succeeds, then R_k gets $\pi_R^{IR} - F_{hk} - F_{ik}^{IR}$, where

$$\pi_R^{IR} \equiv \pi_R^R \left(w_{ik}^{IR}, c, c \right).$$

If instead the negotiation fails, then it gets $\pi_R^{hk} - F_{ik}^{IR}$, where (as I, being unaware of the negotiation's outcome, sticks to $\mathbf{p}_j^R \left(w_{ik}^{IR}, c, c \right)$):

$$\pi_R^{hk} \equiv \max_{p_{ik}} \{ \left(p_{ik} - w_{ik}^{IR} - \gamma \right) D \left(p_{ik}, \infty, \mathbf{p}_j^R (w_{ik}^{IR}, c, c) \right) \}.$$

The fixed fee is therefore $F_{hk}^{IR} = \alpha \Delta_R^{hk}$, where $\Delta_R^{hk} \equiv \pi_R^{IR} - \pi_R^{hk}$.

Finally, consider the negotiation between I and R_k over F_{ik}. If the negotiation succeeds, I gets $\pi_I^{IR} + F_{ik} - F_{hj}^{IR}$ and R_k gets $\pi_R^{IR} - F_{ik} - F_{hk}^{IR}$. If the negotiation fails, both retailers now know that it has failed; hence, I's and R_k's profits become respectively given by $\pi_m^{SE} - F_{hj}^{IR}$ and $\pi_s^{SE} - F_{hk}^{IR}$. The fixed fee is thus equal to

$$F_{ik}^{IR} = \alpha \Delta_R^{ik} - (1 - \alpha) \Delta_I^{ik},$$

where $\Delta_I^{ik} \equiv \pi_I^{IR} - \pi_I^{ik}$ and $\Delta_R^{ik} \equiv \pi_R^{IR} - \pi_R^{ik}$.

• *Equilibrium profits.* The equilibrium profits of the integrated firm, I, the independent manufacturer, M_h, and the independent retailer, R_k, are therefore respectively equal to:

$$\begin{aligned} \hat{\Pi}_I^{IR} &\equiv \pi_I^{ik} + \alpha \left(\pi_R^{IR} - \pi_R^{ik} + \pi_I^{hj} - \pi_I^{ik} \right) \\ \hat{\Pi}_M^{IR} &\equiv \alpha \left(\pi_I^{IR} + \pi_R^{IR} - \pi_I^{hj} - \pi_R^{hk} \right) \\ \hat{\Pi}_R^{IR} &\equiv \pi_R^{IR} - \alpha \left(2\pi_R^{IR} - \pi_R^{hk} - \pi_R^{ik} \right) + (1 - \alpha) \left(\pi_I^{IR} - \pi_I^{ik} \right) \end{aligned}$$

Upstream foreclosure

Suppose now that only the integrated manufacturer is active and sells on both markets.

• *The tariff negotiated between the integrated firm and the independent retailer exhibits a positive margin:* $u_{ik}^e > 0$. The profits of the integrated firm and of the independent retailer (gross of fixed fees) are now respectively given by

$$\begin{aligned}\pi_I^R(w_{ik},\infty,\infty) &= \left[p_{ij}^R(w_{ik},\infty,\infty)-c-\gamma\right]D_{ij}\left(\mathbf{p}^R(w_{ik},\infty,\infty)\right)\\ &\quad+(w_{ik}-c)\,D_{ik}\left(\mathbf{p}^R(w_{ik},\infty,\infty)\right)\end{aligned}$$

and

$$\pi_R^R(w_{ik},\infty,\infty)=\left(p_{ik}^R(w_{ik},\infty,\infty)-w_{ik}-\gamma\right)D_{ik}\left(\mathbf{p}^R(w_{ik},\infty,\infty)\right),$$

where $\mathbf{p}^R(w_{ik},\infty,\infty)=\left(p_{ij}^R(w_{ik},\infty,\infty),\infty,p_{ik}^R(w_{ik},\infty,\infty),\infty\right)$. When negotiating the wholesale price w_{ik}, I and R_k seek to maximize their joint profit, $\Pi^R(w_{ik},\infty,\infty)$, which, using the envelope theorem, satisfies:

$$\begin{aligned}\left.\frac{\partial\Pi^R}{\partial w_{ik}}(\mathbf{w})\right|_{\mathbf{w}=(c,\infty,\infty)} &= \left[p_{ij}^R(\mathbf{w})-c-\gamma\right]\frac{\partial D_{ij}}{\partial p_{ik}}\left(\mathbf{p}^R(\mathbf{w})\right)\frac{\partial p_{ik}^R}{\partial w_{ik}}\left(\mathbf{p}^R(\mathbf{w})\right)\\ &\quad+\left[p_{ik}^R(\mathbf{w})-w_{ik}-\gamma\right]\frac{\partial D_{ik}}{\partial p_{ij}}\left(\mathbf{p}^R(\mathbf{w})\right)\frac{\partial p_{ij}^R}{\partial w_{ik}}\left(\mathbf{p}^R(\mathbf{w})\right)\\ &> 0,\end{aligned}$$

where the inequality stems from product substitution ($\partial D_{ik}/\partial p_{ik}<0$, but $\partial D_{ik}/\partial p_{ij}>0$ and $\partial D_{ij}/\partial p_{ik}>0$), Assumption F3($i.b$), and the fact that both firms' margins are positive (otherwise, the firms – including I, which makes no upstream profit when $w_{ik}=c$ – would make no profit). It follows from Assumption G (the quasi-concavity of Π^R) that the negotiated wholesale price, w_{ik}^{UF}, is strictly above cost. Internalizing the impact of its pricing decisions on the margin $u_{ik}^{UF}>0$, R_j prices less aggressively than pre-merger.

• *Equilibrium profits.* If the negotiation succeeds, I gets $\pi_I^{UF}+F_{ik}$ whereas R_k gets $\pi_R^{UF}-F_{ik}$, where

$$\pi_I^{UF}\equiv\pi_I^R(w_{ik}^{UF},\infty,\infty)\quad\text{and}\quad\pi_R^{UF}\equiv\pi_R^R(w_{ik}^{UF},\infty,\infty).$$

If instead the negotiation fails, I gets π^m and R_k gets nothing. The individual profits of the integrated firm and of the independent retailer are therefore respectively given by:

$$\hat{\Pi}_I^{UF}\equiv\pi^m+\alpha(\pi_I^{UF}+\pi_R^{UF}-\pi^m)\text{ and }\hat{\Pi}_R^{UF}\equiv(1-\alpha)\,(\pi_I^{UF}+\pi_R^{UF}-\pi^m).$$

Single exclusion of channel $h-k$

Suppose now that all channels are active but channel $h-k$.

• *The independent manufacturer's tariff is cost-based:* $w_{hj}^{hk}=c$. The same reasoning as for the case of interlocking relationships can be used to show that (29) still holds, with the caveat that, as q_{hk} is now constrained to be zero (and therefore $\delta_{jk}^{hj}=0$), it directly yields $u_{hj}^{hk}=0$.

