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"The Political Economy of (De)centralization with Complementary Public Goods"

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Abstract: This paper provides a political economy analysis of (de)centralization when local public goods – with spillovers effects – can be substitutes or complements. Depending on the degree of complementarity between local public goods, median voters strategically delegate policy to either 'conservative' or to 'liberal' representatives under decentralized decision-making. In the first case, it accentuates the free-rider problem in public good provision, while it mitigates it in the second case. Under centralized decision-making, the process of strategic delegation results in either too low or too much public spending, with the outcome crucially depending on the sharing of the costs of local public spending relative to the size of the spillover effects. Hence, with a common financing rule, centralization is welfare improving if and only if both public good externalities *and* the degree of complementarity between local public goods are both relatively large.

Keywords: (De)centralization; Local Public Goods; Complements; Strategic Delegation; Spillovers

JEL Classification: D72, H41, H77

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1 Introduction

There is a large literature on fiscal federalism which focuses on the optimal allocation of powers between the central and local governments. The "Decentralization Theorem" of Oates (1972) states that the choice of a centralized system over a decentralized one depends on the benefits of internalizing externalities relative to the costs of policy uniformity. Recently, this trade-off has been re-examined from a political economy perspective with the result that centralization may not be the most efficient system even though centralized policy can be differentiated across localities.

A potential deficiency of a centralized system is that the costs of local public spending are shared through a common budget, thus creating a conflict of interest between citizens in different localities. For instance, Besley and Coate (2003) develop a two-region model of local public spending and show that, with a cooperative legislature, each median voter has an incentive to strategically delegate bargaining power to a representative with higher demand for public spending. The reason is that the appointment of a 'liberal' representative is a commitment device to extract more of the common budget, which in turn results in too much public spending.[1] Their analysis has been extended in several directions. In particular, Dur and Roelfsema (2005) consider that some costs cannot be shared across regions and then focus on the financing rules that eliminate the incentives for strategic delegation under a cooperative legislature.

In the literature on the political economy of fiscal (de)centralization, it is typically assumed that there are no strategic interactions among regions or that the total amount of public good consumption available in a particular region corresponds to a weighted sum of the amounts that are locally provided by all regions – with the weights reflecting the spillovers across regions. This 'summation technology' implies 'perfect' substitutability between local public investments and it originates from the canonical model of private provision of a pure public good, where the good's overall level is defined as the arithmetic sum of individual contributions (see Bergstrom *et al.,* 1986).

However, as first pointed out by Hirshleifer (1983), public goods can take a variety of different forms for which the perfect-substitutes assumption is questionable. He then proposed two other 'social composition functions' for aggregating individual contributions. With the 'weakest-link' technology, the total level of public good is given by the smallest individual contribution, while it is given by the greatest contribution for the 'best-shot' technology. In the first case, Hirshleifer (1983) used the example of a circular island that needs to be protected by a dike and for which the effective level of protection (against floods) is determined by the lowest portion of the dike. For best-shot public goods, one can think of discovering a research breakthrough – e.g. a cure for a degenerative disease – where the payoff is determined by the greatest research effort.

Cornes (1993) and more recently Cornes and Hartley (2007a) went one step further by considering

[1] In fact, strategic delegation can nullify any element of cooperativeness within the legislature (see Cheikbossian, 2000).

a CES 'social composition function' for aggregating individual contributions. Such a technology allows them to consider the intermediate cases of 'weaker-link' and 'better-shot' public goods. For example, in the case of a 'weaker-link' public good, the smallest contribution has the largest marginal impact on the overall level of public good provision but the impact of contributions by others is not equal to zero (due, for example, to differences in the island's topography in Hirshleifer's example). Sandler (1997, 1998) and Arce and Sandler (2001) pointed out that these types of public goods may also be prevalent in many situations involving the provision of 'transnational' public goods stemming from regional public investments, with spillover effects from one region to the other. By way of illustration of transnational 'weaker-link' public goods, they briefly refer to the cases of atmospheric monitoring, cyberspace virus control, disease control or peacekeeping.[2]

In the present analysis, we revisit the issue of the (de)centralization provision of local public goods from a political economy perspective, with the use of a framework that allows for any degree of substitutability/complementarity between local public investments as well as for any degree of public good spillovers. More specifically, we consider a model with two regions, each providing a local public good that benefits the other region. The effective level of public good consumption in each region results from a CES aggregation function of public investments in the two regions. Furthermore, we introduce spillover effects by weighting foreign public investment by an exogenous parameter and, so, one unit of public investment abroad does not entail the same marginal public benefit than when this unit is provided domestically.

The (de)centralization of decision-making is framed by a two-stage policy game. In the first stage, voters in each region elect by majority voting a representative. In the second stage, the elected local representatives choose independently of each other the level of public investment for their own region in case of decentralized decision-making. Under centralization, however, the two representatives choose local public investments so as to maximize joint surplus. We also assume in this case that local public spending can be funded by both a common lump-sum tax and by a local tax within a range between 'pure' common financing and 'pure' local financing. Finally, in our analysis, median voters are decisive and have exactly the same preferences so that we focus exclusively on symmetric equilibria.

The degree of substitutability/complementarity between local public investments relative to the elasticity of the marginal valuation for total public good consumption determine whether local public investments are *strategic* substitutes or complements. We then show that, under decentralization, each median voter strategically delegates policy to a representative with a lower (respectively higher) taste parameter for public good consumption than herself as local public goods are strategic substitutes (respectively

[2] Interestingly, they state that "*weaker-link public goods resemble strategic complements*" (p. 497). For 'better-shot' public goods, they briefly refer to the cases of the development of a vaccine or public policies against international terrorism. They conclude that "*better-shot public goods are akin to strategic substitutes*" (p. 497). However, they do not analyze the nature of strategic interactions between countries as a function of the type of public good. Instead, they analyze correlated strategies for specific numerical examples of 'weaker-link' and 'better-shot' transnational public goods.

complements). In any case, there is underprovision of local public goods under a decentralized system and consequently the delegation process may accentuate or mitigate the free-rider problem in public good provision. Under centralization, however, median voters' incentives to delegate policy to a 'conservative' or to a 'liberal' representative depend only on the share of common funding of local public spending relative to the size of the spillover effects. Depending on which case applies, centralized decision-making results in over or under-spending, but the drawback of centralization is decreasing in the degree of complementarity between local public goods. Finally, we analyze which system dominates the other from the median voters' point of view. For example, under 'pure centralization' with a common budget financing rule, we show that centralization dominates decentralization if and only if both the spillover parameter *and* the degree of complementarity between local public investments are both relatively large.

Our results contribute to the previous traditional and political-economy literatures on the (de)centralization provision of local public goods. Besley and Coate (2003) show that centralized provision is preferable to decentralized provision if spillover effects are sufficiently strong. The conclusion is thus the same as in the standard approach *à la* Oates (1972), with the difference that the costs of centralization result from over-spending decisions due to the appointment of 'liberal' representatives in the central legislature. Dur and Roelfsema (2005) consider that some costs cannot be shared though a common budget, so that the strategic delegation to 'conservative' agents under centralization may also happen. An important feature of these two papers, and of a number of other studies, is that there are no incentives for strategic delegation under decentralized decision-making. The reason is that voters' preferences are separable in the levels of public spending, thus implying that the two regions' allocations are strategically neutral (even though there exists spillover effects).[3]

However, Dur and Roelfsema (2005), in an additional appendix, also consider a 'summation technology' with the public good surplus being given by a (concave) function of the weighed sum of local public goods.[4] This implies that these investments are strategic substitutes and, hence, median voters strategically delegate power to 'conservative' representatives under non-cooperative decision-making. However, they do not investigate the relative performance of the two systems with this extension. In fact, we show that local public investments must not be perfect substitutes for a centralized system to *possibly* dominate a decentralized system even though there are large public good spillover effects.

In other words, the trade-off identified by Besley and Coate (2003) – and others – crucially hinges on the separability assumption of local public goods in the utility of voters. This is a very restrictive

[3] Due to its convenience, this separability assumption has been extensively used in the political economy literature of (de)centralization to investigate a number of issues such as lobbying (see, e.g., Lockwood, 2008), the popular support for centralization (see, e.g., Feld *et al.,* 2008; Feidler and Staal, 2012), or the endogenous choice of the degree of centralization (see, Lorz and Willmann, 2005, 2013).

[4] This corresponds to the standard alternative specification which has also been used to study, for example, the impact of voting rules, such as direct referendum or qualified majority, on the emergence and scope of centralized systems (see, e.g., Redoano and Scharf, 2004; and Alesina *et al.,* 2005); or the effect of lobbying on the performance of a decentralized system (see, e.g., Cheikbossian, 2008).

assumption which is unlikely to be satisfied in a wide variety of contexts. In fact, a number of studies show that there do exist strategic interactions between neighboring jurisdictions. While, most papers focus on the strategic interactions in tax rates, there is a growing body of empirical works that focus on public spending interactions and that conclude, in general, to the presence of a strategic complementarity (see, e.g., Case *et al.*, 1993; Figlio *et al.*, 1999; Baicker, 2005; Foucault *et al.*, 2008).

Our paper is also related to the literature that focuses on the voters' incentives for delegating decision-making to representatives in the context of environmental policies. For example, Buchholz *et al.* (2005) show that median voters support representatives that are less 'green' than they are, both under cooperative (centralized) and non-cooperative (decentralized) decision-making. In a strategic trade policy environment *à la* Brander and Spencer (1985), Roelfsema (2007) raises the possibility of delegating policy to an 'environmental lover' in case the median voter cares sufficiently for the environment. Yet, in a similar manner to the above-mentioned articles, the public bad – i.e. the environmental damage – is modelled as a function of the (weighted) sum of domestic polluting activities.

Last but not least, the use of a more general 'social composition function' raises some technical problems in this kind of two-stage political game. The standard equilibrium concept would be that of a Subgame Perfect Nash Equilibrium (SPNE) – in pure strategies. Yet, in the present study, the existence issue is hard to deal with under this notion, except under very specific assumptions.[5] Indeed, while representatives' payoffs are concave in their own strategies, the payoffs of the decisive voters *induced* by the public good provision subgame between representatives are seemingly not (quasi-)concave, nor is the induced game necessarily supermodular for all the parameter values. Therefore, we introduce the weaker equilibrium concept of a Local Nash Subgame Perfect Equilibrium (LNSPE), which ensures that no median voter benefits from a *small* unilateral deviation from her equilibrium strategy. This concept extends to a two-stage game that of a Second-order Locally Consistent Equilibrium (2-LCE) or that of a Local Nash Equilibrium (LNE) in static models of imperfect competition (see, Gary-Bobo, 1989; Bonanno 1998). This notion of 'local equilibrium' has also been exploited in models of tax competition, where the existence of a Nash equilibrium remains an issue (see, e.g., Bayindir-Upmann and Ziad, 2005; Bucovetsky and Smart, 2006).[6]

[5] In models *à la* Besley and Coate (2003) or Dur and Roelfsema (2005), which can be recovered as (two) cases of our framework, a SPNE always exists. We can also show that such an equilibrium exists under a decentralized system when local public investment are strategic complements (but not too complements). For more details, see the additional appendix, which is available upon request.

[6] In this literature, most authors just assume the existence of a (general) Nash equilibrium and few of them have explicitly focused on the existence problem (see, e.g., Laussel and Le Breton, 1998; Rothstein, 2007). More closely related to the present analysis, Persson and Tabellini (1992) analyze tax competition between two countries and where the policy-maker, in each country, is elected by majority voting. It is thus a two-stage game with strategic delegation effects. However, they implicitly assume the existence of an equilibrium by focusing on the first-order conditions only. This practice is also common in the literature on strategic trade policy with governments choosing (non-cooperatively) their trade policies prior to the time that firms engage in market competition (see, e.g., Brander and Spencer, 1985; Eaton and Grossman, 1986).

The rest of this article is organized as follows. Section 2 outlines the framework for our analysis. Sections 3 and 4 present our political economy analysis of a decentralized and a centralized system, respectively. In Section 5, we evaluate the performance of one system relative to the other and we also compare in details our results with those of previous studies. Finally, Section 6 offers some concluding remarks.

2 The Model

Consider an economy of two equally sized regions, indexed by $j = A, B$, with the region size normalized to 1. There are three goods in the economy, a private good x and two local public goods – or investments – e_A and e_B, each one associated with a particular region. All individuals have identical endowments in private goods y and producing one unit of local public good e_j costs one unit of the private good. There are two additional important features in the model. First, the total amount of public good consumption in region j does not coincide with the amount locally provided because of the existence of cross-regional spillovers. Second, the two local public goods can be substitutes or complements. Specifically, the *effective* level of public good consumption in region j, for a given vector of local public investments $\mathbf{e} \equiv (e_j, e_{-j}) \in \Re^2_+$, is given by

$$G_j(\mathbf{e}) = \left[e_j^{1-\sigma} + \beta e_{-j}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{1}$$

$\beta \in [0, 1]$ represents the spillover parameter and the two special cases where $\beta = 0$ and $\beta = 1$ correspond, respectively, to the two polar cases of 'pure local public goods' and 'pure global public goods' (also referred to as the case of 'perfect spillovers').[7] $\sigma \in \{[0, 1) \cup (1, +\infty)\}$ measures the degree of complementarity between the two local public goods and the elasticity of substitution is $1/\sigma$. For $\sigma = 0$, we have 'perfect substitutability' between local public goods and equation (1) becomes the standard 'summation technology', i.e. $G_j = e_j + \beta e_{-j}$.[8] For $\sigma \to +\infty$ we have 'perfect complementarity' and, in the limit, equation (1) becomes $G_j = \text{Min}\{e_j; e_{-j}\}$ (referred as to the 'weakest-link' function). Finally, if $\sigma > 1$ and $e_j = 0$ or $e_{-j} = 0$, the function is not well defined. Hence, we will also take the limit of (1) as $e_j \to 0$ or $e_{-j} \to 0$, which means $G_j(\mathbf{e}) = 0$ in this case.[9]

[7] Because of the spillover parameter, there are two levels of effective public good consumption – hence two different CES 'social composition functions' – one for each region. For this reason, our model does not belong to the class of "aggregative games", where each player's payoff depends on her own action and the same aggregate of all players' actions (see, Corchón, 1994; Cornes and Hartley, 2007a, 2007b; and Acemoglu and Jensen, 2013).

[8] The denomination of 'perfect-substitutes' for $\sigma = 0$ is actually abusive. Indeed, even in this case, one unit of investment abroad does not entail the same marginal benefit than when this unit is provided domestically because of the spillover parameter. So, the adjective 'perfect' must be understood as referring to an infinite elasticity of substitution between the two levels of public investments.

[9] Note that (1) is also discontinuous at $\sigma = 1$. This case is excluded from our analysis.

Individuals differ in the intensity of their preferences for public good consumption. The preferences of a citizen in region j with taste parameter θ are given by

$$x_j + \theta F\left[G_j\left(\mathbf{e}\right)\right], \tag{2}$$

where θ is symmetrically distributed over the $\left[\underline{\theta}, \overline{\theta}\right]$ interval with $\underline{\theta} > 0$, and with the mean and median values both equal to m_j for region j (for $j = A, B$). Furthermore, $F\left(.\right)$ is increasing and concave with constant index of concavity $\mu = -[F^{''}\left(G_j\left(\mathbf{e}\right)\right).G_j\left(\mathbf{e}\right)]/F'\left(G_j\left(\mathbf{e}\right)\right)$. $F\left(.\right)$ is thus an isoelastic function of the form $F(G_j\left(\mathbf{e}\right)) = \left[G_j\left(\mathbf{e}\right)\right]^{1-\mu}$, where $\mu \in (0, 1)$.[10]

In each region, the cost of providing the local public good is financed by a uniform head tax τ_j on local residents of region j. We will assume throughout that citizens' endowments in private goods y are sufficiently high to always allow positive consumption of the private good x. There are thus no wealth effects and, so, we can focus on the public good surplus.

3 Decentralization

Under decentralization, the level of local public investment in region j is decided by an elected (regional) representative and is financed by local taxation only, so that the uniform head tax in region j is such that $\tau_j = e_j$. The representative is chosen by and amongst the voters of the region. Thus, we have a two-stage policy game to solve. In the first stage, voters in each region elect their representative by majority rule. In the second stage, the two representatives simultaneously choose the level of public spending for their own region.

We work by backward induction. Let the type of the elected representative in region j be θ_j. Given the other representative's policy choice e_{-j}, the representative of region j chooses $e_j \geq 0$ to maximize her own public good surplus

$$v_j\left(\mathbf{e}\right) = \theta_j F\left[G_j\left(\mathbf{e}\right)\right] - e_j. \tag{3}$$

We first establish the following Lemma, which will prove useful in further analysis.[11]

Lemma 1. *For $\sigma \geq \mu$, local public investments are strategic complements – i.e. $\partial^2 v_j(\mathbf{e})/\partial e_j \partial e_{-j} \geq 0$ – while they are strategic substitutes – i.e. $\partial^2 v_j(\mathbf{e})/\partial e_j \partial e_{-j} \leq 0$ – for $\sigma \leq \mu$.*[12]

μ can also be interpreted as the elasticity of the marginal valuation for public good consumption. Thus,

[10] This type of utility function (2), with $F(.)$ being an isoelastic function and $G(.)$ being a CES function, has been used by Ray *et al.* (2007) to analyze voluntary participation in a joint project with imperfect substitution between individual efforts (but without spillover effects). They focus on how (exogenous) share vectors affect joint surplus. Note also that to save on notations, we will work with the $F(.)$ formulation throughout the text and the appendix.

[11] The proofs of the lemma and propositions are given in the appendix.

[12] See Bulow *et al.* (1985) for the definition of strategic complementarity or substitutability.

decisions on local public investments are strategic complements (substitutes) if the inverse of the elasticity of substitution − or the degree of complementarity − between local public goods given by σ is greater (lower) than the elasticity of the marginal valuation for public good consumption. In the special case of $\sigma = \mu$, the cross-derivatives are zero, which means that the two regions' allocations are strategically neutral. This corresponds to the setup considered by Besley and Coate (2003), where voters' preferences are assumed to be separable in the levels of local public spending.

We now characterize the equilibrium outcome in the second stage of the game. We have:

Lemma 2. (i) *Given the types θ_j and θ_{-j} of the two representatives, the non-cooperative game of public good provision admits a pure-strategy Nash equilibrium. In this equilibrium, region j's public good provision (for $j = A, B$) is characterized by the following first-order condition*

$$\theta_j F'(G_j(\mathbf{e}))[G_j(\mathbf{e})]^{\sigma} e_j^{-\sigma} \leq 1, \text{ with equality for } e_j > 0. \tag{4}$$

(ii) *For $\sigma \in \{[0,1) \cup (1,+\infty)\}$, there is a unique interior equilibrium with $e_A^* > 0$ and $e_B^* > 0$. For $\sigma \in (1,+\infty)$, there is another corner equilibrium which involves $e_A^* = e_B^* = 0$.*[13]

In other words, given the types of the two representatives (θ_j, θ_{-j}), there is a unique equilibrium with the property that local public investments are strictly positive in the two regions. In this equilibrium, the system given by (4), with equality for $j = A, B$, implicitly yields the local public good levels under a decentralized system as functions of the types of the two representatives, i.e. $e_A^*(\theta_A, \theta_B)$ and $e_B^*(\theta_A, \theta_B)$. If $\sigma \in (1, +\infty)$ there also exists a corner equilibrium, which we ignore. It is also worth pointing out that the best-response functions of the regions' representatives are equivalent to the (necessary and sufficient) first-order conditions and that with strategic substitutes (complements), best-response functions are decreasing (increasing). The profile of equilibrium response functions are depicted in Figure 1 for $\sigma \in [0, \mu)$, and in Figure 2 for $\sigma \in (\mu, +\infty)$.

INSERT FIGURES

We now turn to the election stage. Citizens, in each region, vote simultaneously to elect their representatives. If the representatives of regions A and B are of types θ_A and θ_B, a type θ citizen in region j will have the following public good surplus $w_j(\theta, \theta_j, \theta_{-j}) = \theta F[G_j(\mathbf{e}^*(\theta_A, \theta_B))] - e_j^*(\theta_j, \theta_{-j})$, with $\mathbf{e}^*(\theta_A, \theta_B) \equiv (e_A^*(\theta_A, \theta_B), e_B^*(\theta_A, \theta_B))$. These preferences over types of representatives determine

[13] See the Appendix. Multiplicity is also possible for $\sigma = 0$ and $\sigma \to +\infty$. In the first case, it happens when there are also 'perfect spillovers' – i.e. $\beta = 1$ – and when the two representatives have identical preferences – i.e. $\theta_A = \theta_B$: there is a continuum of equilibria with the same public good surplus. The latter case corresponds to the 'weakest-link' public good game: there is also a continuum of equilibria with varying public good surplus and in which the representatives' decisions are matched each other.

individuals' voting decisions. To characterize the outcome of the election stage, we first establish the following Lemma.

Lemma 3. *Voters' preferences over types of representatives exhibit the single-crossing property: If* $\theta' > \theta$ and $\theta_j > \theta_j^{'}$, *or if* $\theta' < \theta$ *and* $\theta_j < \theta_j^{'}$, *then* $w_j(\theta, \theta_j, \theta_{-j}) \geq w_j(\theta, \theta_j', \theta_{-j}) \Rightarrow w_j(\theta', \theta_j, \theta_{-j}) \geq w_j(\theta', \theta_j', \theta_{-j})$.[14]

As shown by Rothstein (1990, 1991) and Gans and Smart (1996), the single-crossing property guarantees that a Condorcet winner exists and that it coincides with the preferred candidate of the voter with the median preference given by m_j.

