# Contests with Bilateral Delegation: Unobservable Contracts 

by

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#### Abstract

We study two-player contests in which, in order to win a prize, each player hires a delegate to expend effort on her behalf; neither party's delegation contract is revealed to the rival party when the delegates choose their effort levels. We obtain first the outcomes of this unobservablecontracts case. Next, we perform comparative statics of these outcomes with respect to the higher-valuation player's valuation for the prize. Finally, we compare the outcomes of the unobservable-contracts case with those of the observable-contracts case. We find, among other things, that the unobservability of delegation contracts narrows the gap between the delegates' equilibrium contingent compensation.


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## 1 Introduction

Delegation in a contest - a situation in which, in order to win a prize, each player or contestant hires a delegate to expend effort or resources on the player's behalf - can be readily observed all around us. ${ }^{1}$ Examples include attorney delegation in litigation, lobbyist delegation in rentseeking contests, researcher delegation in patent contests, and strategic managerial delegation. In some contests, delegation occurs because it is compulsory. In others, delegation may occur because players want to use superior ability of delegates, and/or try to achieve strategic commitments.

In contests with delegation, one may well expect that delegates may choose their effort levels without observing the rival parties' delegation contracts. This unobservability of delegation contracts may occur because the parties do not announce their delegation contracts, or because their announced delegation contracts are not verifiable. For example, in litigation between a plaintiff and a defendant in which each litigant hires an attorney to expend effort on the litigant's behalf, the attorney for each side may choose his effort level without observing the contract of the other side.

Accordingly, the purpose of this paper is to study contests with delegation in which delegation contracts are private information. To the best of our knowledge, such contests with delegation have not previously been studied.

We study two-player contests with bilateral delegation in which neither party's delegation contract is revealed to the rival party when the two delegates choose their effort levels. Specifically, we set up and analyze the following game. First, the players hire delegates and independently write contracts with their delegates. Next, the delegates choose their effort levels simultaneously and independently, each choosing his effort level without observing the contract for his counterpart. Finally, the winning player is determined, and each player pays compensation to her delegate according to her contract for him. The players use contracts that
condition delegates' compensation on the outcome, win or lose, of the contest. The probability that a player wins the prize depends on the delegates' effort levels.

We solve the game - called the unobservable-contracts case - treating it as a simultaneous-move game between the two parties. ${ }^{2}$ Then, we examine how the outcomes of the game respond when the ratio of the players' valuations for the prize changes - more specifically, when the higher-valuation player's valuation for the prize changes, ceteris paribus. Finally, we compare the outcomes of the unobservable-contracts case with those of the observable-contracts case, the game that is the same as the one presented above with the exception that each party's delegation contract is observable to the rival party - more specifically, when choosing their effort levels, the delegates know the delegation contracts of both parties. ${ }^{3}$ Here we find that the unobservability of delegation contracts narrows, compared with the observable-contracts case, the gap between the delegates' equilibrium contingent compensation, which in turn leads to several interesting results.

This paper is related to the literature on delegation in contests: See, for example, Baik and Kim (1997), Wärneryd (2000), Schoonbeek (2002, 2004, 2007), Konrad, Peters, and Wärneryd (2004), Lim and Shogren (2004), Kräkel (2005), Kräkel and Sliwka (2006), Baik (2007, 2008), Brandauer and Englmaier (2009), and Baik and Lee (2013). ${ }^{4}$ In this literature, unlike in the current paper, a standard assumption is that, when choosing their effort levels, delegates know the delegation contracts, if any, of the rival parties - in other words, public information is assumed regarding delegation contracts. On the other hand, as in the current paper, most of the above-mentioned papers study delegation in two-player contests, and assume that the delegates' compensation specified in the delegation contracts is conditioned on the outcome of the contest. Baik and Kim (1997) study contests in which the delegation contracts are exogenous and each player has the option of hiring a delegate. Wärneryd (2000) studies contests with bilateral delegation in which the delegation contracts are endogenous. Schoonbeek (2002) considers a contest in which only one player has the option of hiring a delegate and the delegation contract is endogenous. Schoonbeek (2004) considers a contest between two groups
in which each group decides whether or not to hire a delegate and the delegation contracts are endogenous. Schoonbeek (2007) considers a contest with endogenous delegation contracts in which each player has the option of hiring a delegate and the delegates have more instruments than the players. Lim and Shogren (2004) study an environmental conflict with unilateral delegation in which the delegation contract is exogenous. Baik $(2007,2008)$ considers contests with bilateral delegation in which the delegation contracts are endogenous. Baik and Lee (2013) study contests with bilateral delegation in which the delegates decide endogenously when to expend their effort.

This paper is related also to Baik and Lee (2007) and Nitzan and Ueda (2011). These papers assume, as in the main model of this paper, that each party chooses its two sequential moves without observing the other parties' moves. The two papers study collective rent seeking between groups in which the players in each group decide first how to share the rent among themselves if they win, and then choose their effort levels simultaneously and independently without observing the sharing rules to which the players in the other groups agreed.

The paper proceeds as follows. Section 2 formulates a game that models a two-player contest with bilateral delegation in which each party's contract is unobservable to the rival party. In Section 3, solving the game, we obtain the equilibrium contracts and the delegates' equilibrium effort levels. In Section 4, we first obtain all the outcomes of the unobservablecontracts case, and then perform comparative statics of these outcomes with respect to the higher-valuation player's valuation for the prize. In Section 5, we first look at the observablecontracts case. Then we compare the outcomes of the unobservable-contracts case with those of the observable-contracts case. Finally, Section 6 offers our conclusions.

## 2 The Main Model: The Unobservable-Contracts Case

Consider a contest between two players, 1 and 2, in which, in order to win a prize, each player hires a delegate to expend effort on her behalf. The contract between a player and her delegate is
hidden from the rival party when the two delegates, 1 and 2, choose their effort levels. ${ }^{5}$ The probability that a player wins the prize depends on the delegates' effort levels.

This situation is formally modeled as the following game. First, the players hire delegates and independently write contracts with their delegates. ${ }^{6}$ Naturally, each delegate observes the contract for himself. But he cannot observe the contract for his counterpart. Next, the delegates choose their effort levels simultaneously and independently. ${ }^{7}$ Finally, the winning player is determined, and each player (or only the winning player, as will be clear shortly) pays compensation to her delegate according to her contract with him.

