TESTING AXIOMATIZATIONS OF AMBIGUITY AVERSION

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Abstract

The study of choice under uncertainty has made major advances using thought experiments. We implement a thought experiment involving a choice between two ambiguous acts that have three outcomes, one being the certainty equivalent of an embedded lottery. Four prominent theories of ambiguity aversion (multiple priors, rank-dependent, smooth ambiguity preferences, variational preferences) predict indifference. Employing a novel method, we elicit, without deception, a subject's certainty equivalent of the embedded lottery. Three experiments are consistent with indifference being rejected. We show independence is sufficient for indifference, find empirically that Allais consistency is associated with indifference, and use recent theory (recursive ambiguity) to explain our results.

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Keywords: Ellsberg paradox, Machina paradox, uncertainty aversion, independence axiom

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1 Introduction

The development of the normative and positive theory of behavior under uncertainty is characterized by a series of thought experiments to which scholars or laypersons often give a "wrong" answer. The St.-Petersburg-Paradox challenged the notion that a lottery will be evaluated by its expected value (de Montmort 1713), which Bernoulli (1738) accommodated with a concave utility function instead of the payoffs themselves, later put on normative foundations by von Neumann and Morgenstern (1944). Allais (1953) subsequently proposed a thought experiment demonstrating that many people do not exhibit the behavior suggested by Bernoulli and von Neumann and Morgenstern's expected utility theory.[1] Ellsberg (1961) further challenged the notion that decision-makers have a single subjective probability distribution (i.e., are probabilistically sophisticated) with a thought experiment involving choice over ambiguity. Empirical papers (for a survey see Camerer and Weber, 1992) showed that people behave differently than probabilistic sophistication prescribes. New models were proposed to accommodate the ambiguity non-neutrality observed in the Ellsberg experiment, e.g., Schmeidler's (1989) Choquet model (or Rank-Dependent Utility); Gilboa and Schmeidler's (1989) maximin expected utility; Klibanoff et al.'s (2005) smooth ambiguity; and Maccheroni et al.'s (2006) Variational Preferences Model. Ambiguity attitudes are now used to explain puzzles in finance[2] and promote policies in health[3], law[4], and the environment[5], to name a few.

A new thought experiment challenges the prevailing four theories. Machina (2014) proposes

[1]This inconsistency gave rise to prospect theory, rank-dependent expected utility, and regret theory to name a few.

[2]Financial economists, e.g. Erbas and Mirakhor (2007) and Maenhout (2004), attribute part of the equity premium to aversion to ambiguity.

[3]Public health initiatives may base their policies on correlations found between measures of ambiguity aversion and unhealthy behavior (Sutter et al., 2013).

[4]Ambiguity aversion is argued to result in plea bargaining that is too harsh, as defendants are typically more ambiguity averse than the prosecutor who also faces a repeated situation. The criminal process therefore is systematically affected by asymmetric ambiguity aversion, which the prosecution can exploit by forcing defendants into harsh plea bargains, as Segal and Stein (2005) contend. Ambiguity aversion has also been applied to contracts (Talley, 2009) and tax compliance (Lawsky, 2013).

[5]Uncertain risks surrounding environmental protection and medical malpractice have led to calls to provide more scientific data on ambiguity aversion in individuals' policy preferences (Viscusi and Zeckhauser 2006; Farber 2010).

two ambiguous acts, where the four models all predict indifference. The thought experiment involves three outcomes (classic Ellsberg urns never have more than two outcomes) as shown in Figure 1. An urn contains 3 balls, exactly 1 of which is red, while the other two could be both white, both black, or one white and one black ball. The outcomes in this Machina thought experiment are monetary prizes of \$0, \$$c$ and \$100, where $\$c \sim (\frac{1}{2}, \$0; \frac{1}{2}, \$100)$, the certainty equivalent of the lottery of receiving \$100 with probability 50% and else \$0.

Act L			Act H		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
\$0	\$$c$	\$100	\$0	\$$c$	\$100

According to Machina, "If ambiguity aversion somehow involves 'pessimism,' mightn't an ambiguity averter have a strict preference for [Act] H over [Act] L, just as a risk averter might prefer bearing risk about higher rather than lower outcome levels?" Indeed, in our experimental implementation, subjects are not indifferent. However, on average subjects prefer Act L over Act H. We use Dillenberger and Segal (2015) and Segal's (1987) recursive ambiguity in combination with Gul's (1991) disappointment aversion to give conditions under which Act L or Act H is preferred. To the best of our knowledge, this paper is the first one to implement the Machina thought experiment.[6]

We describe the methodological challenges to implementing the thought experiment without deception. First, we cannot directly ask subjects to state their true valuation of a lottery and then ask subjects the Machina thought experiment where that just-elicited valuation appears to increase the values of the acts. It ceases to be optimal to state the true value (for example, using Becker-Degroot-Marshak (BDM)), since overstating it at the first stage increases the value of the second stage decision. Subjects reading the instructions for the entire experiment can see how the two tasks are related. Our use of the PRINCE method avoids deception. Moreover, we raise minimal suspicion from subjects (the two stages are clearly

[6] A google scholar search as of January 3, 2019 finds no article that does so to date.

connected, but via the realization of a random draw contained in an envelope, and not via the certainty equivalent) and without deception (we present the full set of instructions prior to subjects making any decisions). The PRINCE method, which uses a choice list, also yields auxiliary data that corroborates the explanation of recursive ambiguity and disappointment aversion. Namely, with the PRINCE method, we observe the direction of switch (from preferring Act L to Act H or vice versa) in the choice list as the value (in the yet-to-be-opened envelope) increases. The envelope is not opened until the end of the experiment.

We contribute evidence that distinguishes between theoretical foundations of ambiguity aversion. Machina also proposed earlier thought experiments in Machina (2009). Machina distinguishes his 2014 thought experiment, which is based on a single source of purely subjective uncertainty, unlike Machina (2009), which is based on two. Baillon et al. (2011) and L'Haridon and Placido (2010) theoretically and empirically investigated Machina's 2009 thought experiment. Their results complement ours, and together, advance the argument that the Machina paradoxes falsify many ambiguity theories, at least in the Anscombe-Aumann framework adopted by those theories with the independence axiom as central. However, we also show that for decision-makers who satisfy independence (we make precise which independence axiom we mean), the Machina thought experiment is problematic. The remainder of the paper is organized as follows. Section 2 presents a theoretical observation on the thought experiment, Section 3 the online implementation, Section 4 the lab, and Section 5 concluding remarks.

2 Machina thought experiment

By replacing $c with the lottery it is induced, the original Machina choice becomes:

	Act L'				Act H'		
	2 balls		1 ball		1 ball	2 balls	
	Black	White	Red		Red	Black	White
$\frac{1}{2}$	**$0**	**$0**	*$100*	$\frac{1}{2}$	**$0**	$0	$100
$\frac{1}{2}$	$0	$100	*$100*	$\frac{1}{2}$	**$0**	*$100*	*$100*

Figure 1: Machina experiment and reduction

Note that \$0 occurs with one-third probability and \$100 occurs with one-third probability. That is, once we substitute the certainty equivalent c with the underlying lottery, the lotteries are now identical in their objective and subjective aspects. Thus, one view of the Machina thought experiment is whether a finding of a strict preference violates four ambiguity aversion models, or does it show a violation of reduction? By reduction, we mean that a decision-maker is indifferent to replacing the certainty equivalent as the prize by its underlying lottery.

We make two additional observations. First, probabilistically sophisticated non-Expected Utility (non-EU) decision makers (DM) can fail to be indifferent. We present an example (disappointment aversion) where decision makers have a strict preference:

2.1 Example of probabilistically sophisticated DM with $Act\, L \nsim Act\, H$ Let the probabilistic sophisticated DM have: $p_B = \frac{2}{3}, p_W = 0$. Then, suppose the DM has non-EU Gul's (1991) disappointment aversion ($\beta > 0$). Then, for any lottery with 2 outcomes $\underline{x} < \overline{x}$ Gul's functional is simply: $v(lottery) = \frac{(1+\beta)p(\underline{x})u(\underline{x})+p(\overline{x})u(\overline{x})}{1+\beta p(\underline{x})}$. Normalize $u(0) = 0$, $u(100) = 100$. Then, $u(c) = v(\$0; \frac{1}{2}, \$100; \frac{1}{2}) = \frac{\frac{1}{2}100}{1+\frac{1}{2}\beta} = \frac{100}{2+\beta}$. Next, $v(L) = \frac{(1+\beta)\frac{2}{3}u(0)+\frac{1}{3}u(100)}{1+\beta\frac{2}{3}} = \frac{100}{3+2\beta}$ and $v(H) = \frac{(1+\beta)\frac{1}{3}u(0)+\frac{2}{3}u(c)}{1+\beta\frac{1}{3}} = \frac{2u(c)}{3+\beta} = \frac{200}{(2+\beta)(3+\beta)}$. Thus $v(H) < v(L) \Rightarrow Act\ L \succ Act\ H$. This example will be used to also explain our findings.