• *The tariff negotiated between the integrated firm and the independent retailer exhibits a positive margin:* $u_{ik}^{hk}>0$. When negotiating the wholesale price w_{ik}, I and M_k now seek to maximize $\Pi^R(w_{ik}, c, \infty)$, which, using the envelope theorem, satisfies:

$$\begin{aligned}\left.\frac{\partial \Pi^R}{\partial w_{ik}}(\mathbf{w})\right|_{\mathbf{w}=(c,c,\infty)} &= \sum_{g\in\{A,B\}} \left[p_{gj}^R(\mathbf{w}) - c - \gamma\right] \frac{\partial D_{gj}}{\partial p_{ik}}\left(\mathbf{p}^R(\mathbf{w})\right) \frac{\partial p_{ik}^R}{\partial w_{ik}}(\mathbf{w}) \\ &\quad + \left[p_{ik}^R(\mathbf{w}) - c - \gamma\right] \sum_{m\in\{A,B\}} \frac{\partial D_{ik}}{\partial p_{mj}}\left(\mathbf{p}^R(\mathbf{w})\right) \frac{\partial p_{mj}^R}{\partial w_{ik}}(\mathbf{w}) \\ &> 0,\end{aligned}$$

where, as before, the inequality stems from product substitution ($\partial D_{ik}/\partial p_{mj} > 0$ and $\partial D_{gj}/\partial p_{ik} > 0$ for any $g, m \in \{A, B\}$), Assumption F3($i.b$) and the fact that each retailer charges at least one positive downstream margin. Hence, starting from $w_{ik} = c$, the firms wish to increase w_{ik}. Assumption G (the quasi-concavity of Π^R) then implies that the negotiated price is strictly above cost: $w_{ik}^{hk} > c$, and so the integrated R_j prices less aggressively than pre-merger.

• *Equilibrium fixed fees and profits.* Consider first the negotiation between I and R_k over F_{ik}. If the negotiation succeeds, I gets $\pi_I^{hk} + F_{ik} - F_{hj}^{hk}$ and R_k obtains $\pi_R^{hk} - F_{ik}$, where:

$$\pi_I^{hk} = \pi_I^R(w_{ik}^{hk}, c, \infty) \quad \text{and} \quad \pi_R^{hk} = \pi_R^R(w_{ik}^{hk}, c, \infty).$$

If the negotiation fails, R_k is excluded (and its profit is thus equal to 0) whereas I obtains $2\pi^{DF} - F_{hj}^{hk}$. The Nash bargaining rule thus yields:

$$\pi_R^{hk} - F_{ik} = (1-\alpha)\left(\pi_R^{hk} - 2\pi^{FD} + \pi_I^{hk}\right),$$

implying that the fixed fee is equal to

$$F_{ik} = 2\pi^{FD} - \pi_I^{hk} + \alpha\left(\pi_R^{hk} - 2\pi^{FD} + \pi_I^{hk}\right).$$

Consider now the negotiation between I and M_h over F_{hj}. If the negotiation succeeds, I and M_h respectively obtain $\hat{\pi}_I - F_{hj} + F_{ik}^{hk}$ and F_{hj}. If it fails, M_h is excluded and I gets $\hat{\pi}_I^{hj} + F_{ik}^{hk}$, where:

$$\hat{\pi}_I^{hk} = \max_p \pi_I(p, \infty, p_{ik}^R\left(w_{ik}^{hk}, c, \infty\right), \infty; w_{ik}^{hk}, c, \infty).$$

Hence, $F_{hj}^{hk} = \alpha(\hat{\pi}_I - \hat{\pi}_I^{hj})$.

The equilibrium profits are therefore equal to:

$$\begin{aligned}\hat{\Pi}_I^{hk} &\equiv \alpha\left(\hat{\pi}_R + \hat{\pi}_I^{hj}\right) + 2(1-\alpha)\pi^{DF} \\ \hat{\Pi}_M^{hk} &\equiv \alpha\left(\hat{\pi}_I - \hat{\pi}_I^{hj}\right) \\ \hat{\Pi}_R^{hk} &\equiv (1-\alpha)\left(\hat{\pi}_I + \hat{\pi}_R - 2\pi^{DF}\right)\end{aligned}$$

Single exclusion of channel $h-j$

Suppose now that all channels are active but channel $h-j$.

• *The independent manufacturer's tariff is cost-based:* $w_{hk}^{hj}=c$. Once again, the same reasoning as for the case of interlocking relationships can be used to show that (28) still holds, with the caveat that, as q_{hj} is now constrained to be zero (and therefore $\delta_{kj}^{hk}=0$); it directly yields $u_{hk}^{hj}=0$.

• *The tariff negotiated between the integrated firm and the independent retailer exhibits a positive margin:* $u_{ik}^{hj}>0$. When negotiating the wholesale price w_{ik}, I and M_k now seek to maximize $\Pi^R(w_{ik},c,\infty)$, which, using the envelope theorem, satisfies:

$$\begin{aligned}\left.\frac{\partial\Pi^R}{\partial w_{ik}}(\mathbf{w})\right|_{\mathbf{w}=(c,\infty,c)} &= \left[p_{ij}^R(\mathbf{w})-c-\gamma\right]\sum_{m\in\{A,B\}}\frac{\partial D_{ij}}{\partial p_{mk}}\left(\mathbf{p}^R(\mathbf{w})\right)\frac{\partial p_{mk}^R}{\partial w_{ik}}(\mathbf{w}) \\ &\quad+\sum_{g\in\{A,B\}}\left[p_{gk}^R(\mathbf{w})-c-\gamma\right]\sum_{m\in\{A,B\}}\frac{\partial D_{gk}}{\partial p_{mj}}\left(\mathbf{p}^R(\mathbf{w})\right)\frac{\partial p_{mj}^R}{\partial w_{ik}}(\mathbf{w}) \\ &> 0,\end{aligned}$$

where the inequality stems again from product substitution ($\partial D_{ij}/\partial p_{mk}>0$ and $\partial D_{gk}/\partial p_{mj}>0$ for any $g,m\in\{A,B\}$), Assumption F3($i.b$) and $l\in\{1,2\}$) and the fact that each retailer charges at least one positive downstream margin. Hence, starting from $w_{ik}=c$, the firms wish to increase w_{ik}. Assumption G (the quasi-concavity of Π^R) then again implies that the negotiated price is strictly above cost: $w_{ik}^{hj}>c$. Internalizing the impact of its pricing decisions on the upstream margin w_{ik}^{hj}, the integrated R_j prices less aggressively than pre-merger.

• *Equilibrium fixed fees and profits.* Consider first the negotiation between I and R_k over F_{ik}. If the negotiation succeeds, I gets $\pi_I^{hj}+F_{ik}$ and R_k obtains $\pi_R^{hj}-F_{ik}-F_{hk}^{hj}$, where:

$$\pi_I^{hj}=\pi_I^R(w_{ik}^{hj},\infty,c)\quad\text{and}\quad\pi_R^{hj}=\pi_R^R\left(w_{ik}^{hj},\infty,c\right).$$

If the negotiation fails, I and R_k obtain π^{ED} and $\pi^{ED}-F_{hk}^{hj}$ respectively. Nash bargaining thus yields:

$$\pi_I^{hj}+F_{ik}^{hj}-\pi^{ED}=\alpha\left(\pi_I^{hj}+\pi_R^{hj}-2\pi^{ED}\right).$$

Consider now the negotiation between M_h and R_k over F_{hk}. If the negotiation succeeds, M_h gets F_{hk} and R_k gets $\pi_R^{hj}-F_{hk}-F_{ik}^{hj}$. If the negotiation fails, M_h is excluded and R_k obtains $\hat{\pi}_R^{hj}-F_{ik}^{hj}$, where

$$\hat{\pi}_R^{hj}=\max_p\pi_R\left(p_{ij}^R\left(w_{ik}^{hj},\infty,c\right),\infty,p,\infty;w_{ik}^{hj},\infty,c\right).$$

Hence, Nash bargaining yields $F_{hk}^{hj}=\alpha(\pi_R^{hj}-\hat{\pi}_R^{hj})$.

