

Search Diversion and Platform Competition*

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Abstract

Platforms use search diversion in order to trade off total consumer traffic for higher revenues derived by exposing consumers to unsolicited products (e.g. advertising). We show that the entry of a platform competitor leads to higher (lower) equilibrium levels of search diversion relative to a monopoly platform when the degree of horizontal differentiation between platforms is intermediate (low). On the other hand, more intense competition between active platforms (i.e. less differentiation) leads to less search diversion.

When platforms charge consumers fixed access fees, all equilibrium levels of search diversion under platform competition are equal to the monopoly level, irrespective of the nature of competition. Furthermore, platforms that charge positive (negative) access fees to consumers have weaker (stronger) incentives to divert search relative to platforms that cannot charge such fees. Finally, endogenous affiliation on both sides (consumers and advertising) leads to stronger incentives to divert search relative to the one-sided exogenous affiliation (vertical integration) benchmark, whenever the marginal advertiser derives higher profits per consumer exposure relative to the average advertiser.

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1 Introduction

Search diversion occurs when platforms providing access to various products deliberately introduce noise in the search or browsing process through which consumers find the products they are most interested in. This practice is widespread among both offline and online platforms. All advertising-supported media (from magazines to online portals, news sites, and search engines) are purposefully designed to expose users to advertisements, even though they are primarily interested in content. Similarly, retailers often place the most sought-after items at the back or upper floors of their stores (e.g. bread and milk at supermarkets, iPods and iPhones at Apple stores), while shopping malls design their layout to maximize the distance travelled by visitors between anchor stores (Petroski 2003). E-commerce sites (e.g. Amazon, Bing Shopping, eBay, Google Shopping) design their websites in order to divert users' attention from the products they were initially looking for, towards discovering products they might be interested in and eventually buy (unsolicited products or advertising).

On the one hand, search diversion may lead to higher platform revenues per consumer "visit" to the platform. On the other hand, it reduces the overall attractiveness of the platform to consumers and therefore also leads to lower consumer traffic (i.e. total number of visits). All platforms listed above face this fundamental tradeoff.

The basic economic logic of search diversion was first analyzed by Hagiu and Jullien (2011), using a model with a monopoly platform (intermediary) that offers consumers access to two products, whose affiliation with (i.e. availability through) the platform is exogenously given. Here we extend that analysis in two important directions: platform competition with horizontal differentiation and endogenous affiliation on both sides of the market - consumers and an unsolicited product supplier (advertiser).

Our key results are as follows. First, platform competition does not necessarily reduce search diversion incentives relative to monopoly. Specifically, when consumers affiliate exclusively with one platform, competing platforms induce less (more) search diversion than a monopolist if the degree of differentiation between platforms is low (intermediate). When differentiation is large, competing platforms behave like local monopolies and therefore choose the monopoly level of search diversion. One interpretation of the scenario in which competing platforms divert

search more than a monopoly is that, since consumers are more difficult to attract under competition, platforms may prefer to increase revenue per consumer by diverting more search. This result holds whether the advertiser affiliates exclusively or multihomes. Moreover, with exclusive advertising affiliation, each platform takes into account its competitor's incentives to compete for the advertiser. As a result, when competition is effective on both sides, the equilibrium level of search diversion maximizes total industry profit (both platforms and the advertiser). On the other hand, one platform may prefer not to compete for the advertiser if it derives sufficient consumer demand and revenue from the content solicited by consumers. In this case, the equilibrium level of search diversion does not account for the "losing" platform's profits.

Second, if platforms can charge consumers access fees, competing platforms choose the same level of search diversion as a monopoly platform for all parameter values and regardless of the mode of platform competition: all platforms maximize the total surplus per consumer. Furthermore, the ability to charge fixed access fees results in less search diversion if and only if the actual fee charged is positive. Third, endogenous affiliation of the advertiser and incomplete information regarding the advertiser's payoff create a novel source of incentives for platforms to divert search (relative to Hagiu and Jullien 2011). This incentive stems from a distortion reminiscent of Spence (1975)'s distinction between the marginal and the average customer (here, advertiser). As a result, vertical integration of a platform monopolist into advertising leads to more (less) search diversion if and only if the payoff of the average advertiser is greater than the payoff of the marginal advertiser under no integration (endogenous affiliation).

1.1 Model overview and interpretation

In our model, each platform offers consumers access to two products, 1 and 2. Product 1 (content) offers consumers expected utility $u_1 > 0$ and is assumed to be exogenously affiliated with each platform throughout the paper. Product 2 corresponds to unsolicited content, which for convenience we refer to as advertising. It offers consumers expected utility $u_2 = 0$ and is supplied by a third-party seller (advertiser), who must be induced to affiliate by platforms' choices of fees and search diversion. Platforms may derive positive revenues from consumer

exposure to both products. Each product exposure is costly to consumers: it requires time and attention. The platforms' revenues per consumer exposure to product 1 (π_1) could be referral fees paid by an independent seller or the margin made on the sale of product 1 multiplied by the conversion rate (probability that a consumer who sees the product ends up buying it) if the platform supplies product 1 itself; or any type of fees directly tied to usage of product 1 (e.g. pay-per-view). Meanwhile, the platforms' revenues per consumer exposure to product 2 (π_2) can be interpreted as "per-impression" or "per-click" fees paid by its seller. In section 4, we also explore the effect of allowing platforms to charge consumers fixed access fees.

The key decision made by the platform is the amount of search diversion to induce through its service, which we identify with the probability that it exposes consumers to product 2 *before* directing them to product 1. Indeed, although consumers always prefer being immediately exposed to product 1, the platform may find that first diverting them to product 2 maximizes total revenues. We use the term "search" because in a sense consumers are searching for product 1 and the platform chooses how efficient to make this search process. More search diversion leads to higher total exposure costs incurred by consumers.

Our modelling set-up is best interpreted as a stylized representation of advertising-supported media, such as the ones listed in the following table:¹

Platform	Product 1	Product 2	π_1	π_2	Access fee?
Search engines (e.g. Bing, Google)	Organic search results	Sponsored search results	=0	Advertising fees (mostly CPC)	No
Shopping portals (e.g. