Competitive Cross-Subsidization*

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Abstract

Cross-subsidization arises naturally when firms with different comparative advantages compete for consumers with heterogeneous shopping patterns. Firms then face a form of co-opetition, as they offer substitutes for one-stop shoppers and complements for multi-stop shoppers. When intense competition for one-stop shoppers drives total prices down to cost, firms subsidize weak products with the profit made on strong products. Firms have moreover incentives to seek comparative advantages on different products. Finally, banning below-cost pricing increases firms' profits at the expense of one-stop shoppers, which calls for a cautious use of below-cost pricing regulations in competitive markets.

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1 Introduction

Multi-product firms compete through a variety of pricing strategies. One commonly observed strategy is cross-subsidization in which firms price some products below cost and compensating the loss with profits from other products. Competition between Apple and Amazon in e-book and tablet computer markets offered an illustration. In 2010, Amazon was selling the "Kindle Fire" below cost,[1] when Apple pre-loaded 30,000 books free of charge on the iBooks store.[2] It was commonly recognized that Apple's iPad offered more functions than the Kindle Fire, whereas Amazon, with more than two million e-books, provided more variety and thus a higher match value than the iBooks store. Hence, each firm had a comparatively stronger product in relation to its rival. Furthermore, both firms were selling their comparatively weaker products below cost, and deriving profits from their strong products.[3] Moreover, consumers could combine the two firms' strong products, but not the weak ones: iPad users could download a free Kindle Application to access Amazon's e-books, whereas Kindle Fire users had no access to the iBooks store.

These strategies in such competitive markets as tablets and e-books are somewhat at odds with the existing theory. According to this theory, cross-subsidization arises in the context of regulated or monopolistic markets,[4] or in markets characterized by frictions such as consumers' limited information or bounded rationality (see the literature review below). We develop here a new approach, based on the diversity of purchasing patterns.

[1] The Kindle Fire, which offered access to the Amazon Appstore, streaming movies and TV shows, was sold in the U.S. at a retail price of $199. Amazon's hardware cost for a Kindle Fire was estimated at $201.70, not including "additional expenses such as software, licensing, royalties or other expenditures." See https://technology.ihs.com/389433/amazon-kindle-fire-costs-20170-to-manufacture.

[2] See Appleinsider's report, available at http://appleinsider.com/articles/10/03/25/apple_loads_up_new_ibooks_store_with_free_public_domain_ipad_titles.

[3] Before 2010, Amazon was also selling some newly released e-books below cost. However, it raised the prices after Apple proposed the controversial "agency model" for e-books. More recently, Amazon introduced more sophisticated version of its reader (e.g., the Oasis), which offer additional features. Still, the pattern of cross-subsidizing weaker products with stronger ones appears to have persisted from 2010 to 2016.

[4] For instance, Faulhaber (2005, pp.442) asserts that "under competitive conditions, the issue of cross-subsidy simply does not arise."

The literature on competitive multiproduct pricing often assumes that customers engage in "one-stop shopping" and purchase all products from the same supplier. Yet, in practice, many customers engage in multi-stop shopping and rely on several suppliers to fulfill their needs. The choice between these purchasing patterns is driven not only by the diversity and the relative merits of suppliers' offerings, but also by the transaction costs that buyers must bear in order to enjoy the products. As mentioned by Klemperer (1992), these transaction costs include physical costs such as transportation costs, and non-physical costs, such as the opportunity cost of time and the adoption cost of using a new electronic device. Following the terminology of the literature, we will refer to these costs as "shopping costs". Obviously, these costs vary across customers. For example, some consumers may face tighter time constraints and/or dislike shopping, whereas others may be less time-constrained and/or enjoy shopping. Indeed, some users, already familiar with the Kindle system, may be reluctant to switch to the iPad because of the associated learning costs,[5] whereas others may enjoy the adoption of a new device. All other things being equal, customers with high transaction costs tend to favor "one-stop shopping", whereas others are more prone to "multi-stop shopping".

We first note that the diversity of purchasing patterns gives rise to a form of "coopetition": on the one hand, firms offer substitutes for one-stop shoppers, who look for the best basket of products; on the other hand, firms offer complements for multi-stop shoppers, who seek to combine suppliers' best products. We show that this duality drastically affects firms' pricing strategies and can lead to cross-subsidization, even in competitive markets.

Specifically, we consider a setting in which two firms offer the same product line (which consists of two products, for simplicity). Consumers are perfectly informed about prices, as is indeed the case for e-books and tablets. To discard price-discrimination motives, we further assume that consumers have inelastic demands. Altogether, these assumptions allow us to abstract from the motivations already highlighted in the literature on cross-subsidization (see the literature review below). Our key ingredients are instead that: (i) consumers have heterogeneous shopping costs; and (ii) through lower costs and/or higher

[5] Before the launch of the iPad and the Kindle Fire, readers of Amazon's e-books were mainly using the original Kindle device.

consumer value, each firm enjoys a comparative advantage over one product. For the sake of exposition, we initially assume that firms have similar comparative advantages; that is, each firm has a stronger product than its rival, but overall their baskets generate the same surplus. In equilibrium, consumers with high shopping costs engage in one-stop shopping, and competition for these consumers drives firms' aggregate prices down to costs. By contrast, consumers with low shopping costs engage in multi-stop shopping and buy each firm's strong product, by which means the firms make a profit. Cross-subsidization therefore arises naturally, with each firm pricing its weak product below cost and subsidizing the resulting loss with the profit from its strong product.

This provides some insights on the outcome of co-opetition. On the one hand, aggregate price levels are "competitive": firms supply one-stop shoppers at cost. If firms could coordinate their pricing strategies, they would raise total prices in order to exploit one-stop shoppers. At the same time, however, a lack of coordination over the prices charged to multi-stop shoppers leads to "double marginalization", as each firm charges a margin on its strong product. This causes excessive cross-subsidization and results in not enough multi-stop shopping: limiting cross-subsidization would benefit both firms and consumers.

These insights are quite robust and remain valid in more general environments. We extend the analysis to the setting with heterogeneous consumer preferences and show that firms cross-subsidize weak products as long as competition for one-stop shoppers remains sufficiently intense and/or the number of consumers who demand the weak product only is relatively small. We also show that the analysis applies when the dispersion of shopping costs is limited (as long as both shopping patterns arise in equilibrium), or when one firm offers a better basket than the other, thus enjoying market power over one-stop shoppers.

We then explore the implications of these insights for firms' product positioning. We do so by introducing a preliminary stage in which they can improve their offerings (e.g., by investing in quality or cost reduction, or by dedicating more resources to negotiating better conditions with their suppliers). We find that firms have incentives to target different products, which gives rise to asymmetric comparative advantages such as described above – regardless of whether improvement decisions are public or private, and of focussing on pure or mixed strategies.

The prevalence of cross-subsidization in retailing markets has led many countries to adopt specific regulations prohibiting or restricting certain forms of below-cost pricing. These regulations are however quite controversial and have triggered an intense policy debate.[6] To shed some light on this debate, we consider a variant where firms cannot price below cost. The equilibrium then involves mixed strategies: firms sell weak products at cost but randomize the prices of their strong products. Banning below-cost pricing thus results in higher prices for one-stop shoppers (who can no longer purchase the products at cost), and greater profitability for firms (in fact, their expected profits more than double). The impact on multi-stop shoppers is less obvious. However, when weak products offer relatively low value, there are few one-stop shoppers; hence, firms are not too concerned about losing them and, as a result, charge higher prices to multi-stop shoppers as well. Depending on the distribution of shopping costs, this reduction in consumer surplus may exceed the increase in firms' profits and thus result in lower total welfare. This suggests that regulations on below-cost pricing in competitive markets should be carefully evaluated.[7]

Related literature. Cross-subsidization has been extensively studied in the context of regulated markets such as telecommunications, energy, and postal markets, in which historical incumbents may fight entry by pricing below cost in liberalized segments,[8] subsidizing their losses with the profits earned in protected segments. There is a small literature of cross-subsidization in unregulated, competitive markets; however, it typically assumes that consumers engage in one-stop shopping, and relies either on consumers' limited information or on bounded rationality.

In a setting where consumers are initially unaware of prices, Lal and Matutes (1994) show that firms advertise a loss-leader product in order to attract consumers.[9] Rhodes

[6]For instance, OECD (2007) argues that these laws are more likely to harm consumers than benefit them. See Section 5 for a more detailed discussion.

[7]By contrast, Chen and Rey (2012) show that banning below-cost pricing in concentrated markets can discipline the pricing behavior of a dominant firm competing with smaller firms. Such a ban then benefits both consumers and smaller rivals, and enhances social welfare.

[8]Such an exclusionary motive does not appear relevant for the tablet and e-book markets. Amazon can hardly hope to drive the iPad out of the market and, conversely, Apple is probably not primarily aiming to exclude Amazon's e-books.

[9]In equilibrium, consumers stop searching after the first visit, and thus all consumers are one-stop

(2015) develops a multi-product search model where competing firms randomly advertise one product at a low price, and may even set its advertised price below cost. By contrast, when consumers are aware of prices, Ambrus and Weinstein (2008) show that below-cost pricing does not arise when consumers have inelastic demands or when consumers have sufficiently diverse preferences.[10]

Ellison (2005) and Gabaix and Laibson (2006) study add-on pricing and product shrouding. Firms may price a leading product below cost (such as a hotel room fee) to lure consumers and subsidize the loss with the profit from shrouded add-on prices (such as telephone call charges and internet access fees). Grubb (2009) considers consumers with behavioral biases (such as over-confidence about the usage management) in the mobile-phone-service market, and shows that such bias can lead firms to price below cost on some units within a mobile-service plan. Recently, Johnson (2016) considers a setting in which one-stop shoppers may underestimate their needs, and shows that below-cost pricing may arise when consumers have different biases across products.[11]

In the case of tablets and e-books, as already noted, information about Apple and Amazon's prices was readily available to consumers. Furthermore, bounded rationality may be less relevant for simple goods such as e-books than for more complex products such as mobile telephony services. Yet, accounting for the diversity of purchasing patterns enables us to offer a rationale for the observed cross-subsidization, even in the absence of any limitation on consumers' information and rationality.

Chen and Rey (2012) also accounts for heterogeneous purchasing patterns; however, the two papers focus on different situations, and this leads to drastically different policy implications. Our previous paper focused on markets in which a dominant firm (e.g., a platform monopoly or a large retailer carrying a broad range of products) competes with smaller rivals (e.g., applications developers or specialty stores), and showed that

shoppers in their setting.

[10]They find that below-cost pricing arises only when consumers have elastic demands exhibiting a very specific form of complementarity.

[11]There is also a marketing literature on loss leading that focuses on impulsive purchases. For instance, Hess and Gerstner (1987) show that firms can use loss leader products to lure consumers, who will purchase some other products impulsively. Such impulsive purchases are similar to the "unplanned purchases" analyzed by Johnson (2016).

the dominant firm can profitably engage in loss leading by selling competitive products below cost. However, banning below-cost pricing then *hurts the dominant firm*, and it *unambiguously benefits consumers* (as well as smaller rivals) and *increases social welfare.* By contrast, we focus here on firms with similar product ranges; cross-subsidization then arises as a form of "co-opetition", which allows the firms to extract surplus from multi-stop shoppers while competing for one-stop shoppers. In this type of situation, banning below-cost pricing then *benefits the firms*, and *unambiguously harms one-stop shoppers* – it is also likely to harm consumers as a whole, as well as social welfare.

The empirical literature on multi-product pricing and heterogeneous purchasing patterns remains limited. For instance, the empirical literature on platform competition in media or healthcare industries[12] accounts the multiplicity of products (TV channels or doctors & hospitals), but tends to focus on one-stop shopping, whereas the literature on retail competition, where supermarkets offer a large number of products, tends to focus on specific product categories, such as breakfast cereals or mineral water, or on store-level competition (e.g., to assess the impact of a merger), thus ignoring shopping patterns. Recently, however, Thomassen *et al.* (2017) provides an interesting quantitative analysis of supermarket pricing that accounts for price effects across product categories as well as for the heterogeneity (and endogeneity) of shopping patterns. It finds in particular that different product categories exhibit price complementarity within a given retailer, and that this "cross-category complementarity derives from the consumer's shopping costs rather than from any intrinsic complementarity between the categories". It also finds that competition appears to be more intense for one-stop shoppers than for multi-stop shoppers.

The paper is organized as follows. Section 2 develops the baseline framework and presents our main insights. Section 3 extends the baseline model to account for heterogeneous preferences, whereas Section 4 explores its implications for firms' product positioning. Section 5 studies the impact of a ban on below-cost pricing. Finally, Section 6 concludes.

[12] See, e.g., Crawford and Yurukoglu (2012) and Crawford *et al.* (2018) for media, and Gowrisankaran *et al.* (2015) and Ho and Lee (2017) for healthcare.

2 Baseline model and main results

2.1 Setting

There are two product markets, A and B, and two firms, 1 and 2. Consumers are willing to buy one unit of A and one unit of B. Each firm $i = 1, 2$ can produce a variety of each good, A_i and B_i, at constant unit costs c_i^A and c_i^B.[13] Consumers have homogeneous preferences, and derive utility $u_i^h > c_i^h$ from firm i's variety of good $h = A, B$.[14]

Throughout the analysis, we assume that firm 1 enjoys a comparative advantage in the supply of good A, whereas firm 2 enjoys a comparative advantage for good B. This may reflect a specialization in different product lines, and be driven by better product quality (i.e., $u_1^A > u_2^A$), a lower cost (i.e., $c_1^A < c_2^A$), or a combination of both. For the sake of exposition, we initially focus on the case where firms enjoy the same comparative advantage for their strong products:

$$u_1^A - c_1^A - \left(u_2^A - c_2^A\right) = u_2^B - c_2^B - \left(u_1^B - c_1^B\right) \equiv \delta > 0, \tag{1}$$

implying that their baskets offer the same total value:

$$u_1^A - c_1^A + u_1^B - c_1^B = u_2^A - c_2^A + u_2^B - c_2^B \equiv w > \delta. \tag{2}$$

Later on, we consider asymmetric comparative advantages and endogenous specialization (see Section 4).

Our key modelling feature is that consumers incur a shopping cost, s, to visit a firm, and that this cost varies across consumers, reflecting the fact they may be more or less time-constrained, or that they value the shopping experience in different ways. Buying both products from the same firm thus generates one-stop shop benefits, by saving the cost of a second visit. Alternatively, the one-stop shop benefit s may be interpreted as consumption synergies stemming from purchasing both products from the same supplier.

[13] For the sake of exposition, we suppose that these costs are large enough to ensure that relevant prices are all positive.

[14] While we focus here on independent demands for A and B, the analysis carries over when there is partial substitution or complementarity, that is, when the utility derived from enjoying both A_i and B_h is either lower or higher than $u_i^A + u_h^B$.

Intuitively, consumers with high shopping costs favor one-stop shopping, whereas those with lower shopping costs can take advantage of multi-stop shopping. Shopping patterns are, however, endogenous and depend on firms' prices. To ensure that both types of shopping patterns arise, we will assume that the shopping cost s is sufficiently dispersed, namely:

Assumption A: The shopping cost s is distributed according to a cumulative distribution function $F(\cdot)$ with positive density function $f(\cdot)$ over $\mathbb{R}_+$.

Finally, we assume that firms compete in prices; that is, firms simultaneously set their prices, $\left(p_1^A, p_1^B\right)$ and $\left(p_2^A, p_2^B\right)$, and, having observed all prices, consumers then make their shopping decisions. We will look for the subgame-perfect Nash equilibria of this game.

2.2 Competitive cross-subsidization

We first show that, in equilibrium, multi-stop and one-stop shopping patterns coexist, with multi-stop shoppers buying strong products and competition for one-stop shoppers driving firms' basket prices down to cost:

Lemma 1 *Under Assumption A, in equilibrium:*

(i) *there are both multi-stop shoppers and one-stop shoppers;*

(ii) *multi-stop shoppers buy firms' strong products, A_1 and B_2; and*

(iii) *firms sell their baskets at cost.*

Proof. See Online Appendix A. ■

The first two insights are intuitive. Consumers with very low shopping costs (s close to 0) are willing to visit both firms so as to combine products with better value. Conversely, consumers with high shopping costs (s close to w, and thus such that $s > \delta$) are willing to visit one firm at most. The last insight follows directly from firms' symmetry vis-à-vis one-stop shoppers: as their baskets generate the same value w, Bertrand-like competition drives their prices down to cost.

Building on Lemma 1 leads to our main insight:

Proposition 1 *Under Assumption A, in equilibrium firms sell their weak products below cost.*

Proof. See Appendix A. ■

The intuition is fairly simple. From Lemma 1, one-stop shoppers buy firms' baskets at cost, and multi-stop shoppers only buy firms' strong products. Hence, firms either sell both products at cost, or cross-subsidize weak products with strong ones (cross-subsidizing strong products with weak ones would yield negative profits). Suppose now that a firm sells both of its products at cost, and consider the following "cross-subsidization" deviation: the firm slightly raises the price of its strong product, and reducing the price of its weak product by the same amount. This deviation does not affect the total price of the basket, which remains offered at cost to one-stop shoppers, but generates a profit from multi-stop shoppers, who now pay a higher price for the strong product. As the deviation decreases the value of multi-stop shopping, it may also induce some consumers to switch to one-stop shopping; however, this does not affect the firm: it was initially earning zero profit from multi-stop shoppers, and still earns zero profit from one-stop shoppers, regardless of which firm they go to. Hence, cross-subsidization is profitable.

To go further, we introduce the following regularity condition:

Assumption B: The density function $f(\cdot)$ is continuous and the inverse hazard rate $h(\cdot) \equiv F(\cdot)/f(\cdot)$ is strictly increasing.

The following proposition then establishes the existence of a unique equilibrium:

Proposition 2 *Under Assumptions A and B, there exists a unique equilibrium, in which both firms sell their weak products below cost and cross-subsidize them with their strong products. More precisely, defining:*

$$j(x) \equiv x + 2h(x), \tag{3}$$

we have:

(i) *consumers with a shopping cost $s < s^*$, where:*

$$0 < s^* \equiv j^{-1}(\delta) < \delta\,(< w),$$

engage in multi-stop shopping (they visit both firms and buy their strong products), whereas consumers with a shopping cost $s^* < s < w$ engage in one-stop shopping and buy both products from the same firm (either one); and

(*ii*) *both firms offer their baskets at cost, but charge the same margin $\rho^* = h(s^*) > 0$ on their strong products and the same margin $-\rho^* < 0$ on their weak products.*

Proof. See Appendix B. ■

The characterization of this equilibrium builds on Lemma 1. Firms only derive a profit from selling their strong products to multi-stop shoppers, that is, those consumers with a sufficiently low shopping cost, namely:

$$s < \delta - \rho_1 - \rho_2,$$

where $\rho_1 \equiv p_1^A - c_1^A$ and $\rho_2 \equiv p_2^B - c_2^B$ respectively denote firm 1 and 2's margins on their strong products. Hence, firm i's profit can be expressed as:

$$\pi_i(\rho_1, \rho_2) = \rho_i F(\delta - \rho_1 - \rho_2). \tag{4}$$

The monotonicity of the inverse hazard rate $h(\cdot)$ ensures that the profit function $\pi_i(\cdot)$ is strictly quasi-concave in ρ_i. Together with the "aggregative game" nature of $\pi_i(\cdot)$, which depends on ρ_i only through the sum $\rho_1 + \rho_2$, it also ensures that the equilibrium is unique and symmetric. Specifically, both firms charge the same positive margin ρ^* on their strong products,[15] characterized by the first-order condition:

$$\rho^* = h(\delta - 2\rho^*).$$

The equilibrium threshold for multi-stop shopping, s^*, satisfies:

$$s^* = \delta - 2\rho^* = \delta - 2h(s^*),$$

and is therefore given by $s^* = j^{-1}(\delta)$, where $j^{-1}(\cdot)$ is strictly increasing. Finally, in equilibrium, each firm earns a positive profit, equal to:

$$\pi^* = \rho^* F(s^*) = h(s^*) F(s^*).$$

[15] Firms thus sell their weak products with the same negative margin $-\rho^* < 0$. Yet, a firm would not benefit from dropping its weak product (e.g., by charging a prohibitive price): it would no longer serve one-stop shoppers, on which it makes no loss.

As mentioned in the Introduction, firms face a form of co-opetition: they compete for one-stop shoppers, but offer complementary products to multi-stop shoppers. Indeed, the firms' baskets are perfect substitutes for one-stop shoppers, and fierce competition for these consumers drives basket prices down to cost. Firms make instead a profit on multi-stop shoppers, who visit both firms in order to buy their strong products. Furthermore, a *reduction* in the price of one firm's strong product encourages additional consumers to switch from one-stop to multi-stop shopping, thereby *increasing* the other firm's profit. As is usual with complements, the prices of strong products are subject to double marginalization problems. When contemplating an increase in the price of its strong product, firm i balances between the positive impact on its margin ρ_i and the adverse impact on multi-stop shopping, but ignores the negative effect of this reduction in multi-stop shopping activity on the other firm's profit. Firms would therefore benefit from a mutual moderation of the prices charged on these products, e.g., through a bilateral price-cap agreement.[16] Interestingly, while double marginalization is usually associated with excessively high price *levels*, here it yields excessively distorted price *structures*: firms' total prices remain at cost, but they engage in excessive cross-subsidization, compared with what would maximize their joint profit. Keeping total margins equal to zero, firms' joint profit when charging a margin ρ on both strong products is given by:

$$2\rho F\left(\delta - 2\rho\right),$$

and is maximal for some $\hat{\rho} < \rho^*$.[17]

At first glance, that shopping costs generate complementarity in firms' products might not come as a surprise. Indeed, although consumers have independent demands for goods A and B, as one might expect, one-stop shopping introduces a complementarity between the products offered *within a firm*: cutting the price of A_i, say, is likely to steer one-stop shoppers towards firm i, which in turn boosts the sales of the firm's other product, B_i. This form of complementarity is not specific to our setting and is well understood. More interestingly, however, multi-stop shopping introduces here a complementarity *across firms*,

[16] See Rey and Tirole (2018).

[17] A standard revealed preference argument yields $\hat{\rho} F\left(\delta - 2\hat{\rho}\right) > \rho^* F\left(\delta - 2\rho^*\right) > \hat{\rho} F\left(\delta - \rho^* - \hat{\rho}\right)$, implying $\hat{\rho} < \rho^*$.

namely, between their strong products: cutting the price of one firm's strong product induces marginal consumers to switch from one-stop to multi-stop shopping, which boosts the sales of the other firm's strong product.[18]

2.3 Discussion

We now discuss a few robustness checks and variations of the baseline model.

• *Bundling.* As consumers have homogeneous valuations, there is no scope here for tying and (pure or mixed) bundling. For instance, if one firm ties both products together physically, consumers are forced to engage in one-stop shopping, and price competition for one-stop shoppers leads to zero profit. A similar reasoning applies to pure bundling when products are costly, to such an extent that it does not pay to add one's favorite variety to a bundle. In principle, a firm may also engage in mixed bundling, and offer three prices: one for its strong product, one for the weak product, and one for the bundle. However, as one-stop shoppers only purchase the bundle, and multi-stop shoppers only buy the strong product, no consumer will ever pick the weak product on a stand-alone basis. Hence, only two prices matter here: the total price for the bundle, and the stand-alone price for the strong product. As these prices can be implemented using the stand-alone prices for the two products, offering a bundled discount (in addition to these stand-alone prices) cannot generate any additional profit.

• *Multiple firms.* The analysis is unchanged when weak products are supplied by additional firms as well. For example, if weak products are also supplied at cost by competitive fringe(s), and regardless of whether these fringe firms each supply one or both of these products, then each firm $i = 1, 2$ would still undercut the fringe firms and offer its weak product at the same below-cost price. The same applies if each firm $i = 1, 2$ offers both weak products as well as its strong product. For example, if firm 1 can not only offer A_1 and B_1, but also produce the weaker variety A_2 in the same conditions as firm 2, then it would still sell A_1 and B_2 at the same prices as before, and either not offer A_2, or offer it at unattractive prices (e.g., at cost).

[18] A similar complementarity for multi-stop shoppers arises when shopping patterns are driven by heterogeneous preferences rather than transaction cost differences; see Armstrong and Vickers (2010).

• *Bounded Distribution of Shopping Costs.* The baseline model assumes a widespread dispersion of consumers' shopping costs, spanning the entire range from "pure multi-stop shoppers" (consumers with $s = 0$ always choose the best value offered for each product) to "pure one-stop shoppers" (consumers with $s \geq \delta$ never visit a second firm). We show in Online Appendix B.1 that, even with less dispersed distributions of shopping cost, cross-subsidization still occurs as long as one-stop and multi-stop shopping patterns coexist: competition for one-stop shoppers then drives total prices down to cost, but firms obtain a profit by selling their strong products to multi-stop shoppers; hence, they sell their weak products below cost.

• *Non-linear pricing.* For the sake of exposition, we focus on unit demands, and so linear prices are efficient. If instead consumers have an individual elastic demand of the form $q = d(p)$, where $d'(p) < 0$, linear prices are no longer efficient, and firms would have an incentive to offer non-linear prices such as two-part tariffs. For instance, to maximize bilateral gains from trade, firms could use cost-based two-part tariffs, with a constant marginal price reflecting the cost of production and a fixed designed to share the resulting surplus. Yet, the analysis carries over, applying the above analysis to the fixed fees. We show in Online Appendix B.2 that marginal prices are equal to costs, and firms offer their overall baskets at cost, but firms subsidize the fees on their weak products. Interestingly, even if tariffs are individually efficient (in that they induce consumers to buy the efficient quantity, which maximizes the bilateral gains from the transactions), the equilibrium tariffs still feature double marginalization: keeping total fixed fees constant, those charged on strong products exceed the level that would maximize industry profit.

• *Online Retailing.* To analyze the impact of online retailing, suppose that a fraction λ of "internet-savvy" consumers see their shopping costs drop to zero. We show in Online Appendix B.3 that, by modifying the distribution of shopping costs, the development of online sales is not only profitable, but moreover *increases* the prices of strong products: while one-stop shoppers can still buy firms' baskets at cost, multi-stop shoppers (including those buying online) face higher prices as the proportion of online customers increases.

3 Heterogeneous preferences

In the above analysis, cross-subsidization results from two key features: competition for one-stop shoppers drives total margins down to zero, and multi-stop shoppers purchase strong products (and only those ones); as a result, firms cross-subsidize their weak products, in order to make a profit from selling the strong products to multi-stop shoppers. In practice, however, consumers may have heterogeneous preferences over firms' offerings, and/or may be interested only in some of the products; this may relax competition for one-stop shoppers and may also induce some consumers to buy only the weak products. We now show that cross-subsidization still arises, however, as long as competition for one-stop shoppers remains sufficiently intense.

3.1 Horizontal differentiation

Multi-product retailers such as supermarkets often offer differentiated brands (including their own private labels) and consumers may be quite heterogeneous in their valuations over these products. To capture this, we now assume that firms are horizontally differentiated, with consumers' preferences following a classic Hotelling pattern. Specifically, consumers are uniformly distributed along an Hotelling segment of unit length and indexed by their location $x \in [0,1]$, and firms' offerings are located at the two ends of the line: that is, A_1 and B_1 are located at 0, say, whereas A_2 and B_2 are located at 1. Denoting by t the Hotelling differentiation parameter, a consumer located at a distance x from one variety of a product therefore incurs a cost tx when purchasing that variety, and $t(1-x)$ when purchasing the other variety. We also assume that the distribution of shopping costs is independent of consumers' locations.

The location x can be interpreted as consumers' relative preference for the two firms: one-stop shoppers located close to 0 (resp., 1) now favor firm 1 (resp. firm 2). Specifically, a one-stop shopper located at x obtains $w - 2tx - m_1 - s$ from patronizing firm 1 and $w - 2t(1-x) - m_2 - s$ from going instead to firm 2, where m_i denotes firm i's total margin on its basket. Thus, one-stop shoppers favor firm 1 if

$$x < \hat{x} \equiv \frac{1}{2} - \frac{m_1 - m_2}{4t}.$$

Multi-stop shoppers still favor strong products, as in the baseline model. Thus, consumers with $x < \hat{x}$ favor multi-stop shopping over patronizing firm 1 if their shopping cost satisfies

$$s < \lambda_1(x) \equiv \tau_1 - t + 2tx,$$

where $\tau_1 \equiv \delta + \mu_1 - \rho_2$. Likewise, consumers located at $x > \hat{x}$ prefer multi-stop shopping to patronizing firm 2 if

$$s < \lambda_2(x) \equiv \tau_2 + t - 2tx,$$

where $\tau_2 \equiv \delta + \mu_2 - \rho_1$.

Thus, the demand for the bundles $A_1 - B_1$ and $A_2 - B_2$ can be expressed respectively as

$$D_1 \equiv \int_0^{\hat{x}} [1 - F(\lambda_1(x))]\, dx,\ D_2 \equiv \int_{\hat{x}}^1 [1 - F(\lambda_2(x))]\, dx.$$

whereas the demand for multi-stop shopping of two strong products, $A_1 - B_2$, is given by

$$D \equiv \int_0^{\hat{x}} F(\lambda_1(x))dx + \int_{\hat{x}}^1 F(\lambda_2(x))dx.$$

Then, firm 1's total profit can be written as

$$\Pi_1 = m_1 D_1 + \rho_1 D = m_1 (D_1 + D) - \mu_1 D. \tag{5}$$

For simplicity we assume that the distribution of the shopping cost is bounded above by $\bar{s}$, which is however large enough to allow for both types of shopping patters (one-stop and multi-stop); to ensure continuity as the differentiation parameter t tends to vanish, we also assume that the density $f(s)$ is continuously differentiable. We further suppose that the total value w is large enough to ensure full participation (all consumers buy both products). The demand is as then illustrated in Figure 1:

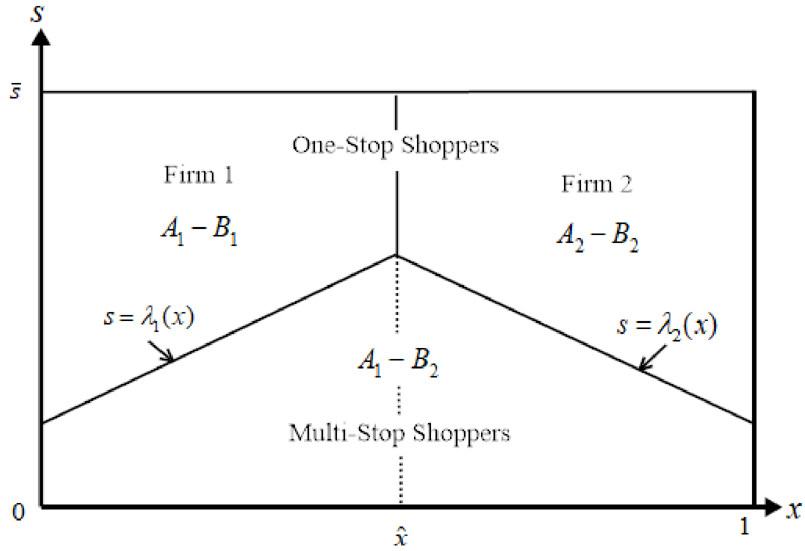

Figure 1: Heterogeneous Preferences

The heterogeneity of consumers' preferences over the two firms relaxes the intensity of competition for one-stop shoppers; as a result, in equilibrium firms charge positive total margins: $\tilde{m}_1(t) = \tilde{m}_2(t) = \tilde{m}(t) > 0$. Yet, the following Proposition shows that, as long as this competition remains sufficiently intense (that is, as long as the differentiation parameter t is not too large), firms keep cross-subsidizing their products. We first show that this feature arises in any symmetric equilibrium with both consumption patterns; we then provide a sufficient condition (namely, that the density $f(s)$ is non-increasing) that ensures the existence of such an equilibrium:

Proposition 3 *Suppose that consumers' preferences follow the Hotelling pattern described above, and focus on symmetric equilibria in which both consumption patterns coexist, as depicted by Figure 1. We have:*

(i) *There exists $\bar{t} > 0$ such that, in the range $t \in (0, \bar{t})$, firms charge a positive total margin over their products (that is, $\tilde{m}(t) > 0$) but keep selling their weak products below cost (that is, $\tilde{\mu}(t) < 0$); in addition, both $\tilde{m}(t)$ and $\tilde{\mu}(t)$ increase with t.*

(ii) *If the density $f(s)$ is non-increasing, then there exists $\hat{t} > 0$ such that, in the range $t \in \left(0, \hat{t}\right)$, there exists a unique such equilibrium.*

Proof. See Online Appendix C.1. ■

Proposition 3 shows that cross-subsidization can still occur when firms offer differentiated brands. However, its magnitude decreases as competition for one-stop shoppers becomes softer. In the particular case where shopping costs are uniformly distributed, it can further be shown that, in equilibrium, firms' total margin increases with t and their margin for weak products is given by

$$\tilde{\mu}(t) = \frac{3\bar{s}t}{3\bar{s} + 2t - \delta} + \frac{t}{6} - \frac{\delta}{3},$$

which also increases with t and is null for some $\bar{t} > 0$;[19] hence, cross-subsidization arises for $t < \bar{t}$, and disappears for $t > \bar{t}$: as competition for one-stop shoppers becomes less and less intense, firms charge them higher total margins, up to the point where they can charge high enough margins on their strong products, to exploit multi-stop shoppers, without selling their weak products below cost.