The equilibrium profits, $\Pi_I=\pi_I^{hj}+F_{ik}^{hj}$, $\Pi_{M_h}=F_{hk}^{hj}$ and $\Pi_{R_k}=\pi_R^{hj}-F_{ik}^{hj}-F_{hk}^{hj}$, are therefore respectively equal to:

$$\begin{aligned}
\hat{\Pi}_I^{hj} &\equiv \pi^{ED} + \alpha\left(\pi_I^{hj} + \pi_R^{hj} - 2\pi^{ED}\right),\\
\hat{\Pi}_M^{hj} &\equiv \alpha\left(\pi_R^{hj} - \hat{\pi}_R^{hj}\right),\\
\hat{\Pi}_R^{hj} &\equiv (1-\alpha)\left(\pi_I^{hj} + \pi_R^{hj} - \pi^{ED}\right) + \alpha\left(\pi^{ED} + \hat{\pi}_R^{hj} - \pi_R^{hj}\right).
\end{aligned}$$

F.3.2 Post-merger distribution network

We now study the CPNE of the network formation game post-merger. To provide a full characterization, in what follows we concentrate on the linear demand example. It can be checked that:

- Equilibrium profits are all positive, implying that no firm can gain from being excluded from the market; hence, bilateral monopoly cannot be an equilibrium structure, as the coalition formed by the two (excluded) independent firms would then benefit from activating their channel.
- In addition, $\pi^{ED} + \pi^m > \max\left\{2\pi^{DF}, \pi_I^{UF} + \pi_R^{UF}\right\}$, which yields:

$$\hat{\Pi}_M^{ED} > \hat{\Pi}_M^{DF}, \tag{30}$$

  and

$$\hat{\Pi}_R^{ED} > \hat{\Pi}_R^{UF}. \tag{31}$$

  Hence, we can rule out two other network structures:

  - Downstream foreclosure can be ruled out, as exclusive dealing would instead enable R_k to earn a positive profit and M_h to increase its profit from $\hat{\Pi}_M^{DF}$ to $\hat{\Pi}_M^{ED}$, their coalition could profitably deviate by activating their own channel (instead of, or in addition to the channel $h-j$).
  - Upstream foreclosure can also be ruled out, as exclusive dealing would enable M_h to earn a positive profit and R_k to increase its profit from $\hat{\Pi}_R^{UF}$ to $\hat{\Pi}_R^{ED}$, their coalition could profitably deviate by activating their own channel (instead of, or in addition to the channel $i-k$).

We now turn to the other five distribution networks. As before, without loss of generality, we can assume that inactive channels, being declared inactive by both involved parties, cannot be activated by unilateral deviations.

Interlocking relationships

We consider the possible deviations, starting with unilateral deviations.

• *Unilateral deviations.* Interlocking relationships constitute a Nash equilibrium of the network formation game if none of the three firms has an incentive to deviate; as (i) the integrated firm always maintains its own channel but can terminate any of the channels $i-k$ and $h-j$, (ii) each independent firm can terminate the channel $h-k$ and one other channel ($h-j$ for M_h and $i-k$ for R_k), and (iii) equilibrium profits are all positive, we must have:

$$\hat{\Pi}_I^{IR} \geq \max\left\{\hat{\Pi}_I^{ED}, \hat{\Pi}_I^{ik}, \hat{\Pi}_I^{hj}\right\}, \hat{\Pi}_M^{IR} \geq \max\left\{\hat{\Pi}_M^{hj}, \hat{\Pi}_M^{hk}\right\} \text{ and } \hat{\Pi}_R^{IR} \geq \max\left\{\hat{\Pi}_R^{ik}, \hat{\Pi}_R^{hk}\right\}.$$

For a linear demand, the relevant conditions are $\hat{\Pi}_I^{IR} \geq \hat{\Pi}_I^{hj}$ for $\alpha > \bar{\alpha}$ and $\mu \in \left(\underline{\mu}^{IR}(\alpha), \bar{\mu}^{IR}(\alpha)\right)$, and $\hat{\Pi}_M^{IR} \geq \hat{\Pi}_M^{hj}$ otherwise, where $\bar{\alpha} \simeq 0.881$ and $\underline{\mu}^{IR}(\alpha)$ (resp., $\bar{\mu}^{IR}(\alpha)$) is a decreasing (resp., increasing) function of α. These conditions amount to:

$$\rho \leq \rho^{IR}(\mu, \alpha),$$

where:

(i) $\rho^{IR}(\mu, \alpha)$ is a strictly decreasing function of μ;

(ii) there exists $\mu^{IR}(\alpha) \in (0, \bar{\mu})$, where $\bar{\mu} \simeq 0.407$,[23] such that: $\rho^{IR}(\mu, \alpha) \geq \rho^*(\mu)$ if and only if $\mu \leq \mu^{IR}(\alpha)$.

• *Deviations by coalitions involving I.* Suppose now that interlocking relationships constitute a Nash equilibrium, and consider deviations by a coalition involving the integrated firm I. As I can unilaterally close the channels $h-j$ and $i-k$, such a coalition can be useful only if it closes the remaining channel, $h-k$. As M_h and R_k can each achieve this, there is no need to consider the grand coalition of all three firms: the smaller coalitions, $I-M_h$ and $I-R_k$, could implement the same deviation and would be subject to fewer self-enforcement constraints. Furthermore, the coalition partner cannot benefit from closing only that channel, (otherwise, interlocking relationships would not constitute a Nash equilibrium), and it cannot benefit either from being completely shut down; it follows that the only relevant options are a coalition with M_h deviating to downstream foreclosure or a coalition with R_k deviating to upstream foreclosure. It can be checked that for, $\rho \leq \rho^{IR}(\mu, \alpha)$, such deviations are never profitable for the coalition partner.

• *Deviations by the coalition $M_h - R_k$.* The only remaining coalition is that of the two independent firms, and the relevant deviations are those that cannot be achieved unilaterally by either M_h or R_k (otherwise, interlocking relationships would not constitute a Nash equilibrium) and do not exclude completely one of the two firms (otherwise, that firm would be harmed by the deviation). Hence, the only relevant deviation is a switch to exclusive dealing (by closing the channels $i-k$ and $h-j$). However, it can be checked

[23] $\mu^{IR}(\alpha)$ actually remains equal to $\bar{\mu}$ as long as $\alpha \leq \bar{\alpha}$, before decreasing and tending towards 0 as α further increases towards 1.

that, whenever $\rho \leq \rho^{IR}(\mu, \alpha)$, following the deviation R_k would have an incentive to re-open its channel with I.

Therefore, we have:

Result 1 *Interlocking relationships constitute a CPNE of the post-merger network formation game if and only if $\rho \leq \rho^{IR}(\mu, \alpha)$.*

Exclusive dealing

As in the pre-merger case, exclusive dealing is always a Nash equilibrium; we thus focus on deviations by coalitions. As the coalition formed by M_h and R_k cannot achieve anything other than closing their only active channel, there are only three relevant coalitions:

- coalition $I - R_k$: as upstream foreclosure is never profitable for R_k, the only relevant option is the exclusion of $h - j$;
- coalition $I - M_h$: as downstream foreclosure is never profitable for M_h, the only relevant option is the exclusion of $i - k$;
- grand coalition: the only options that cannot be achieved by subcoalitions are interlocking relationships and the exclusion of $h - k$.

