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Abstract

In my thesis, I study three important issues in industrial organization.

Chapter 1 studies the impact of vertical integration on innovation in an industry where firms need to undertake risky R&D investments at both production and distribution stages. Vertical integration brings better coordination within the integrated firm, which boosts its investment incentive at both upstream and downstream levels. However, it is only mutually beneficial for firms to integrate when both upstream and downstream innovations are important. When innovation only matters at one level, firms favor instead vertical separation. The analysis provides insights for the wave of mergers and R&D outsourcing observed in the pharmaceutical industry and other vertically related industries.

Chapter 2 studies the effect of quality discrimination on product designs. In the context of Internet, content providers are subject to quality discrimination from the Internet Service Providers. We show that content providers are biased to choose broader designs. This reduces product differentiation in the market, and intervention is necessary to achieve efficiency in the content market. The result brings new insights into the discussion about net neutrality, which mandates equal access to every participant on the Internet.

Chapter 3 studies the role of advertisements in attracting and manipulating attention from consumers. When a product is characterized by several attributes, firms also strategically use advertisements to manipulate the attention of consumers. A monopolist tends to advertise too few attributes, and competition does not necessarily improve the situation. Moreover, in an attention-scarce economy, competition for consumers attention leads firms to advertise fewer attributes and reduces information available to consumers.

Résumé

Dans ma thèse, j'étudie trois questions importantes dans l'économie industrielle.

Chapitre 1 étudie l'impact de l'intégration verticale sur l'innovation dans une industrie où les entreprises doivent entreprendre des R&D investissements risqués à des étapes de production et de distribution. L'intégration verticale permet une meilleure coordination au sein de l'entreprise intégrée, qui renforce son incitation à l'investissement aux niveaux amont et en aval. Cependant, ce n'est que bénéfique pour les entreprises d'intégrer quand innovations à la fois en amont et en aval sont importantes. Quand l'innovation compte qu'à un seul niveau, les entreprises favorisent la séparation verticale. L'analyse donne un aperçu de la vague de fusions et R&D sous-traitance observées dans l'industrie pharmaceutique et d'autres industries verticale.

Chapitre 2 étudie l'effet de la discrimination de la qualité sur la conception des produits. Dans le contexte de l'Internet, les fournisseurs de contenu sont l'objet de discrimination de la qualité des fournisseurs de services Internet. Nous montrons que les fournisseurs de contenu sont biaisées à choisir un design plus larges. Cela réduit la différenciation des produits sur le marché, et l'intervention est nécessaire pour atteindre l'efficacité dans le marché du contenu. Le résultat apporte un nouvel éclairage sur le débat sur la neutralité du net, qui impose l'égalité d'accès à tous les participants sur Internet.

Chapitre 3 étudie le rôle de la publicité pour attirer et manipuler l'attention des consommateurs. Quand un produit est caractérisé par plusieurs attributs, les entreprises utilisent aussi stratégique annonces de manipuler l'attention des consommateurs. Un monopole a tendance à annoncer trop peu d'attributs, et la concurrence n'améliore pas nécessairement la situation. En outre, dans une économie de l'attention-rares, la concurrence pour l'attention des consommateurs conduit les entreprises à annoncer moins d'attributs et réduit l'information à la disposition des consommateurs.

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Chapter 1

Vertical Integration

Innovation is a driving force for most industries, where it moreover affects many stages of the vertical chain. We study the impact of vertical integration on innovation in an industry where firms need to undertake risky R&D investments at both production and distribution stages. Vertical integration brings better coordination within the integrated firm, which boosts its investment incentive at both upstream and downstream levels. However, it is only mutually beneficial for firms to integrate when both upstream and downstream innovations are important. When innovation only matters at one level, firms favor instead vertical separation. The analysis provides insights for the wave of mergers and R&D outsourcing observed in the pharmaceutical industry and other vertically related industries.

1.1 Introduction

In a number of vertically related industries, innovative investment takes place at both upstream and downstream levels. For instance, in the pharmaceutical industry, upstream firms invest into the discovery of new drugs, and downstream firms seek to enhance development and manufacturing. This industry, in which some firms are vertically integrated whereas others are not,[1] has moreover gone through a consolidation process in recent years, where integration with biotech firms plays

[1]Big Pharmaceutical companies are vertically integrated, but many biotech companies and research institutions are only present in the upstream market, and most generic manufacturers are only active in the downstream market.

an important role.[2] At the same time, however, we also witness an outburst of R&D outsourcing,[3] especially in the more traditional segment which relies on chemistry-based technology.

Our purpose in this paper is to study firms' integration decision when innovation matters. We show that it is optimal for firms to integrate vertically only when innovation is important at both upstream and downstream levels. This provides an explanation for the two opposing trends observed in the pharmaceutical industry. Vertical integration is observed in the biotech segment, where biology-based drug development processes requires innovation from both discovery and development, and even from manufacturing; by contrast, outsourcing occurs in the more traditional segment, where chemistry-based processes requires innovation mostly at the discovery stage.[4]

We derive our insights within a simple model where both upstream and downstream markets of a vertically related industry are characterized by a duopoly. Key ingredients are: (i) upstream inputs are homogeneous and downstream firms have unit demand; (ii) at each level, a firm may need to undertake risky investment; and (iii) firms bargain over the terms of supply ex post, if investment has been successful.

Vertical integration changes merging firms' investment incentives in three ways. First, there

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[2]Horizontal mergers between Big Pharmaceutical companies have attracted most of the attention, (For instance, Pfizer has acquired Warner−Lambert, Pharmacia, Wyeth and King Pharmaceuticals since 2000. Other mergers include Merck/Schering-Plough, Teva/Barr, and so forth.) but vertical integration is also an important part of this consolidation process. According to the Global Pharma and Biotech M&A Report 2012 of IMAP, 6 out of the 15 largest transactions in 2011 are R&D driven. The most notable acquisitions include Merck's 5.4 billion euros bid for Millipore and Astellas Pharma's 3.5 billion dollars bid for OSI Pharmaceuticals. More recently, the world third largest pharmaceutical company Roche has attempted to acquire the two leading gene-testing companies Illumina and Life Technologies. Not only does vertical integration happen in developed markets such as North America, Europe and Japan, it is also becoming more and more important in emerging markets such as China. In 2012, China Pharmaceutical Group acquired the research and production capacity of Robust Sun Holdings for 1.2 billion dollars.

[3]For instance, outsourcing of preclinical development in China is projected to increase at an annual rate of 16%, according to the report of JZMed, 2012.

[4]The insights also apply to other industries where innovation plays an important role. For instance, in the satellite navigation industry, Tele Atlas and Navteq are the two main upstream firms for navigable digital map databases. In 2008, Tele Atlas and Navteq were subsequently acquired by two main downstream firms TomTom and Nokia. Also in the smartphone and tablet industry, industry giants Google and Microsoft both integrate to the design stage, rather than remain only as operating system providers.

is a coordination effect. Vertical integration brings better coordination within the integrated firm, by eliminating the hold-up problem. This enhances investment incentives at both upstream and downstream levels. Furthermore, as investments are strategic substitutes, independent firms invest less, which generates additional benefit for the integrated firm. The magnitude of this effect is the greatest when innovation matters both upstream and downstream, whereas it is null when only downstream innovation matters. Second, there is a downstream amplifying effect: the benefit from better coordination in the downstream market augments the benefit from better coordination in the upstream market. This is because for an integrated upstream firm, when it is the only upstream innovator, it captures a larger downstream profit, as integration fosters higher downstream investment. This effect is present only when risky investments take place both upstream and downstream. Third, there is also an insurance effect which lowers the investment incentive of the integrated firm. This effect stems from the risky nature of investment, and reflects the fact that each division of the integrated firm obtains positive profit even if its investment fails, provided that the other division was successful. The overall effect of vertical integration is a boost of both upstream and downstream investments for the integrated firm.

It follows that single integration is always profitable: the joint profit of an upstream firm and a downstream firm is higher when they are integrated than when separated, assuming that the other firms remain separated. The remaining independent firms also have incentive to integrate, however, since a first vertical merger has already taken place. Hence, in a static setting, we would expect pairwise integration, each upstream firm being integrated with a downstream firm. However, the profit of an integrated firm is higher under pairwise integration than under vertical separation only when innovation matters both upstream and downstream. Hence, when innovation matters only upstream, firms fall into a prisoner's dilemma: their joint profit is actually higher under vertical separation. When innovation matters both upstream and downstream, inefficiency stems from under-investment; vertical integration then reduces the inefficiency and benefits all firms. When instead innovation matters only upstream, the inefficiency comes from over-investment; vertical integration thus exacerbates the inefficiency. When firms interact repeatedly, they may thus want and be able to avoid the prisoner's dilemma, in which case firms may remain separated (or outsource upstream production if such outsourcing contract is feasible). This is what happened in the pharmaceutical industry: for traditional drug development processes, the risk of investment concentrates more and more in the discovery stage; firms then favor vertical separation and outsource their R&D activities through contract research organizations(CROs). But for the newer processes based on biology, risk persists from discovery to development, and firms choose to integrate vertically.

Our paper contributes to the literature on vertical integration and foreclosure, which dates back to the seminal paper of Ordover, Saloner and Salop (1990). Most of the literature along this line focuses on downstream foreclosure, which rests on the concern that the integrated upstream firm may restrict its supply to independent downstream firms. In a successive duopoly model, downstream foreclosure could benefit the independent upstream firm: by restricting its supply to the independent downstream firm, the integrated upstream firm grants market power to the independent upstream firm vis-a-vis the independent downstream firm. Our results can be interpreted as the upstream foreclosure effect of vertical integration through the channel of investment. Moreover, when taken upstream investment into consideration, vertical integration instead hurts the independent upstream firm. Most of previous works on the effect of vertical integration on investments tends to focus on investment decisions on one level only, upstream or downstream. Bolton and Whinston (1993), Buehler and Schmutzler (2008) and Allain et al. (2012) focus for instance on downstream investment; whereas Brocas (2003) and Chen and Sappington (2010) study instead upstream investment. Hart and Tirole (1990) consider the case with both upstream and downstream investments, but there the investment is a discrete, riskiness choice. We provide instead a unified model where risky, continuous investment can take place at both levels of a vertically related industry. This allows us to compare the effect of vertical integration under different market configurations, and to understand how the importance of innovation affects industry dynamics. Our paper is also related to the literature on innovation of complementary products. Farrell and Katz (2000) show that integration into a complementary product market allows a monopolist to extract more rent from its core market. Schmidt (2009) studies how vertical integration affects a patent holder's incentive to license its patent to downstream producers. Horizontal complementarity is the main focus of these papers, whereas complementarity is vertical in our paper.

The paper proceeds as follows: We present the basic framework in Section 2. Section 3 studies two benchmark situations, where only upstream innovation or only downstream innovation matters. Section 4 analysis the case when both upstream and downstream innovations are important. We discuss the welfare implications in Section 5. Section 6 provides some extensions and discussion. Section 7 concludes. All proofs are presented in the Appendices.

1.2 The Framework

We consider an industry which consists of an upstream market and a downstream market. There are two upstream firms U_A and U_B, and two downstream firms D_1 and D_2. All firms are risk neutral. Each D_j, for $j = 1, 2$, requires one (non-divisible) unit of input.[5] In order to produce in the market, each firm may need to make a costly investment, the outcome of which is uncertain. We consider the following four-stage game:

- *Stage 1: Upstream Investment.* Upstream firms choose their investments E_i, for $i = A, B$; their outcomes realize and are observed by all firms.

- *Stage 2: Downstream Investment.* Downstream firms choose their investments e_j, for $j = 1, 2$; their outcomes realize and are also observed by all firms.

- *Stage 3: Bargaining.* Successful upstream and downstream firms bargain over supply conditions; inputs are delivered and payments made accordingly.

- *Stage 4: Final Product Market.* Final product market and payoffs to downstream firms realize. Competition in the downstream market determines the profits (gross of payments for input) of D_1 and D_2.

Upstream Technology. We model the upstream investment as follows: by investing $C_U(E_i) = c_u E_i^2/2$, with probability E_i, U_i obtains an innovation that enables it to supply downstream firms. With probability $1 - E_i$, the investment fails and U_i stays out of the market. We assume that there is no marginal cost of production. Thus, the total cost for an upstream firm is the fixed cost of investment. Furthermore, there is no capacity constraint or any other shock that may constrain the production of U_i, and each U_i can supply both downstream firms if it wishes so.

Downstream Technology. Similarly, by investing $C_D(e_j) = c_d e_j^2/2$, each D_j succeeds with probability e_j in becoming able to transform the input into the final product on a one-to-one basis

[5]We make this unit demand assumption for ease of exposition. This is also a natural assumption for the pharmaceutical industry, where downstream prices are heavily regulated, and demand is moreover mainly determined by other factors than prices. However, our main results remain valid with an elastic demand.

at zero cost.[6] In case of failure, which happens with probability $1 - e_j$, D_j is out of the market.

We consider non-channel specific investments, and thus any successful upstream or downstream firm can trade with both firms on the other side. In other words, upstream inputs are perfect substitutes for both downstream firms.[7] This reflects the fact that downstream development methods and resources are not designed for a particular drug, instead they are flexible and can be easily adapted.

Bargaining. To model the bargaining between successful upstream and downstream firms, we adopt a simple procedure which applies to all scenarios with either a monopoly or a duopoly at each level, and with or without vertical integration, namely, with equal probability, either the upstream firms, or the downstream firms, make offers to the other side.[8] More Specifically, the bargaining procedure goes as follows: With probability 1/2, upstream firms make simultaneous offers to downstream firms, which choose whether to accept or reject each offer; With complementary probability 1/2, downstream firms make offers and upstream firms make acceptance or rejection decisions; having observed all offers and acceptance decisions, downstream firms then choose whether to purchase the input and from which upstream firm to purchase.

We do not allow explicit exclusive dealing offers.[9] An offer is therefore simply a price for one unit of input. Note however that, upstream firms can make constructive refusal offers (where the offer to one downstream firm will be rejected), and downstream firms can offer payments that are conditional on actually purchasing the input, both of which may lead to ex post exclusive dealing. All offers and acceptance decisions are publicly observable. For ease of exposition, we assume that whenever a firm is indifferent between accepting and rejecting an offer, it chooses to accept. Moreover, when D_1 is indifferent between purchasing from U_A and U_B, it chooses to purchase from

[6]Zero production cost at both upstream and downstream markets is also a natural assumption for the pharmaceutical industry, where a dominant part of total cost comes from R&D.

[7]The assumption of perfect substitution simplifies our analysis, but is not crucial to our results. In pharmaceutical industries, downstream investments mainly include equipments, clinical tests, human resources and etc, which are not specific to upstream inputs. In other words, a downstream firm can develop any potential drugs discovered by upstream firms as long as it gets access to those drugs. Also in the satellite navigation industry, downstream manufacturers can easily adapt their devices to any upstream map database provider.

[8]That is, whether the offers are made upstream or downstream is channel independent: ex post, the bargaining power is at market level rather than at firm level.

[9]Whether exclusive dealing should be allowed or not is not the focus of this paper. However, we briefly discuss the situation when exclusive offers are allowed in Section 6.

U_A; similarly, D_2 chooses to purchase from U_B in case of indifference.

Final Product Market. Given the unit demand assumption, input prices do not affect downstream firms' gross payoffs in the final product market; these payoffs depend on whether they are competing or not. If D_j is the only active firm in the downstream market, it gets (gross) profit Δ; if both downstream firms are active, each one only gets a profit of δ. Hence, the payoffs for downstream firms are described as Table 1,

Table 1.1: Final Product Market Payoffs

D_1 \ D_2	A	N
A	δ,δ	Δ,0
N	0,Δ	0,0

where "A"and "N"indicate whether a firm is active or not active in the downstream market. We assume that $0 < 2\delta < \Delta$: competition dissipates part of the industry profit, but not all of it. It follows that if U_i is the only upstream innovator, the industry profit is maximized when U_i only sells to one of the two downstream firms.

When conducting welfare analysis, we will interpret the final product market using a simple Hotelling model. The two downstream firms are located at the end points of a segment of length 1. A representative unit demand consumer is randomly located on the line, according to the uniform distribution, and the consumer has valuation v for the product. In addition, the consumer incurs a transportation cost which is t per unit distance. We assume that v is large enough compared to t, i.e. $v > 2t$, to ensure that the market is always fully covered, regardless of the market structure. Hence, when there is only one downstream firm, it charges price $v - t$ and obtains profit $\Delta = v - t$; when there are two downstream firms, they charge the same price $p = t$ and obtain profit $\delta = t/2$.

Finally, to guarantee that profit functions are well-defined and optimal investments are interior solutions, we assume that:

Assumption 1.1. $c_u \geq \Delta$ *and* $c_d \geq \Delta$.

Remarks. We make three remarks about the game. First, we assume that downstream firms invest after observing the outcomes of upstream investments. This simplifies the analysis and allows us to better separate the effect of vertical integration at upstream and downstream level. Moreover, with sequential investments, we avoid socially wasteful downstream investment.[10]

Second, we assume that if the investment failed, the firm is out of the market. That is, innovation is drastic, which is a good approximation for the pharmaceutical industry; yet, our main results hold for non-drastic innovation as well. Similarly, the main insights of our paper still hold when there is a competitive fringe in the upstream and/or the downstream markets, which does not invest but can produce a basic version of the product.

Third, we assume that innovations are protected by patents, and we do not consider issues such as information leakage, reverse engineering, infringement and so forth. These topics are interesting on their own, but we focus here on the effect of vertical integration.

1.3 Two Benchmarks: One-Sided Innovation

To distinguish the main forces at work, we start with two benchmark situations where innovation matters at only one level, either upstream or downstream. When only downstream innovation matters, vertical integration has no effect on downstream investment incentives. This is because upstream competition is always present in this situation, and thus upstream firms supply at cost. When only upstream innovation matters, vertical integration improves the coordination within the integrated firm, which fosters its incentive to invest; as investments are strategic substitutes, the other upstream firm invests less. However, firms may fall into a prisoner's dilemma: each pair of upstream and downstream firms has incentive to integrate, but their joint profit is higher when all firms are separated than when all are integrated.

We proceed backwards. The outcome of the bargaining stage is common in both benchmark situations. Lemma 1.1 summarizes the findings.

Lemma 1.1. *(i) If there is monopoly on one market and a duopoly on the other market, the monopolist obtains Δ and the duopolists obtain zero profit; (ii) if there is bilateral duopoly, upstream firms supply at cost (0), and each downstream firm obtains δ.*[11]

Proof. See Appendix A. □

[10]If both upstream firms failed in investments, there would be no value of downstream investments.

[11]In these two benchmark situations, there is a duopoly in at least one market. The situation of bilateral monopoly only appears when both upstream and downstream innovations matter.

1.3.1 Only Downstream Innovation matters

We begin with the situation when only downstream innovation matters. Clearly, the presence of upstream competition drives the input price down to zero. Therefore, whether or not it is integrated, each downstream firm obtains Δ when it is the sole innovator in downstream market, and δ when instead both downstream firms are successful. Thus the payoffs to downstream firms are as described by Table 1. It follows that D_j chooses its investment e_j so as to maximize, for $j' \neq j \in \{1, 2\}$,

$$e_j e_{j'} \delta + e_j(1 - e_{j'})\Delta - C_D(e_j).$$

As $c_u \geq \Delta$, D_j's best response has slope between -1 and 0, and the unique equilibrium in the investment game is

$$e_1 = e_2 = e_D = \frac{\Delta}{c_d + \Delta - \delta}.$$

This leads to:

Proposition 1.1. ***Downstream Innovation.*** *When only downstream innovation matters, downstream investments are not affected by vertical integration. Firms are indifferent between integration and separation.*

Bolton and Whinston (1993) show instead that an integrated downstream firm invests more than an independent downstream firm, when there is an upstream monopolist and supply uncertainty. Allain et al. (2012) show that vertical integration has a foreclosure effect in case of concerns about information leakage. This neutrality result stems from the assumption that both upstream firms are reliable suppliers of a homogenous input.

1.3.2 Only Upstream Innovation matters

Suppose now that both downstream firms are always active, and only upstream innovation matters.

1.3.2.1 Vertical Separation

When only one upstream firm succeeds, it monopolizes the input market and obtains Δ. When instead both succeed, Bertrand-like competition leads both upstream firms to supply at cost. The payoff matrix for upstream firms is therefore given by:

Table 1.2: Upstream Payoff Under Separation

U_A \ U_B	S	F
S	0,0	Δ,0
F	0,Δ	0,0

where "S"and "F"indicate whether U_i succeeds or fails in investment.

It follows that each U_i chooses an investment level E_i so as to maximize, for $i' \neq i \in \{A, B\}$,

$$E_i(1 - E_{i'})\Delta - C_U(E_i),$$

leading to a best response function $BR_i^{VS}(E_{i'})$, characterized by

$$c_u E_i = (1 - E_{i'})\Delta. \tag{1.1}$$

As $c_u \geq \Delta$, the investment game has a unique equilibrium, given by

$$E_A = E_B = E_U = \frac{\Delta}{c_u + \Delta}.$$

1.3.2.2 Single Vertical Integration

Suppose now that U_A and D_1, say, are vertically integrated whereas U_B and D_2 remain separated. The only difference with the situation of vertical separation is when both upstream firms are successful. As shown in Lemma 1.1, the input price is then driven down to zero no matter who makes the offers; the independent upstream firm thus gets zero profit, but the integrated firm obtains profit δ from its downstream affiliate. The payoff matrix for $U_A - D_1$ and U_B is thus as given by Table 1.3.

Table 1.3: Upstream Payoffs under Integration

$U_A - D_1$ \ U_B	S	F
S	δ,0	Δ,0
F	0,Δ	0,0

Hence U_B's investment E_B^U is still given by the best response (1.1), whereas $U_A - D_1$ chooses E_A^U so as to maximize

$$E_A(1 - E_B^U)\Delta + E_A E_B^U \delta - C_U(E_A),$$

which leads to the best response function $BR_A^{VI}(E_B)$, characterized by

$$c_u E_A^U = (1 - E_B)\Delta + E_B \delta. \tag{1.2}$$

Vertical integration brings better coordination within the integrated firm: the integrated upstream firm U_A now maximizes the joint profit of U_A and D_1, which is positive even if both upstream firms are successful. This improved coordination boosts the investment incentive of the integrated upstream firm. As upstream investments are strategic substitutes, the independent upstream firm invests less. Denote by E_+^U the investment of the integrated upstream firm, and E_-^U that of the independent upstream firm when innovation only matters upstream, we have:

Proposition 1.2. ***Upstream Innovation.*** *When only upstream innovation matters, the integrated upstream firm invests more than the independent upstream firm: $E_+^U > E_U > E_-^U$.*

Proof. See Appendix A. □

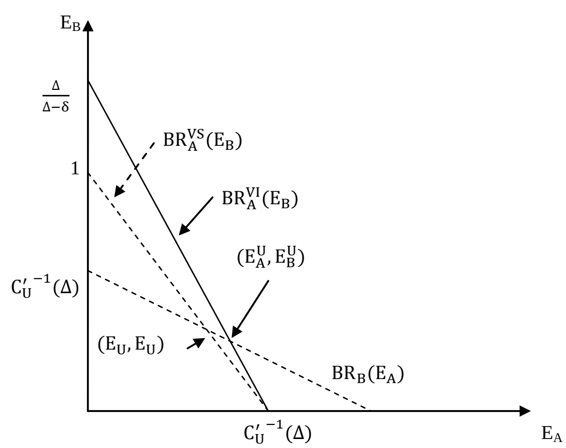

Figure 1.1: Upstream Investment when only Upstream Innovation Matters

The proposition is illustrated by Figure 1.1. The equilibrium investment is determined by the intersection of two best response functions $BR_A(E_B)$ and $BR_B(E_A)$. The best response function for the independent upstream firm U_B is not affected by integration. However, integration of U_A and D_1 leads to a clockwise rotation of the best response function of U_A, which clearly shows that the integrated upstream firm U_A invests more than under vertical separation, whereas the independent upstream firm U_B invests less.

