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“When is Nonfundamentalness in SVARs A Real Problem?”

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When is Nonfundamentalness in SVARs A Real Problem?

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Abstract

Identification of structural shocks can be subject to nonfundamentalness, as the econometrician may have an information set smaller than the economic agentsí one. How serious is that problem from a quantitative point of view? In this work we propose a simple diagnosis statistics for the quantitative importance of nonfundamentalness in structural VARs. The diagnosis is of interest as nonfundamentalness is not an either/or question, but is a quantitative issue which can be more or less severe. Using our preferred strategy for identifying news shocks, we find that nonfundamentalness is quantitatively unimportant and that news shocks continue to generate significant business cycle type fluctuations when adjust the estimating procedure to take into account the potential nonfundamentalness issue.

Key Words : Non-Fundamentalness, Business Cycles, SVARs, News.

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Introduction

Since Sims [1980], Structural Vector AutoRegressions (SVARs) have become a popular tool for macroeconomists, as they allow to identify the structural shocks that affect the macroeconomy as well as the response to those shocks. In a comment on Blanchard and Quah's [1989] SVAR exercise, Lippi and Reichlin [1993] raised the question of the nonfundamentalness of some structural moving average representations. When the econometrician has less information than the agents in the economy, she might not recover the structural shocks from the present and past observations of the economy. In such a case, the structural moving average representation is nonfundamental. The example given by Lippi and Reichlin [1993] and further developed by Lippi and Reichlin [1994] is the one of a technological diffusion process, for which economic agents act knowing the future development of technology while the econometrician does not have such an information.

If one believes that Dynamic Stochastic General Equilibrium (DSGE) models are a good approximation of the true data generating process, then nonfundamentalness might be more than a theoretical curiosity. Indeed, Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007] have shown that DSGE models may not have a fundamental moving average representation in the structural shocks, so that a SVAR cannot recover the structural shocks. With a quantitative perspective, Sims [2012] then shown that nonfundamentalness is not so much of a either/or problem: there are models in which one can pretty well, if not perfectly, recover structural shocks even with nonfundamentalness, as the information of the econometrician "almost" includes the one of the economic agents.

The questions then becomes an empirical one: can we test whether or not a structural representation of the data is fundamental? Forni and Gambetti [2014] and Forni, Gambetti, and Sala [2014] have suggested to answer this question by testing for the orthogonality of SVAR residuals to a large information set that is well captured by the main factors of a Factor Augmented VAR (FAVAR) model – *i.e.* a VAR model to which is added the main factors of a large model with hundreds of macroeconomic variables, that is likely to contain all the information possessed by economic agents. The "sufficient information" test can detect wether or not the SVAR suffers from nonfundamentalness (under the assumption that the factors contain all the information that is used by the economic agents).[1]

[1]Canova and Hamidi Sahneh [2016] show that the Forni and Gambetti [2014] approach may be problematic when VAR variables are cross sectionally aggregated or proxy for non-observables (spurious nonfundamentalness). They

But as for theory, a either/or test for nonfundamentalness is of limited interest, as it does not tell whether the nonfundamentalness problem is severe or not. The paper first proposes a empirical diagnosis of the nonfundamentalness severity by showing that the R^2 of the projection of innovations of the SVAR on the past factors is indeed a proper measure on that severity.[2] Interestingly, this R^2 has some tight connections with some previous literature on VARs and identification, as we will show that it is a measure of the "anticipation rate" discussed in linear rational expectations models by Ljungqvist and Sargent [2004] and Mertens and Ravn [2010]. It is also directly related to the "Poor Man's Invertibility Condition" of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007], more specifically to the largest eigenvalue of $A - BD^{-1}C$ matrix (using the "ABCD" language of these scholars). An additional contribution of the paper is to explicitly characterize the bias in estimated dynamic responses obtained from a misspecified SVARs (in the sense that it omits the relevant state variables, represented as a set of factors) in terms of the coefficient of determination. As an illustration, we determine analytically the R^2 related upper bounds on the relative bias in the estimated impact responses obtained from a Beaudry and Portier [2006] setup.

We then implement our diagnosis in the case of the identification of technological news shocks. The connection between news shocks and nonfundamentalness is tight: if agents receive some information about future technological improvements, this information might not be embedded in the current information set of the econometrician. The running example of Lippi and Reichlin [1994] when they illustrated nonfundamentalness was indeed a technological diffusion. A key insight of the "news" VARs of Beaudry and Portier [2006] is that the use of asset prices might overcome the nonfundamentalness problem, as they are likely to react strongly to agents' changing views of the future. As Forni, Gambetti, and Sala [2014] have questioned this property and shown that such identified technological news might be tested as nonfundamental, it is of interest to implement our R^2 diagnosis in this case. As we will show, relevant R^2 range between 3% and 21% depending on the specification, and nonfundamentalness appears to be of relative minor importance in practice.

Two modeling issues deserve additional comments. First, we consider that factors (observed by

provide an alternative procedure which is robust to aggregation and non-observability problems. This does not change our main point that a test statistic is not really informative and that our R^2 diagnosis is a more appropriate tool to gauge the severity of non-fundamentalness.

[2] Tests of fundamentalness have also been proposed by Chen, Choi, and Escanciano [2012] and Hamidi Sahneh [2015] for non-Gaussian structural shocks. Departure from Gaussian distribution has been recently considered as a way to technically alleviate the identification problem, but this approach may lack of properly identifying structural shocks without additional economic interpretation (see Kilian and Lütkepohl [2016]).

the econometrician) span the true state of the economy. Under this assumption, we are armed with a simple diagnosis about potential misspecification of the SVAR model. In addition, our approach is thus free of any (DSGE) model identification. As shown in Forni, Giannone, Lippi, and Reichlin [2009], this augmented setup is less (if not) affected by the nonfundamentalness problem, because it includes a sufficient amount of information. Second, our procedure assumes that the omitted factors are known and one may wonder why they are not directly included in the VAR model. Our approach has the advantage of maintaining a parsimonious (small-scaled) VAR model and thus does not require estimating a large number of parameters. A small scale VAR model allows minimizing the root mean square errors of the estimated impulse responses. With our pre-test procedure, we can evaluate if small-scale SVARs are a proper approximation of the true economic dynamic economic structure. In addition, the R^2 diagnosis could be employed as an information criterion to properly select a limited set of relevant variables in the VAR model and thus to recover the structural shock of interest.

The paper is organized as follows. In a first section, we expound the $ABCD/AKC\Sigma$ setups and the R^2 diagnosis. We also illustrate the merits of this diagnosis using a simple asset pricing model and simulations experiments with medium-scale DSGE models. In the second section, we connect the bias that arises from a misspecified VAR model to the R^2. In the third section, we implement the diagnosis in the case of the identification of technological news shocks. A last section concludes.

1 $ABCD/AKC\Sigma$ Setups and the R^2 diagnosis

In this section, we introduce notations for a structural model, its VAR representation and the conditions under which that model is invertible, so that its structural moving average representation is fundamental.[3] We will use the "$ABCD$" language of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007]. Then we will introduce a simple economic example and show how a properly defined R^2 statistics can be informative of the nonfundamentalness severity. We will also perform quantitative experiments using the various DSGE models in Sims [2012] and illustrate the usefulness of our R^2 diagnosis.

[3]Fundamentalness is closely related to the concept of invertibility. Invertibility requires that no root of the determinant of the moving-average representation is on or inside the unit circle. Fundalmentalness requires that no root is inside the unit circle.

