

Endogenous Uncertainty and Credit Crunches*

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Abstract

We develop a theory of endogenous uncertainty where the ability of investors to learn about firm-level fundamentals declines during financial crises. At the same time, higher uncertainty reinforces financial distress, causing a persistent cycle of uncertainty, pessimistic expectations, and financial constraints. Through this channel, a temporary shortage of funds can develop into a long-lasting funding problem for firms. Financial crises are characterized by increased credit misallocation, volatile asset prices, high risk premia, an increased cross-sectional dispersion of returns, and high levels of disagreement among forecasters. A numerical example suggests that the proposed channel may significantly delay recovery from financial shocks.

Keywords: Belief traps, credit crunches, endogenous uncertainty, funding freezes, internal persistence of financial shocks, resource misallocation.

JEL Classification: D83, E32, E44, G01.

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1 Introduction

Financial crises often entail deep and long-lasting recessions (Reinhart and Rogoff, 2009; Hall, 2014; Ball, 2014). A common view gives a central role to uncertainty, both as an amplifier of financial distress and a source of slow recovery from financial shocks.¹ This paper explores this idea, developing a theory that formalizes the interaction between financial constraints and uncertainty.

The theory provides a narrative on how a temporary disturbance of the financial sector is reinforced and amplified by endogenously rising uncertainty, developing into a long-lasting crisis of the real economy. The theory is consistent with a number of stylized facts from previous financial crises, such as the one in 2008/09: (i) persistently reduced hiring and output; (ii) volatile asset prices and high risk premia; (iii) an increased cross-sectional dispersion of returns (iv) the contemporaneous increase in measured uncertainty²; and (v) forecasts and expectations marked by high levels of pessimism (relative to the true state of nature) as well as high levels of disagreement among forecasters (Senga, 2016).

In the model, a firm’s funding depends on how investors assess the firm’s business conditions. If investors find it likely that a firm is productive, funding to that firm is generous and the firm achieves its unconstrained optimum. If, however, investors are pessimistic or uncertain regarding a firm’s business potential, firms are financially constrained. When constraints are sufficiently tight, firms are forced to become inactive since they cannot afford to pay fixed operating costs. The key friction in our model is that agents do not learn the productivities of inactive firms even though they perfectly know the productivities of active firms.

In this environment, a temporary tightening of a firm’s financial constraint can trigger a persistent (and perfectly rational) period of increasing uncertainty and inactivity that can persistently disrupt the firm’s access to funds. We refer to such episodes as “funding freezes”. Interestingly, the dynamics of the second moments of agents’ beliefs interact with the first moments: lacking new information during a funding freeze, any initially pessimistic beliefs persist into the future, making it even less likely to exit the funding freeze.

The persistent effects of financial shocks at the firm-level carry over to aggregate financial shocks. In particular, an aggregate shock to firms’ financial constraints increases the fraction of inactive firms. This leads to greater uncertainty among investors and a rise in credit misallocation. The misallocation manifests itself through decreases in output and hours that may persist even after

¹For example, Olivier Blanchard (2009) speculated at the height of the recent financial crisis that “the crisis would largely go away” if it were not for uncertainty, whereas Bloom et al. (2016) document how uncertainty was repeatedly recognized by the Federal Open Market Committee as a driver of both, the recession that followed the dot-com bubble in 2001, and the recent Great Recession. An increasing number of empirical studies further substantiates these ideas, pointing to the Great Recession being likely “an acute manifestation of the toxic interaction between uncertainty and financial shocks” (Caldara et al., 2016; see also Stein and Stone, 2013, Stock and Watson, 2012, and Gilchrist, Sim and Zakrajšek, 2016).

²Unusually high levels of uncertainty during the recent financial crisis have been documented using a variety of different approaches (see, e.g., Jurado, Ludvigson and Ng, 2015, Born, Breuer and Elstner, 2017, and the studies cited in Footnote 1). Most closely related to the concept of uncertainty explored in this paper is the forecast-based evidence given in Senga (2016), which documents a sharp increase in uncertainty regarding firm-level business conditions among financial analysts.

financial stress has subsided. At the same time, the interaction between investors' uncertainty and firms' funding conditions can account for an increase in investors' average disagreement and pessimism (even though all signals are unbiased), an increase in asset price volatility, and a divergence in asset returns.

We explore the quantitative potential of our model in a simple calibration to the U.S. economy. A novel aspect of our calibration is the explicit use of micro data on analyst forecasts, which we use to pin down the information parameters of our model. Our numerical exercise suggests that endogenously rising uncertainty may significantly delay recovery from financial shocks. Specifically, we look at the model's response to an exogenous tightening in financial constraints (with a half-life of 5 quarters), and compare it with counterfactual responses, in which we shut down the impact on uncertainty. While in the fixed-uncertainty counterfactual such as financial shock produces a short-lived recession with the same 5-quarter half-life as the exogenous shock, the same shock produces a long-lasting recession with a half-life of 11 quarters in the economy with endogenous uncertainty.

Finally, we provide direct evidence for the link between uncertainty and financial constraints predicted by the model. Using a combination of firm-level survey data, accounting data, and stock prices, we look at correlations of measures of firms' financial constraint and several proxies for uncertainty. Supporting the main mechanism of our model, we find a significantly positive correlation between financial constraints and all uncertainty proxies that is in line with the model-implied correlations.

Related literature Our paper is related to a large and growing literature that introduces dispersed information into macroeconomics (e.g., Lorenzoni, 2009; Angeletos and La'O, 2010, 2013; Amador and Weill, 2010, 2012; Maćkowiak and Wiederholt, 2015; Hassan and Mertens, 2014, 2017; Acharya, 2013; Hellwig and Venkateswaran, 2014; Chahrour and Gaballo, 2016). La'O (2010) shares with us the combination of information heterogeneities with financial frictions, but considers a static model with a constant level of uncertainty. David, Hopenhayn and Venkateswaran (2016) also analyze information frictions as a source for factor misallocation, but focus on long-run consequences rather than fluctuations driven by financial shocks.

Our paper also contributes to a recent literature that explores the role of endogenous fluctuations in uncertainty for business cycles, including van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), and Fajgelbaum, Schaal and Taschereau-Dumouchel (2017).³ In these papers, the level of aggregate investment determines the amount of information and hence aggregate uncertainty. An important distinction relative to these papers is this paper's focus on uncertainty regarding firm-specific fundamentals rather than economic aggregates (see Senga 2016 for a similar approach).⁴

³Studies of endogenous uncertainty in financial market settings include Veldkamp (2005), Yuan (2005), Albagli (2011), and Sockin (2017). However, none of these papers considers spillovers from financial distress on the real economy that are at the core of this paper.

⁴In a previous version of this paper, we have explored a version of our model where investors are learning about aggregate business conditions instead (Straub and Ulbricht, 2012). While the two versions of the model are qualitatively similar, we argue here that learning at a firm-level is more likely to be quantitatively important.

On the one hand, this allows us to explain the above-mentioned stylized facts relating to the cross-sectional distribution of firms and investor beliefs. On the other hand, this also helps overcoming an important challenge of the endogenous uncertainty literature; namely that often unrealistically large fluctuations in uncertainty are needed to generate a significant amplification. In our model, learning breaks down when *a firm* is constrained, not when aggregate economic activity comes to a stand-still, implying that the aggregate economy scales approximately proportionally with the fraction of firms being constrained. Accordingly, even small variations in average uncertainty, can have severe consequences.

A second difference to the existing endogenous uncertainty literature is that this paper links *financial* crises and uncertainty through a novel mechanism, explaining why high levels of uncertainty are particularly prevalent during financial crises. In our model, it is not the overall level of economic activity that determines how much information about firms' fundamentals is revealed; rather it is the degree to which firms' actions (investments, employment, production, etc) actually reflect these fundamentals. This insight naturally implies that when a firm's behavior is dictated by financial constraints, rather than fundamentals, its actions carry less information.⁵

In our model, the emergence of uncertainty from financial distress interacts with propagation of uncertainty through the financial sector. In support of such a financial transmission channel, [Gilchrist, Sim and Zakrajšek \(2016\)](#) present evidence that uncertainty strongly affects investments *via* increasing credit spreads, but has virtually no impact on investments when controlling for credit spreads. The financial transmission of uncertainty relates our model to a recent literature around [Christiano, Motto and Rostagno \(2014\)](#), [Arellano, Bai and Kehoe \(2016\)](#), and [Gilchrist, Sim and Zakrajšek \(2016\)](#), which stresses the importance of uncertainty (or risk) shocks in the financial sector, but treats these shocks as exogenous.⁶

The predictions of our model are also broadly consistent with a recent empirical literature that measures the effects of financial constraints. [Giroud and Mueller \(2017\)](#) show that establishments of firms that are more likely to be financially constrained were heavily affected by falling collateral values (house prices). In fact, they show that the entire correlation of employment loss and house prices is explained by these arguably financially constrained firms. Similar in spirit, [Chodorow-Reich \(2013\)](#) documents that firms borrowing from less healthy lenders experience relatively steeper declines in employment during the financial crisis, consistent with the interpretation that these firms faced tighter financial constraints. Our model clarifies how an intense but relatively short-lived financial crisis can still translate into *persistent* financial constraints for firms, making it much harder for

⁵In the current draft, we simplify the exposition by looking at the extreme case where a firm's action is either fully informative (it operates) or fully uninformative (it is shut down by the financial market). The insight is, however, more general, as we have shown in previous versions of this paper where information is lost even for constrained but active firms.

⁶Two other related strands of the literature study the propagation of exogenous uncertainty through real options as in [Bloom \(2009\)](#), [Bloom et al. \(2016\)](#), and [Bachmann and Bayer \(2013\)](#), and through risk premia as in the time-varying (disaster) risk literature (e.g., [Gabaix, 2012](#); [Gourio, 2012](#)). Related to the latter, [Kozlowski, Veldkamp and Venkateswaran \(2017\)](#) explore a model where agents learn about tail-risks and where belief revisions after short-lived financial shocks can have long-lasting effects. Similar, [Nimark \(2014\)](#) presents a mechanism that increases uncertainty after rare events, if news selectively focus on outliers.

them to weather such periods and retain their employment and capital.

Outline The plan for the rest of the paper is as follows. The next section introduces the model economy. Section 3 characterizes the equilibrium. Section 4 explains how persistent funding freezes can emerge at the firm level. Section 5 analyzes the model’s response to aggregate shocks. Section 6 tests the mechanism at the core of the paper using micro data. Section 7 concludes and offers a few policy insights.

2 Model

There is a continuum of islands of mass unity, indexed by i . Each island is inhabited by a single firm and a representative household, who interact in a local labor market and a local financial market. The firm uses the labor provided by the household to produce a differentiated consumption good. To finance its wage bill, the firm must obtain working capital funding from the household. The ability of the firm to obtain funding is restricted by a pledgeability constraint, which limits the fraction of revenues that the firm can pledge in exchange for funds. Time is discrete with an infinite horizon and is indexed by t .

Preferences and technology Households have GHH-preferences (Greenwood, Hercowitz and Huffman, 1988), maximizing

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} u_{i,\tau}^{1-\gamma}$$

with

$$u_{i,t} = C_{i,t} - \frac{1}{1+\zeta} \left(L_{i,t}^{1+\zeta} - v \right),$$

where $L_{i,t}$ are hours worked, $C_{i,t}$ is a composite consumption good given by

$$C_{i,t} = \left[\int_0^1 C_{i,j,t}^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}},$$

and $C_{i,j,t}$ is the consumption of good j by the household on island i . The preference parameters satisfy $\gamma, v > 0$, $\zeta \geq 0$, $\beta \in (0, 1)$ and $\xi > 1$.⁷

The output of the firm on island i is given by

$$Y_{i,t} = A_{i,t} \max\{L_{i,t} - \phi, 0\}, \tag{1}$$

where $\phi > 0$ is a fixed amount of overhead labor as in Bartelsman, Haltiwanger and Scarpetta

⁷Throughout we assume that v is sufficiently large so that $u_{i,t} \geq 0$ a.s. at the households’ interior optimal labor supply. This is needed because $C_{i,t}$ will be random at the time where $L_{i,t}$ is chosen by the household.