Brocas (2003) obtains similar effect of vertical integration in a bilateral duopoly model where upstream firms invest in process innovation, but her focus is on the effect of switching cost and technology choice. Chen and Sappington (2010) also consider the effect of vertical integration on upstream investment incentives. However, they only consider the case with an upstream monopolist, and their focus is on how the effect of vertical integration depends on downstream competition. Our focus in the paper is not restricted to the effect of integration on upstream investments, but rather on the difference between different market settings.

1.3.2.3 Incentives to Integrate

Proposition 1.2 indicates that it is profitable for U_A and D_1 to integrate. The joint profit of U_A and D_1 is

$$\pi_{U_A-D_1}(E_A, E_B) = E_A(1-E_B)\Delta + E_A E_B \delta - c_u \frac{E_A^2}{2},$$

and it is easy to show that $\pi_{U_A-D_1}(E_+^U, E_-^U) > \pi_{U_A-D_1}(E_U, E_-^U) > \pi_{U_A-D_1}(E_U, E_U)$. The first inequality reflects the gain from better coordination within the integrated firm, which is the term $E_A E_B \delta$, as well as the fact that the integrated firm adapts to U_B's investment E_B^U. And the second inequality holds because U_A now faces a less aggressive competitor in the upstream market.

Similar reasoning implies that it is profitable for U_B and D_2 to integrate as well, in response to the merger of U_A and D_1. The joint profit of U_B and D_2 is

$$\pi_{U_B-D_2}(E_A, E_B) = E_B(1-E_A)\Delta + E_B E_A \delta - c_u \frac{E_B^2}{2},$$

and, letting E_U^{PI} denote the symmetric equilibrium investment under pairwise integration, we have: $\pi_{U_B-D_2}(E_+^U, E_-^U) < \pi_{U_B-D_2}(E_U^{PI}, E_-^U) < \pi_{U_B-D_2}(E_U^{PI}, E_U^{PI})$. The first inequality holds because U_A becomes less aggressive: $E_U^{PI} < E_A^U$, from strategic substitution. The second inequality follows as U_B and D_2 also coordinate better after the integration, and moreover react to the change in E_A.

Therefore, not only is the first integration profitable, the second integration is also profitable. The outcome is such that firms fall into a prisoner's dilemma. Each pair of upstream and downstream firms has incentive to integrate, but the joint profit of each pair is lower under pairwise integration than under vertical separation:

Proposition 1.3. *When only upstream innovation matters, the joint profit of each pair of upstream and downstream firms is higher under vertical separation than under pairwise integration.*

Proof. See Appendix A. □

When innovation matters only at the upstream level, competition leads to over-investment. Integration further boosts investment incentives, which exacerbates the situation. Hence, in a static game, firms are worse off under pairwise integration. Repeated interaction can provide an easy solution to the prisoner's dilemma: in a game where firms play the above investment game repeatedly over time, and firms choose whether to integrate or not at the beginning of each new period, then patient enough firms could sustain a collusive-like market outcome where all firms remain separated.[12]

1.3.3 Comparing the Two Benchmarks

Vertical integration has different effects in the two benchmark situations. When only upstream innovation matters, vertical integration results in a crowding-out effect; when instead only downstream innovation matters, it has no effect. The divergence is here extreme, due to the assumption of homogeneous products upstream. Still, we show in Section 6 that, more generally, vertical integration has a larger impact on upstream investment than on downstream investment. This is because upstream competition is more intense than downstream competition. Simply speaking, upstream firms compete to sell to each of the two downstream firms, whereas downstream firms would not compete to purchase the input from both upstream firms.

This suggests that it is the upstream firms that benefit more from integration, and that incentives to integrate are higher when innovative investments take place in the upstream market rather than downstream. Our result differs from that of de Fontenay and Gans (2005), who show that either upstream firm or downstream firm may benefit more from integration. Their analysis relies on the Shapley value, and thus in case of a monopoly, the industry profit depends on whether the monopolist is upstream or downstream. By contrast, in our setting, the industry profit is Δ no matter where is the monopolist.

1.4 Two-Sided Innovation

When innovation matters at both levels, upstream and downstream investments are complementary, as there is no value for the final product when innovation fails at either level. It follows

[12]As we show in the extension, if we allow firms to contract on upstream innovation, they can do at least as well as under vertical separation, or may do even better.

that vertical integration affects downstream market as well, which in turn reinforces the impact on upstream investments identified in the previous section. As we will see, vertical integration can now moreover be mutually beneficial for all firms.

1.4.1 Downstream Investment

Vertical integration has no impact on downstream investments when both or none of the upstream firms succeed. In the former situation, downstream firms invest as if only downstream innovation mattered, and Proposition 1.1 shows that vertical integration has no impact. In the the latter situation, downstream innovation is worthless, and thus no downstream firm invests. Vertical integration however has an impact when only one upstream firm succeeds, say U_A.

1.4.1.1 Vertical Separation

If U_A is vertically separated, when only one downstream firm succeeds, there is a bilateral monopoly. Hence, the downstream firm shares the profit with U_A and obtains $\Delta/2$. When both downstream firms succeed, U_A monopolizes the market and each downstream firm obtains 0. The payoff matrix for downstream firms is thus given by:

Table 1.4: Downstream Payoffs under Separation

D_1 \ D_2	S	F
S	0,0	$\frac{\Delta}{2}$,0
F	0,$\frac{\Delta}{2}$	0,0

Then D_j chooses an investment level e_j so as to maximize, for $j' \neq j \in \{1, 2\}$,

$$e_j(1 - e_{j'})\frac{\Delta}{2} - c_d\frac{e_j^2}{2}.$$

Hence, each D_j's best response $BR_j^{VS}(e_{j'})$, is characterized by

$$c_d e_j = (1 - e_{j'})\frac{\Delta}{2}. \tag{1.3}$$

Under Assumption 1, there exists a unique equilibrium : $e_1 = e_2 = e^{VS}$, given by

$$c_d e^{VS} = (1 - e^{VS})\frac{\Delta}{2}.$$

It is obvious that, under vertical separation, downstream firms are subject to serious hold-up problem, and thus their investments are insufficient.

Proposition 1.4. *If only one upstream firm succeeds and it is vertically separated, downstream investment is insufficient: it is lower than the industry profit maximizing and the welfare maximizing level of investment.*

Proof. See Appendix A. □

1.4.1.2 Vertical Integration

If U_A is integrated with one of the two downstream firms, say D_1, then when only the independent downstream firm D_2 succeeds, D_2 and $U_A - D_1$ share the market profit and each obtains $\Delta/2$. When instead only D_1 succeeds, the integrated firm $U_A - D_1$ obtains a profit of Δ. Finally, when both downstream firms succeed, U_A monopolizes the market and the integrated firm $U_A - D_1$ obtains Δ. The payoff matrix is thus given by:

Table 1.5: Downstream Payoffs under Integration

$U_A - D_1$ \ D_2	S	F
S	Δ,0	Δ,0
F	$\frac{\Delta}{2}$,$\frac{\Delta}{2}$	0,0

D_2's best response remains BR_2^{VS}, as given by (1.3). By contrast, D_1 now chooses e_1 so as to maximize

$$e_1\Delta + (1 - e_1)e_2\frac{\Delta}{2} - c_d\frac{e_1^2}{2},$$

leading to D_1's best response BR_1^{VI}, characterized by

$$c_d e_1 = \Delta - e_2\frac{\Delta}{2}.$$

Denote by e_+^{VI} and e_-^{VI} the investment of the integrated downstream firm and the independent downstream firm, respectively, and we have the following result:

Proposition 1.5. *If only one upstream firm succeeds and it is vertically integrated, the integrated downstream firm invests more than the independent downstream firm:* $e_+^{VI} > e^{VS} > e_-^{VI}$.

Proof. See Appendix A. □

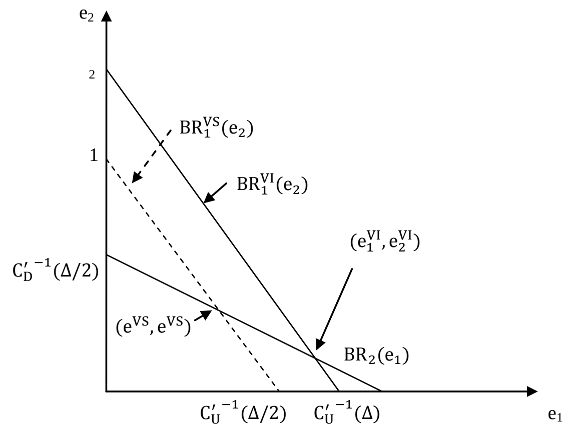

Figure 1.2: Downstream Investment under Upstream Monopoly

Proposition 1.5 is illustrated by Figure 2. Vertical integration does not affect the independent D_2's best response, but now boost that of the integrated D_1, as it eliminates the hold-up problem within the integrated firm. As downstream investments are strategic substitutes, in equilibrium the independent downstream firm invests less. We now show that this effect on downstream investments contributes here to foster upstream investment incentives. Denote by π_{VS} the benefit from its innovation for the only upstream innovator when it is vertically separated, given by

$$\pi_{VS} = 2e^{VS}(1 - e^{VS})\frac{\Delta}{2} + (e^{VS})^2\Delta;$$

and denote by π_{VI} the benefit when it is vertically integrated, given by

$$\pi_{VI} = e_+^{VI}\Delta + e_-^{VI}(1 - e_+^{VI})\frac{\Delta}{2} - c_d\frac{(e_+^{VI})^2}{2}.$$

Lemma 1.2. *If only one upstream firm succeeds, the benefit from its innovation is higher when it is vertically integrated than when it is separated:* $\pi_{VI} > \pi_{VS}$.

Proof. See Appendix A. □

When the upstream monopolist is vertically separated, there is serious under-investment in the downstream market due to hold-up problem. Vertical integration eliminates the problem within the integrated firm, and fosters downstream investment; integration moreover reduces investment of the independent firm, which further increases the benefit of the integrated firm.

1.4.2 Upstream Innovation

Under vertical separation, when only one upstream firm succeeds, the subgame goes as in Section 1.4.1.1, the upstream monopolist gets continuation payoff π_{VS}. When both upstream firms are successful, the subgame goes as Section 1.3.1, where each upstream firm obtains zero profit. Therefore, the payoff matrix for the upstream firms is given by Table 1.6.

Table 1.6: Upstream Payoffs under Separation

U_A \ U_B	S	F
S	0,0	π_{VS},0
F	0,π_{VS}	0,0

Each upstream firm's best response, $BR_i^{VS}(E_{i'})$, is given by

$$c_u E_i = (1 - E_{i'})\pi_{VS}. \tag{1.4}$$

There is a unique equilibrium in the investment game: $E_A = E_B = E^{VS}$, given by

$$E^{VS} = \frac{\pi_{VS}}{c_u + \pi_{VS}}.$$

Suppose now U_A and D_1 integrate. When only the independent upstream firm U_B succeeds, the subgame goes as Section 1.4.1.1, where U_B obtains π_{VS}. In this circumstance, even though U_A does not have a successful upstream innovation, the integrated firm $U_A - D_1$ still gets positive profit from D_1 when it is the only downstream innovator. $U_A - D_1$'s expected profit, π_{VS}^D, which is the profit of a downstream firm when the upstream monopolist is vertically separated, is thus given by

$$\pi_{VS}^D = e^{VS}(1 - e^{VS})\frac{\Delta}{2} - c_d\frac{(e^{VS})^2}{2}.$$

When the integrated upstream firm U_A is the only upstream innovator, the subgame goes as Section 1.4.1.2, and U_A obtains π_{VI}.

When both upstream firms are successful, the subgame goes as Section 1.3.1. However, the profit for the two upstream firms are different: the independent upstream firm U_B obtains zero profit; but the integrated firm obtains positive profit from its downstream affiliate; its expected profit, π_{VI}^D, is given by

$$\pi_{VI}^D = e_D^2\delta + e_D(1 - e_D)\Delta - c_d\frac{e_D^2}{2},$$

where e_D is the downstream investment when only downstream innovation matters.

The payoff matrix for upstream firms is therefore given by

Table 1.7: Upstream Payoffs under Integration

$U_A - D_1$ \ U_B	S	F
S	π_{VI}^D,0	π_{VI},0
F	π_{VS}^D,π_{VS}	0,0

U_B's best response remains given by BR_B^{VS}, characterized by (1.4). The best response of the integrated U_A becomes instead driven by:

$$c_u E_A = (1 - E_B)\pi_{VI} + (\pi_{VI}^D - \pi_{VS}^D)E_B. \tag{1.5}$$

Denote by E_+^{VI} and E_-^{VI} the investment of the integrated upstream firm and the independent upstream firm, respectively, we have:

Proposition 1.6. *The integrated upstream firm invests more than the independent upstream firm:* $E_+^{VI} > E^{VS} > E_-^{VI}$.

Proof. See Appendix A. □

We can rewrite (1.5) as

$$c_u E_A = (1 - E_B)\pi_{VS} + (1 - E_B)(\pi_{VI} - \pi_{VS}) + \pi_{VI}^D E_B - \pi_{VS}^D E_B,$$

and the impact of vertical integration is characterized by the last three terms on the right-hand side: first, the same effect as analyzed in Section 1.3.2 still exists, i.e. $\pi_{VI}^D > 0$, which we refer to as the *Coordination Effect*. The integrated upstream firm takes into account the positive impact of U_A's investment on D_1's profit, which is positive when both upstream firms are successful. This tends to increase the investment of U_A.

Second, the elimination of hold-up problem between U_A and D_1 further increases the investment incentive of the integrated upstream firm, which results from the fact that $\pi_{VI} > \pi_{VS}$. This effect is only present when downstream innovation also matters: integration fosters stronger downstream investment incentive, which increases the benefit from its innovation for an integrated upstream monopolist. We call this second effect as *Downstream Amplifying Effect*.

Third, the combination of upstream innovation and downstream innovation gives rise to an *Insurance Effect* which reduces the investment incentive of the integrated upstream firm. This originates from the fact that the integrated firm obtains positive profit even if it fails in upstream investment, as the downstream affiliate D_1 obtains a profit of $\Delta/2$ when it is the only downstream innovator, that is $\pi^D_{VS} > 0$. This negative effect is dominated, however, and the overall effect of vertical integration is a strengthened crowding-out effect.

1.4.3 Incentives to Integrate

Similar to the arguments made in Section 1.3.2.3, it is profitable for U_A and D_1 to integrate, the benefit of which comes from both the elimination of hold-up problem and lower investments from independent firms. Moreover, given that U_A and D_1 have merged, it is also profitable for U_B and D_2 to integrate. The joint profit of U_B and D_2 under vertical separation is

$$\pi^{VS}_{U_B-D_2}(E_A, E_B) = E_A E_B \pi^D_{VI} + E_B(1-E_A)[\pi_{VS} + \pi^D_{VS}] + E_A(1-E_B)\pi^F_{VI} - c_u \frac{E_B^2}{2},$$

where π^D_{VS} is the profit for a downstream firm when the only upstream innovator is vertically separated; and π^F_{VI} is the profit of the independent downstream firm when the upstream monopolist is vertically integrated, given by

$$\pi^F_{VI} = e^{VI}_-(1-e^{VI}_+)\frac{\Delta}{2} - c_d \frac{(e^{VI}_-)^2}{2}.$$

And the joint profit under pairwise integration is

$$\pi^{PI}_{U_B-D_2}(E_A, E_B) = E_A E_B \pi^D_{VI} + E_B(1-E_A)\pi_{VI} + E_A(1-E_B)\pi^F_{VI} - c_u \frac{E_B^2}{2}.$$

It is easy to check that $\pi^{VS}_{U_B-D_2}(E^{VI}_+, E^{VI}_-) < \pi^{PI}_{U_B-D_2}(E^{VI}_+, E^{VI}_-) < \pi^{PI}_{U_B-D_2}(E^{PI}, E^{PI})$, where E^{PI} is the equilibrium upstream investment when there is pairwise integration. The first inequality reflects the stand-alone gain from the elimination of hold-up problem; and the second inequality shows the gain from being more aggressive, which lowers the investment of U_A. As a result, the integration of U_A and D_1 would be followed by the integration of U_B and D_2. However, the joint profit of U_A and D_1 is still higher than what they would obtain under vertical separation.

Proposition 1.7. *When innovation matters both upstream and downstream, industry profit is higher under pairwise integration than under vertical separation.*

Proof. See Appendix A. □

Therefore, when innovation matters at both upstream and downstream markets, we are likely to observe merger waves, as this leads to the market structure that maximizes industry profit (As each integrated firm gets half of the industry profit, it also maximizes the profit of each integrated firm). This is in contrast to when innovation only matters at one level, where pairwise integration either has no effect (downstream innovation) or hurts both firms (upstream innovation). The reason is intuitive: In all cases, vertical integration eliminates hold-up problems and boosts investments. But when innovation matters only at one level, competition leads to over-investment, which is then further exacerbated by vertical integration. When instead innovation matters at both levels, multiplication of hold-up problems leads to under-investment (Proposition 1.4), which is alleviated by vertical integration.

1.4.4 Industry Overview

The above analysis indicates that in an innovation-driven industry, we are likely to see more integration if both upstream and downstream innovations are important; vertical integration may not be common if investment matters only upstream or downstream, that is, if investment is risky at only one level, firms are better-off with vertical separation.

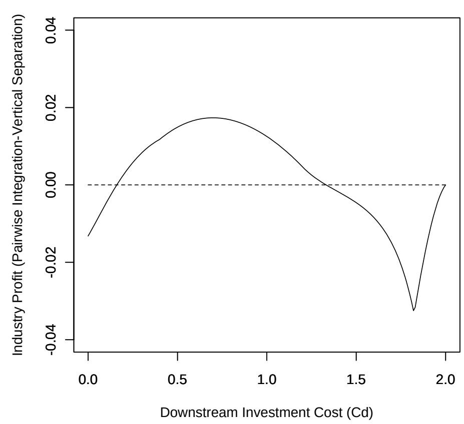

Figure 1.3: Vertical Separation vs Pairwise Integration

We can illustrate this with an example. Downstream product market is characterized by the same Hotelling line as before, with the specification that $v = 2$ and $t = 0.8$. Moreover, suppose that $c_u + c_d = 2$.[13] Figure 1.3 shows the difference between industry profit under pairwise integration and vertical separation, as the cost structure varies. As we can see, as $c_d \to 0$, we are in the situation when only upstream innovation matters, where pairwise integration is strictly dominated. As $c_u \to 0$ ($c_d \to 2$), we approach the situation when only downstream innovation matters, and firms are indifferent between integration and separation. Integration is beneficial and the equilibrium market structure features pairwise integration only when both innovations are important.

From the point view of industry dynamics, at an early stage of the development of an industry, innovation is likely to be important at each stage of the value chain and firms choose to integrate vertically. As the importance of innovation moves more and more to one stage, firms begin to favor vertical separation (or outsourcing when it is feasible). As what happens in the pharmaceutical industry, for the traditional technology, it becomes more and more difficult to make discoveries in the upstream research stage, and firms start to outsource their R&D activities.

1.5 Welfare Implications

We briefly discuss the welfare effect of vertical integration in this section. When only downstream innovation matters, vertical integration does not affect welfare; when only upstream innovation matters, vertical integration improves social welfare if product differentiation in the final product market is strong. Finally, if both upstream and downstream innovations are important, vertical integration generally increases welfare.

1.5.1 One-Sided Innovation

When only downstream innovation matters, vertical integration has no effect on downstream investments, and thus does not affect social welfare. When only upstream innovation matters, consider first single vertical integration, the welfare effect comes from two aspects: on the benefit side, total investment increases; on the cost side, vertical integration may lead to over-investment. This is because the private gain from integration exceeds the social gain. With only one downstream firm, social surplus is $v - t/2$. With two downstream firms, this surplus becomes $v - t/4$.

[13]Thus we do not focus on the situation when optimal investments are interior. Instead, either upstream or downstream investment can be zero or one.

Thus, when one firm has already been successful, the social gain from a second successful firm is $t/4$, whereas the private gain for the second firm is $t/2$.

The welfare function is given by

$$W = (E_A + E_B - E_A E_B)(v - \frac{t}{2}) + E_A E_B \frac{t}{4} - c_u \frac{E_A^2}{2} - c_u \frac{E_B^2}{2}.$$

Thus, if the investment cost is high, welfare is largely determined by total investment, $(E_A + E_B)(v - t/2)$, which is higher under vertical integration. If instead investment cost is low, whether vertical integration increases social welfare critically depends on the extent of downstream product differentiation: when it is weak, the welfare gain from further upstream investment is limited, and the negative effect of over-investment dominates. With stronger product differentiation, however, the positive effect of higher upstream investment dominates and vertical integration increases social welfare. A numerical example in Figure 4 (where we set $v = 1$) confirms this point.

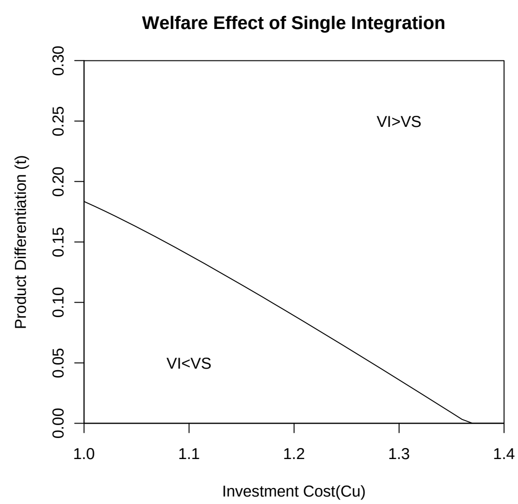

Figure 1.4: Welfare Effect of Single Integration when only Upstream Innovation matters

Now consider pairwise integration, as $c_u > \Delta = v - t$, it is always optimal to have both firms investing, and W is maximized at

$$E_A = E_B = E^* = \frac{v - t/2}{c_u + v - 3t/4},$$

whereas the equilibrium investment levels under separation and pairwise integration are respectively given by:

$$E_U = \frac{v - t}{c_u + v - t} < E^* \text{ and } E_U^{PI} = \frac{v - t}{c_u + v - 3t/2} > E^*.$$

It is easy to check that, if $c_u \geq 3t/2$, we have $E_U + E_U^{PI} \leq 2E^*$, and pairwise integration always increases social welfare compared with vertical separation. This comes from the fact the incentive to over-invest is weaker under pairwise integration than under single integration, and the positive effect of higher total investment dominates.