1.1 $ABCD/AKC\Sigma$ Setups

Let us consider the following state-space representation

$$x_t = Ax_{t-1} + B\varepsilon_t \tag{1}$$

$$y_t = Cx_{t-1} + D\varepsilon_t, \tag{2}$$

where x_t is a vector of state variables, y_t a vector of observed variables and ε_t a vector of white noise structural shocks with normalized variance. Here we assume that D is invertible.[4] The question is then whether or not one can retrieve the true dynamic and stochastic structure of (1)–(2) from the observation of y_t only. We use here an important result from Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007].

Proposition 1 *A sufficient condition for invertibility is that all the eigenvalues of* $(A - BD^{-1}C)$ *are less than one in modulus.*

The condition that all the eigenvalues of $(A - BD^{-1}C)$ are less than one in modulus is the "poor man invertibility" condition given in Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007]. If Proposition 1 is satisfied, the model (1)–(2) has a VAR representation in y_t and it is possible to uncover the structural shocks.[5] Conversely, when this condition is not satisfied, the model (1)–(2) does not admit a VAR representation in the observables y_t. However, using optimal forecasts of the state variables from the vector of observed variables, we can construct a VAR representation of y_t. We denote this representation as the $AKC\Sigma$ innovation representation. K and Σ represent the Kalman gain and the variance of the forecast error $\Sigma = E\left((x_t - \hat{x}_t)(x_t - \hat{x}_t)'\right)$, i.e. the optimal forecast of the state vector x_t given the observations y_t. The matrices K and Σ are given by

$$\begin{aligned} K &= (A\Sigma C' + BD')(C\Sigma C' + DD')^{-1} \\ \Sigma &= (A - KC)\Sigma(A - KC)' + BB' + KDD'K' - BD'K' - KDB'. \end{aligned}$$

From A, C and K, the optimal forecast of x_t is

$$\hat{x}_t = (A - KC)\,\hat{x}_{t-1} + Ky_t.$$

[4] There exist different situations for which the matrix D is noninvertible. For example, if we assume that shocks are imperfectly observed by private agents, i.e. they receive a noisy signal on the fundamentals, they can not disentangle the true shock from the noise and the matrix D is singular.

[5] This should be understood as a theoretical result, that requires an infinite sample as the VAR has typically an infinite number of lags.

Under weak conditions Hansen and Sargent [2013] show that $(A - KC)$ is a stable matrix, so that this new representation writes as an infinite MA representation in terms of innovations. The measurement equation (2) rewrites as

$$y_t = C\hat{x}_{t-1} + u_t,$$

where the innovations vector u_t is then given by

$$u_t = C\left(x_{t-1} - \hat{x}_{t-1}\right) + D\varepsilon_t.$$

The innovations vector u_t is composed of two orthogonal components and the associated covariance matrix Σ_u is immediately deduced to be

$$\Sigma_u = C\Sigma C' + DD'.$$

Using an identification scheme such that $\Sigma_u = SS'$ (for example a Cholesky decomposition of Σ_u), it comes

$$I = S^{-1}C\Sigma C'S^{-1'} + S^{-1}DD'S^{-1'}.$$

Under the assumption that the factors perfectly account for the forecast errors of the state vector[6], i.e. $(x_{t-1} - \hat{x}_{t-1})$, the R^2 resulting from the linear projection of the i^{th} (standardized) residuals of the $AKC\Sigma$ representation on these factors is given by the (i,i) entries of

$$\begin{pmatrix} R_1^2 & 0 & 0 & \cdots & 0 \\ 0 & R_2^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & R_n^2 \end{pmatrix} = S^{-1}C\Sigma C'S^{-1'}(\equiv I - S^{-1}DD'S^{-1'}),$$

where n is the dimension of the vector u_t. When the system is invertible, the observations on y_t perfectly forecast the state vector x_t and $\Sigma = 0$. It follows immediately that all the R^2 are zero. Thus the R^2 yields a simple diagnosis about the severity of nonfundamentalness. In addition, we can inspect which type of shocks is more or less subject to nonfundamentalness.

1.2 An Example

We consider a simple formulation of the Lucas's tree model, with a single (unexpected or news) shock (See Beaudry, Fève, Guay, and Portier [2015] for simulation experiments with this model in

[6] We will always maintain this assumptions that their exists a set of relevant factors that perfectly reveal the state variables of the economy.

a bivariate setup.) The dividend of a tree, denoted a_t, is assumed to follow the process:

$$a_t = \theta_o \varepsilon_t + \theta_1 \varepsilon_{t-1}, \tag{3}$$

where $\theta_o, \theta_1 \in [0,1]$, $E(\varepsilon_t) = 0$ and $V(\varepsilon_t) = \sigma_\varepsilon^2$. Without loss of generality, we omit a relevant constant term in dividend for simplicity. Since we will consider latter an univariate representation, we can normalize the variance of the shock to unity without loss of generality. In what follow, two polar cases are alternatively investigated: *i)* $\theta_o = 1$ and $\theta_1 = 0$, in this case changes in dividends only result in an unexpected shock; *ii)* $\theta_o = 0$ and $\theta_1 = 1$, dividends are governed by a news shock only.

The representative consumer seeks to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i c_{t+i},$$

subject to the constraint

$$c_t + p_t n_{t+1} = (p_t + a_{t-1}) n_t$$

where E_t is the expectation operator conditional on information in period t, $\beta \in (0,1)$ is the subjective discount factor, c_t denotes consumption, p_t is the price of a tree and n_t is the number of trees in period t. The equilibrium asset price is given by $p_t = \beta E_t(p_{t+1} + a_t)$ and using the transversality condition we deduce the present value equation

$$p_t = \beta E_t \sum_{i=0}^{\infty} \beta^i a_{t+i}. \tag{4}$$

From the stochastic process (3), we obtain

$$E_t a_{t+1} = \theta_1 \varepsilon_t \quad \text{and} \quad E_t a_{t+i} = 0 \;\; \forall i > 1.$$

After replacement into (4), we deduce a MA(1) process for the price

$$p_t = \beta(\theta_o + \beta\theta_1)\varepsilon_t + \beta\theta_1 \varepsilon_{t-1}. \tag{5}$$

In terms of the $ABCD$ representation, eq. (3) and (5) rewrite

$$x_t = \begin{pmatrix} 0 & \theta_1 \\ 0 & 0 \end{pmatrix} x_{t-1} + \begin{pmatrix} \theta_0 \\ 1 \end{pmatrix} \varepsilon_t$$

and

$$y_t = (0, \beta\theta_1)\, x_{t-1} + \beta(\theta_o + \beta\theta_1)\varepsilon_t \;\; ,$$

where $x_t = (a_t, \varepsilon_t)'$ and $y_t = p_t$. Here, we assume that the econometrician only observes the price p_t, but does not observe the dividends a_t. This assumption will simplify our computation, while keeping the main idea.

Let us first consider $\theta_o = 1$ and $\theta_1 = 0$. In terms of $ABCD$, we deduce

$$A - BD^{-1}C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

so the two eigenvalues are zero. In this case, there is no information problem (fundamental case) for the econometrician and the observed price perfectly reveals the state of the economy, i.e. $p_t = \beta a_t \equiv \beta \varepsilon_t$.