Player 1's valuation for the prize is $v_{1}$, and player 2's valuation is $v_{2}$, where $v_{1} \geq v_{2}$. We assume that each player's valuation for the prize is positive, measured in monetary units, and publicly known. Each delegate bears the cost of expending his effort. Delegates' effort levels are not verifiable to a third party, so that the players cannot use contracts that base delegates' compensation on their effort levels. We assume that the players use contracts that condition delegates' compensation on the outcome of the contest: Player $i$ pays delegate $i$ compensation of $\alpha_{i} v_{i}$ if she wins the prize, and zero if she loses it, where $0<\alpha_{i}<1 .{ }^{8}$ We call compensation of $\alpha_{i} v_{i}$ delegate $i$ 's contingent compensation.

We assume that the delegates are risk-neutral, and have a reservation wage of 0 . Delegate $i$ signs up for player $i$ only if player $i$ offers him a contract - or, equivalently, a value of $\alpha_{i}$ - under which he can earn an expected payoff greater than or equal to his reservation wage (given his beliefs about the rival party's behavior).

Let $x_{i}$ represent the effort level that delegate $i$ expends. Each delegate's effort level is nonnegative, and measured in monetary units. Let $p_{i}\left(x_{1}, x_{2}\right)$ denote the probability that player $i$ wins the prize when delegate 1 expends $x_{1}$ and delegate 2 expends $x_{2}$. We assume that the contest success function for player $i$ is ${ }^{9}$

$$
\begin{array}{cl}
p_{i}\left(x_{1}, x_{2}\right)= & x_{i} /\left(x_{1}+x_{2}\right)  \tag{1}\\
1 / 2 & \text { for } x_{1}+x_{2}>0 \\
1 / 2 & \text { for } x_{1}=x_{2}=0
\end{array}
$$

This function says that, ceteris paribus, each player's probability of winning is increasing in her delegate's effort level at a decreasing rate; it is decreasing in the rival delegate's effort level at a decreasing rate. Function (1) implies that the delegates have equal ability for the contest.

We assume that the players too are risk-neutral. Let $G_{i}$ represent the expected payoff for player $i$. Then the payoff function for player $i$ is

$$
\begin{equation*}
G_{i}=\left(1-\alpha_{i}\right) v_{i} p_{i}\left(x_{1}, x_{2}\right) . \tag{2}
\end{equation*}
$$

Let $\pi_{i}$ represent the expected payoff for delegate $i$. Then the payoff function for delegate $i$ is

$$
\begin{equation*}
\pi_{i}=\alpha_{i} v_{i} p_{i}\left(x_{1}, x_{2}\right)-x_{i} \tag{3}
\end{equation*}
$$

Expected payoffs to player $i$ and delegate $i$ do not depend directly on $\alpha_{j}$, for $i, j=1,2$ with $i \neq j$. However, they depend indirectly on $\alpha_{j}$ because $\alpha_{j}$ affects $x_{j} .{ }^{10}$ Note that, to choose their optimal strategies at the start of the game, the players and the delegates should compute their expected payoffs at the start of the game, forming their beliefs about both parties' contracts and the delegates' effort levels to be chosen.

Finally, we assume that all of the above is common knowledge among the players and delegates.

## 3 Equilibrium Contracts and Effort Levels

To solve the game, we need to find a quadruple vector ( $\alpha_{1}^{*}, x_{1}^{*}, \alpha_{2}^{*}, x_{2}^{*}$ ) of actions that satisfies the following two requirements. ${ }^{11}$ First, each delegate's effort level is optimal given the contract for him and given the rival party's effort level (or, precisely, the effort level of the delegate in the rival party). Second, each player's contract is optimal given the rival party's effort level and given the behavior of her own delegate that follows. ${ }^{12}$ Note that it makes sense to say that the pair $\left(\alpha_{i}^{*}, x_{i}^{*}\right)$ of actions of party $i$ is a best response to the pair ( $\alpha_{j}^{*}, x_{j}^{*}$ ) of actions of party $j$ because, as mentioned in footnote 7, the two parties play a simultaneous-move game.

To find such a quadruple vector of actions - the equilibrium contracts and effort levels - for the game, we begin by obtaining party $i$ 's best response to a pair ( $\alpha_{j}, x_{j}$ ) of actions of party $j$. Now that party $i$ 's best response to ( $\alpha_{j}, x_{j}$ ) consists of player i's best response to ( $\alpha_{j}, x_{j}$ ) and delegate $i$ 's best response to ( $\alpha_{j}, x_{j}$ ), working backward, we consider first delegate $i$ 's decision on his effort level, and then consider player i's decision on her contract. ${ }^{13}$

Consider delegate $i$ 's decision on his effort level. After observing player i's contract $\alpha_{i}$, delegate $i$ seeks to maximize his expected payoff (3) over his effort level $x_{i}$, taking delegate $j$ 's effort level $x_{j}$ as given. From the first-order condition for maximizing function (3), we obtain

$$
\begin{equation*}
x_{i}\left(\alpha_{i}, x_{j}\right)=\sqrt{\alpha_{i} v_{i} x_{j}}-x_{j} \tag{4}
\end{equation*}
$$

where $x_{i}\left(\alpha_{i}, x_{j}\right)$ denotes delegate $i$ 's best response to delegate $j$ 's effort level $x_{j} .{ }^{14}$
Next, consider player $i$ 's decision on her contract. Taking delegate $j$ 's effort level $x_{j}$ as given, player $i$ seeks to maximize her expected payoff (2) over her contract $\alpha_{i}$, having perfect foresight about $x_{i}\left(\alpha_{i}, x_{j}\right)$ for each value of $\alpha_{i}$. More precisely, player $i$ seeks to maximize

$$
\begin{equation*}
G_{i}\left(\alpha_{i}, x_{j}\right)=\left(1-\alpha_{i}\right) v_{i} x_{i}\left(\alpha_{i}, x_{j}\right) /\left\{x_{i}\left(\alpha_{i}, x_{j}\right)+x_{j}\right\} \tag{5}
\end{equation*}
$$

with respect to $\alpha_{i}$, taking $x_{j}$ as given. Note that we obtain function (5) using functions (1) and (2), and delegate $i$ 's best response (4). The first-order condition for maximizing function (5) reduces to

$$
\begin{equation*}
4 v_{i} \alpha_{i}^{3}-x_{j} \alpha_{i}^{2}-2 x_{j} \alpha_{i}-x_{j}=0 \tag{6}
\end{equation*}
$$

Solving equation (6) for $\alpha_{i}$, we obtain player $i$ 's best response to delegate $j$ 's effort level $x_{j}$, which is denoted by $\alpha_{i}\left(x_{j}\right)$.