We are interested in an equilibrium of the two-stage game, that is a majority preferred pair of local representatives (θ_A^*, θ_B^*) such that $\mathbf{e}^*(\theta_A^*, \theta_B^*)$ solves the system given by Lemma 2 – i.e. equation (4) with equality for $j = A, B$. The equilibrium concept that first comes to mind is that of a Subgame Perfect Nash equilibrium (SPNE) in pure strategies. That is, the median type in region j prefers θ_j^* to any other type $\theta \in [\underline{\theta}, \overline{\theta}]$, given the other region's representative type θ_{-j}^*, and given the resulting subgame of local public good provision. To prove the existence of such an equilibrium in this two-stage game structure is very problematic. Indeed, while representatives' payoffs are concave in their own strategies, the payoffs of the decisive voters *induced* by the equilibrium of the public good provision subgame between representatives are seemingly not (quasi-)concave. Nor is the induced game necessarily supermodular for all the parameter values of the model.[15]

However, in the case of a decentralized system, quasi-concavity of each median voter's payoff can be easily established for $\sigma = 0$ and $\sigma = \mu$. In the first case, local public investments are perfect substitutes while there are no strategic interactions in local public spending in the second case. Again, following Besley and Coate (2003), these two specific cases have been extensively studied in the literature. One can also show that median voters' payoff are quasi-concave in our setup for $\sigma \in [\mu, 1)$. However, for $\sigma \in \{(0, \mu) \cup (1, +\infty)\}$, one need to use a weaker concept than that of a SPNE. We thus introduce the concept of Local Nash Subgame Perfect Equilibrium (LNSPE) – in pure strategies.

Let (again) m_j be the taste parameter of the *median* voter in region j. Her payoff is thus given by $w_j(m_j, \theta_j, \theta_{-j}) = m_j F[G_j(\mathbf{e}^*(\theta_A, \theta_B))] - e_j^*(\theta_j, \theta_{-j})$. We define a LNSPE under decentralization as follows:

Definition 1: *Under decentralization, a vector* $\boldsymbol{\theta}^* \equiv (\theta_j^*, \theta_{-j}^*)$ *is a Local Nash Subgame Perfect Equilibrium* (LNSPE) *if and only if* (i) $(\partial w_j(m_j, \theta_j, \theta_{-j}) / \partial\theta_j)(\boldsymbol{\theta}^*) = 0$, *and* $(\partial^2 w_j(m_j, \theta_j, \theta_{-j}) / \partial\theta_j^2)(\boldsymbol{\theta}^*) < 0$ *for* $j = A, B$; (ii) $(e_j^*(\boldsymbol{\theta}^*), e_{-j}^*(\boldsymbol{\theta}^*))$ *is a Nash equilibrium in pure strategies in the Stage-*2 *subgame – and is characterized by* (4) *in Lemma* 2 *with equality for* $j = A, B$.

[14] See Persson and Tabellini (2000, p. 23). The single-crossing property was formulated by Gans and Smart (1996), which is essentially equivalent to the 'order restriction' of preferences first formulated by Rothstein (1990, 1991).

[15] I am very grateful to the reviewer for pointing out this difficulty.

In words, each median voter's equilibrium strategy ensures a local maximum of her payoff function, given the equilibrium strategy of the other median voter and the resulting equilibrium policy outcome in stage 2. Hence, in a LNSPE, no median voter benefits from a *small* unilateral deviation from her equilibrium strategy. Of course, every LNSPE is a SPNE but the converse is not true. It might indeed be possible that there is a representative type θ_j lying outside the neighborhood of θ_j^* which yields a higher payoff for region j's median voter. In short, the LNSPE is a weaker concept than that of subgame perfection, but is nevertheless characterized by the first-order conditions.

As mentioned in the Introduction, this concept of "equilibrium" is closely related to that of a second-order locally consistent equilibrium (2-LCE), as introduced by Gary-Bobo (1989) in the context of an imperfectly general equilibrium model, or to the equivalent concept of a Local Nash Equilibrium (LNE) used by Bonanno (1988) in a static oligopoly game.[16] The justification is that firms have only local knowledge of their demand curves (and therefore of their profit functions), so that they experiment through small variations of their strategy variable and stop when they reach a local maximum. The notion of LNE has also been exploited in models of tax competition (see, e.g., Bayindir-Upmann and Ziad, 2005, or Bucovetsky and Smart, 2006) – where the existence of (general) Nash equilibria (in pure strategies) has proved difficult to establish.[17]

For simplifying notations, let e_j^* denote $e_j^*(\theta_j, \theta_{-j})$. In a LNSPE, the representative type θ_j for the region j's median voter, given the type of the other region's representative θ_{-j}, must satisfy following first-order condition,

$$m_j F'(G_j(\mathbf{e}^*)) [G_j(\mathbf{e}^*)]^{\sigma} \left[e_j^{*-\sigma} \frac{\partial e_j^*}{\partial \theta_j} + \beta e_{-j}^{*-\sigma} \frac{\partial e_{-j}^*}{\partial \theta_j} \right] - \frac{\partial e_j^*}{\partial \theta_j} = 0. \tag{5}$$

Using (4), this expression can be rewritten as

$$\left[\frac{m_j}{\theta_j} - 1 \right] \frac{\partial e_j^*}{\partial \theta_j} + \beta \frac{m_j}{\theta_j} \left(\frac{e_{-j}^*}{e_j^*} \right)^{-\sigma} \frac{\partial e_{-j}^*}{\partial \theta_j} = 0. \tag{6}$$

Observe that θ_j is larger (respectively lower) than m_j if $\partial e_{-j}^* / \partial \theta_j$ is positive (respectively negative). Therefore, in general, median voters delegate policy to representatives with different preferences than their own.

As mentioned above, we assume that the two median voters have identical preferences – i.e. $m_A = m_B = m$ – and we focus on a symmetric equilibrium with median voters delegating policy to (two) representatives of the same type, thus resulting in a symmetric equilibrium level of local public good

[16] Gary-Bobo (1989) develops the concept of a kth-order locally consistent equilibrium (k-LCE), which is an imperfectly competitive equilibrium allocation at which firms perceive only a kth-order Taylor expansion of their true demand curves (and therefore of their true profit functions). A 2-LCE is thus a strategy profile for which the first derivative of each player's payoff function vanishes and the second derivative is negative.

[17] The idea of local Nash equilibrium is also exploited in probabilistic voting models (see, e.g., Duggan, 2000; Schofield, 2004; and Patty; 2005).

provision – i.e. $e_A^* = e_B^*$. We can then state the following Proposition.[18]

Proposition 1: *Let* $\bar{\sigma} = \sqrt{5} - 2$. *If* $\sigma = 0$ or $\sigma \geq \bar{\sigma}$, *then there exists a unique symmetric* LNSPE *under decentralization for any* $(\mu, \beta) \in (0,1) \times [0,1]$. *In this equilibrium, the election stage is characterized by:*
(i) *The median voter in each region – with taste parameter* m *for public good consumption – delegates policy to a representative with taste parameter*

$$\theta^* = \frac{(1+\beta)\left[\mu(1-\beta) + \sigma\beta\right]}{\mu + \sigma\beta} m. \tag{7}$$

(ii) *The median voter in each region delegates policy to a representative with a lower (respectively higher) taste parameter for public good consumption than herself as* $\sigma = 0$ *or* $\sigma \in [\bar{\sigma}, \mu]$ *(respectively* $\sigma \in \{[\mu, 1) \cup (1, +\infty)\}$*).*

We can also establish the following corollary about the inefficiency of a decentralized system.

Corollary 1: (i) *The unique* LNSPE *under decentralization is always characterized by under-provision of local public investments.* (ii) *The extent of under-provision is increasing in the spillover parameter* β *and decreasing in the degree of complementarity between local public investments given by* σ.

Proof: Let first determine the optimal level of local public investment common to both regions. Recall first that, by assumption, mean and median voters have identical preferences within and across regions. Therefore, the social optimum is given by the maximization of the sum of median voter utilities. With the $F(.)$ function being an isoelastic function – i.e. $F(G) = G^{1-\mu}$ – this optimal level is given by maximizing $\left[mG^{1-\mu} - e\right]$, with $G = (1+\beta)^{1/(1-\sigma)}e$. This yields $\hat{e} = \left[m(1-\mu)(1+\beta)^{\frac{1-\mu}{1-\sigma}}\right]^{1/\mu}$. Moreover, under a decentralized system – and using (4) in Lemma 2 – the symmetric policy outcome is given by $e^* = \left[\theta^*(1-\mu)(1+\beta)^{\frac{\sigma-\mu}{1-\sigma}}\right]^{1/\mu}$. The ratio of these two levels is thus given by $\hat{e}/e^* = \left[m(1+\beta)/\theta^*\right]^{\frac{1}{\mu}}$. Using (7), we then have

$$\frac{\hat{e}}{e^*} = \left[\frac{\mu + \sigma\beta}{\mu(1-\beta) + \sigma\beta}\right]^{\frac{1}{\mu}}, \tag{8}$$

which is clearly greater than 1. Hence, the equilibrium of a decentralized system results in too low levels of local public investments compared to the social optimum, even though median voters delegate policy to representatives who put a higher weight on public good consumption than themselves. Furthermore,

[18] In the proofs of Proposition 1 (for a decentralized system) and 2 (for a centralized system), we assume the existence of a LNSPE and show that, if it the case, this equilibrium is unique and is characterized by the statements given in these two propositions. The proofs of the existence of a LNSPE under decentralization (and under the sufficient condition that $\sigma \geq \bar{\sigma}$) and under centralization are given in a separate appendix. In this second appendix, we also show that, in the case of a decentralized system, there exists a SPNE for $\sigma = 0$ and $\sigma \in [\mu, 1)$. This additional appendix is available upon request.

it is straightforward to verify that the term in [.] of equation (8) – and thus the extent of under-provision – is increasing in β, but decreasing in σ.[19] ■

As shown by part (ii) of Proposition 1, delegating policy to a 'conservative' representative actually arises when decisions are strategic substitutes, i.e., when $\sigma \leq \mu$ (provided that $\sigma \geq \bar{\sigma}$). This is because, in that case, citizens in each region realize when electing their representatives that decreasing domestic provision of public goods will induce the other region's representative to increase its investment. To put it differently, each median voter seeks to place the burden of total public good provision on the citizens of the other region. However, these attempts are self-defeating and hence strategic delegation accentuates the free-rider problem. And this is even more so the case as the spillover parameter – as measured by β – rises (part (ii) of Corollary 1).

If, however, decisions on local public investments are strategic complements – i.e. $\sigma \geq \mu$ – each median voter delegates policy-making a 'liberal' representative, that is to someone who cares more for public goods than she does because this will induce the other region's representative to also raise its own public investment. These incentives are mutually reinforcing and hence the delegation process takes the equilibrium closer to the optimum. And this is even more so the case as the degree of complementarity between local public investments – as measured by σ – rises (part (ii) of Corollary 1).

In the special case of $\sigma = \mu$, the utility of voters is separable in the levels of local public investments, thus eliminating strategic interaction between regions. As a result, there are no incentives for strategic delegation and each median voter prefers a candidate of her own type, i.e. $\theta^*_{|\sigma=\mu} = m$. It follows that the inefficiency of decentralized decision-making is completely characterized by the inability of local governments to internalize externalities. This corresponds to the case analyzed by Besley and Coate (2003).

Furthermore, one can also observe that in the limiting case of perfect complementarity between local public goods – i.e. $\sigma \rightarrow +\infty$ – we have,

$$\theta^*_{|\sigma\rightarrow+\infty} = (1+\beta)\, m. \tag{9}$$

Using e^* and $\hat{e}$ in the Proof of Corollary (1), we observe that $e^* = \hat{e}$, and so the decentralized system yields the social optimum in that case.

If, however, there is perfect substitutability between local public goods – i.e. $\sigma = 0$ – we obtain that

$$\theta^*_{|\sigma=0} = \left(1-\beta^2\right) m. \tag{10}$$

Hence, the 'degree of conservativeness' – as measured by $|\theta^* - m|$ – is increasing in the square of the parameter reflecting the extent of public good externalities when local public investments are perfect

[19] Let $\Lambda(\sigma,\beta)$ the term in [.] of (8). We have $\partial\Lambda(\sigma,\beta)/\partial\beta = \mu^2/\left[\mu(1-\beta)+\sigma\beta\right]^2 > 0$, and $\partial\Lambda(\sigma,\beta)/\partial\sigma = -\mu\beta^2/\left[\mu(1-\beta)+\sigma\beta\right]^2 \leq 0$.

substitutes (see also the appendix of Dur and Roelfsema, 2005, and Buchholz *et al.*, 2005). If, in addition, we have 'perfect spillovers' – i.e. $\beta = 1$ – then median voters delegate policy-making to representatives who do not care to public good consumption at all in order to attempt to put the entire burden of public good provision on the other region. Since the two median voters have the same incentives, no public goods are supplied in equilibrium in this case.

4 Centralization

Under centralization, the policy outcome is also determined by a two-stage policy game. In the first stage, citizens in each region elect their representative in the central legislature. In the second stage, the two representatives bargain over the levels of local public spending. Following Besley and Coate (2003), we assume that the bargaining outcome is given by the maximum sum of the utilities of the two representatives at the bargaining table.[20]

We also assume that, under a centralized regime, the level of public spending e_j in the jth region is funded by both a general lump-sum tax denoted τ, and by a local tax denoted t_j. The funding split between local taxation and general taxation is exogenous and is parameterized by $\lambda \in [0, 1]$, representing the share of general public funding. The head tax in region j is thus $\tau_j = \lambda\tau + (1-\lambda)t_j$, where $\tau = \frac{(e_j + e_{-j})}{2}$ and $t_j = e_j$. Hence, the head tax in region j can be rewritten as

$$\tau_j = \frac{(2-\lambda)e_j + \lambda e_{-j}}{2}, \tag{11}$$

with $\tau_j + \tau_{-j} = e_j + e_{-j}$.

When $\lambda = 0$, we have $\tau_j = e_j$ and local public spending – decided at the central level – is financed by local taxation only. This setup corresponds to the situation analyzed by Buchholz *et al.* (2005) in which elected governments cooperate over environmental policies so as to internalize a pollution externality but without sharing the costs of 'greener' policies. If, however, $\lambda = 1$, then $\tau_j = (e_j + e_{-j})/2$ which corresponds to a 'pure' centralized system with the costs of public goods being shared through a common budget, as it is assumed in Besley and Coate (2003).

Again, let the types of the elected representatives in region j and $-j$ be θ_j and θ_{-j}, respectively. They jointly maximize

$$v_j(\mathbf{e}) + v_{-j}(\mathbf{e}) = \theta_j F\left(G_j\left(\mathbf{e}\right)\right) + \theta_{-j} F\left(G_{-j}\left(\mathbf{e}\right)\right) - (e_j + e_{-j}), \tag{12}$$

[20] Besley and Coate (2003) first analyze the case where decisions are taken by a minimum coalition of representatives – in fact by one representative in their two-region model. Then, they consider the utilitarian solution, which can be motivated by the 'universalism' view in the political science literature on distributive politics. According to this view, the elected representatives develop a norm of reciprocity to overcome the problems associated with minimum coalitions (see, e.g., Weingast, 1979; and Shepsle and Weingast, 1981). Also, decisions in supranational bodies sometimes require unanimity, thus forcing legislators to cooperate. This is the case in the EU for policies falling under the heading of the second pillar – i.e. common foreign and security policy – and third pillar – i.e. police and judicial cooperation in criminal matters.

with respect to both e_j and e_{-j}.

We have the following Lemma.

Lemma 4: *Given the types θ_j and θ_{-j} of the two representatives, there is unique interior equilibrium outcome under centralized decision-making, which is characterized by the following first-order conditions,*

$$\theta_j F'\left(G_j\left(\mathbf{e}\right)\right)\left[G_j\left(\mathbf{e}\right)\right]^{\sigma} e_j^{-\sigma} + \beta\theta_{-j} F'\left(G_{-j}\left(\mathbf{e}\right)\right)\left[G_{-j}\left(\mathbf{e}\right)\right]^{\sigma} e_j^{-\sigma} = 1, \textit{ for } j = A, B. \tag{13}$$

This system implicitly yields the local public good levels under a centralized system, as functions of the types of the two representatives, i.e. $\tilde{e}_A\left(\theta_A, \theta_B\right)$ and $\tilde{e}_B\left(\theta_A, \theta_B\right)$.

We now turn to the election stage. If the representatives in regions A and B are of types θ_A and θ_B, a type θ citizen in region j will have the following public good surplus: $w_j\left(\theta, \theta_j, \theta_{-j}\right) = \theta F\left[G_j\left(\tilde{\mathbf{e}}\left(\theta_A, \theta_B\right)\right)\right] - \left[(2-\lambda)\tilde{e}_j\left(\theta_A, \theta_B\right) + \lambda\tilde{e}_{-j}\left(\theta_A, \theta_B\right)\right]/2$, with $\tilde{\mathbf{e}}\left(\theta_A, \theta_B\right) \equiv \left(\tilde{e}_A\left(\theta_A, \theta_B\right), \tilde{e}_B\left(\theta_A, \theta_B\right)\right)$. Citizens' preferences over types still satisfy the single-crossing property, which is sufficient to apply the median voter result[21]: the representative that is majority-preferred to any other representative is the one that is most preferred by the voter with the median preference m_j in region j.

We are still interested in an equilibrium of this two-stage game, that is a majority preferred pair $(\tilde{\theta}_A, \tilde{\theta}_B)$ such that $\tilde{\mathbf{e}}\ (\tilde{\theta}_A, \tilde{\theta}_B)$ solves the system given by Lemma 4. Under centralization, the proof of the existence of a SPNE in pure strategies is even more problematic than under decentralization, except for $\sigma = 0$ and $\sigma = \mu$. Therefore, we continue to use the weaker concept of a Local Nash Subgame Perfect Equilibrium (LNSPE).

The payoff of the median voter in region j under centralization is $w_j\left(m_j, \theta_j, \theta_{-j}\right) = m_j F\left[G_j\left(\tilde{\mathbf{e}}\left(\theta_A, \theta_B\right)\right)\right] - \left[(2-\lambda)\tilde{e}_j\left(\theta_A, \theta_B\right) + \lambda\tilde{e}_{-j}\left(\theta_A, \theta_B\right)\right]/2$. In a manner similar to the case of decentralization, we thus define a LNSPE under centralization as follows:

Definition 2: *Under centralization, a vector $\tilde{\boldsymbol{\theta}} \equiv (\tilde{\theta}_j, \tilde{\theta}_{-j})$ is a Local Nash Subgame Perfect Equilibrium* (LNSPE) *if and only if* (i) $\left(\partial w_j\left(m_j, \theta_j, \theta_{-j}\right)/\partial\theta_j\right)(\tilde{\boldsymbol{\theta}}) = 0$, *and* $\left(\partial^2 w_j\left(m_j, \theta_j, \theta_{-j}\right)/\partial\theta_j^2\right)(\tilde{\boldsymbol{\theta}}) < 0$ *for* $j = A, B$; (ii) $(\tilde{e}_j(\tilde{\boldsymbol{\theta}}), \tilde{e}_{-j}(\tilde{\boldsymbol{\theta}}))$ *maximizes the joint public good surplus of the two representatives in the Stage-*2 *subgame – and is characterized by* (13) *in Lemma* 4 *for* $j = A, B$.

Again, each median voter's equilibrium strategy ensures a local maximum of her payoff function, given the equilibrium strategy of the other median voter and the resulting equilibrium policy outcome in stage 2.

[21] The Proof is the same as in Lemma 3. Equilibrium public good levels under centralization are implicitly defined by (13) for $j = A, B$. Applying the implicit function theorem together with the fact that aggregate payoff given by (12) is strictly concave in e_j (see the Proof of Lemma 4), we have that e_j and $G_j(\mathbf{e})$ are both increasing in θ_j. It follows that the single-crossing property is satisfied (see the Proof of Lemma 3).