We consider these various deviations in turn.

• *Deviation by the coalition $I - R_k$.* A deviation by the coalition $I - R_k$ to the exclusion of $h - j$ is self-enforcing whenever it is profitable: unilaterally, I can only revert to exclusive dealing, and R_k can only switch to upstream foreclosure, which is less profitable than exclusive dealing. The deviation is moreover strictly profitable for I:

$$\begin{aligned}\hat{\Pi}_I^{hj} \geq \hat{\Pi}_I^{ED} &\iff \pi^{ED} + \alpha\left(\pi_I^{hj} + \pi_R^{hj} - 2\pi^{ED}\right) \geq \pi^{ED} \\ &\iff \pi_I^{hj} + \pi_R^{hj} \geq 2\pi^{ED},\end{aligned}$$

which always holds as I chooses w_{ik} precisely so as to maximize the industry profit, and so $\pi_I^{hj} + \pi_R^{hj} = \Pi^R\left(w_{ik}^{hj}, \infty, c\right) > \Pi^R(\infty, \infty, c) = 2\pi^{ED}$.

Therefore, exclusive dealing constitutes a CPNE if and only if the deviation is not acceptable by R_k, that is, if:

$$\begin{aligned}\hat{\Pi}_R^{hj} < \hat{\Pi}_R^{ED} &\iff (1-\alpha)\left(\pi_I^{hj} + \pi_R^{hj} - \pi^{ED}\right) + \alpha\left(\pi^{ED} + \hat{\pi}_R^{hj} - \pi_R^{hj}\right) < (1-\alpha)\pi^{ED} \\ &\iff (1-\alpha)\left(\pi_I^{hj} + \pi_R^{hj}\right) - \alpha\left(\pi_R^{hj} - \hat{\pi}_R^{hj}\right) < (2-3\alpha)\pi^{ED},\end{aligned}$$

which amounts to

$$\rho > \rho^{ED}(\mu, \alpha),$$

where $\rho^{ED}(0,\cdot) = \rho^{ED}(1,\cdot) = 1$, $\rho^{ED}(\cdot,\alpha) = 1$ for $\alpha \leq \alpha^{ED} = 3/10$ and, for $\alpha > \alpha^{ED}$, $\max\left\{\rho^*(\mu), \rho^{IR}(\mu,\alpha)\right\} < \rho^{ED}(\mu,\alpha) < 1$ for any $\mu \in [0,1]$.[24]

• *Deviations by the coalition $I - M_h$.* A deviation by the coalition $I - M_h$ to the exclusion of $i - k$ is not profitable when $\rho > \rho^{ED}(\mu,\alpha)$; indeed, we then have:

$$\begin{aligned} \hat{\Pi}_I^{ik} &< \hat{\Pi}_I^{ED} \Longleftrightarrow (1-\alpha)\,\pi_m^{SE} + \alpha\hat{\pi}_R < \pi^{ED}, \\ \hat{\Pi}_h^{ik} &< \hat{\Pi}_M^{ED} \Longleftrightarrow \pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R < \pi^{ED}. \end{aligned}$$

• *Deviations by the grand coalition.* A deviation by the grand coalition to interlocking relationships is self-enforcing only if this network constitutes a CPNE outcome, which requires $\rho \leq \rho^{IR}(\mu,\alpha) < \rho^{ED}(\mu,\alpha)$, a contradiction. A deviation to the exclusion of $h - k$ is acceptable by M_h only if

$$\hat{\Pi}_M^{hk} \geq \hat{\Pi}_M^{ED} \Longleftrightarrow \hat{\pi}_I - \hat{\pi}_I^{hj} \geq \pi^{ED},$$

which is never the case for a linear demand.

Therefore, we have:

Result 2 *Exclusive dealing constitutes a CPNE of the post-merger network formation game if and only if $\rho > \rho^{ED}(\mu,\alpha)$.*

Single exclusion of channel $h - j$

• *Unilateral deviations.* Starting from the exclusion of $h - j$, as no firm can benefit from being completely excluded, and R_k prefers exclusive dealing to upstream foreclosure, the only relevant unilateral deviations are for I or R_k to close the channel $i - k$; hence, this distribution network constitutes a Nash equilibrium if and only if

$$\hat{\Pi}_I^{hj} \geq \hat{\Pi}_I^{ED} \text{ and } \hat{\Pi}_R^{hj} \geq \hat{\Pi}_R^{ED}.$$

The above analysis of exclusive dealing shows that these conditions amount to $\rho \leq \rho^{ED}(\mu,\alpha)$.

We now turn to deviations by coalitions. The coalitions $M_h - R_k$ and $I - R_k$ cannot achieve more than what R_k can already achieve through unilateral deviations, and the grand coalition cannot achieve more than what can be achieved by the smaller coalition $I - M_h$; in addition, for that coalition the only deviations that cannot be achieved by

[24] Specifically, it can be checked that $\rho^{ED}(\mu,\alpha)$ always exceeds 0.808, whereas $\max\left\{\rho^{IR}(\mu,\alpha), \rho^*(\mu)\right\} \leq \rho^{IR}(0,\alpha) = \rho^*(0) \simeq 0.612$.

unilateral deviations are those that activate the channel $h-j$: interlocking relationships, downstream foreclosure, and the exclusion of $i-k$.

• *Deviation to interlocking relationships.* For a deviation to interlocking relationships (i.e., adding the channel $h-j$) to be profitable we must have:

$$\hat{\Pi}_I^{IR} \geq \hat{\Pi}_I^{hj} \text{ and } \hat{\Pi}_M^{IR} \geq \hat{\Pi}_M^{hj},$$

with at least one of the two inequalities being strict. As shown above, these conditions amount to $\rho \leq \rho^{IR}(\mu,\alpha)$, in which case the deviation is moreover self-enforcing, as interlocking relationships constitutes a CPNE.

• *Deviation to downstream foreclosure.* A deviation to downstream foreclosure is not self-enforcing, as M_h would have a incentive to deviate unilaterally to exclusive dealing (which is feasible, as in equilibrium R_k is willing to deal with M_h).

• *Deviation to the exclusion of $i-k$.* This deviation is never self-enforcing: as $\hat{\Pi}_I^{ik} \geq \hat{\Pi}_I^{IR}$, I would then re-open its channel with R_k.