We now obtain the reaction functions for the parties. Party $i$ has two reaction functions, one for delegate $i$ and one for player $i$. Delegate $i$ 's reaction function comes from delegate $i$ 's
best response (4), so that it is $x_{i}=x_{i}\left(\alpha_{i}, x_{j}\right)$ or, equivalently, $x_{i}=\sqrt{\alpha_{i} v_{i} x_{j}}-x_{j}$. Player $i$ 's reaction function is $\alpha_{i}=\alpha_{i}\left(X_{j}\right)$, the implicit form of which is given in equation (6).

Finally, we obtain the equilibrium contracts and effort levels, $\left(\alpha_{1}^{*}, x_{1}^{*}, \alpha_{2}^{*}, x_{2}^{*}\right)$, for the game. Because they satisfy all the four reaction functions of the parties simultaneously, we obtain them by solving the following system of four simultaneous equations:

$$
\begin{equation*}
4 v_{1} \alpha_{1}^{3}-x_{2} \alpha_{1}^{2}-2 x_{2} \alpha_{1}-x_{2}=0 \tag{7}
\end{equation*}
$$

$$
\begin{align*}
& x_{1}=\sqrt{\alpha_{1} v_{1} x_{2}}-x_{2}  \tag{8}\\
& 4 v_{2} \alpha_{2}^{3}-x_{1} \alpha_{2}^{2}-2 x_{1} \alpha_{2}-x_{1}=0
\end{align*}
$$

and

$$
\begin{equation*}
x_{2}=\sqrt{\alpha_{2} v_{2} x_{1}}-x_{1} \tag{10}
\end{equation*}
$$

It is computationally intractable to obtain the equilibrium contracts and effort levels, ( $\alpha_{1}^{*}, x_{1}^{*}, \alpha_{2}^{*}$, $x_{2}^{*}$ ), by substituting player 1's reaction function $\alpha_{1}=\alpha_{1}\left(x_{2}\right)$ into equation (8), and player 2's reaction function $\alpha_{2}=\alpha_{2}\left(x_{1}\right)$ into equation (10), then solving the resulting pair of simultaneous equations for $x_{1}$ and $x_{2}$, and so on. Accordingly, we use the following trick that we discovered to get around the intractableness. Let $v_{1}=\theta v_{2}$, where $\theta$ is a parameter greater than or equal to unity. Let $x_{1}^{*}=t x_{2}^{*}$, where $t$ is a positive unknown to be solved for below. Then we have, in equilibrium,

$$
\begin{align*}
& 4 \theta v_{2}\left(\alpha_{1}^{*}\right)^{3}-x_{2}^{*}\left(\alpha_{1}^{*}\right)^{2}-2 x_{2}^{*} \alpha_{1}^{*}-x_{2}^{*}=0  \tag{11}\\
& x_{2}^{*}=\theta v_{2} \alpha_{1}^{*} /(1+t)^{2}  \tag{12}\\
& 4 v_{2}\left(\alpha_{2}^{*}\right)^{3}-t x_{2}^{*}\left(\alpha_{2}^{*}\right)^{2}-2 t x_{2}^{*} \alpha_{2}^{*}-t x_{2}^{*}=0 \tag{13}
\end{align*}
$$

and

$$
\begin{equation*}
x_{2}^{*}=t v_{2} \alpha_{2}^{*} /(1+t)^{2} \tag{14}
\end{equation*}
$$

which come from equations (7) through (10), respectively. Next, using equations (11) through (14), we obtain

$$
\begin{align*}
& \alpha_{1}^{*}=1 /(1+2 t),  \tag{15}\\
& x_{1}^{*}=\theta t v_{2} /(1+t)^{2}(1+2 t),  \tag{16}\\
& \alpha_{2}^{*}=t /(2+t), \tag{17}
\end{align*}
$$

and

$$
\begin{equation*}
x_{2}^{*}=t^{2} v_{2} /(1+t)^{2}(2+t) . \tag{18}
\end{equation*}
$$

As the final step, we solve for the unknown $t$, using the relationship $x_{1}^{*}=t x_{2}^{*}$ and equations (16) and (18), which reduce to

$$
\begin{equation*}
2 t^{3}+t^{2}-\theta t-2 \theta=0 \tag{19}
\end{equation*}
$$

Now, using the above results, we report the equilibrium contracts and effort levels, ( $\alpha_{1}^{*}$, $x_{1}^{*}, \alpha_{2}^{*}, x_{2}^{*}$ ), in Lemma 1 (see Figures 1 and 2 ).

LEMMA 1 We obtain $\alpha_{1}^{*}=1 /(1+2 t), \alpha_{2}^{*}=t /(2+t), x_{1}^{*}=\theta t v_{2} /(1+t)^{2}(1+2 t)$, and $x_{2}^{*}=$ $t^{2} v_{2} /(1+t)^{2}(2+t)$, where $t$ is a unique positive real root of equation (19). ${ }^{15}$

Note that $t$ in Lemma 1 is a positive real for any value of $\theta$, where $\theta \geq 1$; it is equal to unity for $\theta=1$; and it is monotonically increasing in $\theta$. Note also that $\alpha_{1}^{*}$ and $\alpha_{2}^{*}$ do not depend separately on player 1's valuation $v_{1}$ for the prize and player 2 's valuation $v_{2}$, but only on the parameter $\theta$, the ratio of the players' valuations (of player 1 to player 2 ).

## 4 Comparative Statics with respect to the Parameter $\theta$

In this section, we first look at the players' contracts, the delegates' effort levels, the players' probabilities of winning, and the expected payoffs of the delegates and the players in the equilibrium of the game. Then, we examine how these outcomes of the game respond when the asymmetry between the players changes - that is, we perform comparative statics of these outcomes with respect to the parameter $\theta$.

Let $p_{i}\left(x_{1}^{*}, x_{2}^{*}\right)$ represent the probability that player $i$ wins the prize in the equilibrium. Let $\pi_{i}^{*}$ and $G_{i}^{*}$ represent the expected payoff for delegate $i$ and that for player $i$, respectively, in the equilibrium. Then, using Lemma 1 and functions (1) through (3), we obtain Propositions 1 and 2 (see Figures 1 through 4).

PROPOSITION 1 If $\theta=1$, then we obtain $\alpha_{1}^{*}=\alpha_{2}^{*}=1 / 3, x_{1}^{*}=x_{2}^{*}=v_{2} / 12, p_{1}\left(x_{1}^{*}, x_{2}^{*}\right)=1 / 2$, $\pi_{1}^{*}=\pi_{2}^{*}=v_{2} / 12$, and $G_{1}^{*}=G_{2}^{*}=v_{2} / 3$.

