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"A Note on Lobbying a Legislature"

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Abstract

We study a simple influence game, in which a lobby tries to manipulate the decision of a legislature via monetary offers to one or more members. We compute the minimum budget needed for the lobby to pass the bill and the distribution of this budget between the legislators. We also show the connection of the problem to the combinatorial optimization.

Key words: Legislative lobbying; Combinatorial optimization; Knapsack problem

JEL classification: D71, D72, C61

1 Introduction

The aim of this note is to analyze how the complexity of legislative process shapes the special interest politics. To do so we consider a simple influence game, in which a single lobby tries to manipulate the decision of a legislature by making monetary offers to one or more members. Clearly, making contribution to a single legislator (as soon as he/she does not have the veto power) does not guarantee the award of lobbyist's preferred policy. We calculate the minimum budget the lobby needs to secure the required support as well as the distribution of this budget between the legislators. We demonstrate the connection of the problem with the *knapsack problem* from combinatorial optimization. Similar questions are studied in Young (1978). However, contrary to this note, in Young (1978) the problem is considered from the legislators' point of view as they maximize the "bribe" income while the lobbyist is the "price-taker". As a result, at equilibrium the legislators may get strictly more than their reservation

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prices while this can never happen in our setting. The expected incomes of the legislators are then compared to the well-known power measures.

2 Model

The legislature is described by a *simple game*, i.e., a pair $(N, \mathcal{W})$ where $N = \{1, 2, ..., n\}$ is the set of legislators and $\mathcal{W}$ is the set of *winning* coalitions. The set of winning coalitions describes the rules operating in the legislature to make decisions. The legislature can ratify or reject a given proposal. A proposal is ratified if and only if the subset of legislators voting in favor of the proposal forms a winning coalition. A coalition C is *blocking* if $N \backslash C$ is not winning. We denote by $\mathcal{B}$ the subset of blocking coalitions. The status quo is maintained as soon as the set of legislators voting against the proposal forms a blocking coalition. The set of minimal (with respect to inclusion) winning coalitions is denoted by $\mathcal{W}_m$. Similarly, the set of minimal blocking coalitions is denoted by $\mathcal{B}_m$.

Following Young (1978) we assume that there is a minimum price $\alpha_i \geq 0$ (*floor price*) the legislator i accepts as a contribution from the lobby. We denote by $p_i \geq 0$ the equilibrium price of legislator i.

We assume that the lobbyist has a large amount of funds at his disposal and would like to pass the bill at the lowest cost. In the following section we calculate the prices for individual legislators that minimize the total cost $\sum_{i \in N} p_i$ the lobbyist pays to pass the decision.

3 Equilibrium analysis

In order to pass the bill the lobby has to buy the support of a winning coalition. Let us denote it by S, then for each $i \in S$

$$p_i \geq \alpha_i.$$

As the lobby would like to buy the support at the lowest costs, the minimum amount the lobby should pay to legislator $i \in S$ voting in favor of the bill is[1]

$$p_i = \alpha_i.$$

Note, that contrary to Young (1978), the legislators never get more than their floor prices.

[1] We assume that a legislator who is indifferent votes for the bill.

The problem of the lobbyist is to find $S^* \in \mathcal{W}_m$ for which the total contribution is minimal[2]

$$\min_{S \in \mathcal{W}_m} \sum_{i \in S} \alpha_i. \tag{1}$$

The legislators $j \notin S^*$ do not get any offers from the lobbyist, i.e., $p_j = 0$.

Remark. One may notice that if all α_i are identical, problem (1) is equivalent to identifying the minimal winning coalition of the smallest size:

$$\min_{S \in \mathcal{W}_m} |S|,$$

where $|S|$ is the size of coalition S.

3.1 The Knapsack Problem

Suppose that the game $(N, \mathcal{W})$ is a *weighted majority game*, i.e., there exists an n-tuple $w = (w_1, ..., w_n)$ of non-negative weights with $\sum_{i \in N} w_i = 1$ and quota $q \geq 0$ such that any $S \in \mathcal{W}$ if and only if $\sum_{i \in S} w_i \geq q$. Then, the problem of finding S^* can be formulated as the combinatorial problem called a *knapsack problem* (e.g., Pisinger, 1995 and Kellerer et al., 2004):

$$\min_{z_i} \sum_{i=1}^{n} \alpha_i z_i$$

subject to the constraints

$$\sum_{i=1}^{n} w_i z_i \geq q$$

$$z_i \in \{0, 1\}.$$

In this formulation we refer to packing of n items into a knapsack. Each object $i = 1, .., n$ is characterized by a pair (w_i, α_i), where α_i is the weight and w_i is the value of object i. Integer z_i indicates whether the object i is included in the knapsack ($z_i = 1$) or not ($z_i = 0$). The objective is to minimize the total weight of the knapsack $\sum_{i=1}^{n} \alpha_i z_i$ while maintaining the total value $\sum_{i=1}^{n} w_i z_i$ above the threshold q.