This establishes:

Result 3 *The single exclusion of $h-j$ constitutes a CPNE of the post-merger network formation game if and only if $\rho^{IR}(\mu,\alpha) < \rho \leq \rho^{ED}(\mu,\alpha)$.*

Single exclusion of channel $i-k$

For the single exclusion of $i-k$ to constitute a Nash equilibrium, the integrated firm and the independent manufacturer should not want to shut down their channel:

$$\begin{aligned}
\hat{\Pi}_I^{ik} &\geq \hat{\Pi}_I^{ED} \Longleftrightarrow (1-\alpha)\,\pi_m^{CN} + \alpha\hat{\pi}_R \geq \pi^{ED} \\
\text{and } \hat{\Pi}_M^{ik} &\geq \hat{\Pi}_M^{ED} \Longleftrightarrow \pi_m^{CN} + \pi_s^{CN} - \hat{\pi}_R \geq \pi^{ED}.
\end{aligned}$$

The first condition is satisfied if and only if $\rho \leq \rho^{ik}(\mu,\alpha)$, where the threshold $\rho^{ik}(\mu,\alpha)$ is decreasing in α and μ, while the second is equivalent to $\rho \leq \rho^*(\mu)$. However, a deviation by the coalition $I-R_k$ to interlocking relationships is profitable and self-enforceable whenever:

$$\hat{\Pi}_I^{IR} \geq \max\left\{\hat{\Pi}_I^{ED}, \hat{\Pi}_I^{ik}, \hat{\Pi}_I^{hj}\right\} \text{ and } \hat{\Pi}_R^{IR} \geq \max\left\{\hat{\Pi}_R^{ik}, \hat{\Pi}_R^{hk}, 0\right\}.$$

It can be checked that the most stringent of these conditions is $\hat{\Pi}_I^{IR} \geq \hat{\Pi}_I^{hj}$, and that it holds whenever $\rho \leq \min\left\{\rho^{ik}(\mu,\alpha), \rho^*(\mu)\right\}$. Hence, we have:

Result 4 *The single exclusion of $i-k$ cannot constitute a CPNE.*

Single exclusion of channel $h-k$

Suppose that the coalition $M_h - R_k$ deviates and activates the channel $h-k$; three types of deviation are possible:

- A deviation to *interlocking relationships* is profitable if $\hat{\Pi}_f^{IR} \geq \hat{\Pi}_f^{hk}$ for $f = M, R$, with at least one strict inequality, and it is self-enforcing if $\hat{\Pi}_M^{IR} \geq \hat{\Pi}_M^{hj}$ and $\hat{\Pi}_R^{IR} \geq \hat{\Pi}_R^{ik}$. It can be checked that the most stringent of these four conditions is
$$\hat{\Pi}_M^{IR} \geq \hat{\Pi}_M^{hj}. \tag{32}$$

- A deviation to *exclusive dealing* is instead profitable if $\hat{\Pi}_f^{ED} \geq \hat{\Pi}_f^{hk}$ for $f = M, R$, with at least one strict inequality, and it is self-enforcing if $\hat{\Pi}_M^{ED} \geq \max\left\{\hat{\Pi}_M^{ik}, \hat{\Pi}_M^{DF}\right\}$ and $\hat{\Pi}_R^{ED} \geq \max\left\{\hat{\Pi}_R^{hj}, \hat{\Pi}_R^{UF}\right\}$. It can be checked that the most stringent of these six conditions is
$$\hat{\Pi}_R^{ED} \geq \hat{\Pi}_R^{hj}. \tag{33}$$

- Finally, a deviation to the exclusion of $h-j$ is profitable if $\hat{\Pi}_f^{hj} \geq \hat{\Pi}_f^{hk}$ for $f = M, R$, with at least one strict inequality, and it is self-enforcing if $\hat{\Pi}_M^{hj} \geq \hat{\Pi}_M^{IR}$ and $\hat{\Pi}_R^{hj} \geq \max\left\{\hat{\Pi}_R^{ED}, \hat{\Pi}_R^{UF}\right\}$. It can be checked that, out of these five conditions, the relevant ones are
$$\hat{\Pi}_M^{hj} \geq \hat{\Pi}_M^{IR} \text{ and } \hat{\Pi}_R^{hj} \geq \hat{\Pi}_R^{ED},$$
which hold whenever neither (32) nor (33) holds. Hence, we have:

Result 5 *The single exclusion of $h-k$ cannot constitute a CPNE.*

Impact of the vertical merger on the distribution network

The above analysis leads us to conclude that, following a vertical merger between M_i and R_j, the network formation game has a unique CPNE, the equilibrium network structure consisting of:

- exclusive dealing whenever $\rho > \rho^{ED}(\mu, \alpha)$, which is possible if $\alpha > \alpha^{ED} = 0.3$.
- (single) exclusion of $h-j$ whenever $\rho^{IR}(\mu, \alpha) < \rho \leq \rho^{ED}(\mu, \alpha)$; and
- interlocking relationships whenever $\rho \leq \rho^{IR}(\mu, \alpha)$,

where the thresholds $\rho^{IR}(\mu, \alpha)$ and $\rho^{ED}(\mu, \alpha)$ are characterized by:

$$\begin{aligned}
\rho \leq \rho^{IR}(\mu, \alpha) &\iff \{\hat{\Pi}_I^{IR} \geq \hat{\Pi}_I^{hj} \text{ and } \hat{\Pi}_M^{IR} \geq \hat{\Pi}_M^{hj}\}, \\
\rho \geq \rho^{ED}(\mu, \alpha) &\iff \hat{\Pi}_R^{ED} \geq \hat{\Pi}_{ED}^{hj}.
\end{aligned}$$

F.3.3 Impact on consumer surplus and total welfare

When $\rho > \rho^{ED}(\mu, \alpha)$, the merger does not affect the equilibrium distribution network, which consists of exclusive dealing, and it does not affect prices either, as the relevant wholesale prices remain equal to marginal costs; hence, the merger has no impact on consumer surplus and total welfare.

When $\rho \leq \min\left\{\rho^{IR}(\mu, \alpha), \rho^*(\mu)\right\}$, again the merger does not affect the equilibrium distribution network, which there consists of interlocking relationships, and it still does not affect the wholesale prices charged by the independent manufacturer, which remain at cost; however, the merger raises the wholesale price charged by the integrated firm to the independent retailer (i.e., $w_{ik}^e > 0$), and moreover induces the integrated retailer to price less aggressively. As a result, the merger increases equilibrium retail prices, and reduces both consumer surplus and total welfare.

When $\rho^{IR}(\mu, \alpha) < \rho < \rho^*(\mu)$ (which implies that $\mu > \mu^{IR}(\alpha)$), the merger not only raises the wholesale price charged to the independent retailer by the integrated firm (and makes the integrated retailer less aggressive), but it moreover reduces the set of available channels, from four (interlocking relationships) to three (exclusion of the channel $h - j$). It thus reduces again consumer surplus and total welfare.

When instead $\rho^*(\mu) \leq \rho \leq \rho^{ED}(\mu, \alpha)$, the merger expands the set of channels by inducing the integrated firm to supply the rival retailer, although at a wholesale price above cost, but it also makes the integrated retailer less aggressive. Different cases arise:

- if $\rho \leq \rho^{IR}(\mu, \alpha)$ (which implies that $\mu \leq \mu^{IR}(\alpha)$), the merger fully expands the distribution network (interlocking relationships); for a linear demand, this product expansion effect dominates and the merger enhances both consumer surplus and total welfare.

- if instead $\rho > \rho^{IR}(\mu, \alpha)$, the merger only adds the channel $i - k$ to the set of products (exclusion of $h - j$ only); for a linear demand, the softening of the integrated retailer then dominates and the merger reduces consumer surplus. Furthermore, there exists a threshold $\rho^W(\mu, \alpha)$, satisfying

$$\max\left\{\rho^{IR}(\mu, \alpha), \rho^*(\mu)\right\} < \rho^W(\mu, \alpha) < \rho^{ED}(\mu, \alpha),$$

  such that the merger is welfare-improving (resp., welfare-reducing) when $\rho < \rho^W(\mu, \alpha)$ (resp., $\rho > \rho^W(\mu, \alpha)$).