PROPOSITION 2 If $\theta>1$, then we obtain (a) $\alpha_{1}^{*}<1 / 3<\alpha_{2}^{*}$ and $\alpha_{1}^{*} v_{1}>\alpha_{2}^{*} v_{2}$, (b) $x_{1}^{*}>x_{2}^{*}$, (c) $p_{1}\left(x_{1}^{*}, \quad x_{2}^{*}\right)=\theta(2+t) /[\theta(2+t)+t(1+2 t)]>1 / 2$, (d) $\pi_{1}^{*}=\theta t^{2} v_{2} /(1+t)^{2}(1+2 t)$ and $\pi_{2}^{*}=t v_{2} /(1+t)^{2}(2+t), \quad$ so $\quad$ that $\quad \pi_{1}^{*}>\pi_{2}^{*}>0, \quad(e) \quad G_{1}^{*}=2 \theta t^{2} v_{2} /(1+t)(1+2 t) \quad$ and $G_{2}^{*}=2 v_{2} /(1+t)(2+t)$, so that $G_{1}^{*}>G_{2}^{*}>0$, and $(f) G_{i}^{*}>\pi_{i}^{*}$ for $i=1,2$.

Proposition 2 says that, in equilibrium, the lower-valuation player (player 2) offers her delegate a higher value of $\alpha$ than the higher-valuation player, but her delegate (delegate 2) exerts less effort than his counterpart. This can be explained as follows. Player 2 (the lower-valuation player) offers delegate 2 a higher value of $\alpha$ than player 1 . However, delegate 2's contingent compensation is less than delegate 1 's - that is, $\alpha_{2}^{*} v_{2}<\alpha_{1}^{*} v_{1}$ - because player 2's valuation for the prize is lower than player 1's. Consequently, delegate 2 is less motivated, and thus expends less effort than his counterpart.

Another interesting observation from Proposition 2 is that there may be a case where player 2's equilibrium contract $\alpha_{2}^{*}$ is greater than one half - that is, player 2 (the lower-valuation player) pays her delegate more than half her valuation for the prize if she wins the prize. Indeed, using Lemma 1, we confirm that $\alpha_{2}^{*}>1 / 2$ for $\theta>5$. Note, however, that according to part ( $f$ ), player 2's equilibrium expected payoff is always - even for $\theta>5$ - greater than delegate 2's.

Part (d) says that the equilibrium expected payoffs for the delegates differ. The reason for this is not because the delegates possess different bargaining power or ability. Both delegates are identical before they are hired. Intuitively, the delegates' expected payoffs differ because the competition between the players to win the prize drives them to offer different contingent compensation to their delegates. Accordingly, the result that the delegates' expected payoffs differ does not depend on how many potential delegates exist at the start of the game.

Part (d) says also that each delegate's equilibrium expected payoff is greater than his reservation wage. ${ }^{16}$ This economic rent for each delegate is created sheerly by the competition between the players to win the prize - more specifically, by the players' strategic decisions on their delegates' compensation.

Now, using Lemma 1 and Propositions 1 and 2, we examine the effects of increasing the parameter $\theta$ - more specifically, those of increasing player 1 's valuation $v_{1}$ for the prize, ceteris paribus - on the outcomes of the game. Proposition 3 summarizes the comparative statics results (see Figures 1 through 4).

PROPOSITION 3 As $\theta$ increases from unity, (a) $\alpha_{1}^{*}$ decreases while $\alpha_{2}^{*}$ increases, (b) both $\alpha_{1}^{*} \nu_{1}$ and $\alpha_{2}^{*} v_{2}$ increase, but the gap between $\alpha_{1}^{*} v_{1}$ and $\alpha_{2}^{*} v_{2}$ widens, (c) delegate 1 's equilibrium effort level $x_{1}^{*}$ increases while delegate 2 's equilibrium effort level $x_{2}^{*}$ increases for a while but eventually decreases, (d) the gap between $x_{1}^{*}$ and $x_{2}^{*}$ widens, (e) the equilibrium total effort level, $x_{1}^{*}+x_{2}^{*}$, increases, (f) player 1's probability of winning increases while player 2 's decreases, and (g) delegate 1's and player 1's equilibrium expected payoffs each increase while delegate 2's and player 2's equilibrium expected payoffs each decrease.

The following complements Proposition 3. As $\theta$ approaches (plus) infinity, (i) the limit of player 1's equilibrium contract $\alpha_{1}^{*}$ is zero and that of player 2's equilibrium contract $\alpha_{2}^{*}$ is unity, (ii) the limit of delegate 1 's equilibrium contingent compensation $\alpha_{1}^{*} v_{1}$ is infinity, that of delegate 2's equilibrium contingent compensation $\alpha_{2}^{*} v_{2}$ is $v_{2}$, and that of $\alpha_{1}^{*} v_{1}-\alpha_{2}^{*} v_{2}$ is infinity, (iii) the limit of $x_{1}^{*}$ is $v_{2}$, that of $x_{2}^{*}$ is zero, that of $x_{1}^{*}-x_{2}^{*}$ is $v_{2}$, and that of $x_{1}^{*}+x_{2}^{*}$ is $v_{2}$, (iv) the limit of player 1's probability of winning is unity, and (v) the limits of delegate 1's and player 1's equilibrium expected payoffs are infinity, and the limits of delegate 2's and player 2's equilibrium expected payoffs are zero.

Part (b) says that, as the valuation $v_{1}$ of the higher-valuation player (or player 1) increases, ceteris paribus, each player makes her delegate more aggressive or stronger by offering him greater contingent compensation. According to part (a), the lower-valuation player (or player 2) offers her delegate greater contingent compensation by offering a higher value of $\alpha$ than before. By contrast, due to an increase in her valuation, the higher-valuation player offers her delegate greater contingent compensation with a lower value of $\alpha$ than before.