Second, we consider the case where $\theta_o = 0$ and $\theta_1 = 1$. In terms of $ABCD$, we obtain

$$A - BD^{-1}C = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{\beta} \end{pmatrix}$$

In this case, the eigenvalues are 0 and $-1/\beta$. For $\beta < 1$, one eigenvalue exceeds unity and the representation is nonfundamental. Using only the price p_t, the econometrician cannot perfectly uncover the true state of the economy. The question is now to investigate how the severity of the nonfundamentalness problem varies with the value of β. Intuitively, small values of β makes the problem more severe as the largest roots exceeds unity a lot. Conversely, for $\beta \to 1$, the largest root is close to unity and this can mitigate the information problem for the econometrician. We now connect the eigenvalue (or β) to our simple R^2 diagnosis, when the econometrician performs a simple linear regression of the residual onto the past relevant variables (factors, lagged once) and then compute the R^2 associated to this simple regression. If she obtains a large R^2, this indicates that the residuals of the regression are not orthogonal to past realization of the factor. Thus, the information problem may be severe and the identification of shocks be seriously biased. In the following proposition, we connect the value of β to the measurement error on the state variables and the R^2 diagnosis.

Proposition 2 *The R^2 of the projection of u_t on the lagged forecast errors of the state variables, i.e. $(x_{t-1} - \hat{x}_{t-1})$, is a decreasing concave function of β and is given by*

$$R^2 = 1 - \beta^2.$$

With news shock, the process is always nonfundamental as long as $\beta < 1$. We see from Proposition 2 a clear link between the severity of nonfundamentalness (the distance between the unstable root

$1/\beta$ and unity) and the R^2 diagnosis. When β is small, the unstable root is large and the R^2 close to unity. This means in this simple example that the R^2 provides useful information about nonfundamentalness. Conversely, for $\beta \rightarrow 1$, the unstable roots tends to unity and the coefficient of determination tends to zero. Notice that β measure the anticipation rate (see Mertens and Ravn [2010] and Ljungqvist and Sargent [2004]), i.e. the amount of which future information about future dividends are incorporated in today prices. If β is small (or close to zero), the current price contains no future information and this reflects into the high value (close to one) of the R^2 diagnosis. Conversely, if $\beta \rightarrow 1$, future information greatly matters and current prices may contain almost all the useful information about future dividends.

1.3 Quantitative Experiments

We now conduct some quantitative experiments to illustrate how the R^2 can be a useful guide for VAR modeling.

First, we use the DSGE models of Sims [2012] to assess the reliability of SVARs. More precisely, Sims considers four model versions. The first, labeled "Full Model" features both real (under the form of habit formation and investment adjustment costs) and nominal (price rigidity) frictions. A second version ("Sticky Price") shuts down real frictions, but includes price rigidity. A third version ("RBC") is a frictionless representation of the economy. Models are hit by two shocks: an unexpected shock and a news shock on TFP. For these three versions the anticipation lag is equal to three. A last version is the full model but the news shock is specified with one period anticipation. For the first three versions, Sims obtains that the the "poor man's invertibility" condition is not satisfied when the econometrician observes the TFP growth and (the log of) output (in deviation from stochastic trend). Conversely, with one lag in the news shocks, this condition is satisfied. Even though the condition for invertibility is not satisfied for the first three models, Sims shows that SVARs yield reliable impulse responses for the structural shocks. We use the state-space representation and the parametrization as reported in Appendix of Sims [2012] and we compute the corresponding coefficients of determination. Table 1 reports the eigenvalues with modulus greater than one and the R^2 associated to the unexpected and news shock on TFP. As the table makes clear, there exists a strong relationship between the size of the eigenvalue and the R^2. For example, in the case of the full model for which the problem of noninvertibility is the most severe, the coefficients of determination are 0.0837 and 0.1374. Conversely, the RBC model yields eigenvalues close to unity and the coefficients of determination become very close to zero. Note also that in the case of a one period anticipation, the condition for invertibility are satisfied and the R^2 are zero. The small values of the R^2 diagnosis coincides with the accuracy of SVAR impulse response functions as reported in Sims [2012].

Table 1: Eigenvalues and R^2

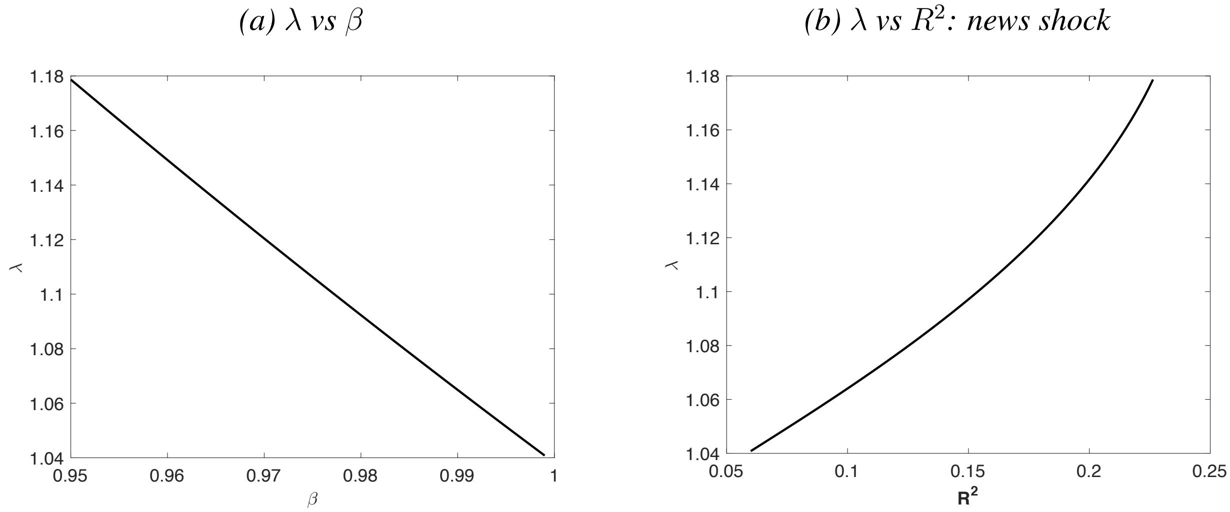
Model	Eigenvalues with modulus greater than one		Coefficient of Determination	
			Unexpected shock	News shock
Full	1.3208	1.3208	0.0837	0.1374
RBC	1.0572	1.0572	0.0365	$1.85\ 10^{-4}$
Sticky Price	1.2397	1.2397	0.0825	0.0143
One Period Anticipation	-	-	0	0

State-space representation and Models in Sims [2012]. The full version is a DSGE model with both real and nominal frictions. The RBC model is a version with no real and nominal frictions. The sticky price model is a version without real friction. For these three versions, the model imposes three period anticipation. One period anticipation is associated to the full model.

Second, we explore the sensitivity of the largest eigenvalue and the R^2 to perturbations in the discount factor β. We conduct simple experiments using a frictionless economy with the same stochastic structure. The model includes two shocks, a news shock and an unexpected shock to TFP. We set three period anticipation for the news shock. The utility is log-log in consumption and leisure. Similar results with other specifications of utility. The calibration of the parameters are the same as in Sims [2012]. As in the simple asset pricing model (see Section 1.2), high values (close to one) for β put more weights on future expectations and thus help the econometrician to better identify news shocks. Conversely, a low value for β will complicate the identification. The range of values for β is $[0.95, 0.999]$. The results are reported in Figures 1. Panel (a) of the Figure shows that the largest eigenvalue of $A - BD^{-1}C$ monotonically decreases as β increases. Panel (b) reports the relationship between this eigenvalue and the R^2 (for the news shock).[7] So, as β increases the value of the coefficient of determination decreases, meaning a diminishing severity of the nonfundalmentalness problem.

[7]A similar shape is obtained for the unexpected shock on TFP.