1.5.2 Two-Sided Innovation

Now consider the situation when innovation matters both upstream and downstream. Denote by W_D the social welfare in the continuation game when both upstream firms succeed; similarly, denote by W_M^{VS} and W_M^{VI} the social welfare in the continuation game when there is only one successful upstream firm, which is vertically separated or vertically integrated, respectively. Total social welfare under vertical separation is then given by

$$W^{VS} = (E^{VS})^2 W_D + 2E^{VS}(1 - E^{VS})W_M^{VS} - 2C_U(E^{VS}),$$

whereas under single vertical integration, it is given by

$$W^{VI} = E_+^{VI} E_-^{VI} W_D + E_+^{VI}(1 - E_-^{VI})W_M^{VI} + E_-^{VI}(1 - E_+^{VI})W_M^{VS} - C_U(E_+^{VI}) - C_U(E_-^{VI}).$$

When investments are substantially lower than the efficient level (which is indeed the case when innovation matters both upstream and downstream), welfare effects are mainly driven by the impact on total investment. Single vertical integration increases social welfare in two ways: First, social welfare is higher when the upstream monopolist is vertically integrated than when it is vertically separated: $W_M^{VS} < W_M^{VI}$. Vertical integration moreover implies that the integrated upstream firm is more likely to be the sole innovator in the upstream market. Second, total investment is higher: $E_+^{VI} + E_-^{VI} > 2E^{VS}$. Indeed, when innovation is important both upstream and downstream, investment incentives are insufficient due to hold-up problems being present at both levels. Vertical integration partially overcomes this problem and pushes investment levels towards social optimum.

This suggests that pairwise integration may further increase social welfare, which becomes

$$W^{PI} = (E^{PI})^2 W_D + 2E^{PI}(1 - E^{PI})W_M^{VI} - 2C_U(E^{PI}).$$

Further welfare improvement from pairwise integration comes from the fact that hold-up problem is now eliminated within each integrated firm, and total investment is even higher: $2E^{PI} >$

$E_{+}^{VI} + E_{-}^{VI}$. The numerical example in Figure 5 (Where we set $v = c_u = c_d = 1$) confirms this welfare effect.

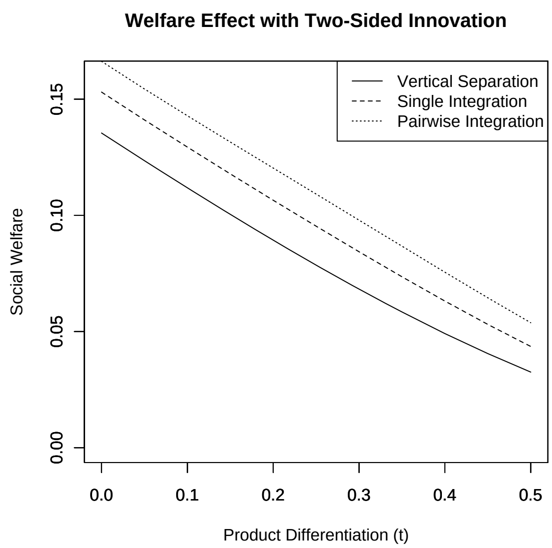

Figure 1.5: Welfare Effect with Two-Sided Innovation

1.6 Discussion and Extension

1.6.1 Robustness

Timing and Observability of Investments. We have assumed that upstream investment takes place before downstream investment. However, the observability and the timing of investments do not affect our main results. This is clearly the case when innovation only matters at one level. When innovation matters both upstream and downstream, the driving force for our results is the better coordination from the elimination of the hold-up problem within the integrated firm. However, by assuming that downstream firms invest after observing the outcomes of upstream investments, we avoid wasteful downstream investments when no upstream firm succeeds. Moreover, sequential investment allows us to better separate the impact of vertical integration on the upstream market from its impact on the downstream market.

Secret Contracting. We have assumed that in the bargaining stage, all offers and acceptance decisions are publicly observable. Secret offers may reduce the profit of an upstream monopolist, due to the classic opportunistic problem. And thus there is additional incentive to integrate vertically if offers are not observable.

Elastic Demand. A simplification in our analysis is that downstream firms have unit demands. When instead downstream firms have an elastic demand, vertical integration generates additional foreclosure for independent downstream firms through a "raising rivals' cost"effect, which has been extensively analyzed in the literature. This foreclosure effect exacerbates crowding-out downstream, which in turn further strengthens crowding-out upstream. Our qualitative results do not change, however, as the resolution of coordination problems within the integrated firm still plays a role in these settings. However, by relying on unit demand and observable offers, we can better focus on the effect of vertical integration through the channel of investment.

Interim Bargaining. In the discussion above, we have assumed that firms bargain after all outcomes of investments have realized and been observed by all firms. The main insights in our paper still hold if bargaining happens in an interim stage, that is, if successful upstream firms bargain with downstream firms before downstream firms make investments. In this situation, there is no hold-up problem for downstream firms, but vertical integration still affects the investment incentives of upstream firms. When both upstream firms are successful, the integrated upstream firm obtains positive profit, whereas the independent upstream firm obtains zero profit. And when only one upstream firm succeeds, this integrated upstream monopolist is able to obtain a higher profit, even if the bargaining outcome does not affect downstream investment incentives. This is because the integrated upstream monopolist holds a stronger bargaining position vis-a-vis the downstream firms, as it benefits from a better outside option.

Exclusive Dealing. Explicit exclusive dealing contracts can have an impact when there is bilateral duopoly. If all firms are separated, then exclusive dealing does not change the payoffs when upstream firms make offers. But when instead downstream firms are the ones making the offers, both firms asking to be supplied at cost (i.e., a zero price) is no longer an equilibrium if exclusive dealing offers are allowed. If D_2 asks for this, D_1 can profitably deviate by making an exclusive dealing offer with a small positive price $\epsilon/2$ to each upstream firm. Both upstream firms are willing to accept the exclusive dealing offer, and D_2 is excluded from the market. D_1 gets profit $\Delta - \epsilon$, which is higher than the profit δ that it would obtain if it also made non exclusive, zero price offers. We show in Appendix B that in this case, downstream firms end up with zero profit.

When U_A and D_1 integrate, the integrated firm can use exclusive dealing offers to exclude the independent upstream firm or the independent downstream firm. As emphasized by Chen and Riordan (2007), the combination of vertical integration and exclusive dealing can lead to ex post cartelization. For instance, when it is downstream firms that make offers, the highest price that D_2 can offer U_B is δ; whereas $U_A - D_1$ can offer U_B an exclusive deal with a price $\delta + \epsilon$; this will be accepted by U_B, and D_2 is then excluded from the market. $U_A - D_1$ gets profit $\Delta - \delta - \epsilon$, which is higher than when exclusive dealing is not allowed (δ). Therefore, when exclusive dealing is allowed, upstream crowding-out effect still exists, and vertical integration still affects downstream investments. Exclusive dealing intensifies downstream competition, however, which contributes to restore symmetry between upstream and downstream markets.

1.6.2 Upstream Differentiation

In our two benchmark situations, the extreme divergence between the effect of integration on upstream and downstream investments stems from the assumption of perfect substitution of inputs. However, the general logic is that upstream competition is more intense than downstream competition, and upstream firms benefit more from vertical integration.

Maintaining the assumption that upstream inputs are not channel specific, consider a simple modification of our basic model to incorporate also upstream differentiation: Both upstream and downstream markets are characterized by a simple Hotelling model, with transportation cost t_u and t_d respectively. The consumer can be two types: he cares only upstream differentiation or only downstream differentiation. All firms observe the type of the consumer, and downstream firms can moreover price discriminate between different types of consumers.[14] When downstream firms purchase from the same supplier, competition for consumers who care only upstream differentiation leads to zero profit, but each firm gets expected profit $\delta_D = t_d/4$ from consumers who care about downstream differentiation. When they instead purchase from different suppliers, each firm gets additional profit $\delta_U = t_u/4$ from consumers who care only upstream differentiation.

Therefore, if downstream firms buy from the same upstream firm, each obtains profit δ_D; if instead they buy from different upstream firms, each gets profit $\delta_D + \delta_U$. Thus, δ_D measures downstream differentiation whereas δ_U instead measures upstream differentiation; as before, we assume that $2(\delta_D + \delta_U) < \Delta$. We prove the following result in the Appendix:

Proposition 1.8. *Vertical integration has a larger impact on investment when innovation matters*

[14] With this modification, all previous analysis remains valid.

only upstream than when it matters only downstream; and vertical integration has the same impact only if there is no differentiation downstream.

Proof. See Appendix A. □

This proposition generalizes the insights of our benchmark case where there is no upstream differentiation, in which vertical integration only affects upstream investments, and not downstream ones. As made clear in the proof, the intuition is that upstream competition is more intense, and thus upstream firms benefit more from integration. Only when all differentiation lies upstream is the effect of vertical integration symmetric on upstream and downstream investments.

1.6.3 Non-Drastic Innovation

We have so far assumed that innovation is "drastic", in that it is necessary to enter a market. When innovation is non-drastic, a failed firm is still active in the market, but with a less effective technology (lower quality and/or higher cost, say).

Consider a slight modification of our framework in which upstream firms invest in order to improve the quality of the input, and downstream firms invest to improve the quality of the final product. The original value of the final product is v as given in our main model. We assume that successful upstream investment can increase the value of the final product by $\alpha\omega$, whereas successful downstream investment further increases this value by $(1-\alpha)\omega$. Notice that if ω is relatively small ($\omega < 3t$ in our Hotelling interpretation), then the innovation is indeed "non-drastic": a downstream firm cannot drive the other one out of the market even if it obtains both upstream and downstream innovation. When $\alpha \to 0$, we are in the situation where only downstream innovation matters; and when $\alpha \to 1$, only upstream innovation matters.

In this case, successful innovation is no longer required to be active in the market. This has two consequences: first, it changes the bargaining procedure as all firms are present no matter how many firms are successful. The presence of an inferior competitor constrains the profit that a successful innovator can extract, but does not qualitatively affect the benefit of better coordination from integration. Second, downstream investment is subject to weaker appropriation by an upstream monopolist. This is because successful investment increases the value of outside option of a downstream firm, which cannot be appropriated by the upstream monopolist. As a result, the hold-up problem is present mostly for upstream firms. This is consistent with our insight that vertical integration has a larger impact at upstream than downstream. It moreover suggests that firms are more likely to find themselves in a prisoner's dilemma situation.

1.6.4 Information Disclosure by Upstream Firms

In our model, a final product requires both upstream and downstream innovation, but downstream innovation does not require any cooperation in the form of information disclosure or actual delivery of upstream innovation. In this subsection, we relax this assumption and assume that downstream firms need information about the upstream innovation in order to make successful investment. We modify the game as follows:

- *Upstream Investment Stage.* Each U_i makes investment E_i; the outcomes of investments realize and are observed; if both firms fail, the game ends;

- *Information Disclosure Stage.* Successful upstream firms decide whether to disclose information about this innovation to any downstream firm; the disclosure decision is observed;

- *Downstream Investment Stage.* Each D_j makes investment e_j if it receives information from an upstream firm; if both firms receive no information or fail, the game ends;

- *Bargaining Stage.* The successful upstream firm(s) and successful downstream firm(s) bargain over the supply condition; Payments are made and inputs are delivered accordingly;

- *Payoff Stage.* Final product market realizes and game ends.

Suppose there is a cost K associated with disclosure. Such cost may be the risk of information leakage, or about how to convey the information correctly to downstream firms. We assume that when an upstream firm is indifferent between disclosing and not disclosing, it chooses to disclose.

When only one upstream firm is successful, the incentive to disclose may differ depending on whether it is vertically integrated or not. Under vertical separation, the profit for the upstream monopolist is π_{VS} if it discloses information to both downstream firms; if it only discloses to one downstream firm, downstream investment is $\Delta/2c_d$, and the profit for the upstream monopolist is $\pi^1_{VS} = \Delta^2/4c_d$. When the upstream monopolist is vertically integrated, the profit is π_{VI} if it also discloses the information to the independent downstream firm. When it does not disclose, downstream investment is given by Δ/c_d and the profit for the integrated firm is $\pi^1_{VI} = \Delta^2/2c_d$. Then an integrated upstream monopolist has less incentive to disclose to both downstream firms.

Proposition 1.9. *There exists a range of value $K \in (\underline{K}, \overline{K})$ such that an monopolist discloses information to both downstream firms when it is independent, but does not disclose to the independent downstream rival when it is vertically integrated.*

Proof. See Appendix A. □

The reason is simple: By disclosing information to both downstream firms, the upstream monopolist may benefit from intensified downstream competition. But an integrated upstream firm has less to gain from such strategy, as competition also deteriorates the profit of its downstream affiliate. This leads to a strengthened downstream amplifying effect, as the difference between the profit of an integrated upstream monopolist and a separated one is even larger for K belonging to the range in the proposition.

When both upstream firms are successful, under vertical separation, neither upstream firm has incentive to disclose: as upstream competition drives input price down to zero, no upstream firm can cover the cost of disclosure if it were to disclose to any downstream firm. Under single integration, no upstream firm discloses to the independent downstream firm,[15] whereas the integrated downstream firm can always get the information from the upstream affiliate (provided that $K \leq \pi^1_{VI}$). Hence, when both upstream firms succeed, the integrated upstream firm generates positive benefit from its innovation, whereas the independent upstream firm gets zero profit. This may further amplify the coordination effect if K is not too large.

1.6.5 Contracting for Innovation–Outsourcing

We have shown that for each pair of upstream and downstream firms, integration is a dominant strategy in the static game. But when only upstream innovation matters, firms face a prisoner's dilemma situation. Hence, when firms interact repeatedly and are patient enough, we can have a collusive-like equilibrium where all firms remain separated.

However, under vertical separation, when there is only one upstream innovator, downstream firms have to compete to be the distributor for the upstream monopolist. Then a downstream firm may want to sign an exclusive contract with an upstream firm, in order to secure the input if the upstream firm succeeds.

[15] The independent upstream firm has no incentive to disclose, as it always gets zero profit. The integrated upstream firm has no incentive to disclose to the independent downstream firm, as this intensifies downstream competition and lowers its profit.

We consider a simple form of such contract between, say, D_1 and U_A: D_1 makes an upfront payment f to U_A, and it commits to pay U_A additionally p_1 if only U_A succeeds, and p_2 if both upstream firms succeed; in return, U_A only supplies to D_1 whenever it succeeds. If D_2 makes a similar offer to U_B, which U_B accepts, D_1's optimal offer is the solution to:

$$
\begin{aligned}
max_{p_1,p_2} \quad & E_A(1-E_B)(\Delta - p_1) + E_A E_B(\delta - p_2) - f, & \\
s.t. \quad & E_A = max_{E_A} E_A(1-E_B)p_1^B + E_A E_B p_2^B - \tfrac{1}{2}c_u E_A^2; & (IC-A) \\
& E_B = max_{E_B} E_B(1-E_A)p_1 + E_A E_B p_2 - \tfrac{1}{2}c_u E_B^2; & (IC-B) \\
& E_A(1-E_B)p_1 + E_A E_B p_2 - \tfrac{1}{2}c_u E_A^2 + f \geq \tfrac{1}{2}c_u(\tfrac{\Delta}{c_u+\Delta})^2; & (IR-A) \\
& E_A(1-E_B)(\Delta - p_1) + E_A E_B(\delta - p_2) - f \geq \delta(\tfrac{\Delta}{c_u+\Delta})^2. & (IR-1)
\end{aligned}
$$

The first two constraints are the Incentive Compatibility constraints which say that upstream firms choose their investments optimally; the last two constraints are the Individual Rationality constraints which require that firms should be better than when they are independent.

Consider a symmetric equilibrium where D_1 and D_2 make the same offer to U_A and U_B respectively. When upfront payment is not allowed, i.e., $f = 0$, for simplicity, consider the case $c_u = \Delta = 1$, the two IC constraints mean that

$$E_A = E_B = \frac{p_1}{1 + p_1 - p_2},$$

and the upstream IR constraint (IR-A) requires that

$$p_1 + p_2 \geq 1.$$

Then downstream firms will make such an offer that $p_1 + p_2 = 1$. Substituting this into the downstream IR constraint (IR-1), it shows that the IR-1 holds with equality. Therefore, with this optimal contract, downstream firms can achieve the same outcome as if they are all vertically separated.

Nonetheless, firms can do better if upfront payment is allowed. In this case, it is easy to check that the profit maximizing investment $E_A = E_B = 1/(3-2\delta)$ can be achieved by setting $f = 1/8 - 1/2(3-2\delta)^2$ and $p_2 + 2(1-\delta)p_1 = 1$. Therefore, depending on the outsourcing contract allowed, firms can do as well as vertical separation or even better when only upstream innovation matters.

1.7 Conclusion

In this paper, we study firms' decision of integration in a vertically related industry where innovation matters. Vertical integration brings better coordination within the integrated firm, and boosts its investment incentive both upstream and downstream. However, it is only mutually beneficial for firms to integrate when innovation is important at both levels, in which case multiplication of hold-up problem leads to under-investment, and integration reduces the inefficiency. When instead innovation only matters at one level, firms prefer to stay vertically separated.

Our analysis provides an explanation for recent developments of the global pharmaceutical industry, where vertical integration and R&D outsourcing happen at the same time. We show that a key determinant of these two trends is the relative importance of innovative investment in the upstream and downstream markets. Vertical integration in the pharmaceutical industry occurs between biotech firms, where innovation matters both at the research and at the development/manufacturing stage; whereas outsourcing takes place for more traditional technologies, where innovation matters mostly at the upstream research stage.

Our insights suggest that when evaluating the impact of vertical integration, especially in industries with intensive innovative investments, the exact nature of the interplay between upstream and downstream investments may be a key point for the decision. Studying the impact of vertical integration in a more general bargaining environment, or in the presence of other forms of complementarity might be interesting avenues for future research. In addition, whereas there has been a number of empirical papers studying the effect of horizontal mergers on innovation, it would also be valuable to study/test empirically the theory predictions developed in this paper.

1.8 Appendix

Most of the propositions in our analysis can be obtained with generic cost function $C(e)$ such that: it is increasing and convex, $C'(0) > 0$, $C'(1) > \Delta$, and $C''(e) > \Delta$ for all $e \in [0, 1]$. However, we provide the proofs with quadratic cost functions, which satisfy Assumption 1.

1.8.1 Appendix A

1.8.1.1 Proof of Lemma 1.1

When only one side innovation matters, at least in one market both firms are active. There are three cases where the market is viable: upstream monopoly with downstream duopoly; upstream duopoly with downstream monopoly; and bilateral duopoly.

Case 1: Upstream Monopoly.

Suppose only one upstream firm succeeds, say U_A. When U_A makes offers to downstream firms, U_A can always guarantee a payoff of Δ. To do so, U_A offers (Δ, p) with $p > \Delta$ to D_1 and D_2 respectively, which will be accepted and only D_1 purchases from U_A.[16]

When D_1 and D_2 make offers to U_A, the only equilibrium is that both propose a price Δ, and U_A accepts one of them. To see this, suppose D_1 proposes a price p_1 and D_2 proposes p_2. First, any prices such that $0 < p_j \leq \delta$ cannot be an equilibrium. In this case, U_A would accept both offers, as it would then be a dominant strategy for both D_j to purchase the input. Then each D_j would deviate to propose a lower price which would still be accepted by U_A. Second, $p_j = 0$ cannot be an equilibrium either. Suppose not, either D_j can propose a price $p'_j = \delta + \epsilon$, which leads U_A to only accept the offer of D_j. This gives D_j a profit of $\Delta - \delta - \epsilon$, which is higher than δ. Third, any offers such that $\delta < p_j < \Delta$ cannot be an equilibrium. In this case, U_A would only accept the offer with higher price or randomize when both downstream firms offer the same price, then at least one downstream firm can profitably deviate by increasing price so as to win. Therefore, the only candidate for equilibrium is that both downstream firms propose Δ. Neither of downstream firms has incentive to increase price, and reducing price leads U_A to accept only the offer of the other firm.

Hence, when there is only one upstream innovator, it obtains the industry profit Δ.

Case 2: Downstream Monopoly.

Suppose only D_1 is successful. Clearly, D_1 obtains Δ when it makes offers. When upstream firms make offers, they both offer zero is the only equilibrium: any positive offer will be undercut by the competitor. Therefore, in this case, the downstream monopolist always obtains Δ.

Case 3: Bilateral Duopoly.

When upstream firms make offers to downstream firms, the only equilibrium is that each U_i

[16] It is a weakly dominant strategy for D_2 not to purchase, then to purchase is a best response of D_1.

offers zero price to both downstream firms, and both downstream firm accept both offers and purchase the input. As each downstream firm only purchases from the firm who offers lower price, any positive offer would be undercut by the other upstream firm, and thus such offers cannot be the equilibrium.

When downstream firms make offers, they will ask for zero price to both upstream firms, and both upstream firms accept both offers. The reason is simple: the only reason that a downstream firm may offer positive price is that both upstream firms choose to only accept the offer from it. However, this cannot be part of the equilibrium. As the downstream firm only purchases from one of them(either from the one with lower price or the preferred one when indifferent), then at least one upstream firm would accept both offers. As there is no way to exclude the other downstream firm from the market, each downstream firm cannot do better by just offering zero prices to upstream firms.

Therefore, when there is bilateral duopoly, upstream firms supply at cost and each downstream firm obtains δ.

1.8.1.2 Proof of Proposition 1.2

Denote by E_+^U and E_-^U the investment of the integrated and the independent upstream firm respectively, when only upstream innovation matters. Rewrite the best response functions as

$$\begin{cases} c_u E_+^U &= \Delta(1 - E_-^U) + \alpha\delta E_-^U, \\ c_u E_-^U &= \Delta(1 - E_+^U). \end{cases}$$

When $\alpha = 0$, the solution corresponds to the investment under vertical separation; when $\alpha = 1$, it is the solution under single vertical integration; and when $\alpha \in (0, 1)$, we have partial integration, where the integrated upstream firm gets a share of the profit of the downstream affiliate. It is clear to see that

$$\frac{\partial E_+^U}{\partial \alpha} = \frac{\delta E_-^U}{c_u},$$

which is always positive. Therefore, we must have $E_+^U > E_U$. Furthermore, we have $\partial E_-^U / \partial E_+^U = -\Delta < 0$, and thus $E_-^U < E_U$. Actually, E_+^U is increasing in α and E_-^U is decreasing, and thus the crowding-out upstream is also increasing in α. The effect of integration on investment is the largest when the integration is full.

1.8.1.3 Proof of Proposition 1.3

The joint profit of each pair $U_i - D_j$ when both upstream firms invest E is given by

$$\pi_{U_i - D_j} = E(1-E)\Delta + E^2\delta - c_u \frac{E^2}{2},$$

which is decreasing if

$$E > \tilde{E} = \frac{\Delta}{c_u + 2(\Delta - \delta)}.$$

As in the above proof of Proposition 1.2, the upstream investment under symmetric market structure (vertical separation or pairwise integration) is determined by

$$c_u E = (1-E)\Delta + \alpha\delta E.$$

$\alpha = 0$ corresponds to the case of vertical separation, whereas $\alpha = 1$ is the case of pairwise integration. It is easy to see that

$$\frac{\partial E}{\partial \alpha} > 0.$$

Therefore, the investment under pairwise integration is higher than under vertical separation. Moreover, when $\alpha = 0$, we have $E_U > \tilde{E}$. Hence, $E_U^{PI} > E_U > \tilde{E}$. Pairwise integration leads to more over-investment, and the profit of the integrated firm is lower than under vertical separation. Moreover, the over-investment problem is increasing in α, and thus the over-investment is more severe under full integration than under partial integration.