2.2 Non-probabilistically sophisticated EU DM with $Act\ L \sim Act\ H$ Next, we show that for any prior, someone who satisfies the independence axiom will be indifferent. First, the purely objective act is:

Act 0		
1 ball	1 ball	1 ball
Black	White	Red
\$0	\$$c$	\$100

Then, two acts that have ambiguity either at the lower two outcomes or at the higher two outcomes are:

Act L			Act H		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
\$0	\$$c$	\$100	\$0	\$$c$	\$100

Now consider two acts that are constructed by replacing the certainty equivalent with the underlying lottery. Note that the acts have an identical mapping from states to outcomes. This inspires our later claim that the Anscombe-Aumann axiom of Substitution together with Ordering (completeness and transitivity) and the classical independence axiom from expected utility theory are sufficient to imply indifference between Machina's acts L and H.

	Act L'				Act H'		
	2 balls		1 ball		1 ball	2 balls	
	black	white	red		red	black	white
$\frac{1}{2}$	\$0	\$0	\$100	$\frac{1}{2}$	\$0	\$0	\$100
$\frac{1}{2}$	\$0	\$100	\$100	$\frac{1}{2}$	\$0	\$100	\$100

2.3 The Anscombe-Aumann Framework These acts have both subjective events and objective ones, which is why we can represent them in the framework by Anscombe and Aumann (1963). We follow the exposition by Machina and Schmeidler (1995):

$\mathscr{X} = \{..., x, ...\}$ set of outcomes (e.g., money)

$\mathscr{S} = \{..., s, ...\}$ set of states

$R = (x_1, p_1; ...; x_m, p_M)$ a roulette lottery (purely objective)

$H = [x_1 \text{ on } E_1; ...; x_n \text{ on } E_n]$ horse race (purely subjective)

$H^R = [R_1 \text{ on } E_1; ...; R_n \text{ on } E_n]$ horse race

$\mathscr{L} = \{..., \text{L}, ...\}$ the combined set of all pure roulette, pure horse, and horse/roulette lotteries

Thus in our context we have:

The set of outcomes is $X = \{0, c, 100\}$.

The prize c is implicitly defined by $c \sim (\frac{1}{2}; 0, \frac{1}{2}; 100)$.

2.4 State space: balls in urn The state space is which balls are in the urn, thus $S = \{BB, BW, WB, WW\}$.

Act 0 (purely objective):

$[(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$on all states$]$

Act L (ambiguity at low outcomes):

$\big[(\frac{2}{3};0,\frac{1}{3};100)$ on $BB; (\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$ on $BW, WB;(\frac{2}{3}; c, \frac{1}{3}; 100)$ on $WW\big]$

Act H (ambiguity at high outcomes)

$\big[(\frac{1}{3};0,\frac{2}{3};$c) on $BB; (\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$ on $BW, WB;(\frac{1}{3}; 0, \frac{2}{3}; 100)$ on $WW\big]$

Act L' = Act H':

$\big[(\frac{2}{3};0,\frac{1}{3};100)$ on $BB; (\frac{1}{2}; 0, \frac{1}{2}; 100)$ on $BW, WB;(\frac{1}{3}; 0, \frac{2}{3}; 100)$ on $WW\big]$

2.5 State space: ball drawn Instead of using as the state space which balls are in the urn, it might be more natural to think of the state as the ball drawn. Here the difficulty is that the ball drawn mixes objective and subjective events. Thus, we can think of the subjective state space as which ball is drawn conditional on that ball not being red, that is, have $S = \{B, W\}$. Another way of thinking about this is that as the red ball is taken out of the urn, one ball is drawn from the urn (horse race), and then a roulette wheel is spun where one third of the fields are red, whereas the rest of the fields have no color but, say, look at the color of the ball drawn from the urn. This approach has the advantage of yielding far shorter expressions, as it has 2 states instead of 4.

Act 0 (purely objective):

$[(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$on all states$]$.

Act L (ambiguity at low outcomes):

$\big[(\frac{2}{3};0,\frac{1}{3};100)$ on $B; (\frac{2}{3}; c, \frac{1}{3}; 100)$ on $W\big]$

Act H (ambiguity at high outcomes):

$\big[(\frac{2}{3};$c$,\frac{1}{3};0)$ on $B; (\frac{2}{3}; 100, \frac{1}{3}; 0)$ on $W\big]$

Act L' and H':

$\left[(\frac{2}{3};0,\frac{1}{3};100) \text{ on } B; (\frac{1}{3}; 0, \frac{2}{3}; 100) \text{ on } W\right]$

2.6 Informational Symmetry We assume that the DM treats the events B and W as informationally symmetric. Ensuring or assuming information symmetry is particularly important in the context of these acts, as White yields a strictly higher prize in both acts. Informational symmetry means $p_w = p_B$ in the ball draw state space, and $p_{BB} = p_{WW}$ in the ball in the urn state space.

2.7 Indifference between the Acts Under what conditions is a DM indifferent between these Acts? First, observe that by informational symmetry, $p_W = p_B$ (resp. $p_{WW} = p_{BB}$), but then the DM effectively views both L and H as the lottery $(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$, and thus $L \sim H$. But more interestingly, what about non-probabilistically sophisticated decision-makers, when are they indifferent?

2.8 Two kinds of independence As Machina and Schmeidler (1995) explain, Anscombe-Aumann has four axioms, in which the first two, Ordering and Mixture Continuity are related to nonstochastic consumer theory, while the latter two, Substitution and Independence, are related to expected utility. All four together imply probabilistic sophistication (and expected utility). Focus here on three of them, abstracting from Mixture Continuity, which we do not need for present purposes.

Axiom (Ordering) *$\succsim$ is a complete, reflexive and transitive binary relation on $\mathscr{L}$.*

The following is what Machina and Schmeidler (1995) name the Substitution Axiom, which Anscombe and Aumann (1963) called the Monotonicity Axiom:

Axiom (Substitution Axiom) *For any pair of pure roulette lotteries P_i and R_i: If $P_i \succcurlyeq R_i$ then $[P_1 \text{ on } E_1; ..; P_i \text{ on } E_i; ..; P_n \text{ on } E_n] \succcurlyeq [R_1 \text{ on } E_1; ..; R_i \text{ on } E_i; ..R_n \text{ on } E_n]$ for all partitions $\{E_1, ..., E_n\}$ and all roulette lotteries $\{R_1, ..., R_{i-1}, R_{i+1}, ..., R_n\}$.*

The next axiom of Anscombe-Aumann, is an independence axiom, but they generalized it to apply to horse race/roulette lotteries, which is why we call it Horse-Race/Roulette-Independence:

Axiom (Horse-Race/Roulette-Independence Axiom) *For any partition* $\{E_1, ..., E_n\}$ *and roulette lotteries* $\{P_1, ..., P_n\}$ *and* $\{R_1, ..., R_n\}$*:*
If $[P_1 \text{ on } E_1; ...; P_n \text{ on } E_n] \succcurlyeq [R_1 \text{ on } E_1; ...; R_n \text{ on } E_n]$
then $[\alpha P_1 + (1-\alpha)Q_1 \text{ on } E_1; ...; \alpha P_n + (1-\alpha)Q_n \text{ on } E_n]$
$\succcurlyeq [\alpha R_1 + (1-\alpha)Q_1 \text{ on } E_1; ...; \alpha R_n + (1-\alpha)Q_n \text{ on } E_n]$
for all probabilities $\alpha \epsilon (0,1]$ *and all roulette lotteries* $\{Q_1, ..., Q_n\}$*.*

By contrast, the classical Independence Axiom (for pure roulette lotteries from expected-utility theory) is the following, and for clarity, we call it Roulette-Independence:

Axiom (Roulette-Independence Axiom) *For all pure roulette-lotteries* R, P, Q*, and all* $\alpha \epsilon (0,1]$
If $R \succcurlyeq P$ *then* $\alpha R + (1-\alpha)Q \succcurlyeq \alpha P + (1-\alpha)Q$*.*

The Horse Race/Roulette-Independence Axiom implies the Roulette-Independence Axiom, while the converse is not true. Indeed the Horse Race/Roulette-Independence Axiom together with the other 3 Anscombe-Aumann axioms implies probabilistic sophistication, while Roulette-Independence does not. Many major theories of ambiguity aversion (as they are theories that allow for ambiguity non-neutrality) violate the Horse-Race/Roulette Independence Axiom, but satisfy Roulette-Independence:

Remark The Multiple Priors, the Rank-Dependent Model, the Smooth Ambiguity Preferences Model, and the Variational Preferences Model satisfy Roulette-Independence.

2.9 Roulette-Independence: Bernoulli without Bayes?

Claim A decision-maker who satisfies the Ordering, Roulette-Independence, and Substitution Axioms is indifferent between Act L and Act H.

Proof: We prove this separately in both state spaces:

1. State space: Balls in Urn:

By Roulette-Independence, we have $(\frac{2}{3};0,\frac{1}{3};100) \sim (\frac{1}{3}$;0,$\frac{2}{3}$;c), and $(\frac{2}{3};c,\frac{1}{3};100) \sim (\frac{1}{3};0,\frac{2}{3};100)$. But then the Substitution Axiom implies that $L \sim H$, since one can substitute these lotteries on BB and WW, respectively.