These insights are illustrated by Figure 4.[25]

[25] Figure 4 has been drawn for $\alpha = 0.5$. The upper area, where the equilibrium market structure is unaffected by the merger, exists for $\alpha > \alpha^{ED} = 0.3$.

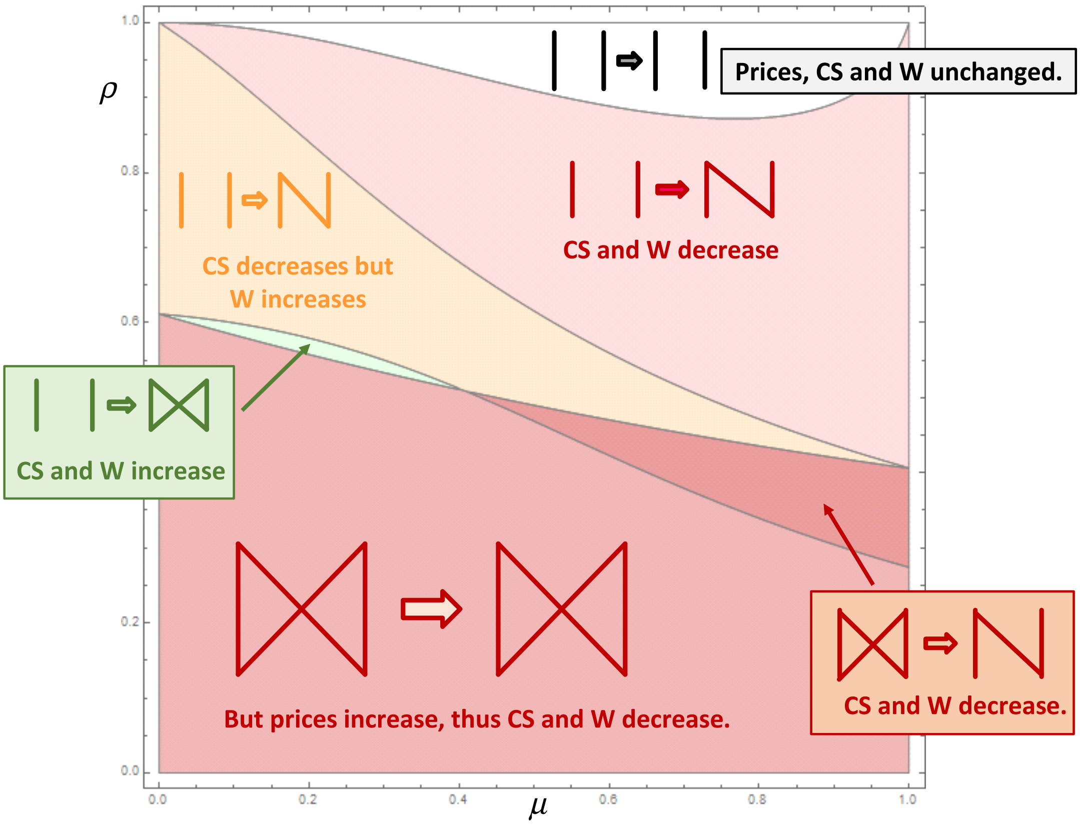

Figure 4: Impact of a vertical merger

G Observable contracting

G.1 Proof of Proposition 12

Part (i). Fix a bargaining equilibrium with observable two-part tariffs in which retail price responses depend only on wholesale prices, $\mathcal{B} = \{\mathbf{p}^R(\mathbf{w}), \mathbf{t}^{\mathbf{e}}, \mathbf{p}^{\mathbf{e}}\}$, and, for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$:

- let

$$\begin{aligned}\Pi^e_{M_i} &\equiv \sum_{k\in\mathcal{J}} [(w_{ik} - c_i) D_{ik}(\mathbf{p}^{\mathbf{e}}) + F^e_{ik}],\\ \Pi^e_{R_j} &\equiv \sum_{h\in\mathcal{I}} [(p^e_{hj} - w^e_{hj} - \gamma_j) D_{hj}(\mathbf{p}^{\mathbf{e}}) - F^e_{hj}],\end{aligned}$$

  denote the equilibrium profits of M_i and R_j;

- let $\mathbf{p}^R(\infty, \mathbf{w}^{\mathbf{e}}_{-i,j}, \mathbf{w}^{\mathbf{e}}_{-j})$ denote the continuation retail price equilibrium in the event that the negotiation between M_i and R_j breaks down, and

$$\begin{aligned}
\Pi_{M_i}^{ij} &\equiv \sum_{k\in\mathcal{J}\setminus\{j\}} \left[(w_{ik}-c_i)\, D_{ik}\left(\mathbf{p}^R\left(\infty,\mathbf{w}^{\mathbf{e}}_{-i,j},\mathbf{w}^{\mathbf{e}}_{-j}\right)\right)+F_{ik}^e\right],\\
\Pi_{R_j}^{ij} &\equiv \sum_{h\in\mathcal{I}\setminus\{i\}} \left[\left(p_{hj}^R\left(\infty,\mathbf{w}^{\mathbf{e}}_{-i,j},\mathbf{w}^{\mathbf{e}}_{-j}\right)-w_{hj}^e-\gamma_j\right) D_{hj}\left(\mathbf{p}^R\left(\infty,\mathbf{w}^{\mathbf{e}}_{-i,j},\mathbf{w}^{\mathbf{e}}_{-j}\right)\right)-F_{hj}^e\right],
\end{aligned}$$

respectively denote the resulting profits for M_i and R_j;

- and for every $\theta_{ij}\in\Theta_{ij}$, let

$$\hat{F}_{ij}^{\theta_{ij}} \equiv \begin{cases} \Pi_{R_j}^e - \Pi_{R_j}^{ij} & \text{if } \theta_{ij}=M_i^j,\\ -\left(\Pi_{M_i}^e-\Pi_{M_i}^{ij}\right) & \text{if } \theta_{ij}=R_j^i,\end{cases}$$

reflect the benefit of the bilateral relationship for θ_{ij}'s firm.

These fees balance each other in expectation:

Lemma G.1 (bargaining fees: observable two-part tariffs) *For every $i\in\mathcal{I}$ and every $j\in\mathcal{J}$:*

$$E_{\theta_{ij}}\left[\hat{F}_{ij}^{\theta_{ij}}\right]=0.$$

Proof. We have:

$$\begin{aligned}
E_{\theta_{ij}}\left[\hat{F}_{ij}^{\theta_{ij}}\right] &= \alpha_{ij}\hat{F}_{ij}^{M_i^j}+(1-\alpha_{ij})\,\hat{F}_{ij}^{R_j^i}\\
&= \alpha_{ij}\left(\Pi_{R_j}^e-\Pi_{R_j}^{ij}\right)-(1-\alpha_{ij})\left(\Pi_{M_i}^e-\Pi_{M_i}^{ij}\right)\\
&= 0,
\end{aligned}$$

where the last equality follows from the Nash bargaining rule (equation (4)). ■

Let $\left(\hat{\mathbf{p}}^{\boldsymbol{\theta}}\right)_{\boldsymbol{\theta}\in\Theta}$ denote the price vector such that $\hat{\mathbf{p}}^{\boldsymbol{\theta}}=\mathbf{p}^{\mathbf{e}}$ for every $\boldsymbol{\theta}\in\Theta$ and, for every $j\in\mathcal{J}$, let $\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j}=(\hat{t}_{ij}^{\boldsymbol{\theta}_{ij}})_{i\in\mathcal{I}}$ denote the tariffs $\hat{t}_{ij}^{\theta_{ij}}=\{w_{ij}^e, F_{ij}^e+\hat{F}_{ij}^{\theta_{ij}}\}$; note that the wholesale and retail prices are the same as in the bargaining equilibrium $\mathcal{B}$: for every $i\in\mathcal{I}$ and every $j\in\mathcal{J}$, $\hat{w}_{ij}^{\theta_{ij}}=w_{ij}^e$ regardless of which agent θ_{ij} is selected to make an offer in the bilateral negotiation between M_i and R_j, and $\hat{p}_{ij}^{\boldsymbol{\theta}}=p_{ij}^e$ regardless of which side gets to make the offer in *any* of the bilateral negotiations. We now show that $(\mathbf{p}^R(\mathbf{w}),(\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta},(\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta})$ constitutes a subgame perfect equilibrium of game Γ^O.