1.8.1.4 Proof of Proposition 1.4

The downstream investment is this case is determined by

$$c_d e^{VS} = (1 - e^{VS})\frac{\Delta}{2}.$$

The industry profit is given by

$$\Pi = (e_1 + e_2 - e_1 e_2)\Delta - c_d\frac{e_1^2}{2} - c_d\frac{e_2^2}{2},$$

which is maximized at e^*, given by

$$c_d e^* = (1 - e^*)\Delta.$$

Social welfare is given by

$$W = (e_1 + e_2 - e_1 e_2)(v - \frac{t}{2}) - c_d\frac{e_1^2}{2} - c_d\frac{e_2^2}{2},$$

which is maximized at e^o, given by

$$c_d e^o = (1 - e^o)(v - \frac{t}{2}).$$

Clearly, as $\Delta/2 < \Delta < v - t/2$, we have $e^{VS} < e^* < e^o$. Hence, downstream investment is insufficient compared with either industry profit maximizing or welfare maximizing investment.

1.8.1.5 Proof of Proposition 1.5

Denote by e_+^{VI} and e_-^{VI} the investment of the integrated and the independent downstream firm respectively, and we can rewrite the best response functions as

$$\begin{cases} c_d e_+^{VI} & = (1 - e_-^{VI})\frac{\Delta}{2} + \alpha\frac{\Delta}{2}, \\ c_d e_-^{VI} & = (1 - e_+^{VI})\frac{\Delta}{2}. \end{cases}$$

When $\alpha = 0$, the solution corresponds to the investment under vertical separation, e^{VS}. When $\alpha = 1$, the solution is the optimal investment under single integration. Clearly, we have

$$\frac{\partial e_+^{VI}}{\partial \alpha} = \frac{\Delta}{2c_d},$$

which is always positive. Therefore, we must have $e_+^{VI} > e^{VS}$; moreover, as $\partial e_-^{VI}/\partial e_+^{VI} < 0$, we have $e_-^{VI} < e^{VS}$.

1.8.1.6 Proof of Lemma 1.2

The benefit for the upstream monopolist when it is vertically separated is

$$\pi_{VS} = 2e^{VS}(1 - e^{VS})\frac{\Delta}{2} + (e^{VS})^2\Delta = e^{VS}\Delta;$$

and the benefit for an integrated upstream monopolist is

$$\pi_{VI} = e_+^{VI}\Delta + e_-^{VI}(1 - e_+^{VI})\frac{\Delta}{2} - c_d\frac{(e_+^{VI})^2}{2}.$$

As shown in the proof of Proposition 1.5, if $\alpha = 0$, we have

$$\pi_{VI} = e^{VS}\Delta + e^{VS}(1 - e^{VS})\frac{\Delta}{2} - c_d\frac{(e^{VS})^2}{2},$$

which is higher than π_{VS}, as the last two terms is the profit for an independent downstream firm, which is positive (we denote this profit as π_{VS}^D). Moreover, we have

$$\frac{\partial \pi_{VI}}{\partial \alpha} = -\frac{\Delta}{2}\frac{\partial e_-^{VI}}{\partial \alpha} > 0,$$

as e_-^{VI} is decreasing in α. Therefore, we must have $\pi_{VI} > \pi_{VS}$, and the difference is larger for larger α.

1.8.1.7 Proof of Proposition 1.6

The best response function for the independent upstream firm is unchanged after integration. The best response function for the integrated upstream firm is given by

$$c_u E_+^{VI} = (1 - E_-^{VI})\pi_{VI} + (\pi_{VI}^D - \pi_{VS}^D)E_-^{VI},$$

which can be rewritten as

$$c_u E_+^{VI} = (1 - E_-^{VI})\pi_{VS} + \alpha(1 - E_-^{VI})(\pi_{VI} - \pi_{VS}) + \alpha(\pi_{VI}^D - \pi_{VS}^D)E_-^{VI}$$

When $\alpha = 0$, this corresponds to the case of vertical separation; and when $\alpha = 1$, it is the case of vertical integration. Lemma 1.2 has shown that $\pi_{VI} > \pi_{VS}$; in addition, we can show that $\pi_{VS}^D < \pi_{VI}^D$:

$$\pi_{VS}^D = \frac{1}{2}c_d(\frac{\Delta}{2c_d + \Delta})^2,$$

and

$$\pi_{VI}^D = \frac{1}{2}c_d(\frac{\Delta}{c_d + \Delta - \delta})^2.$$

Clearly we have $\pi_{VS}^D < \pi_{VI}^D$, as $2c_d + \Delta > c_d + \Delta - \delta$. Therefore, we must have $\partial E_+^{VI}/\partial\alpha > 0$; moreover, we have $\partial E_-^{VI}/\partial E_+^{VI} < 0$. And hence, it must be the case that $E_+^{VI} > E^{VS} > E_-^{VI}$.

1.8.1.8 Proof of Proposition 1.7

We show in this proof that the joint profit of $U_A - D_1$ is higher under pairwise integration than under vertical separation. Denote E^{VS} and E^{PI} as the upstream investments under vertical separation and pairwise integration respectively. The joint profit of $U_A - D_1$ under vertical separation is

$$\begin{aligned}
\pi_{U_A-D_1}^{VS}(E^{VS}) = \quad & (E^{VS})^2 \underbrace{[e_D^2\delta + e_D(1-e_D)\Delta - \frac{1}{2}c_d e_D^2]}_{\pi_{VI}^D} \\
& + E^{VS}(1 - E^{VS})\underbrace{[e^{VS}\Delta + e^{VS}(1-e^{VS})\Delta - \frac{1}{2}c_d(e^{VS})^2 - \frac{1}{2}c_d(e^{VS})^2]}_{\pi_{VS}^{ind}} \\
& - \tfrac{1}{2}c_u(E^{VS})^2,
\end{aligned}$$

and the joint profit under pairwise integration is

$$\begin{aligned}\pi^{PI}_{U_A-D_1}(E^{PI}) = \;& (E^{PI})^2 \underbrace{[e_D^2\delta + e_D(1-e_D)\Delta - \frac{1}{2}c_d e_D^2]}_{\pi^D_{VI}} \\ & + E^{PI}(1-E^{PI})\underbrace{[e_+^{VI}\Delta + e_-^{VI}(1-e_+^{VI})\Delta - \frac{1}{2}c_d(e_+^{VI})^2 - \frac{1}{2}c_d(e_-^{VI})^2]}_{\pi^{ind}_{PI}} \\ & -\tfrac{1}{2}c_u(E^{PI})^2.\end{aligned}$$

First, we have $E^{VS} < E^{PI}$, the upstream investments are higher under pairwise integration than separation. Then, it is easy to check that $\pi^{ind}_{VS} < \pi^{ind}_{PI}$, which says that when there is an upstream monopolist, the industry profit is higher when it is vertically integrated than separated. Thus, if the investment cost is relatively high, the difference between vertical separation and pairwise integration is determined by the first order term: $E^{VS}\pi^{ind}_{VS}$ and $E^{PI}\pi^{ind}_{PI}$. Thus the joint profit is higher under pairwise integration.

Actually, the intuition holds for all $c_u, c_d \geq \Delta$. We have

$$E^{VS} = \frac{\pi_{VS}}{c_u + \pi_{VS}},$$

$$E^{PI} = \frac{\pi_{VI}}{c_u + \pi_{VI} + \pi^F_{VI} - \pi^D_{VI}},$$

where π_{VS}, π_{VI}, π^F_{VI} and π^D_V are as given in the context. We prove for the case where $\delta = 0$ and $\Delta = 1$.[17] We can show that for all $c_u, c_d \geq 1$, the joint profit is higher under pairwise integration than separation, i.e. $\pi^{VS}_{U_A-D_1}(E^{VS}) < \pi^{PI}_{U_A-D_1}(E^{PI})$.

1.8.1.9 Proof of Proposition 1.8

In both cases (when only upstream innovation matters or only downstream innovation matters), if both firms fail in investment, the payoffs for all firms are zero. Similarly, if only one firm succeeds, it is able to extract the industry profit Δ.

If both firms are successful, then we have bilateral duopoly. The case when it is the downstream firms who make offers is simple. Each downstream firm asks $p = 0$ to both upstream firms is an equilibrium, and each downstream firm gets profit $\delta_D + \delta_U$. There is no profitable deviation for either downstream firm. This is because there is no equilibrium such that both upstream firms

[17] As a result of continuity, the result still holds for relatively small δ. Moreover, as all investments are proportional to Δ, it is just a normalization to set Δ equal to 1.

only accept the offer from the same downstream firm, (as the downstream firm would only buy from one of them) there is no gain for either downstream firm to propose a positive price.

The situation is more tricky when it is the upstream firms who make offers. To fix idea, we try to find the symmetric non-discriminatory equilibrium, which is defined in the following way: denote p_{ij} as the offer made by upstream firm $U_i, i = A, B$ to downstream firm $D_j, j = 1, 2$. A symmetric non-discriminatory equilibrium is such that $p_{ij} = p$ for $i = A, B$ and $j = 1, 2$.

Notice that each downstream firm can always choose where to buy the input or not to buy at all, it does not give it any advantage to reject any offer from upstream firms. Therefore, all offers must be accepted by downstream firms. Moreover, in any such equilibrium, when downstream firms make purchase decisions, there is no dominant strategy for each downstream firm. This is because, when upstream inputs are also differentiated, downstream firms would try to avoid purchasing from the same upstream firm. We assume that the two downstream firms play the mix-strategy equilibrium where they randomize between purchasing from U_A or U_B. Moreover, we assume that when this mix-strategy equilibrium gives negative expected profit to downstream firms, both downstream firms choose not to purchase.[18]

Then it is obvious that in any symmetric non-discriminatory equilibrium, we have $p \leq \delta_D + \delta_U/2$. Otherwise, in the mixed strategy equilibrium, each D_j purchases from either upstream firm with equal probability, which results in an expected profit of $\delta_D + \delta_U/2 - p < 0$. Then either upstream firm would deviate to a lower price to attract downstream firms. Now we can show that,

Lemma 1.3. *The unique symmetric non-discriminatory equilibrium is $p = \delta_U$ if $\delta_U \leq 2\delta_D$; and $p = \delta_D + \delta_U/2$ if $\delta_U > 2\delta_D$.*[19]

Proof. Given that the price offered by U_A is $p_{A1} = p_{A2} = p_A$, if U_B offers $p_{B1} = p_{B2} = p_B$, the downstream firms play the mix strategy in the purchase decision such that each D_j purchases from U_A with probability $(\delta_U + p_B - p_A)/2\delta_U$, and purchases from U_B with the complementary probability. And then the profit for U_B is

$$\pi_B = \frac{1}{\delta_U} p_B(\delta_U + p_A - p_B),$$

[18] By assuming this, we allow downstream firms to coordinate their decision to certain extent; however, this is not essential for the result. As the expected profit for each downstream firm is zero if not to buy is played with positive probability in the mixed strategy equilibrium, which is the same if both firms choose to stay out of the market.

[19] In the special case with $\delta_U = 0$, we have $p = 0$ just as our benchmark model.

which gives us the best response function of U_B

$$p_B = \frac{\delta_U + p_A}{2}.$$

Similarly, for U_A, we have

$$p_A = \frac{\delta_U + p_B}{2}.$$

Then the unique solution is given by $p_A = p_B = \delta_U$. However, this leaves the expected profit of downstream firms as $\delta_D - \delta_U/2$. Therefore, if $\delta_U \leq 2\delta_D$, this is the unique equilibrium. If $\delta_U > 2\delta_D$, the upstream price is $\delta_D + \delta_U/2$ so as to keep downstream firms in the market.[20] $\square$

Now we can investigate the effect of vertical integration on investment incentives. If $\delta_U \leq 2\delta_D$, when only upstream innovation matters, the payoff matrix is shown Table 1.8.

Table 1.8: Only Upstream Innovation matters

(a) Vertical Separation

U_A \ U_B	S	F
S	$\frac{\delta_U}{2}, \frac{\delta_U}{2}$	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$U_A - D_1$ \ U_B	S	F
S	$\delta_U + \delta_D, \frac{\delta_U}{2}$	Δ,0
F	0,Δ	0,0

In the left panel (a), each upstream firm gets profit δ_U when it is the upstream firms who make offers to downstream firms, which happens with probability 1/2. Under vertical integration, the independent upstream firm can still ask for δ_U from D_2, since now inputs are differentiated. We measure the impact of vertical integration on investment incentives as the relative gain for the integrated firm when both firms are successful. Then this impact in the case of upstream innovation is $\delta_D + \delta_U/2$.

If only downstream innovation matters, the payoff matrix is given by Table 1.9.

[20]This is so because each upstream firm would want to deviate to a higher price, however such deviating leads both downstream firms to leave the market.

Table 1.9: Only Downstream Innovation matters

(a) Vertical Separation

D_1 \ D_2	S	F
S	$\delta_D + \frac{1}{4}\delta_U$,$\delta_D + \frac{1}{4}\delta_U$	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$D_1 - U_A$ \ D_2	S	F
S	$\delta_U + \delta_D$,$\delta_D + \frac{\delta_U}{2}$	Δ,0
F	0,Δ	0,0

In the left panel, the downstream firms get $\delta_U + \delta_D$ when downstream firms make offers; and they get $\delta_D - \delta_U/2$ when upstream firms make offers. And the effect of vertical integration on downstream investment can be measured by $\delta_U/2$. Therefore, vertical integration has larger impact on upstream innovation than on downstream innovation as $\delta_D + \delta_U/2 > \delta_U/2$. In the special case where $\delta_U = 0$ as in our benchmarks, vertical integration only affects upstream investments but not downstream investments.

Similarly, if $\delta_U > 2\delta_D$, when only upstream innovation matters, the payoff matrix for the innovating firms is given by Table 1.10, and the impact of vertical integration on upstream investment is given by $\delta_D + \delta_U/2$.

Table 1.10: Only Upstream Investment

(a) Vertical Separation

U_A \ U_B	S	F
S	$\frac{\delta_D}{2} + \frac{\delta_U}{4}$,$\frac{\delta_D}{2} + \frac{\delta_U}{4}$	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$U_A - D_1$ \ U_B	S	F
S	$\delta_U + \delta_D$,$\frac{\delta_U}{2}$	Δ,0
F	0,Δ	0,0

When only downstream innovation matters, the payoff matrix for the innovating firms is given by Table 1.11, and the impact of vertical integration on downstream investment is given by $\delta_U/2$. Therefore, vertical integration still has larger impact on upstream investment.

1.8.1.10 Proof of Proposition 1.9

It suffices to show that $\pi_{VS} - \pi_{VS}^1 > \pi_{VI} - \pi_{VI}^1$, which is equivalent to show $\pi_{VI}^1 - \pi_{VS}^1 > \pi_{VI} - \pi_{VS}$. We have

$$\pi_{VI}^1 - \pi_{VS}^1 = \frac{\Delta^2}{4c_d},$$

Table 1.11: Only Downstream Investment

(a) Vertical Separation

D_1 \ D_2	S	F
S	$\frac{\delta_D+\delta_U}{2}$,$\frac{\delta_D+\delta_U}{2}$	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$U_A - D_1$ \ D_2	S	F
S	$\delta_U + \delta_D$,$\delta_D + \frac{\delta_U}{2}$	Δ,0
F	0,Δ	0,0

and

$$\pi_{VI} - \pi_{VS} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(4c_d^3 + \frac{3}{2}c_d\Delta^2 - 4c_d^2\Delta).$$

After simplification, we have

$$(\pi_{VI}^1 - \pi_{VS}^1) - (\pi_{VI} - \pi_{VS}) = \frac{\Delta^2}{4c_d(4c_d^2 - \Delta^2)^2}(16c_d^3\Delta + \Delta^4 - 14c_d^2\Delta^2),$$

which is always positive since $c_d > \Delta$.

Let $\underline{K} = \pi_{VI} - \pi_{VI}^1$ and $\bar{K} = min\{\pi_{VS} - \pi_{VS}^1, \pi_{VS}/2\}$. For $K \in (\underline{K}, \bar{K})$, which is non-empty, we have $\pi_{VS} - \pi_{VS}^1 - K > 0$, and $\pi_{VS} - 2K > 0$, and thus the independent upstream monopolist discloses to both downstream firms; but $\pi_{VI} - \pi_{VI}^1 - K < 0$, so the integrated upstream monopolist does not disclose to the independent downstream firm.

1.8.2 Appendix B

1.8.2.1 The Case of Exclusive Dealing

We study the pure strategy equilibrium in the bilateral duopoly case when it is the downstream firms that make offers. Firstly, we show that simple price offers cannot be an equilibrium, and the equilibrium offers must have exclusive dealing clause. Secondly, we show that there is no symmetric equilibrium such that $O_{A1} = O_{B2}$ and $O_{A2} = O_{B1}$, where O_{ij} is the offer made by D_j to U_i. As before, we make the tie-breaking assumption that when U_A is indifferent between the offers of D_1 and D_2, it chooses the offer of D_1; similarly, U_B prefers the offer of D_2 when it is indifferent.

Claim 1: Equilibrium offers must contain exclusive dealing clauses.

We need to show that both downstream firms making simple price offers cannot be an equilibrium. As any non-negative offers will be accepted by upstream firms, both downstream firms get a profit of δ. Hence, we must have $p_{Aj} + p_{Bj} \leq \delta$, where p_{Aj} and p_{Bj} are the offers made by D_j

to U_A and U_B respectively. Then $D_{j'}$ can profitably deviate by make two exclusive dealing offers $(p_{Aj} + \epsilon/2, E)$ and $(p_{Bj} + \epsilon/2, E)$ to the two upstream firms respectively. Then both U_A and U_B will take the exclusive dealing offer, and the profit for $D_{j'}$ is $\Delta - p_{Aj} - p_{Bj} - \epsilon$, which is higher than δ.(As $\Delta > 2\delta$ and $p_{Aj} + p_{Bj} \leq \delta$)

Claim 2: There exists no symmetric equilibrium.

Consider a symmetric equilibrium with both exclusive dealing offers, i.e. D_1 offers $\{(p_1, E)$ and $(p_2, E)\}$ to U_A and U_B respectively. Then we must have $max\{p_1, p_2\} \leq \delta$. To see this: it cannot be the case that both upstream firms accept the offers from the same downstream firm due to our tie-breaking assumption. And thus each downstream firm must get profit δ, which means that the offered price cannot be higher than δ. Hence, given that D_1 makes such offers, D_2 can deviate by offering $\{(p_1 + \epsilon/2, E), (p_2 + \epsilon/2, E)\}$, which will be accepted by both upstream firms and give D_2 a profit of $\Delta - p_1 - p_2 - \epsilon > \delta - p_1$. Therefore, there is no symmetric equilibrium with both exclusive dealing offers. Similar argument also indicates that there is no symmetric equilibrium where each downstream firm makes an exclusive dealing offer to one upstream firm and a simple price offer to the other upstream firm.

Thus there are only asymmetric equilibria, and any such equilibrium must have the following property.

Claim 3: In any equilibrium, one downstream firm is excluded from the market.

If both downstream firms are active in the market, each of them is supplied by one upstream firm and gets profit δ. And thus, it must be the case that all four prices offered by downstream firms are no higher than δ, which in turn means that each downstream firm has incentive to overbid and exclude the other downstream firm as the argument in Claim 2.

Claim 4: In any equilibrium, we must have $p_{Aj} + p_{Bj} = \Delta$ for the active downstream firm D_j; and $min\{p_{Aj}, p_{Bj}\} \geq \delta$.

Suppose D_1 is active and D_2 is excluded from the market. For the first part, if not, D_2 can profitably deviate by offering slightly higher prices to both upstream firms and thus exclude D_1. For the second part, if not, suppose $p_{B1} < \delta$, then D_2 can profitably deviate by offering $p_{B1} + \epsilon$ to U_B.

Therefore, in any pure strategy equilibrium, both downstream firms earn zero profit. And the industry profit is shared between the two upstream firms. However, pure strategy equilibrium may fail to exist. For instance, suppose D_1 is active in the equilibrium, and D_1 offers $\{(p, E), (\Delta - p, E)\}$

to U_A and U_B, where $\delta \leq p \leq \Delta - \delta$. Then the best response of D_2 is to offer $\{(p_1 \leq p, \cdot), (p_2 < \Delta - p, \cdot)\}$. However, given that D_2 offers $p_2 < \Delta - p$ to U_B, the best response for D_1 is not to offer $\Delta - p$ to U_B but rather to offer $p_2 + \epsilon$ to U_B. To circumvent such circumstances, we can restrict the price quote to be discrete numbers with equal distance ϵ, i.e. prices can only be $\Delta, \Delta - \epsilon, \Delta - 2\epsilon, \ldots$. Then D_1 offers $\{(p, E), (\Delta - p, E)\}$ to U_A and U_B, and D_2 offers $\{(p, \cdot), (\Delta - p - \epsilon, \cdot)\}$ is a pure strategy equilibrium.

Chapter 2

Net Neutrality

Quality discrimination plays an important role in a number of markets. In the context of Internet, content providers are subject to quality discrimination from the Internet Service Providers. We study the effect of quality discrimination on product designs in this paper. We show that content providers are biased to choose broader designs. This reduces product differentiation in the market, and intervention is necessary to achieve efficiency in the content market. The result brings new insights into the discussion about net neutrality, which mandates equal access to every participant on the Internet.

2.1 Introduction

The Internet had been working under the non-discrimination regulation, which requires all contents, sites, and platforms being treated equally. This remained to be the situation until 2005, when the Federal Communication Commission (FCC) changed the classification of Internet transmission from “telecommunication services”to “information services”, which allows the Internet Service Providers to “create different tiers of online service. They (ISP) would be able to sell access to the express lane to deep-pocketed corporations and relegate everyone else to the digital equivalent of a winding dirt road. Worse still, these gatekeepers would determine who gets premium treatment and who doesn’t”[1].

[1] Lawrence Lessig and Robert W. McChesney (8 June 2006). “No Tolls on The Internet”, The Washington Post.

This policy change, backed up and put into practice[2] by large Internet Service Providers, has encountered strong opponent opinions from large content providers such as Google, who has tried to maintain the non-discrimination regulation regime, i.e. a "neutral"network.[3]

We study the economic consequence of such policy change on the content market. Specifically, we analyze the impact on how content providers (hereafter CP) design their products if Internet Service Providers (hereafter ISP) can differentiate the quality of access to the network among different CPs. We show that without net neutrality regulation, if the ISP lacks the ability to commit to how each CP would be treated in the network, the CPs are biased to design their products as broader ones.

To show this, we study the interaction between one ISP and two CPs. The CPs can only reach consumers through the network of the ISP. A consumer values both the match between a content and his taste, and the quality of connection when he consumers the content. That is to say, a consumer's utility from consuming a content depends on both the design of the content and how the ISP treats this CP. Building on the concept of "demand rotation"introduced by Johnson and Myatt (2006), we assume that each CP chooses a design, ranging from the broadest one which is acceptable for a general public, to the most niche one which is tailored to specific consumer taste. And the ISP chooses a network, which can be either neutral or discriminatory. In a neutral network, both CPs are treated equally; whereas in a discriminatory network, the ISP favors one CP and discriminates against the other one.

If the ISP commits to ex ante to a given network priority rule, the the CPs have an incentive to choose the efficient design. When instead the ISP cannot commit ex ante to how it will treat the CPs, then it favors the CP with the broader product. The reason is simple: Broader products attract more consumers, and favoring these products thus increases the demand more than it decreases the demand for niche products. Anticipating this, each CP tends to choose a broader design than its rival so as to avoid being discriminated against. Hence, when the most niche design is efficient, there is prisoner's dilemma: the CPs would be better off with niche products, but each CP has an incentive to design a broader product so as to be favored by the ISP.