2. State space: Ball Drawn:

By Roulette-Independence, we have $(\frac{2}{3};0,\frac{1}{3};100) \sim (\frac{2}{3}$;c,$\frac{1}{3}$;0), and $(\frac{2}{3};c,\frac{1}{3};100) \sim (\frac{1}{3};0,\frac{2}{3};100)$. But then the Substitution Axiom implies that $L \sim H$, since one can substitute these lotteries on B and W respectively. *Q.E.D.*

2.10 Indifference between subjective and objective lottery? Note that Substitution and Roulette-Independence, unlike probabilistic sophistication, do not imply indifference between the horse-race/roulette lotteries L and H on the one hand, and the pure roulette lottery that is Act 0:

EXAMPLE (Multiple Priors) Let us use a simple version of the multiple priors model. Let the priors be $p_W^1 = 0$ and $p_W^2 = 1$. The DM evaluates each Act by the expected utility that nature chooses the worst prior for her. We normalize her Bernoulli utility function with $u(0) = 0$, $u(100) = 100$, which implies $u(c) = 50$. Thus, the DM evaluates the acts as follows: $V(Act\ 0) = \frac{1}{3}0 + \frac{1}{3}c + \frac{1}{3}100 = 50$, $V(Act\ L) = min\left\{\frac{2}{3}0 + 0\cdot c + \frac{1}{3}100, 0\cdot 0 + \frac{2}{3}\cdot c + \frac{1}{3}100\right\} = 33\frac{1}{3}$, $V(Act\ H) = min\left\{\frac{1}{3}0 + \frac{2}{3}\cdot c + 0\cdot 100, \frac{1}{3}0 + 0\cdot c + \frac{2}{3}\cdot 100\right\} = 33\frac{1}{3}$. Thus, while the DM satisfies Roulette-Independence, she still is ambiguity averse as: $Act\ 0 \succ Act\ L \sim Act\ H$.

What about our illustrative Acts L' and H'? Under Substitution and Independence, we have: *Act L'* $\sim$ *Act L* $\sim$ *Act H*.

3 Online Study

The online study used MTurk. This setting allows stakes that can appear low relative to lab settings. MTurkers often do data entry tasks (among other tasks difficult for computers to do but easy for humans to do). A paragraph takes about 100 seconds to enter so a payment of 10 cents per paragraph is equivalent to \$86.40 per day. The current federal minimum wage in the Unites States is \$58/day. In India, payment rate depends on the type of work done, although the "floor" for data entry positions appears to be about \$6.38/day. In one data entry study, one worker emailed (one of the authors) saying that \$0.10 was too high and that the typical payment for this sort of data entry was \$0.03 cents per paragraph. We should see equal proportions for each choice to the extent low stakes bias subjects towards indifference. We use MTurk also to illustrate the intuition for the lab experiment, which is our main contribution.

We had 213 participants in session 1. Instructions are in Appendix A. We replaced $\$c$ with the lottery it is induced by, and asked individuals to choose an urn (lottery). For the purposes of the results discussion and continuity with the theoretical discussion, we refer to Act L' (ambiguity at low outcome) and Act H' (ambiguity at high outcome). The ordering of the urns L' and H' was randomized ("A" and "B" in the instructions were in a fixed order, but assigned arbitrarily) for the subjects. A design choice was the number of balls to put in the urn. Machina parsimoniously fills his opaque urn with 1 known and 2 unknown balls. Experience shows that then some subjects assume some symmetric objective probability distribution is implied, and they mechanically start calculating the resulting distribution of this compound lottery. We avoid this by having 20 known and 40 unknown balls. This serves three purposes. First, it makes the mechanical thoughtless calculation harder. Second, it makes examples better for the experimenter, "for example, 7 black and 33 white balls". Third, Ellsberg also proposed a large number of balls. We found that Act L' was chosen by 123 participants (58%). A two-sided t-test rejects the null hypothesis that this preference for Act L' is random, at a significance level of 5% ($p = 0.0234$).

We had 432 subjects in a second session. Instructions are slightly different and worded in Appendix B. We used oTree (Chen et al. 2016). Among these 432 subjects, 64% preferred Act L'. Appendix C reports demographic correlates of choice for readers who are interested in cross-cultural determinants of ambiguity aversion and demographic determinants of risk aversion (Weber and Hsee 1998; Von Gaudecker et al. 2011). On the basis of the results described thus far, despite the wording being slightly different across the two sessions, subjects appear to act contrary to what Machina thought where 100% of subjects would preference [Act] H over [Act] L. On average, ambiguity at low outcomes was preferred to ambiguity at higher outcomes.

4 Lab Study

4.1 Design We ran the lab experiment at the DeSciL lab following their standard procedures in ETH Zurich using paper-and-pencil, for reasons described below. We had 91 participants across 6 sessions. Rather than replacing $\$c$ with the lottery it is induced by as in Figure 1, we sought to recover $\$c$ through revealed preference. If the decision-maker has a preference relation which satisfies continuity, then a certainty equivalent is guaranteed to exist; strict monotonicity in the monetary outcomes ensures uniqueness. However, the certainty equivalent of a subject is unknown to the experimenter.

The main challenge is to elicit the subject's certainty equivalent prior to conducting the Machina thought experiment. The state-of-the-art method to experimentally elicit willingness to pay for an object is still BDM (Becker et al. 1964). BDM can be implemented by the mechanism itself or a simplified "list" method. In the mechanism, people are asked to state their true valuation, a price is randomly drawn, and they receive the object at the random price if their stated valuation is above it. In the "list" method, people are presented with a list of choices, each consisting of two options, the object and a valuation, and one of the indicated choices is then selected at random. From a formal point of view, the two are close cousins, the difference being that in the list method the valuation one can state is quite coarse.[7] Regardless of the method, subjects are usually told that correctly stating their true valuation is optimal.

However, since the elicited value is later used in the Machina paradox, it ceases to be

[7]Practically, however, there are differences: in the list method, participants may frame each choice as separate, and not view themselves as confronting a big lottery, thus even if independence does not hold, the mechanism would work. The mechanism itself is also quite unusual for non-economists and it is far from obvious to subjects that truth-telling is a dominant strategy. Thus, usually subjects get the opportunity to practice with the mechanism and are explicitly told that correctly stating their true valuation is optimal.

optimal to state the true value, but rather overstating it becomes optimal. Moreover, since the probability of receiving the certainty equivalent in the Machina thought experiment is subjective, it is not possible to correct for that incentive. For these reasons, we use the PRINCE method.[8] The PRINCE (PRior INCEntive system) method is like the list method and formally equivalent to BDM (Johnson et al. 2015). In brief, the choice question (rather than choice options) and implemention is randomly selected before (rather than after) the experiment. It is provided to the subjects in a tangible form (for example in a sealed envelope). Subjects' answers are framed as instructions to the experimenter about the real choice implemented at the end: in the PRINCE method instead of $\$c$, one asks subjects for instructions for which a lottery is preferred for all possible $\$c$ (See Appendix D, especially D.2). It has the advantage over the list method in that it allows any answer, not just an answer on the list (so the valuations are not elicited coarsely). Also, the envelope is already there, and framing as "give us instructions" might lessen concerns of subjects seeing this as a big lottery when eliciting CE. Moreover, reading the instructions makes clear that isolation across tasks is maximally salient. Finally, to further accentuate isolation, the tasks are printed on different colored paper (these colors are reproduced in Appendix D.4). We also offer subjects "indifference" as an option to directly express their indifference rather than infer it from the population (as in the online study).

It is worth highlighting how PRINCE contrasts with the usual BDM for the Machina thought experiment. First, we do not directly ask subjects to state their true valuation of a lottery and then ask subjects the Machina thought experiment where that just-elicited valuation appears to increase the values of the acts. Subjects reading the instructions for the entire experiment would easily realize how the two tasks are related. Note that eliciting valuations of the lottery from subjects without their full awareness of the entire experiment would involve deception. Our use of the PRINCE method avoids deception. The lottery whose valuation is being elicited appears as "Option A" in Task 2. Notice further that the

[8] The PRINCE method was also originally designed to test for endowment effects, so its application to the Machina thought experiment is new.

realization of the random draw, X, is inside an envelope that they hold. This X is then used in the Machina thought experiment. We then ask subjects to choose between the acts for every possible value of X. The connection of the envelope's content across tasks is maximally salient to subjects. What we use, as the experimenter, is the valuation reported in Task 2 to locate the actual comparison of interest among the 20 choice decisions in Task 3. Thus, we raise minimal suspicion from subjects (there is a clear connection between Task 2 and Task 3) and without deception (we present the full set of instructions prior to subjects making any decisions).

To familiarize subjects with PRINCE, we first used it for a first order stochastic dominance (FOSD) task (See Appendix D.1) and then for CE. Since the Machina experiment is implemented with the list method, we can explore if subjects have a unique switching point. A priori it is not clear that people have a unique switching point nor direction.

4.2 Discussion This section discusses the formal conditions under which preferences would imply a threshold. Assume that preferences are strictly monotonic in money. Note that then there should be a certainty equivalent and it should fall between $\$0$ and $\$100$. Now, consider an arbitrary x such that $0 < x < 100$.