We first note that this candidate equilibrium gives all firms the same expected profits as the bargaining equilibrium $\mathcal{B}$: the price responses are the same retail prices as in $\mathcal{B}$, and the equilibrium wholesale prices coincide with the bargaining equilibrium ones; hence equilibrium retail prices (and thus channel profits) are the same as in $\mathcal{B}$. Furthermore, the tariff $\hat{t}_{ij}^{M_i^j}$ gives R_j its disagreement profit in the bargaining equilibrium $\mathcal{B}$, $\Pi_{R_j}^{ij}$, and conversely $\hat{t}_{ij}^{R_j^i}$ gives M_i its disagreement profit in the bargaining equilibrium $\mathcal{B}$, $\Pi_{M_i}^{ij}$; hence, the expected tariff gives each firm the same profit as in $\mathcal{B}$.

In stage 2, for any given negotiated wholesale prices $\mathbf{w}$ the price responses $\mathbf{p}^R(\mathbf{w})$ constitutes a Nash equilibrium; hence, each R_j is willing to stick to its equilibrium price response if the others do so. Turning to stage 1, consider the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. Their agents, M_i^j and R_j^i, expect all other agents to negotiate the equilibrium tariffs, which are of the form $\hat{t}_{hk}^{\boldsymbol{\theta}_{hk}} = \{w_{hk}^e, F_{hk}^e + \hat{F}_{hk}^{\theta_{hk}}\}$, where $E_{\theta_{hk}}[\hat{F}_{hk}^{\theta_{hk}}] = 0$. Hence, when signing a tariff $t_{ij} = \{w_{ij}, F_{ij}\}$ they anticipate the expected joint profit of their two firms to be equal to:

$$\begin{aligned}
\Pi_{M_i-R_j}^R(w_{ij}) \equiv\ & \left[p_{ij}^R\left(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}\right) - c_i - \gamma_j\right] D_{ij}\left(\mathbf{p}^R\left(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}\right)\right) \\
& + \sum_{k \in \mathcal{J} \setminus \{j\}} \left[\left(w_{ik}^e - c_i\right) D_{ik}\left(\mathbf{p}^R\left(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}\right)\right) + F_{ik}^e\right] \\
& + \sum_{h \in \mathcal{I} \setminus \{i\}} \left\{ \begin{array}{l} \left[p_{hj}^R\left(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}\right) - w_{hj}^e - \gamma_j\right] \\ \times D_{hj}\left(\mathbf{p}^R\left(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}\right)\right) - F_{hj}^e \end{array} \right\},
\end{aligned} \tag{34}$$

which coincides with the bilateral joint profit that M_i and R_j seek to maximize in the bargaining equilibrium $\mathcal{B}$. It follows that M_i^j and R_j^i choose $w_{ij} = w_{ij}^e$, regardless of which side gets to make the offer. In addition, the selected agent, θ_{ij}, sets the fixed fee so as to leave the other agent indifferent between accepting or rejecting the offer, which, as noted above, is achieved by charging $F_{ij}^e + \hat{F}_{ij}^{\theta_{ij}}$.

Part (ii). Fix a subgame perfect equilibrium of game Γ^O in two-part tariffs in which retail price responses depend only on wholesale prices, and which side gets to make the offer only affects the equilibrium fixed fees, and not the equilibrium wholesale prices; the equilibrium is thus of the form $\mathcal{E} = \{\hat{\mathbf{p}}_j^R(\mathbf{w}), (\hat{\mathbf{t}}^{\boldsymbol{\theta}} = \{\hat{\mathbf{w}}, \hat{\mathbf{F}}^{\boldsymbol{\theta}}\})_{\boldsymbol{\theta} \in \Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}})_{\boldsymbol{\theta} \in \Theta}\}$. Consider now the fixed fees $\mathbf{F}^{\mathbf{e}} = (F_{ij}^e)_{(i,j) \in \mathcal{I} \times \mathcal{J}}$ where, for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$:

$$F_{ij}^e = E_{\theta_{ij}}\left[\hat{F}_{ij}^{\theta_{ij}}\right] = \alpha_{ij} \hat{F}_{ij}^{M_i^j} + (1 - \alpha_{ij}) \hat{F}_{ij}^{R_j^i}. \tag{35}$$

We now show that $\{\hat{\mathbf{p}}^R(\mathbf{w}), \mathbf{t}^{\mathbf{e}} = \{\mathbf{w}^{\mathbf{e}}, \mathbf{F}^{\mathbf{e}}\}, \mathbf{p}^{\mathbf{e}} = \hat{\mathbf{p}}\}$ constitutes a bargaining equilibrium.

By construction, in stage 2, for any vector of negotiated wholesale prices $\mathbf{w}$, the prices responses $\mathbf{p}^R(\mathbf{w}) = (\mathbf{p}_j^R(\mathbf{w}))_{j \in \mathcal{J}}$ do constitute a Nash equilibrium. We now turn to stage 1, and study the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. In the equilibrium $\mathcal{E}$, the tariff offered by the selected agent, θ_{ij}, maximizes the expected profit of its firm, among those that are acceptable by the other agent. As profits can easily be shared through the fixed fees (which do not affect retailers' pricing decisions), it follows that the wholesale price $\hat{w}_{ij}$ maximizes the expected joint profit of M_i and R_j, given all the other equilibrium tariffs and the retail price responses. As the equilibrium wholesale prices negotiated with the other firms, $(w_{ik})_{k \neq j}$ and $(w_{hj})_{h \neq i}$, do not depend on which side gets to make the offers, and the equilibrium expected fixed fees are equal to $(F^e_{ik})_{k\neq j}$ and $(F^e_{hj})_{h\neq i}$, it follows that $w_{ij} = \hat{w}_{ij}$ maximizes $\Pi^R_{M_i-R_j}(w_{ij})$, given by (34). To conclude the proof, it suffices to note that, by construction, the fixed fees given by (35) share the gains from trade according to the Nash bargaining rule.