Part (e) says that, as the valuation $v_{1}$ of the higher-valuation player increases, ceteris paribus, the equilibrium total effort level increases. This can be explained by the fact that, as $v_{1}$ increases (or, equivalently, as $\theta$ increases), both $\alpha_{1}^{*} v_{1}$ and $\alpha_{2}^{*} v_{2}$ increase, so that the delegates engage in fiercer competition. Part (e) is interesting because it is in contrast with previous results in the literature on the theory of contests. For example, studying two-player asymmetric contests with bilateral delegation in which each party's contract is observable to the rival party and delegates decide endogenously when to expend their effort, Baik and Lee (2013) show that, as the valuation of the higher-valuation player increases, the equilibrium total effort level remains unchanged. ${ }^{17}$

Another interesting result in Proposition 3 is that, as $\theta$ approaches (plus) infinity, the limit of the equilibrium total effort level is $v_{2}$. This implies that the equilibrium total effort level is always less than the valuation $v_{2}$ of the lower-valuation player.

Part ( $f$ ) says that, as the valuation $v_{1}$ of the higher-valuation player increases, ceteris paribus, her probability of winning increases. This follows immediately from part (d), which in turn follows from part (b).

## 5 Comparison with the Case of Observable Contracts

So far we have studied two-player contests with bilateral delegation in which each party's contract is unobservable to the rival party. In this section, we study first contests with bilateral delegation in which each party's contract is observable to the rival party. Then we compare the outcomes of the unobservable-contracts case, which are provided in Lemma 1 and Propositions 1 and 2, with those of the observable-contracts case, which are provided in Lemmas A1 and A2 in the Appendix.

Consider a contest that is the same as the one in Section 2 with the exception that, when choosing their effort levels, the delegates know the contracts of both parties. Specifically, consider the following two-stage game. In the first stage, the players hire delegates and independently write contracts with their delegates, and then they simultaneously announce (and commit to) the contracts - that is, player 1 announces publicly the value of $\alpha_{1}$, and player 2 announces the value of $\alpha_{2} .{ }^{18}$ In the second stage, after knowing both contracts, the delegates choose their effort levels simultaneously and independently. The winning player is determined at the end of the second stage, and only the winning player pays compensation to her delegate according to her contract announced in the first stage.

To solve for a subgame-perfect equilibrium of this standard two-stage game, we work backwards. In the second stage, the delegates know the contracts of both parties: the value of $\alpha_{1}$ and the value of $\alpha_{2}$. Delegate $i$ seeks to maximize his expected payoff (3) over his effort level $x_{i}$, taking delegate $j$ 's effort level $x_{j}$ as given. From the first-order condition for maximizing function (3), we obtain delegate $i$ 's best response $b_{i}\left(x_{j}\right)$ to $x_{j}$. Delegate $i$ 's reaction function,
$x_{i}=b_{i}\left(x_{j}\right)$, which shows his best response to every possible effort level that delegate $j$ might choose, is then

$$
x_{i}=\sqrt{\alpha_{i} v_{i} x_{j}}-x_{j}
$$

for $i, j=1$, 2 with $i \neq j$. Using these reaction functions for the delegates, we obtain the Nash equilibrium of the second-stage subgame: ${ }^{19}$

$$
x_{1}\left(\alpha_{1}, \alpha_{2}\right)=\alpha_{1}^{2} v_{1}^{2} \alpha_{2} v_{2} /\left(\alpha_{1} v_{1}+\alpha_{2} v_{2}\right)^{2}
$$

and

$$
\begin{equation*}
x_{2}\left(\alpha_{1}, \alpha_{2}\right)=\alpha_{1} v_{1} \alpha_{2}^{2} v_{2}^{2} /\left(\alpha_{1} v_{1}+\alpha_{2} v_{2}\right)^{2} \tag{20}
\end{equation*}
$$

Next, consider the first stage in which the players choose their contracts. In this stage, taking player $j$ 's contract $\alpha_{j}$ as given, player $i$ seeks to maximize her expected payoff (2) over her contract $\alpha_{i}$, having perfect foresight about the delegates' optimal strategies - or, equivalently, having perfect foresight about both $x_{1}\left(\alpha_{1}, \alpha_{2}\right)$ and $x_{2}\left(\alpha_{1}, \alpha_{2}\right)$ for any values of $\alpha_{1}$ and $\alpha_{2}$. More precisely, player i seeks to maximize

$$
\begin{equation*}
G_{i}\left(\alpha_{1}, \alpha_{2}\right)=\left(1-\alpha_{i}\right) v_{i} x_{i}\left(\alpha_{1}, \alpha_{2}\right) /\left\{x_{1}\left(\alpha_{1}, \alpha_{2}\right)+x_{2}\left(\alpha_{1}, \alpha_{2}\right)\right\} \tag{21}
\end{equation*}
$$

with respect to $\alpha_{i}$, taking $\alpha_{j}$ as given. Note that we obtain function (21) by substituting the delegates' effort levels, $x_{1}\left(\alpha_{1}, \alpha_{2}\right)$ and $x_{2}\left(\alpha_{1}, \alpha_{2}\right)$, in (20) into function (2). From the first-order condition for maximizing function (21), we obtain player i's best response $r_{i}\left(\alpha_{j}\right)$ to $\alpha_{j}$. Player $i$ 's reaction function, $\alpha_{i}=r_{i}\left(\alpha_{j}\right)$, which shows her best response to every possible value of $\alpha_{j}$ that player $j$ might choose, is then

$$
\alpha_{i}=\left(\sqrt{\alpha_{j} v_{1} v_{2}+\alpha_{j}^{2} v_{j}^{2}}-\alpha_{j} v_{j}\right) / v_{i}
$$

for $i, j=1$, 2 with $i \neq j$. Using these reaction functions for the players, we will obtain the players' contracts, $\alpha_{1}^{* *}$ and $\alpha_{2}^{* *}$, which are specified in the subgame-perfect equilibrium of the
two-stage game. As in Section 3, let $v_{1}=\theta v_{2}$, where $\theta \geq 1$. Then the reaction functions of player 1 and player 2 are rewritten as, respectively,

$$
\begin{equation*}
\alpha_{1}=\left(\sqrt{\alpha_{2} \theta+\alpha_{2}^{2}}-\alpha_{2}\right) / \theta \tag{22}
\end{equation*}
$$

and

The equilibrium contracts of the players satisfy these reaction functions simultaneously. Let $\alpha_{1}^{* *}=k \alpha_{2}^{* *}$, where $k$ is a positive unknown to be solved for below. Then we have, in equilibrium,

$$
\begin{equation*}
\alpha_{1}^{* *}=1 /(2+\theta k) \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
\alpha_{2}^{* *}=\theta k /(1+2 \theta k) . \tag{25}
\end{equation*}
$$

which come from equations (22) and (23), respectively. Next, we solve for the unknown $k$, using the relationship $\alpha_{1}^{* *}=k \alpha_{2}^{* *}$ and equations (24) and (25), which reduce to

$$
\begin{equation*}
\theta^{2} k^{3}+2 \theta k^{2}-2 \theta k-1=0 \tag{26}
\end{equation*}
$$