Act 1			Act 2		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
$\$0$	$\$x$	$\$100$	$\$0$	$\$x$	$\$100$

Figure 2: Machina experiment

4.2.1. *Which preferences imply a threshold x?*

A natural question is whether we can still make the argument that independence would be a sufficient condition for DM to be indifferent between the lotteries. The answer to that question is no. An example to see why, consider the case of a small x close to 0, and a subjective expected utility (SEU) decision-maker who believes that the probability of black is zero and that of white is $\frac{2}{3}$. This decision maker will prefer *Act H* (which gives her \$100 with probability $\frac{2}{3}$) over *Act L* (which gives her \$100 with probability $\frac{1}{3}$). Consider the following simple example to show how people can switch: $EU(L) = \frac{1}{3}100 + p_W u(x)$ and $EU(H) = p_W 100 + p_B u(x)$.

Example 1, $p_W = 0$:

$EU(L) = \frac{1}{3}100$ and $EU(H) = \frac{2}{3}u(x)$, thus $Act\ L \succcurlyeq Act\ H$ iff $x < c$

Example 2, $p_B = 0$:

$EU(L) = \frac{1}{3}100 + \frac{2}{3}u(x)$ and $EU(H) = \frac{2}{3}100$, thus $Act\ H \succcurlyeq Act\ L$ iff $x < c$

Example 3, $p_W = p_B = \frac{1}{3}$:

$EU(L) = \frac{1}{3}100 + \frac{1}{3}u(x)$ and $EU(H) = \frac{1}{3}100 + \frac{1}{3}u(x)$, thus $Act\ L \sim Act\ H$ for all x

4.2.2. *Probabilistic sophistication and SEU*

Slightly more generally, since SEU implies probabilistic sophistication, we assume that $p(White) = p_w$, where $p_b + p_w = \frac{2}{3}$, and assume $u(0) = 0$, $u(\$100) = 1$. Then $u(c) = \frac{1}{2}$ and $0 < u(x) < 1$. Then $SEU(Act\ L) = p_w \cdot u(x) + \frac{1}{3} \cdot 1$ and $SEU(Act\ H) = p_b \cdot u(x) + p_w \cdot 1$.

Thus, the following holds:

$$\begin{cases} \text{for } p_w > p_b: & Act\ L \succcurlyeq Act\ H \Leftrightarrow x \geq c \\ \text{for } p_w = p_b = \frac{1}{3}: & Act\ L \sim Act\ H \\ \text{for } p_w < p_b: & Act\ L \succcurlyeq Act\ H \Leftrightarrow x \leq c \end{cases}$$

Thus, there might be indifference, or there might be a threshold in one direction or the other.

4.2.3. *Probabilistic sophistication and RDU*

Alternatively, under rank dependent utility (RDU), let the probability distortion/weighting function be f. Given this belief,

$$\begin{aligned} RDEU(Act\ L) &= f(p_w) \cdot 0 + (f(p_w + p_b) - f(p_w)) \cdot u(x) + (1 - f(p_w + p_b)) \cdot 1 \\ &= (f(p_w + b) - f(p_w)) \cdot u(x) + 1 - f(p_w + p_b) \end{aligned}$$

and

$$RDEU(Act\ H) = f(\tfrac{1}{3}) \cdot 0 + \left(f\left(p_w + \tfrac{1}{3}\right) - f\left(\tfrac{1}{3}\right)\right) \cdot u(x) + \left(1 - f\left(p_w + \tfrac{1}{3}\right)\right) \cdot 1$$
$$= \left(f\left(p_w + \tfrac{1}{3}\right) - f\left(\tfrac{1}{3}\right)\right) \cdot u(x) + 1 - f\left(p_w + \tfrac{1}{3}\right)$$

Thus, there are three cases:

$$\begin{cases} \text{for } p_w > p_b: & Act\ L \succ Act\ H \\ \text{for } p_w = p_b = \frac{1}{3}: & Act\ L \sim Act\ H \\ \text{for } p_w < p_b: & Act\ H \succ Act\ L \end{cases}$$

4.2.4. *Non-probabilistically sophisticated beliefs/preferences*

Since the event black always yields a worse outcome than the event white, in this situation the multiple priors model is behaviorally identical to a model with probabilistic sophistication and subjective probability of Black equal to $\frac{2}{3}$, that of White equal to 0. Thus, we are in the case of $p_w < p_b : Act\ L \succcurlyeq Act\ H \Leftrightarrow x \leq c$.

4.3 Results Consistent with the online study, we find that subjects prefer the act with ambiguity at the low outcome relative to the act with ambiguity at the high outcome. Figure 3 easily rejects indifference (only 12 out of 91 subjects explicitly express indifference).

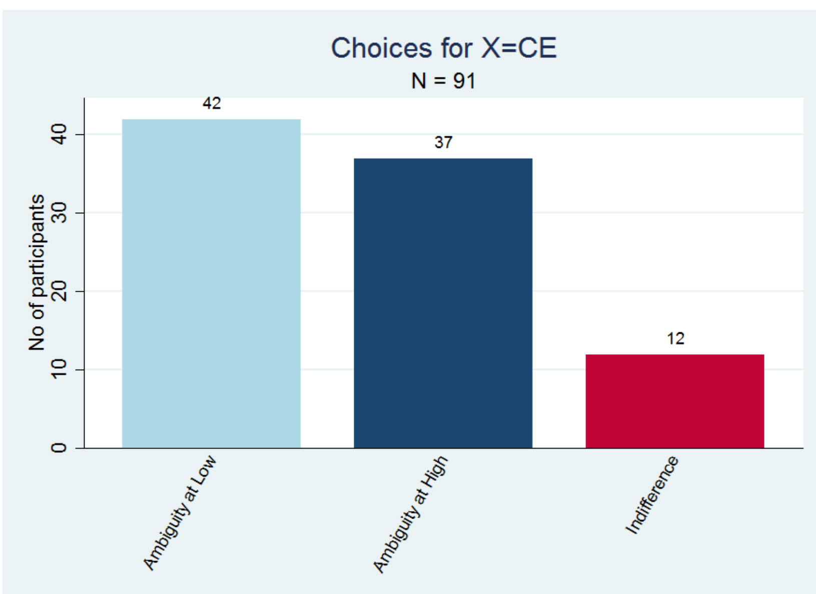


Figure 3: All participants

Next, we use the switching point from the list method to infer indifference between *Act L* and *Act H*. In Figure 3, subjects are classified as indifferent when they are indifferent at their CE (and two neighboring values). Next, we add those who have a clear switching point and their CE lies in the confidence interval of this switching point. In other words, individuals can simply report indifference at $CE \pm 1$. In addition, we can label subjects as indifferent if $CE \in \{S - 1.96 \cdot SD([CE - S]); S + 1.96 \cdot SD([CE - S])\}$, where S is the switching point in Task 3. More precisely, S is the average value between the last A/B and first B/A for single-switchers. SD is calculated for $[CE - S]$.[9] In reality there are people for whom CE strongly differs from S, and thus our confidence interval is too wide. We therefore may overestimate the number of people who are indifferent.

Next, we present an analysis of switching. We present the number of participants who fall into different categories: (i) switch from Ambiguity at Low to Ambiguity at High, (ii) switch from Ambiguity at High to Ambiguity at Low, (iii) always choose Ambiguity at Low, (iv) always choose Ambiguity at High, (v) always indifferent, and (vi) other.

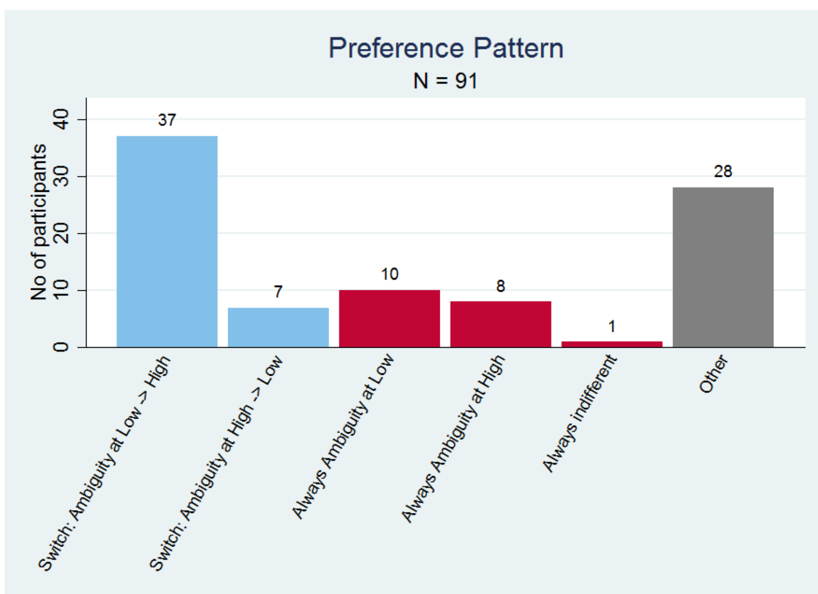


Figure 4: All participants

[9] This means that under the null hypothesis that everyone has $CE = S$, we treat any difference between CE and S as measurement error.

Three results emerge from the tabulation. First, almost a fifth of subjects do not switch. They strictly prefer *Act L* or strictly prefer *Act H*. There is a slight greater preference for ambiguity at low outcomes than for ambiguity at high outcomes. Second, switchers switch from ambiguity at low to ambiguity at high as X increases, which is what one might expect if subjects have a preference for non-ambiguity at high outcomes.[10] Third, there exists many people for whom CE strongly differs from S. Thus even allowing subjects to directly express indifference and inferring as many subjects as possible to be indifferent from their switching points, we can reject indifference in the Machina thought experiment. Appendix E presents additional tabulations that support this claim.

To see additionally how we can reject indifference, we visualize the separation between subjects' CE and switching points. Figure 4 plots the CE on the x-axis and the switching point on the y-axis.

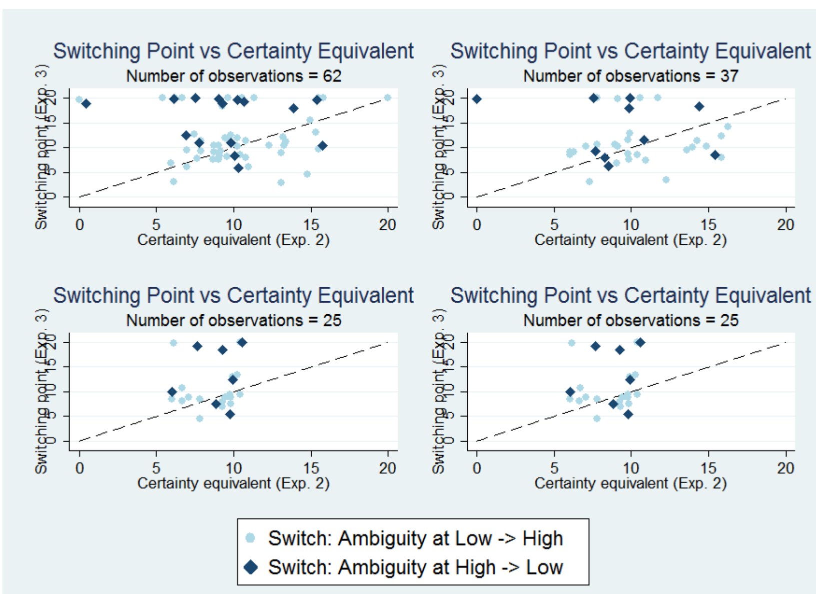


Figure 5: CE vs. Switching point (raw data)

In each subplot, the 45 degree line is the $CE = S$ line. This sample includes people who

[10]This can be seen by considering the extreme case where $X = 20$ and observing that non-ambiguity is now maximized at the high outcomes. The order of the lottery presentation was randomized, but even with the reversed order, the majority of subjects switch from Ambiguity at Low to Ambiguity at High (See Appendix G).

always prefer A or always prefer B (their switching point is represented as 20) and people with single switching points. Each subplot presents a different sample in robustness checks. Clockwise from the upper left: (i) All participants, (ii) $CE \in [4, 10]$, (iii) FOSD, (iv) both. The null hypothesis of indifference at $X = CE$ appears to be rejected because the dots are far away from the 45 degree line. Appendix F visualizes a regression line for "folded" data (we fold the data because we do not want to average the responses of some subjects who switch above their CE and other subjects who switch below their CE) and the confidence interval for the regression line excludes this 45 degree line. A t-test can strongly reject the null that the mean of $abs(CE - S) = 0$ with t-statistic of 7.8.

4.4 Allais and Machina paradoxes Next, we present sub-sample analysis, dividing subjects by whether they are *Allais consistent* (i.e., satisfying independence) or inconsistent. Subjects are classified as indifferent when they express indifference at their CE (and two neighboring values) or when they have a clear switching point and their CE lies in the confidence interval of this switching point. Indifference appears to depend on the answer to Allais (see the questionnaire in Appendix D). Those who are more Allais consistent are more likely to be indifferent, and we previously argued that satisfying the independence axiom is sufficient for indifference.

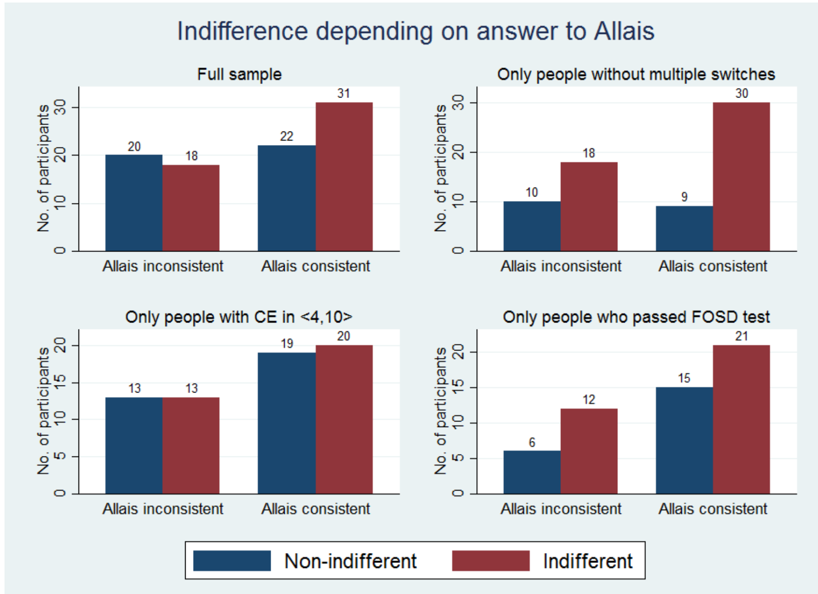


Figure 6: Allais and Machina paradoxes

4.5 Predictions about direction of switch We use Dillenberger and Segal (2015) and Segal's (1987) recursive ambiguity in combination with Gul's (1991) disappointment aversion to give conditions under which *Act L* or *Act H* is preferred. When subjects are disappointment averse, we should observe switching from *Act L* to *Act H*, which is what we found.

The value of Acts are computed as the weighted average of values of first-stage lotteries, with weights being subjective probabilities of different states of the world: BB, BW, WW.

$$W_{Act\ L} = q_{BB} \cdot V_{Act\ L}(BB) + q_{BW} V_{Act\ L}(BW) + q_{WW} V_{Act\ L}(WW)$$

$$W_{Act\ H} = q_{BB} \cdot V_{Act\ H}(BB) + q_{BW} V_{Act\ H}(BW) + q_{WW} V_{Act\ H}(WW)$$

Since terms for state BW are the same for both urns (same payoffs), we may neglect them for comparison purposes. Let's now take Gul's disappointment aversion model with β as the disappointment aversion parameter:

$$V_{Act\ L}(BB) = \frac{\frac{2}{3}(1+\beta)\cdot 0 + \frac{1}{3}\cdot 100}{1+\frac{2}{3}\beta} = \frac{100}{3+2\beta}$$
$$V_{Act\ L}(WW) = \frac{\frac{2}{3}(1+\beta)\cdot X + \frac{1}{3}\cdot 100}{1+\frac{2}{3}\beta} = \frac{100+2(1+\beta)X}{3+2\beta}$$
$$V_{Act\ H}(BB) = \frac{\frac{1}{3}(1+\beta)\cdot 0 + \frac{2}{3}\cdot X}{1+\frac{1}{3}\beta} = \frac{2X}{3+\beta}$$
$$V_{Act\ H}(WW) = \frac{\frac{1}{3}(1+\beta)\cdot 0 + \frac{2}{3}\cdot 100}{1+\frac{1}{3}\beta} = \frac{200}{3+\beta}$$

So *Act L* is preferred to *Act H* if:

$$q_{BB}\tfrac{100(1+\beta)}{3+2\beta} + q_{WW}\tfrac{100+2(1+\beta)X}{3+2\beta} > q_{BB}\tfrac{2X(1+\beta)}{3+\beta} + q_{WW}\tfrac{200}{3+\beta}$$

For $q_{WW} = q_{BB}$ (assuming equal probabilities of having two black balls or two white balls)[11]: $100\beta > 2X\beta$

We now divide by β. Let's first assume that $\beta > 0$:

$$50 > X$$

So if $X < 50$, *Act L* is preferred over *Act H*. Therefore, as X increases we should observe a switch from *Act L* to *Act H*, which is what we find.

If we now go back and assume that $\beta < 0$:

[11] Derivation:

$$\frac{100(1+\beta)}{3+2\beta} + \frac{100+2(1+\beta)X}{3+2\beta} > \frac{2X(1+\beta)}{3+\beta} + \frac{200}{3+\beta} \cdot (3+\beta)(3+2\beta)$$
$$2(1+\beta)(3+\beta)100 + 2(1+\beta)(3+\beta)X > (3+2\beta)2X(1+\beta) + 200(3+2\beta)$$
$$600+300\beta+6X(1+\beta)+200\beta+100\beta^2+2X(1+\beta)\beta > 600+6X(1+\beta)+400\beta+4X(1+\beta)\beta$$

$$50 < X$$

So if $X > 50$, *Act L* is preferred over *Act H*. Therefore, as X increases we should observe a switch from *Act H* to *Act L*.

5 Concluding Remarks

The thought experiment we test is the latest in a series of seminal thought experiments to push the frontiers of both theoretical and empirical research on choice under uncertainty. In this thought experiment, major theories of ambiguity aversion predict indifference. We argue that probabilistically sophisticated non-EU DM can fail to be indifferent. We present an example (disappointment aversion) where decision makers have a strict preference. Second, someone who satisfies the independence axiom will be indifferent. Machina's thought experiment to test of major theories of ambiguity non-neutrality appears at least as much a test of independence as of ambiguity aversion. We overcome a challenge to implementing Machina's thought experiment, which requires knowledge of a subject's certainty equivalent, using the PRINCE method. We also find a strong pattern in which way people shift (in our elicitation of Machina's thought experiment). This shift is used to support recursive ambiguity as an axiomitization of ambiguity aversion.

References

Allais, Maurice, "Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine," *Econometrica: Journal of the Econometric Society*, 1953, pp. 503–546.

Anscombe, F. J. and R. J. Aumann, "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, 1963, *34* (1), pp. 199–205.

Baillon, Aurélien, Olivier L

Haridon, and Laetitia Placido, "Ambiguity models and the Machina paradoxes," *The American Economic Review*, 2011, *101* (4), 1547–1560.

Becker, Gordon M., Morris H. DeGroot, and Jacob Marschak, "Measuring Utility by a Single-Response Sequential Method," *Behavioral Science*, 1964, *9* (3), 226–232.

Bernoulli, Daniel, "Exposition of a New Theory on the Measurement of Risk," *Commentarii Academiae Scientiarum Imperialis Petropolitanae as translated in Econmetrica 22(1), 1954*, 1738, *V*, 175–192.

Borghans, Lex, James J. Heckman, Bart H. H. Golsteyn, and Huub Meijers, "Gender differences in risk aversion and ambiguity aversion," *Journal of the European Economic Association*, 2009, *7* (2/3), 649 – 658.

Camerer, Colin and Martin Weber, "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity," *Journal of Risk and Uncertainty*, 1992, *5* (4), 325–370.

Chen, Daniel L., Martin Schonger, and Chris Wickens, "oTree—An open-source platform for laboratory, online, and field experiments," *Journal of Behavioral and Experimental Finance*, March 2016, *9*, 88 – 97.

de Montmort, Pierre Remond, *Essay d'analyse sur les jeux de hazard* 1713.

Dillenberger, David and Uzi Segal, "Recursive Ambiguity and Machina's Examples," Technical Report, University of Pennsylvania 2015.

Ellsberg, Daniel, "Risk, Ambiguity, and the Savage Axioms," *The Quarterly Journal of Economics*, 1961, *75* (4), pp. 643–669.

Erbas, S. Nuri and Abbas Mirakhor, "The Equity Premium Puzzle, Ambiguity Aversion, and Institutional Quality," Technical Report, IMF Working Paper September 2007.

Farber, Daniel A, "Uncertainty," *Geo. LJ*, 2010, *99*, 901.

Gaudecker, Hans-Martin Von, Arthur Van Soest, and Erik Wengström, "Heterogeneity in risky choice behavior in a broad population," *The American Economic Review*, 2011, *101* (2), 664–694.

Gilboa, Itzhak and David Schmeidler, "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 1989, *18* (2), 141 – 153.

Gul, Faruk, "A Theory of Disappointment Aversion," *Econometrica*, 1991, *59* (3), 667–686.

Johnson, Cathleen A, Aurélien Baillon, Han Bleichrodt, Zhihua Li, Dennie Van Dolder, and Peter P Wakker, "Prince: An improved method for measuring incentivized preferences," *Available at SSRN 2504745*, 2015.

Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji, "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, 2005, *73* (6), pp. 1849–1892.

Lawsky, Sarah B, "Modeling uncertainty in tax law," *Stanford law review*, 2013, *65.*

L'Haridon, Olivier and Laetitia Placido, "Betting on Machinas reflection example: an experiment on ambiguity," *Theory and Decision*, 2010, *69* (3), 375–393.

Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini, "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica*, 2006, *74* (6), pp. 1447–1498.

Machina, Mark J., "Risk, Ambiguity, and the Rank-Dependence Axioms," *The American Economic Review*, 2009, *99* (1), pp. 385–392.

———, "Ambiguity aversion with three or more outcomes," *American Economic Review*, 2014, *104* (12), 3814–3840.

——— **and David Schmeidler**, "Bayes without Bernoulli: Simple Conditions for Probabilistically Sophisticated Choice," *Journal of Economic Theory*, 1995, *67* (1), 106 – 128.

Maenhout, Pascal J., "Robust Portfolio Rules and Asset Pricing," *Review of Financial Studies*, 2004, *17* (4), 951–983.

Schmeidler, David, "Subjective Probability and Expected Utility without Additivity," *Econometrica*, 1989, *57* (3), pp. 571–587.

Segal, Uzi, "The Ellsberg paradox and risk aversion: An anticipated utility approach," *International Economic Review*, 1987, *28* (1), 175–202.

——— **and Alex Stein**, "Ambiguity Aversion and the Criminal Process," *Notre Dame L. Rev.*, 2005, *81*, 1495.

Sutter, Matthias, Martin G. Kocher, Daniela Glaetzle-Ruetzler, and Stefan T. Trautmann, "Impatience and Uncertainty: Experimental Decisions Predict Adolescents' Field Behavior," *American Economic Review*, 2013, *103* (1), 510–31.

Talley, Eric L, "On Uncertainty, Ambiguity, and Contractual Conditions," *Delaware Journal of Corporate Law (DJCL)*, 2009, *34* (3).

Viscusi, W.Kip and RichardJ. Zeckhauser, "The Perception and Valuation of the Risks of Climate Change: A Rational and Behavioral Blend," *Climatic Change*, 2006, *77* (1-2), 151–177.

von Neumann, John and Oskar Morgenstern, *Theory of Games and Economic Behavior* 1944.

Weber, Elke U and Christopher Hsee, "Cross-cultural differences in risk perception, but cross-cultural similarities in attitudes towards perceived risk," *Management science*, 1998, *44* (9), 1205–1217.

Appendix:

A Instructions For Online Study (session 1)

5. There is an urn with 20 red balls and 40 white and black balls. The number of the white and black balls is unknown, and can be anything between 0 and 40 white balls and 0 and 40 black balls. At random, one ball will be drawn from this urn. You can pick either Lottery A or Lottery B to get paid. The payments under the two lotteries are as follows:

Lottery A: If a red ball is drawn, you will be paid 30 cents. If a black ball is drawn, you will be paid nothing. If you pick a white ball, there is a one in two chance that you will be paid 30 cents, and there is a one in two chance that will be paid 0 cents.

Lottery B: If a red ball is drawn, you will be paid 0 cents. If a black ball is drawn, there is a one in two chance that you will be paid 30 cents, and there is a one in two chance that will be paid 0 cents. If a white ball is drawn, you will be paid 30 cents.

Which one will you pick? *

- ◯ Lottery A
- ◯ Lottery B

Appendix Figure A.1

B Instructions For Online Study (session 2)

Pick a lottery

There is an urn containing 60 balls. Exactly 20 of these 60 balls are red. The remaining 40 balls are white or black. The number of the white and black balls is unknown, so there can be anything between 0 and 40 white balls and 0 and 40 black balls.

At random, one ball will be drawn from this urn. At the same time a coin is flipped. The coin can land only on head or tail. The coin is fair, that is to say symmetric. You can pick either Alternative A or Alternative B to get paid. The payments under the two alternatives are as follows:

Alternative A: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid $0.50. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid $0.00. If a white ball is drawn: If the coin has landed on its heads you will be paid $0.50. If it lands on its tails, you will be paid $0.00.

Alternative B: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid $0.00. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid $0.50. If a white ball is drawn: If the coin has landed on its heads you will be paid $0.50. If it lands on its tails, you will be paid $0.00 .

Lottery choice*

A

Next

Pick a lottery

There is an urn containing 60 balls. Exactly 20 of these 60 balls are red. The remaining 40 balls are white or black. The number of the white and black balls is unknown, so there can be anything between 0 and 40 white balls and 0 and 40 black balls.

At random, one ball will be drawn from this urn. At the same time a coin is flipped. The coin can land only on head or tail. The coin is fair, that is to say symmetric. You can pick either Alternative A or Alternative B to get paid. The payments under the two alternatives are as follows:

Alternative A: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid $0.00. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid $0.50. If a white ball is drawn: If the coin has landed on its heads you will be paid $0.50. If it lands on its tails, you will be paid $0.00 .

Alternative B: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid $0.50. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid $0.00. If a white ball is drawn: If the coin has landed on its heads you will be paid $0.50. If it lands on its tails, you will be paid $0.00.