For simplifying notations, let $\tilde{e}_j$ denote $\tilde{e}_j(\theta_j, \theta_{-j})$. In a LNSPE, the representative type θ_j for the region j's median voter, given the type of the other region's representative θ_{-j}, must satisfy the following first-order condition,

$$m_j F'(G_j(\tilde{\mathbf{e}}))\left[G_j(\tilde{\mathbf{e}})\right]^{\sigma}\left[\tilde{e}_j^{-\sigma}\frac{\partial \tilde{e}_j}{\partial \theta_j} + \beta \tilde{e}_{-j}^{-\sigma}\frac{\partial \tilde{e}_{-j}}{\partial \theta_j}\right] - \tfrac{1}{2}\left[(2-\lambda)\frac{\partial \tilde{e}_j}{\partial \theta_j} + \lambda \frac{\partial \tilde{e}_{-j}}{\partial \theta_j}\right] = 0. \tag{14}$$

The two median voters have identical preferences, i.e. $m_A = m_B = m$. We thus focus on a symmetric equilibrium with median voters delegating policy to (two) representatives of the same type, thus resulting in a symmetric equilibrium level of local public good provision – i.e. $\tilde{e}_A = \tilde{e}_B$. We then have the following Proposition.[22]

Proposition 2: *If $\mu \geq 0.5$, then there exists a unique symmetric* LNSPE *under centralization for any $(\lambda, \beta) \in [0,1]^2$ and any $\sigma \in \{[0,1) \cup (1,+\infty)\}$. In this equilibrium, the election stage is characterized by:*
(i) *The median voter in each region – with taste parameter m for public good consumption – delegates policy to a representative with taste parameter*

$$\tilde{\theta} = \frac{2\left[2\sigma\beta + \mu(1-\beta)^2\right]}{4\sigma\beta + \mu(1-\beta)\left[2-\lambda(1+\beta)\right]} m. \tag{15}$$

(ii) *Let $\tilde{\lambda} \equiv 2\beta/(1+\beta)$. Then, the median voter in each region delegates policy to a representative with a higher (respectively lower) taste parameter for public good consumption than herself, thus resulting in over-provision (respectively under-provision) of local public investments as $\lambda \geq \tilde{\lambda}$ (respectively $\lambda \leq \tilde{\lambda}$).*

Again, median voters delegate policy to representatives with different preferences than their own. However, as shown by part (ii) of this Proposition, and in contrast to the decentralized system, whether they choose a 'conservative' representative (i.e. $\tilde{\theta} \leq m$) or a 'liberal' representative (i.e. $\tilde{\theta} \geq m$) does not depend on the degree of complementarity between local public investments (given by σ) relative to the elasticity of the marginal valuation for public good consumption (given by μ). It depends only on the funding split between local taxation and general taxation parameterized by λ relative to the size of public good spillovers given by β. More precisely, the larger the share of general funding and the lower the spillovers are, the more likely median voters delegate policy to representatives who put a higher weigh on public good consumption than themselves. Hence, in the extreme case of pork-barrel spending – i.e. $\lambda = 1$ – the strategic choice of a 'liberal' representative is a commitment device to extract more of the common fiscal resources in the second stage of the game. Conversely, if local public investments generate large spillover effects and are (mainly) financed by local taxation, then the choice of a 'conservative' representative is a way to shift the burden of public good provision to the other region. Obviously, in

[22] Again, in the proof of this Proposition, we assume the existence of a LNSPE. The proof of the existence of a LNSPE (under the sufficient condition that $\mu \geq 0.5$) is given in a separate appendix. In this second appendix, we also state than under a centralized system, there exists a SPNE $\sigma = 0$ and $\sigma = \mu$. This additional appendix is available upon request.

either case, these attempts are self-defeating and the equilibrium results in either over-provision or under-provision of local public goods compared to the social optimum that would be obtained in the absence of delegation. (Recall that the two representatives fully cooperate in the central legislature).

The following Corollary makes further statements about the (potential) distortion induced by the delegation process.

Corollary 2: (i) *The "optimal cost-sharing rule" resulting in a* LNSPE *with sincere delegation – i.e.* $\tilde{\theta} = m$ *– is such that* $\tilde{\lambda} = 2\beta/(1+\beta)$*, which is increasing in* β*.* (ii) *For* $\lambda \geq \tilde{\lambda}$ *(respectively* $\lambda \leq \tilde{\lambda}$*),* $\tilde{\theta}$ *is decreasing (respectively increasing), and hence the extent of over-provision (respectively under-provision) is decreasing in the degree of complementarity given by* σ*. However, the sign of the derivative of* $\tilde{\theta}$ *with respect to* β *is indeterminate.*[23]

The share of general funding given by λ can be interpreted as a budgetary externality. Part (i) of Corollary 2 intuitively shows that larger public good spillovers must be compensated by a larger budgetary externality to induce sincere delegation and hence an efficient policy outcome under a centralized system. Local financing ($\lambda = 0$) is only optimal in case of pure local public goods ($\beta = 0$), while complete cost sharing ($\lambda = 1$) is only optimal in case of global public goods ($\beta = 1$). In general, when the extent of the budgetary externality λ is different from "the optimal cost-sharing rule" given by $\tilde{\lambda}$, centralized decision-making is inefficient as is the case of decentralized decision-making. The difference is that the provision of local public goods can be too low, but also too high compared to the social optimum.

These results are reminiscent to those obtained by Dur and Roelfsema (2005). Again, these authors extend Besley and Coate (2003)'s analysis by considering that some costs cannot be shared among regions and in turn derive the "optimal financing rule" that eliminates the incentives for strategic delegation under a centralized (or cooperative) regime. Essentially, they also show that larger public good externalities must be compensated by a greater sharing of the costs of local public spending.[24] However, as usual in the literature, they model public good surplus either as a (weighted) sum of functions – implying no strategic interactions in local public investments – or as a function of the (weighted) sum of local public investments – implying perfect substitutability between these investments.

The novelty here is that the degree of complementarity between local public investments also affects

[23] The sign of $\partial\tilde{\theta}/\partial\sigma$ is the same as the sign of its numerator, which is given by: $4\beta\mu(1-\beta)[2\beta - \lambda(1+\beta)]\,m$. This term is negative (respectively positive) for λ larger (respectively lower) than $\tilde{\lambda} = 2\beta/(1+\beta)$. The sign of $\partial\tilde{\theta}/\partial\beta$ is the same as the sign of its numerator, which is given by: $-4\mu\left[\mu(1-\lambda)(1-2\beta) + \sigma\lambda + \beta^2\left[\mu(1-\lambda) - \sigma(2-\lambda)\right]\right] m$. This term can be positive or negative depending on the exact values of the triplet (σ, β, λ).

[24] In Dur and Roelfsema (2005), the financing rule of a centralized system is summarized by three parameters: the weight of direct tax costs shared among regions through a common budget; the weight of an indirect utility cost; and the weight of the cross-subsidy from one region to the other. They show that the "optimal" cross-subsidy is increasing in the spillover parameter, which amounts to increase the budgetary externality. Since a linear cost-division rule is imposed, we believe that it is more transparent and clear to describe the financing rule of a centralized system by a unique parameter, as in the present framework.

the extent of the distortion induced by strategic delegation under centralized decision-making (as under decentralized decision-making). Indeed, from point (ii) of Corollary 2, we have that a larger degree of complementarity between local public investments – i.e. a higher σ – decreases the distortion induced by the delegation process in that it decreases the extent of over-provision for $\lambda \geq \tilde{\lambda}$, or the extent of under-provision for $\lambda \leq \tilde{\lambda}$. In the limit case of 'weakest-link' public goods – i.e. $\sigma \rightarrow +\infty$ – we have $\tilde{\theta}_{|\sigma\rightarrow+\infty} = m$, and so a centralized system yields the social optimum as does a decentralized system in this case. This is also the case if there are 'perfect' public good spillovers – i.e. $\beta = 1$ – since then $\tilde{\theta}_{|\beta=1} = m$ independently of the degree of complementarity between local public goods (provided $\sigma > 0$). Under a decentralized system, however, the case of 'perfect spillovers' exacerbates the under-provision of public goods (see Corollary 1).

In case of perfect substitutability between local public goods – i.e. $\sigma = 0$ – we obtain that

$$\tilde{\theta}_{|\sigma=0} = \frac{2(1-\beta)}{2-\lambda(1+\beta)}m. \tag{16}$$

If furthermore public investments are entirely financed by local taxation – i.e. $\lambda = 0$ – then appointed representatives are even more 'conservative' than under a decentralized system since then $\tilde{\theta}_{|\sigma=\lambda=0} = (1-\beta)m \leq \theta^*_{|\sigma=0} = \left(1-\beta^2\right)m$.[25] In the other extreme of complete cost sharing ($\lambda = 1$) and perfect substitutability between local public goods ($\sigma = 0$), we have delegation to an extreme 'liberal' representative, that is $\tilde{\theta}_{|\sigma=0,\lambda=1} = 2m$ independently of the size of spillovers given by β.

5 Centralization versus Decentralization

We now investigate which system dominates the other from a social welfare point of view. Recall that by assumption, in each region, there is a continuum of citizens of mass one and that the median voter has the same preferences than the mean voter. Hence, the payoff of each region's median voter also represents the social welfare of her region.

The general expressions of the levels of welfare as well as the levels of local public investments under the two systems are given in the appendix. In order to simplify the analysis, we now assume that the elasticity of the marginal valuation for public good consumption is such that $\mu = 0.5$, i.e. $F[G_j(\mathbf{e})] = \sqrt{G_j(\mathbf{e})}$.[26] In this case, we have the following levels of welfare,

[25] This result is related to that obtained by Buchholz *et al.* (2005) who show that voters support candidates who are even less green than they are to represent them in the cooperative scenario compared to the isolationist scenario. They also show that in the extreme case of global pollution, elected politicians pay no attention at all to the environment in any scenario. This would correspond to $\beta = 1$ and $\sigma = 0$ in our setup, with no provision of public goods in equilibrium (as under decentralization in this case; see equation (10)).

[26] This is consistent with the sufficient condition imposed for the existence of a LNSPE under centralization (see Proposition 2 and the separate appendix). The condition for the existence of a LNSPE is that $\sigma \geq \bar{\sigma} = \sqrt{5} - 2$, which is assumed to hold in this section.

$$\upsilon^* = \frac{\left[(1+2\sigma\beta)^2 - \beta^2\right](1+\beta)^{\frac{1}{1-\sigma}}}{4\left[1+2\sigma\beta\right]^2} m^2, \tag{17}$$

under a decentralized system, and

$$\tilde{\upsilon} = \frac{\left[4\sigma\beta + (1-\beta)^2\right]\left[4\sigma\beta + (1-\beta^2)(1-\lambda)\right](1+\beta)^{\frac{1}{1-\sigma}}}{\left[8\sigma\beta + (1-\beta)\left[2-\lambda(1+\beta)\right]\right]^2} m^2, \tag{18}$$

in a centralized system.

The following Proposition considers two extreme financing rules under a centralized system: (i) local public spending is entirely financed by local taxation (but yet decided at the centralized level), i.e. $\lambda = 0$; (ii) local public spending is entirely financed by general taxation, i.e. $\lambda = 1$.

Proposition 3: *Suppose that* $F\left[G_j(\mathbf{e})\right] = \sqrt{G_j(\mathbf{e})}$, *then we have:*

(i) When local public investments are financed by local taxation only – i.e. $\lambda = 0$ *– then centralization always dominates decentralization.*

(ii) When local public investments are financed by general taxation only – i.e. $\lambda = 1$ *– then centralization dominates decentralization if* $\beta \geq 3 - 2\sqrt{2}$ *and* $\sigma \geq \tilde{\sigma}$ *with* $\tilde{\sigma} \equiv (1-\beta)^3 / \left[2\beta\left(6\beta - \beta^2 - 1\right)\right]$. *Otherwise, decentralization dominates centralization.*

Again, there is always under-provision under a decentralized system whether local public good decisions are strategic substitutes or complements (Corollary 1). From Proposition 2, this is also the case under a centralized system when local public spending is financed by local taxation only – i.e. $\lambda = 0$ – since in that case $\lambda \leq \tilde{\lambda}$. Actually, it is possible (but not necessary) that, under a centralized system, median voters appoint representatives who are even more conservative than under a decentralized system (which arises, for example, when $\sigma = 0$). However, in this case and according to the first part (i) of Proposition 3, the benefits from cooperation between representatives under a centralized system – in terms of internalizing public good externalities – are larger than the increased cost of the distortion induced by strategic delegation. In any case, the extent of under-provision is lower under a centralized system and thus this system improves welfare relative to decentralization independently of the size of the public good spillovers and of the degree of complementarity between local public goods.

Now let consider the most commonly used assumption of a common financing rule – i.e. $\lambda = 1$ – adopted by Besley and Coate (2003), among others.[27] In this case, a centralized system does not necessarily dominate a decentralized system as shown by the second part (ii) of Proposition 3. Indeed, for $\lambda = 1$, there is over-provision of public goods under centralization (while decentralization is still characterized by under-provision). Thus, centralization is welfare improving if and only if the size of

[27] The prevalence of this assumption in the literature is justified to some extent since most centralized systems of government operate (roughly) according to such a rule. Indeed, equal cost sharing is very often a constitutionally imposed arrangement. For example, in most European countries, uniform tax rules are at the core of budgeting institutions (see, e.g., Von Hagen, 1992).

spillovers *and* the degree of complementarity between the two local public goods are both relatively large. Note, however, that $\tilde{\sigma}$ is a decreasing function of the spillover parameter. Therefore, providing β is larger than $3-2\sqrt{2}$, an increase in its value alleviates the constraint on the minimum degree of complementarity that is required for centralization to be desirable.[28]

Yet, if local public investments are 'perfect' substitutes – i.e. $\sigma = 0$ – then decentralization is most preferred by the median voters irrespective of the size of spillovers. This result contrasts strongly with previous political economy studies on the trade-off between centralized and decentralized provision of local public goods. Indeed, following Oates' *Decentralization Theorem,* the drawback with a decentralized system is typically reflected by the inability of local governments to internalize public good externalities, while the inefficiency of a centralized system stems from political economy considerations. Therefore, in general, there is a threshold level of externalities above which the benefits of improved coordination are larger than the costs of the political inefficiencies of centralization.

In fact, Besley and Coate (2003), Dur and Roelfsema (2005) and a number of other scholars, assume that voters' preferences are separable in the levels of public spending in different regions. It follows that under a decentralized system, there are no strategic interactions across regions: the level of public investment decided by one representative has no effect on that chosen by the representative of the other region.[29] In other words, the outcome of a decentralized system is an equilibrium in dominant strategies and equilibrium best-response curves are two straight lines with 0 slope. It has the important implication that there are no incentives for strategic delegation under decentralization, so that the drawback of decentralization is completely characterized by the free-rider problem in public good provision.

In the present analysis, strategic delegation occurs under both centralization and decentralization as a consequence of the existence of strategic interactions in the public good game. This is the case even though local public investments are assumed to be 'perfect' substitutes, i.e. $\sigma = 0$, so that the public good surplus in each region is a concave function of the (weighted) sum of local public goods. As already explained, this implies that median voters appoint 'conservative' representatives. In turn, this accentuates the classic free-rider problem and the under-provision of public goods, thus making decentralization even more inefficient than in Besley and Coate (2003).

At first sight, this might seem surprising because when $\sigma = 0$ (and $\lambda = 1$), Proposition 3 implies that decentralization dominates centralization even though local public investments correspond very closely

[28] We can also compare the threshold value $\tilde{\sigma}$ to the sufficient condition used for proving the existence of a LNSPE under a decentralized system (see Proposition 1). In fact, for $\mu = 0.5$, the condition in question is given by $\sigma \geq \bar{\sigma} = (1/4)\left[\sqrt{33}-5\right] \simeq 0.19$ (see the separate appendix). Numerically, we have that $\tilde{\sigma} \leq \bar{\sigma}$ for any $\beta \gtrsim 0.42$, in which case centralization dominates decentralization under the sufficient (but not necessary) condition for the existence of a LNSPE in a decentralized system.

[29] For example, the utility from public goods in Besley and Coate (2003) is given by (with our notations): $\theta\left[(1-\beta)\ln(e_j)+\beta\ln(e_{-j})\right]$. The cross-derivatives with respect to e_j and e_{-j} are equal to 0, which means that there are no strategic interactions. Again, this corresponds to the special case of $\sigma = \mu$ in our framework.

to the provision of a pure global public good, i.e. β approaches 1.[30] But, the inefficiency of centralized decision-making is even stronger in our framework with $\sigma = 0$ (and $\lambda = 1$). Under centralization, the separability assumption of Besley and Coate (2003) implies that electing a representative with a higher taste for public goods increases domestic spending but also increases – to a lesser extent – foreign public spending. In the present analysis with $\sigma = 0$, appointing a 'liberal' agent increases domestic spending but reduces foreign spending regardless of λ.[31] Consequently, each median voter has stronger incentives to elect a 'liberal' representative, which accentuates the over-provision of public goods and the inefficiency of centralized decision-making compared to Besley and Coate (2003). Furthermore, we have seen that with complete cost sharing – i.e. $\lambda = 1$ – each median voter appoints an extreme 'liberal' representative, that is a type $2m$ representative independently of the size of the spillover effects (i.e., $\tilde{\theta}_{|\sigma=0,\lambda=1} = 2m$). This can explain why centralization never dominates decentralization in the 'perfect-substitutes' case.

To summarize, the trade-off between centralized and decentralized provision of local public goods as a function of spillovers identified by Besley and Coate (2003) – and others – hinges critically on the separability assumption of local public goods in the utility of voters (which again implies no strategic interactions across regions). Without this assumption, there is no trade-off between the two systems and decentralization always dominate centralization when local public investments are 'perfect' substitutes (i.e. $\sigma = 0$) and are funded from a common budget (i.e. $\lambda = 1$). Dur and Roelfsema (2005) also use the separability assumption in the main body of their analysis and extend – in an additional appendix – their model to the case of a concave function of the weighted sum of local public goods. They properly refer to this case as that of strategic substitutes and show that median voters appoint 'conservative' representatives under decentralization. However, they do not investigate the relative welfare performance of the two systems with this extension. Presumably, this is because there is no trade-off in this case as it is shown here.

As mentioned in the Introduction, there is a number of empirical studies showing that there exist strategic interactions in public spending between neighboring jurisdictions and, moreover, these studies conclude (in general) to the presence of a strategic complementarity. That being said, the present analysis applies to both strategic complementarity and substitutability in public spending.[32] Simply, with a common financing rule, local public investments must not be 'perfect' substitutes for centralization to possibly dominate decentralization. Finally, it is worth pointing out that the welfare difference between the two systems with or without common financing of local public goods – given by (A46) and (A47) in the appendix – is non-monotonic in σ for any $\beta \in [0, 1]$ and non-monotonic in β as well for any

[30] When $\sigma = 0$ and $\lambda = 1$, the equilibrium under centralization is not defined if we further assume $\beta = 1$, as it can be seen from (16). Under decentralization, no public goods are supplied in equilibrium in this case, as shown by (10).

[31] With $\sigma = 0$, we indeed have that $\partial \tilde{e}_B / \partial \theta_A$ and $\partial \tilde{e}_A / \partial \theta_A$ are of opposite signs. This can be seen from equation (A37) in the Appendix.

[32] Notice also that $\sigma \geq \tilde{\sigma}$ – and $\beta \geq 3 - 2\sqrt{2}$ – does not necessarily imply that local public investments must be strategic complements since these two conditions can be satisfied for $\sigma \leq \mu = 0.5$.

$\sigma \in (1, +\infty)$.[33] In other words, an increase in spillovers or in the degree of complementarity between local public investments does not necessarily make centralization relatively more attractive, whether it is welfare-superior to decentralization or not.

6 Conclusion

This paper revisits the traditional analysis of the political economy of (de)centralization by assuming a generalized CES function for aggregating local public investments with spillover effects. Depending on the degree of complementarity between local public goods, median voters strategically delegate policy to either 'conservative' or to 'liberal' representatives under decentralized decision-making. In the first case, it accentuates the free-rider problem in public good provision, while it mitigates it in the second case. Under centralized decision-making, the process of strategic delegation results in either too low or too much public spending, with the outcome crucially depending on the sharing of the costs of local public spending relative to the size of the spillover effects. Finally, we show that, with a common budget financing rule, large public good externalities are not sufficient for a centralized system to dominate a decentralized system from the median voters' point of view. Indeed, it also requires a minimum degree of complementarity between local public investments.

In this study, we tried to generalize previous political economy studies of the trade-off between centralized and decentralized provision of local public goods with spillover effects. However, our formal analysis can be criticized on several fronts. First, we assumed that voters' preferences are quasi-linear in private consumption. This is a very common assumption in the political economy literature partly because it serves to guarantee the existence of a Condorcet winner. Yet, this modeling assumption implies that there are no income effects on public good provision, which may not be an appropriate assumption for all types of (local) public goods. Second, we assumed perfectly identical regions, which greatly simplifies the analysis in that it allows focusing on symmetric equilibria only. But, even in this case, a centralized system need not be more efficient than a decentralized system. We conjecture that it should be also the case in presence of some source of asymmetry across regions, although the equilibrium levels of public spending – under both a decentralized and a centralized system – would be different across regions. It must be remembered also that if regions were heterogeneous, we would lose the symmetry of the model and thus would no be able to obtain closed form solutions.

Finally, we have used the concept of a Local Nash Subgame Perfect Equilibrium (LNSPE) for dealing with the problem of equilibrium existence in the model of decentralized system and that of a centralized system. In such an equilibrium, no median voter faces an incentive to deviate unilaterally from her equilibrium strategy by choosing a slightly different type of representative. If a (symmetric) Subgame Perfect Nash Equilibrium (SPNE) exists in each of the two systems, then it must coincides with the

[33] Indeed, numerical simulations shows that the only exception is for $\sigma < 1$, with the welfare difference being increasing in β.

corresponding LNSPE. Yet, it does not imply that a SPNE actually exists. The problem of existence of such an equilibrium in a two-stage political-economy model of public good provision such as ours remains an open question that is left for future research.