(2013). Log-productivities are given by

$$\log A_{i,t} = \rho \log A_{i,t-1} + \epsilon_{i,t}, \quad (2)$$

with $\rho \in (0, 1)$, and the innovations $\epsilon_{i,t}$ being i.i.d. (across islands and time), normal, with zero mean and variance σ_ϵ^2 .

Pledgeability constraint Each period has two subperiods, a morning and an afternoon. Firms operate subject to a working capital constraint that requires them to finance their wage bill in the morning whereas production realizes in the afternoon. To raise funds, each firm issues claims on current-period revenues to the local household, but is restricted in its ability to raise funds as only a fraction $\chi_{i,t} \in (0, 1]$ of revenues is pledgeable.⁸ Let $Q_{i,t}$ denote the equilibrium valuation of firm i 's expected revenues by the local household. Then the constraint on firm i 's labor input is given by

$$L_{i,t} \leq \bar{L}_{i,t} \equiv \chi_{i,t} Q_{i,t} / W_{i,t}. \quad (3)$$

The pledgeability limit $\chi_{i,t}$ is i.i.d. across islands, has finite support $X \subset (0, 1]$, and follows a Markov process with an aperiodic and irreducible transition matrix Ξ .

The timing of events within each period can be summarized as follows.

- *Morning*: pledgeability limits $\{\chi_{i,t}\}$ realize; labor and financial markets operate; firms issue claims on pledgeable revenues and pay wages to households.
- *Afternoon*: productivities $\{A_{i,t}\}$ realize and production occurs; product markets operate; claims on revenues mature and dividends are paid; households consume.

Households Because the household on island i is the only one trading firm i 's assets, we have that in equilibrium the local household holds a claim on $\chi_{i,t}$ units of the firm's revenues. In addition to its claims on pledgeable revenues, the local household is also residual shareholder to the local firm's profits. Accordingly, the budget constraint in the afternoon is

$$C_{i,t} = Z_{i,t} + \underbrace{W_{i,t} L_{i,t}}_{\text{wage-earnings}} + \underbrace{\chi_{i,t} (P_{i,t} Y_{i,t} - Q_{i,t})}_{\text{claim on pledgeable revenues}} + \underbrace{(1 - \chi_{i,t}) P_{i,t} Y_{i,t} + \chi_{i,t} Q_{i,t} - W_{i,t} L_{i,t}}_{\text{residual dividends}},$$

where $Z_{i,t}$ is the payoff from Arrow-Debreu securities that allow households to complete markets in the morning contingent on all available information (described below). In this environment, households can effectively insure themselves against all consumption losses that arise from the local firm being financially constrained, whereas they continue to be exposed to uncertainty about current operating profits.

⁸Limited pledgeability can be motivated in different ways, including limited commitment, private benefits, or incentives to counter moral hazard (e.g., [Holmstrom and Tirole, 1997](#)). More general, we can think of $\chi_{i,t}$ as a proxy for a firm's dependence or ability to raise external funding.

From the households' optimization problem it follows that local labor supply is

$$W_{i,t} = L_{i,t}^\zeta. \quad (4)$$

Aggregating across islands, the inverse aggregate demand for good i is

$$P_{i,t} = \left(\frac{Y_{i,t}}{Y_t} \right)^{-1/\xi} P_t, \quad (5)$$

where $Y_t \equiv \int_0^1 C_{i,t} di$ is aggregate demand for the composite good and

$$P_t \equiv \left(\int_0^1 P_{i,t}^{1-\xi} di \right)^{1/(1-\xi)}$$

is the composite price index. Henceforth, we normalize $P_t = 1$, defining the composite consumption good to be the numeraire.

Firms There is a single firm on each island. Firms are monopolistic competitors in the goods market, and monopsonists in the local labor market. The latter assumption is made for tractability reasons and ensures that firms always produce as long as they can obtain funding from the financial market.

Subject to (3)–(5), the firm's objective is to maximize the expected present value of its profits, given by

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} m_{i,\tau|t} (P_{i,\tau} Y_{i,\tau} - W_{i,\tau} L_{i,\tau}), \quad (6)$$

where $m_{i,\tau|t} = \beta^{\tau-t} u_{i,\tau}^{-\gamma} / \mathbb{E}_t[u_{i,t}^{-\gamma}]$ is the stochastic discount factor (between date- t mornings and date- τ afternoons) of the local household.⁹

Information We consider a simple information structure where all learning is public and agents have complete information about the realizations of $\chi_{i,t}$. Moreover, there is no aggregate uncertainty; i.e., agents have complete information about the aggregate state including the cross-sectional distribution over $(A_{i,t}, \chi_{i,t})$. The only source of uncertainty is a lack of information about the local productivities $A_{i,t}$ of each *individual* island. Specifically, because labor inputs and outputs are perfectly observable, agents learn the productivities of all operating firms in the afternoon of each period. By contrast, current productivities remain unknown for inactive firms. Let $\mathcal{B}_t = \{i : L_{i,t} > 0\}$ denote the set of active firms. Then the information set available to agents in the morning of date t is

$$\mathcal{I}_t = \{\chi_{i,t}\}_{i \in [0,1]} \cup \{A_{i,t-1}\}_{i \in \mathcal{B}_{t-1}} \cup \mathcal{I}_{t-1}.$$

⁹Firms maximizing profits rather than non-pledged revenues net of costs assumes that holders of non-equity securities have sufficient means to monitor firms and enforce them to maximize the representative households' marginal utility from consuming the proceeds of both equity and non-equity assets.

Discussion Firms in our model are financed each period by pledging a fraction of their revenues that is then traded in a financial market. While this is, of course, overly simplistic, what ultimately matters for our model is that a firm’s funding supply depends on market beliefs about firm fundamentals. Our model achieves this in an admittedly crude but practical way, without having to specify a full-blown banking sector.¹⁰

A crucial assumption in our modeling is that financial markets are unable (or unwilling) to perfectly hedge firm-specific risks. This is consistent with Barber and Odean (2000) and Goetzmann and Kumar (2008), documenting that the vast majority of U.S. investors are under-diversified.¹¹ As usual, under-diversification can be motivated in different ways, including home biases (Coval and Moskowitz, 1999), prior expertise or specialization in information collection (Van Nieuwerburgh and Veldkamp, 2010), and incentive-constraints that require investors to have some “skin in the game” for monitoring purposes (Holmstrom and Tirole, 1997). Alternatively, one can also think of our setting as a short-cut to a class of models where investors are risk-neutral but where entrepreneurs are exposed to idiosyncratic risks due to incentive-considerations and hence need to be compensated for those risk-exposures (e.g., Holmstrom, 1979).

3 Equilibrium Characterization

In this section, we provide a characterization of the equilibrium in the economy. We start by solving an unconstrained firm’s problem and then include the asset market. Finally, we study the dynamics of beliefs, which represent the state variables in this economy. We formally define the notion of equilibrium in the last subsection.

3.1 The Firm Problem

The characterization of the firm problem is facilitated by two facts. First, conditional on households willing to fund the firm ($Q_{i,t} > 0$), it is both feasible and optimal for the firm to operate ($L_{i,t} > \phi$). Second, conditional on operating, the continuation value of the firm is independent of the current scale of production. Accordingly, the firm’s desired level of operation is the solution to a static profit maximization problem. The following lemma summarizes the result.

Lemma 1. *Labor inputs are given by $L_{i,t} = \min\{\bar{L}_{i,t}, L_{i,t}^{\text{opt}}\}$, where*

$$L_{i,t}^{\text{opt}} = \arg \max_{L_{i,t}} \mathbb{E}_t [m_{i,t|t} (P_{i,t} Y_{i,t} - W_{i,t} L_{i,t})].$$

Substituting for (4) and (5), the optimal scale of production $L_{i,t}^{\text{opt}}$, conditional on being active, is

¹⁰Our assumption that firms are funded against cash flows is consistent with Lian and Ma (2017), who document that 80 percent of non-financial corporate debt is based predominately on cash flows from operating revenues rather than being collateralized by physical assets.

¹¹Relatedly, a recent body of evidence documents that idiosyncratic risk significantly affects the cost of capital to firms in stocks (Goyal and Santa-Clara, 2003; Fu, 2009), bonds (Campbell and Taksler, 2003) and in bank loans (Quijano, 2013).

characterized by the unique positive solution to

$$\lambda^\xi (L_{i,t}^{\text{opt}} - \phi)(L_{i,t}^{\text{opt}})^{\xi\zeta} = \theta_{i,t}, \quad (7)$$

where $\lambda \equiv (1 + \zeta)/(1 - 1/\xi)$ defines the markup and $\theta_{i,t} \equiv \mathbb{E}_t[m_{i,t}|t]A_{i,t}^{1-1/\xi}]^\xi Y_t$ is a risk-adjusted measure of expected business conditions.

In the computations in Section 5, it will prove useful to work with a log-linear approximation of $m_{i,t}|t$.¹² This allows us to explicitly express $\theta_{i,t}$ in terms of expected log-productivity, $\mu_{i,t} \equiv \mathbb{E}[\log A_{i,t}|\mathcal{I}_t]$, and the corresponding uncertainty, $\Sigma_{i,t} \equiv \text{Var}[\log A_{i,t}|\mathcal{I}_t]$,

$$\log \theta_{i,t} = (\xi - 1)(\mu_{i,t} - \tilde{\gamma}\Sigma_{i,t}) + \log Y_t, \quad (8)$$

where $\tilde{\gamma}$ defines the “effective degree of risk-aversion” (see Appendix A.2 for details).¹³

3.2 Equilibrium on the Asset Market

Standard asset pricing implies that claims on the firm’s revenue trade at a price

$$Q_{i,t} = \mathbb{E}_t[m_{i,t}|t]P_{i,t}Y_{i,t}. \quad (9)$$

Equations (9) and (3) define the usual loss-spiral between asset prices and financial constraints as in Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). The following lemma states the solution to the resulting fixed-point problem.

Lemma 2. *The funding constraint $\bar{L}_{i,t}$, conditional on being binding, is defined by the largest solution to¹⁴*

$$\bar{L}_{i,t}^{\xi(1+\zeta)} = \chi_{i,t}^\xi \theta_{i,t} \max\{\bar{L}_{i,t} - \phi, 0\}^{\xi-1}. \quad (10)$$

Equipped with Lemma 2, we are now ready to characterize the equilibrium scale of production.

Proposition 1. *There are unique thresholds $\theta^*(\chi) \leq \theta^{**}(\chi)$, such that*

$$L_{i,t} = \begin{cases} \bar{L}_{i,t} = 0 & \text{if } \theta_{i,t} < \theta^*(\chi_{i,t}) \\ \bar{L}_{i,t} > \phi & \text{if } \theta_{i,t} \in [\theta^*(\chi_{i,t}), \theta^{**}(\chi_{i,t})] \\ L_{i,t}^{\text{opt}} > \phi & \text{if } \theta_{i,t} > \theta^{**}(\chi_{i,t}). \end{cases}$$

¹²The approximation is not important for any of the formal characterizations that follow.