3.2 Stand-alone demands

Other sources of consumer heterogeneity could further affect the analysis. For instance, some consumers may be interested in only one of the products rather than in the whole assortment, and among these some may prefer the strong product of a firm but others may prefer its weak product. More generally, even when most consumers prefer one firm's variety of a good over the rival's variety, some consumers may nevertheless have a strong preference for the latter. Intuitively, firms will further engage in cross-subsidization when more consumers are interested in their "strong" products, and will instead reduce the level of cross-subsidization when more consumers are specifically interested in their "weak" products. Suppose for example that, in addition to the multi-product consumers with demand as described in the previous section, a small mass σ of consumers are only interested in the strong products, $A1$ and $B2$, and a mass ω of consumers are interested only in the weak products, $A2$ and $B1$. Firm i's profit is then given by:

$$\begin{aligned} \Pi_i &= m_i D_i + \rho_i D + \sigma \rho_i + \omega \mu_i \\ &= m_i (D_i + D + \sigma) - \mu_i (D + \sigma - \omega). \end{aligned}$$

[19] Equilibrium existence is moreover guaranteed in that range (i.e., $\bar{t} < \hat{t}$) if shopping costs are sufficiently dispersed, e.g., $\bar{s} > 7\delta/2$; see Online Appendix C.1.

As

$$\frac{\partial^2 \Pi_i}{\partial \sigma \partial \mu_i} < 0 < \frac{\partial^2 \Pi_i}{\partial \omega \partial \mu_i},$$

a revealed preference argument shows that, other things being equal, an increase in the mass σ of customers interested in its strong product gives the firm an incentive to sell its weak product further below cost (i.e., to decrease the margin μ_i), whereas an increase in the mass ω of customers specifically interested in its weak product discourages cross-subsidization.

To further explore this, we focus below on the case where $\sigma = 0$ and the shopping cost s is uniformly distributed on $[0, \bar{s}]$; also, to limit firms' market power on the weak products, $A2$ and $B1$, we assume that they are also offered at cost by a competitive fringe – hence, the presence of consumers only interested in these products may limit the scope for cross-subsidization, but does not confer additional market power to the firms. We have:

Proposition 4 *Suppose that a unit-mass of consumers have preferences following the Hotelling pattern previously described, with shopping costs uniformly distributed between* 0 *and* $\bar{s}$*, and that, in addition, for each firm there is a small mass* ω *of consumers only interested in its weak product. There exists* $\hat{t} > 0$*,* $\hat{\omega} > 0$ *and* $\psi(\omega)$ *satisfying* $\psi(0) = 0$ *and* $\psi'(\omega) > 0$*, such that there exists a symmetric equilibrium in pure strategies for any* $(\omega, t) \in [0, \hat{\omega}] \times [\psi(\omega), \hat{t}]$*. Furthermore, there exists* $\phi(\omega)$*, satisfying* $\phi'(\omega) < 0$ *and* $\hat{t} > \phi(\omega) > \psi(\hat{\omega})$ *in the range* $\omega \in [0, \hat{\omega}]$*, such that, in this symmetric equilibrium, firms cross-subsidize their weak products whenever* $\phi(\omega) > t > \psi(\omega)$*.*

Proof. See Online Appendix C.2. ■

These findings are illustrated in Figure 2. It can be checked that, as expected, the equilibrium margins $\tilde{m}(\omega, t)$ and $\tilde{\mu}(\omega, t)$ increase with t: product differentiation softens competition for one-stop shoppers, which reduces the scope for cross-subsidization; the limit case $t = \phi(\omega)$ corresponds to $\tilde{\mu}(\omega, \phi(\omega)) = 0$, where firms stop cross-subsidizing their products. Firms' profits also increase with product differentiation, thanks to reduced competition.

In addition, $\tilde{m}(\omega, t)$ and $\tilde{\mu}(\omega, t)$ also increase with ω: single-product consumers' demand for the weak products reduces the scope of cross-subsidization, which softens competition for one-stop shoppers as well; as $\tilde{\mu}(\omega, t)$ increases in both t and ω, it follows that $\phi(\omega)$ decreases in ω. Firms' profits may however decrease as the demand for weak products increases, as the loss from serving these single-product consumers may more than offset the gain due to reduced competition.

Firms incur a loss from selling at below-cost prices their weak products (and only those) to single-product consumers; they would therefore be tempted to drop these products if this loss were to exceed the gain from serving one-stop shoppers. This happens when ω is sufficiently large compared with t, namely, when $t < \psi(\omega)$.

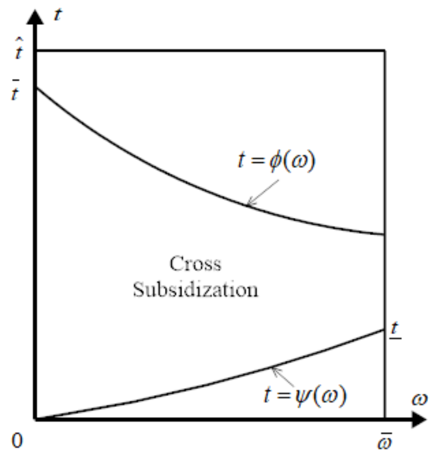

Figure 2: Stand-alone demand
for weak products

4 Product choice

The above analysis relies on the assumption that firms have comparative advantages over different products. We now endogenize firms' product choices and show that, indeed, firms' have an incentive to improve their positions on different products. We first extend the baseline setting by considering arbitrarily given positions in the two markets (Section 4.1). We then endogenize firms' product choice decisions (Section 4.2).

4.1 Asymmetric comparative advantage

We have so far assumed that each firm enjoyed a comparative advantage in one market, and that their advantages were moreover of the same magnitude. We now extend the analysis to arbitrary positions of the firms.

Intuitively, when one firm benefits from a comparative advantage in *both* markets, then the other firm will not attract any consumer; hence, there is no multi-stop shopping, and cross-subsidization becomes a moot issue. The following proposition confirms this intuition and shows that, by contrast, multi-stop shopping and cross-subsidization keep arising as long as each firm enjoys a comparative advantage in one market.

Without loss of generality, we suppose that firm 1 benefits from a comparative advantage $\delta_1 > 0$ in market A, which exceeds firm 2's comparative advantage δ_2 in market B: $\delta_1 > \delta_2$; note that we allow for $\delta_2 < 0$, in which case firm 1 actually enjoys a comparative advantage in both markets.

In the absence of firm 2, firm 1 would sell both of its products as long as its individual margins do not exceed consumers' valuations; by charging m_1, it would attract all consumers with a shopping cost lower than $v_1 = w_1 - m_1$, where w_1 denotes the total surplus generated by firm 1's basket. Hence, it would choose the "monopoly" margin

$$m_1^M \equiv \arg\max_{m_1} \{m_1 F(w_1 - m_1)\},$$

which is uniquely defined under Assumptions A and B. We have:

Proposition 5 *Suppose that firm* 1 *enjoys a weakly larger comparative advantage:* $\delta_1 \geq \max\{\delta_2, 0\}$*; under Assumptions A and B, there exists a unique trembling-hand perfect equilibrium, in which:*

(*i*) *Firm* 2 *sells its basket at cost but firm* 1 *attracts all one-stop shoppers and charges them a total margin* m_1 *reflecting its overall comparative advantage over the two products:*

$$m_1^* = \min\left\{m_1^M, \delta_1 - \delta_2\right\}.$$

(*ii*) *If firm* 2 *does not enjoy a comparative advantage in the other market (i.e.,* $\delta_2 \leq 0$*), then it attracts no consumer; hence, there is no multi-stop shopping, and cross-subsidization need not arise.*

(iii) *If instead firm* 2 *enjoys a comparative advantage in the other market (i.e.,* $0 < \delta_2 < \delta_1$*), then consumers with a shopping cost*

$$s < s^* \equiv j^{-1}(\delta_2),$$

where $j(\cdot)$ *is defined by* (3)*, engage in multi-stop shopping (they visit both firms and buy their strong products), and cross-subsidization arises: both firms sell their weak products below costs, with the same negative margin equal to* $-h(s^*) < 0$*.*

Proof. See Online Appendix D.1. ■

The outcome of competition for one-stop shoppers is intuitive: firm 1's basket offering a greater surplus, it wins the competition for one-stop shoppers and can charge them a total margin as high as its relative comparative advantage, $\delta_1 - \delta_2$; it does so when firm 2 exerts a competitive pressure (i.e., $\delta_1 - \delta_2 < m_1^M$), otherwise it charges the monopoly margin m_1^M.

That firm 2 keeps subsidizing its weak product is not surprising: as its overall basket is less attractive, competition for one-stop shoppers leads firm 2 to offer its basket at cost; as it enjoys market power over multi-stop shoppers, however, it charges a positive margin on its strong product, and must therefore sell the weak product below cost. To understand why firm 1 still subsidizes its weak product even though it now enjoys market power over one-stop shoppers as well, consider again the following thought experiment. Increase firm 1's margin on its strong product by a small amount and decrease the margin on its weak product by the same amount, so as to maintain the total margin m_1. This alteration of the price structure does not affect the profit made on one-stop shoppers (who pay the same total price for the basket) but increases the profit made on multi-stop shoppers (who pay a higher price for the strong product). In addition, this induces some marginal multi-stop shoppers to switch to one-stop shopping and buy firm 1's weak product as well (instead of buying only its strong product). It is therefore profitable for firm 1 to keep altering the price structure as long as it earns a non-negative margin on its weak product; hence, in equilibrium it sells its weak product below cost.

4.2 Endogenous comparative advantage

Our baseline model assumes that firms have comparative advantages over different products. We show now such asymmetric comparative advantages arise endogenously when firms can invest in cost reduction or quality improvement. For the sake of exposition, we suppose that firms initially provide the same value for each product, and add a preliminary stage in which they can improve the value of their products. For tractability, we first focus on a simple setting in which firms can allocate a "value-improvement" endowment Δ among the products A and B; that is, each firm can enhance the value of it products, subject to the constraint that the overall improvement cannot exceed Δ. For instance, firms may have to prioritize the projects of their R&D units so as to target quality-improving and/or cost-reducing innovations across their products. Supermarkets face similar choices for their private labels; in addition, they employ buying agents to negotiate with suppliers, and may concentrate their bargaining efforts so as to obtain better deals on specific products. At the end of this section, we discuss how the insights obtained in this simple setting extend to more general investment environments in which firms choose as well their improvement capability Δ.

We thus consider the following extended game:

- Stage 1: each firm $i = 1, 2$ chooses $(\Delta_{A_i}, \Delta_{B_i}) \in \mathcal{S} \equiv \left\{(\Delta_A, \Delta_B) \in \mathbb{R}_+^2 \mid \Delta_A + \Delta_B \leq \Delta\right\}$; these decisions are simultaneous.
- Stage 2: firms simultaneously set the prices for their products.

Firms' pricing decisions will obviously be driven in part by their own improvement decisions. Whether a firm's pricing decisions can also be contingent on the other firm's improvement decisions depends on the observability of these decisions. For example, quality improvements are more likely to be observed than cost reductions or lower input tariffs. We will consider here both extreme situations, in which improvement decisions are either publicly observed, or remain private, at the end of the first stage. We assume however that consumers observe these decisions before making their purchasing decisions. This is consistent with the quality/cost dichotomy highlighted above: this amounts to assume that consumers can observe the quality of the products offered; in case of reductions in costs or wholesale prices, the assumption is innocuous as consumers' purchasing decisions do not depend on them.

We look for the subgame trembling-hand perfect Nash equilibria of this two-stage game. The following Proposition shows that firms have an incentive to invest in different products, which in turn gives rise to cross-subsidization:

Proposition 6 *In the above two-stage game, and regardless of whether improvement decisions are public or private at the end of the first stage, there exists exactly two subgame trembling-hand perfect Nash equilibria in pure strategies, in which: (i) in stage 1, firms enhance by* Δ *the value of their offerings in different markets; (ii) in stage 2, they sell their baskets at costs but, thanks to cross-subsidization, obtain a positive profit by charging* $\rho^* = h(s^*) > 0$ *on their strong products, where* $s^* = j^{-1}(\Delta)$*.*

In addition:

(i) if improvement decisions remain private at the end of the first stage, then there exists a symmetric mixed-strategy equilibrium in which firms invest Δ *and charge* ρ^* *on either product with equal probability; and*

(ii) if improvement decisions are publicly observed at the end of the first stage, then when $h(\cdot)$ *is weakly concave there is a symmetric mixed-strategy equilibrium in which firms invest* Δ *on either product with equal probability.*

Proof. See Online Appendix D.2. ■

To capture this intuition in its easiest form, consider a simple discrete-choice variant of the above game in which each firm must simply choose which product to target (that is, $(\Delta_{A_i}, \Delta_{B_i}) \in \{(\Delta, 0), (0, \Delta)\}$). If both firms invest in the same market, then their offerings are not differentiated and head-to-head competition leads to zero profit for each firm. If instead they invest in different products, then they sell their baskets at cost but obtain a positive profit equal to $\pi^* = F(s^*) h(s^*)$, where $s^* = j^{-1}(\Delta)$. Hence, there are two pure-strategy equilibria, in which firm 1 invest in one product whereas firm 2 invests in the other product. Whether firms observe each other's improvement decisions at the end of the first stage does not affect these equilibria: if a firm expects its rival to invest entirely on a single product, and to offer its basket at cost (implying that serving one-stop shoppers cannot bring any benefit), it has an incentive to invest on the other product, in order to maximize the value offered to multi-shop shoppers and exploit their demand. The above proposition shows that the argument extends to continuous allocation decisions.

Interestingly, there also exists a mixed-strategy equilibrium in which firms invests in either product with equal probability. Half of the time, they then end-up with similar offerings, in which case all products are supplied at cost. However, the rest of the time each firm ends-up with a strong and a weak product, and cross-subsidizes its weak product with the strong one. It follows that prices are also stochastic, and consistent with each firm's offering random discounts or special offers on one product (either one). Consider, for example, the case of supermarkets negotiating a discount Δ off the regular input costs, c_A or c_B. Half of the time, every product $i = A, B$ is sold below the non-discounted cost c_i by one firm (at price $c_i - \rho^*$), and above the discounted cost $c_i - \Delta$ by the other firm (at price $c_i - \Delta + \rho^*$); this induces multi-stop shoppers to mix-and-match in order to benefit from the lower price, $c - \Delta + \rho^*$, on both products.[20]

As mentioned above, we have also considered more general investment environments in which each firm $i = 1, 2$ can choose any improvements $\Delta_{A_i} \geq 0$ and $\Delta_{B_i} \geq 0$, at total cost $C(\Delta_{A_i} + \Delta_{B_i})$ – see Online Appendix D.3. Under mild regularity conditions ensuring the existence of an equilibrium in which both firms invest (and, for tractability, ensuring that the market is fully covered), the unique subgame (trembling-hand) perfect Nash equilibria in pure strategies are such that firms choose to invest in different products, which leads them to sell again their weaker products below cost. Interestingly, however, it is always the case that one firm invests more than the other, in order to obtain market power over one-stop shoppers as well.

[20]The rest of the time, firms' prices coincide for each product, and multi-stop shopping thus does not arise – all prices are at cost in case of public decisions, whereas cross-subsidization still arises in case of private decisions; in both cases, however, firms obtain the same expected profit, equal to $\pi^*(\Delta)/2$.

5 Resale-below-cost laws

In regulated industries, cross-subsidization has been a well-recognized issue in both theory and practice,[21] and has prompted regulators to impose structural or behavioral remedies.[22] In contrast, in competitive markets, the policy debate is more divided. Although below-cost pricing might be treated as predatory,[23] in many cases (including the Apple vs. Amazon example) there is no such thing as a "predatory phase" followed by a "recoupment phase" (e.g., once rivals have been driven out of the market), which constitute key features of predation scenarios.[24] As mentioned in the Introduction, this has led many countries to adopt specific rules prohibiting or limiting below-cost pricing in retail markets. These rules, known as Resale-Below-Cost (RBC hereafter) laws, have been the subject of heated policy debates. In Ireland, for example, based on evidence that consumers pay more when grocery goods are subject to the prohibition of below-cost sales, in 2005 the Irish Competition Authority recommended terminating the RBC law.[25] However, the Irish Joint Committee on Enterprise and Small Business recommended keeping the RBC law due to concerns about an increased concentration in grocery retailing and predatory pricing. The Irish example highlights the dilemma of antitrust authorities:

[21] The seminal paper of Faulhaber (1975) rigorously defines the concept of cross-subsidy and introduces two tests for subsidy-free pricing, which have been widely applied in both regulation and antitrust enforcement. See Faulhaber (2005) for a recent survey.

[22] Such concerns led, for instance, to the break-up of AT&T and the imposition of lines of business restrictions on local telephone companies (*U.S. v. AT&T* 1982). More recently, the European Commission required the German postal operator to stop cross-subsidizing its parcel services with the profit derived from its legal monopoly on letter services (*Deutsche Post* 2001).

[23] See, for example, Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory pricing tests should be designed. Rao and Klein (1992) and Berg and Weisman (1992) examine the treatment of cross-subsidization under US antitrust laws.

[24] In the US, for instance, the feasibility of recoupment is necessary for a predation case since the Supreme Court decision in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*

[25] The Irish Competition Authority examined pricing trends under the Groceries Order (the RBC law introduced in Ireland in 1987). The authority found that prices for grocery items covered by the Order had been increasing, while prices for grocery items not covered by the Order had been decreasing; it concluded that, on average, Irish families were paying 500 euros more per year because of the Order. See OECD (2007).

RBC laws may prevent dominant retailers from engaging in predatory pricing against smaller or more fragile rivals, but in competitive markets they may also lead to higher prices and thus harm consumers.

We now examine the impact of a ban on below-cost pricing in our baseline setting. We first note that such a ban raises equilibrium basket prices, which benefits firms at the expense of one-stop shoppers:

Proposition 7 *When below-cost pricing is prohibited, in equilibrium each firm obtains a profit at least equal to:*

$$\bar{\pi} \equiv \max_{\rho} \rho F(\delta - \rho) > 2\pi^*.$$

It follows that, compared to the equilibrium that arises in the absence of a ban under Assumptions A and B:

- (i) *firms more than double their profits; and*
- (ii) *one-stop shoppers face higher prices for the firms' baskets.*

Proof. See Appendix C. ■

The intuition is simple. If the rival offers both of its products at cost, a firm cannot make a profit on one-stop shoppers, but can still make a profit by selling its strong product to multi-stop shoppers. Indeed, charging a margin $\rho < \delta$ induces consumers with shopping cost $s < \delta - \rho$ to buy both strong products, thus generating a profit $\rho F(\delta - \rho)$. By choosing the optimal margin:

$$\bar{\rho} \equiv \arg\max_{\rho} \rho F(\delta - \rho), \tag{6}$$

the firm can thus secure $\bar{\pi}$. Hence, in any equilibrium, each firm earns a profit at least equal to $\bar{\pi}$. Furthermore, as the rival can no longer subsidize its weak product, each firm now more than doubles its profit: $\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) > 2\rho^* F(\delta - 2\rho^*) = 2\pi^*$. Finally, equilibrium total margins are positive, as weak products cannot be sold below cost, and strong products are sold with a positive margin. One-stop shoppers thus face higher prices than in the absence of the ban.

Intuitively, banning below-cost pricing should lead the firms to offer their weak products at cost. Furthermore, as a firm can obtain at least $\bar{\pi}$ by charging $\bar{\rho}$ to multi-stop shoppers, it will never charge so low a margin that it would obtain less than $\bar{\pi}$, even if it were to attract all shoppers. That is, no firm will ever charge $\rho < \underline{\rho}$, where $\underline{\rho}$ is the lower solution to:

$$\underline{\rho} F \left(w - \underline{\rho} \right) = \bar{\pi}. \tag{7}$$

The next proposition shows that, while there is no pure-strategy equilibrium when below-cost pricing is banned, there exists an equilibrium in which firms indeed sell their weak products at cost, and obtain an expected profit equal to $\bar{\pi}$ by randomizing the margins on their strong products between $\underline{\rho}$ and $\bar{\rho}$:

Proposition 8 *If s is distributed with positive density over $\mathbb{R}_+$ (Assumption A) or over $[0, \bar{s}]$ with $\bar{s} > w$, then when below-cost pricing is prohibited:*

(i) *there exists no equilibrium in pure strategies; and*

(ii) *there exists a symmetric mixed-strategy equilibrium in which firms obtain an expected profit equal to $\bar{\pi}$ by selling weak products at cost and randomizing the margins on strong products over $\left[\underline{\rho}, \bar{\rho}\right]$.*

Proof. See Online Appendix E.1. ■

As in the sales model of Varian (1980), firms face a dilemma: they are tempted to exploit "captive" customers (the uninformed consumers in Varian's model, and multi-stop shoppers here) but, at the same time, they want to compete for "price-sensitive" customers (the informed consumers in Varian's model, and one-stop shoppers here). To see why there is no pure-strategy equilibrium, note that competition for one-stop shoppers would again drive *total* basket prices down to cost. But as below-cost pricing is banned, this would require selling *both* products at cost. Obviously, this cannot be an equilibrium, as a firm can make a profit on multi-stop shoppers by charging a small positive margin on its strong product.

The characterization of the mixed-strategy equilibrium is similar to that proposed by Varian (1980) and Baye, Kovenock, and de Vries (1992).[26] In this equilibrium, *ex*

[26] Using the analysis of the latter paper, it can moreover be shown that, conditional on pricing weak products at cost, the (mixed-strategy) equilibrium (for the price of strong products) is unique.

post, consumers with a shopping cost below $\tau^b(\rho_1, \rho_2) \equiv \delta - \max\{\rho_1, \rho_2\}$ favor multi-stop shopping and buy both firms' strong products, whereas consumers with a shopping cost in the range $\tau^b(\rho_1, \rho_2) < s < v^b(\rho_1, \rho_2) \equiv w - \min\{\rho_1, \rho_2\}$ are one-stop shoppers and buy from the firm that charges the lowest price for its basket.

Let us now examine the impact of a ban on consumers. We first note that marginal consumers are one-stop shoppers, as $v^b > \tau^b$. As banning below-cost pricing raises prices for one-stop shoppers, it follows that this reduces not only the number of one-stop shoppers, but also the total number of consumers – from $F(w)$ to $F\left(v^b(\rho_1, \rho_2)\right)$. Furthermore, the multi-stop shopping cost threshold τ^b satisfies:

$$\tau^b(\rho_1, \rho_2) \geq \bar{\tau} \equiv \tau^b(\bar{\rho}, \bar{\rho}) = \delta - \bar{\rho} > s^*.$$

Hence, banning below-cost pricing *fosters* multi-stop shopping.

This does not mean that multi-stop shoppers face lower prices, however. In particular, the upper bound $\bar{\rho}$ exceeds the margin ρ^* that arises in the absence of the ban,[27] implying that multi-stop shoppers face higher prices with at least some probability. The next proposition shows that banning below-cost pricing actually harms multi-stop shoppers, as well as one-stop shoppers, when weak products offer relatively little value, that is, when w is close to δ:

Proposition 9 *Suppose that s is distributed with positive density over $\mathbb{R}_+$ (Assumption A) or over $[0, \bar{s}]$ with $\bar{s} > w$. Keeping δ constant, for w close enough to δ:*

(i) every consumer's expected surplus is lower in the equilibrium characterized by Proposition 8 than in the equilibrium that arises in the absence of a ban; and

(ii) total welfare can however be lower or higher, depending on the distribution of shopping costs. For instance, if $F(s) = (s/\bar{s})^k$, then there exists $\hat{k}(w, \delta) > 0$ such that banning below-cost pricing decreases (resp., increases) total welfare if $k < \hat{k}(w, \delta)$ (resp., $k > \hat{k}(w, \delta)$).

[27] To see this, it suffices to note that, from the first-order conditions, ρ^* and $\bar{\rho}$ satisfy respectively, $\rho = h(\delta - \rho^* - \rho)$ and $\rho = h(\delta - \rho)$, where $h(.)$ is an increasing function.

Proof. See Online Appendix E.2. ■

The intuition is that, when weak products are "very" weak, there are relatively few one-stop shoppers. Firms can then raise the prices of their strong products, so as to exploit multi-stop shoppers, without being too concerned about losing one-stop shoppers. Indeed, in the limit case where $w = \delta$, the lower bound $\underline{\rho}$ of the equilibrium margin distribution converges to the upper bound $\bar{\rho}\,(> \rho^*)$, and thus multi-stop shoppers certainly face higher prices. By continuity, multi-stop shoppers face higher *expected* prices, as long as weak products are not too valuable. However, as a ban on below-cost pricing increases firms' profits, the impact on total welfare remains ambiguous, and depends, in particular, on the distribution of shopping costs.

Thus, in competitive markets, RBC laws increase firms' profits but hurt one-stop shoppers. When weak products offer relatively low value, multi-stop shoppers face higher prices as well, in which case banning below-cost pricing increases firms' profits at the expense of consumers. This finding gives support to the conclusion of the OECD (2007) report, which argues that RBC laws are likely to lead to higher prices and thus harm consumers. The reduction in consumer surplus may, moreover, exceed the increase in firms' profits and thus result in lower total welfare. However, when, instead, weak products offer high value, RBC laws may have a positive impact on multi-stop shoppers.[28]

Remark: Upstream margins. In the case of downstream firms (e.g., retailers), their comparative advantages may be mainly driven by differences in wholesale prices rather than in quality or cost. For instance, in the setting developed in Section 4, supermarkets may devote resources to negotiating better conditions from their suppliers, and have an incentive to target different products. Total welfare must also account for the profit of upstream suppliers, which may affect the social impact of RBC laws. For example, in the Online Appendix E.2 we consider a variant along these lines, in which firms initially face the same wholesale price for each product, and negotiate a discount δ on one or the other product; "strong products" then correspond to those on which they negotiated the discount, and "weak products" correspond to those on which they pay the regular wholesale price. To fix ideas, suppose moreover strong products are supplied at cost (that

[28] However, RBC laws reduce total expected consumer surplus when, for instance, the density of the distribution of shopping costs does not increase between s^* and δ; see Online Appendix B.3.

is, the discount corresponds to the supplier's entire margin), so that the suppliers' profit comes only from the sales of "weak" products. As RBC laws reduce the extent of one-stop shopping, and one-stop shoppers are the only ones buying the weak products, on which the firms are paying the regular wholesale price, so does the profit of the upstream suppliers. It follows that banning below-cost pricing hurts upstream suppliers as well, which further degrades the impact on total welfare.

In a setting where consumers are one-stop shoppers who underestimate (some of) their needs, Johnson (2016) finds that banning below-cost pricing has an unambiguously negative impact: it increases the price for potential loss leaders (those products for which consumers do not underestimate their needs) and harms consumers, despite decreasing the prices for the other products. In our setting, a ban on below-cost pricing also raises the price of potential loss leaders (namely, the weak products), but can either increase or decrease the (expected) price of the other products (the strong ones). Also, while one-stop shoppers are worse-off under RBC laws, as in Johnson's paper, we allow for multi-stop shoppers as well, and they can either be worse- or better-off. In spite of these discrepancies, Johnson's paper and this paper both call for the cautious use of below-cost pricing regulations in competitive markets; and where they are implemented, their impact should be carefully evaluated.

6 Conclusion

We have studied competition between multi-product firms in a setting where firms enjoy comparative advantages over different goods or services, and customers have heterogeneous transaction costs. As a result, those with low costs tend to patronize multiple suppliers, whereas those with higher shopping costs are more prone to one-stop shopping. This gives rise to a form of co-opetition, as firms' baskets are substitutes for one-stop shoppers, but their strong products are complements for multi-stop shoppers. As a result, competition for one-stop shoppers drives total basket prices down to total cost but, in order to exploit their market power over multi-stop shoppers, firms price strong products above cost and weak products below cost. Furthermore, the complementarity of firms' strong products generates double marginalization problems, which here take the form of excessive cross-subsidization: indeed, firms would benefit from mutual moderation, for example, by agreeing to put a cap on the prices of strong products. Such bilateral agreements would benefit consumers (competition for one-stop shoppers would still induce firms to offer them their baskets at cost remain, but multi-stop shoppers would benefit from lower prices), and would also increase profits by boosting multi-stop shopping.

These insights highlight the role of the interaction across products in firms' own offerings, and of the diversity in consumers' shopping patterns, for the analysis of competition among multi-product firms. Interestingly, until recently the empirical literature on platform competition in media or healthcare industries, or on retail competition between supermarkets, have instead tended to either ignore multi-stop shopping, or focus on a specific product category. The recent work by Thomassen *et al.* (2017), who account for the multiplicity of product categories and the heterogeneity of shopping patters, constitutes a notable exception, and its findings confirms the importance of these features.

These insights can also shed some light on firms' incentives to invest in improving their offerings or to negotiate better conditions from their suppliers. When endogenizing product choices, we found that firms have indeed incentives to differentiate themselves by targeting different products.

The legal treatment of cross-subsidization in competitive markets has triggered much debate. We find that banning below-cost pricing substantially benefits firms – their profits more than double – at the expense of one-stop shoppers, and it can also reduce total consumer surplus and social welfare, depending on the value offered by weak products and the distribution of shopping patterns. Our analysis thus calls for a cautious use of resale-below-cost laws in competitive markets.