[2] "Comcast Blocks Some Internet Traffic", NBC news, 19/10/2007.

[3] "The broadband carriers should not be permitted to use their market power to discriminate against competing applications or content. Just as telephone companies are not permitted to tell consumers who they can call or what they can say, broadband carriers should not be allowed to use their market power to control activity online."–Guide to Net Neutrality for Google Users.

Therefore, without net neutrality, lack of commitment from the ISP can lead to inefficient design choices, which can significantly reduce production differentiation in the content market. Enforcing net neutrality endows the ISP the commitment power that it lacks, which can eliminate such distortion of designs in the content market.

The analysis brings new insight into the economic debate[4] on net neutrality, which has so far focused on issues related to pricing and investment incentives.[5] For instance, Economides and Tag (2012) and Hermalin and Katz (2007) analyzed the pricing of the ISP in a two-sided market framework; Choi and Kim (2010), Kramer and Wiewiorra (2012), and Bourreau et al (2012) studied the investment incentives of ISPs and CPs in similar frameworks.

The insight that discrimination in quality can have large impact on product designs applies to other markets as well, especially to those where the quality of access to consumers is important. For instance, the shelves in a supermarket differ in their locations and easiness to find, and the owner is likely to locate broader products on those easy-to-find shelves. Thus if the owner lacks the ability to commit, the manufacturers may be biased to choose broader products.

The paper is organized as follows: in section 2, we present the model and main results, where a linear example is also shown; we briefly discuss some extensions and robustness of our results in section 3; and section 4 concludes. All omitted proofs are given in the appendix.

2.2 The Network/Design Model

2.2.1 The Model

Consider a market with a representative consumer/user(U), a monopolistic Internet Service Provider(ISP), and two Content Providers(CP_1 and CP_2). The CPs distribute their products to the user through the network of the ISP.[6]

Consumer The representative consumer values both the content and the quality of connection in an additive way, and only consumes the product with a higher overall valuation.[7] His utility

[4] Schuett(2010) and Kramer et al (2013) provide thorough surveys of the economic literatures on net neutrality.

[5] The ISPs invest in the network capacity, and the CPs invest in the qualities of their contents.

[6] We consider a situation where the network serves as a "bottleneck" for the market, so both the consumer and the CPs cannot bypass the ISP.

[7] In other words, the two CPs compete for the consumer.

function is given by

$$U = max\{v_1 + q_1, v_2 + q_2\} - p,$$

where v_i is the match value between the consumer and the product of CP_i, q_i is the connection quality of CP_i, and p is the price charged by the ISP.

The match value v_i is randomly drawn from a distribution $F(v_i, s_i)$, where s_i is the design chosen by CP_i.[8] We assume that given designs s_1 and s_2, the realizations of v_1 and v_2 are independent. The connection qualities are determined by how the ISP arranges the network. In addition, we normalize the reservation utility of the consumer to zero.

Internet Service Provider The ISP charges a price p for the consumer.[9] More importantly, the ISP can choose how to treat the CPs: The ISP can choose a connection quality $q_1 = \delta \in [-\Delta, \Delta]$ for CP_1, and the connection quality of CP_2 is $q_2 = -\delta$.[10] When $\delta = 0$, the ISP chooses a neutral network where both CPs are treated equally; when $\delta < 0$, the ISP favors CP_2; and when $\delta > 0$, the ISP instead favors CP_1.[11]

Content Providers Each CP chooses a design $s \in S = [B, N]$ which ranges from the broadest (B) to the most niche (N). Each design s induces the consumer's match value v to be distributed according to $F(v; s)$, with density $f(v; s)$ which is positive everywhere in the support $[\underline{v}_s, \overline{v}_s]$. We follow Johnson and Myatt(2006) by assuming that different designs induce demand rotation:

Definition 2.1. ***(Johnson and Myatt 2006):*** *A local change of s leads to a rotation of $F(v; s)$*

[8] In the basic model, we consider the case where the match value is the only valuation that the user derives from the content. The content of CP_i may also have an intrinsic value θ_i that is independent from the match value; in this sense we consider the case of homogeneous CPs where $\theta_1 = \theta_2$ in the basic model.

[9] We do not consider the case when the ISP also charge CPs for connection. Since the main focus in this paper is how the possibility of quality discrimination affects the design on the content market, but not how the ISP sets prices for the user and CPs. For more references on the price setting of the ISP, see the literatures on two-sided market.

In a discriminatory network, the ISP may auction out the priority or charge the favored CP in other manners, we consider this possibility in the next section.

[10] That is, the connection qualities of the two CPs always sum up to zero. Increasing the connection quality of one CP necessarily decreases that of the other one. For instance, giving priority to CP_1 means CP_2 has to wait.

[11] The consumer may value connection quality in a multiplicative way. For instance, $U = max\{v_1 q_1, v_2 q_2\} - p$, and then we can alternatively assume that if the connection quality of CP_1 is $1 + \delta$, then the connection quality of CP_2 is $1 - \delta$, for $\delta \in [-\Delta, \Delta]$. However, this does not change our qualitative result.

if for some v_s^* *and each* $v \in (\underline{v}_s, \overline{v}_s)$

$$v > v_s^* \Leftrightarrow \frac{\partial F(v;s)}{\partial s} < 0 \text{ and } v < v_s^* \Leftrightarrow \frac{\partial F(v;s)}{\partial s} > 0.$$

If this holds for all s*, then* $\{F(v;s)\}$ *is ordered by a sequence of rotations.*

The concept of demand rotation formalizes the idea that some designs induce a wider spread of consumer valuations than others; for instance, some designs cater to a particular group of consumers whereas others cater to a more general public. An increase in s leads to a clockwise rotation of the distribution function around the rotation point v_s^*, so there are more consumers with high valuation, but also more consumers with low valuation. A higher s means a more niche product, and the bounds on s correspond to the broadest and the most niche product.

We focus on a class of rotation-ordered functions such that v_s^* is decreasing in s. As shown in Johnson and Myatt(2006), with such rotation-ordered functions, the monopoly profit is quasi-convex in s, and thus maximized at extreme designs.

Assumption 2.1. $\{F(v;s)\}$ *is rotation-ordered in* s *and the rotation point* v_s^* *is decreasing in* s.

Demands and Profits For given designs s_1 and s_2, network choice δ, and price p, the demand for CP_1 is given by

$$D_1(s_1, s_2; \delta; p) = \int_{p-\delta}^{\infty} F(v + 2\delta; s_2) dF(v; s_1). \tag{2.1}$$

Similarly, the demand for CP_2 is given by

$$D_2(s_1, s_2; \delta; p) = \int_{p+\delta}^{\infty} F(v - 2\delta; s_1) dF(v; s_2). \tag{2.2}$$

The demand for the ISP service equals the sum of the demands for each CP, which is

$$\begin{aligned} D(s_1, s_2; \delta; p) &= D_1 + D_2 \\ &= Prob(max\{v_1 + \delta, v_2 - \delta\} \geq p) \\ &= 1 - F(p - \delta; s_1) F(p + \delta; s_2); \end{aligned} \tag{2.3}$$

Hence the profit of the ISP is

$$\pi_0^\delta(s_1, s_2, p) = p \cdot D(s_1, s_2; \delta; p).$$

We assume that each CP can costlessly choose any design $s \in S$, and the marginal cost of production is zero. Moreover, the CPs do not directly charge the consumer; instead, each CP_i

obtains a profit α from one unit of demand for its product through activities such as advertising.[12] Therefore, the profit of each CP_i is

$$\pi_i^\delta(s_1, s_2, p) = \alpha \cdot D_i(s_1, s_2; \delta; p).$$

2.2.2 Benchmark: Industry Profit Maximization and Commitment

We start with the situation where the ISP can commit to a network characteristic, where we show that CPs are induced to choose the industry profit maximizing designs.

2.2.2.1 Industry Profit Maximization

The industry profit is given by

$$\begin{aligned}\Pi^\delta(s_1, s_2; p) &= \pi_0^\delta(s_1, s_2; p) + \pi_1^\delta(s_1, s_2; p) + \pi_2^\delta(s_1, s_2; p)\\ &= (p + \alpha)D(s_1, s_2; \delta; p).\end{aligned}$$

It is easy to show that the industry profit is quasi-convex in the designs and we focus on the situation where both the ISP and CPs are better-off with the most niche products than the broadest products. Specifically, denote $D^\delta(s, s')$ as the demand for CP_1 in a network δ, when CP_1's design is s and CP_2's design is s', and we assume that

Assumption 2.2. *$\pi_0^\delta(N, N) > \pi_0^\delta(B, N) > \pi_0^\delta(B, B)$, and $D^\delta(N, s) > D^\delta(B, s)$ for $\delta \in [-\Delta, \Delta]$ and $s \in S$.*[13]

Then it is straightforward to show that:

Proposition 2.1. *Under Assumptions 1,2, the industry profit is maximized when both CPs choose the most niche design.*

Proof. See Appendix 2.5.1. □

[12] The result in this paper is not sensitive to this assumption. We can readily make other assumptions on how CPs generate profits. For instance, the ISP and each CP may share the profit that is generated from the demand for the product of the CP, which is the situation in the pay-TV market.

And we briefly discuss the more general case where CPs differentiate in their profitability in section 3.

[13] The price is set at the profit-maximizing level by the ISP, given the network and designs chosen by CPs.

2.2.2.2 Commitment

When the ISP commits to a network characteristic δ, the game goes as follows:

- Stage 0: The ISP announces a network choice $\delta \in [-\Delta, \Delta]$;
- Stage 1: The two CPs choose their designs, s_1 and s_2, simultaneously;
- Stage 2: The ISP sets the price p;
- Stage 3: The consumer observes both match values v_1, v_2, network characteristic δ and price p, then decides whether to connect to the service and which content to consume.

Notice that in the commitment case, the ISP only commits to a network characteristic δ, but not to a price p. Hence, the optimal price depends on the designs chosen by the CPs, which we denote as $\hat{p}(s_1, s_2)$. And we make the following assumption to facilitate our analysis:

Assumption 2.3. $|\frac{\partial D_i}{\partial s_i}| > |\frac{\partial D_j}{\partial s_i}|$,*for* $j \neq i$; *and* $|\frac{\partial D_i}{\partial s_i}| > |\frac{\partial D_i}{\partial p}\frac{\partial \hat{p}}{\partial s_i}|$.[14]

The first part of Assumption 2.3 says that for each CP, keeping the price fixed, changing its product design has a larger impact on its own demand than on the demand of the other CP.[15] The second part of the Assumption says that the indirect effect on demand resulting from the induced change in price is dominated by the direct effect following a change in design. Now we can show the following result:

Proposition 2.2. *Under Assumptions 2.2 and 2.3, if $F(v; s)$ is log-concave in v, when the ISP can commit to a network characteristic, it commits to $\delta \in \{-\Delta, \Delta\}$, and both CPs choose the most niche design.*

Proof. See Appendix 2.5.3 □

So, the ISP always commit to a maximally discriminatory network, and the CPs choose the "efficient"designs that maximize industry profit. The reason (which is shown in detail in the proof) is as follows: Given the network characteristic, under Assumption 2.3, each CP's profit is

[14] A detailed assumption on the demand function is given in the appendix.

[15] Similar assumptions are widely made in industry organization literatures. For example, in Bertrand competition with differentiated goods, we usually assume that the demand for a seller is more sensitive to his own price than to the rival's price.

quasi-convex in its own design and thus maximized at either B or N; and Assumption 2.2 implies that each CP prefers to choose N. Moreover, given that both CPs choose the same design, introducing discrimination increases the demand for the favored CP and decreases the demand for the discriminated CP. The log-concavity assumption ensures that the first effect dominates and the overall demand increases. Therefore, the ISP prefers a discriminatory network. And we assume that in this situation the ISP randomly favors one of the CPs.

2.2.3 No Commitment

Now we turn to the situation when the ISP cannot commit, the CPs have to choose their designs before the ISP makes any arrangement for the network. The game proceeds as follows:

- Stage 1: The two CPs choose their designs, s_1 and s_2, simultaneously;
- Stage 2: The ISP sets the price p and chooses whether to discriminate, i.e. chooses a network characteristic δ;
- Stage 3: The consumer observes both match values v_1, v_2, network characteristic δ, and price p, then decides whether to connect to the service and which content to consume.

We solve the game backwards.

At Stage 3, the consumer's behavior is given by the demand function specified above: he only consumes the content with higher overall value if it is greater than the price. We study the ISP and CP's behavior in the following.

2.2.3.1 Stage 2: ISP's Behavior

For the ISP, two decisions have to be made: what price to charge for the consumer and whether to discriminate any of the CPs. The above Proposition 2.2 has shown that when the two CPs choose the same design s, the ISP will choose a network with maximum discrimination.

The logic also extends to the situation when the two CPs chooses different designs, say CP_1 chooses a product which is broader than that of CP_2, i.e. $s_1 < s_2$, then we have the following result:

Proposition 2.3. *When the designs of the two CPs are different, if $F(v; s)$ is log-concave in v and s, the ISP will maximally favor the CP with broader design.*

Proof. See Appendix 2.5.4. □

The reason why the ISP always favors the broader product is simple. For a given price, choosing a discriminatory network shifts down the distribution function of match value for the favored CP, and shifts up that for the discriminated one. The assumption on $F(v;s)$ ensures that the downward-shift is larger when the broader product is favored and thus demand increases more. Simply speaking, improving the quality of a product that suits more consumers leads to a larger increase in demand than improving the quality of a product which is popular only among a small group of consumers.

Then the problem for the ISP is simply to choose a price to maximize profit under a network with maximum discrimination which favors the broader product. That is to say, for any $s_1 \leq s_2$,[16]

$$ISP\text{'s Problem: } max_p \pi_0^{\Delta}(s_1, s_2; p) = p \cdot (1 - F(p - \Delta; s_1)F(p + \Delta; s_2)).$$

2.2.3.2 Stage 1: CPs' Behavior

When the ISP lacks the ability to commit, the CPs have to make their decisions anticipating how they would be treated ex post. Denote $D(B, B)$ as the demand for each CP when they both choose the broadest design.[17] The following proposition shows that both CPs choosing the most niche design is no longer an equilibrium, and they are biased to choose broader designs.

Proposition 2.4. *Under Assumption 2.2 and 2.3, and $F(v;s)$ is log-concave in both s and v, when the ISP cannot commit to a network, (B, B) is the only equilibrium if $D(B, B) > D^{-\Delta}(N, B)$; and there is no pure strategy equilibrium if $D(B, B) < D^{-\Delta}(N, B)$.*

Proof. See Appendix 2.5.5. □

The intuition for this results is simple: when the ISP cannot commit to a network, under the assumption that $F(v;s)$ is log-concave in s and v, it is equivalent to say that the ISP commits to favor the broader product ex post, and therefore each CP is biased to choose broader designs in order to gain competitive advantage.

When the most niche design is the "efficient"choice for the CP, without uncertainty about the network, each CP can maximize his own profit without considering the design of the other CP.

[16] This is the case subject to a relabeling of the two content providers.

[17] We have $D(B, B) = \frac{D^1(B,B)+D^2(B,B)}{2}$.

When such uncertainty prevails, on one hand, each CP intends to choose a more niche design so as to increase his profit; on the other hand, each CP is inclined to choose a broader design so as not to be discriminated against ex post. The lack of commitment from the ISP creates such strategic concerns, which biases the CPs' choices to broader designs.

Remark When pure-strategy equilibrium does not exist, there exist mixed strategy equilibrium, where each CP randomizes among different designs. Nevertheless, more broader products would emerge in such equilibrium and the result of broad-biased design still holds.

2.2.4 An Example

We consider a linear example with the characteristic that the rotation point is fixed, which enables us to show explicitly how lack of commitment from the ISP could distort the product designs in the content market.

Take the symmetric case $\theta_1 = \theta_2 = \theta$; and the match value v_s for a design s follows a uniform distribution on the interval $[-s, s]$. Thus, for each design s, the user valuation follows a uniform distribution on $[\theta - s, \theta + s]$, and the rotation point is θ for any design s. We assume that the design $s \in S = [\underline{s}, \overline{s}]$, where $s = \underline{s}$ is the most broad design and the user valuation is more concentrated around θ; and $s = \overline{s}$ is the most niche design, the user may get very high or very low value.

In this linear case, we have $F(v; s) = \frac{v+s-\theta}{2s}$ and $f(v; s) = \frac{1}{2s}$, and thus

$$\frac{f}{F}(s, v) = \frac{1}{v + s - \theta},$$

which is indeed decreasing in both s and v. Therefore, if $s_1 < s_2$, and the corresponding optimal price is such that $f(v; s_1)$ is positive, the ISP prefers to give better connection to CP_1; explicitly, at such price, the demand in a network favoring CP_1 is

$$D^1 = 1 - \frac{p + s_1 - \delta - \theta}{2s_1} \cdot \frac{p + s_2 + \delta - \theta}{2s_2}.$$

The demand in a neutral network and a network favoring CP_2 is

$$D^N = 1 - \frac{p + s_1 - \theta}{2s_1} \cdot \frac{p + s_2 - \theta}{2s_2} \text{ and } D^2 = 1 - \frac{p + s_1 + \delta - \theta}{2s_1} \cdot \frac{p + s_2 - \delta - \theta}{2s_2}.$$

Then it is easy to see that $D^1 > D^N$ and $D^1 > D^2$.

Therefore, if the two CPs choose the same design s, the ISP would choose a discriminatory network which randomly favors either one of the two CPs. The profit is then

$$\Pi = p(1 - \frac{p + s - \theta - \delta}{2s} \frac{p + s - \theta + \delta}{2s}),$$

then the optimal price is

$$p^* = \frac{-(2s-2\theta) + \sqrt{(2s-2\theta)^2 + 12s^2 - 3(s-\theta-\delta)(s-\theta+\delta)}}{3}.$$

Thus for a given δ, we have

$$p^* > \theta + \delta \text{ if } s > \hat{s},$$

where $\hat{s} = \frac{2}{3}\theta$ if $\delta = 0$; and $\hat{s}$ increases with δ. We focus on the case where $\underline{s} \geq \hat{s}$, so that when s_1 and s_2 are greater than $\hat{s}$, the optimal price is above the higher rotation point $\theta + \delta$; and thus the industry profit is increasing in s_i and attains maximum at $s_i = \overline{s}$.

Then we need to check if Assumption 2 is satisfied, which is equivalent to check that if increasing s_i can increase the profit of the ISP, then it also increases the profit of CP_i. In this linear example, when $\underline{s} \geq \hat{s}$, it is clear that the profit of the ISP is increasing in s_i. Moreover, increasing s_i also increases the demand for CP_i. Take CP_1 for example, if $s_1 < s_2$, the ISP chooses a network favoring CP_1,

$$D_1 = \int_{p-\delta}^{\theta+s_1} F(s_2, v+2\delta)dF(s_1, v),$$

and

$$D_2 = \int_{p+\delta}^{\theta+s_2} F(s_1, v-2\delta)dF(s_2, v).$$

Note that, for a given price p, we can see D_1 as a weighted summation of $F(v; s_2)$, increasing s_1 leads to a clockwise rotation of $F(v; s_1)$, this rotation shifts more weight to higher v, and thus more weight on higher $F(v; s_2)$. Similarly, for D_2, it's a weighted summation over $F(v; s_1)$, an increase in S_1 lowers the value of $F(v; s_1)$ for each v, and thus lower the demand D_2. Thus, we must have $\frac{\partial D_1}{\partial s_1} > 0 > \frac{\partial D_2}{\partial s_1}$; moreover, as $\frac{\partial D}{\partial s_1} > 0$, we have $|\frac{\partial D_1}{\partial s_1}| > |\frac{\partial D_2}{\partial s_1}|$. Tedious algebra also shows that if $\frac{\partial D_1}{\partial s_1} > 0$, then $\frac{\partial \pi_1}{\partial s_1} > 0$, and vise versa. Hence, Assumption 2 is satisfied in the relevant range we consider.

Therefore, in this linear example, the equilibrium must be extreme ones. Specifically, if the ISP can commit to a network, either neutral or discriminatory, as the profit for each CP increases with more niche designs, $(\overline{s}, \overline{s})$ is the only equilibrium.

However, with limited commitment, $(\overline{s}, \overline{s})$ cannot be an equilibrium. And there exists a threshold $\tilde{s} > \underline{s}$ such that for $\overline{s} < \tilde{s}$, $D(\underline{s}, \underline{s}) > D_{\overline{s}}(\underline{s}, \overline{s})$. So for $\overline{s} < \tilde{s}$, $(\underline{s}, \underline{s})$ is the only equilibrium.

Moreover in this linear example, it is easy to show that the socially efficient designs and network are the most niche designs with a discriminatory network. Thus if the ISP is able to commit, the social optimal designs can be achieved; if not, the social optimal designs are never obtained.

2.3 Discussion and Extension

2.3.1 Bidding for Connection Quality

In the model above, we assume that the ISP does not charge the CP for better connection. If the ISP can auction out the higher connection quality $\overline{q}$, will this change the results? To see this, suppose $s_1 < s_2$, so CP_1 is favored by the ISP if there is no auction. For a given discriminatory policy $\delta \neq 0$, denote D_i^1 and p^1 as the demand for product CP_i and price charged by ISP when CP_1 is favored; similarly D_i^2 and p^2 as the demand and price when CP_2 is favored. Then the maximum amount $CP_i, (i = 1, 2)$ is willing to bid for high connection quality is

$$b_1 = \alpha(D_1^1 - D_1^2) \text{ and } b_2 = \alpha(D_2^2 - D_2^1).$$

Thus, CP_2 cannot outbid CP_1 if $D_1^1 + D_2^1 > D_1^2 + D_2^2$, which is satisfied.

Proposition 2.5. *When the ISP auctions out the high connection quality, if $F(v; s)$ is log-concave in both s and v, the broader CP outbids the other one. And if a pure-strategy symmetric equilibrium exists, it must be that both CPs choose the broadest design.*

Proof. See Appendix 2.5.6. □

Therefore, even we allow the CPs to bid for better connection quality, the CP with the more niche design cannot outbid the broader one; and the bias towards broader designs still exists. However, this result may change if there is a large difference in the profitabilities of CPs. Suppose CP_i generates profit α_i for each unit of demand D_i, then the maximum amount CP_i is willing to bid for high connection quality is

$$b_i = \alpha_i(D_i^i - D_i^j), j \neq i$$

For CP_2 to win the auction and the ISP chooses the network that favors CP_2, we need

$$\alpha_2(D_2^2 - D_2^1) > \alpha_1(D_1^1 - D_1^2) \text{ } (CP_2 \text{ wins}),$$

$$p^2(D_1^2 + D_2^2) + \alpha_2(D_2^2 - D_2^1) > p^1(D_1^1 + D_2^1) + \alpha_1(D_1^1 - D_1^2) \text{ } (ISP \text{ favors } CP_2).$$

When α_2 is larger enough compared to α_1, CP_2 may outbid CP_1 and ISP may actually choose to discriminate the broader product. Thus, if there is systematic difference in the profitability of broad CP and niche CP, the CPs may instead tend to choose niche products.[18]

[18]For example, this may be the case if consumers with higher valuation for the product also create higher profit for

2.3.2 Non-Competing Content Providers

Up to now, we have focused on the case when the two CPs compete for the user in the sense that the user only consumes the product with higher realized match value. In this subsection, we briefly discuss the case when the CPs do not directly compete for the user, i.e. the user can potentially consume both products. For simplicity, we assume that the utility of the consumer is given by

$$U = max\{v_1 + q_1, v_2 + q_2, v_1 + q_1 + v_2 + q_2\} - p,$$

implying that the demand is

$$D(s_1, s_2; q_1, q_2; p) = Prob(max\{v_1 + q_1, v_2 + q_2, v_1 + q_1 + v_2 + q_2\} > p).$$

Suppose $s_1 < s_2$, we show that the ISP still favors the broader CP_1 under a slightly stronger assumption than the previous section.