¹³Technically, $\tilde{\gamma}$ can become negative for small γ . This is because $\mathbb{E}_t[A_{i,t}]$ is increasing in $\Sigma_{i,t}$ due to the convexity of the exponential function, potentially dominating the risk-aversion of households. Throughout we focus on the interesting case where $\tilde{\gamma}$ is positive.

¹⁴Aside from the trivial solution $\bar{L}_{i,t} = 0$, there can be up to two additional solutions. Only the largest of these solutions constitutes a stable equilibrium in the sense that the household does not strictly prefer to offer a marginally higher price than the equilibrium price $Q_{i,t}$. That is, if there exist two solutions $\bar{L}' < \bar{L}''$, then the expected utility gain from extending production to some $\bar{L} \in (\bar{L}', \bar{L}'')$ strictly outweighs the additional cost of buying the claim.

For all $\theta \geq \theta^*(\chi)$, L^{opt} and \bar{L} are defined by (7) and (10), and are increasing in θ . The thresholds are given by

$$\theta^*(\chi) = \chi^{-\xi} \lambda^{\xi-1} \left(\frac{1+\zeta}{1+\xi\zeta} \xi \phi \right)^{1+\xi\zeta} \quad (11)$$

$$\theta^{**}(\chi) = \begin{cases} \chi^{\xi\zeta} \lambda^{\xi(1+\zeta)} \left(\frac{\phi}{\lambda\chi-1} \right)^{1+\xi\zeta} & \text{if } \chi \geq \lambda^{-1} \\ \infty & \text{else.} \end{cases} \quad (12)$$

Proposition 1 provides a complete characterization of the equilibrium level of production on each island as a function of risk-adjusted business expectations $\theta_{i,t}$ and the pledgeability limit $\chi_{i,t}$. Firms are denied funding and are forced to terminate whenever $\theta_{i,t} \leq \theta^*(\chi_{i,t})$. It is worth noting that the threshold of operation enforced by the financial market can be much tighter than what's optimal from the firm's (or a social) perspective. In particular, risk-adjusted expected date- t profits, accounting for overhead labor costs, are positive if and only if $\theta_{i,t} \geq \theta^*(1)$. The difference between $\theta^*(\chi_{i,t})$ and $\theta^*(1)$ therefore defines a wedge on the extensive margin of operation. To get a numerical sense of the magnitude of the wedge, note that from (11), the wedge is given by

$$\frac{\theta^*(\chi_{i,t})}{\theta^*(1)} = \chi_{i,t}^{-\xi}.$$

For $\xi = 7.5$ (implying a markup of 15 percent) and a pledgeability limit of 80% this implies a wedge of roughly a factor 5. Decreasing pledgeability to 50%, the wedge increases to a factor of 181. Introducing limited pledgeability therefore defines a huge region where investors are too uncertain and pessimistic to fund the firm, even though it would be optimal for the firm to operate.

Figure 1 illustrates the characterization. For $\chi_{i,t} = 1$, the household's and firm's objectives are aligned and the firm is either efficiently terminated or operates unconstrained.¹⁵ For all $\chi_{i,t} < 1$, there is a wedge, leading to an increasing region of inefficient termination. Moreover, conditional on operation, firms start out constrained and will operate unconstrained only for sufficiently large $\chi_{i,t}$ and if investors are sufficiently optimistic and have little uncertainty about the firm's business conditions.

¹⁵Here, efficiency is to be understood as a constrained efficiency concept, conditional on the market structure in goods and labor markets (monopolistic competition and monopsony). In particular, we say that a firm is terminated efficiently if risk-adjusted expected current period profits (of the monopolistically competitive and monopsonistic firm) are negative for all $L_{i,t}$, ignoring any value of learning. If the firm's continuation value of being active and learning about its productivity is included in the definition of efficiency, the gap between equilibrium and efficient operation would increase even more.

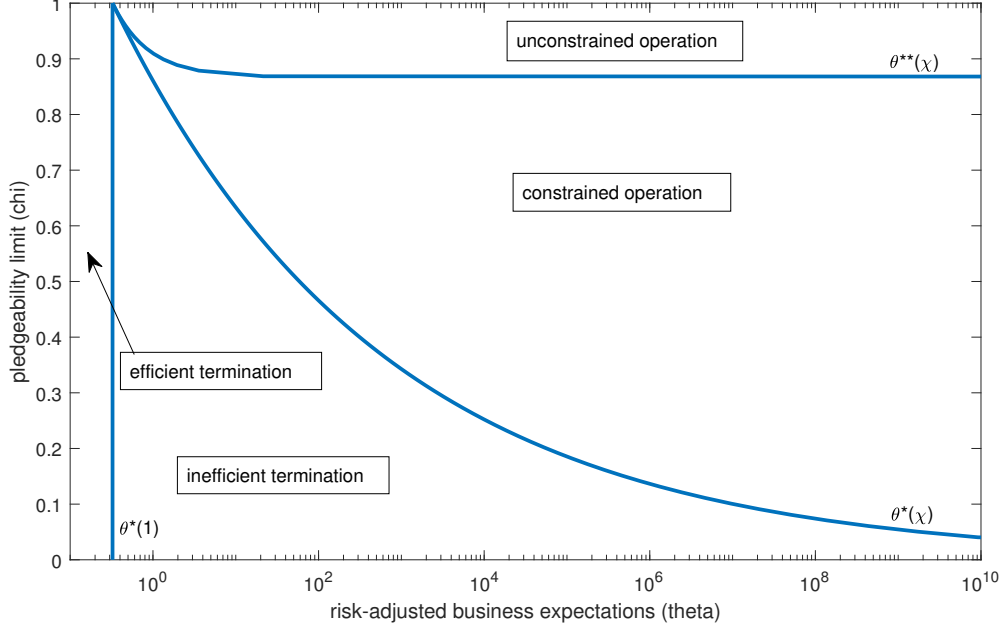


Figure 1: Characterization of funding regimes

3.3 Belief Dynamics

To complete the characterization of equilibrium, we need to characterize how risk-adjusted business expectations $\theta_{i,t}$, defined in (8), evolve over time. From (2), the law of motion of beliefs is

$$\mu_{i,t+1} = \begin{cases} \rho \log A_{i,t} & \text{if } i \in \mathcal{B}_t \\ \rho \mu_{i,t} & \text{if } i \notin \mathcal{B}_t \end{cases} \quad (13)$$

$$\Sigma_{i,t+1} = \begin{cases} \sigma_\epsilon^2 & \text{if } i \in \mathcal{B}_t \\ \rho^2 \Sigma_{i,t} + \sigma_\epsilon^2 & \text{if } i \notin \mathcal{B}_t. \end{cases} \quad (14)$$

As long as a firm is active, learning is perfect and uncertainty only reflects current innovations to productivity. By contrast, uncertainty accumulates for inactive firms, and beliefs converge to the unconditional prior.

3.4 General Equilibrium and Steady State

Imposing market clearing, aggregate output is given by $\int_0^1 C_{i,t} di = Y_t = \left[\int_0^1 Y_{j,t}^{1-1/\xi} dj \right]^{\xi/(\xi-1)}$. From Proposition 1, $Y_{j,t}$ is a function of $\mathcal{S}_{j,t} \equiv (A_{j,t}, \chi_{i,t}, \mu_{j,t}, \Sigma_{j,t})$ and Y_t . Let \mathcal{P}_t be the date- t distribution over $\mathcal{S}_{j,t}$. Since \mathcal{P}_t is predetermined at date t , equilibrium output at date t is the solution to $Y_t = Y(\mathcal{P}_t, Y_t)$. We confirm numerically that the solution exists and is unique for all our simulation experiments.

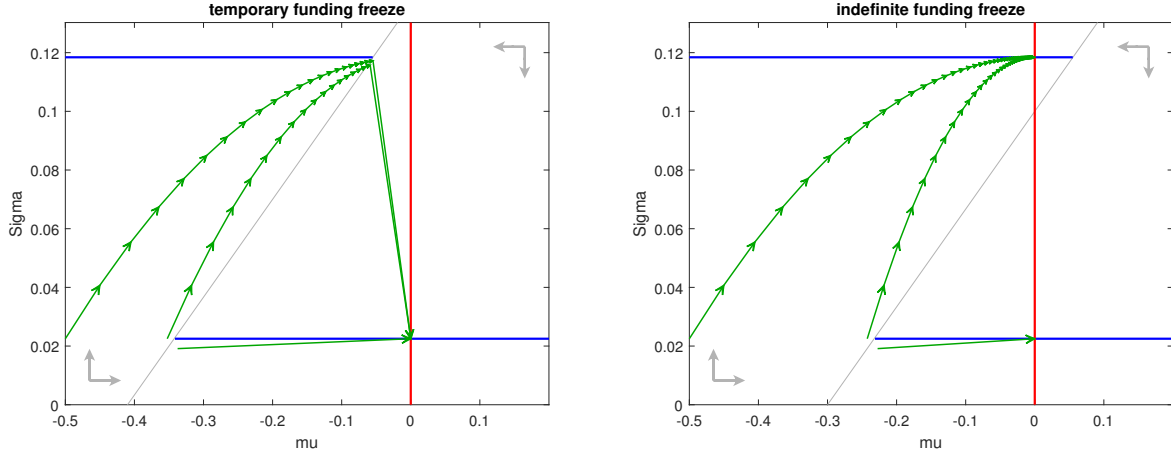


Figure 2: Phase diagram for firm-level beliefs in the absence of shocks. Thin gray lines depict $(\theta = \theta^*)$ -contours; Z-shaped blue lines are $(\Sigma_{i,t} = \Sigma_{i,t-1})$ -loci; vertical red lines are $(\mu_{i,t} = \mu_{i,t-1})$ -loci. Arrowheads represent distinct points in time along the plotted trajectories. *Left:* Case with a unique steady state ($\eta_0 < \bar{\eta}$). *Right:* Case with multiple steady states ($\underline{\eta} < \eta_0 < \bar{\eta}$).

4 Funding Freezes

We are now ready to explore the interaction between funding constraints and learning. In this section, we illustrate the main mechanism of the paper, focusing on the partial equilibrium dynamics of a single firm in isolation. Combining (8) and (11), a firm is denied funding if

$$\mu_{i,t} - \tilde{\gamma}\Sigma_{i,t} \leq \eta(\chi_{i,t}, Y_t), \quad (15)$$

where η is a decreasing, log-linear function in $\chi_{i,t}$ and Y_t . From (13) and (14), it then follows that if a firm is denied funding at t , future uncertainty increases and expectations are anchored around $\mu_{i,t}$ regardless of the realized productivity $A_{i,t}$. Past pessimism and uncertainty thus get reinforced into future periods, creating persistently tight funding constraints \bar{L} .

4.1 Non-stochastic Steady States and Dynamics

Let $\log A_{i,s} = 0$ and fix some $\eta_0 = \eta(\chi_{i,s}, Y_s)$ for all s . Figure 2 shows the resulting phase diagram for two different values of $\bar{\eta}$. The thin gray line depicts the contour where (15) holds with equality, dividing the state space into an active and an inactive region. The red line corresponds to the constant expectations locus $(\mu_{i,t} = \mu_{i,t-1})$, which is given by $\mu_{i,t} = 0$ since $\log A_{i,t}$ is mean-reverting. The blue line corresponds to the constant uncertainty locus $(\Sigma_{i,t} = \Sigma_{i,t-1})$. The latter is “Z”-shaped, because higher levels of uncertainty not only directly increase uncertainty at $t + 1$ but also indirectly by reducing the funds to firms: For sufficiently, optimistic (or pessimistic) expectations, this feedback has no effect as firms’ access to funds will be secured (or denied) regardless of $\Sigma_{i,t}$. For moderate levels of $\mu_{i,t}$, however, uncertainty becomes pivotal to the operation of the firm, hence generating multiple stationary values of uncertainty for a given belief $\mu_{i,t}$. Specifically, for high levels of

uncertainty, the firm is denied funding, reinforcing high levels of uncertainty, and vice versa for low levels of uncertainty. Intersecting the two loci, we can have a unique or multiple (non-stochastic) steady states, depending on the value of η_0 . (In either case, the firm’s dynamics are unique for any aggregate sequence $\{Y_t\}$ as all endogenous components of $\mathcal{S}_{i,t}$ are predetermined at $t - 1$.)