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Appendix

A Proof of Proposition 1

Thanks to Lemma 1, the equilibrium characterization is fairly simple. As firms sell their baskets at cost, one-stop shopping gives consumers the "full" value w. Consumers may, however, prefer buying both strong products, A_1 from firm 1 and B_2 from firm 2; this involves an extra shopping cost s and yields a total value:

$$v_{12} \equiv u_1^A - p_1^A + u_2^B - p_2^B = w + \delta - \rho_1 - \rho_2,$$

where $\rho_1 \equiv p_1^A - c_1^A$ and $\rho_2 \equiv p_2^B - c_2^B$ respectively denote firm 1 and 2's margins on their strong products. Consumers favor multi-stop shopping over one-stop shopping if the additional value from mixing-and-matching exceeds the extra shopping cost, that is, if

$$s \leq \tau \equiv v_{12} - w = \delta - \rho_1 - \rho_2.$$

Hence, consumers with a shopping cost $s < \tau$ engage in multi-stop shopping, whereas those with a shopping cost such that $\tau < s < w$ opt for one-stop shopping (and those with a shopping cost $s > w$ do not shop at all). As firms only derive a profit from selling their strong products to multi-stop shoppers, firm i's profit can be expressed as:

$$\pi_i (\rho_1, \rho_2) = \rho_i F (\tau) = \rho_i F (\delta - \rho_1 - \rho_2) . \tag{4}$$

Furthermore, we know from Lemma 1 that there are some multi-stop shoppers are active in equilibrium; hence, the margins ρ_1 and ρ_2 must satisfy $\rho_1 + \rho_2 < \delta$. If follows that these margins cannot be negative: any firm i offering that $\rho_i < 0$ would make a loss, which it could avoid by charging instead a non-negative margin. Likewise, starting from a candidate equilibrium in which some firm i charges $\rho_i = 0$, that firm could profitably deviate by slightly raising its margin:

$$\left. \frac{\partial \pi_i}{\partial \rho_i} (\rho_1, \rho_2) \right|_{\rho_i = 0} = F (\tau) > 0.$$

Hence, in equilibrium, each firm i must charge a positive margin on its strong product: $\rho_i > 0$. As the basket is offered at cost, this implies that firm i sells its weak product *below cost.*

B Proof of Proposition 2

Thanks to Lemma 1, the equilibrium is interior, and consumers whose shopping cost lies below $\tau = \delta - \rho_1 - \rho_2$ patronize both firms, whereas those whose shopping cost lies between τ and w patronize a single firm. As noted in the text, the monotonicity of the inverse hazard rate $h(\cdot)$ ensures that the profit function $\pi_i(\rho_1, \rho_2)$ is strictly quasi-concave in ρ_i, and its aggregative game nature ensures that any candidate equilibrium is symmetric: $\rho_1 = \rho_2 = \rho$, which satisfies the first-order solution $\rho = h(\delta - 2\rho)$. The monotonicity of $h(\cdot)$ further ensures that this first-order condition characterizes a unique candidate equilibrium, such that:

$$\rho^* = h(\tau^*),$$

where:

$$\tau^* = j^{-1}(\delta).$$

Note that, by construction, $\tau^* > 0$ (as $j(0) = 0 < \delta$) and thus: (i) $\rho^* = h(\tau^*) > 0$; and: (ii) $\rho^* < \delta/2$ (as $\delta - 2\rho^* = \tau^* > 0$).

There is thus a unique candidate equilibrium, in which both firms charge ρ^* on their strong products and a negative margin $\mu^* = -\rho^*$ on their weak products. We now show that firms cannot benefit from any deviation. Suppose, for example, that firm i deviates by charging ρ_i on its strong product and μ_i on its weak product. Obviously, it cannot make a profit from one-stop shoppers, as it would have to sell its basket (weakly) below cost to attract them. Furthermore, it cannot make a profit either by offering its weak product to multi-stop shoppers, as it would have to charge $\mu_i \leq \rho^* - \delta < 0$ to attract them. Thus, it can only make a profit from selling its strong product to multi-stop shoppers, and this profit is equal to $\rho_i F(\tau)$, where $\tau = \min\{\delta + \mu^* - \rho_i, \delta + \mu_i - \rho^*\}$; but then:

$$\rho_i F(\tau) \leq \rho_i F(\delta + \mu^* - \rho_i) = \rho_i F(\delta - \rho^* - \rho_i) \leq \pi^*,$$

where the inequality comes from the fact that the profit function $\rho_i F(\delta - \rho^* - \rho_i)$ is quasi-concave in ρ_i, from the monotonicity of $h(\cdot)$, and, by construction, maximal for $\rho_i = \rho^*$.

C Proof of Proposition 7

We now derive the minmax profit that each firm can earn when below-cost pricing is not allowed. Consider first firm i's response when firm j sets both of its margins to zero, that is, $\mu_j = \rho_j = 0$. Firm i cannot make a profit from one-stop shoppers who can obtain both products at cost from firm j, and thus it can only make a profit by selling its strong product to multi-stop shoppers. The threshold for multi-stop shopping is $\tau = \delta - \rho_i$, and thus the profit from multi-stop shoppers is given by $\rho_i F(\delta - \rho_i)$. Choosing ρ_i so as to maximize this profit gives firm i:

$$\bar{\pi} \equiv \max_{\rho} \rho F(\delta - \rho) > 0,$$

where the inequality stems from $\delta > 0$. The associated margin is given by:

$$\bar{\rho} \in \arg\max_{\rho} \rho F(\delta - \rho).$$

Note that this margin satisfies $\bar{\rho} < (\delta \leq) \bar{w}_i$ for $i \in \{1, 2\}$. [In case there are multiple solutions, then any solution satisfies this property and those that follow below.]

To conclude the argument, it suffices to note that, in response to any rival's margins $\mu_j \geq 0$ and $\rho_j \geq 0$, firm i can always secure at least $\bar{\pi}$ by charging $\mu_i \geq \underline{w}_i$ and $\rho_i = \bar{\rho}$. Choosing $\mu_i \geq \underline{w}_i$ ensures that any multi-stop shoppers will buy both firms' strong products.

Additionally, if $v_j \geq v_i$, then the threshold for multi-stop shopping is given by:

$$\tau = v_{12} - v_j$$

and thus satisfies:

$$\begin{aligned} \tau &= w + \delta - \bar{\rho} - \hat{\rho}_j - \left(w - \hat{\mu}_j - \hat{\rho}_j\right) \\ &= \delta + \hat{\mu}_j - \bar{\rho} \\ &\geq \delta - \bar{\rho}, \end{aligned}$$

where the inequality stems from $\hat{\mu}_j = \min\left\{\mu_j, \underline{w}_j\right\} \geq 0$. It follows that firm i obtains at least $\bar{\pi}$:

$$\pi_i = \rho_i F(\tau) = \bar{\rho} F(\tau) \geq \bar{\rho} F(\delta - \bar{\rho}) = \bar{\pi}.$$

If instead $v_j < v_i$, then firm i sells its strong product to both one-stop and multi-stop shoppers, and thus again obtains at least $\bar{\pi}$:

$$\pi_i = \rho_i F\left(\max\left\{v_i, \frac{v_{12}}{2}\right\}\right) \geq \rho_i F(v_i) = \bar{\rho} F(\bar{w}_i - \bar{\rho}) \geq \bar{\rho} F(\delta - \bar{\rho}) = \bar{\pi},$$

where the second inequality stems from $\bar{w}_i > \delta$.

It follows that, in any candidate equilibrium, firms must obtain a positive profit $\pi_i \geq \bar{\pi}$, and thus charge a positive total margin $m_i > 0$ (as $m_i = 0$ would imply $\mu_i = \rho_i = 0$, and thus $\pi_i = 0$).

Finally, we show that $\bar{\pi} = \bar{\rho} F(\delta - \bar{\rho}) > 2\pi^* = 2\rho^* F(\delta - 2\rho^*)$. The strict inequality follows from $2\rho^* > \bar{\rho}$, or $\bar{\tau} = \delta - \bar{\rho} > \tau^*$ (note that $\tau^* + 2\rho^* = \delta = \bar{\tau} + \bar{\rho}$). To see this, note that $\bar{\tau} = \delta - \bar{\rho} = \delta - h(\bar{\tau})$, which amounts to $\delta = l(\bar{\tau})$, where $l(\tau) \equiv \tau + h(\tau)$ $< j(\tau) = \tau + 2h(\tau)$, and this implies $\bar{\tau} = l^{-1}(\delta) > j^{-1}(\delta) = \tau^*$. **Q.E.D.**

Competitive Cross-Subsidization
Online Appendix (Not for publication)

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1 November 2018

Notation. Throughout the exposition:

- We refer to the two firms as firms i and j, with the convention that $i \neq j \in \{1, 2\}$.
- For each firm $i \in \{1, 2\}$, we denote the social value generated by its strong (resp., weak) product by $\bar{w}_i$ (resp., by $\underline{w}_i$), and denote the margin charged on its strong (resp., weak) product by ρ_i (resp., by μ_i). By assumption, we have $\bar{w}_i, \underline{w}_i > 0$ and:

$$\bar{w}_i + \underline{w}_i = w,$$

$$\bar{w}_i - \underline{w}_j = \delta.$$

- The value offered by firm i is thus equal to:

$$v_i \equiv \max\{\underline{w}_i - \mu_i, 0\} + \max\{\bar{w}_i - \rho_i, 0\},$$

whereas multi-stop shoppers obtain

$$v_{12} = \max\{\bar{w}_1 - \rho_1, 0\} + \max\{\bar{w}_2 - \rho_2, 0\}$$

if they buy both strong products, and obtain instead

$$\underline{v}_{12} = \max\{\underline{w}_1 - \mu_1, 0\} + \max\{\underline{w}_2 - \mu_2, 0\}$$

if they buy both weak products.

- Using the "adjusted" margins, defined as:

$$\hat{\mu}_i \equiv \min\{\mu_i, \underline{w}_i\} \text{ and } \hat{\rho}_i \equiv \min\{\rho_i, \bar{w}_i\},$$

these values can be respectively expressed as:

$$\begin{aligned} v_i &= \underline{w}_i - \hat{\mu}_i + \bar{w}_i - \hat{\rho}_i = w - \hat{\mu}_i - \hat{\rho}_i, \\ v_{12} &= \bar{w}_1 - \hat{\rho}_1 + \bar{w}_2 - \hat{\rho}_2 = w + \delta - \hat{\rho}_1 - \hat{\rho}_2, \\ \underline{v}_{12} &= \underline{w}_1 - \hat{\mu}_1 + \underline{w}_2 - \hat{\mu}_2 = w - \delta - \hat{\mu}_1 - \hat{\mu}_2. \end{aligned}$$

Multi-stop shoppers would not buy strong products unless $\rho_i \leq \bar{w}_i$ for $i = 1, 2$ (the value from multi-stop shopping, even gross of shopping costs, would otherwise be lower than from one-stop shopping), implying $\hat{\rho}_i = \rho_i$ and

$$v_{12} = w + \delta - \rho_1 - \rho_2.$$

Likewise, multi-stop shoppers would not buy both weak products unless $\mu_i \leq \underline{w}_i$ for $i = 1, 2$, implying that:

$$\underline{v}_{12} = w - \delta - \mu_1 - \mu_2.$$

Moreover, for a firm that attracts one-stop shoppers, it is never optimal to charge a margin that exceeds the social value of the product, that is, $\rho_i > \bar{w}_i$ and $\mu_i > \underline{w}_i$ cannot arise in equilibrium where firm i serves some one-stop shoppers. Suppose firm i charges $\mu_i > \underline{w}_i$ and $\rho_i \leq \bar{w}_i$, and one-stop shoppers only buy its strong product. Reducing μ_i such that $\tilde{\mu}_i = \underline{w}_i - \varepsilon > 0$ increases firm i's profit by also selling its weak product to one-stop shoppers and by attracting more one-stop shoppers as $\tilde{v}_i > v_i$. Doing so may also transform some multi-stop shoppers (if indeed there are any multi-stop shoppers buying strong products) into one-stop shoppers, as now $\tilde{\tau} = \delta + \tilde{\mu}_i - \rho_j < \tau$, on which firm i earns a higher profit. Similarly, charging $\rho_i > \bar{w}_i$ is never optimal if firm i attracts some one-stop shoppers. Therefore, without loss of generality we focus on $\mu_i \leq \underline{w}_i$ and $\rho_i \leq \bar{w}_i$ if one-stop shoppers patronize firm i.

The shopping cost thresholds, below which consumers favor picking both strong products rather than patronizing only firm 1 or firm 2, are respectively $\tau_1 = v_{12} - v_1 = \delta + \mu_1 - \rho_2$ and $\tau_2 = v_{12} - v_2 = \delta - \rho_1 + \mu_2$. Likewise, the thresholds for picking weak products are

$\underline{\tau}_1 \equiv \underline{v}_{12} - v_1 = \rho_1 - \mu_2 - \delta$ and $\underline{\tau}_2 \equiv \underline{v}_{12} - v_2 = \rho_2 - \mu_1 - \delta$. Let $\tau \equiv \min\{\tau_1, \tau_2\}$ and $\underline{\tau} \equiv \min\{\underline{\tau}_1, \underline{\tau}_2\}$. Note that $\underline{\tau}_1 = -\tau_2$, $\underline{\tau}_2 = -\tau_1$, and thus $\underline{\tau} = -\tau$. Therefore, in equilibrium, it cannot be the case that some multi-stop shoppers buy strong products, and other buy weak products.

A Proof of Lemma 1

To prove the lemma, we first establish the following claims.

Claim 1 *Some consumers are active in equilibrium.*

Proof. Suppose there is no active consumer. It must be the case that $\max\{v_1, v_2, v_{12}, \underline{v}_{12}\} \leq 0$, and firms make no profit. Consider the following deviation for firm 1: charge $\tilde{\mu}_1 > 0$ and $\tilde{\rho}_1 > 0$ such that $\tilde{m}_1 = \tilde{\rho}_1 + \tilde{\mu}_1 = w - \varepsilon$, for some $\varepsilon \in (0, w)$. Firm 1 then attracts consumers with shopping cost $s \leq \tilde{v}_1 = \varepsilon$ and earns a positive profit, a contradiction. Thus, some consumers must be active in equilibrium. ■

Claim 2 *If there are active one-stop shoppers in equilibrium, then $m_1 = m_2 = 0$.*

Proof. Consider a candidate equilibrium in which some one-stop shoppers are active, which requires $\max\{v_1, v_2\} > 0$. If $m_i < m_j$, then firm i attracts all one-stop shoppers; therefore:

- if $m_i \leq 0$ and firm i makes no profit on multi-stop shoppers (either because there is no multi-stop shopper, or firm i offers them a negative margin), then it would benefit from charging slightly positive margins on both products: this would avoid any loss (if $m_i < 0$) and/or generate a small profit (if $m_i = 0$, implying $m_j > 0$);
- if instead $m_i \leq 0$ but firm i makes a profit on multi-stop shoppers, then it would benefit from raising its margin on the product not purchased by them, so as to charge a slightly positive total margin: this would avoid the loss and/or generate a profit from one-stop shoppers, and would moreover increase the demand from multi-stop shoppers, as it would reduce the value from one-stop shopping without affecting that of multi-stop shopping;

- finally, if $m_i > 0$, then firm j would benefit from offering a total margin slightly below m_i, by reducing as needed its margin on the product not purchased by multi-stop shoppers: firm j would then attract one-stop shoppers and make a positive profit on them, without substantially affecting the profit obtained from multi-stop shoppers, if any.

We thus have $m_1 = m_2 = m$, implying that firms share the demand from one-stop shoppers; let firm i be a firm attracting at least half of them. If $m < 0$, then firm i would benefit from slightly increasing its margin on the product not purchased by multi-stop shoppers: this would avoid the loss made on one-stop shoppers (who then all go to firm j) without substantially affecting any profit obtained from multi-stop shoppers. If instead $m > 0$, then firm j would benefit from slightly decreasing its margin on the product not purchased by multi-stop shoppers: this would attract all one-stop shoppers, without substantially affecting any profit from multi-stop shoppers. Hence, $m = 0$. ■

Claim 3 *In equilibrium, active multi-stop shoppers buy the strong products.*

Proof. Suppose that some multi-stop shoppers buy the weak products. Each firm must then offer better value on its weak product than the rival's strong product; that is, each firm must sell its strong product with a margin that exceeds its rival's "quality-adjusted" margin: $\rho_2 \geq \mu_1 + \delta$ and $\rho_1 \geq \mu_2 + \delta$. We show that such a configuration cannot be an equilibrium. We consider two cases:

- suppose first that there are only multi-stop shoppers (buying the weak products). To make a profit, firms must charge non-negative margins on their weak products, that is, μ_1, $\mu_2 \geq 0$. From the above, this implies that each firm sells its strong product with a margin that exceeds its comparative advantage δ: $\rho_2 \geq \delta$ and $\rho_1 \geq \delta$. But then, any firm could make a profit by reducing the margin on its strong product. For instance, keeping μ_1 unchanged, by charging $\tilde{\rho}_1 = \mu_2 + \delta - \varepsilon > 0$, firm 1 would also sell its strong product to all previously active consumers, as it now offers better value on A: $\tilde{v}_1^A = v_2^A + \varepsilon$. The deviation may also attract additional one-stop shoppers from which the firm makes a profit as $\tilde{\rho}_1 > 0$ and $\mu_1 \geq 0$; and

- suppose, instead, that there are both one-stop shoppers and multi-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. As firms make no profit from one-stop shoppers, they must charge non-negative margins on their weak products, that is, μ_1, $\mu_2 \geq 0$. This implies, however, that margins on strong products are non-positive, say, $\rho_1 = m_1 - \mu_1 \leq 0$, which contradicts the condition $\rho_1 \geq \mu_2 + \delta \geq \delta$.

Therefore multi-stop shoppers must buy strong products in equilibrium. ■

Claim 4 *Some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers. From Claim 2, $m_1 = m_2 = 0$; hence, firms make zero profit and $v_1 = v_2 = w$. As $v_{12} + \underline{v}_{12}$ corresponds to the total value of buying one unit of both products from both firms, we thus have $v_{12} + \underline{v}_{12} = 2w$. However, ruling out multi-stop shopping requires $\max\{v_{12}, \underline{v}_{12}\} \leq w \,(= v_1 = v_2)$; it follows that $v_{12} = \underline{v}_{12} = w$, which implies $\rho_1 + \rho_2 = \delta$. Hence, at least one firm i charges $\rho_i > 0$; it would then be profitable for that firm to encourage some consumers to buy only its strong product, by slightly increasing μ_i and decreasing ρ_i by the same amount: this would trigger some multi-stop shopping, from which firm i would derive a positive profit. ■

Claim 5 *Some one-stop shoppers are active in equilibrium.*

Proof. Suppose there are only multi-stop shoppers who, from Claim 3, buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have:

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\},$$

which implies $\max\{v_1, v_2\} \leq v_{12}/2$, and the demand from multi-stop shoppers is $F(v_{12}/2)$. As consumers only buy strong products, firms must charge non-negative margins on these products. Without loss of generality, suppose $\rho_2 \geq \rho_1 \,(\geq 0)$, and consider the following deviation for firm 1: keeping ρ_1 constant, change μ_1 to:

$$\tilde{\mu}_1 = \frac{w - \delta + \rho_2 - \rho_1}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0,$$

so as to *increase* the value offered to one-stop shoppers to:

$$\tilde{v}_1 = w - \rho_1 - \tilde{\mu}_1 = \frac{w + \delta - \rho_1 - \rho_2}{2} + \varepsilon = \frac{v_{12}}{2} + \varepsilon.$$

This deviation does not affect v_{12} nor τ_2 (which only depends on ρ_1, ρ_2 and μ_2), but it decreases τ_1 to $\tilde{\tau}_1 = \delta + \tilde{\mu}_1 - \rho_2 = v_{12}/2 - \varepsilon$; as initially $\tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 \, (< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some of the initial multi-stop shoppers to buy both products from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), from which firm 1 earns an extra profit by selling its weak product (as $\tilde{\mu}_1 > 0$), and it, moreover, attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), which generates additional profit (as $\rho_1 \geq 0$ and $\tilde{\mu}_1 > 0$). ■

Claims 4 and 5 establish part (i) of the Lemma. Part (ii) then follows from Claim 3, while part (iii) follows from Claim 2.

B Extensions

We consider here two extensions briefly discussed in the text, namely, bounded shopping costs and online retailing.

B.1 Bounded shopping costs

The following propositions confirm that cross-subsidization keeps arising in equilibrium whenever consumers' shopping costs are sufficiently diverse. By contrast, when shopping costs are all low enough, active consumers systematically visit both stores and only buy strong products, which firms price above cost. Conversely, when shopping costs are all high enough, consumers visit at most one firm, and symmetric Bertrand competition leads both firms to offer the basket at cost.

B.1.1 Distribution with upper bound

We first consider the effect of an upper bound on consumers' shopping costs:

Assumption $\bar{A}$: The shopping cost s is distributed according to a cumulative distribution function with positive density over $[0, \overline{s}]$, where $\overline{s} > 0$.

Proposition 10 *Under Assumptions $\bar{A}$ and B:*

- *if $\overline{s} > j^{-1}(\delta)$, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model;*
- *if instead $\overline{s} \leq j^{-1}(\delta)$, there exist multiple equilibria. In each equilibrium: (i) only multi-stop shopping arises; and (ii) weak products are offered at below-cost prices, but firms only sell their strong products, with a positive margin ranging from $h(\overline{s})$ to $\delta - \overline{s} - h(\overline{s})$.*

Proof: Suppose that consumers' shopping costs are distributed over $[0, \overline{s}]$, where $\overline{s} > 0$. It is straightforward to check that the first four claims in the proof of Lemma 1 still hold; that is, in any equilibrium, there exist active multi-stop shoppers who buy the strong products, and in addition, if there are active one-stop shoppers, then $m_1 = m_2 = 0$.

We first note that the equilibrium identified in the baseline model still exists when $\overline{s}$ is large enough:

Claim 6 *When $\overline{s} > j^{-1}(\delta)$, then there exists an equilibrium with both types of shoppers: consumers with a shopping cost lower than $\tau^* = j^{-1}(\delta)$ engage in multi-stop shopping, and face a margin $\rho^* = h(\tau^*)$ on each strong product; whereas those with a higher cost favor one-stop shopping.*

Proof. As shown in the text, there is a unique candidate equilibrium where both types of shopping patterns arise, and is as described in the Claim. The existence of one-stop shopping, however, requires $\overline{s} > \tau^* = j^{-1}(\delta)$. Conversely, when this condition holds, the margins $m_1^* = m_2^* = 0$ and $\rho_1^* = \rho_2^* = h(\tau^*)$ do support an equilibrium: indeed the reasoning of the proof of Proposition 2 ensures that no deviation is profitable. ■

Next, we show that one-stop shopping cannot arise if $\overline{s}$ is too low:

Claim 7 *When $\overline{s} \leq j^{-1}(\delta)$, then one-stop shopping does not arise in equilibrium.*

Proof. Suppose there exist some one-stop shoppers, which requires $\tau < \min\{\max\{v_1, v_2\}, \overline{s}\}$. Competition for these one-stop shoppers leads to $m_1 = m_2 = 0$, and thus $\tau_1 = \tau_2 = \delta - \rho_1 - \rho_2 < \overline{s}$, which implies $\rho_1 + \rho_2 > \delta - \overline{s} > 2h(\overline{s})$. Therefore, at least one of the margins on strong products must exceed $h(\overline{s})$. Suppose $\rho_1 > h(\overline{s})$; then $\rho_1 > h(\overline{s}) > h(\tau)$, as $\overline{s} > \tau$ and $h(\cdot)$ is strictly increasing. Consider now the following deviation: decrease ρ_1 to $\tilde{\rho}_1$ and increase μ_1 by the same amount, so as to maintain the total margin. This does not affect the profit from one-stop shoppers (which remains equal to zero), but yields a profit from multi-stop shoppers, equal to $\tilde{\pi}_1 = \tilde{\rho}_1 F(\tilde{\tau})$, where $\tilde{\tau} = \delta - \tilde{\rho}_1 - \rho_2$. As $d\tilde{\pi}_1/d\tilde{\rho}_1|_{\tilde{\rho}_1=\rho_1} = -f(\tau)(\rho_1 - h(\tau))$, which is strictly negative as $\rho_1 > h(\tau)$, such a deviation is profitable. Hence, one-stop shopping does not arise in equilibrium. ■

Claims 6 and 7 together establish the first part of Proposition 10. We now characterize the equilibria where all consumers are multi-stop shoppers.

Claim 8 *When $\overline{s} \leq j^{-1}(\delta)$, any margin profile such that $\rho_1 \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $\mu_2 = \rho_1 - \delta + \overline{s}$ and $\mu_1 = \rho_2 - \delta + \overline{s}$, constitutes an equilibrium in which all active consumers are multi-stop shoppers.*

Proof. Suppose there are only multi-stop shoppers who, from Claim 3, buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have $\tau \geq \min\{v_{12}/2, \overline{s}\}$, and the demand from multi-stop shoppers is $F(\min\{v_{12}/2, \overline{s}\})$. As consumers only buy strong products, firms must charge non-negative margins on these products: $\rho_1, \rho_2 \geq 0$.

If $\overline{s} < \min\{v_{12}/2, \tau\}$, each firm can profitably deviate by slightly raising the price for its strong product: this increases the margin without affecting the demand, equal to $F(\overline{s})$. Hence, without loss of generality, we can assume $\overline{s} \geq \min\{v_{12}/2, \tau\}$. The condition $\tau \geq \min\{v_{12}/2, \overline{s}\}$ then implies that either $v_{12}/2 \leq \min\{\tau, \overline{s}\}$, or $v_{12}/2 \geq \tau = \overline{s}$. We consider these two cases in turn.

Consider the first case, and note that the condition:

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\}$$

then implies $\max\{v_1, v_2\} \leq v_{12}/2$. Without loss of generality, suppose $\rho_2 \geq \rho_1\ (\geq 0)$, and consider the following deviation for firm 1: keeping ρ_1 constant, reduce μ_1 so as to offer $\tilde{v}_1 = v_{12}/2 + \varepsilon$, which amounts to charging:

$$\tilde{\mu}_1 = \frac{w - \delta + \rho_2 - \rho_1}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0.$$

This deviation does not affect v_{12} or $\tau_2 = v_{12} - v_2$, but it decreases τ_1 to $\tilde{\tau}_1 = v_{12} - \tilde{v}_1 = v_{12}/2 - \varepsilon$; as initially $\tau_2 \geq \tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 \, (< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some multi-stop shoppers to buy everything from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{\mu}_1 > 0$), and it, moreover, attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), generating additional profit (as $\rho_1 \geq 0$ and $\tilde{\mu}_1 > 0$).

Hence, we cannot have an equilibrium of the type $v_{12}/2 \leq \min\{\tau, \overline{s}\}$.

Consider now the second case: $\overline{s} = \tau \leq v_{12}/2$. Note first that if $\tau = \tau_i = v_{12} - v_i < \tau_j = v_{12} - v_j$, then firm i could again profitably deviate by increasing the margin on its strong product without affecting the demand (as τ_i does not depend on ρ_i). Hence, we must have $\overline{s} = \tau = \tau_1 = \tau_2$, and thus $v_1 = v_2$, or $m_1 = m_2 = m$.

We now show that firms' margins on weak products must satisfy $\mu_1, \mu_2 \leq -h(\overline{s})$, and margins on strong products must satisfy $\rho_1, \rho_2 \geq h(\overline{s})$. To see this, note that firm 1, say, could induce some multi-stop shoppers to buy its weak product B as well, by reducing the margin on its weak product, so that $\tilde{\tau}_1 = \delta + \tilde{\mu}_1 - \rho_2 < \tau_1 \, (= \delta + \mu_1 - \rho_2) = \overline{s}$, keeping the total margin constant: $\tilde{\rho}_1 + \tilde{\mu}_1 = m_1$. By so doing, firm 1 would earn a profit equal to:

$$\begin{aligned} \pi_1 &= \tilde{\rho}_1 F(\tilde{\tau}_1) + m_1(F(\overline{s}) - F(\tilde{\tau}_1)) \\ &= m_1 F(\overline{s}) - \tilde{\mu}_1 F(\delta + \tilde{\mu}_1 - \rho_2) . \end{aligned}$$

To rule out such a deviation, μ_1 must satisfy:

$$\mu_1 \in \arg\max_{\tilde{\mu}_1 \leq \mu_1} -\tilde{\mu}_1 F(\delta + \tilde{\mu}_1 - \rho_2) ,$$

which, given the monotonicity of $h(\cdot)$, amounts to:

$$\mu_1 \leq -h(\overline{s}).$$

Alternatively, firm 1 could discourage some multi-stop shoppers by increasing $\tilde{\rho}_1$, so that $\tilde{\tau}_2 = \delta + \mu_2 - \tilde{\rho}_1 < \tau_2 \, (= \delta + \mu_2 - \rho_1) = \overline{s}$, keeping $\tilde{\mu}_1$ unchanged. Doing so yields a profit equal to:

$$\pi_1 = \tilde{\rho}_1 F(\tilde{\tau}_2) .$$

Ruling out this deviation thus requires:

$$\rho_1 \in \arg\max_{\tilde{\rho}_1 \geq \rho_1} \tilde{\rho}_1 F(\delta + \mu_2 - \tilde{\rho}_1),$$

or:

$$\rho_1 \geq h(\overline{s}).$$

The conditions $\mu_2 \leq -h(\overline{s})$ and $\rho_2 \geq h(\overline{s})$ can be derived using the same logic.

Therefore, the margins for any candidate equilibria must satisfy (using $\tau = \delta + \mu_1 - \rho_2 = \overline{s}$): $-h(\overline{s}) \geq \mu_1 = \rho_2 - \delta + \overline{s} \geq h(\overline{s}) - \delta + \overline{s}$, implying $\overline{s} + 2h(\overline{s}) \leq \delta$. Hence, an equilibrium with only multi-stop shopping exists only when $\overline{s} \leq j^{-1}(\delta)$. Conversely, when this condition holds, any margins satisfying $\rho_1, \rho_2 \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $\mu_2 = \rho_1 - \delta + \overline{s}$ and $\mu_1 = \rho_2 - \delta + \overline{s}$ constitute an equilibrium in which all consumers are multi-stop shoppers.
■

Claims 7 and 8 together establish the second part of Proposition 10. **Q.E.D.**

Hence, while firms always price their weak products below cost, it is only when some consumers have high enough shopping costs, namely, when $\overline{s} > j^{-1}(\delta)$, that cross-subsidization actually occurs. Otherwise, all consumers patronize both firms and only buy strong products. Indeed, in the limit case $\overline{s} = 0$, where consumers incur no shopping costs, each firm earns a margin of up to δ on its strong product, reflecting its comparative advantage, as standard asymmetric Bertrand competition suggests.

B.1.2 Distribution with lower bound

We now turn to the impact of a lower bound on shopping costs:

Assumption $\underline{A}$: The shopping cost s is distributed according to a cumulative distribution function with positive density over $[\underline{s}, +\infty)$, where $\underline{s} < w$.[1]

Proposition 11 *Under Assumptions $\underline{A}$ and B:*

- *if $\underline{s} < \delta/3$, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model;*

[1] This assumption is needed for the viability of the markets, as consumers with shopping costs exceeding w never visit any firm.