7 Appendix

7.1 Proof of Lemma 1

The first derivative of $v_j(\mathbf{e})$ given by (3) with respect to e_j is given by

$$\frac{\partial v_j(\mathbf{e})}{\partial e_j} = \theta_j F'(G_j(\mathbf{e}))\frac{\partial G_j(\mathbf{e})}{\partial e_j} - 1. \tag{A1}$$

From (1), we have that $\partial G_j(\mathbf{e})/\partial e_j = [G_j(\mathbf{e})]^{\sigma} e_j^{-\sigma}$, and hence

$$\frac{\partial v_j(\mathbf{e})}{\partial e_j} = \theta_j F'(G_j(\mathbf{e}))[G_j(\mathbf{e})]^{\sigma} e_j^{-\sigma} - 1. \tag{A2}$$

Now, calculating the derivative of (A2) with respect to e_{-j}, we have

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j \partial e_{-j}} = \theta_j e_j^{-\sigma}\left\{F''(G_j(\mathbf{e}))\frac{\partial G_j(\mathbf{e})}{\partial e_{-j}}[G_j(\mathbf{e})]^{\sigma} + \sigma F'(G_j(\mathbf{e}))[G_j(\mathbf{e})]^{\sigma-1}\frac{\partial G_j(\mathbf{e})}{\partial e_{-j}}\right\}, \tag{A3}$$

which can be rewritten as

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j \partial e_{-j}} = \theta_j e_j^{-\sigma}\frac{\partial G_j(\mathbf{e})}{\partial e_{-j}}[G_j(\mathbf{e})]^{\sigma-1}\left\{F''(G_j(\mathbf{e}))G_j(\mathbf{e}) + \sigma F'(G_j(\mathbf{e}))\right\}. \tag{A4}$$

Now, let $\mu \equiv -[F''(G_j(\mathbf{e})).G_j(\mathbf{e})]/F'(G_j(\mathbf{e}))$ be the elasticity of the marginal utility for public good consumption and factorizing by $F'(G_j)$, we obtain

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j \partial e_{-j}} = \theta_j e_j^{-\sigma}\frac{\partial G_j(\mathbf{e})}{\partial e_{-j}}[G_j(\mathbf{e})]^{\sigma-1}F'(G_j(\mathbf{e}))[\sigma-\mu], \tag{A5}$$

We also have $\partial G_j(\mathbf{e})/\partial e_{-j} = \beta[G_j(\mathbf{e})]^{\sigma} e_{-j}^{-\sigma}$. Hence, (A5) can be rewritten as

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j \partial e_{-j}} = \beta\theta_j e_j^{-\sigma} e_{-j}^{-\sigma}[G_j(\mathbf{e})]^{2\sigma-1}F'(G_j(\mathbf{e}))[\sigma-\mu], \tag{A6}$$

Therefore, $\partial^2 v_j(\mathbf{e})/\partial e_j \partial e_{-j}$ is positive (respectively negative) and local public investments are strategic complements (respectively substitutes) for σ larger (respectively lower) than μ.

7.2 Proof of Lemma 2

(i) Existence: We first show that the game of public good provision admits a pure strategy Nash equilibrium. First, each region's representative can at most invest its private endowment, y, in the public good so that the strategy space of each representative is a compact interval, $S = [0, y]$. We now show that the maximization problem of each representative is strictly concave. Using (A1), the second derivative of $v_j(\mathbf{e})$ with respect to e_j is given by

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j^2} = \theta_j \left\{ F''(G_j(\mathbf{e})) \left[\frac{\partial G_j(\mathbf{e})}{\partial e_j}\right]^2 + F'(G_j(\mathbf{e})) \frac{\partial^2 G_j(\mathbf{e})}{\partial e_j^2} \right\}. \tag{A7}$$

The first term in $\{.\}$ is negative since $F''(.) < 0$. The sign of the second term is the same as the sign of $\partial^2 G_j(\mathbf{e}) / \partial e_j^2$ since $F'(.) > 0$. We have $\partial G_j(\mathbf{e}) / \partial e_j = [G_j(\mathbf{e})]^\sigma e_j^{-\sigma}$ and hence

$$\frac{\partial^2 G_j(\mathbf{e})}{\partial e_j^2} = \sigma [G_j(\mathbf{e})]^{\sigma-1} \frac{\partial G_j(\mathbf{e})}{\partial e_j} e_j^{-\sigma} - \sigma [G_j(\mathbf{e})]^\sigma e_j^{-\sigma-1}. \tag{A8}$$

Again, $\partial G_j(\mathbf{e}) / \partial e_j = [G_j(\mathbf{e})]^\sigma e_j^{-\sigma}$, so that (A8) can be rewritten as follows

$$\frac{\partial^2 G_j(\mathbf{e})}{\partial e_j^2} = \sigma [G_j(\mathbf{e})]^{2\sigma-1} e_j^{-2\sigma} \left[1 - [G_j(\mathbf{e})]^{1-\sigma} e_j^{-(1-\sigma)}\right]. \tag{A9}$$

This (second) derivative is strictly negative since $[G_j(\mathbf{e})]^{1-\sigma} e_j^{-(1-\sigma)} = 1 + \beta(e_{-j}/e_j)^{1-\sigma} > 1$. It follows that $\partial^2 v_j(\mathbf{e}) / \partial e_j^2$ given by (A7) is strictly negative. Specifically (and for future use), substituting $\partial^2 G_j(\mathbf{e}) / \partial e_j^2 = -\sigma\beta [G_j(\mathbf{e})]^{2\sigma-1} e_j^{-2\sigma}(e_{-j}/e_j)^{1-\sigma}$ into (A7) yields

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j^2} = \theta_j \left\{ F''(G_j(\mathbf{e})) [G_j(\mathbf{e})]^{2\sigma} e_j^{-2\sigma} - \sigma\beta F'(G_j(\mathbf{e})) [G_j(\mathbf{e})]^{2\sigma-1} e_j^{-2\sigma}(e_{-j}/e_j)^{1-\sigma} \right\}. \tag{A10}$$

Factorizing by $F'(G_j(\mathbf{e})) [G_j(\mathbf{e})]^{2\sigma-1} e_j^{-2\sigma}$ and using $\mu = -[F''(G_j(\mathbf{e})) . G_j(\mathbf{e})]/F'(G_j(\mathbf{e}))$, (A10) can be rewritten as

$$\frac{\partial^2 v_j(\mathbf{e})}{\partial e_j^2} = -\theta_j F'(G_j(\mathbf{e})) [G_j(\mathbf{e})]^{2\sigma-1} e_j^{-2\sigma} \left[\mu + \sigma\beta(e_{-j}/e_j)^{1-\sigma}\right] < 0. \tag{A11}$$

As a result $v_j(\mathbf{e})$ is strictly concave and continuous in e_j for $\sigma \in [0, 1)$ or $\sigma \in (1, +\infty)$, which guarantees the existence of a pure strategy Nash equilibrium. In this equilibrium, the first-order condition given by (4) is both necessary and sufficient for characterizing the best-response function of region j's representative.

(ii) Uniqueness: We first observe that there does not exist an equilibrium in which for one region a corner solution at zero public investments is obtained while an interior solution holds for the other region for any $\sigma \in \{[0, 1) \cup (1, +\infty)\}$. First, for $\sigma \in [0, 1)$, the first-order condition (4) cannot be satisfied for $e_j = 0$ and $e_{-j} > 0$ because in that case $G_j(\mathbf{e}) > 0$ and the left-hand term of (4) approaches infinity. Second, if $\sigma \in (1, +\infty)$ and $e_j = 0$ then, as mentioned in the text, we take the limit of (1), i.e. $G_j(\mathbf{e}) = 0$ *and* $G_{-j}(\mathbf{e}) = 0$. Hence, $v_{-j}(\mathbf{e})$ is strictly decreasing in e_{-j}, and so $e_j = 0$ and $e_{-j} = 0$ are mutually best responses.

Next, we show that there exists a unique equilibrium with $e_j^* > 0$, for $j = A, B$, when local public investments are strategic substitutes – i.e. $\sigma \in [0, \mu)$ – and when they are strategic complements – i.e. $\sigma \in (\mu, +\infty)$. For $\sigma \in [0, \mu)$, the proof proceeds by contradiction (in the spirit of Bloch and Zenginobuz, 2007). Suppose that there exists two distinct equilibria $\mathbf{e} \equiv (e_j, e_{-j}) \in \Re_+^{*2}$ and $\mathbf{e}' \equiv (e'_j, e'_{-j}) \in \Re_+^{*2}$. Suppose further, without loss of generality, that $e'_j < e_j$. We first show that this implies $G_j(\mathbf{e}') > G_j(\mathbf{e})$. From the first-order condition (4), we have $\theta_j F'(G_j(\mathbf{e})) [G_j(\mathbf{e})]^\sigma =$

e_j^σ. The right-hand term (RHT) is lower with e_j' than with e_j, which implies that the left-hand term (LHT) must also be lower with e_j'. The derivative of this term with respect to $G_j(\mathbf{e})$ is given by $\partial (LHT) / \partial G_j(\mathbf{e}) = \theta_j F''(G_j(\mathbf{e}))[G_j(\mathbf{e})]^\sigma + \sigma\theta_j F'(G_j(\mathbf{e}))[G_j(\mathbf{e})]^{\sigma-1}$. This can be rewritten as $\partial (LHT) / \partial G_j(\mathbf{e}) = \theta_j [G_j(\mathbf{e})]^{\sigma-1} F'(G_j(\mathbf{e}))[\sigma - \mu]$, which is negative for $\sigma < \mu$ (so that the LHT is decreasing in $G_j(\mathbf{e})$). Hence, we must have $G_j(\mathbf{e}') > G_j(\mathbf{e})$ for satisfying the first-order condition in region j when $e_j' < e_j$. However, $G_j(\mathbf{e}') > G_j(\mathbf{e})$ and $e_j' < e_j$ necessarily imply $e_{-j}' > e_{-j}$ and $G_{-j}(\mathbf{e}') > G_{-j}(\mathbf{e})$. But for region $-j$, the RHT of the first-order condition is (also) increasing in e_{-j} and the LHT is (also) decreasing $G_{-j}(\mathbf{e})$. Therefore, one cannot have another equilibrium $\mathbf{e}' \neq \mathbf{e}$, which satisfies the two first-order conditions. To summarize, there is a unique equilibrium which involves $e_A^* > 0$ and $e_B^* > 0$ for $\sigma \in [0, \mu)$.

Suppose now that $\sigma \in (\mu, +\infty)$. In an interior equilibrium, public good provision is still characterized by the first-order condition (4) with equality. This equation implicitly defines $e_j = \varphi(\theta_j, e_{-j})$. By the implicit function theorem, $\varphi(.)$ is continuous and furthermore $\partial e_j / \partial e_{-j} = -\left[\partial^2 v_j(\mathbf{e}) / \partial e_j \partial e_{-j}\right] / [\partial^2 v_j(\mathbf{e}) / \partial e_j^2]$, with $\partial^2 v_j(\mathbf{e}) / \partial e_j^2$ being strictly negative (from Lemma 2). Using (A6) and (A11), we have

$$\frac{\partial e_j}{\partial e_{-j}} = \frac{\beta(\sigma - \mu)}{\mu [e_{-j}/e_j]^\sigma + \sigma\beta [e_{-j}/e_j]}, \tag{A12}$$

which is positive for $\sigma > \mu$ (and negative for $\sigma < \mu$). When $\sigma > \mu$, one can also observe that $\partial^2 e_j / \partial e_{-j}^2 < 0$, so that best-response functions are increasing at a decreasing rate. We also have that $(\partial e_j / \partial e_{-j})_{|e_{-j} \to 0} = +\infty$, which means that near the origin, each best-response function must be on the upper side of the 45° line. At the other extreme, we have $(\partial e_j / \partial e_{-j})_{|e_{-j} \to +\infty} = 0$, so that each best-response function must cross the 45° line. Since its slope is always decreasing, each best-response function cross the 45° line only once. There is thus a unique equilibrium which involves $e_A^* > 0$ and $e_B^* > 0$ for $\sigma \in (\mu, +\infty)$.

For $\sigma = \mu$ − which corresponds to the analysis of Besley and Coate (2003) − there is a unique equilibrium in dominant strategies, and best-response functions are two straight lines with 0 slope.

7.3 Proof of Lemma 3

We first show that e_j and $G_j(\mathbf{e})$ are increasing in θ_j. Equilibrium public good provision is characterized by the first-order condition (4). Again, this equation implicitly defines $e_j = \varphi(\theta_j, e_{-j})$. By the implicit function theorem, $\partial e_j / \partial \theta_j = -\left[\partial^2 v_j(\mathbf{e}) / \partial e_j \partial \theta_j\right] / [\partial^2 v_j(\mathbf{e}) / \partial e_j^2]$. Using (A2), we have $\partial^2 v_j(\mathbf{e}) / \partial e_j \partial \theta_j = F'(G_j(\mathbf{e}))[G_j(\mathbf{e})]^\sigma e_j^{-\sigma} > 0$. Again, we also have $\partial^2 v_j(\mathbf{e}) / \partial e_j^2 < 0$ (Lemma 2). It follows that $\partial e_j / \partial \theta_j > 0$. This also implies that $\partial G_j(\mathbf{e}) / \partial \theta_j > 0$ since $G_j(\mathbf{e})$ is increasing in e_j.

Next suppose that $\theta' > \theta$ and $\theta_j > \theta_j'$. The inequality $w_j(\theta, \theta_j, \theta_{-j}) \geq w_j(\theta, \theta_j', \theta_{-j})$ can be rewritten as $\theta \left[F(G_j(\theta_j, \theta_{-j})) - F(G_j(\theta_j', \theta_{-j}))\right] \geq e_j(\theta_j, \theta_{-j}) - e_j(\theta_j', \theta_{-j})$. The right-hand term and the term in [.] in the left-hand side are both strictly positive (since $F(.)$ is also an increasing function). So, if this inequality is verified for a type θ citizen in region j, it is also obviously verified for a type θ' citizen with $\theta' > \theta$, i.e., $w_j(\theta', \theta_j, \theta_{-j}) \geq w_j(\theta', \theta'_j, \theta_{-j})$. Suppose now that $\theta' < \theta$ and $\theta_j < \theta'_j$. The inequality $w_j(\theta, \theta_j, \theta_{-j}) \geq w_j(\theta, \theta'_j, \theta_{-j})$ can be rewritten as $\theta \left[F(G_j(\theta'_j, \theta_{-j})) - F\left(G_j\left(\theta_j, \theta_{-j}\right)\right)\right] \leq e_j(\theta'_j, \theta_{-j}) - e_j(\theta_j, \theta_{-j})$. Again, the right-hand term and the term in [.] in the left-hand side are both strictly positive. So if this inequality is verified for a type θ citizen in region j, it is also obviously verified for a type θ' citizen with $\theta' < \theta$, i.e., $w_j(\theta', \theta_j, \theta_{-j}) \geq w_j(\theta', \theta'_j, \theta_{-j})$.

7.4 Proof of Proposition 1

Here, we just assume the existence of a LNSPE (under the sufficient condition that $\sigma \geq \bar{\sigma}$) and characterize the properties of this equilibrium. We also show that if a symmetric LNSPE exists, then this equilibrium is unique. The proof of the existence of such an equilibrium under decentralization is given in a separate appendix. In this second appendix, we also show that, in the case of a decentralized system, there exists a SPNE for $\sigma = 0$ and $\sigma \in [\mu, 1)$. This additional appendix is available upon request.

(i) in a LNSPE, the preferred representative of region j's median voter is given by the first-order condition (6). We first derive the expression for $\partial e^*_{-j}/\partial\theta_j$. For expositional convenience only let $j \equiv A$ and $-j \equiv B$, so that we first determine $\partial e^*_{-B}/\partial\theta_A$. Using (4), e^*_B must satisfy

$$\theta_B F'\left(G_B\left(\mathbf{e}^*\right)\right)\left[G_B\left(\mathbf{e}^*\right)\right]^{\sigma} e_B^{*-\sigma} - 1 = 0. \tag{A13}$$

Differentiating this expression with respect to θ_A yields

$$\begin{aligned}
&-\theta_B \sigma e_B^{*-\sigma-1} F'\left(G_B\left(\mathbf{e}^*\right)\right)\left[G_B\left(\mathbf{e}^*\right)\right]^{\sigma} \frac{\partial e^*_B}{\partial\theta_A} \\
&+\theta_B e_B^{*-\sigma} \sigma \left[G_B\left(\mathbf{e}^*\right)\right]^{\sigma-1} F'\left(G_B\left(\mathbf{e}^*\right)\right) \frac{\partial G_B\left(\mathbf{e}^*\right)}{\partial\theta_A} \\
&+\theta_B e_B^{*-\sigma}\left[G_B\left(\mathbf{e}^*\right)\right]^{\sigma} F''\left(G_B\left(\mathbf{e}^*\right)\right) \frac{\partial G_B\left(\mathbf{e}^*\right)}{\partial\theta_A} = 0.
\end{aligned} \tag{A14}$$

Factorizing by $\theta_B e_B^{*-\sigma-1}\left[G_B\left(\mathbf{e}^*\right)\right]^{\sigma-1}$, the equality (A14) reduces to

$$\begin{aligned}
&-\sigma F'\left(G_B\left(\mathbf{e}^*\right)\right)\left[G_B\left(\mathbf{e}^*\right)\right] \frac{\partial e^*_B}{\partial\theta_A} \\
&+\sigma e^*_B F'\left(G_B\left(\mathbf{e}^*\right)\right) \frac{\partial G_B\left(\mathbf{e}^*\right)}{\partial\theta_A} \\
&+e^*_B\left[G_B\left(\mathbf{e}^*\right)\right] F''\left(G_B\left(\mathbf{e}^*\right)\right) \frac{\partial G_B\left(\mathbf{e}^*\right)}{\partial\theta_A} = 0
\end{aligned} \tag{A15}$$

Now, factorizing by $F'\left(G_B\left(\mathbf{e}^*\right)\right)$ and use $\mu = -[F''\left(G_j\left(\mathbf{e}\right)\right).G_j\left(\mathbf{e}\right)]/F'\left(G_j\left(\mathbf{e}\right)\right)$, then the equality (A15) reduces to

$$-\sigma\left[G_B\left(\mathbf{e}^*\right)\right] \frac{\partial e^*_B}{\partial\theta_A} + \sigma e^*_B \frac{\partial G_B\left(\mathbf{e}^*\right)}{\partial\theta_A} - \mu e^*_B \frac{\partial G_B\left(\mathbf{e}^*\right)}{\partial\theta_A} = 0. \tag{A16}$$

We also have $G_B\left(\mathbf{e}^*\right) = \left[e_B^{*1-\sigma} + \beta e_A^{*1-\sigma}\right]^{\frac{1}{1-\sigma}}$ and hence

$$\frac{\partial G_B(\mathbf{e}^*)}{\partial \theta_A} = [G_B(\mathbf{e}^*)]^\sigma \left(e_B^{*-\sigma} \frac{\partial e_B^*}{\partial \theta_A} + \beta e_A^{*-\sigma} \frac{\partial e_A^*}{\partial \theta_A} \right). \tag{A17}$$

Substituting this expression into (A16), we have

$$-\sigma [G_B(\mathbf{e}^*)] \frac{\partial e_B^*}{\partial \theta_A} + e_B^* [G_B(\mathbf{e}^*)]^\sigma \left(e_B^{*-\sigma} \frac{\partial e_B^*}{\partial \theta_A} + \beta e_A^{*-\sigma} \frac{\partial e_A^*}{\partial \theta_A} \right)(\sigma - \mu) = 0. \tag{A18}$$

Factorizing by $[G_B(\mathbf{e}^*)]^\sigma$ and observing that $[G_B(\mathbf{e}^*)]^{1-\sigma} = \left[e_B^{*1-\sigma} + \beta e_A^{*1-\sigma}\right]$, the equality (A18) reduces to

$$-\sigma \left(e_B^{*1-\sigma} + \beta e_A^{*1-\sigma} \right) \frac{\partial e_B^*}{\partial \theta_A} + e_B^* \left(e_B^{*-\sigma} \frac{\partial e_B^*}{\partial \theta_A} + \beta e_A^{*-\sigma} \frac{\partial e_A^*}{\partial \theta_A} \right)(\sigma - \mu) = 0. \tag{A19}$$

This implies

$$\frac{\partial e_B^*}{\partial \theta_A} = \left[\frac{\beta(\sigma - \mu) e_B^* e_A^{*-\sigma}}{\mu e_B^{*1-\sigma} + \sigma \beta e_A^{*1-\sigma}} \right] \frac{\partial e_A^*}{\partial \theta_A}. \tag{A20}$$

Now, assuming that the median voters in both regions have the same taste parameter m**,** this implies that $\theta_A = \theta_B = \theta$ and $e_A^* = e_B^* = e^*$. In a symmetric equilibrium, (A20) then reduces to

$$\frac{\partial e_B^*}{\partial \theta_A} \Big|_{e_B^* = e_A^*} = \frac{\beta(\sigma - \mu)}{\mu + \sigma\beta} \frac{\partial e_A^*}{\partial \theta_A} \Big|_{e_A^* = e_B^*}. \tag{A21}$$

Substituting into the first-order condition (6) then yields

$$\left(\frac{m}{\theta} - 1 \right) + \frac{m\beta}{\theta} \frac{(\sigma - \mu)\beta}{\mu + \sigma\beta} = 0. \tag{A22}$$

The solution of this equation is thus given by (7) in Proposition 1.

(ii) We have $\theta^* \gtreqless m$, if $(1+\beta)[\mu(1-\beta) + \sigma\beta] \gtreqless \mu + \sigma\beta$, which reduces to $\beta^2(\sigma - \mu) \gtreqless 0$ or $\sigma \gtreqless \mu$.

7.5 Proof of Lemma 4

We show that aggregated payoff given by (12) is strictly concave, so that a unique solutions results. From Lemma 2 – equation (A11) – we have that $\partial^2 v_j(\mathbf{e}) / \partial e_j^2 < 0$. It is then sufficient to show that $\partial^2 v_{-j}(\mathbf{e}) / \partial e_j^2 < 0$. We have

$$\frac{\partial v_{-j}(\mathbf{e})}{\partial e_j} = \theta_{-j} F'(G_{-j}(\mathbf{e})) \frac{\partial G_{-j}(\mathbf{e})}{\partial e_j} - 1. \tag{A23}$$

Now, calculating the second derivative of $v_{-j}(\mathbf{e})$ with respect to e_j, we obtain

$$\frac{\partial^2 v_{-j}(\mathbf{e})}{\partial e_j^2} = \theta_{-j} \left\{ F''(G_{-j}(\mathbf{e})) \left[\frac{\partial G_{-j}(\mathbf{e})}{\partial e_j} \right]^2 + F'(G_{-j}(\mathbf{e})) \frac{\partial^2 G_{-j}(\mathbf{e})}{\partial e_j^2} \right\}. \tag{A24}$$

The first term in $\{.\}$ is negative since $F''(.) < 0$. The sign of the second term in $\{.\}$ is the same as the sign of $\partial^2 G_{-j}(\mathbf{e}) / \partial e_j^2$ since $F'(.) > 0$. We have $\partial G_{-j}(\mathbf{e}) / \partial e_j = \beta [G_{-j}(\mathbf{e})]^\sigma e_j^{-\sigma}$, and hence

$$\frac{\partial^2 G_{-j}(\mathbf{e})}{\partial e_j^2} = \beta \left\{ \sigma \left[G_{-j}(\mathbf{e})\right]^{\sigma-1} \frac{\partial G_{-j}(\mathbf{e})}{\partial e_j} e_j^{-\sigma} - \sigma \left[G_{-j}(\mathbf{e})\right]^{\sigma} e_j^{-\sigma-1} \right\}. \tag{A25}$$

Again, $\partial G_{-j}(\mathbf{e}) / \partial e_j = \beta \left[G_{-j}(\mathbf{e})\right]^{\sigma} e_j^{-\sigma}$ so that (A25) can be rewritten as follows

$$\frac{\partial^2 G_{-j}(\mathbf{e})}{\partial e_j^2} = \beta\sigma \left[G_{-j}(\mathbf{e})\right]^{2\sigma-1} e_j^{-2\sigma} \left[\beta - \left[G_{-j}(\mathbf{e})\right]^{1-\sigma} e_j^{-(1-\sigma)}\right], \tag{A26}$$

which is strictly negative since $\left[G_{-j}(\mathbf{e})\right]^{1-\sigma} e_j^{-(1-\sigma)} = \beta + (e_{-j}/e_j)^{1-\sigma} > \beta$. It follows that $\partial^2 v_{-j}(\mathbf{e}) / \partial e_j^2 < 0$. Together with Lemma 2, we have that $v_j(\mathbf{e}) + v_{-j}(\mathbf{e})$ is strictly concave in e_j. There is thus a unique solution which is given by the (necessary and sufficient) first-order condition (13).

7.6 Proof of Proposition 2

Again for this proof, we just assume the existence of a LNSPE (under the sufficient condition that $\mu \geq 0.5$) and characterize the properties of this equilibrium. The proof of existence of such an equilibrium under centralization is given in a separate appendix, which is available upon request.

In a LNSPE, the preferred representative of region j's median voter is given by the first-order condition (14). We then need to derive the expression of $\partial \tilde{e}_{-j} / \partial \theta_j$. Again, for expositional convenience only, we derive the outcome of the delegation stage for region A keeping in mind that the same reasoning will apply for region B. Hence, we first characterize $\partial \tilde{e}_B / \partial \theta_A$.

Using (13), $\tilde{e}_A$ and $\tilde{e}_B$ are such that

$$\begin{aligned} &\frac{\theta_A F'(G_A(\tilde{\mathbf{e}}))\left[G_A(\tilde{\mathbf{e}})\right]^{\sigma} + \beta\theta_B F'(G_B(\tilde{\mathbf{e}}))\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma}}{\tilde{e}_A^{\sigma}} \\ &= \frac{\theta_B F'(G_B(\tilde{\mathbf{e}}))\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma} + \beta\theta_A F'(G_A(\tilde{\mathbf{e}}))\left[G_A(\tilde{\mathbf{e}})\right]^{\sigma}}{\tilde{e}_B^{\sigma}}. \end{aligned} \tag{A27}$$

We then obtain

$$\theta_A F'(G_A(\tilde{\mathbf{e}}))\left[G_A(\tilde{\mathbf{e}})\right]^{\sigma} = \frac{\theta_B F'(G_B(\tilde{\mathbf{e}}))\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma}(\tilde{e}_A^{\sigma} - \beta\tilde{e}_B^{\sigma})}{(\tilde{e}_B^{\sigma} - \beta\tilde{e}_A^{\sigma})}. \tag{A28}$$

Substituting into (13) – with $j \equiv B$ and $-j \equiv A$ – and simplifying, we have

$$\theta_B F'(G_B(\tilde{\mathbf{e}}))\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma}(1-\beta^2) = \tilde{e}_B^{\sigma} - \beta\tilde{e}_A^{\sigma}. \tag{A29}$$

Differentiating this expression with respect to θ_A yields

$$\begin{aligned} &\theta_B(1-\beta^2)\left[\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma} F''(G_B(\tilde{\mathbf{e}}))\frac{\partial G_B(\tilde{\mathbf{e}})}{\partial\theta_A} + \sigma\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma-1}\frac{\partial G_B(\tilde{\mathbf{e}})}{\partial\theta_A}F'(G_B(\tilde{\mathbf{e}}))\right] \\ &= \sigma\tilde{e}_B^{\sigma-1}\frac{\partial\tilde{e}_B}{\partial\theta_A} - \sigma\beta\tilde{e}_A^{\sigma-1}\frac{\partial\tilde{e}_A}{\partial\theta_A}. \end{aligned} \tag{A30}$$

We also have

$$\frac{\partial G_B(\tilde{\mathbf{e}})}{\partial\theta_A} = \left[G_B(\tilde{\mathbf{e}})\right]^{\sigma}\left(\tilde{e}_B^{-\sigma}\frac{\partial\tilde{e}_B}{\partial\theta_A} + \beta\tilde{e}_A^{-\sigma}\frac{\partial\tilde{e}_A}{\partial\theta_A}\right). \tag{A31}$$

Substituting into (A30) and factorizing by $[G_B(\tilde{\mathbf{e}})]^{2\sigma-1}$, we have

$$\theta_B(1-\beta^2)\left[G_B(\tilde{\mathbf{e}})\right]^{2\sigma-1}\left[F''\left(G_B(\tilde{\mathbf{e}})\right)G_B(\tilde{\mathbf{e}})+\sigma F'\left(G_B(\tilde{\mathbf{e}})\right)\right]\left(\tilde{e}_B^{-\sigma}\frac{\partial\tilde{e}_B}{\partial\theta_A}+\beta\tilde{e}_A^{-\sigma}\frac{\partial\tilde{e}_A}{\partial\theta_A}\right)$$
$$=\sigma\tilde{e}_B^{\sigma-1}\frac{\partial\tilde{e}_B}{\partial\theta_A}-\sigma\beta\tilde{e}_A^{\sigma-1}\frac{\partial\tilde{e}_A}{\partial\theta_A}. \tag{A32}$$

Now, factorizing by $F'(G_B(\tilde{\mathbf{e}}))$ and using $\mu=-[F''(G_j(\mathbf{e})).G_j(\mathbf{e})]/F'(G_j(\mathbf{e}))$ gives

$$\theta_B(1-\beta^2)\left[G_B(\tilde{\mathbf{e}})\right]^{2\sigma-1}F'\left(G_B(\tilde{\mathbf{e}})\right)[\sigma-\mu]\left(\tilde{e}_B^{-\sigma}\frac{\partial\tilde{e}_B}{\partial\theta_A}+\beta\tilde{e}_A^{-\sigma}\frac{\partial\tilde{e}_A}{\partial\theta_A}\right)$$
$$=\sigma\tilde{e}_B^{\sigma-1}\frac{\partial\tilde{e}_B}{\partial\theta_A}-\sigma\beta\tilde{e}_A^{\sigma-1}\frac{\partial\tilde{e}_A}{\partial\theta_A}. \tag{A33}$$

Now, using (A29), we have

$$\left(\tilde{e}_B^{\sigma}-\beta\tilde{e}_A^{\sigma}\right)\left[G_B(\tilde{\mathbf{e}})\right]^{\sigma-1}[\sigma-\mu]\left(\tilde{e}_B^{-\sigma}\frac{\partial\tilde{e}_B}{\partial\theta_A}+\beta\tilde{e}_A^{-\sigma}\frac{\partial\tilde{e}_A}{\partial\theta_A}\right)$$
$$=\sigma\tilde{e}_B^{\sigma-1}\frac{\partial\tilde{e}_B}{\partial\theta_A}-\sigma\beta\tilde{e}_A^{\sigma-1}\frac{\partial\tilde{e}_A}{\partial\theta_A}. \tag{A34}$$

Observing that $[G_B(\tilde{\mathbf{e}})]^{\sigma-1}=1/\left[\tilde{e}_B^{1-\sigma}+\beta\tilde{e}_A^{1-\sigma}\right]$, we have

$$\left(\tilde{e}_B^{\sigma}-\beta\tilde{e}_A^{\sigma}\right)(\sigma-\mu)\left(\tilde{e}_B^{-\sigma}\frac{\partial\tilde{e}_B}{\partial\theta_A}+\beta\tilde{e}_A^{-\sigma}\frac{\partial\tilde{e}_A}{\partial\theta_A}\right)=\left(\tilde{e}_B^{1-\sigma}+\beta\tilde{e}_A^{1-\sigma}\right)\left(\sigma\tilde{e}_B^{\sigma-1}\frac{\partial\tilde{e}_B}{\partial\theta_A}-\sigma\beta\tilde{e}_A^{\sigma-1}\frac{\partial\tilde{e}_A}{\partial\theta_A}\right). \tag{A35}$$

We then have

$$\frac{\partial\tilde{e}_B}{\partial\theta_A}=\left[\frac{\sigma\beta\left(\tilde{e}_B^{1-\sigma}\tilde{e}_A^{\sigma-1}+\beta\right)+\beta(\sigma-\mu)\left(\tilde{e}_B^{\sigma}\tilde{e}_A^{-\sigma}-\beta\right)}{(\sigma-\mu)\left(\beta\tilde{e}_A^{\sigma}\tilde{e}_B^{-\sigma}-1\right)+\sigma\left(1+\beta\tilde{e}_A^{1-\sigma}\tilde{e}_B^{\sigma-1}\right)}\right]\frac{\partial\tilde{e}_A}{\partial\theta_A}. \tag{A36}$$

As for the decentralization system, we focus on a symmetric equilibrium i.e. $m_A=m_B=m$, which implies $\theta_A=\theta_B=\theta$ and $\tilde{e}_A=\tilde{e}_B=\tilde{e}$. The above expression then reduces to

$$\frac{\partial\tilde{e}_B}{\partial\theta_A}|_{\tilde{e}_B=\tilde{e}_A}=\frac{\beta\left[2\sigma-\mu(1-\beta)\right]}{2\sigma\beta+\mu(1-\beta)}\frac{\partial\tilde{e}_A}{\partial\theta_A}|_{\tilde{e}_A=\tilde{e}_B}. \tag{A37}$$

In addition, from (13), we also have in a symmetric equilibrium $F'(G(\tilde{\mathbf{e}}))[G(\tilde{\mathbf{e}})]^{\sigma}\tilde{e}^{-\sigma}=1/[\theta(1+\beta)]$. Substituting this last expression into (14) (with $m_j=m$) yields

$$\left(\frac{m}{\theta(1+\beta)}-\frac{2-\lambda}{2}\right)\frac{\partial\tilde{e}_A}{\partial\theta_A}|_{\tilde{e}_A=\tilde{e}_B}+\left(\frac{\beta m}{\theta(1+\beta)}-\frac{\lambda}{2}\right)\frac{\partial\tilde{e}_B}{\partial\theta_A}|_{\tilde{e}_B=\tilde{e}_A}=0. \tag{A38}$$

Finally, substituting (A37) into (A38) yields

$$[2m-\theta(1+\beta)(2-\lambda)][2\sigma\beta+\mu(1-\beta)]+\beta[2\beta m-\theta(1+\beta)\lambda][2\sigma-\mu(1-\beta)]=0. \tag{A39}$$

The solution of this equation in θ is thus given by (15) in Proposition 2.

(ii) It is immediately verified that $\tilde{\theta}\gtreqless m$ if $\lambda\gtreqless\tilde{\lambda}$ with $\tilde{\lambda}\equiv 2\beta/(1+\beta)$.

7.7 Welfare under decentralization and centralization

From (4) with $F(G) = G^{1-\mu}$, we can obtain the level of public investment in each region in the symmetric equilibrium of a decentralized system, that is $e^* = \left[\theta^* (1-\mu)(1+\beta)^{\frac{\sigma-\mu}{1-\sigma}}\right]^{\frac{1}{\mu}}$. Substituting (7) into this expression, we then obtain

$$e^* = \left[\frac{(1-\mu)\left[\mu(1-\beta)+\sigma\beta\right](1+\beta)^{\frac{1-\mu}{1-\sigma}}}{\mu+\sigma\beta} m\right]^{\frac{1}{\mu}}. \tag{A40}$$

We then obtain the following level of welfare for the median voter under a decentralized system,

$$v^* = \mu\left[\beta(1+\sigma)+\mu(1-\beta)\right]\left[\frac{\left[(1-\mu)\left[\mu(1-\beta)+\sigma\beta\right]\right]^{1-\mu}(1+\beta)^{\frac{1-\mu}{1-\sigma}}}{\mu+\sigma\beta} m\right]^{\frac{1}{\mu}}. \tag{A41}$$

Assuming that $\mu = 0.5$, we obtain (17) in the text.

Using (13) with $F(G) = G^{1-\mu}$, we obtain the level of public investment in each region in the symmetric equilibrium of a centralized system, that is $\tilde{e} = \left[\tilde{\theta}(1-\mu)(1+\beta)^{\frac{1-\mu}{1-\sigma}}\right]^{\frac{1}{\mu}}$. Using (15), we then have

$$\tilde{e} = \left[\frac{2(1-\mu)\left[2\sigma\beta+\mu(1-\beta)^2\right](1+\beta)^{\frac{1-\mu}{1-\sigma}}}{4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]} m\right]^{\frac{1}{\mu}}. \tag{A42}$$

We then obtain the following level of welfare for the median voter under a centralized system.

$$\tilde{v} = \mu\left[4\sigma\beta+2(1-\beta)\left[\beta+\mu(1-\beta)\right]-\lambda\left(1-\beta^2\right)\right]\left[\frac{\left[2(1-\mu)\left[2\sigma\beta+\mu(1-\beta)^2\right]\right]^{1-\mu}(1+\beta)^{\frac{1-\mu}{1-\sigma}}}{4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]} m\right]^{\frac{1}{\mu}}. \tag{A43}$$

Assuming that $\mu = 0.5$, we obtain (18) in the text.

7.8 Proof of Proposition 3

(i) When $\lambda = 0$, we have under a centralized system

$$\tilde{v}_{|\lambda=0} = \frac{\left[4\sigma\beta+(1-\beta)^2\right]\left[4\sigma\beta+(1-\beta^2)\right](1+\beta)^{\frac{1}{1-\sigma}}}{4\left[4\sigma\beta+(1-\beta)\right]^2} m^2, \tag{A44}$$

while for $\lambda = 1$, we have

$$\tilde{v}_{|\lambda=1} = \frac{4\sigma\beta\left[4\sigma\beta+(1-\beta)^2\right](1+\beta)^{\frac{1}{1-\sigma}}}{\left[8\sigma\beta+(1-\beta)^2\right]^2} m^2. \tag{A45}$$

The welfare of each region under a decentralized system is still given by (17).

Hence, when $\lambda = 0$, the welfare difference between a centralized and a decentralized system is given by

$$\tilde{\upsilon}_{|\lambda=0} - \upsilon^* = \frac{\sigma\beta^3 \left[\sigma\beta\left(3+2\beta-\beta^2\right)+(1-\beta^2)\right](1+\beta)^{\frac{1}{1-\sigma}}}{\left[4\sigma\beta+(1-\beta)\right]^2\left[1+2\sigma\beta\right]^2} m^2. \tag{A46}$$

This expression is clearly positive for any $\sigma \in \{[0,1), (1,+\infty)\}$ and $\beta \in [0,1]$.

When $\lambda = 1$, the welfare difference between a centralized and a decentralized system is given by

$$\tilde{\upsilon}_{|\lambda=1} - \upsilon^* = \frac{\left[4\sigma^2\beta^2(1+\beta)(6\beta-\beta^2-1)+4\sigma\beta(1-\beta)^2(3\beta-1)-(1-\beta)^5\right](1+\beta)^{\frac{2-\sigma}{1-\sigma}}}{4\left[8\sigma\beta+(1-\beta)^2\right]^2\left[1+2\sigma\beta\right]^2} m^2. \tag{A47}$$

The sign of $\tilde{\upsilon}_{|\lambda=1} - \upsilon^*$ is the same as the sign of

$$\Gamma(\sigma,\beta) = 4\sigma^2\beta^2(1+\beta)(6\beta-\beta^2-1)+4\sigma\beta(1-\beta)^2(3\beta-1)-(1-\beta)^5, \tag{A48}$$

which is quadratic in σ. Thus $\Gamma(\sigma,\beta) = 0$ has two solutions given by

$$\hat{\sigma} = -\frac{(1-\beta)^2}{2\beta(1+\beta)} \text{ and } \tilde{\sigma} = \frac{(1-\beta)^3}{2\beta(6\beta-\beta^2-1)}. \tag{A49}$$

Clearly, $\hat{\sigma}$ is negative while $\tilde{\sigma}$ is positive if only if $6\beta-\beta^2-1 \geq 0$ or $\beta \geq 3-2\sqrt{2}$.

Furthermore, the second derivative of $\Gamma(\sigma,\beta)$ with respect to σ is also positive if $\beta \geq 3-2\sqrt{2}$ and negative if $\beta \leq 3-2\sqrt{2}$. Then, for $\beta \leq 3-2\sqrt{2}$, $\Gamma(\sigma,\beta)$ reaches a global maximum and furthermore $0 \geq \tilde{\sigma} \geq \hat{\sigma}$, which necessarily implies that $\Gamma(\sigma,\beta) < 0$ for any $\sigma \geq 0$. When $\beta \geq 3-2\sqrt{2}$, $\Gamma(\sigma,\beta)$ reaches a global minimum and hence $\Gamma(\sigma,\beta)$ is positive only if $\sigma \geq \tilde{\sigma} \geq 0 \geq \hat{\sigma}$.