Choose an alternative:

A

Next

Appendix Figure A.2: Choice of lottery

C Demographic Correlates of Choice

We also had demographic characteristics for 333 subjects. In linear probability models, Republicans were 22 percentage points more likely to prefer Act L'. Americans were 48 percentage points and Asians were 27 percentage points more likely to prefer Act H'. Marginal effects from logit and probit models were similar. We did not see significant differences in choice of ambiguity at high or low outcomes by gender (which is the focal demographic heterogeneity of a recent study on gender differences in ambiguity aversion (Borghans et al. 2009)).

Correlates of Urn A Choice		
	(1)	(2)
	chooseA	chooseA
Mean dep. Var.	0.36	0.37
Male		0.0564
		(0.0559)
Age		0.00200
		(0.00249)
Republican		-0.215**
		(0.102)
Democrat		-0.0398
		(0.0842)
American		0.475*
		(0.280)
Indian		0.438
		(0.290)
Black		0.112
		(0.120)
Hispanic		0.116
		(0.116)
Native American		-0.0419
		(0.173)
Asian		0.270**
		(0.107)
Hindu		0.0489
		(0.115)
Catholic		-0.0594
		(0.0934)
Religious Services		0.00468
		(0.0218)
Constant	0.359***	-0.260
	(0.0231)	(0.291)
N	432	333
R-sq	0.000	0.107

Standard errors in parentheses

* p<0.10 ** p<0.05 *** p<0.01

Appendix Figure A.3: Regression analysis

D Instructions For Lab Study

The first task is the first order stochastic dominance task. The second task is the CE task. The third task is the Machina task. The fourth task is a short survey questionnaire shown at the end.

D.1 First Order Stochastic Dominance Task Note that first order stochastic dominance implies that option B is always preferred when X is less than 7.

Option A	
Balls in drum	Money you get when a ball of this color is drawn
4 red balls	CHF X
3 white balls	CHF 9
3 black balls	CHF 7

Option B	
Balls in drum	Money you get when a ball of this color is drawn
2 red balls	CHF X
3 white balls	CHF 9
5 black balls	CHF 7

Appendix Figure A.4: Envelope content - FOSD

□ If X is below or equal to CHF _ _, _ _ then I want **Option A,** else I will receive **Option B.**

□ If X is below or equal to CHF _ _, _ _ then I want **Option B,** else I will receive **Option A.**

Appendix Figure A.5: Answer sheet - FOSD

D.2 Certainty Equivalent Task (PRINCE method) Note that someone who is risk averse would write down X less than 10.

The drum is filled with 20 balls, 10 of which are white and 10 of which are black. The experimenter will draw one ball from the drum at random.

Option A
If the ball is black you get CHF 20. If the ball is white you get nothing.

Option B
Regardless of what color is drawn, you get **CHF X**.

Appendix Figure A.6: Envelope content - CE

IF X is below or equal to CHF [__ __,__ __] I want Option A, otherwise I will receive Option B.

Appendix Figure A.7: Answer sheet - CE

D.3 Machina Task Note that someone who satisfies SEU would have a unique switching point when X is CE.

The ball will be drawn from a drum with 20 red balls and 40 balls which may be any combination of white and black balls. You do not know exactly how many white/black balls are in the drum. There are 60 balls in total in the drum. You can choose one of two options. The option payoff is dependent on the color of the ball drawn from the drum:

	20 balls	40 balls	
	Red ball drawn	Black ball drawn	White ball drawn
Option A	CHF 0	CHF **X**	CHF 20
Option B	CHF 20	CHF 0	CHF **X**

Appendix Figure A.8: Envelope content - Machina

If X is....	...I want Option A	...I want Option B	I am indifferent
CHF 0			
CHF 1			
CHF 2			
CHF 3			
CHF 4			
CHF 5			
CHF 6			
CHF 7			
CHF 8			
CHF 9			
CHF 10			
CHF 11			
CHF 12			
CHF 13			
CHF 14			
CHF 15			
CHF 16			
CHF 17			
CHF 18			
CHF 19			
CHF 20			

Appendix Figure A.9: Answer sheet - Machina

D.4 Complete Instructions For completeness, we include all relevant information seen by the subjects. The original colors for the experiment tasks are reproduced.

EXPERIMENT 1 Participant number: _ _

Please now enter your participant number in the space above.

In this first of 3 experiments you will decide between two options. In both options your payoff depends on the result of a random draw from a drum filled with balls and some unknown amount CHF X.

On the table in the middle of the room there is a drum. We will conduct 2 draws. First we will fill the drum as shown in the table "Option A" below and randomly draw a single ball. Then we will fill the drum as shown in the table for "Option B" below, and randomly draw a single ball.

When asked by the experimenter to do so, draw one sealed white envelope. Each participant will draw a white envelope. Each of these envelopes contains a note. The notes are identical except for the random value X which differs. The CHF X is a random number between CHF 0,00 and CHF 20,00. All possible numbers have equal probability.

The value of X is printed on the note inside the sealed envelope. You will only learn X once experiment 1 is over. Thus DO NOT OPEN YOUR ENVELOPE. It will be opened later by an experimenter in your presence. If you open your envelope yourself, you will not get paid for this experiment.

Here is what the note in each of the envelopes looks like:

Option A	
Balls in drum	Money you get when a ball of this color is drawn
4 red balls	CHF X
3 white balls	CHF 9
3 black balls	CHF 7

Option B	
Balls in drum	Money you get when a ball of this color is drawn
2 red balls	CHF X
3 white balls	CHF 9
5 black balls	CHF 7

You can choose whether you want to get Option A or Option B. But since your envelope may contain any value of X between 0,00 and 20,00 please give us general instructions whether you want Option A or Option B depending on the value of X.

Do so by selecting **one** of the following options and indicating a threshold:

□ If X is below or equal to CHF _ _, _ _ then I want **Option A**, else I will receive **Option B**.

□ If X is below or equal to CHF _ _, _ _ then I want **Option B**, else I will receive **Option A**.

Please give this sheet to the experimenter when asked to do so. He will later return it to you, and in the end you have to hand it to the cashier to get paid.

Appendix Figure A.10: FOSD Task

EXPERIMENT 2 Participant number: _ _

Please now enter your participant number in the space above.

In this experiment you will decide whether you prefer to play a lottery, in which the payoff is dependent on the color of a ball randomly drawn from a drum, or to receive a guaranteed amount of money.

When asked by the experimenter to do so, draw one sealed green envelope. Each participant will draw a green envelope. Each of these envelopes contains a note. The notes are identical except for the random value X which differs. The CHF X is a random number between CHF 0,00 and CHF 20,00. All possible numbers have equal probability.

The value of X is printed on the note inside the sealed envelope. You will only learn X once experiment 2 is over. Thus DO NOT OPEN YOUR ENVELOPE. It will be opened later by an experimenter in your presence. If you open your envelope yourself, you will not get paid for this experiment.

In this experiment 2 the note has two options: participating in the lottery or receiving a guaranteed amount of money. The guaranteed amount of money is CHF X.

Here is what the note in each of the envelopes looks like:

> The drum is filled with 20 balls, 10 of which are white and 10 of which are black. The experimenter will draw one ball from the drum at random.
>
> **Option A**
> If the ball is black you get CHF 20. If the ball is white you get nothing.
>
> **Option B**
> Regardless of what color is drawn, you get **CHF X**.

Since your envelope may contain any value of X between 0,00 and 20,00 please give us general instructions whether you want Option A or Option B. Do so by specifying a threshold:

IF X is below or equal to CHF [_ _,_ _] I want Option A, otherwise I will receive Option B.

Please give this sheet to the experimenter when asked to do so. He will later return it to you, and in the end you have to hand it to the cashier to get paid.

Appendix Figure A.11: CE Task

EXPERIMENT 3 **Participant number: _ _**

Please now enter your participant number in the space above.

In experiment 3 you will choose one of two options. In both options your payoff depends on the result of a random draw from a drum filled with balls and some unknown amount CHF X.

On the table in the middle of the room there is a drum. We will conduct a single draw. The experimenter will show you the drum with 20 red balls already in it. He will also show you a box with 40 white balls, and a box with 40 black balls. Later, in secret he will add exactly 40 white and black balls to the drum in addition to the 20 red ones already in the drum. The additional 40 balls may be any combination of black and white balls. The experimenter could for example add 7 black balls and 33 white balls or just add 40 white balls, or he could add 12 white balls and 28 black balls, or.... He can do whatever he likes as long as the sum of white and black balls in the drum is exactly 40, and the number of red balls in the drum remains at 20.

When asked by the experimenter to do so, draw one sealed blue envelope. Each participant will draw a blue envelope. Each of these envelopes contains a note. The notes are identical except for the random value X which differs. The CHF X is a random number between CHF 0 and CHF 20. In this experiment these numbers will be only whole Swiss Francs, so CHF 0, CHF 1, CHF 2,............, CHF 19, CHF 20. Thus there are 21 possible numbers, and they have equal probability.

The value of X is printed on the note inside the sealed envelope. You will only learn X once experiment 3 is over. Thus DO NOT OPEN YOUR ENVELOPE. It will be opened later by an experimenter in your presence. If you open your envelope yourself, you will not get paid for this experiment.