- *if instead* $\underline{s} > \delta$, *there exist multiple equilibria in which: (i) only one-stop shopping arises, and (ii) firms make zero profit;*

- *finally, if* $\delta/3 \leq \underline{s} \leq \delta$, *both types of equilibria coexist.*[2]

Proof: Suppose that consumers' shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. We first show that part of Lemma 1 still applies:

Lemma 2 *Suppose that consumer shopping costs are distributed over* $[\underline{s}, +\infty)$, *where* $\underline{s} < w$. *Then, in equilibrium:*

- *some one-stop shoppers are active;*

- $m_1 = m_2 = 0$; *and*

- *active multi-stop shoppers buy strong products.*

Proof. It is straightforward to check that the first three claims of the proof of Lemma 1 in the baseline model remain valid: in equilibrium, some consumers are active (Claim 1); $m_1 = m_2 = 0$ whenever there are active one-stop shoppers (Claim 2), and active multi-stop shoppers buy the strong products (Claim 3). This last claim establishes part (iii) of Lemma 2, whereas Claim 2 implies that part (ii) of Lemma 2 follows from part (i). To complete the proof, it suffices to note that the proof of Claim 5 also remains valid, which yields part (i). ■

We now proceed to establish the proposition. We first note that multi-stop shopping must arise when some consumers have low enough shopping costs:

Lemma 3 *If* $\underline{s} < \delta/3$, *some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. Ruling out multi-stop shopping requires $v = w \geq \underline{v}_{12} - \underline{s} = w - \delta - \mu_1 - \mu_2 - \underline{s}$, or (using $m_1 = m_2 = 0$) $\rho_1 + \rho_2 \leq \delta + \underline{s}$. If firm 2, say, is the one that charges less on its strong product (i.e.,

[2] In the limit case $\underline{s} = \delta$, however, only those consumers with a shopping cost equal to δ may opt for multi-stop shopping.

$\rho_2 \leq \rho_1$), then we must have $\rho_2 \leq (\delta + \underline{s})/2$. Consider the following deviation for firm 1: charge $\tilde{\rho}_1 = \varepsilon > 0$ and $\tilde{\mu}_1 = -\varepsilon$ such that the total margin remains zero. The multi-stop shopping threshold becomes:

$$\tilde{\tau} = \delta - \tilde{\rho}_1 - \rho_2 \geq \delta - \varepsilon - \frac{\delta + \underline{s}}{2} = \frac{\delta - \underline{s}}{2} - \varepsilon.$$

As $\delta > 3\underline{s}$ (implying $(\delta - \underline{s})/2 > \underline{s}$), it follows that $\tilde{\tau} > \underline{s}$ for ε sufficiently small. Hence, firm 1 can induce some consumers to engage in multi-stop shopping and make a profit on them. ■

Next, we show that there indeed exists an equilibrium with multi-stop shopping as long as some consumers' shopping costs are not too large:

Lemma 4 *If $\underline{s} < \delta$, there exists an equilibrium exhibiting both types of shopping patterns, in which firms' total margins are zero ($m_i^* = 0$) and the margins on their strong products are equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.*

Proof. Suppose $\underline{s} < \delta$. As discussed in the text, the unique candidate equilibrium exhibiting both types of shopping patterns is such that: (i) both firms charge zero total margins ($m_i^* = 0$) and a positive margin on their strong products equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$; and (ii) consumers with a shopping cost lying between $\underline{s}$ and τ^* engage in multi-stop shopping, whereas those with a shopping cost lying between τ^* and w are one-stop shoppers. Therefore, this type of equilibrium exists when $\underline{s} < \tau^* = j^{-1}(\delta)$. As the function $j(\cdot)$ is strictly increasing and satisfies $j(\underline{s}) = \underline{s} + 2h(\underline{s}) = \underline{s}$, the condition $\underline{s} < \tau^*$ amounts to $\underline{s} < \delta$.

Conversely, these margins indeed constitute an equilibrium. By construction, given the equilibrium prices charged by the other firm, a firm cannot make a profit on one-stop shoppers, and charging ρ^* on the strong product maximizes the profit that a firm earns from multi-stop shoppers. ■

It follows that the analysis of the baseline model still applies when the lower bound is small enough, namely, when $\underline{s} < \delta/3$. From Lemmas 2 and 3, both types of shopping patterns must arise in equilibrium; Lemma 4 then ensures that the unique candidate identified in the text is indeed an equilibrium. This establishes the first part of the Proposition.

We now turn to the second part of the Proposition, and first note that multi-stop shopping cannot arise when all consumers have high shopping costs:

Lemma 5 *If $\underline{s} > \delta$, there are no multi-stop shoppers in equilibrium.*

Proof. Suppose, to the contrary, there are some active multi-stop shoppers. From Lemma 2, $m_1 = m_2 = 0$ and multi-stop shoppers must buy strong products; hence, $\tau = \delta - \rho_1 - \rho_2 > \underline{s}$. As $\underline{s} > \delta$, it follows that $\rho_1 + \rho_2 < 0$; hence, at least one firm must charge a negative margin on its strong product and incur a loss from serving multi-stop shoppers. But this cannot be an equilibrium, as that firm could avoid the loss by increasing its prices. ■

Finally, we show that when all consumers have large enough shopping costs, there exists equilibria with no multi-stop shoppers.

Lemma 6 *There exist equilibria with one-stop shopping if and only if $\underline{s} \geq \delta/3$. In these equilibria, margins satisfy: (i) $\rho_1 + \mu_1 = \mu_2 + \rho_2 = 0$; (ii) $\delta - \underline{s} \leq \rho_1, \rho_2, \rho_1 + \rho_2 \leq \delta + \underline{s}$; and (iii) $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$.*

Proof. Consider a candidate equilibrium with only one-stop shopping. From Lemma 2, $m_1 = m_2 = 0$ and thus $\tau = \delta - \rho_1 - \rho_2$. For firm 1, say, it cannot be profitable to deviate by attracting one-stop shoppers, as this would require a negative total margin $\tilde{m}_1 < 0$. Firm 1 could, however, deviate so as to induce some consumers to engage in multi-stop shopping; more specifically:

- it could induce some consumers to buy both strong products by charging $\tilde{\rho}_1$ such that $\tilde{\tau}_2 = \delta - \tilde{\rho}_1 + \mu_2 = \delta - \tilde{\rho}_1 - \rho_2 > \underline{s}$, or $\tilde{\rho}_1 < \delta - \underline{s} - \rho_2$; and

- alternatively, it could induce some consumers to buy both weak products by charging $\tilde{\mu}_1$ such that $\tilde{\underline{\tau}}_2 = -\delta + \rho_2 - \tilde{\mu}_1 > \underline{s}$, or $\tilde{\mu}_1 < \rho_2 - \delta - \underline{s}$.

Ruling out the first type of deviation requires $\rho_2 \geq \delta - \underline{s}$, while preventing the second type of deviation requires $\rho_2 \leq \delta + \underline{s}$. Therefore, the equilibrium margin ρ_2 must lie between $\delta - \underline{s}$ and $\delta + \underline{s}$. Applying the same logic to rule out firm 2's deviations requires the equilibrium margin ρ_1 to lie between $\delta - \underline{s}$ and $\delta + \underline{s}$ as well. Moreover, the margins cannot exceed the social values, which requires $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$.

Conversely, any margins that satisfy: (i) $\rho_1 + \mu_1 = \mu_2 + \rho_2 = 0$; (ii) $\delta - \underline{s} \leq \rho_1, \rho_2, \rho_1 + \rho_2 \leq \delta + \underline{s}$; and (iii) $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$ constitute an equilibrium in which all active consumers are one-stop shoppers and both firms earn zero profit.

The above analysis shows that equilibrium margins must satisfy: (i) $\delta - \underline{s} \leq \rho_1, \rho_2$, implying $\rho_1 + \rho_2 \geq 2\delta - 2\underline{s}$; and (ii) $\rho_1 + \rho_2 \leq \delta + \underline{s}$. These two conditions then lead to $2\delta - 2\underline{s} \leq \delta + \underline{s}$, which amounts to $\delta/3 \leq \underline{s}$. It thus follows that such an equilibrium exists if and only if $\delta/3 \leq \underline{s}$. ■

Combining Lemmas 5, 6 and 2 yields the second part of the Proposition, whereas Lemmas 4 and 6 together yield the last part. **Q.E.D.**

Thus, cross-subsidization arises in equilibrium as long as some consumers have a shopping cost lower than the extra value δ offered by combining both strong products, and it does arise for certain when some consumers have a low enough shopping cost (namely, lower than $\delta/3$).

B.2 Non-linear pricing

We show here that our insights carry over when consumers have elastic individual demands. We consider the following variant of the baseline model.

- *Demand.* Consumers obtain a gross utility $u_i^h\left(q_i^h\right)$ from purchasing a quantity q_i^h of good $h = A, B$ from firm $i = 1, 2$; their individual demand is thus given by

$$d_i^h\left(p_i^h\right) = \arg\max_{q_i^h} \left\{u_i^h\left(q_i^h\right) - p_i^h q_i^h\right\},$$

where $d_i^h\left(p_i^h\right)$ is decreasing in p_i^h, and the associated surplus is:

$$s_i^h\left(p_i^h\right) = \max_{q_i^h} \left\{u_i^h\left(q_i^h\right) - p_i^h q_i^h\right\} = u_i^h\left(d_i^h\left(p_i^h\right)\right) - p_i^h d_i^h\left(p_i^h\right).$$

- *Demand.* Each firm $i = 1, 2$ can supply any quantity q_i^h of good $h = A, B$ at total cost $C_i^h\left(q_i^h\right) = k_i^h + c_i^h q_i^h$, where c_i^h denotes as before a constant marginal cost of production, and k_i^h now denotes a fixed cost of supplying the good to a given consumer. For the sake of exposition, we suppose that these fixed costs are large enough to ensure that relevant fixed fees are all positive.[3]

[3] Alternatively, firms impose exclusivity provisions preventing consumers to buy the same good from

• *Non-linear pricing.* It is optimal for each firm i to offer each good h using a cost-based two-part tariff of the form $T_i^h\left(q_i^h\right) = f_i^h + c_i^h q_i^h$, where f_i^h denotes the fixed fee that a consumer must pay to obtain good h from firm i. A consumer buying this good from the firm then obtains $v_i^h = s_i^h\left(c_i^h\right) - f_i^h$.

• *Comparative advantages.* As in the baseline model, we assume that each firm enjoys a comparative advantage on one of the products, and that these comparative advantages are of the same magnitude:

$$\begin{aligned} w_1^A - w_2^A &= w_2^B - w_1^B \equiv \delta > 0, \\ w_1^A + w_1^B &= w_2^A + w_2^B \equiv w > \delta, \end{aligned}$$

where

$$w_i^h \equiv s_i^h\left(c_i^h\right) - k_i^h$$

now denotes the maximal surplus that can be generated by firm i's variety of good h.

No consumer has an incentive to buy a given good from both firms: he would have to pay both fixed fees, but would only buy from the firm with the lower marginal cost. One-stop shoppers, who buy both goods from the same firm, thus pay both fixed fees, whereas multi-stop shoppers, who only buy one product from each firm, only pay one fixed to each firm. Hence, as in the baseline model, each firm $i = 1, 2$ offers $v_i = w - \rho_i - \mu_i$ to one-stop shoppers, whereas multi-stop shoppers obtain $v_{12} = w + \delta - \rho_1 - \rho_2$ if they buy strong products, and $\underline{v}_{12} = w - \delta - \mu_1 - \mu_2$ if instead they buy the weak products, where $\rho_1 = f_1^A - k_1^A$ (resp., $\rho_2 = f_2^B - k_2^B$) and $\mu_1 = f_1^B - k_1^B$ (resp., $\mu_2 = f_2^A - k_2^A$) denote here firm 1's (resp., firm 2's) "fixed fee margins" on its strong and weak products. The same analysis as before then shows that, in equilibrium, firms sell their baskets at cost (i.e., $f_i^A + f_i^B = k_i^A + k_i^B$, and thus $\mu_i = -\rho_i$) but derive a profit from their strong products: consumers with $s < \tau = \delta - \rho_1 - \rho_2$ engage in multi-stop shopping and buy both firms' strong products, giving firm i a profit equal to

$$\pi_i = \rho_1 F\left(\delta - \rho_1 - \rho_2\right).$$

another firm. In that situation, the analysis applies even in the absence of any fixed cost of servicing a consumer.

Hence, in equilibrium, both firms charge $\rho_i = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.

Interestingly, although each tariff is individually efficient (namely, $p_i^h = c_i^h$, which induces consumers to buy the efficient quantity that yields the maximal surplus w_i^h), the equilibrium tariffs still feature double marginalization: keeping total fees equal to total fixed costs, the fixed fees charged on strong products exceed the level that would maximize industry profit.

B.3 Online retailing

The development of online retailing offers consumers an alternative way of fulfilling their needs, but has also an impact on retail competition and on retailers' pricing strategies. To explore some of these implications, consider the following variant of the baseline model, where a fraction λ of "internet-savvy" consumers see their shopping costs drop to zero. That is, the distribution of shopping costs is then characterized by a cumulative distribution function $F_\lambda(s)$ and a density $f_\lambda(s)$, where $F_\lambda(0) = \lambda$ and, for $s > 0$:

$$f_\lambda(s) = (1-\lambda) f(s) \text{ and } F_\lambda(s) = \lambda + (1-\lambda) F(s).$$

The inverse hazard rate becomes:

$$h_\lambda(s) = h(s) + \frac{\lambda}{1-\lambda} \frac{1}{f(s)}.$$

Hence:

(*i*) this hazard rate still increases with s if $f(s)$ does not increase with s, or if λ is not too large;[4] and

(*ii*) the hazard rate moreover increases with the proportion λ of "internet-savvy" consumers.

[4]If $f'(s) > 0$, then $h_\lambda(s)$ still increases with s in the relevant range $s \in [0, \delta]$ if:

$$\frac{\lambda}{1-\lambda} < \max_{s\in[0,\delta]} \{f^2(s) \frac{h'(s)}{f'(s)}\}.$$

Condition (*i*) ensures that the equilibrium characterization of Proposition 2 remains valid; condition (*ii*) then implies that the equilibrium prices charged on strong products increase with λ.

More generally, the following proposition shows that the development of online retailing leads to an increase in the prices of strong products whenever it inflates the inverse hazard rate:

Proposition 12 *Suppose that the development of online retailing affects the distribution of shopping costs in such a way that: (i) the distribution still satisfies Assumptions A and B; and (ii) the inverse hazard rate is inflated. Then there exists a unique equilibrium, in which firms sell their baskets at cost but charge a positive margin on their strong products (and thus a negative margin on their weak products); furthermore, the equilibrium prices of strong products increase with the development of online retailing.*

Proof. Let us index the development of online retailing by a parameter λ and suppose that the associated distribution of shopping costs, characterized by a cumulative distribution function $F(s;\lambda)$ with density $f(s;\lambda)$, satisfies Assumptions A and B, and is, moreover, such that:

$$h(s;\lambda) \equiv \frac{F(s;\lambda)}{f(s;\lambda)},$$

increases with λ. The analysis developed for the baseline model carries over: the equilibrium margin, ρ_λ^*, and the associated multi-stop shopping threshold, $\tau_\lambda^* = \delta - 2\rho_\lambda^*$, are now such that $\rho_\lambda^* = h(\tau_\lambda^*;\lambda)$. Hence, the margin ρ_λ^* satisfies:

$$\rho_\lambda^* = h(\delta - 2\rho_\lambda^*;\lambda).$$

As $h(s;\lambda)$ increases with both s and λ, it follows that ρ_λ^* increases with λ. Conversely, the threshold τ_λ^* is such that:

$$\frac{\delta - \tau_\lambda^*}{2} = h(\tau_\lambda^*;\lambda).$$

Hence, as $h(s;\lambda)$ increases with both s and λ, τ_λ^* decreases as λ increases. ■

Proposition 12 points out that the development of online sales is not only profitable, but also consistent with an *increase* in the prices of strong products: while one-stop shoppers can still buy firms' baskets at cost, multi-stop shoppers (including those buying online) face higher prices as the proportion of online customers increases. The intuition is straightforward: an increase in the development of online activity, as measured, for instance, by the proportion λ of "internet-savvy" consumers, boosts multi-stop shopping, which benefits the firms but also encourages them to take advantage of this shift in demand by raising the prices of their strong products – at the expense of the less internet-savvy multi-stop shoppers.

C Heterogeneous preferences

C.1 Horizontal differentiation

C.1.1 Setting

Consumers are uniformly distributed along the Hotelling unit-length segment and indexed by their location $x \in [0,1]$, whereas the firms' offerings are located at the two ends of the segment: A_1 and B_1 are located at 0, say, whereas A_2 and B_2 are located at 1. A consumer located at a distance x from one variety of a product incurs a cost tx when purchasing that variety, and $t(1-x)$ when purchasing the other variety. We assume consumers shopping costs are distributed independently of their locations, according to a cumulative distribution function $F(s)$ with positive density over $[0,\bar{s}]$, where $\bar{s} > \delta$ (to allow for one-stop shopping as well as multi-stop shopping), and further assume that w is large enough, relatively to $\bar{s}$, to ensure the market is fully covered in equilibrium.

A one-stop shopper located at x obtains a net value $w - 2tx - m_1 - s$ from patronizing firm 1 and $w - 2t(1-x) - m_2 - s$ from going instead to firm 2. Thus, one-stop shoppers favor firm 1 if

$$x < \hat{x} \equiv \frac{1}{2} - \frac{m_1 - m_2}{4t}.$$

In addition, consumers with $x < \hat{x}$ favor one-stop shopping (at firm 1) to multi-stop shopping (and purchasing strong products),[5] if their shopping cost is sufficiently large, namely, if

$$s > \lambda_1(x) \equiv \tau_1 - t + 2tx,$$

[5] We will check ex post that multi-stop shoppers indeed favor the strong products.

where $\tau_1 = \delta + \mu_1 - \rho_2$. Likewise, consumers located at $x > \hat{x}$ prefer one-stop shopping (at firm 2) if

$$s \geq \lambda_2(x) \equiv \tau_2 + t - 2tx,$$

where $\tau_2 = \delta + \mu_2 - \rho_1$.

Thus, the demand for the bundles $A_1 - B_1$ and $A_2 - B_2$ are given respectively as

$$\begin{aligned} D_1 &\equiv \int_0^{\hat{x}} [1 - F(\lambda_1(x))]\, dx, \\ D_2 &\equiv \int_{\hat{x}}^1 [1 - F(\lambda_2(x))]\, dx. \end{aligned}$$

whereas multi-stop shoppers' demand for the two strong products, A_1 and B_2, is given by

$$D \equiv \int_0^{\hat{x}} F(\lambda_1(x))dx + \int_{\hat{x}}^1 F(\lambda_2(x))dx.$$

Firm i's total profit can then be written as

$$\Pi_i = m_i D_i + \rho_i D = m_i (D_i + D) - \mu_i D.$$

C.1.2 Equilibrium analysis

To characterize the equilibrium margins, we focus on firm 1, say, and consider the impact of a small change in ρ_1 and μ_1, and evaluate it at a symmetric candidate equilibrium.

Consider first a modification of ρ_1 by dr together with a change of μ_1 by $-dr$, so that the total margin for firm 1's basket remains unchanged. Such modification does not affect the behavior of one-stop shoppers; in particular, the threshold $\hat{x}$ is not affected. Yet (see Figure 1):

- On the one hand, firm 1 obtains a larger margin from multi-stop shoppers, who only buy A_1 from it; the associated gain is Ddr.
- On the other hand, it induces some one-stop shoppers, on which firm 1 was earning the margin ρ_1, to switch to multi-stop shopping; hence:
  - for $x < \hat{x}$, these marginal consumers (namely, those with $s = \lambda_1(x)$) now buy both products from firm 1; the resulting gain for firm 1 is equal to $\mu_1 \int_0^{\hat{x}} f(\lambda_1(x))dxdr$;

– for $x > \hat{x}$, consumers with $s = \lambda_2(x)$ now patronize firm 2; the resulting loss for firm 1 is equal to $-\rho_1 \int_{\hat{x}}^{1} f(\lambda_2(x))dxdr$.

In equilibrium, the overall impact on firm 1's profit must be zero, which (using symmetry) leads to the following first-order condition:

$$\mu_1 \int_0^{\hat{x}} f(\lambda_1(x))dx - \rho_1 \int_{\hat{x}}^{1} f(\lambda_2(x))dx + D = 0.$$

Evaluating this condition for symmetric equilibrium margins $\mu_1 = \mu_2 = \mu$ and $\rho_1 = \rho_2 = m - \mu$, where m denotes the total margin charged by both firms (hence, we have $\tau_1 = \tau_2 = \tau = \delta + 2\mu - m$ and $\hat{x} = 1/2$),[6] we obtain

$$\mu = \frac{m}{2} - \tilde{h}(\tau; t), \tag{1}$$

where, letting $\Phi(s) \equiv \int_0^s F(x)dx$ denote the primitive of $F(s)$:

$$\tilde{h}(\tau; t) \equiv \frac{\int_0^{1/2} F(\lambda_1(x))dx}{\int_0^{1/2} f(\lambda_1(x))dx} \equiv \frac{\int_{\tau-t}^{\tau} F(s)ds}{\int_{\tau-t}^{\tau} f(s)ds} = \frac{\Phi(\tau) - \Phi(\tau - t)}{F(\tau) - F(\tau - t)}$$

converges towards the inverse hazard rate $h(\tau)$ as t goes to zero: $\lim_{t\to 0} \tilde{h}(\tilde{\tau}; t) = h(\tau)$. Likewise, its derivative converges towards $h'(\tau)$ as t goes to zero:

$$\tilde{h}'(\tau; t) = 1 - \tilde{h}(\tau; t) \frac{f(\tau) - f(\tau - t)}{F(\tau) - F(\tau - t)},$$

leading to:

$$\lim_{t\to 0} \tilde{h}'(\tau; t) = 1 - h(\tau) \frac{f'(\tau)}{f(\tau)} = h'(\tau).$$

It follows that $\tilde{h}(\tau; t)$ is strictly increasing in τ when t is small enough.[7]

[6] In addition, $\lambda_2(x) = \lambda_1(1 - x)$ and thus:

$$\begin{aligned} \int_0^{\hat{x}} f(\lambda_1(x))dx &= \int_{\hat{x}}^{1} f(\lambda_2(x))dx = \int_0^{1/2} f(\lambda_1(x))dx, \\ \int_0^{\hat{x}} F(\lambda_1(x))dx &= \int_{\hat{x}}^{1} F(\lambda_2(x))dx = \int_0^{1/2} F(\lambda_1(x))dx. \end{aligned}$$

[7] If the density $f(s)$ weakly decreases in s, then $\tilde{h}'(\tau) \geq 1 > 0$.

Using $\tau = \delta + 2\mu - m$, (1) yields $\tau = \delta - 2\tilde{h}(\tau; t)$. The equilibrium threshold, $\tilde{\tau}$, thus solves $\tilde{\tau} = \phi^{-1}(\delta; t)$, where

$$\phi(\tau; t) \equiv \tau + 2\tilde{h}(\tau; t) \tag{2}$$

is increasing in τ. Hence, the equilibrium threshold $\tilde{\tau}(t)$ is uniquely defined by

$$\tilde{\tau}(t) \equiv \phi^{-1}(\delta; t). \tag{3}$$

Consider now a small increase in μ_1 by dr, keeping ρ_1 constant. This does not affect consumers' choices between multi-stop shopping and patronizing firm 2; yet:

- On the one hand, firm 1 charges a larger margin to the one-stop shoppers who patronize it; the associated gain is $D_1 dr$.
- On the other hand, it induces some one-stop shoppers that were patronizing to switch to either multi-stop shopping, or to visiting firm 2 instead; hence:
  - for $x \in [0, 1/2]$, consumers with $s = \lambda_1(x)$ switch to multi-stop shopping; the resulting loss for firm 1 is equal to $-\mu_1 \int_0^{1/2} f(\lambda_1(x)) dx dr$;
  - for $x = \hat{x} = 1/2$, one-stop shoppers switch to firm 2; the resulting loss for firm 1 is equal to $-\frac{m_1}{4t}(1 - F(\tau_1)) dr$.

In equilibrium, these effects must cancel out, which leads to a second first-order condition:

$$\mu_1 \int_0^{1/2} f(\lambda_1(x)) dx + \frac{m_1}{4t}(1 - F(\tau_1)) = D_1.$$

Evaluating this condition for symmetric equilibrium margins yields

$$\mu = \frac{4t \displaystyle\int_0^{1/2} [1 - F(\lambda_1(x))] \, dx - m(1 - F(\tau))}{4t \displaystyle\int_0^{1/2} f(\lambda_1(x)) dx} = \frac{2t - m(1 - F(\tau))}{2[F(\tau) - F(\tau - t)]} - \tilde{h}(\tau; t).$$

Comparing with (1) yields the equilibrium total margin:

$$\tilde{m}(t) \equiv \frac{2t}{1 - F(\tilde{\tau}(t) - t)}. \tag{4}$$

The equilibrium margins for weak and strong products are then respectively given by

$$\tilde{\mu}(t) = \frac{t}{1 - F(\tilde{\tau}(t) - t)} - \tilde{h}(\tilde{\tau}(t); t), \tag{5}$$

$$\tilde{\rho}(t) = \frac{t}{1 - F(\tilde{\tau}(t) - t)} + \tilde{h}(\tilde{\tau}(t); t). \tag{6}$$

C.1.3 Proof of Proposition 3

Comparative statics. Note that $\tilde{\mu}(t)$ is continuous in t and $\lim_{t \to 0} \tilde{\mu}(0) = -h(\tau) < 0$. Therefore, there exists $\bar{t} > 0$ such that $\tilde{\mu}(t) < 0$ for $t \in [0, \bar{t}]$. We now show that $\tilde{m}(t)$ and $\tilde{\mu}(t)$ both increase in t for sufficiently small t. For this purpose, we first show that $\tilde{\tau}(t)$ increases in t. Using $\tilde{\tau}(t) = \delta - 2\tilde{h}(\tilde{\tau}(t); t)$, we obtain

$$\tilde{\tau}'(t) = -2\frac{d}{dt}\left[\tilde{h}(\tilde{\tau}(t); t)\right] = -2\tilde{h}'(\tilde{\tau}(t); t)\tilde{\tau}'(t) - 2\frac{\partial \tilde{h}}{\partial t}(\tilde{\tau}(t); t).$$

Hence, we have

$$\tilde{\tau}'(t) = \frac{-2\frac{\partial \tilde{h}}{\partial t}(\tilde{\tau}(t); t)}{1 + 2\tilde{h}'(\tilde{\tau}(t); t)}.$$

Note that:

$$\frac{\partial \tilde{h}(\tau; t)}{\partial t} = \frac{f(\tau - t)\left[h(\tau - t) - \tilde{h}(\tau; t)\right]}{F(\tau) - F(\tau - t)} = \frac{f(\tau - t)}{F(\tau) - F(\tau - t)} \frac{\displaystyle\int_{\tau - t}^{\tau} [h(s) - h(\tau - t)] f(s) ds}{\displaystyle\int_{\tau - t}^{\tau} f(s) ds} > 0.$$

Hence, $\tilde{\tau}'(t) > 0$, which further implies

$$\frac{d}{dt}\left[\tilde{h}(\tilde{\tau}(t); t)\right] = -\frac{\tilde{\tau}'(t)}{2} < 0.$$

From (4), we have

$$\tilde{m}'(t) = \frac{2}{1 - F(\tilde{\tau}(t) - t)} + 2t\frac{f(\tilde{\tau}(t) - t)\left[\tilde{\tau}'(t) - 1\right]}{\left[1 - F(\tilde{\tau}(t) - t)\right]^2},$$

which is positive for t sufficiently small.

Finally, from (5), we have:

$$\tilde{\mu}'(t) = \frac{\tilde{m}'(t)}{2} - \frac{d}{dt}\left[\tilde{h}(\tilde{\tau}(t); t)\right] > \frac{\tilde{m}'(t)}{2}.$$

Therefore, $\tilde{\mu}'(t)$ is also positive for t sufficiently small.

Existence. We show now the above margins constitute indeed an equilibrium when t is close to 0. For this purpose, it suffices to show that firm i's profit function is quasi-concave in m_i and ρ_i, in the relevant range, when t approaches zero. The following assumption is sufficient to establish quasi-concavity:

Assumption B': The density function $f(\cdot)$ is non-increasing.

As the firms are symmetric, we focus on firm 1 and rewrite its profit function as follows (taking firm 2's margins $\tilde{m}(t)$ and $\tilde{\rho}(t)$ as given):

$$\Pi_1(m_1, \rho_1) = m_1 D_1(m_1, \rho_1) + \rho_1 D(m_1, \rho_1),$$

where

$$\begin{aligned} D_1(m_1, \rho_1) &\equiv \int_0^{\hat{x}(m_1)} [1 - F(\lambda_1(x; m_1 - \rho_1))]\, dx, \\ D(m_1, \rho_1) &\equiv \int_0^{\hat{x}(m_1)} F(\lambda_1(x; m_1 - \rho_1))\, dx + \int_{\hat{x}(m_1)}^1 F(\lambda_2(x; \rho_1))\, dx, \end{aligned}$$

and

$$\begin{aligned} \hat{x}(m_1) &\equiv \min\{\max\{\frac{1}{2} - \frac{m_1 - \tilde{m}(t)}{4t}, 0\}, 1\}, \\ \lambda_1(x; m_1 - \rho_1) &\equiv \max\{\delta + m_1 - \rho_1 - \tilde{\rho}(t) - t + 2tx, 0\}, \\ \lambda_2(x; \rho_1) &\equiv \max\{\delta + \tilde{\mu}(t) - \rho_1 + t - 2tx, 0\}. \end{aligned}$$

Differentiating D and D_1 with respect to m_1 and ρ_1, we obtain:

$$\begin{aligned} \frac{\partial D}{\partial \rho_1}(m_1, \rho_1) &= -\int_0^{\hat{x}} f(\lambda_1(x; m_1 - \rho_1))\, dx - \int_{\hat{x}}^1 f(\lambda_2(x; \rho_1))\, dx < 0, \\ \frac{\partial D}{\partial m_1}(m_1, \rho_1) &= \frac{\partial D_1}{\partial \rho_1}(m_1, \rho_1) = \int_0^{\hat{x}} f(\lambda_1(x; m_1 - \rho_1))\, dx \geq 0, \\ \frac{\partial D_1}{\partial m_1}(m_1, \rho_1) &= -\int_0^{\hat{x}} f(\lambda_1(x; m_1 - \rho_1))\, dx - \frac{1 - F(\lambda_1(\hat{x}; m_1 - \rho_1))}{4t} < 0. \end{aligned}$$

Obviously, charging $\rho_1 < 0$ *and* $m_1 < 0$ is never optimal. Suppose now that firm 1 charges $\rho_1 < 0$ and $m_1 \geq 0$, and consider raising ρ_1 to 0 while maintaining m_1. It avoids the loss from selling the strong product to multi-stop shoppers, without reducing the demand from one-stop shoppers, as $\partial D_1/\partial \rho_1 \geq 0$. Suppose instead that firm 1 charges $m_1 < 0$ and $\rho_1 > 0$, and consider raising m_1 to 0. This avoids the loss on one-stop shoppers, without reducing the demand for the strong product from multi-stop shoppers, as $\partial D/\partial m_1 \geq 0$.

Hence, without loss of generality, we can restrict attention to firm 1's deviations such that $\rho_1 \geq 0$ and $m_1 \geq 0$. Furthermore, if firm 1 deviates $m_1 \geq \tilde{m}(t) + 2t$, then it does not attract any one-stop shopper and its profit remains the same as when charging $m_1 = \tilde{m}_2 + 2t$, where $\hat{x} = 0$:

$$\Pi_1(m_1, \rho_1) = \Pi_1(\tilde{m}(t) + 2t, \rho_1) = \rho_1 \int_0^1 F(\lambda_2(x; \rho_1))\, dx.$$

In addition, for $m_1 = \tilde{m}(t) + 2t$, $\hat{x} = 0$ implies $D_1 = \partial D/\partial m_1 = 0$, and thus:[8]

$$\left.\frac{\partial \Pi_1}{\partial m_1}\right|_{m_1 = \tilde{m}(t) + 2t} = -m_1 \frac{1 - F(\tau_2 + t)}{4t} < 0.$$

It follows that it is never optimal for firm 1 to deviate to $m_1 \geq \tilde{m}(t) + 2t$.