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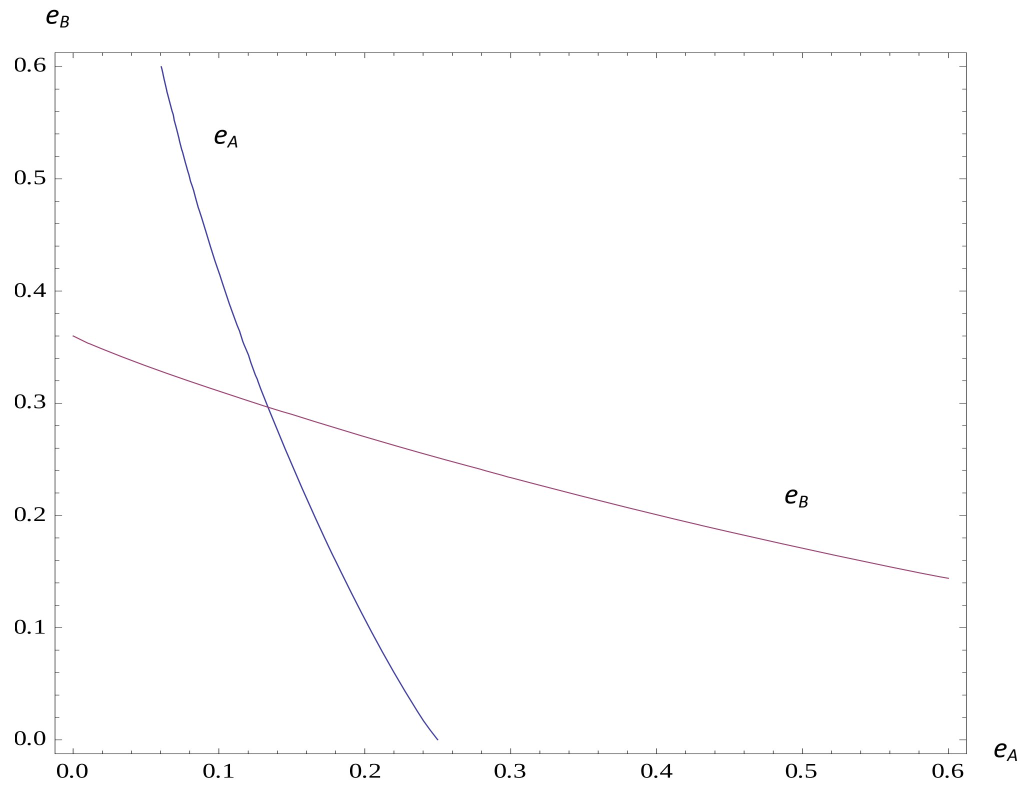

Figure 1 : The shape of reaction functions for σ < μ

(Numerical values used: μ = 0.5; σ = 0.1; β = 0.5; θ_A = 1; θ_B =1.2)

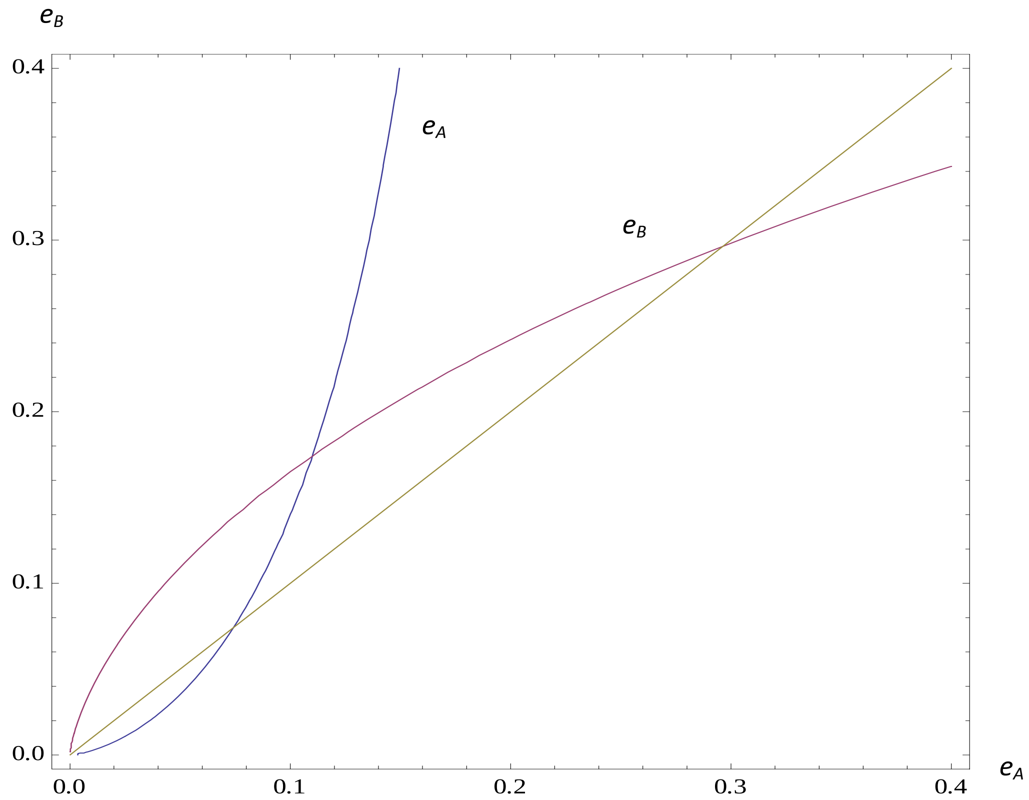

Figure 2 : The shape of reaction functions for σ > μ

(Numerical values used: μ = 0.5; σ = 2; β = 0.5; θ_A = 1; θ_B = 2)

The political Economy of (De)centralization with Complementary Public Goods: Equilibrium Existence

(Not for publication: Available upon request)

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Abstract: In this appendix, we prove the existence of a (symmetric) Local Nash Perfect Equilibrium (LNSPE) in pure strategies under both a decentralized system and a centralized system. We also show the existence of a Subgame Perfect Nash Equilibrium (SPNE) in pure strategies under decentralization and for a specific range of the parameters of the model.

1 Existence of a (Symmetric) Local Nash Subgame Perfect Equilibrium (LNSPE)

1.1 Decentralization

The payoff for the region A's median voter in the first stage of the game under decentralization is given by $w_A = mF\left[G_A\left(\mathbf{e}^*\left(\theta_A, \theta_B\right)\right)\right] - e_A^*\left(\theta_A, \theta_B\right)$. The first derivative of w_A with respect to θ_A after substituting the equilibrium conditions of the policy stage (given by (4)) – and eliminating the "$*$" in order to reduce the amount of notation – is given by (6):

$$\left(\frac{m}{\theta_A} - 1\right)\frac{\partial e_A}{\partial \theta_A} + \frac{\beta m}{\theta_A} e_A^{\sigma} e_B^{-\sigma} \frac{\partial e_B}{\partial \theta_A}. \tag{B1}$$

The second derivative of w_A with respect to θ_A is thus given by

$$-\frac{m}{\theta_A^2}\frac{\partial e_A}{\partial \theta_A} + \left(\frac{m}{\theta_A} - 1\right)\frac{\partial^2 e_A}{\partial \theta_A^2}$$
$$+\beta m\left\{-\frac{1}{\theta_A^2} e_A^{\sigma} e_B^{-\sigma}\frac{\partial e_B}{\partial \theta_A} + \frac{\sigma}{\theta_A} e_A^{\sigma-1} e_B^{-\sigma}\frac{\partial e_A}{\partial \theta_A}\frac{\partial e_B}{\partial \theta_A} - \frac{\sigma}{\theta_A} e_A^{\sigma} e_B^{-\sigma-1}\left(\frac{\partial e_B}{\partial \theta_A}\right)^2 + \frac{1}{\theta_A} e_A^{\sigma} e_B^{-\sigma}\frac{\partial^2 e_B}{\partial \theta_A^2}\right\}. \tag{B2}$$

In a symmetric equilibrium with $e_A = e_B = e$, we have

$$-\frac{m}{\theta_A^2}\frac{\partial e_A}{\partial \theta_A}|_e + \left(\frac{m}{\theta_A} - 1\right)\frac{\partial^2 e_A}{\partial \theta_A^2}|_e$$
$$+\beta m\left\{-\frac{1}{\theta_A^2}\frac{\partial e_B}{\partial \theta_A}|_e + \frac{\sigma e^{-1}}{\theta_A}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)\left(\frac{\partial e_B}{\partial \theta_A}|_e\right) - \frac{\sigma e^{-1}}{\theta_A}\left(\frac{\partial e_B}{\partial \theta_A}|_e\right)^2 + \frac{1}{\theta_A}\frac{\partial^2 e_B}{\partial \theta_A^2}|_e\right\}. \tag{B3}$$

We have (from (A21)) $\left(\partial e_B/\partial \theta_A\right)|_e = \left[\beta\left(\sigma - \mu\right)/\left(\mu + \sigma\beta\right)\right]\left(\partial e_A/\partial \theta_A\right)|_e$. Substituting into (B3), we have

$$-\frac{m}{\theta_A^2}\frac{\partial e_A}{\partial \theta_A}|_e + \left(\frac{m}{\theta_A} - 1\right)\frac{\partial^2 e_A}{\partial \theta_A^2}|_e$$

$$+\beta m\left\{-\frac{1}{\theta_A^2}\frac{\beta\left(\sigma-\mu\right)}{\mu+\sigma\beta}\frac{\partial e_A}{\partial\theta_A}\left|_e+\frac{\sigma e^{-1}}{\theta_A}\frac{\beta\left(\sigma-\mu\right)\mu(1+\beta)}{\left(\mu+\sigma\beta\right)^2}\left(\frac{\partial e_A}{\partial\theta_A}\right|_e\right)^2+\frac{1}{\theta_A}\frac{\partial^2 e_B}{\partial\theta_A^2}\left|_e\right\}. \tag{B4}$$

Rearranging the terms, we have

$$\left(\frac{m-\theta_A}{\theta_A}\right)\frac{\partial^2 e_A}{\partial\theta_A^2}|_e+\frac{\beta m}{\theta_A}\frac{\partial^2 e_B}{\partial\theta_A^2}|_e$$

$$-\frac{m}{\theta_A^2}\left[\frac{(1+\beta)\left[\mu\left(1-\beta\right)+\sigma\beta\right]}{\mu+\sigma\beta}\right]\frac{\partial e_A}{\partial\theta_A}\left|_e+\frac{\beta m\sigma e^{-1}}{\theta_A}\frac{\mu\beta\left(\sigma-\mu\right)\left(1+\beta\right)}{\left(\mu+\sigma\beta\right)^2}\left(\frac{\partial e_A}{\partial\theta_A}\right|_e\right)^2. \tag{B5}$$

We now determine $(\partial^2 e_B/\partial\theta_A^2)$. We have (from (A20))

$$\frac{\partial e_B}{\partial\theta_A}=\mathbf{X}(\theta_A)\frac{\partial e_A}{\partial\theta_A},\text{ with }\mathbf{X}(\theta_A)\equiv\frac{\beta\left(\sigma-\mu\right)e_B e_A^{-\sigma}}{\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}}. \tag{B6}$$

We then have

$$\frac{\partial^2 e_B}{\partial\theta_A^2}=\frac{\partial\mathbf{X}(\theta_A)}{\partial\theta_A}\frac{\partial e_A}{\partial\theta_A}+\mathbf{X}(\theta_A)\frac{\partial^2 e_A}{\partial\theta_A^2}. \tag{B7}$$

We have

$$\frac{\partial\mathbf{X}(\theta_A)}{\partial\theta_A}=\frac{\beta\left(\sigma-\mu\right)\mathbf{Z}(\theta_A)}{\left[\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}\right]^2}\text{ with }\mathbf{Z}(\theta_A)\equiv Z_1(\theta_A)-Z_2(\theta_A),$$

$$Z_1\equiv\frac{\partial U(\theta_A)}{\partial\theta_A}V(\theta_A),\ Z_2\equiv U(\theta_A)\frac{\partial V(\theta_A)}{\partial\theta_A},\ U(\theta_A)\equiv e_B e_A^{-\sigma}\text{ and }V(\theta_A)\equiv\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}. \tag{B8}$$

We have,

$$\frac{\partial U(\theta_A)}{\partial\theta_A}=e_A^{-\sigma-1}\left\{e_A\frac{\partial e_B}{\partial\theta_A}-\sigma e_B\frac{\partial e_A}{\partial\theta_A}\right\}. \tag{B9}$$

Substituting (B6) into this expression and rearranging the terms, we obtain

$$\frac{\partial U(\theta_A)}{\partial\theta_A}=e_A^{-\sigma-1}e_B\left\{\frac{\beta\left[\sigma(1-\sigma)-\mu\right]e_A^{1-\sigma}-\sigma\mu e_B^{1-\sigma}}{\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}}\right\}\frac{\partial e_A}{\partial\theta_A}. \tag{B10}$$

We thus have

$$Z_1(\theta_A)=e_A^{-\sigma-1}e_B\left[\beta\left[\sigma(1-\sigma)-\mu\right]e_A^{1-\sigma}-\sigma\mu e_B^{1-\sigma}\right]\frac{\partial e_A}{\partial\theta_A}. \tag{B11}$$

We also have

$$\frac{\partial V(\theta_A)}{\partial\theta_A}=(1-\sigma)\left\{\mu e_B^{-\sigma}\frac{\partial e_B}{\partial\theta_A}+\sigma\beta e_A^{-\sigma}\frac{\partial e_A}{\partial\theta_A}\right\}. \tag{B12}$$

Substituting (B6) into this expression and rearranging the terms, we obtain

$$\frac{\partial V(\theta_A)}{\partial\theta_A}=\beta(1-\sigma)e_A^{-\sigma}\left\{\frac{\mu\left(2\sigma-\mu\right)e_B^{1-\sigma}+\beta\sigma^2 e_A^{1-\sigma}}{\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}}\right\}\frac{\partial e_A}{\partial\theta_A}. \tag{B13}$$

We thus have

$$Z_2(\theta_A) = \beta(1-\sigma)e_B e_A^{-2\sigma}\left\{\frac{\mu\left(2\sigma-\mu\right)e_B^{1-\sigma}+\beta\sigma^2 e_A^{1-\sigma}}{\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}}\right\}\frac{\partial e_A}{\partial\theta_A}. \tag{B14}$$

Using (B11) and (B14), we find (after some tedious calculations)

$$\mathbf{Z}(\theta_A) = -\frac{\mu\sigma e_B e_A^{-\sigma-1}\left[\beta(1+\mu)e_A^{1-\sigma}e_B^{1-\sigma}+\beta^2 e_A^{2-2\sigma}+\mu e_B^{2-2\sigma}\right]}{\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}}\frac{\partial e_A}{\partial\theta_A}. \tag{B15}$$

It follows that

$$\frac{\partial^2 e_B}{\partial\theta_A^2} = \frac{\beta\left(\sigma-\mu\right)\mathbf{Z}(\theta_A)}{\left[\mu e_B^{1-\sigma}+\sigma\beta e_A^{1-\sigma}\right]^2}\frac{\partial e_A}{\partial\theta_A}+\mathbf{X}(\theta_A)\frac{\partial^2 e_A}{\partial\theta_A^2}. \tag{B16}$$

with $\mathbf{X}(\theta_A)$ and $\mathbf{Z}(\theta_A)$ given by (B6) and (B15) respectively.

We now evaluate this derivative in a symmetric equilibrium. We first have

$$\mathbf{X}(\theta_A)\,|_e = \frac{\beta\left(\sigma-\mu\right)}{\mu+\sigma\beta} \text{ and } \mathbf{Z}(\theta_A)\,|_e = -\frac{\mu\sigma e^{1-2\sigma}\left[\beta(1+\mu)+\beta^2+\mu\right]}{\mu+\sigma\beta}\frac{\partial e_A}{\partial\theta_A}\,|_e\,. \tag{B17}$$

Hence, we have

$$\frac{\partial^2 e_B}{\partial\theta_A^2}\,|_e = -\frac{\mu\sigma\beta\left(\sigma-\mu\right)\left[\beta(1+\mu)+\beta^2+\mu\right]e^{-1}}{\left[\mu+\sigma\beta\right]^3}\left(\frac{\partial e_A}{\partial\theta_A}\,|_e\right)^2+\frac{\beta\left(\sigma-\mu\right)}{\mu+\sigma\beta}\frac{\partial^2 e_A}{\partial\theta_A^2}\,|_e \tag{B18}$$

Substituting into (B5) and rearranging the terms, we can obtain after some tedious manipulations

$$\left(\frac{m(1+\beta)\left[\mu(1-\beta)+\sigma\beta\right]-\theta_A\left(\mu+\sigma\beta\right)}{\theta_A\left(\mu+\sigma\beta\right)}\right)\frac{\partial^2 e_A}{\partial\theta_A^2}\,|_e$$

$$-\frac{m}{\theta_A^2}\left[\frac{(1+\beta)\left[\mu\left(1-\beta\right)+\sigma\beta\right]}{\mu+\sigma\beta}\right]\frac{\partial e_A}{\partial\theta_A}\,|_e+\frac{m\mu\sigma\beta^3(\sigma-1)(\sigma-\mu)(1+\beta)e^{-1}}{\theta_A\left(\mu+\sigma\beta\right)^3}\left(\frac{\partial e_A}{\partial\theta_A}\,|_e\right)^2. \tag{B19}$$

In the symmetric equilibrium, we have $\theta_A = \theta^* = (1+\beta)\left[\mu(1-\beta)+\sigma\beta\right]m/\left(\mu+\sigma\beta\right)$, and hence the first term is equal to 0. Observe also that the second term reduces to $-(1/\theta_A)\left(\partial e_A/\partial\theta_A\right)|_e$. As a result the second derivative of w_A with respect to θ_A in the symmetric equilibrium is finally given by

$$\frac{1}{\theta_A}\left[\frac{m\mu\sigma\beta^3(\sigma-1)(\sigma-\mu)(1+\beta)e^{-1}}{\left(\mu+\sigma\beta\right)^3}\frac{\partial e_A}{\partial\theta_A}\,|_e-1\right]\frac{\partial e_A}{\partial\theta_A}\,|_e\,. \tag{B20}$$

We also need to determine $\left(\partial e_A/\partial\theta_A\right)|_e$. Rewrite the equilibrium conditions of the policy stage (given by (4)) as – and still eliminating the "$*$" superscripts – $F(e_A, e_B(e_A), \theta_A) = 0$. Again, in the first stage, the median voter of each region anticipates the other-region representative's best-response in the second stage. Applying the implicit function theorem, we have $\partial e_A/\partial\theta_A = -\partial F/\partial\theta_A/\left[\partial F/\partial e_A + \left(\partial F/\partial e_B\right)\left(\partial e_B/\partial e_A\right)\right]$. $F(e_A, e_B(e_A), \theta_A) = 0$ is given by

$$\theta_A(1-\mu)\left[e_A^{1-\sigma}+\beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}}e_A^{-\sigma}-1=0. \tag{B21}$$

Hence, we have

$$\frac{\partial F}{\partial \theta_A} = (1-\mu)\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}} e_A^{-\sigma}. \tag{B22}$$

Furthermore, we can easily obtain that

$$\frac{\partial F}{\partial e_A} = -\theta_A(1-\mu)\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_A^{-\sigma-1}\left[\mu e_A^{1-\sigma} + \sigma\beta e_B^{1-\sigma}\right]. \tag{B23}$$

We also have

$$\frac{\partial F}{\partial e_B} = \theta_A(1-\mu)(\sigma-\mu)\beta\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_A^{-\sigma} e_B^{-\sigma}. \tag{B24}$$

From (B6), we have $\partial e_B/\partial e_A = \beta(\sigma-\mu)e_B e_A^{-\sigma}/\left[\mu e_B^{1-\sigma} + \sigma\beta e_A^{1-\sigma}\right]$, and then

$$\frac{\partial F}{\partial e_B}\frac{\partial e_B}{\partial e_A} = \frac{\theta_A(1-\mu)(\sigma-\mu)^2\beta^2\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_B^{1-\sigma} e_A^{-2\sigma}}{\mu e_B^{1-\sigma} + \sigma\beta e_A^{1-\sigma}}. \tag{B25}$$

Hence, we have $\partial e_A/\partial\theta_A = -\partial F/\partial\theta_A/\left[\partial F/\partial e_A + (\partial F/\partial e_B)(\partial e_B/\partial e_A)\right]$, or

$$\frac{\partial e_A}{\partial \theta_A} = \frac{e_A\left(e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right)\left(\mu e_B^{1-\sigma} + \sigma\beta e_A^{1-\sigma}\right)}{\mu\theta_A\left[e_A^{1-\sigma}e_B^{1-\sigma}(\mu(1-\beta^2) + 2\sigma\beta^2) + \sigma\beta(e_A^{2-2\sigma} + e_B^{2-2\sigma})\right]}. \tag{B26}$$

In the symmetric equilibrium, we have

$$\frac{\partial e_A}{\partial \theta_A}|_e = \frac{e(\mu+\sigma\beta)}{\mu\theta_A\left[\mu(1-\beta) + 2\sigma\beta\right]}. \tag{B27}$$

Substituting into (B20), we have

$$\frac{1}{\theta_A}\left[\frac{m\sigma\beta^3(\sigma-1)(\sigma-\mu)(1+\beta)}{\theta_A(\mu+\sigma\beta)^2\left[\mu(1-\beta)+2\sigma\beta\right]} - 1\right]\frac{\partial e_A}{\partial\theta_A}|_e\ . \tag{B28}$$

In the symmetric equilibrium, we have $\theta_A = \theta^* = (1+\beta)\left[\mu(1-\beta)+\sigma\beta\right]m/(\mu+\sigma\beta)$. Thus the above expression reduces to

$$\frac{1}{\theta_A}\left[\frac{\sigma\beta^3(\sigma-1)(\sigma-\mu)}{(\mu+\sigma\beta)\left[\mu(1-\beta)+2\sigma\beta\right]\left[\mu(1-\beta)+\sigma\beta\right]} - 1\right]\frac{\partial e_A}{\partial\theta_A}|_e\ . \tag{B29}$$

This expression is negative, which implies that $\partial^2 w_A/\partial\theta_A^2 < 0$ in $\theta_A = \theta^*$, if

$$\Psi(\sigma,\beta,\mu) \equiv \frac{\sigma\beta^3(\sigma-1)(\sigma-\mu)}{(\mu+\sigma\beta)\left[\mu(1-\beta)+2\sigma\beta\right]\left[\mu(1-\beta)+\sigma\beta\right]} < 1. \tag{B30}$$

Note first that this inequality is always verified for $\sigma \in [\mu, 1)$. We also have

$$\frac{\partial\Psi}{\partial\beta} = \frac{\mu\sigma\beta^2(\sigma-1)(\sigma-\mu)\left[\mu\sigma\beta(8-5\beta) + 5\sigma^2\beta^2 + \mu^2(1-\beta)(3-\beta)\right]}{(\mu+\sigma\beta)^2\left[\mu(1-\beta)+2\sigma\beta\right]^2\left[\mu(1-\beta)+\sigma\beta\right]^2}. \tag{B31}$$

which is strictly positive for $\sigma \notin [\mu, 1)$. In this case, Ψ is increasing in β and reaches a maximum in $\beta = 1$, i.e. $\Psi|_{\beta=1} = (\sigma-1)(\sigma-\mu)/[2\sigma(\mu+\sigma)]$. We have that $\Psi|_{\beta=1} < 1$ for any $\sigma > \bar{\sigma} \equiv (1/2)\left[-(3\mu+1)+\sqrt{9\mu^2+10\mu+1}\right]$. $\bar{\sigma}$ is itself increasing in μ and reaches a maximum in $\mu = 1$ in which case $\bar{\sigma}|_{\mu=1} = \sqrt{5}-2$. Therefore a sufficient condition for $\Psi < 1$ is that $\sigma \geq \sqrt{5}-2 \simeq 0.24$. This implies that the second derivative of w_A with respect to θ_A in $\theta_A = \theta^*$ is strictly negative, thus proving the existence of a symmetric Local Nash Subgame Perfect Equilibrium (LNSPE) in pure strategies under a decentralized system. (Note that when $\mu = 0.5$, as it is assumed in Section 5, we have $\bar{\sigma}|_{\mu=1} = (1/4)\left[\sqrt{33}-5\right]$, as stated in Footnote 28).