Finally, substituting a unique positive real root of equation (26) into equations (24) and (25), respectively, we obtain the equilibrium contracts, $\alpha_{1}^{* *}$ and $\alpha_{2}^{* *}$, of the players (see Figure 1 ). ${ }^{20}$

Let $x_{i}^{* *}$ represent the equilibrium effort level of delegate $i$. Let $p_{i}\left(x_{1}^{* *}, x_{2}^{* *}\right)$ represent the probability that player $i$ wins the prize in the subgame-perfect equilibrium. Let $\pi_{i}^{* *}$ and $G_{i}^{* *}$ represent the equilibrium expected payoff for delegate $i$ and that for player $i$, respectively. Then, substituting the equilibrium contracts, $\alpha_{1}^{* *}$ and $\alpha_{2}^{* *}$, of the parties into (20) and using functions (1) through (3), we obtain Lemmas A1 and A2 in the Appendix (see Figures 1 through 4).

Now, using Propositions 1 and 2 and Lemmas 1, A1, and A2, we compare the outcomes of the unobservable-contracts case with those of the observable-contracts case. First, if $\theta=1$, then the two cases yield the same outcomes. That is, the unobservability of delegation contracts
makes no difference, as compared to the observable-contracts case, if the players have the same valuation for the prize. Next, Proposition 4 compares the outcomes of the two cases when $\theta>1$. The superscripts * and ${ }^{* *}$ in Proposition 4 indicate the outcomes of the unobservable-contracts case and those of the observable-contracts case, respectively.

PROPOSITION 4 If $\theta>1$, then we obtain (a) $\alpha_{1}^{*}<\alpha_{1}^{* *}$ and $\alpha_{2}^{*}>\alpha_{2}^{* *}$, (b) $x_{1}^{*}<x_{1}^{* *}$ unless $\theta$ is large, $x_{2}^{*}>x_{2}^{* *}$, and $x_{1}^{*}+x_{2}^{*}>x_{1}^{* *}+x_{2}^{* *}$, (c) $p_{1}\left(x_{1}^{*}, x_{2}^{*}\right)<p_{1}\left(x_{1}^{* *}, x_{2}^{* *}\right)$, (d) $\pi_{1}^{*}<\pi_{1}^{* *}$ and $\pi_{2}^{*}>\pi_{2}^{* *}$, and $(e) G_{1}^{*}<G_{1}^{* *}$ and, unless $\theta$ is extremely large, $G_{2}^{*}>G_{2}^{* *} \cdot{ }^{21}$

Proposition 4 is illustrated in Figures 1 through 4. Proposition 4 says that, in equilibrium, the higher-valuation player (or player 1) offers her delegate greater contingent compensation in the observable-contracts case than in the unobservable-contracts case, whereas the lower-valuation player (or player 2) offers her delegate greater contingent compensation in the unobservable-contracts case than in the observable-contracts case: $\alpha_{1}^{*} v_{1}<\alpha_{1}^{* *} v_{1}$ and $\alpha_{2}^{*} v_{2}>\alpha_{2}^{* *} v_{2} .{ }^{22}$ This, together with Proposition 2, yields $\alpha_{2}^{* *} v_{2}<\alpha_{2}^{*} v_{2}<\alpha_{1}^{*} v_{1}<\alpha_{1}^{* *} v_{1}$. Clearly, as compared to the observable-contracts case, the unobservability of contracts narrows the gap between the delegates' equilibrium contingent compensation: $\alpha_{1}^{*} v_{1}-\alpha_{2}^{*} v_{2}<\alpha_{1}^{* *} v_{1}-\alpha_{2}^{* *} v_{2} .{ }^{23}$

Proposition 4 says that the equilibrium total effort level is greater in the unobservablecontracts case than in the observable-contracts case. ${ }^{24}$ Not surprisingly, this arises because the delegates' equilibrium contingent compensation gets closer - so that their competition gets fiercer - in the unobservable-contracts case than in the observable-contracts case. On the basis of this result, we may argue that the contest organizer or the decision-maker, if any, who wants to induce more effort from contestants prefers the unobservable-contracts case to the observable-contracts case). On the other hand, in two-player rent-seeking contests with bilateral delegation, the observable-contracts case reduces social costs associated with rent seeking, compared with the unobservable-contracts case. ${ }^{25}$ We may then argue that one way to lower
social costs in such contests is to create an environment in which both parties release the information on their contracts.

Another interesting result in Proposition 4 is that the equilibrium expected payoffs of the higher-valuation player and her delegate, respectively, are less in the unobservable-contracts case than in the observable-contracts case, whereas, unless $v_{1}$ is very much greater than $v_{2}$ (or, equivalently, unless $\theta$ is extremely large), the equilibrium expected payoffs of the lowervaluation player and her delegate, respectively, are greater in the unobservable-contracts case than in the observable-contracts case. ${ }^{26}$ On the basis of this result, we argue that the highervaluation player and her delegate prefer the observable-contracts case (to the unobservablecontracts case), while the lower-valuation player and her delegate prefer the unobservablecontracts case. We argue also that it is beneficial to the lower-valuation player and her delegate, but harmful to the higher-valuation player and her delegate, to enact or establish policies or regulations or institutions that require both parties to release the information on their contracts in a two-player contest with bilateral delegation.

## 6 Conclusions

We have studied two-player contests with bilateral delegation in which each party's delegation contract is not revealed to (or is hidden from) the rival party when the two delegates choose their effort levels.

In Section 3, we solved the game, treating it as a simultaneous-move game between the two parties. In Section 4, we obtained first the players' contracts, the delegates' effort levels, the players' probabilities of winning, and the expected payoffs of the delegates and the players, in the equilibrium of the game. Then, we examined how these outcomes of the game respond when the valuation parameter $\theta$ changes. In the same section, we showed that, in equilibrium, the higher-valuation player offers her delegate greater contingent compensation than her rival does, the expected payoff of the delegate hired by the higher-valuation player is greater than that of his
counterpart, and economic rent exists for each delegate. We showed also that, as the valuation for the prize of the higher-valuation player increases, ceteris paribus, each player offers her delegate greater contingent compensation and the equilibrium total effort level increases. Finally, we showed that the equilibrium total effort level is always less than the valuation for the prize of the lower-valuation player.