If instead firm 1 deviates to $m_1 < \tilde{m}(t) - 2t$, it attracts all one-stop shoppers and its profit is equal to

$$\begin{aligned} \Pi_1(m_1, \rho_1) &= \int_0^1 [1 - F(\lambda_1(x; m_1 - \rho_1))]\, dx + \rho_1 \int_0^1 F(\lambda_1(x; m_1 - \rho_1))\, dx \\ &= m_1 - (m_1 - \rho_1) \int_0^1 [1 - F(\lambda_1(x; m_1 - \rho_1))]\, dx. \end{aligned}$$

It follows that a simultaneous increase in both m_1 and ρ_1 would increase firm 1's profit. As the profit function $\Pi_1(m_1, \rho_1)$ is continuous at the boundary $m_1 = \tilde{m}(t) - 2t$, it is never optimal for firm 1 to deviate to $m_1 < \tilde{m}(t) - 2t$.

Hence, without loss of generality we can restrict attention to firm 1's deviations such that $m_1 \in [\tilde{m}(t) - 2t, \tilde{m}(t) + 2t)$, for which $\hat{x}(m_1) = \frac{1}{2} - \frac{m_1 - \tilde{m}(t)}{4t} \in (0, 1]$. We now show that, in that range, the profit function $\Pi_1(m_1, \rho_1)$ is strictly concave when t is close to 0. The second-order derivatives of the demand functions are given by:

$$\begin{aligned} \frac{\partial^2 D}{\partial \rho_1^2} &= \int_0^{\hat{x}} f'(\lambda_1(x; m_1 - \rho_1))\, dx + \int_{\hat{x}}^1 f'(\lambda_2(x; \rho_1))\, dx, \\ \frac{\partial^2 D}{\partial m_1^2} &= \int_0^{\hat{x}} f'(\lambda_1(x; m_1 - \rho_1))\, dx - \frac{f(\lambda_1(\hat{x}; m_1 - \rho_1))}{4t} \\ \frac{\partial^2 D}{\partial \rho_1 \partial m_1} &= -\int_0^{\hat{x}} f'(\lambda_1(x; m_1 - \rho_1))\, dx, \end{aligned}$$

[8] Note that $\tau_2 + t = \delta + \tilde{\mu}(t) - \rho_1 + t < \delta + t$. The assumption $\bar{s} > \delta$ then implies $\bar{s} \geq \delta + t$ for t small enough, which ensures that some one-stop shoppers remain active following firm 1's deviation.

and

$$
\begin{aligned}
\frac{\partial^2 D_1}{\partial \rho_1^2} &= -\int_0^{\hat{x}} f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) dx, \\
\frac{\partial^2 D_1}{\partial m_1^2} &= -\int_0^{\hat{x}} f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) dx + \frac{3f\left(\lambda_1\left(\hat{x}; m_1 - \rho_1\right)\right)}{8t}, \\
\frac{\partial^2 D_1}{\partial m_1 \partial \rho_1} &= \int_0^{\hat{x}} f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) dx - \frac{f\left(\lambda_1\left(\hat{x}; m_1 - \rho_1\right)\right)}{4t}.
\end{aligned}
$$

Differentiating Π_1 with respect to ρ_1, we obtain

$$
\frac{\partial \Pi_1}{\partial \rho_1} = D + m_1 \frac{\partial D_1}{\partial \rho_1} + \rho_1 \frac{\partial D}{\partial \rho_1}
$$

and:

$$
\begin{aligned}
\frac{\partial^2 \Pi_1}{\partial \rho_1^2} &= 2\frac{\partial D}{\partial \rho_1} + m_1 \frac{\partial^2 D_1}{\partial \rho_1^2} + \rho_1 \frac{\partial^2 D}{\partial \rho_1^2} \\
&= -2\left[\int_0^{\hat{x}} f\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) dx + \int_{\hat{x}}^1 f\left(\lambda_2\left(x; \rho_1\right)\right) dx\right] - m_1 \int_0^{\hat{x}} f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) dx \\
&\quad + \rho_1 \left[\int_0^{\hat{x}} f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right)) dx + \int_{\hat{x}}^1 f'\left(\lambda_2\left(x; \rho_1\right)\right) dx\right] \\
&= -\left(\Psi_1 + \Psi_2\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi_1 &\equiv \int_0^{\hat{x}} \left[2f\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) + \left(m_1 - \rho_1\right) f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right)\right] dx, \\
\Psi_2 &\equiv \int_{\hat{x}}^1 \left[2f\left(\lambda_2\left(x; \rho_1\right)\right) - \rho_1 f'\left(\lambda_2\left(x; \rho_1\right)\right)\right] dx.
\end{aligned}
$$

We show now this second-order derivative is negative when t approaches zero. As firm 1's relevant deviations are such that $\rho_1 \geq 0$ and $m_1 \leq \tilde{m}(t) + 2t$, and $f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) \leq 0$ under Assumption B', we have:

$$
\begin{aligned}
\Psi_1 &\geq \int_0^{\hat{x}} \left\{2f\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) + \left[\tilde{m}(t) + 2t\right] f'\left(\lambda_1\left(x; m_1 - \rho_1\right)\right)\right\} dx \\
&\geq \int_0^{\hat{x}} 2f\left(\lambda_1\left(x; m_1 - \rho_1\right)\right) dx + \left[\tilde{m}(t) + 2t\right] \max_{s \in [0, \bar{s}]} f'(s) \\
&\geq \int_0^{\hat{x}} 2f\left(\delta + \tilde{m} + 3t\right) dx + \left[\tilde{m}(t) + 2t\right] \max_{s \in [0, \bar{s}]} f'(s), \qquad (7)
\end{aligned}
$$

where the second inequality uses $\hat{x} \leq 1$ and the last one follows from Assumption B' and (using $m_1 \leq \tilde{m}(t) + 2t$ and $\rho_1 \geq 0$):

$$\lambda_1 (x; m_1 - \rho_1) = \delta + m_1 - \rho_1 - \tilde{\rho} - t + 2tx \leq \delta + \tilde{m}(t) + 3t. \tag{8}$$

Likewise, we have:

$$\begin{aligned} \Psi_2 &\geq \int_{\hat{x}}^{1} 2f (\lambda_2 (x; \rho_1))\, dx \\ &\geq \int_{\hat{x}}^{1} 2f (\delta + t)\, dx, \end{aligned}$$

where the second inequality follows from Assumption B' and $\lambda_2 (x; \rho_1) = \delta + \tilde{\mu} - \rho_1 + t - 2tx \leq \delta + t$. Using Assumption B' and $\tilde{m}(t) + 2t \geq 0$, we thus have:

$$\frac{\partial^2 \Pi_1}{\partial \rho_1^2} \leq -\underline{\Psi}_1 (t),$$

where

$$\underline{\Psi}_1 (t) \equiv \int_0^1 2f (\delta + \tilde{m}(t) + 3t)\, dx + [\tilde{m}(t) + 2t] \max_{s \in [0, \bar{s}]} f'(s),$$

and

$$\lim_{t \to 0} \underline{\Psi}_1 (t) = 2f(\delta) > 0.$$

It follows that $\frac{\partial^2 \Pi_1}{\partial \rho_1^2}$ is negative when t is close to zero.

Differentiating Π_1 with respect to m_1, we obtain

$$\frac{\partial \Pi_1}{\partial m_1} = D_1 + m_1 \frac{\partial D_1}{\partial m_1} + \rho_1 \frac{\partial D}{\partial m_1}$$

and:

$$\begin{aligned} \frac{\partial^2 \Pi_1}{\partial m_1^2} &= 2\frac{\partial D_1}{\partial m_1} + m_1 \frac{\partial^2 D_1}{\partial m_1^2} + \rho_1 \frac{\partial^2 D}{\partial m_1^2} \\ &= -2 \left[\int_0^{\hat{x}} f (\lambda_1 (x; m_1 - \rho_1))\, dx + \frac{1 - F (\lambda_1 (\hat{x}; m_1 - \rho_1))}{4t} \right] \\ &\quad + m_1 \left[- \int_0^{\hat{x}} f' (\lambda_1 (x; m_1 - \rho_1))\, dx + \frac{3f (\lambda_1 (\hat{x}; m_1 - \rho_1))}{8t} \right] \\ &\quad + \rho_1 \left[\int_0^{\hat{x}} f' (\lambda_1 (x; m_1 - \rho_1))\, dx - \frac{f (\lambda_1 (\hat{x}; m_1 - \rho_1))}{4t} \right] \\ &= -\Psi_1 - \Psi_3, \end{aligned}$$

where

$$\Psi_3 \equiv \frac{1 - F(\lambda_1(\hat{x}; m_1 - \rho_1))}{2t} - \frac{3m_1 f(\lambda_1(\hat{x}; m_1 - \rho_1))}{8t} + \frac{\rho_1 f(\lambda_1(\hat{x}; m_1 - \rho_1))}{4t}.$$

We have:

$$\begin{aligned}
\Psi_3 &\geq \frac{1 - F(\delta + \tilde{m}(t) + 3t)}{2t} - 3f(0)\frac{\tilde{m}(t) + 2t}{8t} \\
&= \frac{1 - F(\delta + \tilde{m}(t) + 3t)}{2t} - \frac{3f(0)}{4}\left[1 + \frac{1}{1 - F(\tilde{\tau}(t) - t)}\right] \\
&\geq \frac{1 - F(\delta + \tilde{m}(t) + 3t)}{2t} - \frac{3f(0)}{4}\frac{2 - F(\delta)}{1 - F(\delta)}.
\end{aligned}$$

where the first inequality follows from $m_1 \leq \tilde{m}(t) + 2t$, $\rho_1 \geq 0$, Assumption B' and (8), the equality stems from (4), and the last inequality follows from $\tilde{\tau}(t) - t = \delta - 2\tilde{h}(\tilde{\tau}(t); t) - t < \delta$. Using (7), we then have:

$$\frac{\partial^2 \Pi_1}{\partial m_1^2} \leq -\underline{\Psi}_2(t),$$

where

$$\underline{\Psi}_2(t) \equiv [\tilde{m}(t) + 2t] \max_{s \in [0, \bar{s}]} f'(s) + \frac{1 - F(\delta + \tilde{m}(t) + 3t)}{2t} - \frac{3f(0)}{4}\frac{2 - F(\delta)}{1 - F(\delta)},$$

and

$$\lim_{t \to 0} \underline{\Psi}_2(t) = +\infty.$$

It follows that $\frac{\partial^2 \Pi_1}{\partial m_1^2}$ is negative and moreover goes to $-\infty$ as t tends to zero.

Finally, the remaining second-order derivative of the profit function is given by:

$$\begin{aligned}
\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} &= \frac{\partial D}{\partial m_1} + \frac{\partial D_1}{\partial \rho_1} + m_1 \frac{\partial^2 D_1}{\partial \rho_1 \partial m_1} + \rho_1 \frac{\partial^2 D}{\partial \rho_1 \partial m_1} \\
&= 2\int_0^{\hat{x}} f(\lambda_1(x; m_1 - \rho_1))\, dx + m_1 \left[\int_0^{\hat{x}} f'(\lambda_1(x; m_1 - \rho_1))\, dx - \frac{f(\lambda_1(\hat{x}; m_1 - \rho_1))}{4t}\right] \\
&\quad -\rho_1 \int_0^{\hat{x}} f'(\lambda_1(x; m_1 - \rho_1))\, dx \\
&= \Psi_1 - \frac{m_1 f(\lambda_1(\hat{x}; m_1 - \rho_1))}{4t}.
\end{aligned}$$

We have:

$$\begin{aligned}
\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} &\geq \Psi_1 - \frac{[\tilde{m}(t) + 2t] f(0)}{4t} \\
&= \Psi_1 - \frac{f(0)}{2}\left[1 + \frac{1}{1 - F(\tilde{\tau}(t) - t)}\right] \\
&\geq \Psi_1 - \frac{f(0)}{2}\frac{2 - F(\delta)}{1 - F(\delta)},
\end{aligned}$$

where the first inequality follows from $m_1 \leq \tilde{m}(t) + 2t$ and Assumption B', whereas the other two steps stem again from (4) and $\tilde{\tau}(t) - t < \delta$. Using (7), we thus have:

$$\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} \geq \underline{\Psi}_3(t),$$

where

$$\underline{\Psi}_3(t) \equiv [\tilde{m}(t) + 2t] \max_{s \in [0,\bar{s}]} f'(s) - \frac{f(0)}{2} \frac{2 - F(\delta)}{1 - F(\delta)}$$

and

$$\lim_{t \to 0} \underline{\Psi}_3(t) = -\frac{f(0)}{2} \frac{2 - F(\delta)}{1 - F(\delta)}.$$

Conversely:

$$\begin{aligned}
\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} &\leq \Psi_1 \\
&= \int_0^{\hat{x}} \left[2f(\lambda_1(x; m_1 - \rho_1)) + (m_1 - \rho_1) f'(\lambda_1(x; m_1 - \rho_1))\right] dx \\
&\leq \int_0^{\hat{x}} \left[2f(0) - \rho_1 \min_{s \in [0,\bar{s}]} f'(s)\right] dx \\
&\leq 2f(0) - w_1^A \min_{s \in [0,\bar{s}]} f'(s),
\end{aligned}$$

where the first two inequalities follow from $m_1 \geq 0$ and Assumption B', whereas the last one stems from $\hat{x} \leq 1$, Assumption B' and the fact that, without loss of generality, we can restrict attention to deviations such that $\rho_1 \leq w_1^A$, the surplus generated by firm 1's variety of product A. We thus have:

$$\left|\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}\right| \leq \underline{\Psi}_4(t),$$

where

$$\underline{\Psi}_4(t) = \max\left\{\left|\frac{f(0)}{2} \frac{2 - F(\delta)}{1 - F(\delta)} - [\tilde{m}(t) + 2t] \max_{s \in [0,\bar{s}]} f'(s)\right|, 2f(0) - w_1^A \min_{s \in [0,\bar{s}]} f'(s)\right\}$$

and

$$\lim_{t \to 0} \underline{\Psi}_4(t) = \underline{\Psi} \equiv \max\left\{\frac{f(0)}{2} \frac{2 - F(\delta)}{1 - F(\delta)}, 2f(0) - w_1^A \min_{s \in [0,\bar{s}]} f'(s)\right\}.$$

It follows that $|\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}|$ remains bounded as t goes to zero.

Summarizing the above analysis, we can conclude that as $t \to 0$, in the relevant range the second-order derivatives $\frac{\partial^2 \Pi_1}{\partial \rho_1^2}$ and $\frac{\partial^2 \Pi_1}{\partial m_1^2}$ are both negative and the Hessian, $H = \frac{\partial^2 \Pi_1}{\partial \rho_1^2} \frac{\partial^2 \Pi_1}{\partial m_1^2} - \left(\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}\right)^2$ is positive. More precisely, fix $\hat{t}_0 > 0$ and the associated relevant range $\mathcal{R} \equiv \left\{m_1 \in [0, \tilde{m}(t) + 2t], \rho_1 \in \left[0, w_1^A\right]\right\}$. We have:

- as $\frac{\partial^2 \Pi_1}{\partial \rho_1^2}(m_1, \rho_1) \leq -\underline{\Psi}_1(t)$ for any $(m_1, \rho_1) \in \mathcal{R}$, and $\lim_{t\to 0} \underline{\Psi}_1(t) = 2f(\delta) > 0$, there exists $\hat{t}_1$ such that $\frac{\partial^2 \Pi_1}{\partial \rho_1^2}(\cdot) < -f(\delta) < 0$ in the range $\mathcal{R}$;

- as $\frac{\partial^2 \Pi_1}{\partial m_1^2}(m_1, \rho_1) \leq -\underline{\Psi}_2(t)$ for any $(m_1, \rho_1) \in \mathcal{R}$, and $\lim_{t\to 0} \underline{\Psi}_2(t) = +\infty$, there exists $\hat{t}_2$ such that $\frac{\partial^2 \Pi_1}{\partial m_1^2}(\cdot) < -4\underline{\Psi}^2/f(\delta)$ in the range $\mathcal{R}$;

- as $\left|\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}(m_1, \rho_1)\right| \leq \underline{\Psi}_4(t)$ for any $(m_1, \rho_1) \in \mathcal{R}$, and $\lim_{t\to 0} \underline{\Psi}_4(t) = \underline{\Psi}(> 0)$, there exists $\hat{t}_3$ such that $\left|\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}(\cdot)\right| < 2\underline{\Psi}$ in the range $\mathcal{R}$;

It follows that, for $t < \hat{t} \equiv \min\{\hat{t}_0, \hat{t}_1, \hat{t}_2, \hat{t}_3\}$, $\frac{\partial^2 \Pi_1}{\partial \rho_1^2}(m_1, \rho_1) < 0$, $\frac{\partial^2 \Pi_1}{\partial m_1^2}(m_1, \rho_1) < 0$ and $H(m_1, \rho_1) > 0$ for any $(m_1, \rho_1) \in \mathcal{R}$. Hence, for $t < \hat{t}$, the profit function $\Pi(m_1, \rho_1)$ is thus strictly concave in the range $\mathcal{R}$.

C.1.4 Illustration: uniform distribution

To further characterize the scope for cross-subsidization, we consider here the case where the shopping cost is uniformly distributed over $[0, \bar{s}]$. We thus have $f(s) = 1/\bar{s}$, $F(s) = s/\bar{s}$, $h(s) = s$ and

$$\tilde{h}(\tau; t) = 2\int_0^{1/2} (\tau - t + 2tx)dx = \tau - \frac{t}{2}.$$

Equilibrium outcome. The equilibrium threshold is given by:

$$\tilde{\tau}(t) = \frac{\delta + t}{3}. \tag{9}$$

We focus on the case where $t \leq \delta/2$, which ensures that $t \leq \tilde{\tau}(t)$, and moreover assume that $\bar{s} > 7\delta/2$; as we will see, this ensures that the profit functions are strictly concave in the relevant range.

The equilibrium margins are then:

$$\begin{aligned}
\tilde{m}(t) &= \frac{2t\bar{s}}{\bar{s} - [\tilde{\tau}(t) - t]} = \frac{6t\bar{s}}{3\bar{s} + 2t - \delta}, \\
\tilde{\mu}(t) &= \frac{t\bar{s}}{\bar{s} - [\tilde{\tau}(t) - t]} - \left[\tilde{\tau}(t) - \frac{t}{2}\right] = \frac{3t\bar{s}}{3\bar{s} - \delta + 2t} - \frac{2\delta - t}{6}.
\end{aligned}$$

It can be checked that $\tilde{\mu}(t)$ increases with t:

$$\tilde{\mu}'(t) = \frac{\bar{s}[\bar{s} - \tilde{\tau}(t)] + \frac{t\bar{s}}{3}}{\{\bar{s} - [\tilde{\tau}(t) - t]\}^2} + \frac{1}{6} > 0.$$

Moreover, $\tilde{\mu}(0) = -\tau^* = -\frac{\delta}{3} < 0$ and $\tilde{\mu}(\delta/2) = t/2 > 0$. It follows that there exists a threshold $\bar{t} \in (0, \delta/2)$ such that $\tilde{\mu}(t) < 0$ if and only if $t < \bar{t}$. Solving for $\tilde{\mu}(t) = 0$, or

$$2\delta^2 - 6\bar{s}\delta - (5\delta - 21\bar{s})\,t + 2t^2 = 0,$$

yields:

$$\bar{t} = \frac{3\sqrt{\delta^2 - 18\bar{s}\delta + 49\bar{s}^2} - (21\bar{s} - 5\delta)}{4}.$$

Existence. We now check for the concavity of the profit functions. Note that

$$\tilde{m}(t) = \frac{2t}{1 - \frac{\tilde{\tau}(t) - t}{\bar{s}}} > 2t.$$

Conversely, the condition $\delta < 2\bar{s}/7$ yields:

$$\tilde{m}(t) = \frac{6t\bar{s}}{3\bar{s} + 2t - \delta} \leq \frac{6t\bar{s}}{3\bar{s} - \delta} < \frac{6t\bar{s}}{3\bar{s} - \frac{2\bar{s}}{7}} = \frac{42}{19}t.$$

Using $F(s) = s/\bar{s}$, we have

$$\Psi_1 = \frac{2\hat{x}}{\bar{s}},\;\; \Psi_2 = \frac{2(1 - \hat{x})}{\bar{s}},$$

and thus

$$\frac{\partial^2 \Pi_1}{\partial \rho_1^2} = -\frac{2}{\bar{s}}.$$

Furthermore:

$$\Psi_3 \equiv \frac{\bar{s} - \lambda_1(\hat{x}; m_1 - \rho_1)}{2t\bar{s}} - \frac{3m_1}{8t\bar{s}} + \frac{\rho_1}{4t\bar{s}}.$$

Therefore:

$$\frac{\partial^2 \Pi_1}{\partial m_1^2} = -(\Psi_1 + \Psi_3) = -\left[\frac{2\hat{x}}{\bar{s}} + \frac{\bar{s} - \lambda_1(\hat{x}; m_1 - \rho_1)}{2t\bar{s}} - \frac{3m_1}{8t\bar{s}} + \frac{\rho_1}{4t\bar{s}}\right].$$

Using

$$\lambda_1(\hat{x}; m_1 - \rho_1) = \delta + \frac{m_1 + \tilde{m}(t)}{2} - \rho_1 - \tilde{\rho}(t)$$

and

$$\hat{x} = \frac{1}{2} - \frac{m_1 - \tilde{m}(t)}{4t},$$

we have:

$$\frac{\partial^2 \Pi_1}{\partial m_1^2} = -\frac{1}{2\bar{s}}\left[\frac{\bar{s} - \delta}{t} + \frac{6\rho_1 + 8t - 9m_1 + 2\tilde{m}(t) + 4\tilde{\rho}(t)}{4t}\right].$$

As $m_1 \leq \tilde{m}(t) + 2t$ and $\rho_1 \geq 0$ in the relevant range, and focusing on $\tilde{\mu}(t) \leq 0$ (i.e., $\tilde{\rho}(t) \geq \tilde{m}(t)$), we have:

$$\begin{aligned}\frac{\partial^2 \Pi_1}{\partial m_1^2} &\leq -\frac{1}{2\bar{s}}\left(\frac{\bar{s}-\delta}{t} - \frac{3\tilde{m}(t)+10t}{4t}\right) \\ &< -\frac{1}{2\bar{s}}\left(\frac{\bar{s}-\delta}{t} - \frac{79}{19}\right),\end{aligned}$$

where the last inequality follows from $\tilde{m}(t) < 42t/19$. Furthermore, we have:

$$\begin{aligned}\frac{\partial^2 \Pi_1}{\partial \rho_1^2}\frac{\partial^2 \Pi_1}{\partial m_1^2} &= (\Psi_1 + \Psi_2)(\Psi_1 + \Psi_3) \\ &> \frac{1}{\bar{s}^2}\left(\frac{\bar{s}-\delta}{t} - \frac{79}{19}\right).\end{aligned}$$

Finally,

$$\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} = \frac{2\hat{x}}{\bar{s}} - \frac{m_1}{4t\bar{s}} = \frac{4t - 3m_1 + 2\tilde{m}(t)}{4t\bar{s}}.$$

Using $(0 <)\, \tilde{m}(t) - 2t \leq m_1 \leq \tilde{m}(t) + 2t$, lower and upper bounds for the above expression are given by

$$\frac{-\tilde{m}(t) - 2t}{4t\bar{s}} \leq \frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} \leq \frac{10t - \tilde{m}(t)}{4t\bar{s}}.$$

Therefore:

$$\begin{aligned}\left|\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}\right| &\leq \frac{\max\{\tilde{m}(t) + 2t, 10t - \tilde{m}(t)\}}{4t\bar{s}} \\ &= \frac{10t - \tilde{m}(t)}{4t\bar{s}},\end{aligned}$$

where the last equality comes from the fact that $\tilde{m}(t) < 42t/19$. As in addition $\tilde{m}(t) > 2t$

$$\left(\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1}\right)^2 \leq \left(\frac{8t}{4t\bar{s}}\right)^2 = \frac{4}{\bar{s}^2}.$$

Therefore, $H > 0$ if

$$\frac{1}{\bar{s}^2}\left(\frac{\bar{s}-\delta}{t} - \frac{79}{19}\right) \geq \frac{4}{\bar{s}^2},$$

which holds when

$$t < \hat{t} \equiv \frac{19}{155}(\bar{s} - \delta).$$

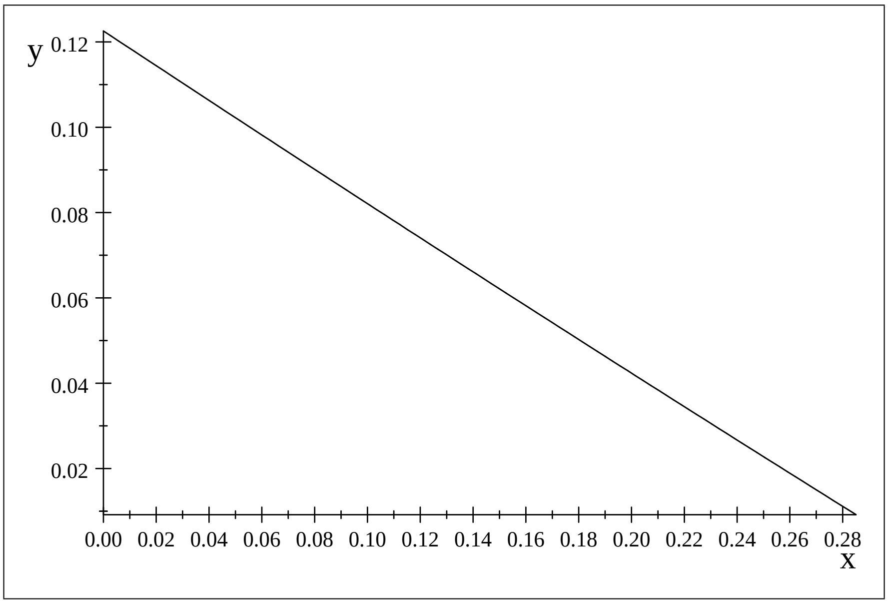
Finally, it can be checked that $\hat{t} > \bar{t}$:

$$\frac{\hat{t} - \bar{t}}{\bar{s}} = \chi\left(\frac{\delta}{\bar{s}}\right),$$

where

$$\chi(z) \equiv \frac{3331}{620} - \frac{3}{4}\sqrt{z^2 - 18z + 49} - \frac{851}{620}z$$

is positive in the relevant range $\delta/\bar{s} < 2/7 \simeq 0.285$:



C.2 Stand-alone demand for weak products

We extend the previous model by introducing some consumers interested only in specific products. Intuitively, the scope for cross-subsidization decreases as more consumers are interested in weak products, as this discourages firms from selling these products below cost. To explore this further, we thus assume here that, in addition to the unit-mass population of consumers interested in both products, with heterogeneous preferences à la Hotelling described above and shopping costs uniformly distributed over $[0, \bar{s}]$, a mass ω of consumers are only interested in the weak products.

For the sake of exposition, we assume that weak products are also supplied at cost by a competitive fringe. This deters firms from deviating and exploit solely the stand-alone demand for weak products. We also assume that ω is sufficiently small, so that in equilibrium firms are willing to serve one-stop shoppers as well as multi-stop shoppers – that is, they are not willing to drop their weak products. Firm i's profit profit is then given by:

$$\Pi_i = m_i D_i + \rho_i D + \omega \mu_i = m_i (D_i + D) - \mu_i (D - \omega).$$

To ensure that both types of shopping patterns arise, we assume that the distribution of shopping cost is sufficiently dispersed and that t and ω are sufficiently small:

$$t < \frac{\delta}{3}, \omega < \frac{\delta}{3\bar{s}} \text{ and } \delta < \frac{2\bar{s}}{7}. \tag{10}$$

As we will see, these restrictions indeed ensure that the equilibrium threshold for multi-stop shopping satisfies $\tilde{\tau} < \delta$.

C.2.1 Comparative statics

We follow the same approach as above, and first consider a small deviation in the margins of firm 1, say, keeping constant the total margin for its basket. Compared with the previous analysis, the only change is that this deviation now reduces the margin earned on those consumers who are only interested in weak products; as a result, the first-order condition becomes

$$\mu = \frac{m}{2} - \tilde{h}(\tau) + \bar{s}\omega, \tag{11}$$

where as before:

$$\tilde{h}(\tau; t) = \frac{\int_0^{1/2} F(\lambda_1(x))dx}{\int_0^{1/2} f(\lambda_1(x))dx} = \tau - \frac{t}{2}.$$

Using $\tau = \delta + 2\mu - m$, the first-order condition (11) yields $\tau + 2\tilde{h}(\tau; t) = \delta + 2\bar{s}\omega$, leading to

$$\tilde{\tau}(t, \omega) = \frac{t + \delta + 2\bar{s}\omega}{3}, \tag{12}$$

where, under (10):[9]

$$\tilde{\tau}(t, \omega) < \frac{2\delta}{3} < \frac{4\bar{s}}{21}. \tag{13}$$

Consider in turn a small deviation in μ_1, keeping ρ_1 constant. This deviation now affects the margin earned on consumers only interested in weak products; as a result, the

[9] Under (10), we have:

$$\tilde{\tau}(t, \omega) = \frac{t + \delta + 2S\omega}{3} < \frac{1}{3}\left(\frac{\delta}{3} + \delta + 2S\frac{\delta}{3S}\right) = \frac{2\delta}{3}.$$

associated first-order condition becomes:

$$\mu = -\frac{m}{2t}(\bar{s}-\tau)+\bar{s}-\left(\tau-\frac{t}{2}\right)+2\bar{s}\omega. \tag{14}$$

Combining this condition with (11) yields $\tilde{m}(t,\omega)=\hat{m}(t,\omega,\tilde{\tau}(t,\omega))$, where:

$$\hat{m}(t,\omega,\tau)\equiv\frac{2t\bar{s}(1+\omega)}{\bar{s}-\tau+t},$$

where $\tilde{m}(t,\omega)>0$, as (13) implies $\bar{s}-\tilde{\tau}(t,\omega)+t>0$, and:

$$\frac{\partial\hat{m}}{\partial t}(t,\omega,\tau)=\frac{2\bar{s}(1+\omega)(\bar{s}-\tau)}{(\bar{s}+t-\tau)^2},\ \frac{\partial\hat{m}}{\partial\omega}(t,\omega,\tau)=\frac{2t\bar{s}}{\bar{s}+t-\tau},\ \frac{\partial\hat{m}}{\partial\tau}(t,\omega,\tau)=\frac{2t\bar{s}(1+\omega)}{(\bar{s}+t-\tau)^2}.$$

Hence:

$$\begin{aligned}
\frac{\partial\tilde{m}}{\partial t}(t,\omega) &= \frac{\partial\hat{m}}{\partial t}(t,\omega,\tilde{\tau}(t,\omega))+\frac{\partial\hat{m}}{\partial\tau}(t,\omega,\tilde{\tau}(t,\omega))\frac{\partial\tilde{\tau}(t,\omega)}{\partial t}(t,\omega)\\
&= \frac{2\bar{s}(1+\omega)(\bar{s}-\tilde{\tau}(t,\omega))}{(\bar{s}+t-\tau)^2}+\frac{2t\bar{s}(1+\omega)}{3(\bar{s}+t-\tau)^2}\\
&> 0,\\
\frac{\partial\tilde{m}}{\partial\omega}(t,\omega) &= \frac{\partial\hat{m}}{\partial\omega}(t,\omega,\tilde{\tau}(t,\omega))+\frac{\partial\hat{m}}{\partial\tau}(t,\omega,\tilde{\tau}(t,\omega))\frac{\partial\tilde{\tau}(t,\omega)}{\partial\omega}(t,\omega)\\
&= \frac{2t\bar{s}}{\bar{s}+t-\tau}+\frac{4t\bar{s}^2(1+\omega)}{3(\bar{s}+t-\tau)^2}\\
&> 0,
\end{aligned}$$

where the inequalities follows from (13). These conditions moreover impose an upper bound on the equilibrium total margin:

$$\tilde{m}(t,\omega)=\frac{6\bar{s}t(1+\omega)}{3\bar{s}+2t-\delta-2\bar{s}\omega}\leq\frac{138}{53}t. \tag{15}$$

Using (11) yields the equilibrium margin on weak products:

$$\tilde{\mu}(t,\omega)=\frac{\tilde{m}(t,\omega)}{2}+\frac{t}{2}-\tilde{\tau}(t,\omega)+\bar{s}\omega,$$

and thus:

$$\begin{aligned}
\frac{\partial\tilde{\mu}}{\partial t}(t,\omega) &= \frac{1}{2}\frac{\partial\tilde{m}}{\partial t}(t,\omega)+\frac{1}{2}-\frac{\partial\tilde{\tau}}{\partial t}(t,\omega)>0,\\
\frac{\partial\tilde{\mu}}{\partial\omega}(t,\omega) &= \frac{1}{2}\frac{\partial\tilde{m}}{\partial\omega}(t,\omega)+\bar{s}>0,
\end{aligned}$$

where the inequalities follow from the comparative statics for $\tilde{m}(t,\omega)$ and from $\partial\tilde{\tau}/\partial t = 1/3 < 1/2$.

As the equilibrium margin $\tilde{\mu}$ increases in t and ω, there is a boundary curve of the form $t = \phi(\omega)$, where $\phi(\omega)$ is decreasing in ω, such that $\tilde{\mu}(\phi(\omega), \omega) = 0$, and $\tilde{\mu}(t, \omega) < 0$ for $t < \phi(\omega)$. Furthermore, for $\omega = 0$ we have seen in the previous subsection and showed that $\tilde{\mu}(0, t) < 0$ as long as

$$t < \phi(0) = \bar{t} = \frac{3\sqrt{\delta^2 - 18\bar{s}\delta + 49\bar{s}^2} - (21\bar{s} - 5\delta)}{4},$$

where $\bar{t} \in (0, \delta/2)$.

Finally, for $\omega = \bar{\omega} = \delta/3\bar{s}$, $\tilde{\mu}(\bar{\omega}, t) < 0$ as long as

$$t < \phi(\bar{\omega}) = \underline{t} \equiv \frac{\sqrt{(63\bar{s} + 5\delta)^2 + 32\delta(9\bar{s} - 5\delta)} - (63\bar{s} + 5\delta)}{12}.$$

As $\phi(\omega)$ is decreasing in ω, $\underline{t} < \bar{t}$.

C.2.2 Existence

By construction, marginal deviations from the margins characterized above are not profitable. To establish existence, we now check that larger deviations are not profitable either. The arguments previously used for the case $\omega = 0$ still ensure that, without loss of generality, we can restrict attention to non-negative margins m_i and ρ_i. In addition, no firm i has an incentive to charge $m_i < \tilde{m}(t, \omega) - 2t$. Furthermore, as firms' profits are linear in the mass ω of consumers interested only in weak products, the second-order derivatives of these profit functions with respect to m_i and ρ_i are the same as before. For example, for firm 1 we have:

$$\begin{aligned}
\frac{\partial^2 \Pi_1}{\partial \rho_1^2} &= -\frac{2}{\bar{s}} < 0, \\
\frac{\partial^2 \Pi_1}{\partial m_1^2} &= -\frac{1}{2\bar{s}} \left[\frac{\bar{s} - \delta}{t} + \frac{6\rho_1 + 8t - 9m_1 + 2\tilde{m}(t, \omega) + 4\tilde{\rho}(t, \omega)}{4t} \right], \\
\frac{\partial^2 \Pi_1}{\partial \rho_1 \partial m_1} &= \frac{4t - 3m_1 + 2\tilde{m}(t, \omega)}{4t\bar{s}}.
\end{aligned}$$

We now check that $\partial^2 \Pi_1 / \partial m_1^2 < 0$ and $H > 0$ in the range $\tilde{m}(t, \omega) + 2t \leq m_1 \leq \tilde{m}(t, \omega) + 2t$ and $\rho_1 \geq 0$. Focusing on $\tilde{\mu}(t, \omega) \leq 0$ (i.e., $\tilde{\rho}(t, \omega) \geq \tilde{m}(t, \omega)$), we have:

$$\begin{aligned}
\frac{\partial^2 \Pi_1}{\partial m_1^2} &\leq -\frac{1}{2\bar{s}} \left(\frac{\bar{s} - \delta}{t} - \frac{3\tilde{m}(t, \omega) + 10t}{4t} \right) \\
&< -\frac{1}{2\bar{s}} \left(\frac{\bar{s} - \delta}{t} - \frac{236}{53} \right),
\end{aligned}$$

where the last inequality follows from (15). Furthermore, we have:

$$\begin{aligned}\frac{\partial^2\Pi_1}{\partial\rho_1^2}\frac{\partial^2\Pi_1}{\partial m_1^2} &= (\Psi_1+\Psi_2)(\Psi_1+\Psi_3)\\ &> \frac{1}{\bar{s}^2}\left(\frac{\bar{s}-\delta}{t}-\frac{236}{53}\right).\end{aligned}$$

Finally,

$$\frac{\partial^2\Pi_1}{\partial\rho_1\partial m_1}=\frac{2\hat{x}}{\bar{s}}-\frac{m_1}{4t\bar{s}}=\frac{4t-3m_1+2\tilde{m}(t)}{4t\bar{s}}.$$

Using $(0<)\,\tilde{m}(t,\omega)-2t\le m_1\le\tilde{m}(t,\omega)+2t$, lower and upper bounds for the above expression are given by

$$\frac{-\tilde{m}(t,\omega)-2t}{4t\bar{s}}\le\frac{\partial^2\Pi_1}{\partial\rho_1\partial m_1}\le\frac{10t-\tilde{m}(t,\omega)}{4t\bar{s}}.$$

Therefore:

$$\begin{aligned}\left|\frac{\partial^2\Pi_1}{\partial\rho_1\partial m_1}\right| &\le \frac{\max\{\tilde{m}(t,\omega)+2t,10t-\tilde{m}(t,\omega)\}}{4t\bar{s}}\\ &= \frac{10t-\tilde{m}(t,\omega)}{4t\bar{s}},\end{aligned}$$

where the last equality comes from (15). As in addition $\tilde{m}(t)>2t$, we have:

$$\left(\frac{\partial^2\Pi_1}{\partial\rho_1\partial m_1}\right)^2\le\left(\frac{8t}{4t\bar{s}}\right)^2=\frac{4}{\bar{s}^2}.$$

Therefore, $H>0$ if

$$\frac{1}{\bar{s}^2}\left(\frac{\bar{s}-\delta}{t}-\frac{236}{53}\right)\ge\frac{4}{\bar{s}^2},$$

which amounts to:

$$t<\hat{t}\equiv\frac{53}{448}(\bar{s}-\delta).$$

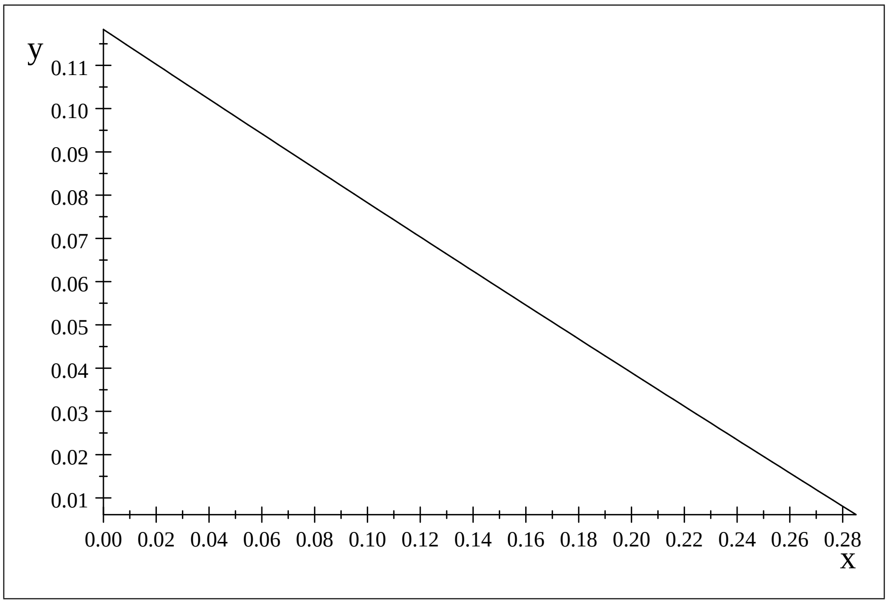
It can be checked that $\hat{t}>\bar{t}$:

$$\frac{\hat{t}-\bar{t}}{\bar{s}}=\chi\left(\frac{\delta}{\bar{s}}\right),$$

where

$$\chi(z)\equiv\frac{2405}{448}-\frac{3}{4}\sqrt{x^2-18x+49}-\frac{613}{448}x$$

is positive in the relevant range $\delta/\bar{s}<2/7\simeq 0.285$:



It follows that firm 1 has no profitable deviations in the range $m_1 \leq \tilde{m}(t,\omega) + 2t$. In the particular case where $\omega = 0$, we saw in the previous section that there is no profitable deviation either in the range $m_1 \geq \tilde{m}(t,0) + 2t$. However, when $\omega > 0$, firm 1 may seek to avoid the loss on consumers interested only in weak products; as a result, it may have an incentive to increase μ_1, and thus m_1, in the range $m_1 > \tilde{m}(t,\omega) + 2t$. Indeed, for $\rho_1 > \tilde{m}(t,\omega) + 2t$ and $\tilde{m}(t,\omega) + 2t < m_1 < \rho_1$, firm 1 does not attract any one-stop shopper and its profit thus boils down to

$$\Pi_1 = \rho_1 D + \mu_1 \omega = \rho_1 D + (m_1 - \rho_1)\,\omega = \rho_1 (D - \omega) + m_1 \omega,$$

which thus increase with m_1 when $\omega > 0$. It follows that a relevant deviation consists in charging $\rho_1 > 0$ and $m_1 \geq \max\{\tilde{m}(t,\omega) + 2t, \rho_1\}$, so as not sell to one-stop shoppers and to consumers only interested in weak products, and focus solely on multi-stop shoppers, that is, consumers with a location x and a shopping cost s such that

$$s \leq \lambda_2(x) = \delta + \tilde{\mu} - \rho_1 + t - 2tx.$$

The resulting profit is equal to

$$\Pi_1 = \rho_1 D = \frac{\rho_1}{\bar{s}} \int_0^1 \left(\delta + \tilde{\mu}(t,\omega) - \rho_1 + t - 2tx\right) dx = \frac{\rho_1}{\bar{s}} \left(\delta + \tilde{\mu}(t,\omega) - \rho_1\right).$$

It follows that the maximal profit from such deviation is given by:

$$\Pi^D(t,\omega) \equiv \frac{1}{\bar{s}} \left(\frac{\delta + \tilde{\mu}(t,\omega)}{2}\right)^2.$$

By construction, this deviation is not profitable for $t = \phi(\omega)$, as $\tilde{\mu}(\phi(\omega), \omega) = 0$, and firms thus incur no loss on the stand-alone demand for their weak products. By contrast, for $t = 0$, where $\tilde{m}(0, \omega) = 0$ and, using (11) and (12), the equilibrium margin on weak products and the demand from multi-stop shoppers are given by:

$$\begin{aligned} \tilde{\mu}(0, \omega) &= -\tilde{\tau}(0, \omega) + \bar{s}\omega = -\frac{\delta - \bar{s}\omega}{3}, \\ D(0, \omega) &= \frac{\tilde{\tau}(0, \omega)}{\bar{s}} = \frac{\delta + 2\bar{s}\omega}{3\bar{s}}, \end{aligned}$$

the deviation profit exceeds the equilibrium profit whenever $\omega > 0$:

$$\Pi^D(0, \omega) - \Pi^*(0, \omega) = \frac{1}{\bar{s}}\left(\frac{\delta + \tilde{\mu}(0, \omega)}{2}\right)^2 + \tilde{\mu}(0, \omega)[D(0, \omega) - \omega] = \frac{1}{12}\omega(4\delta - \bar{s}\omega),$$

where the right-hand side is positive for $\omega \leq \bar{\omega} = \delta/3\bar{s}$. As the equilibrium and deviation profits are both continuous in t, it follows that there exists $\psi(\omega) \in [0, \phi(\omega)]$, where moreover $\psi(\omega) > 0$ for $\omega > 0$, such that, for any $\omega \leq \bar{\omega}$, the deviation is not profitable for $t \in [\psi(\omega), \phi(\omega)]$.

D Product Choice

D.1 Proof of Proposition 5

We suppose here that firm 1 benefits from a comparative advantage $\delta_1 > 0$ in market A, which exceeds firm 2's comparative advantage δ_2 in market B.

Consider first the case where firm 1 enjoys a comparative advantage over *both* products; that is, $\delta_2 \leq 0$. Intuitively, firm 1 then wins the competition in both markets; indeed, a standard asymmetric Bertrand competition argument shows that there exists a unique trembling-hand perfect equilibrium, in which firm 2 offers both products at cost, whereas firm 1 supplies all consumers and charges on each product a margin reflecting its comparative advantage over that product.

Consider now the case where firms have asymmetric comparative advantages in the two markets; that is, $\delta_1 = \bar{\delta}$ and $\delta_2 = \underline{\delta} > 0$, where:

$$\bar{\delta} \equiv u_1^A - c_1^A - \left(u_2^A - c_2^A\right) > \underline{\delta} \equiv u_2^B - c_2^B - \left(u_1^B - c_1^B\right) > 0.$$

Firm 1 therefore enjoys market power over one-stop shoppers, as it offers a more attractive basket:

$$w_1 - w_2 = \bar{\delta} - \underline{\delta} > 0,$$

where $w_1 \equiv u_1^A - c_1^A + u_1^B - c_1^B$ and $w_2 \equiv u_2^A - c_2^A + u_2^B - c_2^B$ denote the surpluses generated by the two firms' offerings.

The following Lemma extends the insights of Lemma 1 and shows that, in equilibrium, firm 2 then still offers its basket at cost whereas firm 1 attracts all one-stop shoppers and charge them a positive total margin reflecting its competitive advantage:

Lemma 7 *Under Assumption A, in equilibrium:*

(i) there are both multi-stop shoppers and one-stop shoppers;

(ii) multi-stop shoppers buy firms' strong products, A_1 and B_2; and

(iii) in a trembling-hand perfect equilibrium, $m_1 = \bar{\delta} - \underline{\delta} > m_2 = 0$.

Proof. The proof follows the same steps as for Lemma 1.

• *Claim 1bis: Some consumers are active in equilibrium.* The proof of Claim 1 holds unchanged.

• *Claim 2bis: If there are active one-stop shoppers in equilibrium, then $m_1 = \bar{\delta} - \underline{\delta} > m_2 = 0$.* The same arguments as for Claim 2 can be used to show that $m_1 = m_2 + \bar{\delta} - \underline{\delta}$, $m_2 \leq 0$ and $m_1 \geq 0$. Furthermore, firm 1 must attract all one-stop shoppers: otherwise, if $m_2 < 0$ then firm 2 would benefit from slightly increasing its margin on the product not purchased by multi-stop shoppers, so as to avoid the loss made on one-stop shoppers without substantially affecting any profit obtained from multi-stop shoppers; and if instead $m_2 = 0$ (implying $m_1 > 0$), then firm 1 would benefit from slightly decreasing its margin on the product not purchased by multi-stop shoppers, so as to attract all one-stop shoppers, without substantially affecting any profit from multi-stop shoppers. Finally, the only equilibrium surviving trembling hand perfection is the one where $m_2 = 0$, and thus $m_1 = \bar{\delta} - \underline{\delta}$.

• *Claim 3bis: In equilibrium, active multi-stop shoppers buy the strong products.* The reasoning underlying Claim 3 can be used with some adjustment.[10] If there are no one-stop shoppers, then each firm i must charge $\mu_i \geq 0$ (otherwise, it would make a loss) and $\rho_i \geq \delta_i$ (otherwise, consumers would buy both products from it); keeping μ_i unchanged, firm i would however increase its profit by charging $\tilde{\rho}_i = \mu_j + \delta_i - \varepsilon > 0$, so as to sell its strong product to all previously active consumers (and possibly attract additional, profitable one-stop shoppers). If instead there are some one-stop shoppers as well, then $m_2 = 0$ (from Claim 2bis) and thus $\mu_2 \geq 0$ (otherwise, firm 2 would make a loss), $\rho_2 = -\mu_2 \leq 0$ and $\mu_1 \leq \rho_2 - \underline{\delta} < 0$; firm 1 could then increase its profit by offering its weak product at cost and charging a total margin slightly below its overall comparative advantage, so as to avoid the loss on multi-stop shoppers without substantially affecting the profit from one-stop shoppers.

• *Claim 4bis: Some multi-stop shoppers are active in equilibrium.* The reasoning used for Claim 4 still applies, noting that, if all active consumers were one-stop shoppers, then we would have (using Claim 1bis) $v_1 = v_2 = w_2 \geq \max\{v_1, v_2\}$ and $v_{12} + \underline{v}_{12} = v_1 + v_2 = 2w_2$, leading to $v_{12} = \underline{v}_{12} = w_2$ and $\rho_1 + \rho_2 = \underline{\delta}$. It would then again be profitable for any firm i charging $\rho_i > 0$ to encourage consumers to buy only its strong product, by slightly increasing μ_i and decreasing ρ_i by the same amount.

• *Claim 5bis: Some one-stop shoppers are active in equilibrium.* The reasoning underlying Claim 5 can be used with some adjustment.[11] Starting from a candidate equilibrium with multi-stop shoppers only, the same reasoning shows that firm i would profitably deviate by reducing the margin on its weak product, so as to convert consumers to one-stop shopping, whenever $\tilde{\mu}_i > 0$, where

$$\tilde{\mu}_i \equiv \frac{w_i - \delta_j + \rho_j - \rho_i}{2}.$$

By construction, we have:

$$\tilde{\mu}_1 + \tilde{\mu}_2 = \frac{w_1 + w_2 - \delta_1 - \delta_2}{2} = u_2^A - c_2^A + u_1^B - c_1^B > 0.$$

[10] For the case where there are one-stop shoppers as well, the original argument relies on $m_1 = m_2 = 0$, which no longer holds.

[11] For the case where $\rho_2 < \rho_1$, the original argument requires $w_2 \geq \bar{\delta}$, which may not hold.

Hence, $\tilde{\mu}_i > 0$ for at least one firm i, and that firm could profitably deviate.

Claims 4bis and 5bis establish part (i) of the Lemma; part (ii) then follows from Claim 3bis, while part (iii) follows from Claim 2bis. ■

Hence, in equilibrium, one-stop shoppers obtain a consumer value

$$v_1 = w_1 - m_1 = w_2,$$

whereas multi-stop shoppers obtain (noting that $u_1^A - c_1^A + u_2^B - c_2^B = w_1 + \underline{\delta} = w_2 + \bar{\delta}$):

$$v_{12} = w_2 + \bar{\delta} - \rho_1 - \rho_2.$$

The multi-stop shopping threshold thus becomes:

$$\tau = v_{12} - v_1 = \bar{\delta} - \rho_1 - \rho_2 = \underline{\delta} + \mu_1 - \rho_2,$$

where $\mu_1 = m_1 - \rho_1 = \bar{\delta} - \underline{\delta} - \rho_1$ denotes firm 1's margin on its weak product.

As firm 1 sells both products to one-stop shoppers and, in addition, sells its strong product to multi-stop shoppers, its profit can be expressed as:

$$\begin{aligned} \pi_1 &= \rho_1 F(\tau) + m_1 \left[F(v_1) - F(\tau)\right] \\ &= m_1 F(w_1 - m_1) - \mu_1 F(\underline{\delta} + \mu_1 - \rho_2). \end{aligned}$$

This profit is additively separable and quasi-concave in m_1 and μ_1. Hence, maximizing it with respect to $m_1 \leq \bar{\delta} - \underline{\delta}$ and to $\mu_1$leads firm 1 to charge

$$m_1 = \min\left\{m_1^M, \bar{\delta} - \underline{\delta}\right\},$$

where

$$m_1^M \equiv \arg\max_{m_1} \left\{m_1 F(w_1 - m_1)\right\},$$

and to subsidize its weak product: its best response is such that:

$$\mu_1 = -h(\tau) < 0.$$

As firm 2 only supplies multi-stop shoppers, to whom it sells its strong product, its profit is given by:

$$\pi_2 = \rho_2 F(\underline{\delta} + \mu_1 - \rho_2),$$

and thus chooses

$$\rho_2 = h(\tau) > 0.$$

Hence, the unique candidate equilibrium is such that: $-\mu_1^* = \rho_2^* = h(\tau^*)$, where $\tau^* = j^{-1}(\underline{\delta})$. Following the same steps as in the proof of Proposition, it is straightforward to check that these strategies (together with $m_1 = \min\{m_1^M, \bar{\delta} - \underline{\delta}\}$ and $m_2 = 0$) constitute indeed an equilibrium. Obviously, no firm i can make a profit by offering its weak product to multi-stop shoppers, as (noting that $\rho_1^* - \bar{\delta} = \rho_2^* - \underline{\delta} = -\tau^* - h(\tau^*)$) it would have to charge $\mu_i \leq -\tau^* - h(\tau^*) < 0$ to attract them. Furthermore, firm 2 cannot make a profit from one-stop shoppers either, as it would have to sell its basket (weakly) below cost to attract them. Hence, firm 2's deviation profit cannot exceed $\rho_2 F(\tau)$, where $\tau = \min\{\underline{\delta} + \mu_1^* - \rho_2, \bar{\delta} + \mu_2 - \rho_1^*\}$; but then:

$$\rho_2 F(\tau) \leq \rho_2 F(\underline{\delta} + \mu_1^* - \rho_2) \leq \pi^*,$$

where the inequality comes from the fact that the profit function $\rho_2 F(\delta + \mu_1^* - \rho_2)$ is quasi-concave in ρ_2, from the monotonicity of $h(\cdot)$, and, by construction, maximal for $\rho_2 = \rho_2^*$. As for firm 1, its deviation profit cannot exceed

$$m_1 [F(v_1) - F(\tau)] + \rho_1 F(\tau) = m_1 F(w_1 - m_1) - \mu_1 F(\tau),$$

where $\tau = \min\{\underline{\delta} + \mu_1 - \rho_2^*, \bar{\delta} + \mu_2^* - \rho_1\}$. Hence, it is optimal to choose $\mu_1 < 0$, which in turn implies that this profit cannot exceed

$$m_1 F(w_1 - m_1) - \mu_1 F(\underline{\delta} + \mu_1 - \rho_2^*),$$

which is separable in m_1 and μ_1, and by construction maximal for $m_1 = \min\{m_1^M, \bar{\delta} - \underline{\delta}\}$ and $\mu_1 = -h(\tau^*)$.

D.2 Proof of Proposition 6

We consider here the setting with endogenous improvement decisions in which: (i) initially, firms' offerings generate the same surplus w_A in market A and w_B in market B; and (ii) in a first stage, each firm $i = 1, 2$ chooses to improve its products A_i and B_i by $\Delta_{A_i} \geq 0$ and $\Delta_{B_i} \geq 0$, respectively, subject to the constraint that the total improvement cannot exceed Δ: $\Delta_i \equiv \Delta_{A_i} + \Delta_{B_i} \leq \Delta$. Firms then choose their prices.

We consider two variants, depending on the observability of improvement decisions made in the first stage. In both situations, we however assume that consumers are fully aware of firms' moves before making their own shopping decisions. When improvement decisions are publicly observed, one firm's prices can be contingent on both firms' improvement decisions; the game is thus as follows:

- Stage 1: each firm $i = 1, 2$ chooses $(\Delta_{A_i}, \Delta_{B_i}) \in \mathcal{S} \equiv \left\{(\Delta_A, \Delta_B) \in \mathbb{R}_+^2 \mid \Delta_A + \Delta_B \leq \Delta\right\}$; these decisions are simultaneous and publicly observed by the rival firm and consumers.
- Stage 2: having observed all value-improvement decisions, firms simultaneously set the prices for their products; these price decisions are public.

When instead firms' improvement decisions are private, each firm can adjust its prices as a function of its own product improvements, but cannot respond to the other firm's improvement decisions; the above two stages then boil down to a single stage, as follows:

- Each firm $i = 1, 2$ chooses $(\Delta_{A_i}, \Delta_{B_i}) \in \mathcal{S} \equiv \left\{(\Delta_A, \Delta_B) \in \mathbb{R}_+^2 \mid \Delta_A + \Delta_B \leq \Delta\right\}$ and its prices for its products; all decisions are simultaneous and publicly observed by consumers.

In both variants, we look for the subgame trembling-hand perfect Nash equilibria of the game,[12] and first focus on pure strategies, before turning to mixed strategies.

D.2.1 Pure strategies

We first assume that improvement decisions are public, before addressing the case of private improvement decisions.

Public improvement decisions. We first note that, in equilibrium, each firm obtains a comparative advantage in one of the markets. To see this, suppose instead that firm i, say, ends up with no comparative advantage in any market. From Proposition 5, it then obtains zero profit. However, as $\Delta_{A_j} + \Delta_{B_j} \leq \Delta$, there is at least one market in which

[12] In case of private improvement decisions, subgame perfection still applies to consumers' response.

firm j does not improve its product by Δ; hence, by using its own endowment Δ in that market, firm i could obtain a net advantage and earn a positive profit.

Hence, in equilibrium, one firm obtains a comparative advantage in one market, whereas the other firm obtains a comparative advantage in the other market. Without loss of generality, suppose that firm 1 obtains its comparative advantage in market A, and firm 2 obtains it in market B. We thus have:

$$\delta_1 = \Delta_{A1} - \Delta_{A2} > 0 \text{ and } \delta_2 = \Delta_{B1} - \Delta_{B2} > 0.$$

Without loss of generality, suppose that firm i, say, ends up with a weakly greater comparative advantage (that is, $\delta_i = \bar{\delta} \geq \underline{\delta} = \delta_j$); from Proposition 5, the profits of the two firms can then be expressed as:

$$\begin{aligned} \pi_i^* \left(\bar{\delta}, \underline{\delta}\right) &= m^* \left(\bar{\delta}, \underline{\delta}\right) F \left(w_A + w_B + \bar{\delta} - m^* \left(\bar{\delta}, \underline{\delta}\right)\right) + \pi^* \left(\underline{\delta}\right), \\ \pi_j^* \left(\underline{\delta}\right) &= \pi^* \left(\underline{\delta}\right), \end{aligned}$$

where:

- $\pi^* (\delta) \equiv \rho^* (\delta) F (\tau^* (\delta))$, where $\tau^* (\delta) = j^{-1} (\delta)$ and $\rho^* (\delta) = h (\tau^* (\delta))$, and thus:
$$\frac{d\pi^*}{d\delta} (\cdot) = \frac{1 + h' (\tau^*)}{1 + 2h' (\tau^*)} F (\tau^*) > 0. \tag{16}$$
- $m^* \left(\bar{\delta}, \underline{\delta}\right) = \min \left\{m^M \left(\bar{\delta}\right), \bar{\delta} - \underline{\delta}\right\}$, where
$$m^M (\delta) \equiv \arg\max_m \{mF (w_A + w_B + \delta - m)\},$$
and thus (using $v_i^* = w_A + w_B + \bar{\delta} - m^* \left(\bar{\delta}, \underline{\delta}\right)$):
$$\frac{\partial \pi_i^*}{\partial \bar{\delta}} = F (v_i^*) > 0.$$

It follows that both firms wish to increase their own comparative advantage; hence, in equilibrium:

- Both firms exhaust their endowments: $\Delta_1 = \Delta_2 = \Delta$; indeed, any firm l that does not exhaust its endowment (i.e., $\Delta_l < \Delta$) could increase its profit by allocating the unused part of it ($\Delta - \Delta_l$) to the product on which it already enjoys a comparative advantage, so as to increase δ_l.

- Each firm targets a single product: $\Delta_{A1} = \Delta_{B2} = \Delta$ (and thus $\Delta_{A2} = \Delta_{B1} = 0$). For example, starting from $\Delta_{A1} < \Delta$ and $\Delta_{B1} = \Delta - \Delta_{A1} > 0$, firm 1 could increase its profit by reducing Δ_{B1} and allocating the saved endowment to product A. A similar reasoning applies to firm 2.

It follows that, in equilibrium, one firm uses its full endowment to improve product A, whereas the other uses it to improve product B. We thus have $\delta_1 = \delta_2 = \Delta$ and the resulting equilibrium profits are

$$\pi_1^* = \pi_2^* = \pi^* (\Delta) .$$

Private improvement decisions. We first note that, again, in equilibrium each firm obtains a comparative advantage in one of the markets. To see this, consider a candidate equilibrium in which firm i, say, ends up with no comparative advantage in any market. A standard Bertrand argument then ensures that firm i then obtains zero profit. However, as $\Delta_{A_j} + \Delta_{B_j} \leq \Delta$, there is at least one market in which firm j does not improve its product by Δ; hence, by using its own endowment Δ in that market, firm i could obtain a net advantage in that market and, by increasing its own margin in that market accordingly, it could earn a positive profit.

Second, in equilibrium both firms fully use their improvement capability (i.e., $\Delta_{Ai} + \Delta_{Bi} = \Delta$ for $i = 1, 2$). Suppose instead that $\Delta_{Ai} + \Delta_{Bi} < \Delta$ for some $i = 1, 2$. Firm i could then increase its improvement in the market in which it has a comparative advantage, and increase its margin in that market by the same amount (or slightly less than that, in case of ties). This would have no impact on consumers' demands, and would increase firm i's profit from that product.

It follows that, in equilibrium, one firm invests Δ in one product, and the other firm invest Δ in the other product. The associated price equilibrium is then such that each firm charges $\rho = \rho^* (\Delta)$ on its strong product and $\mu = -\rho^* (\Delta)$ on its weak product.

Conversely, the above candidate equilibrium constitutes indeed an equilibrium. To see this, suppose that firm j invests Δ in market B, say, charges a margin $\rho^* (\Delta)$ in that market, and offers its basket at cost. Suppose further that firm i deviates by investing δ in market A and (without loss of generality, in the light of the above remarks) $\Delta - \delta$ in market B. As firm j offers its basket at cost, firm i cannot earn any profit from one-stop shoppers. Hence, it can make a profit only from selling one product to multi-stop shoppers. Obviously that product must be firm i's strong product, that is, product A (as firm j invests Δ in product B). Furthermore, to avoid any losses, we can restrict attention to deviations in which firm i only offers product A. Thus, without loss of generality, consider a deviation in which firm i invests δ in product A, charges a margin ρ on that product, and does not offer product B (or offers it at a prohibitively high price). Firm i's profit is then given by:

$$\rho F\left(\delta-\rho-\rho^*\left(\Delta\right)\right).$$

Maximizing this profit leads to $\delta=\Delta$ and $\rho=\rho^*\left(\Delta\right)$.