Proposition 2. *There exist two thresholds $\underline{\eta} < \bar{\eta}$, such that for all $\underline{\eta} \leq \eta_0 \leq \bar{\eta}$ there are two (non-stochastic) steady states at the firm-level, and otherwise there is a unique (non-stochastic) steady state. When $\theta_{i,t}$ is approximated using (8), the thresholds are given by $\underline{\eta} = -\tilde{\gamma}\sigma_\epsilon^2$ and $\bar{\eta} = -\tilde{\gamma}\sigma_\epsilon^2/(1 - \rho^2)$.*

For intermediate levels of η_0 , funding freezes are infinitely persistent in the absence of shocks. Accordingly, a one-time disruption in a firms’ funds (e.g., through a shock to $\chi_{i,t}$) can indefinitely cut off the firm from future funding.¹⁶

However, even when the steady state is unique, funding freezes may emerge as a persistent (though not indefinite) disruption in a firm’s access to funding. This is illustrated by the gray trajectories in the left panel of Figure 2. Along these trajectories, each arrowhead represents a distinct point of time, so that the distance between two consecutive arrowheads can be viewed as an inverse measure of the speed at which the state is evolving.

The three trajectories differ in the persistence of beliefs and the amount of uncertainty induced along the path. Along the path starting to the right of the black contour line, the firm is initially funded and beliefs thus immediately adjust to the unique steady state. The two other paths, however, behave distinctly different: Starting to the left of the dashed contour line, households are sufficiently pessimistic to deny funding so that learning breaks down. Accordingly, expectations only slowly converge to the unconditional prior, whereas uncertainty accumulates to higher and higher levels (since information about past levels of $a_{i,t}$ becomes less and less useful for predicting current productivity), reinforcing tight funding constraints. Even though the steady state is unique, a firm can find itself in a persistent “funding freeze” (lasting more than 20 periods along the left-most trajectory).

4.2 A Temporary Financial Shock to the Pledgeability Limit

We next illustrate how a temporary reduction in the local pledgeability limit can disrupt a firm’s access to funding, triggering funding freezes as discussed above. Fix some initial productivity and pledgeability limit, $A_{i,0} = A_0$ and $\chi_{i,0} = \chi_0$ such that the firm is active at $t = 0$. Now suppose that at $t = 1$, the pledgeability is reduced to $\chi_1 < \chi_0$ and then mechanically reverts back to χ_0 at $t = 3$. Figure 3 illustrates the dynamics using the phase diagram developed above. In the diagram, the drop in pledgeability results in a rightward-shift of the $(\theta = \theta^*)$ -contour (depicted by the dashed gray line). For sufficiently small $\chi_{i,0}$, the firm is denied funding, setting in motion a downward-spiral

¹⁶See Section 4.2 for how temporary shocks to $\chi_{i,t}$ can trigger funding freezes. Alternatively, the starting two leftmost starting beliefs depicted in Figure 2 can be implemented by a sequence of productivities $a_{i,t+s} = 0$ for all $s \neq -1$ and $a_{t-1} = \rho^{-1}\mu_0$. Similarly, one could derive independent fluctuations in $\mu_{i,t}$ by endowing agents with an additional signal that communicates a noisy version of $a_{i,t}$ in the mornings of each date.

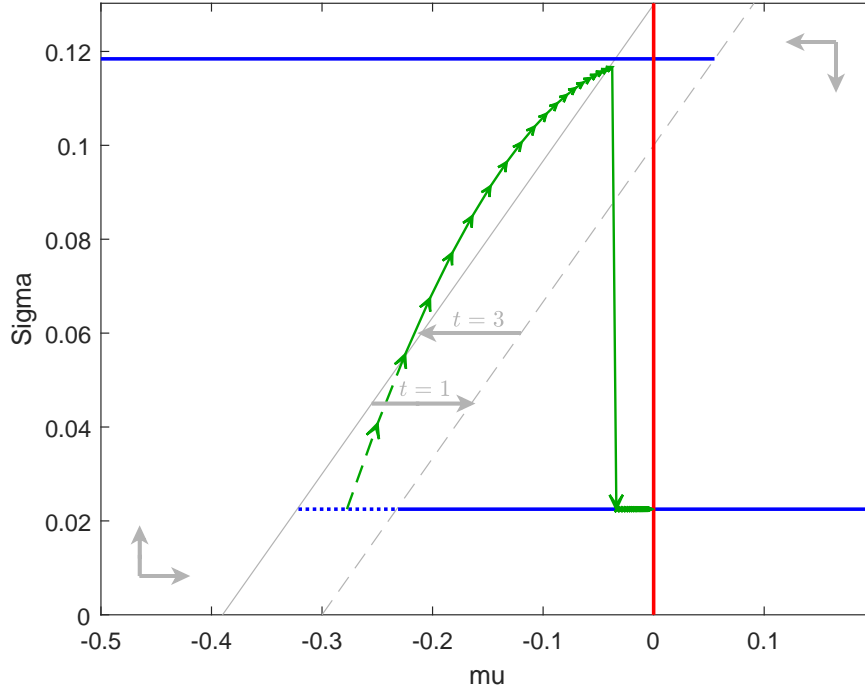


Figure 3: Dynamic response to a drop in χ at $t = 1$ and a subsequent recovery at $t = 3$. Arrowheads depict distinct points in time along the plotted trajectory.

between uncertainty and reinforcement of the funding constraint. Once uncertainty has passed the original ($\theta = \theta^*$)-contour line (depicted by the solid gray line), even a recovery of the pledgeability limit will not end the spiral, resulting in funding freezes that may significantly outlast the exogenous disruption in pledgeability.

The response of local output along with the responses of $(\chi_{i,t}, \mu_{i,t}, \Sigma_{i,t})$ can be seen in Figure 4. To isolate the contribution of the endogenous-uncertainty channel, we contrast the model's response (starred green lines) with a counterfactual response where the firm suffers the same loss in pledgeability but uncertainty is fixed at its lower bound, $\Sigma = \sigma_\epsilon^2$ (dashed black lines).¹⁷ In both cases, output initially drops to zero as long as pledgeability is reduced. The difference between the model and the counterfactual emerges at $t = 3$. Whereas output recovers in the counterfactual once pledgeability is restored, the firm continues to be denied funding in the presence of endogenously increased uncertainty. Since the parameters imply a unique steady state at χ_0 , beliefs and funding eventually recover when $\mu_{i,t}$ crosses the ($\theta = \theta^*$)-contour line in Figure 3. At this point, the firm continues operation, uncertainty drops to σ_ϵ^2 and $\mu_{i,t} = \rho \log A_{i,t-1}$.¹⁸

Comment on misallocation There are two sources of misallocation here (aside from the two monopoly distortions). First, the financial constraint drives a discrepancy between the equilibrium

¹⁷The parametrization of the model is detailed in Section XX. Throughout the response, Y_t is fixed at its steady state level.

¹⁸Since there are no shocks to $A_{i,t}$ along the response path, expectations are correct throughout ($\mu_{i,t} = \log A_{i,t}$).

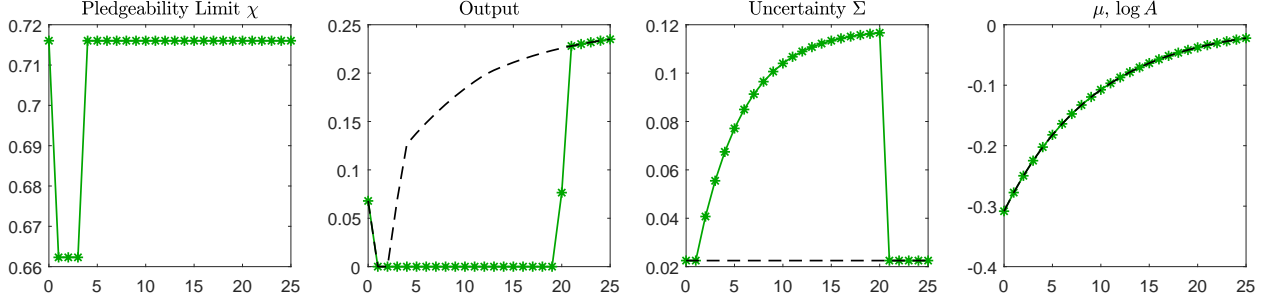


Figure 4: Dynamic responses to a drop in χ at $t = 1$ and a subsequent recovery at $t = 3$. Starred green lines are model responses; dashed black lines are counterfactual responses where uncertainty is exogenously fixed at $\Sigma = \sigma_\epsilon^2$.

use of labor and the optimal use of labor given $\theta_{i,t}$ as illustrated in Figure 1. Second, an inefficient response due to *undetected* changes in productivity, reflecting that competitive financial markets have no incentives to experiment to identify productive firms as all potential gains are arbitrated away under perfect competition in financial markets. Along the simulated response path, this manifests itself by the fact that, as long as the firm is denied funding, any change in $A_{i,t}$ goes undetected, potentially implying huge efficiency losses from forgone business opportunities.

5 Aggregate Financial Shock

In this section, we explore how an aggregate disruption in firms' access to funding propagates through our model.

5.1 Extended Model and Parametrization

Extended model So far, agents did not receive any information on inactive firms. For our numerical experiment we depart from this strong assumption and provide agents with an additional noisy signal, $s_{i,t} = \log A_{i,t-1} + u_{i,t}$, disclosing past productivities subject to a Gaussian noise term $u_{i,t}$ with zero mean and variance σ_u^2 . The signal $s_{i,t}$ plays two roles: First, it gives agents a mean to learn about a firm's business potential (to a limited degree) even if the firm is inactive. Second, it introduces some independent source of variation in agents' beliefs regarding inactive firms which, as we show below, has interesting implications.

The information set available to agents in the morning of date t is now

$$\mathcal{I}_t = \{\chi_{i,t}, s_{i,t}\}_{i \in [0,1]} \cup \{A_{i,t-1}\}_{i \in \mathcal{B}_{t-1}} \cup \mathcal{I}_{t-1}.$$

Using the Kalman filter to recursively filter through information about inactive firms, agents' beliefs about firm i in the morning of date t are given by

$$\mu_{i,t} = \begin{cases} \rho \log A_{i,t-1} & \text{if } i \in \mathcal{B}_{t-1} \\ \rho [\delta_{i,t} \mu_{i,t-1} + (1 - \delta_{i,t}) s_{i,t}] & \text{else} \end{cases} \quad (16)$$

and

$$\Sigma_{i,t} = \begin{cases} \sigma_\epsilon^2 & \text{if } i \in \mathcal{B}_{t-1} \\ \rho^2 \delta_{i,t} \Sigma_{i,t-1} + \sigma_\epsilon^2 & \text{else,} \end{cases} \quad (17)$$

where $\delta_{i,t} = (1 + \sigma_u^{-2} \Sigma_{i,t-1})^{-1}$. Replacing (13) and (14) by (16) and (17), the equilibrium characterization given in Section 3 remains fully valid.