In Section 5, to make comparisons, we first studied two-player contests with bilateral delegation in which each party's contract is observable to the rival party. Then, we compared the outcomes of the unobservable-contracts case with those of the observable-contracts case. Comparing the delegates' equilibrium contingent compensation, we found that the highervaluation player offers her delegate greater contingent compensation in the observable-contracts case than in the unobservable-contracts case, whereas the lower-valuation player offers her delegate greater contingent compensation in the unobservable-contracts case than in the observable-contracts case. We found also that the gap between the delegates' equilibrium contingent compensation is narrower in the unobservable-contracts case than in the observablecontracts case. Comparing the equilibrium total effort levels, we found that the equilibrium total effort level is greater in the unobservable-contracts case than in the observable-contracts case. Finally, comparing the equilibrium expected payoffs, we found that the equilibrium expected payoffs of the higher-valuation player and her delegate are less in the unobservable-contracts case than in the observable-contracts case, whereas, unless $v_{1}$ is very much greater than $v_{2}$, the equilibrium expected payoffs of the lower-valuation player and her delegate are greater in the unobservable-contracts case than in the observable-contracts case. On the basis of this result, we have argued that the higher-valuation player and her delegate prefer the observable-contracts case (to the unobservable-contracts case), while the lower-valuation player and her delegate prefer the unobservable-contracts case. We have argued also that it is beneficial to the lowervaluation player and her delegate, but harmful to the higher-valuation player and her delegate, to enact or establish policies or regulations or institutions that require both parties to release their contracts in a two-player contest with bilateral delegation.

In the models that we have studied in this paper, public or private information is exogenously assumed regarding contracts between the players and the delegates. It would be interesting to study an extended model in which the parties decide first whether they will release the information on their delegation contracts. However, that must involve a sizable analysis of four distinct subgames including two unilateral-release subgames in which one party releases the information on its delegation contract, but the other party does not. Furthermore, it must focus on the equilibrium decisions on releasing the information on the delegation contracts, which is beyond the purpose of this paper. Hence, we leave this extension for future research.

In the models that we have studied in this paper, potential delegates have equal ability for the contest. It would be interesting to study corresponding models in which potential delegates have different ability for the contest, and each potential delegate's reservation wage depends on his ability. In this paper, we have assumed that both players hire their delegates. It would be interesting to study corresponding extended models in which the players each have the option of hiring a delegate. We leave them for future research.

## Footnotes

1. A contest is defined as a situation in which players or contestants compete with one another to win a prize. The literature on the theory of contests - which deals with rent-seeking contests, tournaments, litigation, patent contests, sporting contests, all-pay auctions, etc. - is enormous and growing. Important work in this literature includes Tullock (1980), Rosen (1986), Appelbaum and Katz (1987), Dixit (1987), Hillman and Riley (1989), Ellingsen (1991), Nitzan (1991), Baik and Shogren (1992), Baye, Kovenock, and de Vries (1993), Clark and Riis (1998), Hurley and Shogren (1998), Moldovanu and Sela (2001), Hvide (2002), Che and Gale (2003), Szymanski (2003), Corchón (2007), Epstein and Nitzan (2007), Morgan and Várdy (2007), Congleton, Hillman, and Konrad (2008), Konrad (2009), Siegel (2009), and Gürtler and Münster (2010).
2. We treat the game as a simultaneous-move game between the two parties because, in the game, each party chooses its two sequential moves without observing the other party's moves.
3. In this paper, by the situation of "observable contracts," we mean a situation in which the players announce publicly their delegation contracts and, furthermore, are committed to their delegation contracts.
4. Important work in the literature on delegation that does not study contests includes Schelling (1960), Fershtman and Judd (1987), Sklivas (1987), Fershtman, Judd, and Kalai (1991), and Katz (1991), (2006).
5. Katz (1991) shows that strategic effects may not be present with delegation if contracts are unobservable. Thus one may well say that, in this model, a main motive for delegation is not for the players to achieve strategic commitments through delegation. Delegation may occur here because the players want to use superior ability of their delegates, and/or delegation is compulsory.
6. Specifically, player $i$ designs and offers a contract, which delegate $i$ accepts.
7. Overall, the two parties play a simultaneous-move game, whatever the chronological timing of their decisions may be: Each party chooses its two sequential actions - specifically, a contract and then an effort level - without observing those chosen by the rival party. According to Baik and Lee (2007), the game is classified as a simultaneous-move game with sequential moves. Clearly, it differs from a standard two-stage game in which the first move of each party is observed by the rival party before the second moves of the parties are made.
8. Instead, we could assume that player $i$ pays delegate $i$ compensation of $\alpha_{i} v_{i}$ if she wins the prize, and $\beta_{i} v_{i}$ if she loses it, where $\beta_{i} \geq 0$ and $\beta_{i}<\alpha_{i}<1$ (see Baik, 2007, 2008). In this alternative contract specification, one may consider $\beta_{i} v_{i}$ as fixed compensation (or a fixed fee) that is paid to delegate $i$, regardless of the outcome of the contest, and $\left(\alpha_{i}-\beta_{i}\right) v_{i}$ as contingent compensation (or a contingent fee) which is paid to delegate $i$ only if she wins the prize. Using the alternative contract specification, however, we obtain exactly the same (main) results, because the value of $\beta_{i}$ - subject to the nonnegativity constraint on $\beta_{i}$ - that player $i$ offers and delegate $i$ accepts in equilibrium is zero.
9. This contest success function is extensively used in the literature on the theory of contests. Examples include Tullock (1980), Appelbaum and Katz (1987), Hillman and Riley (1989), Nitzan (1991), Baik and Kim (1997), Hurley and Shogren (1998), Schoonbeek (2002, 2004, 2007), Lim and Shogren (2004), Baik (2004, 2007, 2008), Baik and Lee (2007), and Baik and Lee (2013). One may be tempted to assume that $p_{i}\left(x_{1}, x_{2}\right)=x_{i}^{r} /\left(x_{1}^{r}+x_{2}^{r}\right)$ for $x_{1}+x_{2}>0$, where $0<r \leq 1$. In this case, however, except for the case where $r=1$, it is not computationally tractable to analyze the model.
10. Throughout the paper, when we use $i$ and $j$ at the same time, we mean that $i \neq j$.
11. We find each player's contract and each delegate's effort level that are specified in a sequential equilibrium and also in a perfect Bayesian Nash equilibrium.
12. Each player's decision on her contract is not directly affected by her beliefs about the rival party's contract. This is because the player's expected payoff does not depend directly on the rival party's contract and because the two parties play a simultaneous-move game.
13. Note that player $i$ 's best response to ( $\alpha_{j}, x_{j}$ ) and delegate $i$ 's best response to ( $\alpha_{j}, x_{j}$ ) amount to their best responses to only delegate $j$ 's effort level $x_{j}$, because their expected payoffs do not depend directly on player $j$ 's contract $\alpha_{j}$ and because the two parties play a simultaneousmove game.
14. It is straightforward to see that $\pi_{i}$ in (3) is strictly concave in $x_{i}$, and thus the secondorder condition for maximizing (3) is satisfied. Incidentally, the second-order condition is satisfied for every maximization problem in the paper; for brevity that is not stated explicitly in each case.
15. We use the computer program Mathematica to solve for the unknown $t$.
16. Schoonbeek (2002), Baik (2007, 2008), and Baik and Lee (2013) show that economic rent for the delegate or delegates may exist.
17. On the other hand, Baik (2004) shows in two-player simultaneous-move asymmetric contests without delegation that, as the valuation of the higher-valuation player increases, the equilibrium total effort level increases.
18. The contracts may not have strategic effects if they are not perfectly observed by the delegates (see, for example, Bagwell, 1995).
19. The Nash equilibrium of the second-stage subgame satisfies the delegates' reaction functions simultaneously. Geometrically speaking, it occurs at the intersection of the delegates' reaction functions.
20. We use the computer program Mathematica to solve for the unknown $k$.
21. If $\theta$ is less than approximately 4.0755, then we obtain $x_{1}^{*}<x_{1}^{* *}$ (see Figure 2). If $\theta$ is less than approximately 9.5, then we obtain $G_{2}^{*}>G_{2}^{* *}$ (see Figure 4).
22. In the literature on strategic delegation, there have been debates on whether strategic effects are present with delegation when contracts are unobservable. For an excellent survey of this and other related issues, see Gal-Or (1997). In this paper, player 1 makes her delegate more aggressive or motivated in the observable-contracts case than in the unobservable-contracts case, which may indicate that strategic effects may not be present with delegation when contracts are
unobservable. On the other hand, player 2 makes her delegate less aggressive or motivated in the observable-contracts case than in the unobservable-contracts case, which may indicate that strategic effects may be present with delegation when contracts are unobservable.
23. Using Propositions 2 and 4, we obtain also the fact that the unobservability of contracts narrows, compared with the observable-contracts case, the gap between the delegates' equilibrium expected payoffs and the gap between the players' equilibrium expected payoffs: In terms of the symbols, $\pi_{2}^{* *}<\pi_{2}^{*}<\pi_{1}^{*}<\pi_{1}^{* *}$ and, unless $\theta$ is extremely large, $G_{2}^{* *}<G_{2}^{*}<G_{1}^{*}<G_{1}^{* *}$.
24. Fershtman and Judd (1987) and Sklivas (1987) study duopolies with strategic managerial delegation in which, when choosing their actions, the two managers know the delegation contracts of both firms. They find that, as compared with the no-delegation situation, bilateral delegation increases outputs if the firms engage in quantity competition, and reduces prices if the firms engage in price competition. In the models, outputs are regarded as strategic substitutes, while prices are regarded as strategic complements. In this paper, bilateral delegation reduces total effort both in the unobservable-contracts case and in the observable-contracts case, as compared with the no-delegation situation. However, both players do not regard their effort as strategic complements.
25. In the literature on contests, effort expended in rent-seeking contests is interpreted as social costs.
26. One may say that ignorance is bliss to the lower-valuation player and her delegate.