D.2.2 Mixed strategies

We now turn to mixed strategies. We will rely on the following Lemma:

Lemma 8 *Let $\hat{\pi}\left(\delta\right)\equiv\max_\rho \rho F\left(\delta-\rho\right)$ and $\pi^*\left(\delta\right)\equiv\rho^*\left(\delta\right)F\left(\tau^*\left(\delta\right)\right)$ respectively denote the monopoly profit obtained from a comparative advantage δ, and the equilibrium profit described above. We have:*

(i) $\hat{\pi}\left(0\right)=0$, $\hat{\pi}'\left(\delta\right)>0$ and $\hat{\pi}''\left(\delta\right)>0$; and

(ii) $\hat{\pi}\left(0\right)=0$ and $\hat{\pi}'\left(\delta\right)>0$; if in addition $h\left(s\right)$ is weakly concave in s, then $\pi^*\left(\delta\right)$ is convex in δ.

Proof. We start with the comparative statics for $\hat{\pi}\left(\cdot\right)$. Obviously, $\hat{\pi}\left(0\right)=0$. Furthermore, letting $\tau\equiv\delta-\rho$, it is useful to express $\hat{\pi}\left(\cdot\right)$ as:

$$\hat{\pi}\left(\delta\right)=\max_{\tau}\left(\delta-\tau\right)F\left(\tau\right),$$

where, from the monotonicity of $h\left(\cdot\right)$, the expression $\left(\delta-\tau\right)F\left(\tau\right)$ is strictly quasi-concave in τ and maximal for $\hat{\tau}\left(\delta\right)=l^{-1}\left(\delta\right)$, where $l\left(s\right)\equiv s+h\left(s\right)$ is strictly increasing in s. Using the envelope theorem, we have:

$$\begin{aligned}\hat{\pi}'\left(\delta\right) &= F\left(\hat{\tau}\left(\delta\right)\right),\\ \hat{\pi}''\left(\delta\right) &= f\left(\hat{\tau}\left(\delta\right)\right)\hat{\tau}'\left(\delta\right)=\frac{f\left(\hat{\tau}\left(\delta\right)\right)}{1+h'\left(\hat{\tau}\left(\delta\right)\right)}>0.\end{aligned}$$

We now turn to $\pi^*(\delta)$, which can be expressed as:

$$\pi^*(\delta) = \hat{\pi}(\delta - \rho^*(\delta)).$$

Hence:

$$\frac{d\pi^*}{d\delta}(\delta) = \hat{\pi}'(\delta - \rho^*(\delta))\left[1 - \frac{d\rho^*}{d\delta}(\delta)\right].$$

Using $\rho^*(\delta) = h(\tau^*(\delta))$ and $\tau^*(\delta) + 2h(\tau^*(\delta)) = \delta$, this derivative can be expressed as:

$$\begin{aligned}
\frac{d\pi^*}{d\delta}(\delta) &= \hat{\pi}'(\delta - \rho^*(\delta))\left[1 - h'(\tau^*(\delta))\frac{d\tau^*}{d\delta}(\delta)\right] \\
&= \hat{\pi}'(\delta - \rho^*(\delta))\left[1 - \frac{h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))}\right] \\
&= \hat{\pi}'(\delta - \rho^*(\delta))\frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))}.
\end{aligned}$$

It follows that:

$$\begin{aligned}
\frac{d^2\pi^*}{d\delta^2}(\delta) &= \hat{\pi}''(\delta - \rho^*(\delta))\left[1 - \frac{d\rho^*}{d\delta}(\delta)\right]\frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))} \\
&\quad + \hat{\pi}'(\delta - \rho^*(\delta))\left[\frac{-h''(\tau^*(\delta))}{[1 + 2h'(\tau^*(\delta))]^2}\right]\frac{d\tau^*}{d\delta}(\delta) \\
&= \hat{\pi}''(\delta - \rho^*(\delta))\frac{[1 + h'(\tau^*(\delta))]^2}{[1 + 2h'(\tau^*(\delta))]^2} - \hat{\pi}'(\delta - \rho^*(\delta))\frac{h'(\tau^*(\delta))\,h''(\tau^*(\delta))}{[1 + 2h'(\tau^*(\delta))]^3},
\end{aligned}$$

where, in the last expression, the first term is positive (as $\hat{\pi}''(\cdot) > 0$) and the second term is non-negative if $h''(\cdot) \leq 0$ (as $\hat{\pi}'(\cdot) > 0$ and $h'(\cdot) > 0$). ■

We now show that there exists an equilibrium in which firms invests Δ in either product with equal probability. We first consider private improvement decisions, before addressing the case of public improvement decisions.

Private improvement decisions. Suppose that improvement decisions are private, and consider the following symmetric candidate equilibrium: firms always offer their baskets at cost, invest Δ and charge $\rho^*(\Delta)$ on product A with probability 1/2, and do the same on product B with complementary probability 1/2. Suppose that firm j adopts this strategy, and that in response firm i invests $\Delta_A \in [0, \Delta]$ in product A and $\Delta_B \in [0, \Delta - \Delta_A]$ in product B. With probability 1/2, firm j invests Δ on product A, in which case it obtains a comparative advantage $\Delta - \Delta_A$ on that product, whereas firm

i obtains a comparative advantage Δ_B on product B. With complementary probability 1/2, firm j invests Δ on product B, in which case it obtains a comparative advantage $\Delta - \Delta_B$ on product B, whereas firm i obtains a comparative advantage Δ_A on product A. As firm j always invests Δ in total, and offers its basket at cost, firm i cannot obtain a profit from one-stop shoppers. Hence, it can only earn a profit by selling a single product to multi-stop shoppers. Furthermore, charging $\rho_h < 0$ on some product $h = A, B$ is weakly dominated by selling product h at cost: (i) whenever some multi-stop shoppers buy product h from firm i, the change improves firm i's profit; and (ii) whenever some one-stop shoppers buy both products from firm i, implying that firm i's total margin must be negative, the change again improves firm i's profit, by deterring these one-stop shoppers. Hence, without loss of generality, we can restrict attention to firm i's charging $\rho_h \geq 0$ on product $h = A, B$. As firm j always charges $\rho^*(\Delta) > 0$ on its strong product, it follows that multi-stop shoppers always combine firms' strong products. Hence, firm i's expected profit is at most equal to:

$$\begin{aligned}\Pi_i^e &= \frac{1}{2} \times \rho_B F(\Delta_B - \rho_B - \rho^*(\Delta)) + \frac{1}{2} \times \rho_A F(\Delta_A - \rho_A - \rho^*(\Delta)) \\ &\leq \frac{\hat{\pi}(\Delta_B - \rho^*(\Delta)) + \hat{\pi}(\Delta_A - \rho^*(\Delta))}{2} \\ &\leq \frac{\hat{\pi}(\Delta_B - \rho^*(\Delta)) + \hat{\pi}(\Delta - \Delta_B - \rho^*(\Delta))}{2},\end{aligned}$$

where the first inequality stems from the definition of $\hat{\pi}(\cdot)$ and the second one follows from $\Delta_A \leq \Delta - \Delta_B$. The convexity of $\hat{\pi}(\cdot)$ then implies that firm i cannot do better than investing Δ in either product (i.e., either $\Delta_B = 0$, or $\Delta_B = \Delta$). Furthermore, in case it invests Δ in product $h \in \{A, B\}$, it cannot do better than offering its basket at cost and charging $\rho^*(\Delta)$ on product h.

Public improvement decisions. Suppose now that improvement decisions are public, and consider the following symmetric candidate equilibrium in which, in the first stage, firms invest Δ in either product with equal probability; along the equilibrium path, they then offer their baskets at cost, and charge $\rho^*(\Delta)$ on their strong products.

Suppose that firm j invests Δ in either product with equal probability in the first stage, and that in response firm i invests $\Delta_A \in [0, \Delta]$ in product A, and $\Delta_B \in [0, \Delta - \Delta_A]$ in product B. With probability 1/2, firm j invests Δ on product A, in which case it obtains a comparative advantage $\Delta - \Delta_A$ on that product, whereas firm i obtains a weaker comparative advantage $\Delta_B \leq \Delta - \Delta_A$ on product B. With complementary probability 1/2, firm j invests Δ on product B, in which case it obtains a comparative advantage $\Delta - \Delta_B$ on product B, whereas firm i obtains a weaker comparative advantage $\Delta_A \leq \Delta - \Delta_B$ on product A. From Proposition 5, firm i's expected profit is therefore equal to:

$$\begin{aligned}\Pi_i^e &= \frac{\pi^*(\Delta_B) + \pi^*(\Delta_A)}{2} \\ &\leq \frac{\pi^*(\Delta_B) + \pi^*(\Delta - \Delta_B)}{2},\end{aligned}$$

where the inequality follows from $\Delta_A \leq \Delta - \Delta_B$. The convexity of $\pi^*(\cdot)$ then implies that firm i cannot do better than investing Δ in either product (i.e., either $\Delta_B = 0$, or $\Delta_B = \Delta$).

D.3 Endogenous investments

We consider here a variant of the previous setting in which, in stage 1, each firm $i = 1, 2$ chooses to improve its products A_i and B_i by Δ_{A_i} and Δ_{B_i}, respectively, at cost $C(\Delta_{A_i} + \Delta_{B_i})$. To ensure the existence of an equilibrium in which both firms invest, we introduce the following regularity assumptions:

Assumption C: $C(0) = C'(0) = 0$, $C''(0) < 2f(0)/9$, $C''(\cdot) \geq 0$ and there exists $\bar{\delta}$ such that $C'(\bar{\delta}) = 1$.

Also, for tractability purposes we replace Assumption A with the following assumption:

Assumption A':

(i) The shopping cost s is distributed according to a cumulative distribution function $F(\cdot)$ with positive, continuously differentiable density function $f(\cdot)$ over $[0, \bar{s}]$.

(ii) The surplus initially offered by the firms (i.e., absent value improvements) is large enough to ensure full participation: $w \equiv w_A + w_B > \bar{s}$.

We first note that at least one firm invests:

Claim 9 *In any equilibrium, at least one firm invests.*

Proof. Consider a candidate equilibrium in which no firm invests. In the second stage, head-to-head competition then yields zero profit for both firms. Suppose now that one firm deviates and invests $\delta > 0$ in one of the products. From Proposition 5, the other firm offers its products at cost but attracts no consumer, whereas the investing firm charges a total margin equal to $m = \delta$ (doing so still enables it to serve all consumers, as it offers $w + \delta - m = w > \bar{s}$, implying that the monopoly margin $m^M(w + \delta)$ would be even higher) and thus obtains a profit equal to:

$$\tilde{\pi}(\delta) \equiv \delta F(w) = \delta.$$

As $\tilde{\pi}'(0) = 1 > 0 = C'(0)$, it follows that a small deviation is profitable, a contradiction. ■

Next, we note that firms never invest on the same products. To see this, suppose that both firms invest on some product $h \in \{A, B\}$, and suppose without loss of generality that firm i, say, invests weakly less than its rival (that is, $\Delta_{hi} \geq \Delta_{hj} > 0$), implying that firm i has no comparative advantage on this market. Firm i would then profitably stop investing on product h: this would save on investment costs, without affecting the profit it achieves in any market. Therefore, in equilibrium, either the two firms invest in different markets, or only one firm invests. In both cases, firm i's investment Δ_i in its product gives it a comparative advantage $\delta_i = \Delta_i$ in that product; hence, we can say that firm i invests in comparative advantage δ_i at cost $C(\delta_i)$.

Consider the first type of equilibrium, and suppose without loss of generality that firm i, say, invests weakly more than firm j, so that $\delta_i \geq \delta_j > 0$. From Proposition 5, the profits of the two firms are then given by:

$$\begin{aligned} \pi_i &= \bar{\pi}(\delta_i, \delta_j) \equiv \delta_i - \delta_j + \pi^*(\delta_j), \\ \pi_j &= \pi^*(\delta_j), \end{aligned}$$

where $\pi^*(\delta) \equiv \rho^*(\delta) F(\tau^*(\delta))$, where $\tau^*(\delta) \equiv j^{-1}(\delta)$ and $\rho^*(\delta) \equiv h(\tau^*(\delta))$, and the expression of $\bar{\pi}(\delta_i, \delta_j)$ relies on the observation that firm i finds it optimal to charge the full value of its net comparative advantage (i.e., $m_i = \delta_i - \delta_j$), as doing so still allows it to serve all consumers (as it offers one-stop shoppers $w + \delta_i - m_i = w + \delta_j > \bar{s}$), implying that the monopoly margin $m^M(w + \delta_j)$ would be even higher.

We have:

Claim 10 *Under Assumptions A', B and C, the function* $\phi(\delta) \equiv \pi^*(\delta) - C(\delta)$ *is maximal for some* $\underline{\delta} \in (0, \bar{\delta})$*.*

Proof. From the previous analysis, we have:

$$\frac{d\pi^*}{d\delta}(\delta) = \frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))} F(\tau^*(\delta)) < 1.$$

Hence, $\phi(\delta)$ is decreasing for $\delta \geq \bar{\delta}$, where $C'(\delta) \geq 1$. Furthermore, for $\delta = 0$, we have $\rho^*(0) = \tau^*(0) = h(0) = 0$, $h'(0) = (1 - h(0) f(0) =) 1$; therefore:

$$\begin{aligned} \phi(0) &= \pi^*(0) - C(0) = 0; \\ \phi'(0) &= \frac{1 + h'(0)}{1 + 2h'(0)} F(0) - C'(0) = 0, \end{aligned}$$

and:[13]

$$\begin{aligned} \phi''(0) &= \frac{d}{d\delta}\left(\frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))}\right)\bigg|_{\delta=0} F(\tau^*(0)) + \frac{1 + h'(0)}{1 + 2h'(0)} f(\tau^*(\delta)) \frac{d\tau^*}{d\delta}(\delta)\bigg|_{\delta=0} - C''(0) \\ &= \frac{d}{d\delta}\left(\frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))}\right)\bigg|_{\delta=0} F(0) + \frac{1 + h'(0)}{1 + 2h'(0)} \frac{f(0)}{1 + 2h'(0)} - C''(0) \\ &= \frac{2}{9} f(0) - C''(0), \end{aligned}$$

which is positive from Assumption C. Hence, there exists $\underline{\delta} \in (0, \bar{\delta})$ such that $\phi(\delta)$ is maximal for $\underline{\delta}$. ■

We now establish the existence of two equilibria of the first type:

Claim 11 *Under Assumptions A', B and C, in the above two-stage game there exists two subgame (trembling-hand) perfect Nash equilibria in which: (i) in stage 1, one firm invests*

[13] We assume here that $f''(0)$ exists and is finite; this ensures that

$$\begin{aligned} & \frac{d}{d\delta}\left(\frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))}\right)\bigg|_{\delta=0} \\ = & \left.\frac{h'(\tau)\left[-h'(\tau) f'(\tau) - h(\tau) f''(\tau)\right]}{\left[1 + 2h'(\tau)\right]^3}\right|_{\tau=0} \\ = & -f'(0) \end{aligned}$$

is also finite.

$\bar{\delta}$ *in one market (and* 0 *in the other market), whereas the other firm invests* $\underline{\delta} \in (0, \bar{\delta})$ *in the other market (and* 0 *in the first market); in stage 2, the second firm offers its basket at cost whereas the first firm supplies one-stop shoppers with a total margin reflecting its net comparative advantage,* $m = \bar{\delta} - \underline{\delta} > 0$*, and both firms sell their weaker products below cost. Furthermore, there is no other pure-strategy equilibrium in which both firms invest.*

Proof. Consider a candidate equilibrium in which both firms invest, and obtain comparative advantages over different products. We first note that the two firms cannot invest to the same extent. Indeed, starting from a candidate equilibrium in which $\delta_1 = \delta_2 = \delta$, giving the same profit $\pi^*(\delta)$ to both firms, any firm could obtain $\bar{\pi}(\delta', \delta)$ by deviating to $\delta' > \delta$, and could obtain instead $\pi^*(\delta')$ by deviating to $\delta' < \delta$. Ruling out the first type of deviation requires:

$$0 > \partial_1 \bar{\pi}(\delta, \delta) = 1 - C'(\delta),$$

where $\partial_1 \bar{\pi}(\cdot, \cdot)$ denotes the partial derivative of the function $\bar{\pi}(\cdot, \cdot)$ with respect to its first argument; ruling the second type of deviation requires instead:

$$0 < \frac{d\pi^*}{d\delta}(\delta) = \frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))} F(\tau^*(\delta)) - C'(\delta).$$

Combining both conditions yields:

$$1 < \frac{1 + h'(\tau^*(\delta))}{1 + 2h'(\tau^*(\delta))} F(\tau^*(\delta)),$$

a contradiction (as $h'(\cdot) > 0$ and $F(\cdot) \leq 1$).

Therefore, one firm, say firm i, must invest more than the other: $\delta_i > \delta_j > 0$, implying that firm i obtains $\pi_i = \bar{\pi}(\delta_i, \delta_j)$ whereas firm j obtains $\pi_j = \pi^*(\delta_j)$. The first-order condition for firm i then yields $\delta_i = \bar{\delta}$, and firm j's best-response in the range $\delta_j \leq \bar{\delta}$ is given by $\delta_j = \underline{\delta}$. The two firms thus obtain $\pi_i^* = \bar{\delta} - \underline{\delta} + \pi^*(\underline{\delta})$ and $\pi_j^* = \pi^*(\underline{\delta})$. To complete the proof, it suffices to check that no firm can benefit from a "large deviation" which would lead to $\delta_i \leq \delta_j$. For firm i, a deviation to $\delta_i \leq \underline{\delta}$ is not profitable, as:

$$\pi^*(\delta_i) - C(\delta_i) \leq \max_{\delta} \{\pi^*(\delta) - C(\delta)\} = \pi^*(\underline{\delta}) < \bar{\delta} - \underline{\delta} + \pi^*(\underline{\delta}) = \pi_i^*.$$

Likewise, for firm j, a deviation to $\delta_j \geq \bar{\delta}$ is not profitable either, as:

$$\delta_j + \pi^*(\bar{\delta}) - C(\delta) \leq \max_{\delta} \{\delta_j - C(\delta)\} + \pi^*(\bar{\delta}) - \bar{\delta} = \pi^*(\bar{\delta}) - C(\bar{\delta}) \leq \max_{\delta} \{\pi^*(\delta) - C(\delta)\} = \pi_j^*.$$

■

Finally, we provide a condition ensuring that no other pure-strategy equilibrium exists.

Assumption D:

$$\max_{\delta}\left\{\pi^*\left(\delta-\frac{\bar{\delta}}{2}\right)-C\left(\delta\right)\right\}>0.$$

We have:

Claim 12 *Under Assumptions A', B, C and D, in the above two-stage game there is no other subgame perfect Nash equilibrium in pure strategies than the ones described in Claim 11.*

Proof. Given the above analysis, it suffices to rule equilibria of the second type, where only one firm invests. Thus, without loss of generality, consider a candidate equilibrium in which firm 1, say, invests $\Delta_{A_1}>0$ and $\Delta_{B_1}\geq 0$, whereas firm 2 does not invest: $\Delta_{A_2}=\Delta_{B_2}=0$. Firm 2 thus obtains zero profit, whereas firm 1 obtains a comparative advantage $\delta_{hi}=\Delta_{hi}$ in each product $h=A,B$, and supplies all consumers with a total margin reflecting its overall competitive advantage: $m_1=\delta_{A_1}+\delta_{B_1}$; its profit is thus equal to

$$\pi_1=\delta_{A_1}+\delta_{B_1}-C\left(\delta_{A_1}+\delta_{B_1}\right),$$

leading it to choose δ_A^* and δ_B^* such that $\delta_A^*+\delta_B^*=\bar{\delta}$.

By construction, given that firm 2 does not invest, firm 1 has no incentive to deviate from these investment levels. We now consider possible deviations by firm 2, and denote by δ_A and δ_B the deviating investment levels.

It is never optimal for firm 2 to invest in a product if it does not obtain a comparative advantage in that product. Furthermore, it cannot pay for firm 2 to invest in both products: by choosing $\delta_A>\delta_A^*$ and $\delta_B>\delta_B^*$, firm 2 obtains:

$$\hat{\phi}\left(\delta\right)\equiv\delta-\bar{\delta}-C\left(\delta\right)<0,$$

where $\delta=\delta_A+\delta_B>\bar{\delta}$ and the inequality follows from $\hat{\phi}\left(\bar{\delta}\right)=0$ and $\hat{\phi}'\left(\delta\right)=1-C'\left(\delta\right)<0$ for $\delta>\bar{\delta}$. Hence, without loss of generality, we can restrict attention to deviations that aim at obtaining a comparative advantage in a single market; that is, $\delta_h=\delta>\delta_h^*$ and

$\delta_k = 0$, for some $h \neq k \in \{A, B\}$. In addition, it would not be optimal for firm 2 to obtain a larger comparative advantage than firm 1: by choosing $\delta > \bar{\delta}$, firm 2 obtains

$$\delta - \bar{\delta} + \pi^* \left(\delta_k^*\right) - C\left(\delta\right),$$

which, as $C'\left(\delta\right) > 1$ for $\delta > \bar{\delta}$, is lower than what it would obtain for $\delta = \bar{\delta}$. Hence, we can further restrict attention to $\delta \leq \bar{\delta}$; firm 2 then obtains:

$$\pi^* \left(\delta - \delta_h^*\right) - C\left(\delta\right).$$

It follows that such an equilibrium exists if and only if:

$$\max_{\delta} \left\{\pi^* \left(\delta - \min\left\{\delta_A^*, \delta_B^*\right\}\right) - C\left(\delta\right)\right\} \leq 0,$$

where $\delta_A^* + \delta_B^* = \bar{\delta}$. As this condition is the least demanding for $\delta_A^* = \delta_B^* = \bar{\delta}/2$, condition D rules out the existence of such an equilibrium. ■

Together, Claims 9, 11 and 12 yield:

Proposition 13 *In the above two-stage game:*

- (i) *Under Assumptions A', B, C, there are two subgame (trembling-hand) perfect Nash equilibria, in which: (i) in stage 1, one firm invests* $\bar{\delta}$ *in one market (and* 0 *in the other market), whereas the other firm invests* $\underline{\delta} < \bar{\delta}$ *in the other market (and* 0 *in the first market); in stage 2, the second firm offers its basket at cost whereas the first firm supplies one-stop shoppers and charges them a total margin* $m = \bar{\delta} - \underline{\delta}$*, and both firms sell their weaker products below cost.*

- (ii) *If Assumption D also holds, then there is no other equilibrium in pure strategies.*

E RBC laws

We first characterize the mixed-strategy equilibrium that arise under RBC laws and then examine the impact of RBC laws on consumer surplus and welfare. For the sake of exposition, we assume that Assumption A holds. Building on the analysis of Section B.1.1, it is straightforward to show that the analysis carries over when the distribution of the shopping cost is sufficiently dispersed, e.g., if it is distributed over $[0, \bar{s}]$ with $\bar{s} > w$.

E.1 Proof of Proposition 8

We first show that there is no pure strategy Nash equilibrium under RBC laws. We note that in any pure strategy Nash equilibrium each firm $i = 1, 2$ would have to charge $\rho_i, \mu_i \geq 0$, so as to satisfy the RBC laws, and from Proposition 3 we have:

Corollary 1 *Under RBC laws, in any equilibrium, each firm must obtain a positive profit; therefore, each firm should attract some consumers and sell them at least one product with a positive margin.*

Proof. This follows directly from Proposition 7, which implies that under RBC laws, in any equilibrium, each firm i must obtain a profit at least equal to $\pi_i \geq \bar{\pi} > 0$. ■

It follows that, in any equilibrium with pure strategies, some consumers must be active. We successively consider the cases in which one-stop shoppers would be supplied by both firms, one firm, or none (that is, only multi-stop shoppers would be active).

Case (1): Both firms supply one-stop shoppers. This case can only arise when the two firms offer one-stop shoppers the same positive value, $v_1 = v_2 > 0$, implying $\hat{m}_1 = \hat{m}_2$. By construction, at least one firm, say firm i, attracts only a fraction of these one-stop shoppers; and from Corollary 1, firm i must sell at least one good with a positive margin. Suppose firm i deviates by reducing that margin by ε:

- this deviation enables firm i to attract all active one-stop shoppers; and
- in addition, the relevant thresholds for multi-stop shopping, which can initially be expressed as:

$$\begin{aligned} \tau &= v_{12} - \max\{v_1, v_2\} = v_{12} - v_i = \delta - \hat{\rho}_j + \hat{\mu}_i, \\ \underline{\tau} &= \underline{v}_{12} - \max\{v_1, v_2\} = \underline{v}_{12} - v_i = -\delta - \hat{\mu}_j + \hat{\rho}_i, \end{aligned}$$

  can only be lowered by the reduction of firm i's margin.[14] Therefore:

  - if initially there are only one-stop shoppers, then the deviation does not transform any of them into multi-stop shoppers; and

[14]More precisely, as $\tilde{v}_i > v_j$, these thresholds either become $\tilde{\tau} = \tilde{v}_{12} - \tilde{v}_i = \tau - \varepsilon$ and $\tilde{\underline{\tau}} = \tilde{\underline{v}}_{12} - \tilde{v}_i = \underline{\tau}$ (if $\tilde{\mu}_i = \mu_i - \varepsilon$ and $\tilde{\rho}_i = \rho_i$), or $\tilde{\tau} = \tau$ and $\tilde{\underline{\tau}} = \tilde{\underline{\tau}} - \varepsilon$ (if $\tilde{\rho}_i = \rho_i - \varepsilon$ and $\tilde{\mu}_i = \mu_i$).

– if instead there are initially multi-stop shoppers as well, then the deviation can only transform marginal multi-stop shoppers into one-stop shoppers, from which firm i makes a higher profit.

It follows that, for ε small enough, the deviation is profitable.

Case (2): One firm supplies one-stop shoppers. This case arises when, for instance, $v_i > v_j\,(> 0)$, implying $\hat{m}_j > \hat{m}_i$, in which case firm i attracts all one-stop shoppers. From Corollary 1, firm j must also obtain a profit, implying that some multi-stop shoppers must also be active. For this to be the case, firm i must offer a positive value, $v_i^{ms} > 0$, on the product they target. It is moreover straightforward to see that firm i must offer a positive value, $v_i^{os} \equiv v_i - v_i^{ms} > 0$, on its other product as well. Starting from a situation where it would offer no value on this other product, reducing its margin so as to offer a slightly positive value on that product (e.g., $\tilde{v}_i^{os} = \varepsilon > 0$) would not only enable firm i to sell both of its products to one-stop shoppers (with an almost "full" margin on the other product) but, by slightly increasing its overall value, from v_i to $\tilde{v}_i = v_i + \varepsilon$, it would also transform marginal multi-stop shoppers into (more profitable) one-stop shoppers, buying both products from firm i. Therefore, we can restrict attention to firm i's margins such that $\rho_i < \bar{w}_i$ and $\mu_i < \underline{w}_i$. As from Corollary 1, firm i must sell at least one good with a positive margin. We thus have $(m_j \geq)\,\hat{m}_j > \hat{m}_i = m_i > 0$ and firm i's profit can be expressed as:

$$\begin{aligned} \pi_i &= m_i \left[F(v_i) - F(\tau)\right] + m_i^{ms} F(\tau^{ms}) \\ &= m_i F(v_i) - m_i^{os} F(\tau^{ms}), \end{aligned}$$

where τ^{ms} denotes the threshold for multi-stop shopping, whereas m_i^{ms} and $m_i^{os} = m_i - m_i^{ms}$ respectively denote firm i's margins on the product bought by multi-stop shoppers (as well as by one-stop shoppers), and on the other product (bought only by one-stop shoppers).[15] Note that charging a zero margin on the product bought by multi-stop shoppers is never optimal: starting from $m_i^{ms} = 0$, deviating to $\tilde{m}_i^{ms} = \varepsilon$ (where ε is

[15] If multi-stop shoppers buy strong products, we thus have $\tau^{ms} = \tau = v_{12} - v_i$, $m^{ms} = \rho_i$ and $m^{os} = \mu_i$; if instead multi-stop shoppers buy weak products, we have $\tau^{ms} = \underline{\tau} = \underline{v}_{12} - v_i$, $m^{ms} = \mu_i$ and $m^{os} = \rho_i$.

positive but "small") and $\tilde{m}_i^{os} = m_i - \varepsilon$ allows firm i to earn the same profit from one-stop shoppers, but, in addition, it now derives a positive profit from multi-stop shoppers. Moreover, the deviation keeps v_i unchanged but reduces the multi-stop shopping threshold τ^{ms} to $\tilde{\tau}^{ms} = \tau^{ms} - \varepsilon$, and thus transforms marginal multi-stop shoppers into one-stop shoppers, from which firm i makes more profit. Thus, in what follows, we focus on $m_i^{ms} > 0$ and distinguish two cases, depending on whether or not firm i charges the monopoly profit-maximizing margin $m^* \equiv \arg\max_m mF(w-m)$ (which, given the monotonicity of the inverse hazard rate $h(\cdot)$, is uniquely defined by $h(w - m^*) = m^*$):

- if $m_i \neq m^*$, then suppose that firm i adjusts its margin on the product bought by multi-stop shoppers to $\tilde{m}_i^{ms} = m_i^{ms} + \varepsilon(m^* - m_i)$, where $\varepsilon > 0$ is small enough to ensure that $\tilde{m}_i < \hat{m}_j$ and $\tilde{m}_i^{ms} > 0$. Such a deviation does not change the threshold τ^{ms} (which depends on firm i's prices only through m_i^{os}), and firm i's profit becomes:

  $$\tilde{\pi}_i = (m_i + \varepsilon(m^* - m_i)) F(v_i - \varepsilon(m^* - m_i)) - m_i^{os} F(\tau^{ms}).$$

  The monotonicity of the inverse hazard rate $h(\cdot)$ ensures that the first term increases with ε as long as $\tilde{m}_i < m^*$, implying that such a deviation is profitable;

- if $m_i = m^*$, then firm j can benefit from undercutting its rival. Firm j's profit is given by:

  $$\pi_j = m_j^{ms} F(\tau^{ms}),$$

  where m_j^{ms} denotes firm j's margin on the product bought by multi-stop shoppers. Using:

  $$\tau^{ms} = v_i^{ms} + v_j^{ms} - v_i \leq v_j^{ms} < w - m_j^{ms},$$

  where the first inequality stems from $v_i = v_i^{os} + v_i^{ms} \geq v_i^{ms}$,[16] and the second follows from the fact that the surplus generated by any single product cannot exceed w, we have:

  $$\pi_j = m_j^{ms} F(\tau^{ms}) < m_j^{ms} F\left(w - m_j^{ms}\right) \leq \pi^* \equiv m^* F(w - m^*). \tag{17}$$

  That is, the maximum profit that firm j can earn from multi-stop shoppers is strictly lower than the monopoly profit derived from one-stop shoppers. Consider now firm j's deviation to $\tilde{\mu}_j = \max\{\rho_i - \delta - \varepsilon/2, 0\}$ and $\tilde{\rho}_j = m^* - \varepsilon - \tilde{\mu}_j$, for some $\varepsilon > 0$:

[16] From the remarks above, this inequality is actually strict, as $v_i^{os} > 0$.