Outside observers For the upcoming exploration it will also be useful to introduce a “background layer” of outside observers (or forecasters) $j \in [0, 1]$. In addition to \mathcal{I}_t , these forecasters each observe a private signal $\omega_{ij,t} = \log A_{i,t} + \psi_{ij,t}$, where ψ is normally distributed with zero mean and variance σ_ψ^2 . For simplicity, at each date t , the previous generation of forecasters is replaced by a new one. The belief of forecaster j about firm i ’s productivity at date t is given by

$$\tilde{\mu}_{ij,t} = \frac{\Sigma_{i,t}^{-1} \mu_{i,t} + \sigma_\psi^{-2} \omega_{ij,t}}{\Sigma_{i,t}^{-1} + \sigma_\psi^{-2}}. \quad (18)$$

In Section 6, we use these forecasters’ beliefs to compare the model’s predictions with micro data from a survey of professional forecasters. To keep the model tractable, we assume that forecasters do not interact with the rest of the economy.

Parametrization We interpret one period as a quarter. The inverse Frisch elasticity of labor supply ζ is set to 0.5, the elasticity of substitution between consumption goods is set to 7.5, and the effective degree of relative risk aversion $\tilde{\gamma}$ is set to 3.¹⁹ These parameters are within the range typically used by the literature. The productivity parameters are set to $\rho = 0.9$ and $\sigma_\epsilon = 0.15$ (e.g., Gilchrist, Sim and Zakrajšek, 2016, Appendix 4). Next, we set ϕ to match a 20 percent share of overhead labor at the steady state. This is consistent with Valerie A. Ramey (1991) who reviews the empirical literature and considers 20 percent a reasonable consensus estimate.²⁰

The process of firm-specific pledgeability limits is modeled as a two-state Markov process on $X = \{\underline{\chi}, \bar{\chi}\}$ with a financially fragile state $\underline{\chi}$ and a financially sound state $\bar{\chi}$. We fix the latter at $\bar{\chi} = 1$, implying full pledgeability in the good state. The transition matrix is given by

$$\Xi = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}.$$

The financially fragile state $\underline{\chi}$ and the transition probability p are chosen to roughly match the 8-quarter, 16-quarter and 24-quarter survival rates of newly active firms to 0.66, 0.5 and 0.4, respectively, consistent with the empirical 2-year, 4-year and 6-year survival rates of new firms (c.f., Headd, 2003). This yields $\underline{\chi} = 0.63$ and $p = 0.07$.

¹⁹While with our utility specification the coefficient of relative risk aversion depends on γ , ζ and v and fluctuates over time as a function of $C_{i,t}$ and $L_{i,t}$, its effect on risk-perceptions remains constant given the log-linear approximation to $m_{i,t|t}$. Moreover, fixing $\tilde{\gamma}$, there is no need to take a separate stand on γ and v .

²⁰See also the discussion in Bartelsman, Haltiwanger and Scarpetta (2013) who conclude that the reasonable range for overhead labor is between 10 and 30 percent.

Table 1: Parameters used for numerical experiment.

Parameter	ζ	ξ	$\tilde{\gamma}$	ϕ	p	$\underline{\chi}$	$\bar{\chi}$	ρ	σ_ϵ	σ_u	σ_ψ
Value	0.50	7.50	3.0	0.052	0.07	0.63	1.00	0.90	0.15	0.80	0.97

Finally, we set σ_u to match $\text{Corr}(\int_0^1 \tilde{\mu}_{i,j,t} dj, \log A_{i,t}) = 0.859$, consistent with the empirical pre-crisis correlation between the average professional firm-specific forecasts and realized returns on assets (see Section 6 for a description of the data). Similarly, we set σ_ψ to match the average belief dispersion among forecasters regarding each firm, $\int_0^1 \sqrt{\text{Var}_j[\tilde{\mu}_{i,j,t}]} di = 0.023$.

All calibration targets are matched at the steady state.²¹ Table 1 summarizes the calibrated parameters.

Computation strategy In order to solve for the equilibrium distribution \mathcal{P}_t , we discretize the state space $\mathcal{S}_{i,t}$ using a combination of Rouwenhorst’s method (c.f., Kopecky and Suen, 2010) and a discrete numerical approximation to the agent’s equilibrium beliefs. See Appendix B for details on our computation strategy.

5.2 Simulation of an Aggregate Financial Shock

We are now ready to explore the economy’s response to an aggregate disruption in firms’ access to funding. The economy is initialized at its stochastic steady state where $\mathcal{P}_t = \mathcal{P}_{t-1}$.²² We consider a one-shot perturbation to Ξ at date 0 that increases the transition probability $\Pr(\chi_{i,0} = \underline{\chi} | \chi_{i,-1} = \bar{\chi})$ to $(p + 0.1)$, while keeping the reverse transition probability at $p = 0.07$. The disruption effectively reallocates a random mass 0.05 of firms from the financially sound state into the financially fragile state (reflecting a 10 percent increase in financially fragile firms). At $t = 1$, Ξ is reverted back to its calibrated value and the economy starts converging back to the steady state. Throughout we assume that the perturbation to Ξ is common knowledge so that $Y_t \in \mathcal{I}_t$ for all t . With this assumption, our equilibrium characterization applies without change.

Figure 5 displays the model’s response to the perturbation at $t = 0$, depicted by the starred green lines. To isolate the role of the endogenous amplification through rising uncertainty from the exogenous impact of tighter funding requirements, we contrast the model’s responses with two counterfactuals. First, dashed black lines display the direct impact of tighter pledgeability limits, keeping uncertainty evolving according to its steady state dynamics. Second, solid red lines show the propagation through rising uncertainty, eliminating the direct impact of tighter pledgeability limits.²³ All responses are in percentage deviations from their respective steady state values.

²¹Due to overidentification and computational restrictions, we match the targeted moments with a small numerical error. At our parametrization, the overhead labor share is 19.97 percent, the survival rates are, respectively, 0.643, 0.505 and 0.405, the correlation between beliefs and realized productivities is 0.851, and the average belief dispersion is 0.023.

²²We confirm numerically that our model has a unique steady state distribution for the chosen parameter values.

²³Formally, we compute the counterfactuals as follows. Let $\Delta = \begin{bmatrix} 0 & 0 \\ .1 & -.1 \end{bmatrix}$ denote the initial perturbation of Ξ at date 0. Then the distribution over $\{\chi_{i,t}\}$ at date t can be derived directly from the steady state distribution over

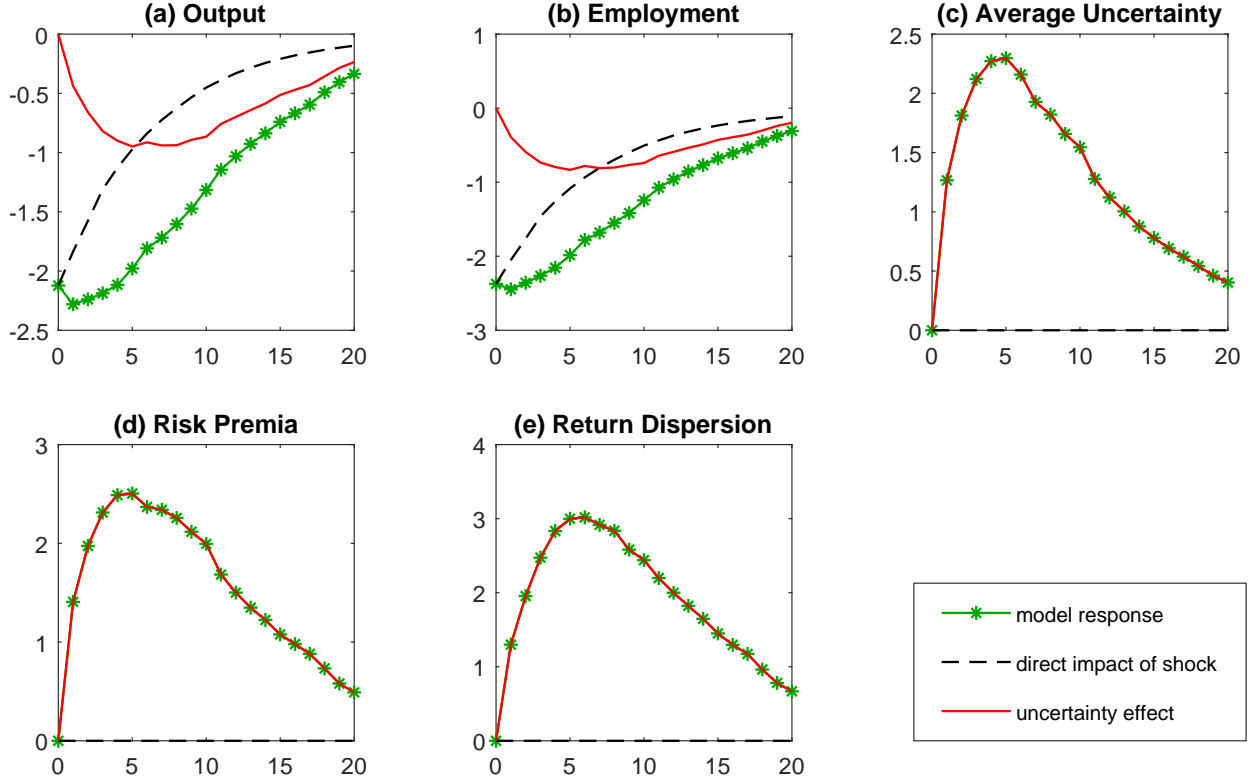


Figure 5: Dynamic responses to a one-shot perturbation in Ξ at $t = 0$. All responses are in percentage deviations from the steady state. Starred green lines are model responses; dashed black lines are “pure financial shock” counterfactuals where uncertainty evolves according to its steady state dynamics; solid red lines are “pure uncertainty” counterfactuals where the direct impact of the financial shock is shut down.

Amplification and persistence Because uncertainty is predetermined, the impact response at date 0 is fully explained by a tightening of pledgeability limits. Starting at date 1, however, the adverse effects of rising uncertainty starts to both amplify and prolong the crisis relative to the case where uncertainty evolves according to its steady state dynamics. At $t = 5$, output in the counterfactual isolating the direct impact of the financial shock recovers by more than 50 percent to 0.97 percent below steady state. By comparison, at $t = 5$, output is still 1.98 percent below steady in the endogenous uncertainty economy, amounting to a recovery of less than 7 percent. In terms of half-lives, recovery takes more than twice as long in the model economy (11 quarters) compared to the counterfactual isolating the direct impact of the financial shock (5 quarters).²⁴

$\chi_{i,t}$ using a one-time perturbation of Ξ to $\hat{\Xi}_t = \Xi + \Delta\Xi^t$. Correspondingly, the direct impact of tighter pledgeability limits is derived by initializing the economy at *each date* at its steady state distribution \mathcal{P} and perturbing Ξ to $\hat{\Xi}_t$. Effectively, we induce the counterfactual to follow the same path for $\{\chi_{i,t}\}$ as in the equilibrium response. However, by re-initializing the economy at its steady state in every period, we fix the path for uncertainty (and the ability of agents to learn from past production) *as if* the economy would have been in its steady state until $t - 1$. Conversely, we define the counterfactual isolating the role of uncertainty by initializing the economy at *each date* at the equilibrium distribution \mathcal{P}_t copied from the model’s equilibrium response and perturb Ξ to $\hat{\Xi}_{t-1}^{-1}$ to undo the perturbation of $\{\chi_{i,t}\}$. Effectively, this keeps the distribution of $\{\chi_{i,t}\}$ at its steady state path, while inducing the same path of uncertainty (and agent’s ability to learn from past production) as in the model’s equilibrium response.