G.2 Proof of Proposition 13

Fix a symmetric equilibrium in which wholesale prices are all equal to w^O, and retail prices are thus equal to $p^O = p^R\left(w^O\right)$. By construction, this price satisfies, for any $i \in \mathcal{I}$ and any $j \in \mathcal{J}$:

$$p^O \in \arg\max_{p_{ij}} \left\{ \begin{array}{c} \left(p_{ij} - w^O - \gamma\right) D_{ij}\left(p_{ij}, \mathbf{p}^O_{-i,j}, \mathbf{p}^O_{-j}\right) \\ +\sum_{h\in\mathcal{I}\setminus\{i\}} \left(p^O - w^O - \gamma\right) D_{hj}\left(p_{ij}, \mathbf{p}^O_{-i,j}, \mathbf{p}^O_{-j}\right) \end{array} \right\}.$$

Letting $q^O = D_{ij}\left(\mathbf{p}^O\right)$ and $d^O = p^O - w^O - \gamma$ respectively denote the symmetric equilibrium quantity and the symmetric downstream margin, we thus have:

$$\begin{aligned} 0 &= q^O + d^O \sum_{h\in\mathcal{I}} \frac{\partial D_{hj}}{\partial p_{ij}}\left(\mathbf{p}^O\right) \\ &= q^O + d^O \sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^O\right), \end{aligned}$$

where the first equality stems from the optimality condition for p_{ij} and the second one follows from symmetry. As $q^O > 0$ and an increase in all of R_j's prices reduces the demand D_{ij}, it follows that the equilibrium downstream margins are positive:

$$d^O > 0. \tag{36}$$

Consider now the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. Following a unilateral deviation in w_{ij}, R_j chooses its prices so as to maximize its variable profit, which (ignoring fixed fees) is given by:

$$\begin{aligned} \pi_j = &\left(p_{ij} - w_{ij} - \gamma\right) D_{ij}\left(\mathbf{p}_j, \mathbf{p}^R_{-j}\left(w_{ij}, \mathbf{w}^O_{-i,j}, \mathbf{w}^O_{-j}\right)\right) \\ &+ \sum_{h\in\mathcal{I}\setminus\{i\}} \left(p_{hj} - w^O - \gamma\right) D_{hj}\left(\mathbf{p}_j, \mathbf{p}^R_{-j}\left(w_{ij}, \mathbf{w}^O_{-i,j}, \mathbf{w}^O_{-j}\right)\right). \end{aligned}$$

Using the envelope theorem, the resulting profit, $\pi^R_j\left(w_{ij}, w^O_{-i,j}, w^O_{-j}\right)$, satisfies:

$$\frac{\partial \pi^R_j}{\partial w_{ij}}\left(w^O\right) = -q^O + d^O \sum_{h\in\mathcal{I}} \sum_{k\in\mathcal{J}\setminus\{j\}} \sum_{g\in\mathcal{I}\setminus\{i\}} \frac{\partial D_{hj}}{\partial p_{gk}}\left(\mathbf{p}^O\right) \frac{\partial p^R_{gk}}{\partial w_{ij}}\left(\mathbf{w}^O\right). \tag{37}$$

In their bilateral negotiation, M_i and R_j choose w_{ij} so as to maximize their (variable) joint profit, which can be expressed as:

$$\begin{aligned}\pi_i + \pi_j &= \pi_j^R\left(w_{ij}, \mathbf{w}_{-i,j}^O, \mathbf{w}_{-j}^O\right) + \left(w_{ij} - c\right) D_{ij}\left(\mathbf{p}^R\left(w_{ij}, \mathbf{w}_{-i,j}^O, \mathbf{w}_{-j}^O\right)\right) \\ &\quad + \sum_{k \in \mathcal{J}\setminus\{j\}} \left(w^O - c\right) D_{ik}\left(\mathbf{p}^R\left(w_{ij}, \mathbf{w}_{-i,j}^O, \mathbf{w}_{-j}^O\right)\right).\end{aligned}$$

Hence, letting $u^O = w^O - c$ and $Q_{M_i}^R(\mathbf{w})$ denote the symmetric equilibrium upstream margin and the total quantity sold by M_i (through all retailers) in the continuation Nash equilibrium $\mathbf{p}^R(\mathbf{w})$, the equilibrium wholesale price $w_{ij} = w^O$ satisfies:

$$\frac{\partial \pi_j^R}{\partial w_{ij}}\left(\mathbf{w}^O\right) + q^O + u^O \frac{\partial Q_{M_i}^R}{\partial w_{ij}}\left(\mathbf{w}^O\right) = 0.$$

From (37), the sum of the first two terms is positive; as M_i's total quantity decreases when it increases any of its wholesale prices, it follows that upstream margins are positive:

$$u^O > 0 \iff w^O > c. \tag{38}$$

As $p^R(w)$ increases in w, this in turn implies that the equilibrium retail prices are above the competitive level:

$$p^O = p^R\left(w^O\right) > p^R(c) = p^*.$$

Alternatively, the (variable) joint profit of M_i and R_j can be expressed as:

$$\begin{aligned}\Pi - \sum_{h \in \mathcal{I}\setminus\{i\}} \pi_h - \sum_{k \in \mathcal{J}\setminus\{j\}} \pi_k &= \Pi^R\left(w_{ij}, \mathbf{w}_{-i,j}^O, \mathbf{w}_{-j}^O\right) - u^O \sum_{h \in \mathcal{I}\setminus\{i\}} Q_{M_h}^R\left(w_{ij}, \mathbf{w}_{-i,j}^O, \mathbf{w}_{-j}^O\right) \\ &\quad - d^O \sum_{k \in \mathcal{J}\setminus\{j\}}^{R} Q_{R_k}^R\left(w_{ij}, \mathbf{w}_{-i,j}^O, \mathbf{w}_{-j}^O\right),\end{aligned}$$

where $\Pi^R(\mathbf{w})$, $Q_{M_h}^R(\mathbf{w})$ and $Q_{R_j}^R(\mathbf{w})$ respectively denote the industry profit, the total quantity sold by M_h (through all retailers) and the total quantity sold by R_j (on all brands) in the continuation Nash equilibrium $\mathbf{p}^R(\mathbf{w})$. It follows that the equilibrium wholesale price $w_{ij} = w^O$ satisfies:

$$\frac{\partial \Pi^R}{\partial w_{ij}}\left(\mathbf{w}^O\right) = u^O \sum_{h \in \mathcal{I}\setminus\{i\}} \frac{\partial Q_{M_h}^R}{\partial w_{ij}}\left(\mathbf{w}^O\right) + d^O \sum_{k \in \mathcal{J}\setminus\{j\}}^{R} \frac{\partial Q_{R_k}^R}{\partial w_{ij}}\left(\mathbf{w}^O\right) > 0,$$

where the inequality stems from (36) and (38), together with the property that, from Assumption A^O, an increase in w_{ij} leads to an increase in the total quantities sold by every other manufacturer M_h, for $h \neq i$, and by every other retailer R_k, for $k \neq j$.

To conclude the proof, let $\hat{\Pi}^R(p)$ and $\tilde{\Pi}(w)$ respectively denote the industry profit obtained when all retail prices are equal to p, and the industry profit in the continuation equilibrium when all wholesale prices are equal to w. By construction, we have:

$$\hat{\Pi}^R\left(p^R(w)\right) = \tilde{\Pi}^R(w)$$

and, by symmetry:

$$\frac{d\hat{\Pi}^R}{dp}\left(p^O\right)\frac{dp^R}{dw}\left(w^O\right) = \frac{d\tilde{\Pi}^R}{dw}\left(w^O\right) = \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\frac{\partial\Pi^R}{\partial w_{ij}}\left(\mathbf{w}^O\right) > 0.$$

As $p^R(w)$ increases with w, it follows that:

$$\frac{d\hat{\Pi}^R}{dp}\left(p^O\right) > 0.$$

The assumed concavity of this industry profit function then implies that p^O lies below the monopoly level:

$$p^O < p^M.$$