Figure 1: Discount factor (β), largest eigenvalue (λ) and coefficient of determination (R^2)



2 The R^2 diagnosis in the SVAR setup

This previous section has set the scene by showing the relevance of our R^2 measure in order to asses the severity of nonfundamentalness in the estimation context of VAR models. In this section, we formally make the connection. To do so, we recast the state-space representation (1) and (2) as a FAVAR model by considering that the factors are a proper approximation of the true state variables. We investigate a misspecified VAR representation that would omit the factors and thus not fully capture dynamics of the state variables. Using that misspecified model to identify structural shocks, we show that the bias in recovering these shocks is of the size of the coefficient of determination (aka R^2). The R^2 is obtained from the projection of (misspecified) structural shocks on the past of the factors, which corresponds to the regression proposed by Forni and Gambetti [2014]. We also show that a small R^2 is compatible with a clear rejection of fundamentalness. In such a case, nonfundamentalness is of little quantitative importance.

2.1 The Econometric Setup

Assume that data are generated according to the following FAVAR(1) model,[8] labeled as $\mathcal{M}_0$:

$$\begin{aligned} Y_t &= B_y Y_{t-1} + B_f f_{t-1} + \epsilon_{y,t}, \\ f_t &= C_f f_{t-1} + \epsilon_{f,t}, \end{aligned} \qquad (\mathcal{M}_0)$$

where the vector Y_t contains n variables of interest and f_t is a vector of (observed) relevant q factors. We assume that the variance of each factor is normalized to unity.[9] Our goal is to assess the quantitative effects of omitting the relevant set of factors f_t at the estimation stage and thus when identifying the true structural shocks. The analysis is then similar to a standard omitting variable problem in linear regression.

Assume that there is a unique linear transformation that maps innovations $\epsilon_{y,t}$ into structural shocks η_t according to

$$\epsilon_{y,t} = A_0 \eta_t,$$

where A_0 is a nonsingular matrix. As usual, we impose the normalization assumption that $E(\eta_t \eta_t') = I_n$. This orthogonality/normalization assumption is not sufficient (except if $n = 1$) to identify the structural shocks, since $E(\epsilon_{y,t}\epsilon_{y,t}') = A_0 A_0'$ is symmetric. At least $n(n-1)/2$ restrictions must be imposed to identify A_0. To fix ideas, we assume that identification is achieved by imposing point restrictions in the form

$$\Gamma \text{ vec}\,(A_0) = \gamma, \qquad (6)$$

where Γ is a $(m \times n^2)$ selection matrix and γ a $(m \times 1)$ vector of m restrictions with a dimension of m greater of equal to $n(n-1)/2$. These restrictions can be generalized to other identification schemes. Combining with the covariance matrix, these additional restrictions allow to identify each elements of A_0. $\gamma = 0$ corresponds to a case of zero impact restriction, which is often assumed in the SVAR literature. When $n = 2$, a single zero restriction in A_0 is sufficient to uncover structural shocks. This is for example the case of Beaudry and Portier [2006], in which technological news are identified by imposing that they have no contemporaneous effect on the level of TFP. In what follows, we do not need to be explicit about the identifying restrictions Γ, and will keep the matrix A_0 unspecified.

[8] For the clarity of the presentation, we consider a FAVAR model with one lag only. Results can be easily extended to a more general lags structure.

[9] This representation adds factors in a VAR representation of the data and thus differs from a more general representation of dynamic factor models (see Stock and Watson [2005]).

2.2 The misspecified model

Now, suppose that the econometrician estimates the following misspecified VAR(1) model $\mathcal{M}_1$:

$$Y_t = \tilde{B}_y Y_{t-1} + \tilde{\epsilon}_{yt}, \qquad (\mathcal{M}_1)$$

whereas $\mathcal{M}_0$ constitutes the Data Generating Process of Y_t. We further assume that the econometrician uses the restrictions (6) to identify the structural shocks.

This model improperly ignores the role played by the factors f_t. We are in a typical case of missing relevant variables in VARs.[10] The omitted variables problem will affect the misspecified VAR model $\mathcal{M}_1$ in various ways. First, by omitting the factor f_{t-1}, the VAR(1) model will not properly uncover the size of the shocks, because part of the identified structural shocks will be polluted by the missing factors f_{t-1}. Second, the omitting factor f_{t-1} will affect the dynamics of y_t and the matrix $\tilde{B}_y$ does not properly summarize the true dynamic structure of the economy. Third, the covariance structure of the variables y_t and f_t can affect the proper measurement of the auto-regressive matrix $\tilde{B}_y$ at the estimation stage. In what follows, we explicitly measure the bias of the identified structural shocks by omitting the factors.

The restricted structural shocks (the ones obtained from $\mathcal{M}_1$) are denoted $\tilde{\epsilon}_{yt} = \tilde{A}_0 \tilde{\eta}_t$. We impose the same normalization assumption $E(\tilde{\eta}_t \tilde{\eta}_t') = I_n$ and the same additional restrictions

$$\Gamma \text{ vec}\left(\tilde{A}_0\right) = \gamma,$$

where Γ and γ are the same as in model $\mathcal{M}_0$. Denoting $\tilde{\Sigma} = E(\tilde{\epsilon}_{yt} \tilde{\epsilon}_{yt}')$ and $\Sigma = E(\epsilon_{y,t} \epsilon_{y,t}')$, we deduce $\tilde{A}_0 \tilde{A}_0' = \tilde{\Sigma} \geq \Sigma = A_0 A_0'$ in the matrix sense and $\|\tilde{A}_0\| \geq \|A_0\|$, because the canonical residual omits the factor f_{t-1}. We now examine in more details the effects of omitting f_{t-1} in the estimation of model $\mathcal{M}_1$ and for the identification of structural shocks.

2.3 Testing for nonfundamentalness

Testing for nonfundamentalness for a particular structural shock can be achieved by regressing this structural shock of interest $\tilde{\eta}_{it}$ on the lags of the factors and then performing an orthogonality test as proposed by Forni and Gambetti [2014]. As previously discussed in the introduction, the testing procedure implies that the econometrician observes the factor. It is thus legitimate to wonder why these factors are not directly included in the VAR model. Our approach does not requires

[10] See Stock and Watson [2001], [2005], Canova [2006] and Lütkepohl [2005] for a discussion of this issue.

estimating a large number of parameters and thus allows to minimize the root mean square errors of the estimated impulse responses of interest. All the proofs are in the appendix.

Proposition 3 *For a given sample of size T, the following relation holds between the Wald statistics W_T of the orthogonality test and the R_i^2 of the projection of the (misspecified) structural shocks $\tilde{\eta}_{it}$ on the lags of the factors orthogonal to Y_{t-1}:*

$$W_T = T\frac{R_i^2}{(1-R_i^2)}.$$

The Wald statistic is composed of two terms. The first term T (the size of the sample) refers to the precision of the estimation, since the covariance matrix of the factors has been normalized to identity. The second term R_i^2 accounts for the explanatory power of the lagged factors for the (misspecified) structural shocks $\tilde{\eta}_{it}$.

Corollary 1 *Suppose that the Wald statistics is greater than its critical value, so that the test rejects the fundamentalness of the residuals (or of identified structural shocks from the wrong model). Such a a rejection is compatible with arbitrarily low level of the R^2, and therefore with little quantitative importance on the nonfundamentalness problem, as long as the sample size T is large enough.*

Let us illustrate Corollary 1. Consider a single factor ($q = 1$) in the regression and a sample of size $T = 200$, as very usual in applied time series macroeconomics. In this case, the limiting distribution of the Wald statistic under the null hypothesis that the identified shock $\tilde{\eta}_{it}$ is orthogonal to the lagged factor is a chi-square statistic with one degree of freedom. Its critical value at 5% is 3.84. This implies an associated critical R^2 equals to 0.0192. In words, it is possible to reject fundamentalness even though the lagged factor explain only less than 2% of the variance of the identified structural shock.