²⁴Another way to measure the direct impact of the financial shock is to look at the mass of firms with $\chi_{i,t} = \underline{\chi}$ relative to the steady state. The response (not plotted) looks similar to the counterfactual isolating the direct impact

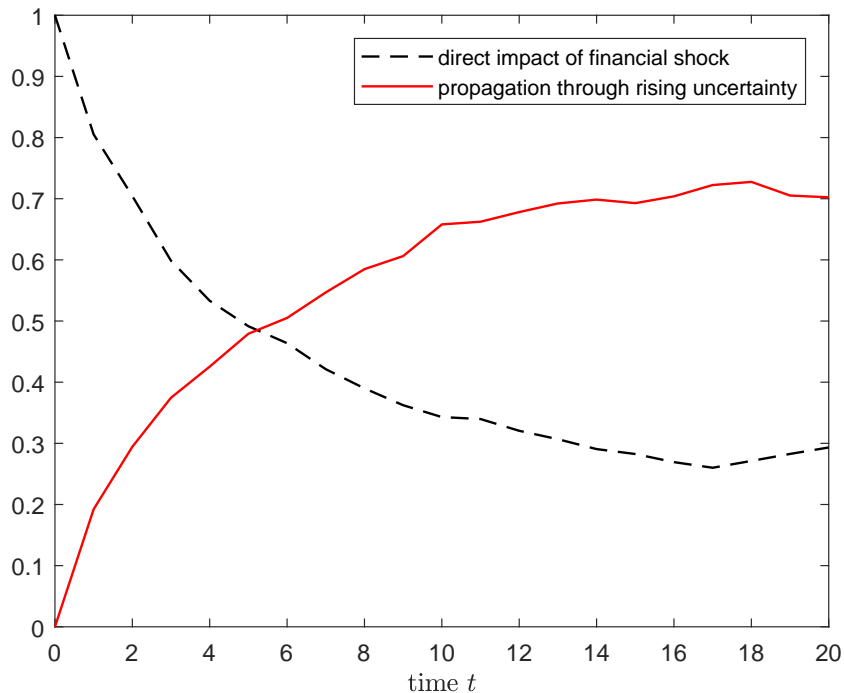


Figure 6: Decomposition of output response.

Figure 6 illustrates the relative contributions of the financial shock itself relative to the endogenously rising uncertainty.²⁵ In line with the above explanations, the contribution of endogenously rising uncertainty increases over the course of the crisis, becoming the main driver after about 5 quarters.

Risk premia, pessimism, volatility and dispersion Rising uncertainty also helps explaining a few financial market characteristics typically associated with financial crises. First, rising uncertainty increases risk premia. Let $R_{i,t} = \log(P_{i,t}Y_{i,t}/Q_{i,t})$ define log-returns from pledgeable claims. In equilibrium, we have²⁶

$$R_{i,t} = (1 - 1/\xi) (\log A_{i,t} - \mu_{i,t} + \tilde{\gamma}\Sigma_{i,t}). \quad (19)$$

Conditional on date- t information, expected returns are hence given by

$$\mathbb{E}[R_{i,t}|\mathcal{I}_t] = (1 - 1/\xi)\tilde{\gamma}\Sigma_{i,t}, \quad (20)$$

illustrating how risk-premia rise in uncertainty.

Interestingly, (20) understates the risk-premia conditional on realized returns. This is because of a selection effect introduced by the exogenous signal $s_{i,t}$: Realizations in $s_{i,t}$ that induce overly

of the financial shock, sharing the same 5-quarter half-life.

²⁵The two contributions add up to slightly less than 1 due to the model's non-linear response.

²⁶Note that returns are well-defined in the limit as $L \rightarrow \phi$ due to L'Hospital's rule (despite payouts converging to zero).

optimistic beliefs are likely to result in firm i being funded. This allows agents to effectively learn about the firm’s true business potential, quickly correcting overly optimistic beliefs. By contrast, overly pessimistic beliefs are more likely to result in firm i being denied funding and are therefore endogenously more persistent. This discrepancy in the persistence of optimism and pessimism results in agents having, on average, more pessimistic beliefs about business conditions, thereby increasing the realized returns relative to (20). Panel (d) of Figure 5 plots average returns along the simulated response path. Caused by both rising uncertainty and an increased pessimism (not plotted), average returns increase by up to 2.5 percent relative to their steady state level throughout the crisis.²⁷ The response is exclusively driven by endogenous variations in agent’s ability to learn; i.e., the direct impact of the financial shock on risk premia is zero.

Relatedly, the model also predicts an increased return volatility during financial crisis. To see this, consider the one-step ahead volatility conditional on date- $(t - 1)$ realizations, $\text{Var}[R_{i,t}|t - 1] = (1 - 1/\xi)^2 \text{Var}[\log A_{i,t} - \mu_{i,t}]$.²⁸ Substituting for (16), we have

$$\text{Var}[R_{i,t}|t - 1] = (1 - 1/\xi)^2 \begin{cases} \sigma_\epsilon^2 & \text{if } i \in \mathcal{B}_{t-1} \\ \sigma_\epsilon^2 + \rho^2(1 - \delta_{i,t})^2 \sigma_u^2 & \text{else.} \end{cases}$$

Since $\delta_{i,t}$ is decreasing in $\Sigma_{i,t-1}$, we have that higher uncertainty results in higher realized volatility. Intuitively, as uncertainty increases, agents place increasingly more weight on the exogenous signal $s_{i,t}$ for the purpose of forming their beliefs. As a result, their beliefs, and hence asset prices, become increasingly exposed to the exogenous noise term $u_{i,t}$.

Finally, increased volatility of firm-level returns also translates into an increased cross-sectional dispersion of returns.²⁹ Panel (e) of Figure 5 plots the cross-sectional dispersion over $R_{i,t}$ relative to its steady state level. As can be seen, rising uncertainty also induces the dispersion to rise along the response path. Again, the response is zero throughout the counterfactual which isolates the direct impact of the financial shock.

Disagreement A final prediction of our model is with respect to the beliefs of market observers. From (18), the degree of “disagreement” among market observers is given by

$$\text{Var}_j[\tilde{\mu}_{ij,t}] = \frac{\sigma_\psi^{-1}}{\Sigma_{i,t}^{-1} + \sigma_\psi^{-2}}. \quad (21)$$

²⁷In our simulation, the contribution of pessimism to return premia is small (2.34 percent at the steady state). Accordingly, the response of the average risk premium closely mirrors the response of uncertainty even though pessimism significantly increases along the return path (up to 20 percent at the peak relative to its steady state level).

²⁸Alternatively, one may also be interested in the variance of returns conditional on date- t information. Computing the conditional variance over (19), we have $\text{Var}[R_{i,t}|\mathcal{I}_t] = (1 - 1/\xi)^2 \Sigma_{i,t}$. Clearly, higher uncertainty about $\log A_{i,t}$ directly translates into a higher *expected* variability of returns.

²⁹Generally, the (unconditional) cross-sectional dispersion will be larger than the one-step ahead volatility, as it accumulates idiosyncratic volatility over time and is further amplified by a feedback from beliefs into funding, which results in variation over the $\Sigma_{i,t}$ -term in (19).

Clearly, disagreement rises with uncertainty. Intuitively, as less can be learned about firms that are denied funding, market observers rely more on their own private information. As a result, (average) disagreement among market observers increases during financial crisis as an increasing number of firms becomes constrained.³⁰ These predictions are in line with [Carlin, Longstaff and Matoba \(2014\)](#) who document that increased disagreement is associated with higher expected returns and higher return volatility.

6 A Test Using Micro Data

At the core of our model is a two-way interaction between uncertainty and financial constraints. In this section, we attempt to test for the predicted link between a firms’ access to funding and uncertainty among market participants using a combination of firm-level survey data, accounting data and stock price data.

6.1 Data

In the following we describe the data sources and define the main variables used for our empirical exploration. Technical details regarding the construction of our dataset are contained in [Appendix C](#).

We use forecasts about earnings per share (EPS) by financial analysts from the IBES database to construct a proxy for uncertainty among financial market participants about firm-level fundamentals. To make these forecasts comparable to our model, we follow [Senga \(2016\)](#) and transform EPS-forecasts into forecasts regarding returns on assets (ROA). In our dataset, the median productivity as measured by ROA is 3.7 percent (−9.5 percent at the 10th percentile, 13 percent at the 90th percentile). Let $\mu_{ij,t}^{\text{EPS}}$ denote analyst j ’s expectation about firm i ’s EPS at date t .³¹ Beliefs regarding returns on assets are computed as

$$\mu_{ij,t}^{\text{ROA}} = \mu_{ij,t}^{\text{EPS}} \times \frac{\text{number of outstanding shares}_{i,t}}{\text{total assets}_{i,t-1}}.$$

As our primary proxy for firm-level uncertainty, we look at the cross-analysts dispersion of forecast errors, defined by

$$\sigma_{i,t}^{\text{fce}} \equiv \sqrt{\text{Var}_j \left[\mu_{ij,t}^{\text{ROA}} - \text{ROA}_{i,t} \right]}.$$

Noting that $\text{ROA}_{i,t}$ is constant across all forecasters j , $\sigma_{i,t}^{\text{fce}}$ can be equivalently interpreted as disagreement among forecasters.

For the purpose of measuring financial constraints, we follow the corporate finance literature and combine various balance sheet data to proxy for firms’ access to funds using the “KZ-index”

³⁰For simplicity, all active agents in our model share the same information set \mathcal{I}_t , sparing us from dealing with infinite regress ([Townsend, 1983](#)). Nevertheless it is worth pointing out that our predictions about disagreement are not limited to professional forecasters, but would more generally carry over to any set of agents with exogenous private signals.

³¹As in [Senga \(2016\)](#), we extract those forecasts 8 months prior to each firm’s fiscal-year end month. See [Appendix C](#) for details on the timing of our variables.

Table 2: Financial constraints and uncertainty

	(1)	(2)	(3)	(4)
Panel a: Dependent variable is forecast-error dispersion $\sigma_{i,t}^{\text{fce}}$				
Financially constrained	.081 (.012)	.079 (.012)	.079 (.012)	.031 (.008)
Observations	47 342	47 342	47 335	46 141
Adj. R-sq.	.010	.023	.078	.709
Panel b: Dependent variable is stock returns $R_{i,t}$				
Financially constrained	.030 (.025)	.034 (.024)	.034 (.024)	.146 (.036)
Observations	51 993	51 993	51 990	50 787
Adj. R-sq.	.000	.116	.117	.130
Year \times month FE	no	yes	yes	yes
Sector FE (4 digit)	no	no	yes	no
Firm FE	no	no	no	yes

Note: Standard errors clustered at the firm-level are in parenthesis.

developed by Kaplan and Zingales (1997) and Lamont, Polk and Saá-Requejo (2001).³² Based on a firm’s $kz_{i,t}$ -score, we classify it as likely to be constrained if its current score is at or above the 95th percentile in a given calendar year. To check the robustness of our results, we have also implemented a variety of alternative classifications based on dividend payments and debt-to-capital ratios, obtaining similar results.

Finally, we augment our dataset with data on stock returns from CRSP in order to test the model’s prediction regarding risk premia and return dispersion. The resulting dataset is an unbalanced yearly panel, ranging from 1976 to 2016, covering on average 1979 firms per year.

6.2 Financial Constraints and Uncertainty

To explore whether the predicted link between financial constraints and uncertainty is present in the data, we run a simple OLS regression of forecast-error dispersion $\sigma_{i,t}^{\text{fce}}$ on the $(kz_{i,t-1} \geq 0.95\%)$ -indicator.³³ Panel (a) of Table 2 reports the estimated coefficients, controlling for different combinations of fixed effects. The estimated effect is roughly constant over the first three specifications where we control for a combination of year, fiscal-end year month and 4-digit sector fixed effects. In all three specifications, the forecast-error dispersion is increased by about 0.08 for firms that are classified as financially constrained. Controlling for firm-level fixed effects, the estimated difference between financially constrained and unconstrained firms is reduced to 0.031.