There is strong theoretical evidence that for the knapsack problem no polynomial time algorithm exists for computing its optimal solution (e.g., Kellerer et al., 2004). In fact, the knapsack problem belongs to a class of so-called $\mathcal{NP}$-hard optimization problems, for which there does not exist any polynomial time algorithm to find an optimal solution. However, if we consider the linear relaxation

[2] The problem can be reformulated if the lobbyist is willing to block the bill instead of seeking to pass it. Then we substitue $S^* \in \mathcal{W}_m$ for $T^* \in \mathcal{B}_m$.

$$z_i \in [0,1] \text{ for all } i = 1, ..., n,$$

things become simpler. Indeed, let us consider the impact of a small change (dz_i, dz_j) leaving the constraint unchanged, i.e., such that $w_i dz_i + w_l dz_l = 0$. The change in the objective is equal to

$$\alpha_i dz_i + \alpha_j dz_j = dz_i w_i \left(\frac{\alpha_i}{w_i} - \frac{\alpha_j}{w_j} \right).$$

For $\frac{\alpha_i}{w_i} > \frac{\alpha_i}{w_l}$ the change is positive if dz_i is positive and negative otherwise. This suggests the following optimal solution. Order the numbers $\left(\frac{\alpha_i}{w_i}\right)_{1 \leq i \leq n}$ in increasing order. Let σ be that order. Then, define

$$z_{\sigma(i)} = 1 \text{ for all } i = 1, .., i^* - 1$$

and

$$z_{\sigma(i^*)} = q - \sum_{i=1}^{i^*-1} w_{\sigma(i)} z_{\sigma(i)},$$

where

$$i^* = \inf_{1 \leq i \leq n} \left\{ i : \sum_{i=1}^{i^*-1} w_{\sigma(i)} z_{\sigma(i)} \geq q \right\}.$$

This algorithm, called *greedy algorithm*, is simple but its performance under the integer constraints is not clear. Thus, Kellerer et al. (2004) show that greedy solutions can be arbitrary bad as compared to the optimal solution. Of course, for small n one can find the solution by elementary checking as we illustrate in the examples below.

The problem has a straightforward solution in the symmetric case, when $w_i = 1$ for all $i = 1, ..., n$. Suppose for simplicity that $\alpha_1 \leq \alpha_2 \leq ...\alpha_n$. In such a case:

$$z_i = \begin{cases} 1 \text{ if } i = 1, 2, ..., q, \\ 0 \text{ otherwise.} \end{cases}$$

In general, the determination of a closed-form solution may be complicated because of the trade-off between the voting weight w_i of player i and his reservation price α_i.

4 Examples

Let us demonstrate the solution of problem (1) for some important voting bodies.

Example 1: EU Council of Ministers (1958 - 1972). In that period the Council consisted of representatives from six countries. The three "big" countries (Germany, Italy and France) held four votes each, the two "medium" countries (Belgium and the Netherlands) held two votes each and the "little" country (Luxembourg) held one vote. A qualified majority was set at 12 out of 17, i.e., passing a decision required at least 12 votes in favor of the decision.

There are two types of minimal winning coalitions in this case: three big countries or two big countries together with two medium ones. There is one coalition of the first type and six coalitions of the second. Luxembourg is never a part of a minimum winning coalition.

One can conclude from the Remark that if all floor prices are equal to 1 then $S^* = \{1, 2, 3\}$ and the total cost paid by the lobbyist is 3.

Assume that the weights of three big countries are $\alpha_1 \leq \alpha_2 \leq \alpha_3$ and the weights of two medium countries are α_4 and α_5. Then, the coalition S^* is defined as

$$S^* = \begin{cases} \{1, 2, 3\} & \text{if } \alpha_3 \leq \alpha_4 + \alpha_5 \\ \{1, 2, 4, 5\} & \text{otherwise} \end{cases}.$$

As a result, the minimum cost paid by the lobbyist is defined as

$$\sum_{i \in S^*} \alpha_i = \begin{cases} \alpha_1 + \alpha_2 + \alpha_3 & \text{if } \alpha_3 \leq \alpha_4 + \alpha_5 \\ \alpha_1 + \alpha_2 + \alpha_4 + \alpha_5 & \text{otherwise} \end{cases}.$$

Example 2: The U.S. Federal Legislative System.

The members of the House of Representatives (R) and the Senate (S), together with the Vice-President (V) and the President (P) are the players in this voting game. A coalition is winning if it contains either more than half the house and more than half the senate (with the vice president playing the role of tie-breaker in the senate), together with the support of the president or two-thirds of both the house and the senate (to override a veto by the president): $\{218R, 50S, V, P\}$, $\{218R, 51S, P\}$, $\{290R, 67S\}$[3].

Following Example 7 (Young, 1978), we assume that all the floor prices are equal to 1. Then, the first two winning coalitions are the cheapest and the minimal total cost is 270. Suppose $\alpha_R =$

[3]This game cannot be represented as a weighted majority game (e.g.,Young, 1978).

$\alpha_S = \alpha_V = 1$ and $\alpha_P = 88$ (the equilibrium prices obtained in the example), then the three coalitions become equally costly and the total cost incurred by the lobbyist is 357.

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