– if $\rho_i > \delta$, then for ε small enough, $\tilde{\mu}_j = \rho_i - \delta - \varepsilon/2\left(< \bar{w}_i - \delta = \underline{w}_j\right)$ and $\tilde{\rho}_j = \mu_i + \delta - \varepsilon/2\,(< \underline{w}_i + \delta = \bar{w}_j)$, implying $\tilde{v}_{12} = \tilde{\underline{v}}_{12} = v_i + \varepsilon/2 < \tilde{v}_j = v_i + \varepsilon$, and thus $\tilde{\tau} = \tilde{\underline{\tau}} = -\varepsilon/2 < 0$. Therefore, firm j transforms all multi-stop shoppers into one-stop shoppers, and attracts all one-stop shoppers to whom it charges a total margin of $\tilde{m}_j = m^* - \varepsilon$; and

– if, instead, $\rho_i \leq \delta$, which implies that multi-stop shoppers buy strong products,[17] then $\tilde{\mu}_j = 0$ and $\tilde{\rho}_j = m^* - \varepsilon$ (note that $\rho_i \leq \delta$ then implies $\tilde{\rho}_j < m^* = \rho_i + \mu_i \leq \delta + \underline{w}_i = \bar{w}_j$); firm j then attracts all one-stop shoppers and also serves any remaining multi-stop shoppers (who still buy strong products, as $\tilde{\tau} = \tilde{v}_{12} - \tilde{v}_j = \delta - \rho_i \geq 0$), but makes the same margin $\tilde{m}_j = \tilde{\rho}_j = m^* - \varepsilon$ on both types of shoppers; and

- in both cases, the deviation yields a profit:

$$\tilde{\pi}_j = \tilde{m}_j F\left(\tilde{v}_j\right) = \left(m^* - \varepsilon\right) F\left(w - m^* + \varepsilon\right),$$

which, from (17), makes the deviation profitable for ε small enough.

Case (3): There only exist multi-stop shoppers. This case arises when $v^{ms} \equiv v_1^{ms} + v_2^{ms} \geq 2\max\{v_1, v_2\}$,[18] where, as before, v_i^{ms} denotes the value offered by firm i on the product targeted at multi-stop shoppers. By construction, however, $v_i = v_i^{ms} + v_i^{os}$, where, as before, v_i^{os} denotes the value offered by firm i on its other product. The first

[17] Multi-stop shoppers would buy weak products only if $\underline{v}_{12} > v_i$, or $\underline{\tau} = \underline{v}_{12} - v_i = -\delta - \hat{\mu}_j + \hat{\rho}_i > 0$, which (using $\hat{\rho}_i = \rho_i$, as noted above) implies $\rho_i > \delta + \hat{\mu}_j \geq \delta$.

[18] We must have:

$$v^{ms} - 2s \geq 0 \Longrightarrow v^{ms} - 2s \geq \{v_1, v_2\} - s,$$

which amounts to:

$$s \leq v^{ms}/2 \Longrightarrow s \leq v^{ms} - \max\{v_1, v_2\},$$

or $\max\{v_1, v_2\} \leq v^{ms} - v^{ms}/2 = v^{ms}/2$.

condition therefore implies:[19]

$$v_1^{ms} = v_2^{ms} = \frac{v^{ms}}{2} > v_1^{os} = v_2^{os} = 0.$$

But then any firm i can profitably deviate by charging a positive but non-prohibitive margin on its other product, leaving a positive value $\tilde{v}_i^{os} > 0$. This deviation does not affect the value offered to multi-stop shoppers, v^{ms}, but it *increases* the value offered to one-stop shoppers to:

$$\tilde{v}_i = v_i^{ms} + \tilde{v}_i^{os} = \frac{v^{ms}}{2} + \tilde{v}_i^{os} > \frac{v^{ms}}{2}.$$

This deviation thus induces some of the initial multi-stop shoppers (namely, those whose shopping costs lie between $\tilde{\tau}^{ms} = v^{ms} - \tilde{v}_i$ and $v^{ms}/2$) to buy both products from firm i, enabling firm i to earn an additional profit from selling its other product, and it, moreover, attracts more one-stop shoppers (namely, those whose shopping cost lies between $v^{ms}/2$ and $\tilde{v}_i$), generating yet another profit.

To summarize, no pure-strategy satisfying $\rho_i \geq 0$ and $\mu_i \geq 0$ for $i \in \{1, 2\}$ can form a Nash equilibrium in any of the above configurations; hence, there is no pure-strategy Nash equilibrium when below-cost pricing is prohibited.

We now characterize the mixed-strategy equilibrium. Firm i's profit, as a function of the two firms' margins on their strong products, ρ_1 and ρ_2, is given by:

$$\pi_i^b\left(\rho_i, \rho_j\right) \equiv \begin{cases} \rho_i F\left(w - \rho_i\right) & \text{if} \quad \rho_i < \rho_j, \\ \rho_i F\left(\delta - \rho_i\right) & \text{if} \quad \rho_i > \rho_j. \end{cases}$$

In the first case ($\rho_i < \rho_j$), firm i sells its strong product to both one-stop and multi-stop shoppers, whereas in the second case ($\rho_i > \rho_j$), it sells its strong product only to multi-stop shoppers.

[19]To see this, note that the condition $v^{ms} \geq 2v_j$ amounts to:

$$\begin{aligned} v_1^{ms} + v_2^{ms} \quad &\geq \quad 2\left(v_j^{ms} + v_j^{os}\right) \\ &\iff \quad v_i^{ms} - v_j^{ms} \geq 2v_j^{os}. \end{aligned}$$

As v_j^{os} cannot be negative (consumers can always opt out), and the condition $v^{ms} \geq 2v_j$ must hold for $j \in \{1, 2\}$, it follows that $0 \geq v_1^{ms} - v_2^{ms} \geq 0$, or $v_1^{ms} = v_2^{ms}$; this, in turn, implies $0 \leq v_j^{os} \leq 0$, or $v_j^{os} = 0$, for $j \in \{1, 2\}$.

Consider a candidate equilibrium in which each firm i: (i) sells its weak product at cost; (ii) randomizes the margin ρ_i on its strong product according to a distribution $G(\rho)$ over some interval with continuous density $g(\rho)$; and (iii) obtains an expected profit equal to the minmax, $\bar{\pi}$. By construction, the bounds of the support of the distribution must be given by $\bar{\rho} \equiv \arg\max_\rho \rho F(\delta - \rho)$ and $\underline{\rho} F\left(w - \underline{\rho}\right) = \bar{\pi}$.

Consider consumers' responses to given margins ρ_i and ρ_j:

- consumers buy both goods from firm i if:
  - firm i undercuts its rival:
  $$\rho_j \geq \rho_i;$$
  - one-stop shopping is valuable:
  $$s \leq v_i = w - \rho_i;$$
  - and is more valuable than multi-stop shopping:
  $$s \geq v_{12} - v_i = \delta - \rho_j; \text{ and}$$
- consumers instead engage in multi-stop shopping if:
  $$s \leq v_{12} - \max\{v_1, v_2\},$$
  which amounts to:
  $$s \leq \delta - \rho_i \text{ and } s \leq \delta - \rho_j.$$

Figure 1 depicts the consumers' response.

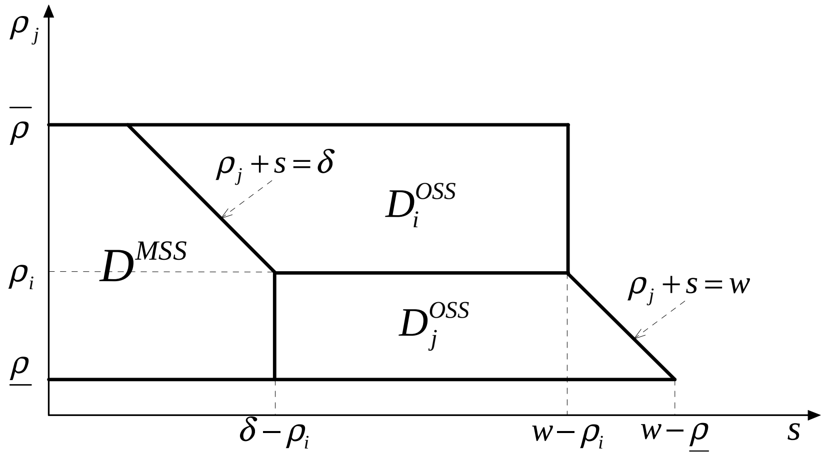


Figure 1

Firm i's expected profit can then be expressed as:

$$\rho_i E\left(D_i^{OSS} + D^{MSS}\right),$$

where D_i^{OSS} represents the demand from one-stop shoppers going to firm i, and D^{MSS} is the demand from multi-stop shoppers. As firm j's margin is distributed according to the distribution function $G\left(\rho_j\right)$, firm i's expected profit can be written as:

$$\begin{aligned}\pi\left(\rho_i\right) &= \rho_i\left[\left(1 - G\left(\rho_i\right)\right) F(w-\rho_i) + G\left(\rho_i\right) F\left(\delta - \rho_i\right)\right] \\ &= \rho_i\left\{F(w-\rho_i) - G\left(\rho_i\right)\left[F(w-\rho_i) - F\left(\delta-\rho_i\right)\right]\right\}.\end{aligned}$$

Hence, for a firm to obtain its minmax profit $\bar{\pi}$, we must have, for all ρ:

$$\rho\left\{F(w-\rho) - G\left(\rho\right)\left[F(w-\rho) - F\left(\delta-\rho\right)\right]\right\} = \bar{\pi},$$

or:

$$G\left(\rho\right) \equiv \frac{\rho F\left(w-\rho\right) - \bar{\pi}}{\rho F\left(w-\rho\right) - \rho F\left(\delta-\rho\right)}. \tag{18}$$

By construction, the function $G\left(\cdot\right)$ defined by (18) is such that $G\left(\underline{\rho}\right) = 0$ and $G\left(\bar{\rho}\right) = 1$; it remains to confirm that it is increasing in ρ in the range $\left[\underline{\rho}, \bar{\rho}\right]$. Differentiating (18) with respect to ρ, we have:

$$G'\left(\rho\right) = \frac{\left[\bar{\pi} - \rho F\left(\delta-\rho\right)\right]\left[F(w-\rho) - \rho f(w-\rho)\right] + \left[\rho F(w-\rho) - \bar{\pi}\right]\left[F\left(\delta-\rho\right) - \rho f\left(\delta-\rho\right)\right]}{\left[\rho F(w-\rho) - \rho F\left(\delta-\rho\right)\right]^2}.$$

As $w > \delta$, and given the definition of $\bar{\rho}$ and $\underline{\rho}$, the functions $\rho F(w-\rho)$ and $\rho F(\delta-\rho)$ are both increasing in the range $\left[\underline{\rho}, \bar{\rho}\right]$, and moreover satisfy $\underline{\rho}F(w-\underline{\rho}) = \bar{\rho}F(\delta-\bar{\rho}) = \bar{\pi}$ and $\rho F(w-\rho) > \bar{\pi} > \rho F(\delta-\rho)$ for $\underline{\rho} < \rho < \bar{\rho}$. It follows that $G'(\bar{\rho}) = 0$ and $G'(\rho) > 0$ for $\underline{\rho} \leq \rho < \bar{\rho}$.

We now show that the function $G(\cdot)$ supports a symmetric mixed strategy equilibrium. To see this, consider firm i's best response when its rival, firm j, adopts the above strategy. If firm i were to charge a total margin $m_i > \bar{\rho}$, one-stop shoppers would go to the rival and multi-stop shoppers become those consumers whose shopping cost is lower than $v_{12}-v_j = \delta-\rho_i$; hence, firm i would earn a profit equal to $\rho_i F(\delta-\rho_i) \leq \bar{\pi}$. Thus, without loss of generality, we can restrict attention to deviations that are such that $m_i \leq \bar{\rho}$.

Suppose first that firm i prices its weak product above cost (i.e., its total margin satisfies $m_i > \rho_i$), and consider the impact of an increase in the margin on the strong product, ρ_i, keeping constant the total margin m_i. We distinguish between two cases, depending on which firm offers the best prices.

- When the realization of the rival's margin is such that $m_j \left(= \rho_j\right) > m_i$, one-stop shoppers (if any) favor firm i, and thus the multi-stop shopping threshold is $\tau = v_{12} - v_i = \delta + m_i - \rho_i - \rho_j$. Two cases may then arise:

  - if $\tau = v_{12} - v_i \leq v_i$, which amounts to $v_i \geq v_{12}/2$, consumers whose shopping costs lie below τ engage in multi-stop shopping and buy strong products, whereas those with s between τ and v_i buy both products from firm i. Hence, increasing ρ_i:

    - increases the profit earned by selling the strong product to all active consumers (that is, those with $s \leq v_i = w - m_i$); and

    - also induces some multi-stop shoppers to buy firm i's weak product as well, which further enhances firm i's profit.

  - if instead $v_i < v_{12}/2$, consumers whose shopping costs lie below $v_{12}/2$ engage in multi-stop shopping and buy strong products, and all other consumers are inactive. Hence, firm i's profit is equal to:

$$\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right) = \rho_i F\left(\frac{w+\delta-\rho_1-\rho_2}{2}\right),$$

which increases with ρ_i: the derivative is equal to:

$$\pi_i'(\rho_i) = F\left(\frac{v_{12}}{2}\right) - \frac{\rho_i f\left(\frac{v_{12}}{2}\right)}{2} = \left[2h\left(\frac{v_{12}}{2}\right) - \rho_i\right]\frac{f\left(\frac{v_{12}}{2}\right)}{2},$$

where the term in brackets is positive, as $v_i < v_{12}/2$ implies $2h(v_{12}/2) > h(v_{12}/2) > h(v_i) = h(w - m_i) > m_i > \rho_i$ (where the penultimate inequality stems from $m_i \leq \bar{\rho}$, the function $m_i F(w - m_i)$ being increasing in m_i in that range).

• When, instead, the realization of the rival's margin is such that $m_j \left(= \rho_j\right) < m_i$, one-stop shoppers (if any) favor firm j; hence, firm i only sells (its strong product) to multi-stop shoppers, and the multi-stop shopping threshold is $\tau = v_{12} - v_j = \delta - \rho_i$. Two cases may again arise:

- if $\tau = v_{12} - v_j \leq v_i$, which amounts to $v_j \geq v_{12}/2$, all consumers whose shopping costs lie below τ engage in multi-stop shopping, and so firm i's profit is equal to:

$$\pi_i(\rho_i) = \rho_i F(\tau) = \rho_i F(\delta - \rho_i),$$

  which increases with ρ_i on the relevant range $\rho_i \leq \bar{\rho}$; and

- if instead $v_j < v_{12}/2$, only those consumers with s below $v_{12}/2$ engage in multi-stop shopping, and so firm i's profit is equal to $\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right)$. The same reasoning as above then shows that this profit again increases with ρ_i.

Therefore, it is never optimal for a firm to price its weak product above cost: starting from $\rho_i < m_i$, raising ρ_i would always increase firm i's *ex post* profit, and would thus increase its expected profit as well.

Suppose now that firm i sells its weak product at cost: $m_i = \rho_i$. By construction, choosing any ρ_i in the range $\left[\underline{\rho}, \bar{\rho}\right]$ yields the same expected profit, $\bar{\pi}$. It remains to confirm that it is not profitable to pick a margin ρ_i outside the support of G:

- choosing $\rho_i < \underline{\rho}$ attracts all one-stop shoppers and thus yields an expected profit equal to $\pi_i(\rho_i) = \rho_i F(w - \rho_i)$, which increases in ρ_i for $\rho_i \leq \bar{\rho}$, and is thus lower than $\pi_i\left(\underline{\rho}\right) = \bar{\pi}$; and

- choosing $\rho_i > \bar{\rho}$ attracts no one-stop shoppers, and thus the expected profit must be lower than $\rho_i F(\delta - \rho_i) \leq \max_\rho \rho F(\delta - \rho) = \bar{\pi}$.

This establishes the first part of the proposition; the rest has been established in the main text.

E.2 Proof of Proposition 9

We now analyze the impact of banning below-cost pricing on consumer surplus. When below-cost pricing is not prohibited, the equilibrium consumer surplus can be expressed as:

$$\begin{aligned} S^* &= \int_0^w (w-s) f(s) ds + \int_0^{\tau^*} (\tau^* - s) f(s) ds \\ &= \int_0^w F(s) ds + \int_0^{\tau^*} F(s) ds, \end{aligned}$$

where the second expression relies on integration by parts. The first term in that expression represents the surplus that would be generated if all consumers were one-stop shoppers (and thus bought the bundle at cost), and the second term represents the extra surplus from multi-stop shopping. When, instead, below-cost pricing is banned, *ex post* (i.e., for a given realization of the margins ρ_1 and ρ_2) consumer surplus can be written as:

$$\begin{aligned} S^b(\rho_1, \rho_2) &= \int_0^{v^b(\rho_1,\rho_2)} \left[v^b(\rho_1, \rho_2) - s\right] f(s) ds + \int_0^{\tau^b(\rho_1,\rho_2)} \left[\tau^b(\rho_1, \rho_2) - s\right] f(s) ds \\ &= \int_0^{v^b(\rho_1,\rho_2)} F(s) ds + \int_0^{\tau^b(\rho_1,\rho_2)} F(s) ds. \end{aligned}$$

Thus, the resulting change in *ex post* consumer surplus is given by:

$$\Delta S(\rho_1, \rho_2) = S^b(\rho_1, \rho_2) - S^* = \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} F(s) ds - \int_{v^b(\rho_1,\rho_2)}^{w} F(s) ds.$$

Banning below-cost pricing generates two opposite effects on consumer surplus. On the one hand, the increase in multi-stop shopping (recall that $\tau^b > \tau^*$) has a positive effect, represented by the first term in the above expression; on the other hand, one-stop shoppers face higher prices than before, causing a loss of consumer surplus represented by the second term. The net effect depends on the value of w, δ, and the distribution of shopping costs, which contribute to determining equilibrium prices.

To explore this further, we fix the parameter δ and examine the sign of ΔS as a function of the social value w. Note that τ^* and $\bar{\rho}$ do not depend on w, whereas $\underline{\rho}(w)$ is the lower solution to $\underline{\rho} F\left(w - \underline{\rho}\right) = \bar{\pi} = \bar{\rho} F(\delta - \bar{\rho})$, and thus decreases in w.

In the limit case where $w = \delta$, the lower bound $\underline{\rho}(w)$ coincides with $\bar{\rho}$; that is, both firms charge $\rho = \bar{\rho}$ with probability one. As $\bar{\rho} > \rho^*$ (and weak products are priced at cost, instead of being subsidized), all prices are higher than before, and thus every consumer's (expected) surplus goes down. By continuity, this remains the case as long as weak products offer sufficiently low value (i.e., as long as w is close enough to δ).

We now examine the impact of a ban on total welfare, that is, on the sum of consumer surplus and firms' profits. When w is close to δ, the equilibrium margin distribution tends to assign a probability mass of 1 on $\bar{\rho}$, and the impact of a ban on *expected* welfare then becomes:

$$\begin{aligned}
\Delta W &= \Delta S(\bar{\rho}, \bar{\rho}) + 2(\bar{\pi} - \pi^*) \\
&= \int_{\tau^*}^{\tau^b(\bar{\rho},\bar{\rho})} F(s)ds - \int_{v^b(\bar{\rho},\bar{\rho})}^{w} F(s)ds + 2(\bar{\pi} - \pi^*) \\
&= 2\Phi(\delta - \bar{\rho}) - \Phi(\delta - 2\rho^*) - \Phi(\delta) + 2(\bar{\pi} - \pi^*),
\end{aligned}$$

where:

$$\Phi(x) = \int_0^x F(s)ds.$$

The sign of ΔW can be either positive or negative, depending on the distribution of shopping costs. To see this, we consider the case where shopping costs are distributed according to $F(s) = (s/\bar{s})^k$, where $\bar{s} > \delta$. The hazard rate assumption is satisfied for any $k > 0$, and:

$$f(s) = \frac{k}{\bar{s}^k} s^{k-1}, \;\; \Phi(x) = \frac{s^{k+1}}{(k+1)\bar{s}^k} \text{ and } h(s) = \frac{F(s)}{f(s)} = \frac{s}{k}.$$

When below-cost pricing is not prohibited, the equilibrium is characterized by:

$$\begin{aligned}
\rho^* &= h(\delta - 2\rho^*) = \frac{\delta - 2\rho^*}{k} \Leftrightarrow \rho^* = \frac{\delta}{2+k}, \\
\tau^* &= \delta - 2\rho^* = \frac{k\delta}{2+k}, \\
\pi^* &= \rho^* F(\tau^*) = \frac{\delta}{2+k} \frac{\left(\frac{k\delta}{2+k}\right)^k}{\bar{s}^k} = \frac{1}{\bar{s}^k} \delta \frac{k^k \frac{\delta^k}{(k+2)^k}}{k+2} = \frac{k^k}{\bar{s}^k} \frac{\delta^{k+1}}{(k+2)^{k+1}}, \\
v^* &= w = \delta.
\end{aligned}$$

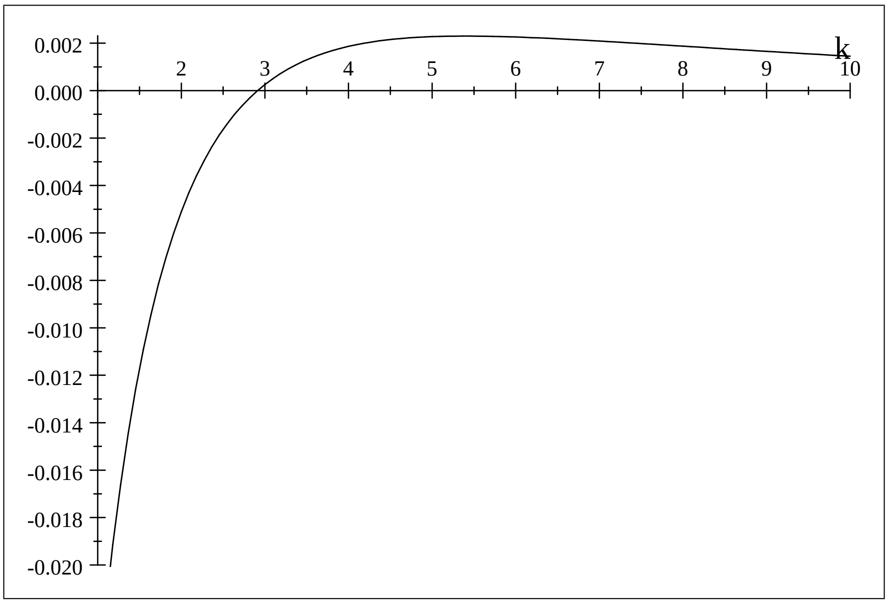
Instead, when below-cost pricing is banned, the equilibrium is characterized as follows:

$$\begin{aligned}\bar{\rho} &= h(\delta - \bar{\rho}) = \frac{\delta - \bar{\rho}}{k} \Leftrightarrow \bar{\rho} = \frac{\delta}{1+k}, \\ \bar{\tau} &= v^b(\bar{\rho}, \bar{\rho}) = \delta - \bar{\rho} = \frac{k\delta}{1+k}, \\ \bar{\pi} &= \bar{\rho}F(\delta - \bar{\rho}) = \frac{k^k}{\bar{s}^k}\frac{\delta^{k+1}}{(1+k)^{k+1}}.\end{aligned}$$

Thus, banning below-cost pricing results in the following change of total welfare:

$$\begin{aligned}\Delta W(k;\delta,\bar{s}) &= \frac{2k^{k+1}}{\bar{s}^k}\frac{\delta^{k+1}}{(1+k)^{k+2}} - \frac{k^{k+1}}{\bar{s}^k}\frac{\delta^{k+1}}{(k+1)(2+k)^{k+1}} - \frac{1}{\bar{s}^k}\frac{\delta^{k+1}}{(k+1)} \\ &\quad +2\frac{k^k}{\bar{s}^k}\delta^{k+1}\left[\frac{1}{(1+k)^{k+1}} - \frac{1}{(k+2)^{k+1}}\right].\end{aligned}$$

This expression is continuous in k and, as k goes to 0, it tends to 0 and its derivative tends to $-\infty$; hence, banning below-cost pricing reduces total welfare when the distribution is not too convex. The following graph, which represents $\Delta W(k;\delta,\bar{s})$ for $\delta = 1$ and $\bar{s} = 1.1$, shows that banning below-cost pricing instead increases total welfare when the distribution of shopping cost is sufficiently convex (namely, for $k > \hat{k} \simeq 2.9$):



By continuity, for w close enough to δ, there exists $\hat{k}(w,\delta)$ such that banning below-cost pricing reduces total welfare when $k < \hat{k}(w,\delta)$.

Remark: upstream margins.

As already mentioned, in the case of downstream firms (e.g., retailers), their comparative advantages may be mainly driven by differences in wholesale prices rather than in quality or cost. Consider for instance the setting developed in Section 4, where supermarkets can devote resources to negotiating better conditions from their suppliers and, in equilibrium, target different products. Total welfare must then account for the profit of upstream suppliers. To explore this further, suppose that which firms initially face the same wholesale price for each product, and negotiate a discount δ on one or the other product; "strong products" then correspond to those on which they negotiated the discount, and "weak products" correspond to those on which they pay the regular wholesale price.

To fix ideas, suppose firm 1 obtained the discount on product A, which reduces the wholesale price to from c to $c-\delta$, whereas it still faces the regular wholesale price c on product B. By contrast, firm 2 benefits from the discounted wholesale price $c-\delta$ on product B, but faces the regular wholesale price c on product A. Suppose further that upstream suppliers face the same marginal cost of production, γ, for both products A and B. Finally, to fix ideas, assume that the discount erases the supplier's margin, so that strong products are supplied at cost: $c-\delta=\gamma$. The suppliers' profit then comes solely from the sales of the "weak" products, and is equal to

$$\Pi_U \equiv (c-\gamma)\left[F(v)-F(\tau)\right] = \delta\left[F(v)-F(\tau)\right].$$

Let $\Delta\Pi_U$ denote the change in suppliers' brought by RBC laws; we have:

$$\begin{aligned} \Delta\Pi_U &= \delta\left[F\left(v^b\right)-F\left(\tau^b\right)\right]-\delta\left[F\left(v^*\right)-F\left(\tau^*\right)\right] \\ &= -\delta\left[\left(F\left(v^*\right)-F\left(v^b\right)\right)+F\left(\tau^b\right)-F\left(v^*\right)\right]. \end{aligned}$$

As $v^* > v^b$ and $\tau^b > v^*$, it follows that $\Delta\Pi_U < 0$: the upstream suppliers earn less profits under RBC laws, as they reduce the number of one-stop shoppers. As a result, RBC laws have a negative impact on the upstream industry, which degrades further its impact on total welfare.

E.3 Impact on expected consumer surplus

We conclude by noting that RBC laws necessarily decrease (expected) consumer surplus when the density of the distribution of shopping costs does not increase between τ^* and δ. The impact of RBC laws on total expected consumer surplus can be expressed as the impact on expected social welfare, minus the impact on expected industry profit:

$$E\left[\Delta S\left(\rho_1,\rho_2\right)\right] = E\left[\Delta W\left(\rho_1,\rho_2\right)\right] - E\left[\Delta \Pi\left(\rho_1,\rho_2\right)\right],$$

where:

$$E\left[\Delta \Pi\left(\rho_1 \rho_2\right)\right] = 2\left(\bar{\pi} - \pi^*\right),$$

and $\Delta W\left(\rho_1 \rho_2\right)$ can be obtained by comparing the two regimes:

- when firms are allowed to price below-cost, social welfare is equal to:

$$W^* = \int_0^w \left(w - s\right) dF\left(s\right) + \int_0^{\tau^*} \left(\delta - s\right) dF\left(s\right),$$

  where the first term is the social welfare that would be generated if all consumers were one-stop shoppers, and the second term represents the additional welfare from multi-stop shopping; and

- under RBC laws, *ex post* social welfare is equal to:

$$W^b\left(\rho_1,\rho_2\right) = \int_0^{v^b(\rho_1,\rho_2)} \left(w - s\right) dF\left(s\right) + \int_0^{\tau^b(\rho_1,\rho_2)} \left(\delta - s\right) dF\left(s\right),$$

  where:

$$v^b\left(\rho_1,\rho_2\right) = w - \min\left\{\rho_1,\rho_2\right\} \text{ and } \tau^b\left(\rho_1,\rho_2\right) = \delta - \max\left\{\rho_1,\rho_2\right\}.$$

  Hence, the impact of a ban on *ex post* social welfare is given by:

$$\Delta W\left(\rho_1,\rho_2\right) = \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} \left(\delta - s\right) dF\left(s\right) - \int_{v^b(\rho_1,\rho_2)}^{w} \left(w - s\right) dF\left(s\right), \tag{19}$$

and the impact of RBC laws on total expected consumer surplus can thus be expressed as:

$$\begin{aligned} E\left[\Delta S\left(\rho_1,\rho_2\right)\right] &= E\left[\Delta W\left(\rho_1,\rho_2\right)\right]-2\left(\bar{\pi}-\pi^*\right) \\ &= E\left[\Delta W\left(\rho_1,\rho_2\right)-2\left(\bar{\pi}-\pi^*\right)\right] \\ &= E\left[\int_{\tau^*}^{\tau^b(\rho_1,\rho_2)}\left(\delta-s\right)dF\left(s\right)-\int_{v^b(\rho_1,\rho_2)}^{w}\left(w-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right] \\ &\leq E\left[\int_{\tau^*}^{\tau^b(\rho_1,\rho_2)}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right] \\ &= E\left[\int_{\delta-2\rho^*}^{\delta-\max\{\rho_1,\rho_2\}}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right] \\ &= E\left[\phi\left(\max\left\{\rho_1,\rho_2\right\}\right)\right], \end{aligned}$$

where:

$$\phi\left(\rho\right)\equiv\int_{\delta-2\rho^*}^{\delta-\rho}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right).$$

It follows that RBC laws reduce expected consumer surplus whenever $E\left[\phi\left(\rho\right)\right]<0$, where the function $\phi\left(\rho\right)$ decreases as ρ increases:

$$\phi'\left(\rho\right)=-\rho f\left(\delta-\rho\right)<0.$$

We have:

Proposition 14 *If $f\left(s\right)$ is non-increasing for $s\in\left[\tau^*,\delta\right]$, then RBC laws reduce total expected consumer surplus.*

Proof. It suffices to show that $\phi\left(0\right)\leq 0$. Using $\tau^*=\delta-2\rho^*$, we have:

$$\begin{aligned} \phi\left(0\right) &= \int_{\tau^*}^{\delta}\left(\delta-s\right)f\left(s\right)ds-2\left(\bar{\pi}-\pi^*\right) \\ &\leq \int_{\tau^*}^{\delta}\left(\delta-s\right)f\left(\tau^*\right)ds-2\pi^* \\ &= \left[-\frac{\left(\delta-s\right)^2}{2}\right]_{\tau^*}^{\delta}\times\frac{F\left(\tau^*\right)}{\rho^*}-2\rho^*F\left(\tau^*\right) \\ &= \left[\frac{\rho^2}{2}\right]_0^{2\rho^*}\times\frac{F\left(\tau^*\right)}{\rho^*}-2\rho^*F\left(\tau^*\right) \\ &= 0, \end{aligned}$$

where the first inequality stems from the assumed monotonicity of $f(\cdot)$ on the range $[\tau^*, \delta]$ and from the fact that:

$$\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) \geq 2\rho^* F(\delta - 2\rho^*) = 2\pi^*,$$

and the equality that follows uses the first-order condition characterizing ρ^*, namely:

$$\rho^* f(\tau^*) = F(\tau^*).$$

It follows that $\phi(\rho) < 0$ for any $\rho > 0$, and thus:

$$E[\Delta S(\rho_1, \rho_2)] \leq E[\phi(\max\{\rho_1, \rho_2\})] < 0.$$

■