³²The KZ-score is a weighted combination of a firm’s cash flow to total capital, its market to book ratio, its debt to capital, dividends to total capital, and cash holdings to capital (see Appendix C for details). The weighting coefficients are based on an ordered logit regression relating those accounting variables to an explicit classification of firms into categories of financial constraints (Kaplan and Zingales, 1997; Lamont, Polk and Saá-Requejo, 2001). Firms with a higher $kz_{i,t}$ are more likely to be constrained.

³³The constraint-indicator is lagged by one year to align it as closely as possible to the date at which forecasts about t are formed by analysts. Similar results hold when we use $kz_{i,t}$ instead.

Table 3: Conditional heteroskedasticity model for $R_{i,t}$

	(1)	(2)	(3)	(4)
Financially constrained	1.045	1.015	.958	.481
	(.063)	(.059)	(.057)	(.064)
Avg. marginal effect on $\sigma_{i,t}$.122	.116	.113	.061
Observations	51 993	51 993	51 990	50 787
Adj. R-sq.	0.007	0.075	.107	.190
Year \times month FE	no	yes	yes	yes
Sector FE (4 digit)	no	no	yes	no
Firm FE	no	no	no	yes

Note: Table reports results for the second-stage regression $\hat{\sigma}_{i,t}^2 = \exp(x_{i,t}\beta + \epsilon_{i,t})$ where $\hat{\sigma}_{i,t}$ are estimated from the residuals of linear first stage estimations of $R_{i,t}$ on the same set of independent variables as in the second stage. Standard errors are clustered at the firm level.

These results lend support to the model’s prediction, as the coefficient is positive and statistically significant across all specifications. To check the quantitative significance of our results, we compute the corresponding statistic at the steady state of the calibrated model. Using (21), we find that for firms that are denied funding, $\sigma_{i,t}^{\text{fce}}$ is on average increased by 0.043, roughly in line with our empirical estimates.

Closely related to the predictions regarding uncertainty, our model also predicts higher risk premia, an increased return volatility and greater dispersion in returns for financially constrained firms. In panel (b) of Table 2, we regress CRSP stock returns on the financial constrainedness indicator. For specifications (1)–(3), we estimate financially constrained firms to have about 3 percent higher returns than financially unconstrained firms, but cannot reject that the effect is zero. This may be due to the fact that funding is, in part, determined based on *observable* characteristics, so that firms that are *known* to be unproductive also receive less funding. Specification (4) partially controls for this by controlling for firm-specific fixed effects. Indeed, the estimated effect is now increased to 0.146 and is found to be statistically significant. By comparison, our calibrated model predicts a risk premia that is increased by 0.129 for firms that are denied funding.

Finally, we use a conditional heteroskedasticity model using an exponential variance function to test the model’s prediction regarding return volatility and return dispersion. Table 3 reports the estimated coefficient of the variance function. As predicted by the model, we find a positive and statistically significant effect on the return dispersion across all specifications, with average marginal effects on the standard deviation of returns ranging from 6.1 percent to 12.2 percentage points. By comparison, the model predicts an increase in the return dispersion by 12.3 percentage points for firms without funding.

7 Concluding Remarks

In this paper, we propose a theory of endogenous uncertainty and its interaction with firms' financial conditions. In the model, firms require access to external finance in order to produce. When uncertainty about a firm's fundamentals is high, access to finance is limited, and the firm's production is reduced, or even halted. This endogenously limits production of *information* about the firm as well, increasing uncertainty and perpetuating the funding problems. While present even in normal times, this feedback loop becomes especially powerful in an aggregate setting, where a temporary financial shock can significantly increase average uncertainty and create a prolonged economic downturn.

We have so far refrained from policy analysis in this paper. There are, however, several policy insights that merit further discussion. First, recapitalizing banks (investors) is not an effective policy to restore lending in the model. In fact, investments are done by households, who are unconstrained at all times, so such a policy would not have any effects. The critical friction in bad times is an informational one and cannot easily be undone by transfers to banks. This, however, suggests a second policy action: direct transfers to firms. Even if the government has access to the same information as everyone else in the economy, providing transfers or cheap loans to inactive firms can crowd *in* lending in future periods due to an information externality: a re-activated firm produces information that lets future private investors decide whether to resume private lending. Finally, if the government can somehow increase transparency, for instance by improving reporting standards, this could be useful during a financial crisis.

In this paper, we have mainly focused on the effects of financial shocks. However, the model's internal propagation mechanism applies similarly to other types of economic shocks, such as aggregate demand shocks. For example, by reducing revenues, a fall in aggregate demand exacerbates the financial friction at the heart of the model, increasing the fraction of inactive firms (cf., equation 15). This raises uncertainty going forward, so that, even as demand recovers, a supply problem (uncertainty and tight financial conditions) remains. Conversely, strong demand (e.g. due to accommodative monetary policy) can help shift firms into business, and thereby reduce uncertainty and increase the allocative efficiency of the economy.

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A Mathematical Appendix (for online publication)

A.1 Proof of Lemma 1

First note that for all $Q_{i,t} > 0$ it is *feasible* for the firm to operate ($\bar{L}_{i,t} > \phi$). To see this, suppose it were not. Then $L_{i,t} \leq \bar{L}_{i,t} \leq \phi$ and, hence, $Y_{i,t} = 0$. But then, from (9), $Q_{i,t} = 0$, a contradiction.

Next, substituting (3) into (9) and rearranging, we get

$$\chi_{i,t} \mathbb{E}_t[m_{i,t|t} P_{i,t} Y_{i,t}] - W_{i,t} \bar{L}_{i,t} = 0. \quad (22)$$

By contrast, from (6), a sufficient condition for the firm to operate is

$$\mathbb{E}_t[m_{i,t|t} P_{i,t} Y_{i,t}] - W_{i,t} L_{i,t} \geq 0. \quad (23)$$

Comparing (22) and (23), we have that whenever operation is feasible, it is also optimal for the firm to operate. To complete the proof, it suffices to note that conditional on producing, the continuation value of the firm is constant in $L_{i,t}$.

A.2 Log-linearization of $m_{i,t|t}$

By definition, $m_{i,t|t} = u_{i,t}^{-\gamma} / \mathbb{E}_t[u_{i,t}^{-\gamma}]$. Taking a log-linear approximation to $u_{i,t}$, we have

$$\log u_{i,t} \approx f(Z_{i,t}, L_{i,t}, Y_t) + \kappa(1 - 1/\xi) \log(A_{i,t})$$

for $\kappa = \bar{Y} / [\bar{Y} - (\bar{L}^{1+\zeta} - v) / (1 + \zeta)]$ and some log-linear function f . Noting that $Z_{i,t}, L_{i,t}, Y_t \in \mathcal{I}_t$, we get

$$m_{i,t|t} \approx \frac{A_{i,t}^{-\gamma\kappa(1-1/\xi)}}{\mathbb{E}_t[A_{i,t}^{-\gamma\kappa(1-1/\xi)}]}.$$

Substituting into $\theta_{i,t} = \mathbb{E}_t[m_{i,t|t} A_{i,t}^{1-1/\xi}] \xi Y_t$, we have

$$\begin{aligned} \log \theta_{i,t} &= \xi \log \left(\frac{\mathbb{E}_t[A_{i,t}^{(1-\gamma\kappa)(1-1/\xi)}]}{\mathbb{E}_t[A_{i,t}^{-\gamma\kappa(1-1/\xi)}]} \right) + \log Y_t \\ &= (\xi - 1) (\mu_{i,t} - \tilde{\gamma} \Sigma_{i,t}) + \log Y_t \end{aligned}$$

where $\tilde{\gamma} \equiv (\gamma\kappa - 1/2)(1 - 1/\xi)$.

A.3 Proof of Lemma 2

Combining (1), (3), (4), (5) and (9), we get

$$\bar{L}_{i,t} = \chi_{i,t} \mathbb{E}_t[m_{i,t|t} A_{i,t}^{1-1/\xi}] Y_t^{1/\xi} \max\{L_{i,t} - \phi, 0\}^{1-1/\xi} / L_{i,t}^\zeta$$

or, substituting out $\theta_{i,t} = \mathbb{E}_t[m_{i,t}|t]A_{i,t}^{1-1/\xi}\xi Y_t$,

$$\bar{L}_{i,t} = \chi_{i,t}\theta_{i,t}^{1/\xi} \max\{\bar{L}_{i,t} - \phi, 0\}^{1-1/\xi}/L_{i,t}^\zeta.$$

Conditional on $\bar{L}_{i,t}$ being binding, we obtain

$$\bar{L}_{i,t}^{\xi(1+\zeta)} = \chi_{i,t}^\xi \theta_{i,t} \max\{\bar{L}_{i,t} - \phi, 0\}^{\xi-1}. \quad (24)$$

Tracing out the steps above, the left-hand side of (24) corresponds to the asset price $Q_{i,t}$, whereas the right-hand side corresponds to the weighted expected value $\mathbb{E}_t[m_{i,t}|t]P_{i,t}Y_{i,t}$. For the equilibrium to be stable in the sense that no single investor could profitably extend lending by paying a price $Q_{i,t}$ above the equilibrium price, it therefore must be that

$$g(\bar{L}) \equiv \bar{L}^{\xi(1+\zeta)} - \chi_{i,t}^\xi \theta_{i,t} (\bar{L} - \phi)^{\xi-1}$$

is strictly positive for all \bar{L} larger than the equilibrium level. Let \bar{L}'' define the largest solution to $g(\bar{L}) = 0$. Clearly, $\lim_{\bar{L} \rightarrow \infty} g''(\bar{L}) > 0$ so that for all $\bar{L} > \bar{L}''$, we have $g(\bar{L}) > 0$. Moreover, by continuity, for any solution $\bar{L}' < \bar{L}''$ it must hold that there exist a $\bar{L} > \bar{L}'$ such that $g(\bar{L}) < 0$, violating stability.

A.4 Proof of Proposition 1

Consider the threshold for operation, $\theta^*(\chi)$, first. Following the proof of Lemma 2, equation (10) has a solution $\bar{L} > \phi$ if and only if there exists a solution to $g(\bar{L}) = 0$. Rearranging, the condition reads

$$\tilde{g}(\bar{L}_{i,t}) \equiv (\bar{L}_{i,t} - \phi)^{1-\xi} \bar{L}_{i,t}^{\xi(1+\zeta)} = \chi_{i,t}^\xi \theta_{i,t}. \quad (25)$$

The left-hand side of (25) defines a quasi-convex function \tilde{g} in \bar{L} with the unique minimum

$$\bar{L}_0 = \frac{1+\zeta}{1+\xi\zeta} \xi \phi.$$

Hence, (25) has a solution if and only

$$\theta_{i,t} \geq \theta^*(\chi_{i,t}) \equiv \chi_{i,t}^{-\xi} \tilde{g}(\bar{L}_0) = \chi_{i,t}^{-\xi} \lambda^{\xi-1} \left(\frac{1+\zeta}{1+\xi\zeta} \xi \phi \right)^{1+\xi\zeta}.$$

Moreover, since the largest solution to (10) corresponds to the upward-sloping arm of \tilde{g} , we have that for all $\theta_{i,t} \geq \theta^*(\chi_{i,t})$, \bar{L} is increasing in $\theta_{i,t}$.