## Appendix: The Outcomes of the Observable-Contracts Case

LEMMA A1 If $\theta=1$, then we obtain $\alpha_{1}^{* *}=\alpha_{2}^{* *}=1 / 3, x_{1}^{* *}=x_{2}^{* *}=v_{2} / 12, p_{1}\left(x_{1}^{* *}, x_{2}^{* *}\right)=1 / 2$, $\pi_{1}^{* *}=\pi_{2}^{* *}=v_{2} / 12$, and $G_{1}^{* *}=G_{2}^{* *}=v_{2} / 3$.

LEMMA A2 If $\theta>1$, then we obtain (a) $\alpha_{1}^{* *}=1 /(2+\theta k)$ and $\alpha_{2}^{* *}=\theta k /(1+2 \theta k)$, (b) $x_{1}^{* *}=\theta^{3} k^{3} v_{2} /(1+\theta k)^{2}(1+2 \theta k) \quad$ and $\quad x_{2}^{* *}=\theta^{2} k^{2} v_{2} /(1+\theta k)^{2}(1+2 \theta k), \quad$ (c) $\quad p_{1}\left(x_{1}^{* *}\right.$, $\left.x_{2}^{* *}\right)=\theta k /(1+\theta k), \quad$ (d) $\quad \pi_{1}^{* *}=\theta^{2} k v_{2} /(1+\theta k)(2+\theta k)-\theta^{3} k^{3} v_{2} /(1+\theta k)^{2}(1+2 \theta k) \quad$ and $\pi_{2}^{* *}=\theta k v_{2} /(1+\theta k)(1+2 \theta k)-\theta^{2} k^{2} v_{2} /(1+\theta k)^{2}(1+2 \theta k)$, and (e) $G_{1}^{* *}=\theta^{2} k v_{2} /(2+\theta k)$ and $G_{2}^{* *}=v_{2} /(1+2 \theta k)$, where $k$ is the unique positive real root of equation (26).

Note that $k$ in Lemma A2 is a positive real for any value of $\theta$, where $\theta \geq 1$; it is equal to unity for $\theta=1$; and $\theta k$ is monotonically increasing in $\theta$. Note also that $\alpha_{1}^{* *} \leq \alpha_{2}^{* *}<1 / 2$ holds, and $\alpha_{1}^{* *}$ and $\alpha_{2}^{* *}$ do not depend separately on player 1's valuation $v_{1}$ for the prize and player 2's valuation $v_{2}$, but only on the parameter $\theta$, the ratio of the players' valuations (of player 1 to player 2). Lemmas A1 and A2 are also reported, but without providing their full derivations, in Appendix B in Baik and Lee (2013).

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Figure 1. The Equilibrium Contracts in the Two Cases


Figure 2. The Delegates’ Equilibrium Effort Levels in the Two Cases



Figure 3. The Delegates' Equilibrium Expected Payoffs in the Two Cases



Figure 4. The Players' Equilibrium Expected Payoffs in the Two Cases