Here is what the note in each of the envelopes looks like:

The ball will be drawn from a drum with 20 red balls and 40 balls which may be any combination of white and black balls. You do not know exactly how many white/black balls are in the drum. There are 60 balls in total in the drum. You can choose one of two options. The option payoff is dependent on the color of the ball drawn from the drum:

	20 balls	40 balls	
	Red ball drawn	Black ball drawn	White ball drawn
Option A	CHF 20	CHF 0	CHF X
Option B	CHF 0	CHF X	CHF 20

PLEASE NOW ANSWER THE QUESTIONS ON THE OTHER SIDE OF THIS PAPER

Appendix Figure A.12: Machina Task (page 1)

Please give us instructions, for each possible value of X that your envelope may contain, whether you want Option A or B. Do so by ticking the option you prefer for every possible value of X (so put exactly one tick in each of the 21 rows). If you are indifferent, you will get the payoff from Option A or Option B – it will be randomly determined which one.

If X is....	...I want Option A	...I want Option B	I am indifferent
CHF 0			
CHF 1			
CHF 2			
CHF 3			
CHF 4			
CHF 5			
CHF 6			
CHF 7			
CHF 8			
CHF 9			
CHF 10			
CHF 11			
CHF 12			
CHF 13			
CHF 14			
CHF 15			
CHF 16			
CHF 17			
CHF 18			
CHF 19			
CHF 20			

Note: This question does not have a "correct" answer. So just think row by row which option you feel is better.

Please give this sheet to the experimenter when asked to do so.
He will later return it to you, and in the end you have to hand it to the cashier to get paid.

Appendix Figure A.13: Machina Task (page 2)

QUESTIONNAIRE **Participant number:___**

Please now enter your participant number in the space above

Please answer the following questions.

QUESTIONNAIRE PART 1

A doctor gives you 3 pills, and tell you to take 1 pill every 30 minutes starting right away. After how many minutes will you run out of pills? ____________minutes

A meal, including a beverage costs CHF 12 in total. The food costs 5 times as much as the beverage. How much does the food cost? _________

A population of a town halves every month due to a plague. 1 000 people are still alive after 10 months. After how many months were 2 000 people alive? _________

QUESTIONNAIRE PART 2

In this part of the questionnaire we will ask you to make a choice between pairs of lotteries. These lotteries will NOT be paid out. Please answer as you think you would if the choice were real rather than hypothetical. Note that in neither of these questions there is a unique correct answer.

QUESTION: Suppose you got offered a choice between these 2 lotteries. Suppose you would not have to pay anything for either of them, and you could choose exactly one lottery.
Which one would you choose?

O Lottery A: CHF 1 Million for sure

O Lottery B: 1% Chance of Nothing.
89% Chance of CHF 1 Million.
10% Chance of CHF 5 Million

Appendix Figure A.14: Questionnaire (page 1)

QUESTION: Now imagine you did not get the choice offered above, but instead got offered a choice between the 2 lotteries below for free. Suppose you got offered a choice between these 2 lotteries for free. And you could choose exactly one.
Which one would you choose?

O Lottery C: 89% Chance of Nothing
11% Chance of 1 Million CHF

O Lottery D: 90% Chance of Nothing.
10% Chance of 5 Million CHF

QUESTIONNAIRE PART 3

1. What is your country of citizenship: ______________________________

2. What is your mother tongue (native language): ______________________________

3. What is your age? ______years

4. What is your gender? O Male O Female

5. In what kind of program are you currently enrolled?

O Bachelor's program O Master's program O Not a student

6. In which year do you think you will graduate from your current program?

O 2014 O 2015 O 2016 O 2017 O 2018 O Later O I am not a student.

7. What is your field of study? ______________________________

8. Have many times have you participated in experiments before today (in this ETH laboratory or at the University of Zurich) ?

O Never O Once O 2times O 3 times O _______

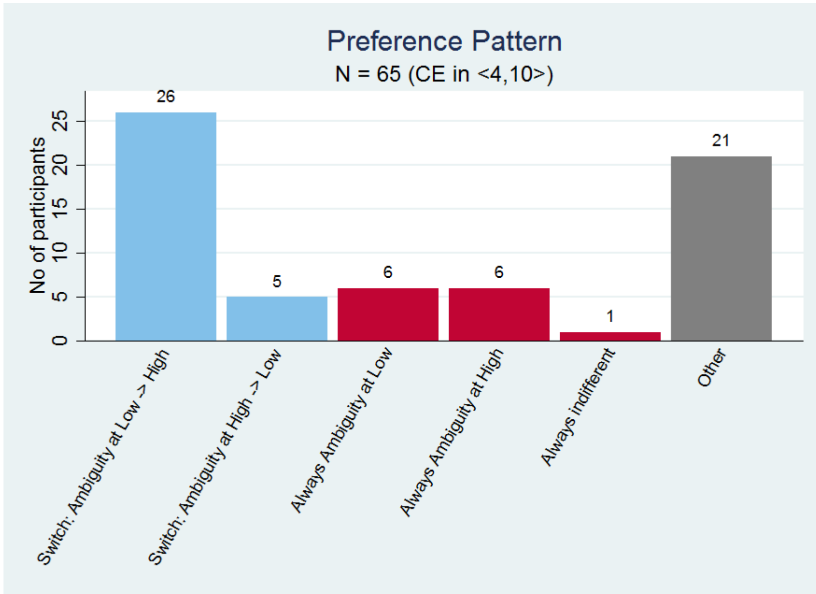
9. How hard to understand were today's experiments? For each experiment choose the most appropriate option. Please note that this will not influence your payoff and will not be linked to your personal data.

	I didn't understand the instructions	I understood the instructions, but I didn't know what answer to give	Everything was clear	Other (provide extra details)
Experiment 1				
Experiment 2				
Experiment 3				

Appendix Figure A.15: Questionnaire (page 2)

E Additional Analysis of Switching Points

Next, we restrict to participants with a certainty equivalent between 4 and 10, inclusive. The results are similar as without the restriction.



Appendix Figure A.16: Participants with reasonable CE

The following tabulation indicates there exists many people for whom CE strongly differs from S:

	Count	Share in %
CE inside switch interval	20	46.5
CE outside switch interval	23	53.5
Total	43	100

Appendix Figure A.17: Whether CE is inside Machina switching point interval

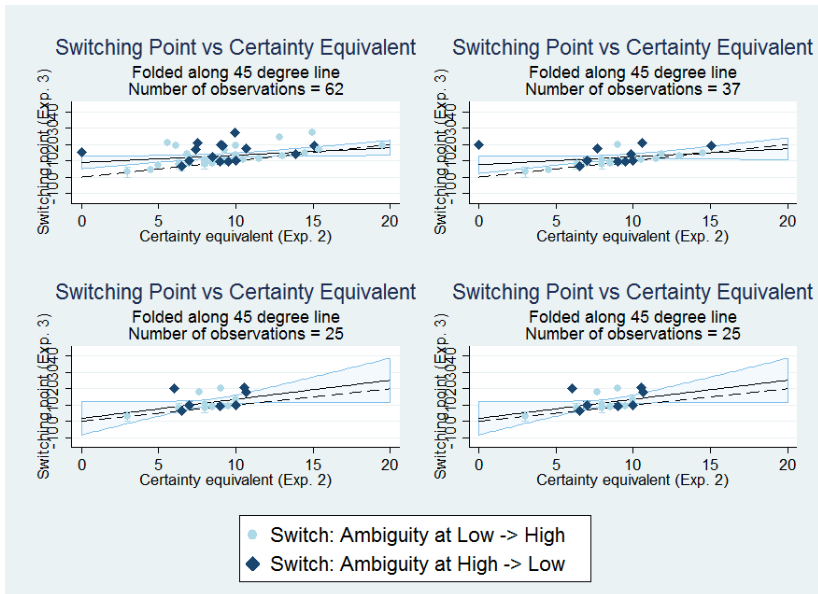
We also present the number of observations for specific combinations of CE and S values:

	CE<10	CE=10	CE>10
S<10	14	4	4
S=10	1	1	0
S>10	5	6	9

Appendix Figure A.18: 2x2 table of CE vs. Switching point

F Additional Regression Analysis of CE and Switching Points

This figure visualizes a regression line and replaces the some dots with bars when subjects report indifference for a range rather than the data indicating a switching point. On this evidence, the confidence interval for the regression line excludes the 45 degree line for the entire set of participants. Smaller samples of the data would not reject the null.



Appendix Figure A.19: CE vs. Switching point (folded, with regression line)

G Order Effects

The order of the lottery presentation was randomized, but we can check if the order influenced the switch direction. We find that the answer is yes, but people still generally switch from Ambiguity at Low to Ambiguity at High.

Fraction of switches from Risk at Low Outcome to Risk at High Outcome depending on the order of options on the answer sheet (normal order lists Risk at High Outcome first).

Group	Obs	Mean	Std Dev
Normal Order	32	.13	.34
Reversed order	11	.18	.4

H0: means are equal; p-value for two-sided test: 0.648

Appendix Figure A.20: Order and switch direction

The tabulation indicates that the fraction of switches from Ambiguity at High to Ambiguity at Low depends on the order of options on the answer sheet (normal order lists Ambiguity at Low Outcome first). But even with the reversed order, the majority of subjects switch from Ambiguity at Low to Ambiguity at High.