Conditional on operating, the firm is constrained whenever $L_{i,t}^{\text{opt}} > \bar{L}_{i,t}$. From (7) and (10),

define the inverse functions

$$\begin{aligned}(L^{\text{opt}})^{-1}(L) &\equiv \lambda^\xi(L - \phi)L^{\xi\zeta} \\ \bar{L}^{-1}(L) &\equiv \chi_{i,t}^{-\xi}(L - \phi)^{1-\xi}L^{\xi(1+\zeta)}.\end{aligned}$$

The two functions intersect on (ϕ, ∞) if and only if $\chi_{i,t} > \lambda^{-1}$. Because both functions are continuous on (ϕ, ∞) , it thus must hold that in the case where $\chi_{i,t} \leq \lambda^{-1}$, active firms are either always constrained or always unconstrained. To see which one is the case, compare $\bar{L}(\theta^*) = L_0$ with $L^{\text{opt}}(\theta^*)$. Specifically, since L^{opt} is increasing in θ , we have that $L^{\text{opt}}(\theta^*) \geq L_0$ if and only if

$$(L^{\text{opt}})^{-1}(L_0) \leq \theta^*$$

or, substituting for L_0 and θ^* , if and only if $\chi \leq 1$, which holds by assumption. We conclude that for $\chi_{i,t} < \lambda^{-1}$, the firm is always constrained.

In the case where $\chi_{i,t} \geq \lambda^{-1}$, a unique intersection, $\bar{L}(\theta) = L^{\text{opt}}(\theta)$, exists and is given by

$$L^{**} = \frac{\phi}{1 - (\lambda\chi_{i,t})^{-1}} \quad \theta^{**} = \lambda^{\xi(1+\zeta)} \chi_{i,t}^{\xi\zeta} \left(\frac{\phi}{\lambda\chi_{i,t} - 1} \right)^{1+\zeta\xi}.$$

Using the implicit function theorem, we have

$$\begin{aligned}\frac{\partial L^{\text{opt}}}{\partial \theta} &= \frac{\lambda^{-\xi}L^{-\xi\zeta}}{1 + \xi\zeta(1 - \phi/L)} \\ \frac{\partial \bar{L}}{\partial \theta} &= \frac{\chi_{i,t}^\xi(L - \phi)^\xi L^{-\xi(1+\zeta)} \frac{1}{\xi-1}}{-1 + \lambda(1 - \phi/L)}.\end{aligned}$$

Comparing slopes at L^{**} , $\partial L^{\text{opt}}(\theta^{**})/\partial \theta < \partial \bar{L}(\theta^{**})/\partial \theta$, and hence $\bar{L}(\theta) \geq L^{\text{opt}}(\theta)$ if and only if $\theta \geq \theta^{**}$.

Summarizing the two cases, $\bar{L}_{i,t} \geq L_{i,t}^{\text{opt}}$ if and only if

$$\theta_{i,t} \geq \theta^{**}(\chi_{i,t}) = \begin{cases} \lambda^{\xi(1+\zeta)} \chi_{i,t}^{\xi\zeta} \left(\frac{\phi}{\lambda\chi_{i,t} - 1} \right)^{1+\zeta\xi} & \text{if } \chi_{i,t} \geq \lambda^{-1} \\ \infty & \text{else.} \end{cases}$$

A.5 Proof of Proposition 2

From (15), the contour line is upward-sloping in $\mu_{i,t}$, implying that the upper arm of the Σ -locus ($\bar{\Sigma} = \sigma_\epsilon^2/(1 - \rho^2)$) is overlapping with the lower arm ($\underline{\Sigma} = \sigma_\epsilon^2 < \bar{\Sigma}$) for some $\mu \in [\underline{\mu}, \bar{\mu}]$. Specifically, from (15), $\underline{\mu} = \tilde{\gamma}\underline{\Sigma} + \eta$ and $\bar{\mu} = \tilde{\gamma}\bar{\Sigma} + \eta$. Accordingly, both arms of the Σ -locus intersect the μ -locus ($\mu = 0$), if and only if

$$\tilde{\gamma}\underline{\Sigma} + \eta \leq 0 \leq \tilde{\gamma}\bar{\Sigma} + \eta$$

or

$$-\tilde{\gamma}\bar{\Sigma} \leq \eta \leq -\tilde{\gamma}\underline{\Sigma}.$$

Summarizing, there are two steady states whenever $\eta \in [\underline{\eta}, \bar{\eta}]$ with $\underline{\eta} = -\tilde{\gamma}\sigma_\epsilon^2$ and $\bar{\eta} = -\tilde{\gamma}\sigma_\epsilon^2/(1-\rho^2)$. Otherwise, there is a unique steady state at either $\underline{\Sigma}$ (for $\eta < \underline{\eta}$) or at $\bar{\Sigma}$ (for $\eta > \bar{\eta}$).

B Computational Appendix (for online publication)

In order to efficiently discretize the state space, we rewrite expectations (16) for all $i \notin \mathcal{B}_{t-1}$ in terms of the last observed productivity, $a_{i,t}^{last} \equiv \log A_{i,t-k-1}$, and the number of consecutive periods k prior to t that the firm was denied funding. Noting that (17) defines a deterministic sequence $\{\Sigma_k\}$, we have

$$\mu_{i,t}|_{k \geq 1} = \rho^{k+1} a_{i,t}^{last} + \rho(1-\delta_k)(\tilde{s}_{i,t}^\epsilon + \tilde{s}_{i,t}^u), \quad (26)$$

where $\delta_k = (1 + \sigma_u^{-2}\Sigma_{k-1})^{-2}$,

$$\tilde{s}_{i,t}^\epsilon = \rho \frac{1-\delta_{k-1}}{1-\delta_k} \tilde{s}_{i,t-1}^\epsilon + \epsilon_{i,t-1} \quad (27)$$

$$\tilde{s}_{i,t}^u = \rho \delta_k \frac{1-\delta_{k-1}}{1-\delta_k} \tilde{s}_{i,t-1}^\epsilon + u_{i,t}, \quad (28)$$

and where $\tilde{s}_{i,t}^\epsilon$ and $\tilde{s}_{i,t}^u$ are initialized at zero for $k = 1$.

Our numerical implementation exploits that δ_k converges to a constant as $k \rightarrow \infty$, yielding

$$\tilde{s}_{i,t}^\epsilon|_{k \rightarrow \infty} = \rho \tilde{s}_{i,t-1}^\epsilon + \epsilon_{i,t-1} \quad (29)$$

$$\tilde{s}_{i,t}^u|_{k \rightarrow \infty} = \rho \delta_\infty \tilde{s}_{i,t-1}^\epsilon + u_{i,t}. \quad (30)$$

To obtain a simple representation of the state space, we approximate (27) and (28) using (29) and (30) for all $k \geq 1$. To evaluate the loss from our approximation, we compute the expected approximation error, finding that is quantitatively insignificant.³⁴ Given the approximation, we have

$$\tilde{s}_{i,t}^\epsilon|_{k \rightarrow \infty} = a_{i,t-1} - \rho^k a_{i,t}^{last}, \quad (31)$$

allowing us to rewrite $(\mu_{i,t}, \Sigma_{i,t})$ in terms of $(a_{i,t-1}, a_{i,t}^{last}, k_{i,t}, \tilde{s}_{i,t}^u)$.

Adding $\chi_{i,t}$, we can then solve our model on the following state space

$$\{\chi_{i,t}\} \times \{a_{i,t-1}\} \times \{a_{i,t}^{last}\} \times \{\tilde{s}_{i,t}^u\} \times \{k_{i,t}\}. \quad (32)$$

While the dimensionality of (32) is increased by 1 relative to $\{S_{i,t}\}$, its complexity is significantly

³⁴By construction, the error is zero for $k \in \{0, 1\}$. For $k \geq 2$, the error peaks at $k = 2$ where the approximation results in an expected 2.09 percent change in $\mu_{i,t}$. For larger k , the error quickly diminishes, falling to less than 0.5 percent for $k \geq 7$.

reduced for two reasons. First, (26) and (31) converge in k at a rate of ρ and $\{\Sigma_k\}$ converges at a rate of ρ^2 . Accordingly, we can truncate k at some large number \bar{k} without changing any of the dynamics. In our simulation, we set $\bar{k} = 70$. Second, $\tilde{s}_{i,t}^u$ evolves exogenously to any of the other variables (conditionally on being initialized at 0 for $k = 1$). Accordingly, we can efficiently approximate both $\{a_{i,t}\}$ and $\{\tilde{s}_{i,t}^u\}$ on a small grid using the usual Rouwenhorst approximation (c.f., Kopecky and Suen, 2010). In our simulation, we use a 5-point grid for $\{a_{i,t}\}$ and a 3-point grid for $\{\tilde{s}_{i,t}^u\}$. Using finer grids has no significant impact on our results. Overall, this yields a manageable grid consisting of $2 \times 5 \times 5 \times 3 \times 70 = 10500$ states.

C Data Appendix (for online publication)

Our dataset is a yearly panel of public US firms. Forecast data is extracted from the Institutional Brokers Estimate System (IBES). Returns are obtained from the CRSP database and are adjusted for splits and dividends. All balance sheet data is from the COMPUSTAT database. From the original sample, we exclude all financial firms (SIC codes between 6000 and 6799) and firms in the electricity sector (SIC codes between 4900 and 4999). The resulting dataset ranges from 1976 to 2016 and covers, on average, 1979 firms per year.³⁵ All variables are winsorized at the 1 percent level.

A unit observation is defined by firm i and year t , where t refers to the year in which earnings are realized. Let $m_{i,t}$ denote the fiscal-year end month of firm i . All balance sheet data and realized earnings per share (EPS) for observation (i, t) are extracted at $m_{i,t}$. As in Senga (2016), we match each observation (i, t) with analysts' EPS-forecasts, $\mu_{i,j,t}^{\text{EPS}}$, extracted 8 months prior to $m_{i,t}$. That is, if in 2007, firm i 's fiscal-year ends in March, then $\mu_{i,j,2007}^{\text{EPS}}$ would be extracted in July 2006. Similarly, returns are synced with the fiscal-year end month of each observation, computed from 12 months prior to $m_{i,t}$ to $m_{i,t}$.

The variables used for our empirical exploration are defined in the main body of the paper. The KZ-index underlying our classification of financial constraints is based on Kaplan and Zingales (1997) and Lamont, Polk and Saá-Requejo (2001). Specifically,

$$\begin{aligned} \text{kz}_{i,t} = & -1.001909 \times \frac{\text{cashflow}_{i,t}}{k_{i,t-1}} + 0.2826389 \times Q_{i,t} + 3.139193 \times \frac{\text{debt}_{i,t}}{\text{total capital}_{i,t}} - \\ & - 39.3678 \times \frac{\text{dividends}_{i,t}}{k_{i,t-1}} - 1.314759 \times \frac{\text{cash}_{i,t}}{k_{i,t-1}}, \end{aligned}$$

where $\text{cashflow}_{i,t}$ is the sum of COMPUSTAT items “income before extraordinary items” and “depreciation and amortization”, $Q_{i,t}$ is (“market capitalization” + “total shareholder’s equity” – “book value of common equity” – “deferred tax assets”)/“total shareholder’s equity”, $\text{debt}_{i,t}$ is “long-term debt” + “debt in current liabilities”, $\text{total capital}_{i,t}$ is “long-term debt” + “debt in current liabilities” + “stockholders equity”, and $k_{i,t}$ is “total property, plants and equipment” (see the

³⁵Due to incomplete balance sheet data and firms with less than 2 forecasters for which $\sigma_{i,t}^{\text{fce}}$ is not defined, the effective number of observations is reduced for some of our empirical tests.

Appendix to Lamont, Polk and Saá-Requejo, 2001 for a listing of the corresponding COMPUSTAT items).