Proposition 4 formalizes the relationship between the R^2 of the projection of (misspecified) structural residuals and the distance to the true model.

Proposition 4 *The R^2 statistics are :*

(i) a consistent estimator of the distance between the misspecified impact matrix of structural shocks $\tilde{A}_0$ and the true one A_0. Indeed, when R^2 is small, $\tilde{A}_0$ is close to A_0 and at the limit when $R^2 \to 0$, $\tilde{A}_0$ tends to the true one A_0.

(ii) a consistent estimator of the distance between the misspecified variance decomposition on impact and the true one.

The meaning of Proposition 4 is that if R^2 is small, the distance between the well-specified model $\mathcal{M}_0$ and the misspecified model $\mathcal{M}_1$ is small even if the Wald test rejects fundamentalness.

2.4 Characterization of biases in the canonical bivariate model of Beaudry and Portier [2006]

Consider the identification of technological news shocks in the bivariate model with Total Factor Productivity (TFP) and a measure of Stock Prices (SP). Following Beaudry and Portier [2006], the technological news $\eta_{2,t}$ is the shock that is orthogonal to current TFP. The true model is $\mathcal{M}_0$ with $Y = (TFP, SP)'$, while the econometrician is estimating $\mathcal{M}_1$ with only these two variables. According to the structural assumption, A_0 is lower triangular and given by

$$A_0 = \begin{bmatrix} a_{0,11} & 0 \\ a_{0,21} & a_{0,22} \end{bmatrix}.$$

Under the misspecified model $\mathcal{M}_1$, we maintain the same identifying restriction, such that the misspecified impact matrix $\tilde{A}_0$ is given by

$$\tilde{A}_0 = \begin{bmatrix} \tilde{a}_{0,11} & 0 \\ \tilde{a}_{0,21} & \tilde{a}_{0,22} \end{bmatrix}.$$

Using the same logic than before, we can derive proposition 5.

Proposition 5 *For small R_1^2 and R_2^2 , lhe relative biases of the impact response for the identified structural shocks satisfy:*

$$\begin{aligned} \frac{\widehat{\tilde{a}}_{0,11} - a_{0,11}}{a_{0,11}} &\simeq \frac{1}{2} R_1^2, \\ \frac{\widehat{\tilde{a}}_{0,22} - a_{0,22}}{a_{0,22}} &\leq \frac{1}{2} R_2^2. \end{aligned}$$

The proposition characterizes the relative biases for the estimated impact responses of the TFP to an unexpected technology shock and the stock prices to a news shock. In particular, the relative bias impact response of the stock price to a news shock is smaller than half of the R_2^2. This proposition makes explicit that from a quantitative point of view, it is not the value of the Wald statistics but the size of the R^2 that matters for the bias caused by nonfundamentalness.

3 Application to the identification of TFP news shocks in U.S. data

In this section, we apply the R^2 diagnosis to the results of Beaudry and Portier [2006] and [2014].

3.1 Baseline results

In the following, we use the same sample as used by Forni, Gambetti, and Sala [2014] and use the data described in Beaudry and Portier [2014]. Note that the results of our VARs are robust to a longer sample (1946-2013), but the factors are only available on the shorter sample. TFP is corrected for utilisation, consumption is total consumption (including durable) and investment is total investment (see the data appendix).

We first consider the basic Beaudry and Portier's [2006] bivariate VAR. Whereas the small dimension of the VAR might be a weakness, this VAR has the advantage of being simple and, as discussed in Beaudry and Portier [2014], gives results that are robust to various extensions. The two variables in the system are TFP and Stock Prices. The single identifying restriction is that the identified news has no impact effect on TFP, which correspond to a Choleski decomposition in which TFP is the first variable and the news shock the second shock. Figure 2 shows that we indeed identify a diffusion news. TFP does not increase for about 10 quarters [11], but does in the long run.

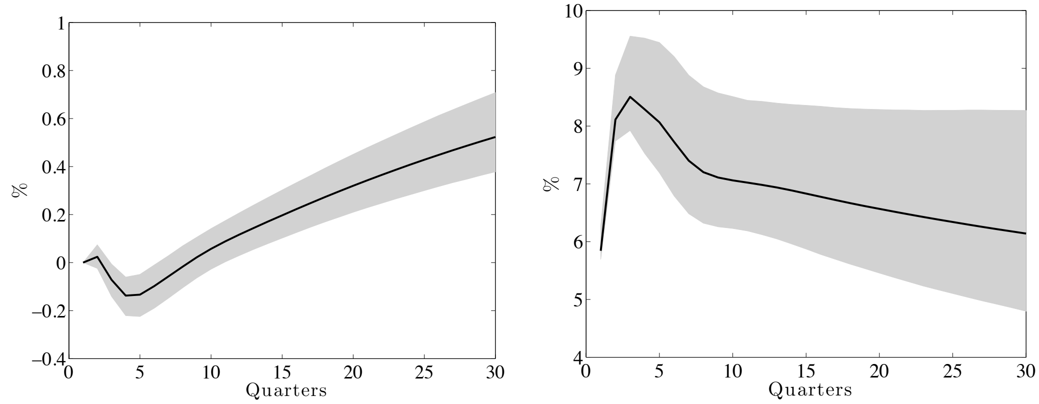
We now extend the VAR to add three extra variables: consumption, investment and hours. To identify a TFP news shock, we follow the identification strategy set out in Beaudry and Portier [2014] which is a natural extension to that introduced in Beaudry and Portier [2006]. This identification strategy only identifies a and an unrestricted technology shock, while the other shocks remain unnamed. The identifying restrictions are the following: *(i)* all the shocks but the unrestricted technology shock have zero impact effect on TFP, *(ii)* the news and the unrestricted technology shock are the only permanent shocks to TFP. In Beaudry and Portier [2014] it is shown that this identification gives robust results when one varies either the information set, the sample period and the specification.

Impulse responses are presented in Figure 3. The plain line shows the point estimates. We observe all the characteristic of a news driven economic expansion. TFP does not move in the

[11] TFP actually decreases, which might be the consequence of an excessive correction for utilization.

Figure 2: Response to a news shock in the Beaudry and Portier's [2006] VAR 2

(a) TFP *(b) Stock Prices*



Data are described in the appendix and the sample period in 1960Q1-2012Q2. The news shock is the one that does not affect TFP on impact. The VAR include 4 lags. The unit of the vertical axis is percentage deviation from the situation without shock. Grey areas correspond to the 66% confidence band. The distribution of IRF is the Bayesian simulated distribution obtained by Monte-Carlo integration with 10,000 replications, using the approach for just-identified systems discussed in Doan [1992].