Proposition 2.6. *When the content providers do not compete for the user, if $f(v; s)$ is log-concave in both s and v, then the ISP maximally favors the broader product. And therefore, if a pure-strategy symmetric equilibrium exists, it must be that both CPs choose the broadest design.*

Proof. See Appendix 2.5.7. □

The result of broad-biased design in the previous section still holds when the two CPs do not directly compete for the user. The intuition remains the same: an increase in the connection quality of a broader product leads to a larger increase in demand than a niche product, as the valuation of the user is more concentrated around the center.

2.3.3 Asymmetric Content Providers

In the previous section, we studied the case of symmetric content providers, i.e. the intrinsic value of CP_i is the same for $i = 1, 2$; and the user valuations only differ in the match value. When the two CPs are asymmetric in the sense that one CP has a higher intrinsic value, the result of favoring the broader CP may be weakened.

Denote the two CPs as CP_L and CP_H, CP_i has an intrinsic value to the user θ_i, we suppose $\theta_L < \theta_H$. The user's valuation for the product of CP_i consists of three parts: the intrinsic value

the CP in the advertising market. In this case, the niche product catering to particular high valuation consumers may indeed obtain higher profit than a broader product.

θ_i, the match value v_i, and the connection quality q_i. Following the model of competing CPs in section 3, for a given price p, the demand for the service of the ISP becomes

$$D = 1 - F(p - \theta_L - q_L; s_L)F(p - \theta_H - q_H; s_H).$$

If $s_L = s_H = s$, the effect of favoring CP_H in a discriminatory network δ on the demand is

$$\frac{\partial D^H}{\partial \delta} = f(p - \delta - \theta_H; s)F(p + \delta - \theta_L; s) - f(p + \delta - \theta_L; s)F(p - \delta - \theta_H; s). \tag{2.4}$$

Thus

$$sign\frac{\partial D^H}{\partial \delta} = sign\{\frac{f(p - \delta - \theta_H; s)}{F(p - \delta - \theta_H; s)} - \frac{f(p + \delta - \theta_L; s)}{F(p + \delta - \theta_L; s)}\}.$$

Clearly, under the assumption that $\frac{f}{F}$ is decreasing in v, $\frac{\partial D^H}{\partial \delta} > 0$. Therefore, if the two CPs have the same design, the ISP strictly prefer to favor the CP with higher intrinsic value; furthermore, even if CP_H has a more niche design $s_H > s_L$, the ISP still prefers to favor CP_H as long as θ_H is large enough compared to θ_L.

The reason why the ISP would prefer favoring the more valuable CP to favoring the CP with broader design is simple. When the two CPs compete for the user, CP_H has a larger chance of winning; and the price is closer to θ_H. Thus by giving CP_H a better connection quality, the ISP is able to charge a higher price for the user.

Moreover, with asymmetric CPs, the high-value CP will generally chooses a broader design than the low-value one. The reason is quite obvious: when the two CPs compete for the user, the ISP would charge a price which is closer to θ_H; then the low-value CP_L has to choose a more niche design so that there is a larger chance that the user will generate high match value from the low intrinsic value product. If CP_L chooses a broad product, it may happen that the product never deliver high enough utility to the user in order to compete with CP_H. For instance, in the linear example above, if $\theta_L < \theta_H - 2\delta$, the rotation point for CP_L is always lower than the rotation point of CP_H. Hence, if the two CPs choose different designs, it must be the high value CP_H that chooses the broader product. This observation is consistent with the result of Bar-Isaac et al(2012), where they show that high valuation firms choose broad design and low valuation firms choose niche design in a search model.

2.3.4 Competing ISPs

The results above are derived in the situation with a monopolistic ISP, and the results still hold under the presence of competition in the ISP market as long as consumers are not hugely differentiated in their tastes. In the simplest scenario, two ISPs locate at the end points of a Hotelling line, and consumers are uniformly distributed along the line. If consumers are only differentiated in their location, each ISP would still favor the CP with broader product, because any other network would reduce the utility a consumer would generate from connecting to his service.

When consumers are highly differentiated in their tastes, for example, some consumers strongly prefer the product of CP_1 while the others strongly prefer that of CP_2, then each ISP would have incentive to differentiate himself from the other. In this case, we may have each ISP favors a different CP and each CP chooses his design without distortion.

2.3.5 Investments of CPs

So far, we have been focusing on the situation where there is no cost for CPs to choose any design, nor is there any cost for them to enter the market. Now suppose that each CP has to pay a fixed cost E to enter the market.[19]

When the ISP cannot commit to a network characteristic, we focus on the case of $D(B, B) > D^{-\Delta}(N, B)$, so (B, B) is the only equilibrium if both CPs chose to enter. And thus, if $E \leq \alpha D(B, B)$, both CPs choose to enter with the broadest design.

When the ISP can commit to a network characteristic $\delta \geq 0$,[20] if a CP decides to enter, it must choose to enter with the most niche product. However, if the profit of the CP_2, who is discriminated against, is not enough to cover the entry cost, then it won't enter. That is, if $E \geq D^{-\delta}(N, N)$, only one CP enters under network characteristic δ. Hence, the ISP may want to commit to a less discriminatory network so as to induce both CPs to enter, which is beneficial for both consumers and the ISP. In the situation where the ISP is constrained in the networks that it can choose, it may be that when the ISP can commit, it commits to a neutral network. For instance, when the ISP can only choose a neutral network and a network with maximal

[19] In other words, the entry cost does not depend on the design chosen by the CP. Therefore, Assumption 2.2 is still verified in this situation. However, we can also assume that the entry cost is increasing in the design chosen (as it demands more information to design a niche product), and the analysis goes through as long as Assumption 2.2 is satisfied.

[20] That is, the ISP either chooses a neutral network or a network favoring CP_1. And this is the situation only subject to a relabeling of CPs.

discrimination,[21] if the profit of the discriminated CP is too low, then the ISP prefers to commit to the neutral network.

2.4 Concluding Remarks on Net Neutrality

In this paper we studied a simple model where the Internet Service Provider and content providers interact. The main result shows that if the ISP cannot commit to a network structure, the CPs are biased to choose broader designs. In some circumstances such as our linear example, such biases take an extreme form. The social optimal designs are achieved with commitment, while they are never achieved with limited commitment. The underlying reason is simple, with limited commitment, ex post the ISP prefers a network which favors the broader product; and thus each CP is inclined to design its product as broader ones in fear that it might be discriminated against ex post.

This raises the concerns about the effect of net neutrality regulation if we take into account such impact it may have on the content market. Enforcing such regulation may reduce the biases on the designs of products, but it may also increase the biases on the network structure in case where a discriminatory network may be socially beneficial. Abandoning such regulation may not do much harm if the ISP is able to commit. However, if the ISP behaves opportunistically, the content markets may end up with many similar products, which the consumers like but don't love. Especially, this may be the case in emerging content markets, where content providers are less differentiated and there is a larger chance that the ISP would be opportunistic.

Although this paper proceeds in the framework of net neutrality, we believe the intuition that lack of commitment may have serious consequences on the upstream designs works in other environments also. For instance, a search engine is more likely to put a link which everybody may click at a higher click-through rate position rather than a link which only a few consumers might be interested in; a supermarket is likely to put a product that most consumers like at an easy-to-find shelf rather than a product that attracts only consumers with special tastes. The key insight is that if the downstream resource owner can potentially practice certain forms of discrimination, lack of commitment may seriously distort upstream investments.

[21] This would be the case where one CP gets priority, and the other CP has to wait.

2.5 Appendix

2.5.1 Proof of Proposition 2.1

To prove proposition 1, we start with the following lemma:

Lemma 2.1. *Under Assumption 1, the industry profit is quasi-convex in the designs and thus maximized at extreme designs.*

Proof. The result follows immediately from proposition 1 of Johnson and Myatt(2006). First, it is obvious that $F(v;s)$ is rotation-ordered implies that $F(v;s_1)F(v;s_2)$ is rotation-ordered in both s_1 and s_2. Then with a given network characteristic δ, for any designs $s_1 < s_2$,[22] suppose the price that maximizes Π^δ is above the highest rotation point, i.e. $p^* > v^*_{s_1}$, so that the profit maximizing quantity is below $1 - F(v^*_{s_1}; s_1) \cdot F(v^*_{s_1}; s_2)$, then by the definition of rotation-ordering, an increase in either s_1 or s_2 would shift down $F(v^*_{s_1}; s_1, \cdot F(v^*_{s_1}; s_2)$ as both $v^*_{s_i}$ are decreasing in s_i, and thus demand increases even if the price is unchanged. Therefore a higher s_i always leads to higher profit. Similarly, if $p^* < v^*_{s_2}$, decreasing both s_1 and s_2 would increase demand when price is fixed. If $v^*_{s_2} < p^* < v^*_{s_1}$, increasing s_2 and decreasing s_1 at the same time would increase profit. Thus the profit maximizing designs must be extreme ones. □

Then by Assumption 2.2, both the ISP and CPs prefer the most niche design to the broadest design, so we have the industry profit is maximized with the most niche designs.

2.5.2 Details of Assumption 2.3

Assumption 2.3 can be obtained from two assumptions on the demand function $D(s_1, s_2; p)$ and $D_i(s_1, s_2; p)$.

- (2.1) $\frac{\partial^2 D}{\partial P^2} \leq 0$ and $|\frac{\partial D}{\partial p}| > p|\frac{\partial^2 D}{\partial p^2}|$ for any p;
- (2.2) $\frac{|\partial D_i/\partial s_i|}{|\partial D_i/\partial p|} \geq \frac{1}{2}\frac{|\partial^2 D/\partial p \partial s_i|}{|\partial^2 D/\partial p^2|} \geq \frac{|\partial D_j/\partial s_i|}{|\partial D_j/\partial p|}$;

We need to show that the above two conditions imply that $sign\{\partial\pi_i/\partial s_i\} = sign\{\partial D_i/\partial s_i\}$. First, we have

$$\frac{\partial \pi_i}{\partial s_i} = \alpha[\frac{\partial D_i}{\partial s_i} + \frac{\partial D_i}{\partial p}\frac{\partial p}{\partial s_i}]$$

[22] Without loss of generality, we suppose $s_1 < s_2$, so that $v^*_{s_1} > v^*_{s_2}$.

then what we need to show is simply

$$|\frac{\partial D_i}{\partial s_i}| \geq |\frac{\partial D_i}{\partial p}\frac{\partial p}{\partial s_i}|$$

The profit maximization problem for the ISP gives us the following first order condition

$$D(s_1, s_2; p) + p(s_1, s_2)\frac{\partial D(s_1, s_2; p)}{\partial p(s_1, s_2)} = 0$$

and thus we have

$$\frac{\partial p}{\partial s_i} = -\frac{\frac{\partial D}{\partial s_i} + p\frac{\partial^2 D}{\partial p \partial s_i}}{2\frac{\partial D}{\partial p} + p\frac{\partial^2 D}{\partial p^2}}$$

Thus we need to show that

$$\frac{|\frac{\partial D}{\partial s_i} + p\frac{\partial^2 D}{\partial p \partial s_i}|}{|2\frac{\partial D}{\partial p} + p\frac{\partial^2 D}{\partial p^2}|} \leq \frac{|\frac{\partial D_i}{\partial s_i}|}{|\frac{\partial D_i}{\partial p}|}$$

From (2.1) we have

$$\frac{|\frac{\partial D}{\partial s_i} + p\frac{\partial^2 D}{\partial p \partial s_i}|}{|2\frac{\partial D}{\partial p} + p\frac{\partial^2 D}{\partial p^2}|} \leq \frac{|\frac{\partial D}{\partial s_i}| + p|\frac{\partial^2 D}{\partial p \partial s_i}|}{2|\frac{\partial D}{\partial p}| + p|\frac{\partial^2 D}{\partial p^2}|} \leq \frac{|\frac{\partial D}{\partial s_i}| + p|\frac{\partial^2 D}{\partial p \partial s_i}|}{|\frac{\partial D}{\partial p}| + 2p|\frac{\partial^2 D}{\partial p^2}|} \leq \frac{|\frac{\partial D_i}{\partial s_i}| + |\frac{\partial D_j}{\partial s_i}| + p|\frac{\partial^2 D}{\partial p \partial s_i}|}{|\frac{\partial D_i}{\partial p}| + |\frac{\partial D_j}{\partial p}| + 2p|\frac{\partial^2 D}{\partial p^2}|}$$

The right hand side of the above equation is smaller than $|\frac{\partial D_i}{\partial s_i}|/|\frac{\partial D_i}{\partial p}|$ if

$$\frac{|\frac{\partial D_j}{\partial s_i}| + p|\frac{\partial^2 D}{\partial p \partial s_i}|}{|\frac{\partial D_j}{\partial p}| + 2p|\frac{\partial^2 D}{\partial p^2}|} \leq \frac{|\frac{\partial D_i}{\partial s_i}|}{|\frac{\partial D_i}{\partial p}|}$$

which is implied by (2.2), as

$$\frac{|\frac{\partial D_j}{\partial s_i}| + p|\frac{\partial^2 D}{\partial p \partial s_i}|}{|\frac{\partial D_j}{\partial p}| + 2p|\frac{\partial^2 D}{\partial p^2}|} \leq \frac{1}{2}\frac{|\frac{\partial^2 D}{\partial p \partial s_i}|}{|\frac{\partial^2 D}{\partial p^2}|} \leq \frac{|\frac{\partial D_i}{\partial s_i}|}{|\frac{\partial D_i}{\partial p}|}$$

2.5.3 Proof of Proposition 2.2

We first show that given the network characteristic δ and the design chosen by CP_j, s_j, the profit of CP_i is quasi-convex in s_i and thus maximized at one of the extreme designs, i.e. $s_i \in \{B, N\}$.

Lemma 2.2. *If Assumption 2.3 is satisfied, then for $i \in \{1, 2\}$, $sign\{\partial \pi_0^\delta / \partial s_i\} = sign\{\partial \pi_i^\delta / \partial s_i\}$ for a given network characteristic $\delta \in [-\Delta, \Delta]$.*

Proof. It suffices to show that $sign\{\partial \pi_0^\delta / \partial s_i\} = sign\{\partial D_i^\delta / \partial s_i\}$. First, we have

$$\pi_0^\delta(s_i, s_j) = p(s_i, s_j)D(s_i, s_j; \delta; p(s_i, s_j))$$

Envelop theorem immediately implies that

$$\frac{\partial \pi_0^\delta}{\partial s_i} = p\frac{\partial D}{\partial s_i} = p(\frac{\partial D_i}{\partial s_i} + \frac{\partial D_j}{\partial s_i})$$

From assumption 2, $|\partial D_i/\partial s_i| > |\partial D_j/\partial s_i|$,for $j \neq i$.

If $\partial \pi_0^\delta/\partial s_i > 0$, then either $\partial D_i/\partial s_i > \partial D_j/\partial s_i > 0$ or $\partial D_i/\partial s_i > 0 > \partial D_j/\partial s_i$. So we must have $\partial D_i/\partial s_i > 0$. Similarly, if $\partial \pi_0^\delta/\partial s_i < 0$, we must have $\partial D_i/\partial s_i < 0$. □

Therefore, under Assumption 2.3, for a given network characteristic δ, changing s_i changes the profit of the ISP and CP_i in the same direction, and thus both are quasi-convex and maximized at extreme designs.

Moreover, Assumption 2.2 says that the profit of CP_i is higher when it chooses the most niche rather than the broadest design. Hence, given the network characteristic δ, each CP prefers to choose N.

Then the ISP chooses δ so as to maximize its profit, given that both CPs will choose the most niche design. And the following lemma shows that the ISP will choose to maximally favor one of the CPs.

Lemma 2.3. *If $F(v; s)$ is log-concave in v, and both CPs choose the same design s, the ISP will choose a network with maximum discrimination $\delta \in \{-\Delta, \Delta\}$.*

Proof. The profit function of the ISP is given by

$$\pi_0^\delta = p \cdot [1 - F(p-\delta; s)F(p+\delta; s)] \tag{2.5}$$

Keeping p fixed, differentiating with respect to δ yields

$$\frac{\partial \pi_0^\delta(\delta)}{\partial \delta} = p(f(p-\delta; s)F(p+\delta; s) - f(p+\delta; s)F(p-\delta; s)). \tag{2.6}$$

Therefore, $\partial \pi_0^\delta(\delta)/\partial \delta = 0$ at $\delta = 0$; and $\partial \pi_0^\delta(\delta)/\partial \delta > 0$ for any $\delta \in [-\Delta, \Delta]$ if

$$\frac{f}{F}(p-\delta; s) > \frac{f}{F}(p+\delta; s),$$

which is satisfied if $F(v; s)$ is log-concave in v. □

2.5.4 Proof of Proposition 2.3

If $s_1 < s_2$, we first show the profit is higher when the ISP favors CP_1 than when it chooses a neutral network. The profit of the ISP is

$$\pi_0^\delta = p(1 - F(p-\delta; s_1)F(p+\delta; s_2))$$

If $\delta = 0$, the network is neutral; if $\delta > 0$, the ISP favors CP_1. Fix p, take FOC w.r.t. δ,

$$\frac{\partial \pi_0^\delta}{\partial \delta} = p(f(p-\delta; s_1)F(p+\delta; s_2) - f(p+\delta; s_2)F(p-\delta; s_1))$$

$$sign\{\frac{\partial \pi_0^\delta}{\partial \delta}\} = sign\{\frac{f}{F}(p-\delta; s_1) - \frac{f}{F}(p+\delta; s_2)\}$$

If $F(v;s)$ is log-concave in s and v, we have

$$\frac{f}{F}(p-\delta; s_1) > \frac{f}{F}(p-\delta; s_2) > \frac{f}{F}(p+\delta; s_2) \text{ for all } \delta \in [0, \Delta]$$

Thus, we have $\partial \pi_0^\delta / \partial \delta > 0$ for all $\delta \in [0, \Delta]$. Therefore, the ISP prefers a maximum discriminatory network which favors CP_1 to a neutral network.

Then we show that a discriminatory network favoring CP_1 is preferred to a network favoring CP_2. It suffices to show that for a given p and $\delta \geq 0$,

$$F(p-\delta; s_1)F(p+\delta; s_2) < F(p+\delta; s_1)F(p-\delta; s_2)$$

which is equivalent to

$$\frac{F(p-\delta; s_1)}{F(p-\delta; s_2)} < \frac{F(p+\delta; s_1)}{F(p+\delta; s_2)}$$

which in turn is satisfied if $F(x;s_1)/F(x;s_2)$ is increasing in x,

$$sign\{\frac{\partial \frac{F(x;s_1)}{F(x;s_2)}}{\partial x}\} = sign\{\frac{f}{F}(x; s_1) - \frac{f}{F}(x; s_2)\}$$

as $F(v;s)$ is log-concave in s, we have $f/F(x;s_1) > f/F(x;s_2)$. Thus, favoring CP_1 is better for the ISP than favoring CP_2.

2.5.5 Proof of Proposition 2.4

It is easy to show that (N, N) is not an equilibrium anymore. Suppose not, at $s_1 = s_2 = N$, the ISP will randomly favor one of the two CPs. Then by choosing a design slightly broader

$s_i' = N - \epsilon$, CP_i induces a negligible change in the price charged by the ISP and the total demand from the consumer; however, this slight change of design induces the ISP to favor CP_i with probability 1, and thus CP_i can get a larger share of the demand. Therefore this constitutes a profitable deviation for CP_i. The same logic implies that any $s_1 = s_2 \neq B$ cannot be an equilibrium.

Moreover, any $B < s_1 < s_2 < N$ cannot be an equilibrium. Suppose not, CP_2 would be discriminated against by the ISP. Lemma 2.2 implies that the profit of CP_2 is quasi-convex in s_2 on the interval $(s_1, N]$, and thus maximized either at $s_2 = N$ or $s_2 \to s_1$. In the former case, CP_2 has a profitable deviation to N. And in the latter case, CP_2 has a profitable deviation to $s_2 = s_1$, as this would induce the ISP to favor CP_2 with probability 0.5.

Therefore, if the two CPs choose different designs, the one with the more niche design must be choosing the most niche design. Suppose CP_2 chooses N, then the profit of CP_1 is quasi-convex in s_1 on the interval $[B, N)$, where CP_1 is favored by the ISP. By Assumption 2.2, $D^\delta(N, B) > D^\delta(B, B)$ for any δ, and thus CP_1 would choose $s_1 \to N$, in which case there is no pure-strategy equilibrium. Hence, (B, N) cannot be an equilibrium.

Thus, the only candidate for pure-strategy equilibrium is (B, B). If both CPs choose the broadest design, each CP is favored by the ISP with the same probability; any CP deviating to a more niche design will be discriminated against for sure. Hence, the deviating CP's profit is quasi-convex on the interval $s \in (B, N]$. By Assumption 2.2, the profit is maximized at N. Thus, if $D(B, B) < D^{-\Delta}(N, B)$, such deviation is profitable, and then no pure strategy equilibrium exists; on the contrary, if $D(B, B) > D^{-\Delta}(N, B)$, there is no profitable deviation and (B, B) constitutes the only equilibrium.

2.5.6 Proof of Proposition 2.5

Given s_1 and s_2, the profit of the ISP is given by

$$\pi_0^\delta = p(1 - F(p - \delta; s_1)F(p + \delta; s_2))$$

Envelop theorem implies that

$$\frac{\partial \pi_0^\delta}{\partial \delta} = p\frac{\partial D^\delta}{\partial \delta}$$

where $D^{\delta} = 1 - F(p-\delta; s_1)F(p+\delta; s_2)$, under the assumption that $F(v; s)$ is log-concave in s and v, Proposition 2.3 has shown that

$$\frac{\partial \pi_0^{\delta}}{\partial \delta} = p\frac{\partial D(\delta)}{\partial \delta} > 0$$

so the demand is higher in a network which favors the broader product CP_1.

2.5.7 Proof of Proposition 2.6

First, we show that if $f(v; s)$ is log-concave in s and v, then $F(v; s)$ is also log-concave in s and v. And therefore the condition in the proposition is stronger than that in the main model. Bagnoli and Bergstrom (2005) shows that if the p.d.f $f(x)$ is log-concave, then the c.d.f. $F(x)$ is also log-concave, which proves the "log-concave in v"part. If $f(v; s)$ is log-concave in s, then it is easy to show that $f(v; s_1)/f(v; s_2)$ is increasing in v for $s_1 < s_2$. Moreover, we have

$$\frac{f(v; s_1)}{F(v; s_1)} - \frac{f(v; s_2)}{F(v; s_2)} = \frac{\int_{-\infty}^{v}(f(v; s_1)f(x; s_2) - f(v; s_2)f(x; s_1))dx}{F(v; s_1)F(v; s_2)}$$

as $f(v; s_1)/f(v; s_2) > f(x; s_1)/f(x; s_2)$ for any $x < v$, we must have $f(v; s_1)/F(v; s_1) > f(v; s_2)/F(v; s_2)$. So $F(v; s)$ is also log-concave in s.