1.2 Centralization

The payoff for the region A's median voter in the first stage of the game under decentralization is given by $w_A = mF\left[G_j\left(\tilde{\mathbf{e}}\left(\theta_A,\theta_B\right)\right)\right] - \left[(2-\lambda)\tilde{e}_j\left(\theta_A,\theta_B\right)+\lambda\tilde{e}_{-j}\left(\theta_A,\theta_B\right)\right]/2$. From the equilibrium conditions in the second stage we have using (A29) – and omitting the "tilde" to reduce the amount of notation: $F'\left(G_A\left(\mathbf{e}\right)\right)\left[G_A\left(\mathbf{e}\right)\right]^{\sigma} = \left(e_A^{\sigma}-\beta e_B^{\sigma}\right)/\left[\theta_A(1-\beta^2)\right] \equiv \mathbf{K}(\theta_A)$ Using (14), the first derivative of w_A with respect to θ_A can be rewritten as .

$$m\mathbf{K}(\theta_A)\left[e_A^{-\sigma}\frac{\partial e_A}{\partial\theta_A}+\beta e_B^{-\sigma}\frac{\partial e_B}{\partial\theta_A}\right]-\tfrac{1}{2}\left[(2-\lambda)\frac{\partial e_A}{\partial\theta_A}+\lambda\frac{\partial e_B}{\partial\theta_A}\right]=0. \tag{B32}$$

The second derivative of w_A with respect to θ_A is thus given by

$$\begin{aligned}
&\mathbf{Y}_1(\theta_A)+\mathbf{Y}_2(\theta_A)+\mathbf{Y}_3(\theta_A), \text{ with}\\
&\mathbf{Y}_1(\theta_A) = m\frac{\partial\mathbf{K}(\theta_A)}{\partial\theta_A}\left[e_A^{-\sigma}\frac{\partial e_A}{\partial\theta_A}+\beta e_B^{-\sigma}\frac{\partial e_B}{\partial\theta_A}\right],\\
&\mathbf{Y}_2(\theta_A) = m\mathbf{K}(\theta_A)\left[-\sigma e_A^{-\sigma-1}\left(\frac{\partial e_A}{\partial\theta_A}\right)^2+e_A^{-\sigma}\frac{\partial^2 e_A}{\partial\theta_A^2}-\sigma\beta e_B^{-\sigma-1}\left(\frac{\partial e_B}{\partial\theta_A}\right)^2+\beta e_B^{-\sigma}\frac{\partial^2 e_B}{\partial\theta_A^2}\right],\\
&\mathbf{Y}_3(\theta_A) = -\frac{1}{2}\left[(2-\lambda)\frac{\partial^2 e_A}{\partial\theta_A^2}+\lambda\frac{\partial^2 e_B}{\partial\theta_A^2}\right].
\end{aligned} \tag{B33}$$

We first have

$$\frac{\partial\mathbf{K}(\theta_A)}{\partial\theta_A} = \frac{1}{(1-\beta^2)\theta_A^2}\left\{\left[\sigma e_A^{\sigma-1}\frac{\partial e_A}{\partial\theta_A}-\sigma\beta e_B^{\sigma-1}\frac{\partial e_B}{\partial\theta_A}\right]\theta_A-\left(e_A^{\sigma}-\beta e_B^{\sigma}\right)\right\}. \tag{B34}$$

In a symmetric equilibrium, we have $\mathbf{K}(\theta_A)|_e = e^{\sigma}/\left[\theta_A(1+\beta)\right]$ and

$$\frac{\partial\mathbf{K}(\theta_A)}{\partial\theta_A}|_e = \frac{1}{(1-\beta^2)\theta_A^2}\left\{\sigma e^{\sigma-1}\theta_A\left[\frac{\partial e_A}{\partial\theta_A}|_e-\beta\frac{\partial e_B}{\partial\theta_A}|_e\right]-e^{\sigma}(1-\beta)\right\}. \tag{B35}$$

We have (from (A37)) $(\partial e_B/\partial\theta_A)|_e = \left\{\beta\left[2\sigma-\mu(1-\beta)\right]/\left[2\sigma\beta+\mu(1-\beta)\right]\right\}(\partial e_A/\partial\theta_A)|_e$. Substituting into the above expression, we can obtain

$$\frac{\partial\mathbf{K}(\theta_A)}{\partial\theta_A}|_e = \frac{\sigma e^{\sigma-1}\left[2\sigma\beta+\mu\left(1+\beta^2\right)\right]}{\theta_A(1+\beta)\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\frac{\partial e_A}{\partial\theta_A}|_e-\frac{e^{\sigma}}{\theta_A^2(1+\beta)}. \tag{B36}$$

We also have in the symmetric equilibrium

$$\left(e_A^{-\sigma}\frac{\partial e_A}{\partial \theta_A}+\beta e_B^{-\sigma}\frac{\partial e_B}{\partial \theta_A}\right)|_e = \frac{e^{-\sigma}(1+\beta)\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{2\sigma\beta+\mu\left(1-\beta\right)}\frac{\partial e_A}{\partial \theta_A}|_e . \tag{B37}$$

Substituting (B36) and (B37) into $\mathbf{Y}_1(\theta_A)$ and simplifying, we have

$$\mathbf{Y}_1(\theta_A)|_e = \frac{m\sigma e^{-1}\left[2\sigma\beta+\mu\left(1+\beta^2\right)\right]\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^2}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2$$

$$-\frac{m\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A^2\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\frac{\partial e_A}{\partial \theta_A}|_e . \tag{B38}$$

Regarding $\mathbf{Y}_2(\theta_A)$ in the symmetric equilibrium, we have

$$\mathbf{Y}_2(\theta_A) = \frac{m}{\theta_A(1+\beta)}\left[-\sigma e^{-1}\left[\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2+\beta\left(\frac{\partial e_B}{\partial \theta_A}|_e\right)^2\right]+\frac{\partial^2 e_A}{\partial \theta_A^2}|_e+\beta\frac{\partial^2 e_B}{\partial \theta_A^2}|_e\right]. \tag{B39}$$

Using $(\partial e_B/\partial\theta_A)|_e = \{\beta\left[2\sigma-\mu(1-\beta)\right]/\left[2\sigma\beta+\mu(1-\beta)\right]\}(\partial e_A/\partial\theta_A)|_e$, we can obtain

$$\mathbf{Y}_2(\theta_A)|_e = -\frac{m\sigma e^{-1}}{\theta_A(1+\beta)}\frac{\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^2+\beta^3\left[2\sigma-\mu\left(1-\beta\right)\right]^2}{\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^2}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2$$

$$+\frac{m}{\theta_A(1+\beta)}\left[\frac{\partial^2 e_A}{\partial \theta_A^2}|_e+\beta\frac{\partial^2 e_B}{\partial \theta_A^2}|_e\right]. \tag{B40}$$

Regarding $\mathbf{Y}_3(\theta_A)$ in the symmetric equilibrium is given by

$$\mathbf{Y}_3(\theta_A)|_e = -\frac{1}{2}\left[(2-\lambda)\frac{\partial^2 e_A}{\partial \theta_A^2}|_e+\lambda\frac{\partial^2 e_B}{\partial \theta_A^2}|_e\right]. \tag{B41}$$

We therefore need to characterize $(\partial^2 e_B/\partial\theta_A^2)$. We have (from (A36))

$$\frac{\partial e_B}{\partial \theta_A} = \mathbf{X}(\theta_A)\frac{\partial e_A}{\partial \theta_A}, \text{ with } \mathbf{X}(\theta_A)\equiv\frac{\sigma\beta\left(e_B^{1-\sigma}e_A^{\sigma-1}+\beta\right)+\beta\left(\sigma-\mu\right)\left(e_B^{\sigma}e_A^{-\sigma}-\beta\right)}{\left(\sigma-\mu\right)\left(\beta e_A^{\sigma}e_B^{-\sigma}-1\right)+\sigma\left(1+\beta e_A^{1-\sigma}e_B^{\sigma-1}\right)}. \tag{B42}$$

We then have

$$\frac{\partial^2 e_B}{\partial \theta_A^2} = \frac{\partial \mathbf{X}(\theta_A)}{\partial \theta_A}\frac{\partial e_A}{\partial \theta_A}+\mathbf{X}(\theta_A)\frac{\partial^2 e_A}{\partial \theta_A^2}. \tag{B43}$$

We have

$$\frac{\partial \mathbf{X}(\theta_A)}{\partial \theta_A} = \frac{\mathbf{Z}(\theta_A)}{\left[\left(\sigma-\mu\right)\left(\beta e_A^{\sigma}e_B^{-\sigma}-1\right)+\sigma\left(1+\beta e_A^{1-\sigma}e_B^{\sigma-1}\right)\right]^2}$$

with $\mathbf{Z}(\theta_A)\equiv Z_1(\theta_A)-Z_2(\theta_A)$, $Z_1(\theta_A)\equiv\dfrac{\partial U(\theta_A)}{\partial\theta_A}V(\theta_A)$, $Z_2(\theta_A)\equiv U(\theta_A)\dfrac{\partial V(\theta_A)}{\partial\theta_A}$,

and where $U(\theta_A)$ and $V(\theta_A)$ correspond to the numerator and to the denominator of $\mathbf{X}(\theta_A)$ respectively.(B44)

We have

$$\frac{\partial U(\theta_A)}{\partial \theta_A} = \sigma\beta\left[(1-\sigma)\, e_B^{-\sigma} e_A^{\sigma-1} + (\sigma-\mu)\, e_B^{\sigma-1} e_A^{-\sigma}\right] \frac{\partial e_B}{\partial \theta_A}$$

$$-\sigma\beta\left[(1-\sigma)\, e_A^{\sigma-2} e_B^{1-\sigma} + (\sigma-\mu)\, e_A^{-\sigma-1} e_B^{\sigma}\right] \frac{\partial e_A}{\partial \theta_A}. \tag{B45}$$

In a symmetric equilibrium, we have

$$\frac{\partial U(\theta_A)}{\partial \theta_A}\,|_e = \sigma\beta e^{-1}(1-\mu)\left[\frac{\partial e_B}{\partial \theta_A}\,|_e - \frac{\partial e_A}{\partial \theta_A}\,|_e\right]. \tag{B46}$$

Again, we have $(\partial e_B/\partial \theta_A)\,|_e = \{\beta\,[2\sigma - \mu(1-\beta)]\,/\,[2\sigma\beta + \mu(1-\beta)]\}\,(\partial e_A/\partial \theta_A)\,|_e$. It follows that

$$\frac{\partial U(\theta_A)}{\partial \theta_A}\,|_e = -\frac{\mu\sigma\beta\,(1-\mu)\left(1-\beta^2\right) e^{-1}}{2\sigma\beta + \mu\,(1-\beta)} \frac{\partial e_A}{\partial \theta_A}\,|_e\,. \tag{B47}$$

We also have $V(\theta_A)\,|_e = [2\sigma\beta + \mu\,(1-\beta)]$. It follows that

$$Z_1(\theta_A)\,|_e = -\left[\mu\sigma\beta\,(1-\mu)\left(1-\beta^2\right) e^{-1}\right] \frac{\partial e_A}{\partial \theta_A}\,|_e\,. \tag{B48}$$

We also have

$$\frac{\partial V(\theta_A)}{\partial \theta_A} = \sigma\beta\left[(\sigma-\mu)\, e_A^{\sigma-1} e_B^{-\sigma} + (1-\sigma)\, e_A^{-\sigma} e_B^{\sigma-1}\right] \frac{\partial e_A}{\partial \theta_A}$$

$$-\sigma\beta\left[(\sigma-\mu)\, e_A^{\sigma} e_B^{-\sigma-1} + (1-\sigma)\, e_A^{1-\sigma} e_B^{\sigma-2}\right] \frac{\partial e_B}{\partial \theta_A}. \tag{B49}$$

In a symmetric equilibrium, we have

$$\frac{\partial V(\theta_A)}{\partial \theta_A}\,|_e = \sigma\beta e^{-1}(1-\mu)\left[\frac{\partial e_A}{\partial \theta_A}\,|_e - \frac{\partial e_B}{\partial \theta_A}\,|_e\right]. \tag{B50}$$

It follows that

$$\frac{\partial V(\theta_A)}{\partial \theta_A}\,|_e = \frac{\mu\sigma\beta\,(1-\mu)\left(1-\beta^2\right) e^{-1}}{2\sigma\beta + \mu\,(1-\beta)} \frac{\partial e_A}{\partial \theta_A}\,|_e\,. \tag{B51}$$

We also have $U(\theta_A)\,|_e = \beta\,[2\sigma - \mu\,(1-\beta)]$. It follows that

$$Z_2(\theta_A)\,|_e = \frac{\mu\sigma\beta\,(1-\mu)\left(1-\beta^2\right)[2\sigma\beta - \mu\beta\,(1-\beta)]\, e^{-1}}{2\sigma\beta + \mu\,(1-\beta)} \frac{\partial e_A}{\partial \theta_A}\,|_e \tag{B52}$$

As a result, we have

$$\mathbf{Z}(\theta_A)\,|_e = -\frac{\mu\sigma\beta\,(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta + \mu\,(1-\beta)^2\right] e^{-1}}{2\sigma\beta + \mu\,(1-\beta)} \frac{\partial e_A}{\partial \theta_A}\,|_e\,. \tag{B53}$$

And then, we have

$$\frac{\partial \mathbf{X}(\theta_A)}{\partial \theta_A}\,|_e = -\frac{\mu\sigma\beta\,(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta + \mu\,(1-\beta)^2\right] e^{-1}}{[2\sigma\beta + \mu\,(1-\beta)]^3} \frac{\partial e_A}{\partial \theta_A}\,|_e\,. \tag{B54}$$

Finally, we have in a symmetric equilibrium

$$\frac{\partial^2 e_B}{\partial \theta_A^2}|_e = -\frac{\mu\sigma\beta(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta+\mu(1-\beta)^2\right]e^{-1}}{\left[2\sigma\beta+\mu(1-\beta)\right]^3}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2 + \frac{\beta\left[2\sigma-\mu(1-\beta)\right]}{2\sigma\beta+\mu(1-\beta)}\frac{\partial^2 e_A}{\partial \theta_A^2}|_e . \text{(B55)}$$

Substituting into $\mathbf{Y}_2(\theta_A)$ given by (B40) and simplifying, we can obtain after some tedious manipulations,

$$\mathbf{Y}_2(\theta_A)|_e = -\frac{m\sigma e^{-1}}{\theta_A(1+\beta)}\frac{\mathbf{\Delta}(\sigma,\beta,\mu)}{\left[2\sigma\beta+\mu(1-\beta)\right]^3}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2 + \frac{m\left[2\sigma\beta+\mu(1-\beta)^2\right]}{\theta_A\left[2\sigma\beta+\mu(1-\beta)\right]}\frac{\partial^2 e_A}{\partial \theta_A^2}|_e$$

where

$$\mathbf{\Delta}(\sigma,\beta,\mu) \equiv \left[2\sigma\beta+\mu(1-\beta)\right]\left[\left[2\sigma\beta+\mu(1-\beta)\right]^2+\beta^3\left[2\sigma-\mu(1-\beta)\right]^2\right]$$

$$+\mu\beta^2(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta+\mu(1-\beta)^2\right]. \quad \text{(B56)}$$

Substituting (B55) into $\mathbf{Y}_3(\theta_A)$ given by (B41) and simplifying, we can also obtain (still after some tedious calculations),

$$\mathbf{Y}_3(\theta_A)|_e = \frac{\lambda}{2}\frac{\mu\sigma\beta(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta+\mu(1-\beta)^2\right]e^{-1}}{\left[2\sigma\beta+\mu(1-\beta)\right]^3}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2$$

$$-\frac{1}{2}\frac{\left[4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]\right]}{2\sigma\beta+\mu(1-\beta)}\frac{\partial^2 e_A}{\partial \theta_A^2}|_e . \quad \text{(B57)}$$

Now, using (B38), (B56) and (B57), we have that the second derivative of w_A with respect to θ_A is thus given by

$$\frac{m\sigma e^{-1}\left[2\sigma\beta+\mu\left(1+\beta^2\right)\right]\left[2\sigma\beta+\mu(1-\beta)^2\right]}{\theta_A\left[2\sigma\beta+\mu(1-\beta)\right]^2}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2 - \frac{m\left[2\sigma\beta+\mu(1-\beta)^2\right]}{\theta_A^2\left[2\sigma\beta+\mu(1-\beta)\right]}\frac{\partial e_A}{\partial \theta_A}|_e$$

$$-\frac{m\sigma e^{-1}}{\theta_A(1+\beta)}\frac{\mathbf{\Delta}(\sigma,\beta,\mu)}{\left[2\sigma\beta+\mu(1-\beta)\right]^3}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2 + \frac{m\left[2\sigma\beta+\mu(1-\beta)^2\right]}{\theta_A\left[2\sigma\beta+\mu(1-\beta)\right]}\frac{\partial^2 e_A}{\partial \theta_A^2}|_e$$

$$\frac{\lambda}{2}\frac{\mu\sigma\beta(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta+\mu(1-\beta)^2\right]e^{-1}}{\left[2\sigma\beta+\mu(1-\beta)\right]^3}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2$$

$$-\frac{1}{2}\frac{\left[4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]\right]}{2\sigma\beta+\mu(1-\beta)}\frac{\partial^2 e_A}{\partial \theta_A^2}|_e . \quad \text{(B58)}$$

Rearranging the terms, we have

$$\left\{\begin{array}{c}\dfrac{m\sigma e^{-1}\left[2\sigma\beta+\mu\left(1+\beta^2\right)\right]\left[2\sigma\beta+\mu(1-\beta)^2\right]}{\theta_A\left[2\sigma\beta+\mu(1-\beta)\right]^2} - \dfrac{m\sigma e^{-1}\mathbf{\Delta}(\sigma,\beta,\mu)}{\theta_A(1+\beta)\left[2\sigma\beta+\mu(1-\beta)\right]^3} \\ +\dfrac{\lambda}{2}\dfrac{\mu\sigma\beta(1-\mu)\left(1-\beta^2\right)\left[4\sigma\beta+\mu(1-\beta)^2\right]e^{-1}}{\left[2\sigma\beta+\mu(1-\beta)\right]^3}\end{array}\right\}\left(\frac{\partial e_A}{\partial \theta_A}|_e\right)^2$$

$$\left[\frac{m\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}-\frac{1}{2}\frac{\left[4\sigma\beta+\mu\left(1-\beta\right)\left[2-\lambda(1+\beta)\right]\right]}{2\sigma\beta+\mu\left(1-\beta\right)}\right]\frac{\partial^2 e_A}{\partial\theta_A^2}\mid_e$$

$$-\frac{m\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A^2\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\frac{\partial e_A}{\partial\theta_A}\mid_e . \tag{B59}$$

Simplifying, we have

$$\left\{\frac{m\sigma e^{-1}\mathbf{\Psi}\left(\sigma,\beta,\mu\right)}{\theta_A(1+\beta)\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^3}+\frac{\lambda}{2}\frac{\mu\sigma\beta\left(1-\mu\right)\left(1-\beta^2\right)\left[4\sigma\beta+\mu\left(1-\beta\right)^2\right]e^{-1}}{\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^3}\right\}\left(\frac{\partial e_A}{\partial\theta_A}\mid_e\right)^2$$

$$\left[\frac{2\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]m-\theta_A\left[4\sigma\beta+\mu\left(1-\beta\right)\left[2-\lambda(1+\beta)\right]\right]}{2\theta_A\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\right]\frac{\partial^2 e_A}{\partial\theta_A^2}\mid_e$$

$$-\frac{m\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A^2\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\frac{\partial e_A}{\partial\theta_A}\mid_e ,$$

$$\text{where } \mathbf{\Psi}\left(\sigma,\beta,\mu\right)=(1+\beta)\left[2\sigma\beta+\mu\left(1+\beta^2\right)\right]\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]\left[2\sigma\beta+\mu\left(1-\beta\right)\right]-\mathbf{\Delta}\left(\sigma,\beta,\mu\right). \tag{B60}$$

In the symmetric equilibrium, we have $\theta_A=\tilde{\theta}=2\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]m/\left[4\sigma\beta+\mu\left(1-\beta\right)\left[2-\lambda(1+\beta)\right]\right]$, and hence the second term of (B60) in front of $\left[\left(\partial^2 e_A/\partial\theta_A^2\right)\mid_e\right]^2$ is equal to 0.