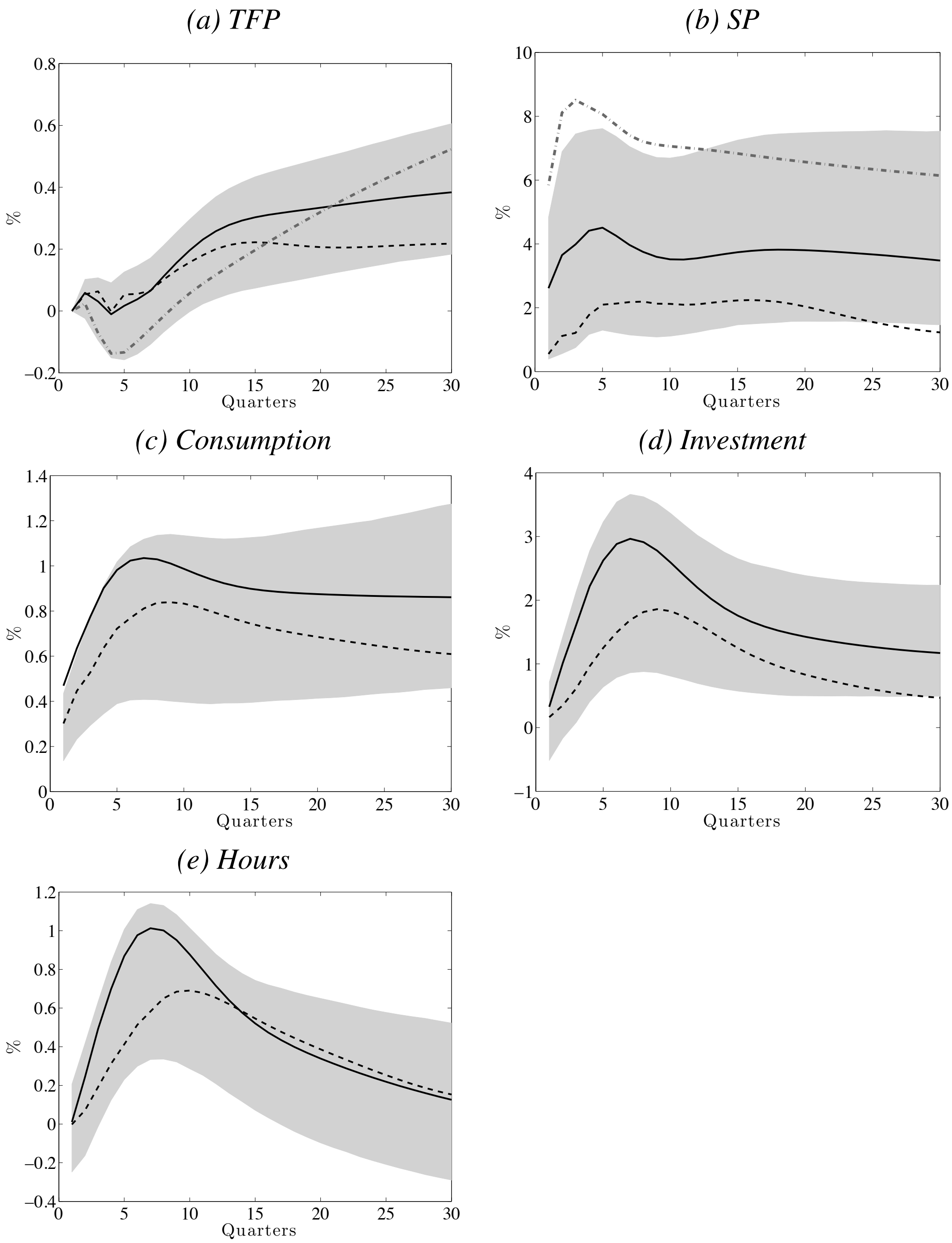
short run, the stock market reacts instantaneously to the news, consumption, investment and hours do increase on impact and subsequently, before any sizable increase in TFP. In panels (a) and (b), we also represent the responses of TFP and SP obtained from the VAR 2 (dashed-dotted gray line). Note that the response of TFP is very similar, while the response of SP is now purged from some non-news related variations.

These results suggest that there are indeed news in the business cycle, but it might be the case, as pointed out by Forni, Gambetti, and Sala [2014], that the estimation suffers from nonfundamentalness. This is what we check now.

3.2 The quantitative unimportance of nonfundamentalness

In order to test for nonfundamentalness, we follow Forni, Gambetti, and Sala [2014]. The authors use a dataset composed of 107 US quarterly macroeconomic series, and estimate the principal components of this data set. They show that essentially all the information is contained in the first three factors. We therefore use these first three factors. We project the estimated news shock of the VAR 2 and of the VAR 5 on one lag or four lags of the first three factors, and test for the orthogonality of our news shocks to the factors. The test is a F-test, and the p-values are reported in Table 2. In all cases, the p-value is less that 5%. We therefore do agree with Forni,

Figure 3: Comparison of the VAR 5 responses with the ones of the VAR 5 augmented with the first three factors



Data are described in the appendix and the sample period is 1960Q1-2012Q2. In the VAR 5 (the plain line), the news shock is restricted to have no impact effect on TFP but is not restricted in the long run. The dotted lines correspond to the VAR 8, that is the VAR 5 augmented with the first three factors of Forni, Gambetti, and Sala [2014]. The dashed-dotted gray lines of panels (a) and (b) are the responses to a news shock in the VAR 2 of Figure 2. The VARs include 4 lags. The unit of the vertical axis is percentage deviation from the situation without shock. Grey areas correspond to the 66% confidence band of the VAR 5. The distribution of IRF is the Bayesian simulated distribution obtained by Monte-Carlo integration with 10,000 replications, using the approach for just-identified systems discussed in Doan [1992].

Gambetti, and Sala [2014] that our identified strategy is likely subject to the nonfundamentalness problem. However, does it matter for the estimation of the impulse response functions to a news shock? The answer we find is no, or at least not very much. A first element suggestive of this negative answer comes from the inspection of the R^2s associated with specification test. These are displayed in Table 2. The R^2s are never larger than .2: even though our estimated news shocks are not orthogonal to the factors, those factors explain less than 20% of the variance of the news. The simulation and theoretical results of the previous section suggest that in such a case, the nonfundamentalness should not be much of a quantitative problem.

We then re-estimate our VAR 5 by adding the three factors, so that we end up estimating a VAR 8. We use the same identification strategy, that is, : *(i)* all the shocks but the unrestricted technology shock have zero impact effect on TFP, *(ii)* the news and the unrestricted technology shock are the only permanent shocks to TFP. The estimated responses to the newly identified news shock are the black dashed lines of Figure 3. Except for the Stock Price whose response has a similar shape but is divided by a factor two, the responses of TFP, consumption, investment and hours are all very similar to that obtained in the absence of including the factors. This contrasts with Forni, Gambetti, and Sala [2014] finding that is based on the identification strategy of Barsky and Sims [2011], which itself is not very supportive of the news shocks view of business cycles. Hence, these results suggests that our chosen means of identifying news shocks generate impulse responses with properties that are robust to the nonfundamentalness critique. There may remain debate about how best to identify new shocks, but that is an issue entirely different form the issue of nonfundamentalness emphasized in Forni, Gambetti, and Sala [2014].[12] We therefore infer that nonfundamentalness is not likely an important factor in evaluating whether or not news shocks are relevant for business cycles.

4 Conclusion

In this paper, we have proposed a simple R^2 diagnosis to asses the severity of nonfundamentalness in SVARs. Building on the $ABCD$ and $AKC\Sigma$ setups, we have shown how the coefficient of determination allows to measure the distance between the structural model and VAR. Using a simple Lucas's tree model with news shock, we have connected the R^2 with the discount factor.

[12] See Beaudry, Nam, and Wang [2011] for some answers to that question.

Table 2: Test for nonfundamentalness and associated R^2s

Model	One lag		Four lags	
	R^2	F-test p-value	R^2	F-test p-value
VAR 2	.03	.04	.18	.05
VAR 5	.09	.01	.21	.01

This Table presents the results of the sufficient information test proposed by Forni and Gambetti [2014]. For each VAR, the news shock is projected on one or four lags of the first three factors of Forni, Gambetti, and Sala [2014]. Table includes the p-value for the orthogonality test, as well as the R^2 of those regressions. Data are described in the appendix and the sample period in 1960Q1-2012Q2. In the VAR 2, the news shock is the one that does not affect TFP on impact.In the VAR 5 , the news shock is only restricted to have no impact effect on TFP but is not restricted in the long run. The VARs are estimated in levels and with 4 lags.