For a given price p, the user will connect to the services in a neutral network if either $v_1 > p$, or $v_2 > p$ or $v_1 + v_2 > p$; in a discriminatory network which favors CP_1, the user connects if either $v_1 + \delta > p$, or $v_2 - \delta > p$ or $v_1 + v_2 > p$. Hence, if the ISP switches from neutral network to the discriminatory network favoring CP_1, that is, if $\delta \geq 0$, the change in demand is

$$\Delta D(\delta) = \int_{p-\delta}^{p} F(p-x; s_2)dF(x; s_1) - \int_{p}^{p+\delta} F(p-x; s_1)dF(x; s_2),$$

Clearly, $\Delta D(\delta) = 0$ for $\delta = 0$; then the ISP prefers a network favoring CP_1 to a neutral network if

$$\frac{\partial \Delta D(\delta)}{\partial \delta} = F(\delta; s_2)f(p-\delta; s_1) - F(-\delta; s_1)f(p+\delta; s_2) > 0$$

we have

$$sign\{\frac{\partial \Delta D(\delta)}{\partial \delta}\} = sign\{\frac{f(p-\delta; s_1)}{F(-\delta; s_1)} - \frac{f(p+\delta; s_2)}{F(\delta; s_2)}\}$$

then it is easy to show that

$$\frac{f(p-\delta; s_1)}{F(-\delta; s_1)} > \frac{f(p+\delta; s_1)}{F(\delta; s_1)} > \frac{f(p+\delta; s_2)}{F(\delta; s_2)}$$

hence $\partial \Delta D(\delta)/\partial \delta > 0$ for all δ, so the ISP prefers a maximum discriminatory network favoring CP_1 to a neutral network. Similarly, when the ISP favors CP_2, the demand increases and the change in demand is

$$\Delta D(-\delta) = \int_{p-\delta}^{p} F(p-x; s_1) dF(x; s_2) - \int_{p}^{p+\delta} F(p-x; s_2) dF(x; s_1)$$

Therefore, we have

$$\Delta D(\delta) - \Delta D(-\delta) = \int_{p-\delta}^{p+\delta} (f(x; s_1)F(p-x; s_2) - f(x; s_2)F(p-x; s_1))dx$$

Then as long as δ is small enough($\delta < p - \delta$), we must have $\Delta D(\delta) > \Delta D(-\delta)$ for all δ. So the ISP always prefers to favor CP_1 rather than CP_2.

Chapter 3

Limited Attention

An important feature of advertisements is that they attract attention from consumers. When a product is characterized by several attributes, firms also strategically use advertisements to manipulate the attention of consumers. A monopolist tends to advertise too few attributes, and competition does not necessarily improve the situation. Moreover, in an attention-scarce economy, competition for consumers attention leads firms to advertise fewer attributes and reduces information available to consumers.

3.1 Introduction

Two important roles of advertisements have been well analyzed in the literature: informational and persuasive. The former one refers to the fact that advertisements inform consumers the existence of a product; and the latter one improves the perceived value of a product and persuades consumers to purchase. In this paper, we try to explore the role of advertisements in attracting and manipulating the attention of consumers.

It is a well-documented fact that consumers have limited attention,[1] i.e. they don't pay attention to all available information, instead they sample a subset of information and base their decisions on the sample. A consumer may not know the existence of a product unless he sees an advertisement of the product; in addition, the consumer may not pay attention to specific features of the product unless they are evoked in the advertisement. Therefore, an advertisement not only

[1]See DellaVigna (2009) for a survey on empirical evidences of limited attention.

indicates the existence and valuation of a product, it also attracts consumers' attention to the advertised features of the product.

For instance, a car manufacturer advertises its new model, in addition to features that consumers know well such as color and fuel efficiency, it can also decide whether and how much to advertise its new internal design, of which consumers differentiate in their tastes. If this new design draws the attention of a consumer, his valuation for the new model will be more dispersed compared to a consumer who does not pay attention to the new design, as this new design introduces additional variation. Therefore, when this new design is more heavily advertised, more consumers will pay attention to this new attribute, and valuations for the new model becomes more dispersed. In other words, increasing the advertising intensity of more attributes brings more information into consumers' attention, which results in more consumers with higher valuations (when they like the new design), but also more consumers with lower valuations (when they do not like the new design). In the terminology of Johnson and Myatt (2006), this leads to a rotation of the demand function for the product.

Thus, for a monopolist whose product having a mass market position and targeting the general public, its main objective is to reduce the number of consumers who have low valuations. Hence, it advertises as less attributes as possible, so as to limit the attention that consumers pay to those attributes and reduce the dispersion of valuations. This leads to distortions in consumers' choices and reduces consumer welfare, as all purchase decisions are based on limited information. However, competition does not necessarily improve the information available to consumers.

Consider first the situation where consumers know the existence of competing firms, and firms only decides how intensive to advertise new attributes of their products. In the case of symmetric firms, the effect of competition on advertising strategies crucially depends on the form of advertising. If advertisements are out-of-store, any advertised attributes will be brought to each consumer's attention. This not only affects how consumers evaluate the advertised product, but also how they evaluate other products in the same industry. In this case, firms do not advertise more attributes than a monopolist. This is because each firm's advertising changes the way how consumers evaluate the whole industry, which hurts all firms. On the contrary, if advertisements are in-store, they have a larger effect on the attention paid to the advertised product and a weaker effect on how consumers evaluate other products. In this situation, firms have incentives to advertise more so as to induce more high-value consumers, which improves their competitive advantages.

And consumers are better-off as well due to improved information. When we take into account that advertising also attract consumers' attention to the existence of a product, our analysis shows that firms advertise more out-of-store and less in-store in an attention-scarce economy, i.e. when most consumers are attention limited. This is because firms' primary goal is to attract consumers' attention to the existence of their products, and this is done by out-of-store advertising, which is accompanied with less information.

If firms are asymmetric, i.e. each firm holds a competitive advantage over others on certain attributes, then each firm has an incentive to advertise its advantageous attributes even if advertisements are out-of-store. However, this incentive may be dominated by the incentive to manipulate consumers' attention, when consumers are heterogeneous in their attention capacity. We show that firms may advertise too little if there are a large number of consumers who have high attention capacity. The reason is straightforward: each firm advertising its advantageous attribute leads to more consumers who pay attention to all attributes of a product, and this reduces total demand. The incentive to manipulate the attention of those high attention consumers dominates the incentive to build competitive advantage when there are a large number of such consumers. Therefore, consumer welfare could be reduced when there are more rational consumers with high attention capacity, as this induces firms to advertise less and reduces the amount of information available.

Our paper belongs to the growing literature on psychology and economics, especially on effects of limited attention. Several papers have studied the consequence of limited attention on hierarchy (Geanakoplos and Milgrom (1991)), product quality (Armstrong and Chen (2009)), information disclosure (Hirshleifer and Teoh (2003)) etc. In terms of limited attention and advertising, most papers (Van Zandt (2004), Falkinger (2007, 2008), Anderson and De Palma (2012), Hefti (2012), etc) have explored the informational role of advertising, i.e. they assume that advertising is to inform consumers the existence of products and firms have to compete for such awareness. We take a step further in assuming that advertisements not only inform consumers the existence but also specific attributes of products. In other words, our focus is not on the intensity of advertising, but on the "content"of advertising (Anderson and Renault (2006)). Secondly, our paper is related to the literature on competition and information disclosure, especially on the disclosure of horizontal match information. Among others, Sun (2011), Hotz and Xiao (2013) have shown that competition may not lead to full disclosure when there is also vertical differentiation.

Anderson and Renault (2009) also explored the incentive to disclose horizontal match information in the context comparative advertising. In their model, quality difference between firms plays a key role in determining firms' disclosure incentives. In addition, consumers have access to the same set of information in their model; whereas consumers may have access to different information due to their attention limitations in our model. In our paper, the intention to manipulate consumers' attention provides another reason why competition does not improve information availability in a market even without vertical differentiation. Another closely related paper is Persson (2012), where the incentive to manipulate attention is derived in a principal-agent framework.

The paper is organized as follows: We present the basic model in Section 2, where we study the strategy of a monopolist and the effect of competition with out-of-store and in-store advertising; Section 3 shows several extensions and discussions; Section 4 concludes. All omitted proofs are presented in the Appendix.

3.2 The Model

3.2.1 Benchmark: The Monopoly Case

A monopolistic firm sells a product to a unit mass of consumers. Each consumer demands only one unit of the product. The product is characterized by two attributes: a "default"attribute 0 and an additional attribute a. The valuation that a consumer generates from the product is given by

$$v = v_0 + \gamma v_a,$$

where v_0 and v_a are the values that a consumer generates from attribute 0 and attribute a respectively, and γ measures how much attention a consumer pays to attribute a. The outside option is $\underline{u}$, if a consumer chooses not to buy the product.

The idiosyncratic match value v_0 and v_a are independently drawn from distribution $F_0(\cdot)$ and $F_a(\cdot)$, and the values drawn by different consumers are independent. We assume that both distributions have zero mean[2], with variance σ_0 and σ_a respectively, and they are symmetric

[2]This is without loss of generality: we can write the utility function as $v = E(v_0) + (v_0 - E(v_0)) + \gamma v_a$, where $E(v_0)$ is the expectation of v_0. Clearly, $v_0 - E(v_0)$ has zero mean. And our main result is not affected by such variation.

around 0. Then the distribution of v is given by

$$F(v;\gamma) = \int\int_{v_0+\gamma v_a \le v} dF_0(v_0)dF_a(v_a).$$

The attention level γ captures the fact that consumers have limited attention, which is well documented in a lot of experiments. Attention as a resource is constrained in capacity either physically or psychologically. For instance, when a consumer shops for some wine in a supermarket, she may not pay attention to all information available (either the information on the price tag, or the information printed on the bottle) due to time constraint or lack of knowledge.

We assume that γ can be either 0 or 1. A consumer with $\gamma = 0$ only cares about the default attribute, and pays no attention to the additional attribute a, and thus his valuation for the product is simply drawn from $F(v;0) = F_0(\cdot)$. Instead, a consumer with $\gamma = 1$ pays full attention to attributes a, and his valuation is drawn from $F(v;1)$, which has mean zero, and variance $\sigma_0 + \sigma_a$.

When there is a proportion s of consumers paying attention to attribute a, and the rest $1-s$ only paying attention to the default attribute, the valuation for the firm's product in the whole population is drawn from

$$G(v;s) = (1-s)F(v;0) + sF(v;1).$$

Clearly, we have

$$\frac{\partial G(v;s)}{\partial s} = F(v;1) - F(v;0).$$

And it is straightforward to show that

$$\frac{\partial G(v;s)}{\partial s} = F(v;1) - F(v;0) \begin{cases} < 0 & \text{if } v > 0; \\ > 0 & \text{if } v < 0. \end{cases} \tag{3.1}$$

i.e. $G(v;s)$ is increasing in s for $v < 0$ and decreasing in s for $v > 0$. Following the definition of Johnson and Myatt (2006):

Definition 3.1. ***(Johnson and Myatt 2006):*** *A local change of s leads to a rotation of $G(v;s)$ if for some v^* and any $v \in (-\infty, \infty)$,*

$$v > v^* \Leftrightarrow \frac{\partial G(v;s)}{\partial s} < 0 \textit{ and } v < v^* \Leftrightarrow \frac{\partial G(v;s)}{\partial s} > 0.$$

We easily see that:

Lemma 3.1. *$G(v;s)$ is rotation-ordered in s with rotation point 0.*

Higher attention level in the population leads to a more dispersed valuation distribution: more consumers have very high valuation, and more consumers have very low valuation as well. Simply speaking, high attention consumers pay attention to more details of a product, and thus their valuation distributions have a larger variance. In the terminology of Johnson and Myatt, higher s makes the firm's product a more niche one. A simple example is when both $F_0(v)$ and $F_a(v)$ are the standard normal distribution $\phi(\cdot)$, then $F(v;1)$ is a normal distribution with expectation 0, and standard variation $\sqrt{2}$. Therefore, $G(v;s)$ gradually rotates from $\Phi(0,1)$ to $\Phi(0,\sqrt{2})$ as s increases from 0 to 1.[3]

Given that the outside option is $\underline{\mathrm{u}}$, the demand for the product is

$$D(s) = 1 - G(\underline{\mathrm{u}}; s).$$

We assume that the firm makes a profit of 1 from one unit of demand, then the above equation also represents total profit of the firm. It is straightforward to show that:[4]

Lemma 3.2. *The firm's profit is quasi-convex in s. Moreover, if the outside option $\underline{u} < 0$, then the profit is decreasing in s.*

Proof. This is a direct consequence of the Proposition 1 of Johnson and Myatt (2006). As the demand function is rotation-ordered with rotation point being 0, if the outside option is such that $\underline{\mathrm{u}} < 0$, it is the same situation as in Johnson and Myatt where the optimal price is below the rotation point, so the profit is decreasing in s. To be more specific, the demand is $D(s) = 1 - G(\underline{\mathrm{u}}; s)$. As $\underline{\mathrm{u}} < 0$, $G(\underline{\mathrm{u}}; s)$ is increasing in s from Lemma 1, and thus $D(s)$ is decreasing in s. $\square$

When the outside option is below 0, the product has a mass product position. And thus to maximize profit, the firm needs to minimize the number of consumers with low valuations, which is achieved when the variance of the valuation distribution is minimized, that is when $s = 0$. In

[3]Another example is the linear distribution: If $F_0(v)$ and $F_a(v)$ are uniform distributions on $[-\theta, \theta]$, we can also show that changes in s also lead to a rotation of $G(v;s)$.

[4]We focus on the advertising strategy of the firm, so the firm only chooses s but not the price. The result still holds even when the firm also makes price decisions. To incorporate this, we need to assume that $E(v_0) > 0$, and the optimal price when the firm does not advertise the additional attribute is $p < E(v_0)$. The important thing is that the product features a mass-market position. This also facilitates our comparison between monopoly and competition.

other words, when the product features a mass market, profit is maximized with the broadest design.

Lemma 3.2 implies that consumers with higher attention level actually hurts the firm if the firm's product features a mass-market position. More attentive consumers care about more details of the product, which implies that it is more likely that they would find out some negative facts of the product and thus choose not to purchase.

In the following, we focus on the situation where the outside option is negative.

Assumption 3.1. $\underline{u} < 0$.

In reality, a firm has a number of ways to affect the attention level of consumers, i.e. s depends on strategies chosen by the firm. Here, we focus on advertising and the information content of advertisements. The firm has free access to one unit of resource for advertising. For now, we suppose that informing consumers about the existence of a product is not a concern, the firm rather advertises to keep their products imposed to consumers. So the firm always fully uses its advertising resource, and it chooses whether to spend any of this resource on the additional attribute a. If the firm advertises attribute a with intensity s, a proportion s of consumers pay attention to a, and the rest do not.[5] Then it is straightforward to have the following proposition:

Proposition 3.1. *Under Assumption 3.1, the firm does not advertise the additional attribute a, i.e. $s = 0$.*

When the firm's product features a mass market, information hurts the firm and thus it chooses to minimize the information provided to consumers, that is to advertise no additional attribute except the default one which every consumer pays attention to.[6] In other words, the firm only informs consumers about the existence of its product, but not any information about the product in addition to what consumers have already known.

[5] We can think of the situation where each consumer has a capacity of attention equal to 1: this unit of attention is randomly drawn by one of the advertising messages. Hence, if the attention is drawn by a message containing the information about attribute a, this consumer pays attention to a; otherwise, he does not pay attention to a. When the firm sends out s messages that contains information about a, the proportion of consumers that eventually pay attention to a is then s.

[6] This result does not depend on whether consumers have limited attention or not. As long as the attention level is manipulatable, the firm has incentive to divert consumers' attention to the default attribute.

3.2.2 The Case of Competition

As shown in the previous section, a monopolist advertises "too little". And consumers make suboptimal decisions based only on information about the default attribute. A natural question to ask is whether competition can mitigate such problem, i.e. would competition induce firms to advertise the additional attribute, and bring everything into consumers' attention?

Suppose now there are two firms: firm 1 and firm 2. As before, each firm has one unit of advertising resource to allocate. We distinguish between two cases: the first is out-of-store advertising where each firm's advertising affects consumers' attention allocation when evaluating both firms' products; the second is in-store advertising where one firm's advertising only affects how consumers evaluate its own product. We start with the out-of-store advertising case.

3.2.2.1 Out-of-Store Advertising

Advertisements such as outdoor billboards advertisements draw attention of all consumers, and thus an advertisement of one product may also affect how a consumer thinks about other products in the same market. For instance, an outdoor advertisement of BMW which introducing some new features also induces consumers to think about whether Benz offers the same features. In others words, any information contained in such advertisements not only attracts consumers attention when they visit the store of the advertised product, but also has long-lasting effect when they are out-of-store.[7] To capture this effect, we assume that an attribute advertised by one firm attracts consumers attention when they evaluate both firms' products. Specifically, suppose each firm i has one unit of advertising resource, and it decides a share s_i of this resource to be spent on the additional attribute a. And then the proportion of consumers who pay attention to attribute a is

$$s = \frac{s_1 + s_2}{2},$$

which is increasing in both s_1 and s_2. And we have the following result:[8]

[7] This type of out-of-store advertising not only includes various types of outdoor advertising, but also some information pamphlets which consumers can take away in the sense that the information contained in the pamphlets may still affect how consumers think when they evaluate other products.

[8] As in the monopoly case, we assume that firms only choose advertising strategies but not prices. In the competition case, firms' prices will be constrained by the outside option. And the following results hold as long as the constrained prices are below $E(v_0)$.

Proposition 3.2. *When advertising is out-of-store, both firms do not advertise the additional attribute, i.e.* $s_1 = s_2 = 0$.

Proof. Given that firm j chooses s_j, the demand for firm i is

$$D_i(s_i, s_j) = \frac{1}{2}(1 - G^2(\underline{\mathrm{u}}; \frac{s_i + s_j}{2})),$$

as two firms share the total demand which is $1 - G^2(\underline{\mathrm{u}}; \frac{s_i+s_j}{2})$. Therefore, given s_j, any positive s_i leads to a wider dispersion of consumers' valuations, which lowers the total demand and thus the demand for firm i. Hence, we have $s_i = 0$. □

When advertising is out-of-store, any information in advertising affects consumers' valuation for the products symmetrically, i.e. consumers' valuations for both products are drawn from the same distribution. Hence, any advertising that increases the dispersion of valuations will hurt both firms. And thus competition does not improve the information available to consumers with out-of-store advertising. In addition, the joint profit of two firms attains its maximum with out-of-store advertising, which acts as a coordinating device for the two firms.

Remark Out-of-store advertising does not improve the information available to consumers, but consumers can still benefit from competition: they have more firms to choose, which increases the probability that they find something better than the outside option.

3.2.2.2 In-Store Advertising

Now we turn to the case of in-store advertising: this type of advertising is mostly carried out through sales talks, information contained in such advertisements would attract consumers' attention only when they visit the store, and thus only affect consumers' evaluation for the advertised product. In other words, such advertising has strong effects in-store, but has limited long lasting effect when consumers are out-of-store. This distinction between out-of-store and in-store advertising may be too extreme, however, we emphasize that in-store advertising has a weaker "spillover"effect than out-of-store advertising in affecting consumers' attention towards products other than the advertised one.[9]

As before, each firm has one unit of advertising resource to allocate, if firm 1 and 2 allocates s_1 and s_2 of their advertising resource to attribute a, then the attention level that a consumer pays

[9]For instance, the information from a salesman of BMW may still affect a consumer's evaluation for a Benz, but this effect is weaker than a take-away pamphlet due to short-memory, or manipulation from a salesman of Benz.

to attribute a of product 1 is s_1, and s_2 is the attention level paid to the additional attribute of product 2. Hence, consumers' valuations for products of firm 1 and 2 are drawn from distribution $G(v_1; s_1)$ and $G(v_2; s_2)$ respectively.

The demands for firm 1 and firm 2 are then given by

$$D_1 = \int_{\underline{\mathrm{u}}}^{\infty} G(v_1; s_2) dG(v_1; s_1)$$

and

$$D_2 = \int_{\underline{\mathrm{u}}}^{\infty} G(v_2; s_1) dG(v_2; s_2)$$

And we have the following proposition:

Proposition 3.3. *When advertising is in-store, each firm fully advertises the additional attribute, i.e.* $s_1 = s_2 = 1$, *if the outside option is not too small.*

Proof. See Appendix 3.5.1. □

When advertising is in-store, each firm has incentive to induce more consumers to pay attention to every detail of its product: this leads to a higher probability that a consumer has higher valuation for its product than that of the opponent, and thus improves its competitive advantage. However, both firms are worse-off than in the case of out-of-store advertising.

Corollary 3.1. *The demands for both firms are lower with in-store advertising than with out-of-store advertising.*

Proof. Under in-store advertising, each firm gets a demand of

$$D(1,1) = \frac{1}{2}(1 - G(\underline{\mathrm{u}}; 1) G(\underline{\mathrm{u}}; 1))$$

and the demand under out-of-store advertising is

$$D(0,0) = \frac{1}{2}(1 - G(\underline{\mathrm{u}}; 0) G(\underline{\mathrm{u}}; 0))$$

And clearly $D(1,1) < D(0,0)$ as $G(\underline{\mathrm{u}}; 1) > G(\underline{\mathrm{u}}; 0)$ for $\underline{\mathrm{u}} < 0$. □

The equilibrium under in-store advertising is a form of prisoner's dilemma: each firm wants to draw consumers' attention to more details of its product, but in the end this reduces the total demand. However, consumers benefit from such in-store advertising: each consumer makes the right choice based on all information available under in-store advertising, whereas with out-of-store advertising they might make the wrong choice.

Corollary 3.2. *Consumers are better-off under in-store advertising than under out-of-store advertising.*

Remark We have focused on the two extreme cases of out-of-store advertising and in-store advertising. In reality, firms normally use a mixture of these two. The above analysis indicates that the information available to consumers decreases with the share of out-of-store advertising. In addition, this implies that if firms could cooperate on their advertising strategy, they would favor out-of-store advertising and this again reduces information availability.

3.2.3 Attention Attraction vs. Attention Manipulation

The above two extreme cases show us the incentive for attention manipulation when consumers have already known the existence of both firms. And they are the building blocks when we take into account the fact that advertisements attract consumers' attention to the advertised product at the first place. Specifically, consumers get to notice the existence of a product when they see advertisements of the product, and then they will pay additional attention to details of the product.

In our model, we can simply incorporate this feature of advertising by assuming that a consumer gets aware of a product when he sees an out-of-store advertisement of the product; and when he enters the store, he gets further information from in-store advertisement in addition to what has been contained in the out-of-store advertisement. Under this assumption, out-of-store advertising serves to attract consumers' attention, and both out-of-store and in-store advertising serve to manipulate consumers' attention.