Furthermore, simplifying $\mathbf{\Psi}\left(\sigma,\beta,\mu\right)$, we find (after long and tedious calculations)

$$\mathbf{\Psi}\left(\sigma,\beta,\mu\right)=\mu\beta\left(1+\beta\right)\left[4\sigma\beta+\mu\left(1-\beta\right)^2\right]\left[2\sigma\beta+\mu\left(1-\beta^2\right)-\beta(1-\beta)\right]. \tag{B61}$$

Therefore, (B60) reduces to

$$\left\{\begin{array}{c}\frac{m\sigma e^{-1}\mu\beta\left[4\sigma\beta+\mu\left(1-\beta\right)^2\right]\left[2\sigma\beta+\mu\left(1-\beta^2\right)-\beta(1-\beta)\right]}{\theta_A\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^3}\\+\frac{\lambda}{2}\frac{\mu\sigma\beta\left(1-\mu\right)\left(1-\beta^2\right)\left[4\sigma\beta+\mu\left(1-\beta\right)^2\right]e^{-1}}{\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^3}\end{array}\right\}\left(\frac{\partial e_A}{\partial\theta_A}\mid_e\right)^2$$

$$-\frac{m\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A^2\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\frac{\partial e_A}{\partial\theta_A}\mid_e . \tag{B62}$$

Simplifying again, we have

$$\left\{\frac{\mu\sigma\beta e^{-1}\left[4\sigma\beta+\mu\left(1-\beta\right)^2\right]\mathbf{\Omega}\left(\sigma,\beta,\mu,\lambda,m,\theta_A\right)}{2\theta_A\left[2\sigma\beta+\mu\left(1-\beta\right)\right]^3}\frac{\partial e_A}{\partial\theta_A}\mid_e-\frac{m\left[2\sigma\beta+\mu\left(1-\beta\right)^2\right]}{\theta_A^2\left[2\sigma\beta+\mu\left(1-\beta\right)\right]}\right\}\frac{\partial e_A}{\partial\theta_A}\mid_e ,$$

$$\text{where } \mathbf{\Omega}\left(\sigma,\beta,\mu,\lambda,m,\theta_A\right)=2m\left[2\sigma\beta+\mu\left(1-\beta^2\right)-\beta\left(1-\beta\right)\right]+\lambda\theta_A\left(1-\mu\right)\left(1-\beta^2\right). \tag{B63}$$

Therefore, we need to determine $\left(\partial e_A/\partial\theta_A\right)\mid_e$. Rewrite the equilibrium conditions of the policy stage (given by (13)) as $F(e_A, e_B(e_A), \theta_A) = 0$. Again, in the first stage, the median voter of each region anticipates the other-region representative's best-response in the second stage. Applying the implicit function theorem, we have $\partial e_A/\partial\theta_A = -\partial F/\partial\theta_A/\left[\partial F/\partial e_A + (\partial F_A/\partial e_B)(\partial e_B/\partial e_A)\right]$. $F(e_A, e_B(e_A), \theta_A) = 0$ is given by

$$\theta_A(1-\mu)\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}} e_A^{-\sigma} + \theta_B\beta(1-\mu)\left[e_B^{1-\sigma} + \beta e_A^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}} e_A^{-\sigma} - 1 = 0. \qquad \text{(B64)}$$

We have

$$\frac{\partial F}{\partial\theta_A} = (1-\mu)\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}} e_A^{-\sigma}. \qquad \text{(B65)}$$

In a symmetric equilibrium, it reduces to

$$\frac{\partial F}{\partial\theta_A}|_e = (1-\mu)(1+\beta)^{\frac{\sigma-\mu}{1-\sigma}} e^{-\mu}. \qquad \text{(B66)}$$

We also have,

$$\frac{\partial F}{\partial e_A} = \theta_A(1-\mu)\left\{(\sigma-\mu)\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_A^{-2\sigma} - \sigma\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}} e_A^{-\sigma-1}\right\}$$

$$+\theta_B\beta(1-\mu)\left\{\beta(\sigma-\mu)\left[e_B^{1-\sigma} + \beta e_A^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_A^{-2\sigma} - \sigma\left[e_B^{1-\sigma} + \beta e_A^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}} e_A^{-\sigma-1}\right\}. \qquad \text{(B67)}$$

In a symmetric equilibrium, it reduces to

$$\frac{\partial F}{\partial e_A}|_e = -\theta(1-\mu)(1+\beta)^{\frac{\sigma-\mu}{1-\sigma}-1}\left[2\sigma\beta + \mu\left(1+\beta^2\right)\right] e^{-\mu-1}. \qquad \text{(B68)}$$

We also have,

$$\frac{\partial F}{\partial e_B} = \theta_A\beta(1-\mu)(\sigma-\mu)\left[e_A^{1-\sigma} + \beta e_B^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_B^{-\sigma} e_A^{-\sigma}$$

$$+\theta_B\beta(1-\mu)(\sigma-\mu)\left[e_B^{1-\sigma} + \beta e_A^{1-\sigma}\right]^{\frac{\sigma-\mu}{1-\sigma}-1} e_B^{-\sigma} e_A^{-\sigma}. \qquad \text{(B69)}$$

In a symmetric equilibrium, it reduces to

$$\frac{\partial F}{\partial e_B}|_e = 2\theta\beta(1-\mu)(\sigma-\mu)(1+\beta)^{\frac{\sigma-\mu}{1-\sigma}-1} e^{-\mu-1}. \qquad \text{(B70)}$$

Furthermore, from (B42), we have $(\partial e_B/\partial e_A)|_e = \left[\beta\left[2\sigma - \mu(1-\beta)\right]\right]/\left[2\sigma\beta + \mu(1-\beta)\right]$. Using this expression with (B68) and (B70), we have that

$$\frac{\partial F}{\partial e_A}|_e + \left(\frac{\partial F}{\partial e_B}|_e\right)\left(\frac{\partial e_B}{\partial e_A}|_e\right) = -\frac{\theta(1-\mu)(1+\beta)^{\frac{\sigma-\mu}{1-\sigma}-1} e^{-\mu-1}\Phi(\sigma,\beta,\mu)}{2\sigma\beta + \mu(1-\beta)},$$

$$\text{where } \Phi(\sigma,\beta,\mu) = \left[2\sigma\beta + \mu\left(1+\beta^2\right)\right]\left[2\sigma\beta + \mu(1-\beta)\right] - 2\beta^2(\sigma-\mu)\left[2\sigma - \mu(1-\beta)\right]. \qquad \text{(B71)}$$

Simplifying, we obtain $\Phi(\sigma,\beta,\mu) = \mu(1+\beta)\left[4\sigma\beta+\mu(1-\beta)^2\right]$. Hence using (B66) and (B71), we obtain that $(\partial e_A/\partial\theta_A)|_e = -(\partial F/\partial\theta_A)|_e / [(\partial F/\partial e_A)|_e + (\partial F_A/\partial e_B)|_e (\partial e_B/\partial e_A)|_e]$, or

$$\frac{\partial e_A}{\partial\theta_A}|_e = \frac{[2\sigma\beta+\mu(1-\beta)]\,e}{\theta\mu\left[4\sigma\beta+\mu(1-\beta)^2\right]}. \tag{B72}$$

Substituting this expression in (B63), we can obtain

$$\left\{\frac{\sigma\beta\mathbf{\Omega}(\sigma,\beta,\mu,\lambda,m,\theta_A)}{2\theta^2[2\sigma\beta+\mu(1-\beta)]^2} - \frac{m\left[2\sigma\beta+\mu(1-\beta)^2\right]}{\theta^2[2\sigma\beta+\mu(1-\beta)]}\right\}\frac{\partial e_A}{\partial\theta_A}|_e\,. \tag{B73}$$

Using the definition of $\mathbf{\Omega}(\sigma,\beta,\mu,\lambda,m,\theta_A)$ in (B63) – with $\theta_A=\theta$ – the above expression reduce

$$\frac{\Gamma(\sigma,\beta,\mu,\lambda m,\theta)}{2\theta^2[2\sigma\beta+\mu(1-\beta)]^2}\frac{\partial e_A}{\partial\theta_A}|_e\,,$$

where $\Gamma(\sigma,\beta,\mu,m,\theta) \equiv \theta\lambda\sigma\beta(1-\mu)(1-\beta^2)+$

$$2m\left\{\sigma\beta\left[2\sigma\beta+\mu\left(1-\beta^2\right)-\beta(1-\beta)\right]-\left[2\sigma\beta+\mu(1-\beta)^2\right][2\sigma\beta+\mu(1-\beta)]\right\}. \tag{B74}$$

Simplifying, we obtain

$$\Gamma(\sigma,\beta,\mu,m,\theta) = -2m\left[\sigma\beta^2(2\sigma+1-\beta)+\mu^2(1-\beta)^3+3\mu\sigma\beta(1-\beta)^2\right]+\theta\lambda\sigma\beta(1-\mu)(1-\beta^2). \tag{B75}$$

Substituting $\theta = \tilde{\theta} = 2\left[2\sigma\beta+\mu(1-\beta)^2\right]m/\left[4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]\right]$ into this expression, it becomes negative which implies that $\partial^2 w_A/\partial\theta_A^2 < 0$ in $\theta=\tilde{\theta}$, if

$$\left[\sigma\beta^2(2\sigma+1-\beta)+\mu^2(1-\beta)^3+3\mu\sigma\beta(1-\beta)^2\right]\left[4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]\right] >$$

$$\lambda\sigma\beta(1-\mu)(1-\beta^2)\left[2\sigma\beta+\mu(1-\beta)^2\right]. \tag{B76}$$

A sufficient condition for this inequality to be satisfied is that the two terms in [.] of the left-hand side (LHS) of this inequality are larger than the two terms in [.] of the right-hand side (RHS). First observe that $\left[4\sigma\beta+\mu(1-\beta)\left[2-\lambda(1+\beta)\right]\right] \geq \left[2\sigma\beta+\mu(1-\beta)^2\right]$, since it reduces to $2\sigma\beta+\mu(1-\lambda)(1-\beta^2) \geq 0$, which is always satisfied. Now, a sufficient condition for the first term of the LHS to be larger than that of the RHS is that $\left[\sigma\beta^2(2\sigma+1-\beta)+\mu^2(1-\beta)^3+3\mu\sigma\beta(1-\beta)^2\right] \geq \sigma\beta(1-\mu)(1-\beta^2)$ since $\lambda \leq 1$. Rearranging the terms, this inequality can be rewritten as $\mu^2(1-\beta)^3+\sigma\beta\left[2\sigma\beta+(1-\beta)\left[2\mu(2-\beta)-1\right]\right] \geq 0$. A sufficient condition for this last inequality to be satisfied is that $\mu \geq \bar{\mu} = 1/\left[2(2-\beta)\right]$. $\bar{\mu}$ is increasing in β and hence reach a maximum in $\beta=1$, in which case $\bar{\mu}=1/2$. We exclude the situation with $\sigma=0$ together with $\beta=1$, since no public goods are supplied in this case. Hence, for $\mu \geq 0.5$ the two inequalities are strict. Hence, a sufficient condition for the second derivative of w_A with respect to θ_A in $\theta_A=\tilde{\theta}$ to be strictly negative is that $\mu \geq 0.5$, thus proving the the existence of a symmetric Local Nash Subgame Perfect Equilibrium (LNSPE) in pure strategies under a centralized system.

2 Existence of a (Symmetric) Subgame Perfect Nash Equilibrium (SPNE) under a Decentralized System

Under a decentralized system, one can nevertheless show the existence of a (symmetric) Subgame Perfect Nash Equilibrium (SPNE) in pure strategies for $\sigma = 0$, $\sigma = \mu$ and $\sigma \in (\mu, 1)$.

2.1 Decentralization

2.1.1 The case of $\sigma = 0$ or $\sigma = \mu$

$\sigma = \mu$ corresponds to the model analyzed by Besley and Coate (2003) and others. In this case, the utility of a citizen in region j is separable in the levels of local public investments and is given by $x_j + \theta \left[e_j^{1-\sigma} + \beta e_{-j}^{1-\sigma}\right]$. As explained in the paper, this implies that there are no strategic interactions in public spending. In other words, in the second stage of the game, the equilibrium is in dominant strategies and the level of local public investment in a particular region only depends on the identity of its representative. In turn, in the first stage of the game, each median voter has (also) a dominant strategy which is to appoint herself as the region's representative.

$\sigma = 0$ corresponds to the model analyzed by Dur and Roelfsema (2005) in their appendix, or Buccholz *et al.* (2005), among many others. Under this specific assumption, the utility of a citizen θ in region j is given by $x_j + \theta \left[e_j + \beta e_{-j}\right]^{1-\mu}$. This 'summation technology' implies that there is 'perfect substitutability' between local public investments. The first-order (and sufficient) conditions for an equilibrium in the second stage of the game are given by $\theta_j (1-\mu) \left[e_j + \beta e_{-j}\right]^{-\mu} = 1$ for $j = A, B$. Solving this system, we have $e_j^* = \left[(1-\mu)^{1/\mu} / \left(1-\beta^2\right)\right] \left[\theta_j^{1/\mu} - \beta \theta_{-j}^{1/\mu}\right]$. We then have that $G_j^* = e_j^* + \beta e_{-j}^* = \left[(1-\mu)\,\theta_j\right]^{1/\mu}$. Substituting e_j^* and G_j^* into the utility of the median voter given by $mG_j^{*1-\mu} - e_j^*$, we obtain that it is separable in θ_j and in θ_{-j}. It follows that each median has a dominant strategy in this case as well. Therefore, the SPNE always exists for $\sigma = 0$ or $\sigma = \mu$.

2.1.2 The case of $\sigma \in (\mu, 1)$: Strategic Complementarity

The payoff for the region A's median voter in the first stage of the game is given by $w_A = mF\left[G_A\left(\mathbf{e}^*\left(\theta_A, \theta_B\right)\right)\right] - e_A^*\left(\theta_A, \theta_B\right)$. The first derivative of w_A with respect to θ_A after substituting the equilibrium conditions of the policy stage (given by (4)) is given by (6) or

$$\frac{\partial w_A}{\partial \theta_A} = \frac{1}{\theta_A}\left[(m-\theta_A)\frac{\partial e_A^*}{\partial \theta_A} + \beta m e_A^{*\sigma} e_B^{*-\sigma}\frac{\partial e_B^*}{\partial \theta_A}\right].$$

We also have

$$\frac{\partial e_B^*}{\partial \theta_A} = \left[\frac{(\sigma-\mu)\,\beta e_B^* e_A^{*-\sigma}}{\mu e_B^{*1-\sigma} + \sigma \beta e_A^{*1-\sigma}}\right]\frac{\partial e_A^*}{\partial \theta_A}.$$

Substituting into the first expression, we obtain

$$\frac{\partial w_A}{\partial \theta_A} = \frac{Y(e_A^*, e_B^*, m, \theta_A)}{\theta_A\left[\mu e_B^{*1-\sigma} + \sigma \beta e_A^{*1-\sigma}\right]}\frac{\partial e_A^*}{\partial \theta_A},$$

with $Y(e_A^*, e_B^*, m, \theta_A) = \left[\mu(m - \theta_A) + m\beta^2(\sigma - \mu)\right] e_B^{*1-\sigma} + \sigma\beta\,(m - \theta_A)\, e_A^{*1-\sigma}$

In the symmetric equilibrium $e_A^* = e_B^*$ and $Y(e_A^*, e_B^*, m, \theta_A) = 0$ – for satisfying the first-order conditions in the policy stage – yields θ^* given by (7), i.e. $\theta^* = (1+\beta)\,[\mu(1-\beta) + \sigma\beta]\, m/\,(\mu + \sigma\beta)$. Suppose that $\theta_B = \theta^*$. For the quasi-concavity of w_A with respect to θ_A, the plan is to show that $Y(e_A^*, e_B^*, m, \theta_A) > 0$ for $\theta_A < \theta^*$ and $Y(e_A^*, e_B^*, m, \theta_A) < 0$ for $\theta_A > \theta^*$.

When $\sigma > \mu$, we have that $\theta^* > m$. Suppose first hat $\theta_A < \theta^*$ which leads to $e_A^* < e_B^*$. Indeed, as shown in the previous section of this appendix, the equilibrium level of public investment in one region is increasing in the type of its representative (see equation (B26)). When $\sigma > \mu$, local public investments are strategic complements and, so, the level of public investment in the other region decreases as well. But it decreases by a lower extent since the slope of each best-response function is lower than 1 at the equilibrium point (which can be seen in Figure 2).

If $\theta_A < m < \theta^*$, then $Y(e_A^*, e_B^*, m, \theta_A)$ is obviously positive. If $m < \theta_A < \theta^*$, then $(m - \theta_A)$ is negative and $Y(e_A^*, e_B^*, m, \theta_A)$ can be positive only if $\mu(m - \theta_A) + m\beta^2(\sigma - \mu) > 0$, that is only if $\theta_A < \hat{\theta}$ where $\hat{\theta} = (m/\mu)\left[\mu\left(1 - \beta^2\right) + \sigma\beta^2\right]$. For any $\sigma > \mu$, we can easily verify that $\hat{\theta} > \theta^*$, which implies that $\mu(m - \theta_A) + m\beta^2(\sigma - \mu)$ is positive for any $\theta_A < \theta^*$. Now, observe that $Y(e_A^*, e_B^*, m, \theta_A) > \left[\mu(m - \theta_A) + m\beta^2(\sigma - \mu) + \sigma\beta\,(m - \theta_A)\right] e_A^{*1-\sigma}$ when $\sigma \in (\mu, 1)$ since $e_A^* < e_B^*$. The term in [.] in the previous expression is positive if

$$(\theta_A - m) < \frac{m\beta^2(\sigma - \mu)}{\mu + \sigma\beta} = (\theta^* - m)$$

which is always verified for $\theta_A < \theta^*$. It follows that $Y(e_A^*, e_B^*, m, \theta_A)$ is positive for any $\theta_A < \theta^*$ when $\sigma \in (\mu, 1)$.

Suppose now that $\theta_A > \theta^*$ – and $\theta^* > m$ since $\sigma > \mu$ – which leads to $e_A^* > e_B^*$. If $\theta_A > \hat{\theta} > \theta^* > m$, then $(m - \theta_A)$ and $\mu(m - \theta_A) + m\beta^2(\sigma - \mu)$ are both negative and $Y(e_A^*, e_B^*, m, \theta_A)$ is thus negative as well. If $\hat{\theta} > \theta_A > \theta^* > m$, then $\mu(m - \theta_A) + m\beta^2(\sigma - \mu)$ is positive (and $(m - \theta_A)$ is still negative). Now, observe that $Y(e_A^*, e_B^*, m, \theta_A) < \left[\mu(m - \theta_A) + m\beta^2(\sigma - \mu) + \sigma\beta\,(m - \theta_A)\right] e_A^{*1-\sigma}$ when $\sigma \in (\mu, 1)$ since $e_A^* > e_B^*$. The term in [.] in the previous expression is negative if

$$(\theta_A - m) > \frac{m\beta^2(\sigma - \mu)}{\mu + \sigma\beta} = (\theta^* - m)$$

which is always verified for $\theta_A > \theta^*$. It follows that $Y(e_A^*, e_B^*, m, \theta_A)$ is negative for any $\theta_A > \theta^*$ when $\sigma \in (\mu, 1)$.

As result each median voter's payoff is quasi-concave, which implies that a symmetric SPNE exists for $\sigma \in (\mu, 1)$. Unfortunately, the same type of reasoning does not apply for $\sigma \notin (\mu, 1)$.

2.2 Centralization

Showing the existence of a SPNE in pure strategies in the case of a centralized system is even more problematic, except for the very specific cases of $\sigma = \mu$ – i.e. separability of local public goods in the utility of voters – and $\sigma = 0$ – i.e. perfect substitutability between local public investments.

For $\sigma = 0$, the joint public good surplus is given by $\theta_j \left[e_j + \beta e_{-j}\right]^{1-\mu} + \theta_{-j} \left[e_{-j} + \beta e_j\right]^{1-\mu} - (e_j + e_{-j})$. The first-order (and sufficient) conditions for an equilibrium in the second stage of the game are thus given by $\theta_j (1-\mu) \left[e_j + \beta e_{-j}\right]^{-\mu} + \theta_{-j} \beta (1-\mu) \left[e_{-j} + \beta e_j\right]^{-\mu} = 1$ for $j = A, B$. Solving this system, we can obtain $\tilde{e}_j = \left[\left[(1-\mu)(1+\beta)\right]^{1/\mu} / \left(1-\beta^2\right)\right] \left[\theta_j^{1/\mu} - \beta \theta_{-j}^{1/\mu}\right]$. We then have that $\tilde{G}_j = \tilde{e}_j + \beta \tilde{e}_{-j} = \left[(1-\mu)(1+\beta)\theta_j\right]^{1/\mu}$. Substituting $\tilde{e}_j$ and $\tilde{G}_j$ into the utility of the median voter given by $m\tilde{G}_j^{1-\mu} - (1/2)\left[(2-\lambda)\tilde{e}_j + \lambda \tilde{e}_{-j}\right]$, we obtain that it is separable in θ_j and in θ_{-j}. It follows that each median has a dominant strategy and therefore a SPNE in pure strategies exists in this case.

For $\sigma = \mu$, the payoff of each median induced by the public good provision subgame is no longer separable in the types of the two representatives. Yet, as shown by Besley and Coate (2003), there exists a SPNE in pure strategies in this case as well (see the Proof of Lemma 3, pp. 2631-35).