We have also performed quantitative experiments to highlight how the R^2 can be a useful guide for VAR modelling. We then have developed a FAVAR setup and characterized the bias when factors are omitted. We have notably shown hat the relative bias in recovering the true structural shocks is of the order of half the R^2 of the projection of the misspecified structural shocks on the true ones. An application to news shock with US data indicates that nonfundamentalness is present, but it does not appear to matter quantitatively (the R^2 is small). This is not of course a general result that would apply to all SVARs exercises. In fact, the test proposed by Forni and Gambetti [2014] is a useful one that macro-econometricians should systematically perform when nonfundamentalness may be present. However, this test should be accompanied with the computation of the R^2 associated to the test, in order to assess whether nonfundamentalness is likely to be quantitative important.

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Appendix

A Proofs

Proof of Proposition 2 : Using A, B, C and D, we have to solve for the Kalman gain K and the matrix Σ of forecast errors:

$$K = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \equiv (A\Sigma C' + BD')(C\Sigma C' + DD')^{-1}$$

and

$$\Sigma = \begin{pmatrix} V(a_t - \hat{a}_t) & Cov(a_t - \hat{a}_t, \varepsilon_t - \hat{\varepsilon}_t) \\ Cov(\varepsilon_t - \hat{\varepsilon}_t, a_t - \hat{a}_t) & V(\varepsilon_t - \hat{\varepsilon}_t) \end{pmatrix} \equiv \begin{pmatrix} \sigma_{aa} & \sigma_{ae} \\ \sigma_{ea} & \sigma_{ee} \end{pmatrix}$$

Using the innovation representation, the matrix Σ is given by:

$$\Sigma = (A - KC)\Sigma(A - KC)' + BB' + KDD'K' - BD'K' - KDB'$$

Let us first consider the vector K. Using A, B, C and D and given Σ, we deduce

$$k_1 = \frac{\sigma_{ee}}{\beta(\sigma_{ee} + \beta^2)} \quad \text{and} \quad k_2 = \frac{1}{\sigma_{ee} + \beta^2}$$

Now, we insert K into the covariance matrix Σ and we use again A, B, C and D. We obtain

$$\begin{aligned}
\sigma_{aa} &= \frac{\beta^4 \sigma_{ee}}{(\sigma_{ee} + \beta^2)^2} + \frac{\beta^2 \sigma_{ee}^2}{(\sigma_{ee} + \beta^2)^2} \\
\sigma_{ae} &= \sigma_{ea} = -\frac{\beta \sigma_{ee}}{(\sigma_{ee} + \beta^2)^2} \\
\sigma_{ee} &= \frac{\beta^2 \sigma_{ee}}{(\sigma_{ee} + \beta^2)^2} + 1 + \frac{\beta^4}{(\sigma_{ee} + \beta^2)^2} - \frac{2\beta^2}{\sigma_{ee} + \beta^2}
\end{aligned}$$

σ_{aa} and σ_{ae} depends on σ_{ee}, whereas σ_{ee} can be solved independently. After some algebra, we obtain $\sigma_{ee} = 1 - \beta^2$ and we immediately deduce the covariance matrix of forecast errors:

$$\Sigma = (1 - \beta^2) \begin{pmatrix} \beta^2 & -\beta \\ -\beta & 1 \end{pmatrix}$$

and the Kalman gain K

$$K = \begin{pmatrix} \frac{1-\beta^2}{\beta} \\ 1 \end{pmatrix}$$

Now the variance of the residual u_t in the infinite order AR representation of the observed variable y_t is given by

$$\begin{aligned}
\sigma_u^2 &= C\Sigma C' + DD' \\
&= \beta^2(1 - \beta^2) + \beta^4 \\
&= \beta^2
\end{aligned}$$

So the linear regression of u_t on $x_{t-1} - \hat{x}_{t-1}$ (or a factor that accurately represents the state vector) yields a coefficient of determination

$$R^2 = \frac{C\Sigma C'}{\sigma_u^2} \equiv 1 - \beta^2$$

This completes the proof.

Proof of Proposition 3 : The vector of the residuals from the estimation of model $\mathcal{M}_1$ is given by

$$\widehat{\tilde{\epsilon}}_y = M_Y \tilde{\epsilon}_y = M_Y Y,$$

where $\tilde{\epsilon}_y$ is the $T \times n$ of error terms for each of the n equations, Y is the $T \times n$ matrix of the corresponding Y_t and $M_Y = I - Y_{-1}\left(Y_{-1}'Y_{-1}\right)^{-1}Y_{-1}'$ is the orthogonal projection matrix to Y_{-1}, *i.e.* the matrix containing the lagged values of Y. Using model $\mathcal{M}_0$ and the same notations, we deduce

$$M_Y Y \;\; = \;\; M_Y F_{-1} B_f' + M_Y \epsilon_y,$$

where F_{-1} is a $T \times q$ matrix containing the lagged values of the factors. This implies for the estimated misspecified structural shocks

$$\widehat{\tilde{\eta}}\widehat{\tilde{A}_0}' = M_Y F_{-1} B_f' + M_Y \epsilon_y.$$

Since $\widehat{\tilde{\epsilon}}_{yt} = \tilde{\epsilon}_{yt} + o_p(1)$, $\widehat{\tilde{\eta}} = \tilde{\eta} + o_p(1)$ and $\widehat{\tilde{A}_0} = \tilde{A}_0 + o_p(1)$, we obtain [13]

$$\tilde{\eta}\tilde{A}_0' = M_Y F_{-1} B_f' + M_Y \epsilon_y + o_p(1).$$

By $M_Y \epsilon_y = \epsilon_y + o_p(1)$, we can write

$$\tilde{\eta} = M_Y F_{-1} B_f' \left(\tilde{A}_0'\right)^{-1} + \epsilon_y \left(\tilde{A}_0'\right)^{-1} + o_p(1).$$

Using the linear relation between the canonical residuals and the structural shocks, this finally yields

$$\tilde{\eta} = M_Y F_{-1} B_f' \left(\tilde{A}_0'\right)^{-1} + \eta A_0 \left(\tilde{A}_0'\right)^{-1} + o_p(1).$$

Now suppose for the sake of exposition that we are interested in one specific structural shock η_{it}. By the above expression, one gets

$$\tilde{\eta}_{it} = e_i'\tilde{\eta}_t \equiv e_i'\tilde{A}_0^{-1} B_y \hat{f}_{t-1} + e_i'\tilde{A}_0^{-1} A_0 \eta_t + o_p(1), \qquad \text{(A.1)}$$

[13] This holds for each element of the matrix $\tilde{A}_0$. The expression $o_p(1)$ means that this term converges in probability to zero

with e_i a selecting vector that is composed of zeros and one at the ith element. The variable $\hat{f}_{t-1}$ is the orthogonal projection of the lags of the f_{t-1} onto the space generated by Y_{t-1}, namely

$$\hat{f}'_{t-1} = f'_{t-1} - Y'_{t-1}\left(\sum_{t=2}^{T} Y_{t-1}Y'_{t-1}\right)^{-1}\sum_{t=2}^{T} Y_{t-1}f'_{t-1}.$$

We can rewrite equation (A.1) under the form

$$\tilde{\eta}_{it} = e'_i\tilde{\eta}_t = \delta'_i\hat{f}_{t-1} + e'_i\tilde{A}_0^{-1}A_0\eta_t + o_p(1). \tag{A.2}$$

We now define $v_{it} = e'_i\tilde{A}_0^{-1}A_0\eta_t$. In a matrix form, equation (A.2) rewrites:

$$\tilde{\eta}_i = M_Y F_{-1}\delta_i + v_i + o_p(1).$$

with v_i the vector containing individual v_{it}.