As before, each firm i has one unit of advertising resource, which can be allocated between out-of-store advertising s_i^{out} and in-store advertising s_i^{in}. Furthermore, firms also decide what information to include in the out-of-store and in-store advertising. For simplicity, we also assume that a proportion of $1 - \alpha$ consumers are attention limited, they only pay attention to one out-of-store advertising message and thus get aware of at most one product; the rest α proportion of consumers are attention unlimited and they get aware of both products as long as they are advertised. Then it is easy to show that:

Proposition 3.4. *In the symmetric equilibrium, there exist $\bar{\alpha}$ such that, if $\alpha < \bar{\alpha}$, both firms only use out-of-store advertising and they do not advertise the additional attribute.*

Proof. See Appendix 3.5.3. □

A firm faces a trade-off between exploiting limited attention consumers and competing for full attention consumers. On one hand, a firm wants to advertise more out-of-store and disclose limited information in order to attract those limited attention consumers (each firm has monopoly power over limited attention consumers who are aware of only its product, and thus it does not advertise any additional attribute); on the other hand, a firm wants to advertise more in-store and disclose more information in order to win those full attention consumers. Hence, in an attention-scarce economy (α small), the first incentive dominates and competing to attract consumers attention leads firms to rely more on out-of-store advertising, which correspondingly results in less information availability.

The analysis in the section resembles that of Section III.B in Johnson and Myatt (2006), where they differentiated between advertisements consisting of hype information and real information. In their analysis, advertisements with hype information do not change the dispersion of value distribution, whereas advertisements with real information lead to demand rotation. Under their assumption, a monopolist prefers either full information (real information advertising) or total ignorance (hype information advertising).

We depart from their analysis in several ways: First, in our model advertisements always provide real information, but firms can choose how much real information to advertise. Second, we explicitly model the role of advertising in attracting and manipulating attention of consumers. Third, and most importantly, we show how competition affects firms' choice between out-of-store advertising (which provides less real information) and in-store advertising (which provides more real information). Particularly, we show that this effect depends on the attention level of consumers. In other words, we present a simple situation where firms indeed prefer hype information advertising: when firms have to compete for consumer awareness in an attention-scarce economy.

Anderson and Renault (2009) also studies whether firms disclose horizontal information in the presence of vertical differentiation. In their model, once a firm discloses some horizontal information, all consumers get aware of that information (although they differentiate in their valuation for that information). We take a different approach and emphasize that consumers may evaluate different products based on different information sets, due to the fact that attention of a consumer is limited and manipulable.

3.3 Extensions and Discussions

3.3.1 Multiple Attributes

So far, we have focused on a single attribute in addition to the default one. The situation is basically the same when there are possibly more than one attribute to advertise, as long as consumers evaluate the products in the same way (they have the same attention level).

Suppose there are two additional attributes a and b, the valuations of which are drawn from the same distribution. All consumers are attention limited: they pay attention to at most one additional attribute. It is clear that in the out-of-store advertising case, no firm advertises any of the additional attributes. In the in-store advertising case, each firm fully advertises the additional attributes.[10]

The situation is slightly different when the additional attributes are asymmetric: Suppose that the valuation of one attribute, say a, is drawn from a distribution with a larger variance than the other attribute b.[11] It is obvious that no firm advertises if advertising is out-of-store. In the in-store advertising case, the only equilibrium is that both firms fully advertise attribute b. The reason is the same as Proposition 3.3: each firm wants to induce more high value consumers to build its competitive advantage over the other firm. Even though competition in this case does not lead to complete advertising, it does induce firms to provide the more valuable information to consumers.

3.3.2 Heterogeneous Consumers

In the above analysis, we have assumed that consumers are homogeneous. Now we briefly study the situation when consumers are heterogeneous in their attention levels: a proportion p of consumers have their attention capacity equal to one, so they randomly draw one advertising message and pay attention to the attribute contained in this message if there were any; the rest $1 - p$ of the consumers have larger attention capacity, for simplicity, say two units of attention. So they draw two advertising messages and pay attention to what is contained in the messages. Heterogeneous consumers does not bring any difference when there is only one additional attribute

[10] A firm can divide the advertising space between the two additional attributes, as a consumer's valuation is drawn from the same distribution no matter whether he pays attention to a or b.

[11] Both distributions still have zero mean.

or in the out-of-store advertising case, so we focus on in-store advertising with two attributes a and b.

Consider a simple extension of the above model: Each firm i can spend s_i^a proportion of its advertising space on attribute a, and s_i^b on attribute b. Then the proportion of consumers who only pay attention to attribute a is

$$\alpha_a(s_i^a, s_i^b) = s_i^a p + ((s_i^a)^2 + 2s_i^a(1 - s_i^a - s_i^b))(1 - p),$$

where the first term is those consumers who have one attention capacity and draw an advertising message about a; the second term is those consumers who have two attention capacity and draw two messages about a, or one message about a and one with no additional attribute. Similarly, the proportion of consumers who only pay attention to attribute b is

$$\alpha_b(s_i^a, s_i^b) = s_i^b p + ((s_i^b)^2 + 2s_i^b(1 - s_i^a - s_i^b))(1 - p);$$

the proportion of consumers who pay attention to both attributes is

$$\alpha_{a,b}(s_i^a, s_i^b) = 2s_i^a s_i^b(1 - p).$$

And the rest of consumers only pay attention to the default attribute.[12]

The valuation of a consumer who pays attention to both attribute is

$$u = v + v_a + v_b.$$

It is clear that these consumers have a valuation distribution that is more dispersed than those who only pay attention to one attribute.

Therefore, in the in-store advertising case, if the two attributes are symmetric, each firm has an incentive to induce more consumers to pay attention to both attributes. Hence, both firms must choose to allocate equally the advertising resource to both attributes, that is

$$s_i^a = s_i^b = \frac{1}{2}, \text{ for } i = 1, 2.$$

[12]It is easy to see that $\alpha_a(1, 0) = 1$ and $\alpha_b(0, 1) = 1$. When the firm only advertise one attribute, those consumers with higher attention level also only pay attention to the advertised attribute. In this setup, if the attention level of a consumer goes to infinity, then this consumer pays attention to everything that is advertised, no matter what is the advertising intensity.

Competition for high attention capacity consumers, who are also more valuable, leads firms to advertise more about their products, and this in turn benefit these high attention consumers, as they now evaluate the product more thoroughly.

Remark When the two attributes are asymmetric, it might be the case that each firm only advertises one attribute, and we present such a case in Appendix 3.5.2.

3.3.3 Asymmetric Firms

In the previous analysis, we have assumed that the two firms are symmetric, i.e. the valuations for the same attribute of the two products are drawn from the same distribution. Suppose now the two firms are differentiated, and firm 1 has a competitive advantage over firm 2 on attribute a: the valuation for attribute a of firm 1 is drawn from a distribution that first-order-stochastically dominates that of firm 2. For simplicity, we assume that the distribution of attribute a for firm 1 is drawn from $F_a(v)$ with zero mean and variance σ; whereas for firm 2 it is drawn from $F^\delta(v)$ with mean $\delta < 0$ and variance σ, that is $F^\delta(v) = F_a(v - \delta)$.

We have seen that when firms are symmetric, no firm advertises the additional attribute in the out-of-store advertising case. This may not be the case when firms are asymmetric, as firm 1, who has a competitive advantage over firm 2, has an incentive to advertise attribute a. To see this, if firm 2 does not advertise, the demand of firm 1 when it advertises a with intensity s is given by

$$D(s, 0) = s \int_{\underline{u}}^{\infty} F(v, \delta; 1) dF(v; 1) + (1 - s) \int_{\underline{u}}^{\infty} F(v; 0) dF(v; 0),$$

where the first term on the right hand side is the demand from consumers who pay attention to a, and

$$F(v, \delta; 1) = \int \int_{v_0 + v_a \leq v} dF_0(v_0) dF_a(v_a - \delta),$$

is the valuation distribution for product 2 when consumers pay attention to attribute a; and the second term is the demand from those who do not pay attention to a.

Therefore, firm 1 will advertise attribute a if

$$D^+ = \int_{\underline{u}}^{\infty} F(v, \delta; 1) dF(v; 1) > \int_{\underline{u}}^{\infty} F(v; 0) dF(v; 0) = D_0.$$

Clearly, the inequality does not satisfy when $\delta = 0$, which is the case of symmetric firms. It is easy to see that the left-hand-side is decreasing in δ, and thus the inequality is satisfied for δ small enough, i.e. if the competitive advantage of firm 1 over attribute a is high enough (as $\delta < 0$). On the other hand, firm 2 has no incentive to advertise a, as this always decreases its demand.

This implies that if there are multiple attributes, and firms have their competitive advantage over different attributes, then it might be the case that all attributes would be advertised even with out-of-store advertising. This is true when all consumers are attention limited, i.e. they only pay attention to at most one attribute. Specifically, suppose there are two attributes a and b: firm 1 has competitive advantage over a as defined above, and firm 2 has competitive advantage over b which can be similarly defined. The above analysis has shown that firm 1 will advertise attribute a if the competitive advantage is high enough. The same reasoning indicates that firm 2 also has an incentive to advertise b.

However, this may not necessarily be the case if consumers are heterogeneous, i.e. when some have higher attention level.[13] Suppose as before a proportion p consumers have one unit of attention capacity, and the rest $1 - p$ have two units of attention capacity. We have the following result:

Proposition 3.5. *In the asymmetric firm case, firms' advertising intensity on the additional attributes decreases with the proportion of consumers with high attention capacity.*

Proof. See Appendix 3.5.4, where we also give a proof when the high attention capacity consumers have infinite attention capacity. □

The logic of the above proposition is straightforward: each firm has an incentive to advertise the attribute over which it has a competitive advantage. However, by doing so, all full attention consumers will pay attention to both attributes and this hurts both firms. Therefore, when there is a large number of such consumers, firms instead choose to advertise less.

The proposition shows that when a lot of consumers are attention-limited, competition induces firms to advertise both attributes which benefit full attention consumers; however, when a large proportion of consumers have full attention, the information available is reduced. And thus consumer welfare may not be monotonic with the attention level p: as p decreases, firms gradually change their strategy to "hide"some attributes of their products. In other words, the presence of low attention consumers exerts positive externality on full attention consumers. Individually, each

[13]Here, we abstract from attention attraction as in Section 3.2.3, and assume that all consumers know the existence of both products.

consumer would be better off if he pays full attention to firms' products; however, collectively it would be better for the whole population to maintain some level of attention limitation. This also relates to some discussion about consumer knowledge: if most consumers only understand a small subset of aspects of a product, firms may choose to disclose all information so as to win those knowledgeable consumers; however, if more and more consumers get to understand every detail of the product, firms may choose to disclose less information which in turn hurt consumers.

This may also shed light on the difference between online shopping and off-line shopping: online shopping is characterized by information overload, which can be a result of the fact that online shoppers are more time-constrained than off-line shoppers, and thus online shoppers exhibit more attention limitation.

3.3.4 Costly Advertising

The above analysis has focus on costless advertising where firms have free access to advertising resource. This can be thought of as the situation in the short-run, where firms have sunk their costs of acquiring advertising space such as billboards and they only decide what information to be included in the advertisements. In the long-run, firms may also need to decide how much advertising resource to invest, especially as in-store advertising is probably less costly than out-of-store advertising.[14]

Incorporating costly advertising introduces additional trade-offs between marginal benefits and marginal costs of out-of-store and in-store advertising. Compared with our benchmark costless advertising model, firms use relatively more in-store advertising due to its low cost, however, this does not alter our basic intuition. In an attention-scarce economy, the marginal benefit of out-of-store advertising would be higher than that of in-store advertising, which is the dominating effect as long as the cost difference between two types of advertising is not too large. And thus firms tends to advertise more out-of-store and information available to consumers is limited.[15]

[14]The cost of out-of-store advertising includes, for instance, bidding for billboards, designing advertisements, etc. The cost of in-store advertising includes training salesmen and providing pamphlets, etc.

[15]A firm may incur additional costs when it includes more information in advertising, however such costs are likely to be less significant than bidding for advertising space and training employees.

3.4 Conclusion

We study how firms may use advertisements to manipulate consumers' attention. A monopolist advertises too few attributes and this reduces consumer welfare. Whether competition improves this situation depends on the form of advertising. When advertisements are out-of-store and affect how consumers evaluate products at the industry level, competition does not increase the information available to consumers; when advertisements are in-store and mainly affect consumers' evaluation at the product level, competition may lead to higher information availability. We also show that consumer heterogeneity plays a key role in shaping firms' advertising strategy when they are asymmetric. Our results suggest that instead of information overload, attention manipulation and the resulting information insufficiency may also be a serious concern.

3.5 Appendix

3.5.1 Proof of Proposition 3.3

Given s_2, we have

$$\begin{aligned}
\tfrac{\partial D_1}{\partial s_1} &= \textstyle\int_{\underline{\mathrm{u}}}^{\infty} G(v;s_2)d\tfrac{\partial G(v;s_1)}{\partial s_1} \\
&= -G(\underline{\mathrm{u}};s_2)\tfrac{\partial G(\underline{\mathrm{u}};s_1)}{\partial s_1} - \textstyle\int_{\underline{\mathrm{u}}}^{\infty} \tfrac{\partial G(v;s_1)}{\partial s_1} dG(v;s_2) \\
&= -(\textstyle\int_{-\infty}^{\underline{\mathrm{u}}} \tfrac{\partial G(\underline{\mathrm{u}};s_1)}{\partial s_1} dG(v;s_2) + \textstyle\int_{\underline{\mathrm{u}}}^{\infty} \tfrac{\partial G(v;s_1)}{\partial s_1} dG(v;s_2)) \\
&= -\textstyle\int_{-\underline{\mathrm{u}}}^{\infty}(\tfrac{\partial G(v;s_1)}{\partial s_1} - \tfrac{\partial G(-\underline{\mathrm{u}};s_1)}{\partial s_1})dG(v;s_2). \text{ (use the symmetry of } G(v;s) \text{ and } g(v;s))
\end{aligned}$$

If $\underline{\mathrm{u}} = 0$, we have

$$\frac{\partial G(v;s_1)}{\partial s_1} < 0 \text{ for all } v > 0, \text{ and } \frac{\partial G(0;s_1)}{\partial s_1} = 0.$$

Therefore, $\partial D_1/\partial s_1 > 0$. Hence, for all $\underline{\mathrm{u}}$ that is close to zero, we have that the demand for firm 1 is increasing in s_1. So we must have $s_1 = 1$ in equilibrium. The same reasoning indicates that $s_2 = 1$ also.

Notice that

$$\partial \frac{\partial G(v;s_1)}{\partial s_1} / \partial v = \frac{\partial g(v;s_1)}{\partial s_1}$$

Due to the symmetry of $g(v;s)$, there must exist $\bar{v}$ such that

$$\frac{\partial g(v;s_1)}{\partial s_1} < 0 \text{ for } 0 < v < \bar{v} \text{ and } \frac{\partial g(v;s_1)}{\partial s_1} > 0 \text{ for } v > \bar{v}.$$

Hence, if $\underline{\mathrm{u}} = -\bar{v}$, we have

$$\frac{\partial G(v; s_1)}{\partial s_1} \geq \frac{\partial G(-\underline{\mathrm{u}}; s_1)}{\partial s_1} \text{for all } v \geq -\underline{\mathrm{u}},$$

and then $\partial D_1/\partial s_1 < 0$. Therefore, there exists $\tilde{u} \in (-\bar{v}, 0)$ such that

$$\frac{\partial D_1}{\partial s_1}(\underline{\mathrm{u}}; s_1)|_{\underline{\mathrm{u}}=\tilde{u}} = 0.$$

And $\partial D_1/\partial s_1 > 0$ for all $\underline{\mathrm{u}} \in (\tilde{u}, 0]$.

3.5.2 Heterogeneous Consumers with Asymmetric Attributes

When consumers are heterogeneous in their attention capacity, and the valuation of attribute b is drawn from a more dispersed distribution, both firms advertising only attribute b may also be an equilibrium.

Suppose firm 2 only advertise b, if firm 1 advertises both attributes, the profit is given by

$$\pi^p_{a,b} = \alpha_a(s_a, s_b)D(a, b) + \alpha_b(s_a, s_b)D(b, b) + (1 - \alpha_a - \alpha_b)(s_a, s_b)D(ab, b)$$

where $D(k, b)$ denotes the demand for firm 1 when the consumer only pays attention to attribute b of firm 2 and pays attention to attribute k of firm 1. As we can see, there is a gain from full attention consumers, as $D(ab, b) > D(b, b)$; but there is a loss from partial attention consumers who only pay attention to attribute a, as $D(a, b) < D(b, b)$. Therefore, if it is difficult to induce consumers to pay attention to both attributes, firm 1 may find it profitable to only advertise attribute b; specifically, this is satisfied if

$$\frac{\alpha_{a1} - \alpha_{a2}}{\alpha_{b2} - \alpha_{b1}} > \frac{D(ab, b) - D(b, b)}{D(ab, b) - D(a, b)}$$

which means that the marginal effect of advertising attribute a on the proportion of consumers who only pay attention to a is relatively large compared to the proportion of consumers who pay attention to both attributes. [16]

[16] As we can see, whether this condition holds depends on the shape of $\alpha(s_a, s_b)$. In the quadratic interpretation, this condition is not satisfied.

3.5.3 Proof of Proposition 3.4

In the symmetric equilibrium, $s_1^{out} = s_2^{out} = s^{out}$ and $s_1^{in} = s_2^{in} = s^{in}$. If $s^{out} = 1$ and $s^{in} = 0$, the profit of each firms is given by

$$\pi = \frac{1-\alpha}{2} D(0) + \alpha D(0;0).$$

In order for this to be an equilibrium, both firms have no incentive to reduce out-of-store advertising and use the resource for in-store advertising. Precisely, the condition means that

$$(1-\alpha)[\frac{\partial D(s)}{\partial s}|_{s=0} - \frac{1}{4}] + \alpha \frac{\partial D(s_1;0)}{\partial s_1}|_{s_1=0} < 0,$$

which is satisfied if $\alpha < \bar{\alpha}$, where $\bar{\alpha}$ satisfies

$$\bar{\alpha} = \frac{(1-\bar{\alpha})(\frac{\partial D(s)}{\partial s}|_{s=0} - \frac{1}{4})}{\frac{\partial D(s_1;0)}{\partial s_1}|_{s_1=0}}.$$

It can be easily seen that $\bar{\alpha} \in (0,1)$, as the right hand side is positive when $\bar{\alpha} = 0$ (as $\frac{\partial D(s)}{\partial s}|_{s=0} < 0$), and equals zero when $\bar{\alpha} = 1$ (as $\frac{\partial D(s_1;0)}{\partial s_1}|_{s_1=0} > 0$) .

Moreover, in this equilibrium, both firms do not advertise the additional attribute, as this corresponds to the pure out-of-store advertising case and the result follows directly from Proposition 3.2.

3.5.4 Proof of Proposition 3.5

If one firm decides to advertise some additional attributes, it only advertise the one that it has competitive advantage. Advertising the attribute in which it is disadvantageous either increases the proportion of consumers who only pay attention to this attribute, or reduces the proportion of those who pay attention to no additional attributes, both of which hurt the firm. Hence, firm 1 chooses to advertise attribute a with intensity s_a and firm 2 chooses s_b to advertise attribute b.

Given s_a, the demand of firm 2 is given by

$$\begin{aligned} D_2(s_a, s_b) &= [\tfrac{p}{2} s_a + (1-p)((\tfrac{s_a}{2})^2 + s_a(1 - \tfrac{s_a+s_b}{2}))]D^- \\ &+[\tfrac{p}{2} s_b + (1-p)((\tfrac{s_b}{2})^2 + s_b(1 - \tfrac{s_a+s_b}{2}))]D^+ \\ &+\tfrac{1}{2} s_a s_b (1-p) D_f \\ &+[p(1 - \tfrac{s_a+s_b}{2}) + (1-p)(1 - \tfrac{s_a+s_b}{2})^2]D_0 \end{aligned}$$

where D^+ and D_0 are given in the main text; D^- is demand for a firm from consumers who only pay attention to its disadvantageous attribute, that is

$$D^- = \int_{\underline{u}}^{\infty} F(v;1) dF(v,\delta;1);$$

and D_f is the demand from those consumers who pay attention to both attributes, that is

$$D_f = \int_{\underline{u}}^{\infty} F(v,\delta;1,1) dF(v,\delta;1,1),$$

where

$$F(v,\delta;1,1) = \int\int\int_{v_0+v_a+v_b\leq v} dF_0(v_0)dF_a(v_a)dF_a(v_b-\delta),$$

is the valuation distribution of those consumers who pay attention to both attributes.

The first order condition is given by

$$\frac{\partial D_2}{\partial s_b} = \frac{1-p}{2}s_a(D_f - D^-) + (1-p)(1-\frac{s_a+s_b}{2})(D^+ - D_0) + \frac{p}{2}(D^+ - D_0).$$

If $p = 1$, i.e. all consumers are attention limited, then

$$\frac{\partial D_2}{\partial s_b} = \frac{1}{2}(D^+ - D_0) > 0,$$

which means that both firms fully advertise their advantageous attribute. If $s_a = 0$, all consumers only pay attention to b if $s_b > 0$, and we have

$$\frac{\partial D_2}{\partial s_b} = ((1-p)(1-\frac{s_b}{2}) + \frac{p}{2})(D^+ - D_0) > 0,$$

so firm 2 fully advertises attribute b.

If $p < 1$, and $s_a > 0$, the best response function of firm 2 is such that

$$\frac{\partial s_a}{\partial s_b} = \frac{D_f - D^-}{D^+ - D_0} - 1;$$

therefore, an symmetric equilibrium exists if $D_f - D^- < 0$, and the symmetric equilibrium is given by

$$s_a = s_b = s = \frac{(2-p)(D^+ - D_0)}{(1-p)(2(D^+ - D_0) - (D_f - D^-))}.$$

Then it is easy to check that $s < 1$ if

$$p < \frac{D^- - D_f}{(D^+ - D_0) + (D^- - D_f)}.$$

Otherwise, when p is big enough, the only equilibrium is $s_a = s_b = 1$. Moreover, it is obvious that s is increasing in p. Therefore, when there are more consumers with high attention capacity, the equilibrium intensity of advertising on additional attribute is lower.

The case of infinite attention capacity

The proof is basically the same as above except that: For those consumers with infinity attention capacity, they pay attention to both attribute as long as $min(s_a, s_b) > 0$. If $s_a, s_b > 0$, the demand of firm 1 is given by

$$D_1(s_a, s_b) = p\frac{s_a}{2}D^+ + p\frac{s_b}{2}D^- + p\frac{2 - s_a - s_b}{2}D_0 + (1-p)D_f,$$

As $D^+ > D_0$, firm 1 chooses $s_a = 1$ if it were to advertise a. If $s_a = 0$, the demand of firm 1 is

$$D_1(0, s_b) = (p\frac{s_b}{2} + 1 - p)D^- + p\frac{2 - s_b}{2}D_0.$$

Therefore, firm 1 does not advertise a if

$$D_1(0, s_b) > D_1(1, s_b) \Leftrightarrow (1-p)(D^- - D_f) > \frac{p}{2}(D^+ - D_0);$$

That is if

$$p < \frac{2(D^- - D_f)}{2(D^- - D_f) + (D^+ - D_0)};$$

and such p exists if $D^- > D_f$.

Thus, if p is small enough, firm 1 chooses not to advertise its advantageous attribute if firm 2 has already advertised the other attribute. However, if $s_b = 0$, then firm 1 fully advertise attribute a as if all consumers are attention limited. Hence, there are two pure-strategy equilibrium when p is small, either $s_a = 1, s_b = 0$ or $s_a = 0, s_b = 1$, in both of which only one attribute is advertised.

There is also a mix-strategy equilibrium, where each firm randomize between fully advertising its advantageous attribute and not advertising at all. However, in such an equilibrium, the probability that both attributes are advertised is still smaller than one.

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