Testing for nonfundamentalness for a particular structural shock is achieved by regressing this structural shock of interest $\tilde{\eta}_i$ on the lags of the factors and then performing an orthogonality test on equation (A.2), as proposed by Forni and Gambetti [2014]. As Y_{-1} is correlated with the factors F_{-1} (a natural result, because factors are extracted from macroeconomic variables in Y_t), this does not yield a consistent estimator of δ_i. Therefore, we regress the structural shocks $\tilde{\eta}_i$ on the lags of the factors orthogonal to Y_{t-1}, namely $M_Y F_{-1}$.

The corresponding Wald statistic W_T for the orthogonality test is:

$$W_T = \frac{\hat{\delta}'_i\left(F'_{-1}M_Y F_{-1}\right)\hat{\delta}_i}{\hat{\sigma}^2_{vi}},$$

where $\hat{\delta}_i$ is the consistent estimator of δ_i and $\hat{\sigma}^2_{vi}$ is the estimator of the variance of v_{it}, *i.e.* the error term of the regression of $\hat{\tilde{\eta}}_{it}$ on $\hat{f}_{t-1}$. The coefficient of determination R^2_i associated to the linear regression of $\hat{\tilde{\eta}}_{it}$ on $\hat{f}_{t-1}$ is given by:

$$R^2_i = \frac{\hat{\delta}'_i\left(F'_{-1}M_Y F_{-1}\right)\hat{\delta}_i}{\hat{\delta}'_i\left(F'_{-1}M_Y F_{-1}\right)\hat{\delta}_i + \hat{v}'_i\hat{v}_i}.$$

Using $\hat{v}'_i\hat{v}_i = T \times \hat{\sigma}^2_{vi}$, we obtain the desired result.

Proof of Proposition 4 : The first part of proposition 4 is obtained by using that $V(\widehat{\tilde{\eta}}_{it}) = 1 = \hat{\delta}'_i\left(F'_{-1}M_Y F_{-1}\right)\hat{\delta}_i + \hat{\sigma}^2_{vi}$, so that variance of $\hat{\tilde{\eta}}_{it}$ can be rewritten as [14] $V(\widehat{\tilde{\eta}}_{it}) = R^2_i + \hat{\sigma}^2_{vi}$. The estimator $\hat{\sigma}^2_{vi} = (1 - R^2_i)$ is a consistent estimator of the expression $e'_i\left(\tilde{A}_0\right)^{-1}A_0A'_0\left(\tilde{A}_0\right)^{-1'}e_i$ using the fact that $v_{it} = e'_i\tilde{A}_0^{-1}A_0\eta_t$ and equation (A.2). Thus, the R^2 is a consistent estimator of the distance between the misspecified $\tilde{A}_0$ and the true one A_0. To prove the second part of proposition 4, consider the variance of a variable j attributable to structural shock η_{it}. On impact, it is given by: $e'_j\tilde{A}_0e_iVar(\tilde{\eta}_{it})e'_i\tilde{A}'_0e'_j = e'_j\tilde{A}_0e_iR^2_ie'_i\tilde{A}'_0e'_j + e'_j\tilde{A}_0e_i(1-R^2_i)e'_i\tilde{A}'_0e'_j + o_p(1)$ with $(1-R^2)$ a consistent estimator of $e'_i\left(\tilde{A}_0\right)^{-1}A_0A'_0\left(\tilde{A}_0\right)^{-1'}e_i$ as aforementioned. The R^2 is then a consistent empirical measure of the discrepancy between the misspecified variance on the impact attributable to a particular shock and its true one.

[14] The unit variance of $\widehat{\tilde{\eta}}_{it}$ is just the consequence of the normalization assumption of the structural shocks.

Proof of Proposition 5 : Applying the previous computations to this simple two–variable example yields the following expression for the first structural shock : $\tilde{\eta}_{1t} = \delta_1' \hat{f}_{t-1} + e_1' \tilde{A}_0^{-1} A_0 \eta_t + o_p(1)$ $= \delta_1' \hat{f}_{t-1} + \frac{a_{0,11}}{\tilde{a}_{0,11}} \eta_{1t} + o_p(1)$ and $e_1' \left(\tilde{A}_0\right)^{-1} A_0 A_0' \left(\tilde{A}_0\right)^{-1\prime} e_1 = \left(\frac{a_{0,11}}{\tilde{a}_{0,11}}\right)^2$. Consequently, $(1 - R_1^2)$ is a consistent estimator of this term. Hence, $V\left(\widehat{\tilde{\eta}}_{1t}\right) = \hat{\delta}_1' \left(F_{-1}' M_Y F_{-1}\right) \hat{\delta}_1 + \widehat{\left(\frac{a_{0,11}}{\tilde{a}_{0,11}}\right)}^2 \equiv R_1^2 + (1 - R_1^2)$. For R_1^2 small, a first order expansion this implies that $\widehat{\tilde{a}}_{0,11} \simeq \left(1 + \frac{1}{2} R_1^2\right) a_{0,11}$. So, the relative bias is $\frac{\widehat{\tilde{a}}_{0,11} - a_{0,11}}{a_{0,11}} \simeq \frac{1}{2} R_1^2$. Consider now the second structural shock η_{2t}. Again, using our calculations above yields $\tilde{\eta}_{2t} = \delta_2' \hat{f}_{t-1} + e_2' \tilde{A}_0^{-1} A_0 \eta_t + o_p(1) = \delta_2' \hat{f}_{t-1} + \left[\frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}} \frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}}\right] \eta_{1t} + \frac{a_{0,22}}{\tilde{a}_{0,22}} \eta_{2t} + o_p(1)$. The expression for $\tilde{\eta}_{2t}$ is a function of the relative bias for the three terms in the matrix A_0. This implies the following variance of the second structural shock $V\left(\widehat{\tilde{\eta}}_{2t}\right) = \hat{\delta}_2' \left(F_{-1}' M_Y F_{-1}\right) \hat{\delta}_2 + \widehat{\Theta}^2 + \widehat{\left(\frac{a_{0,22}}{\tilde{a}_{0,22}}\right)}^2 \equiv R_2^2 + (1 - R_2^2)$ where $\Theta = \left[\frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}} \frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}}\right]$. This implies that $\widehat{\Theta}^2 + \widehat{\left(\frac{a_{0,22}}{\tilde{a}_{0,22}}\right)}^2 = (1 - R_2^2)$ Consequently, $\widehat{\tilde{a}}_{0,22} \leq (1 - R_2^2)^{-1/2} a_{0,22} \leq \left(1 + \frac{1}{2} R_2^2\right) a_{0,22} + o\left(R_2^2\right) a_{0,22}$ and $\widehat{\Theta} \leq (1 - R_2^2)^{1/2}$.

B Data

- Hours: BLS, Series Id: PRS85006033, Nonfarm Business sector, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012

- Consumption: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012

- Investment: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012

- TFP: Utilization-adjusted quarterly-TFP series for the U.S. Business Sector, produced by John Fernald, series ID: dtfp_util, 1947Q1-2012Q3, downloaded: 12/2012

- Stock Prices: S&P500 index deflated by CPI, obtained from the homepage of Robert J. Shiller.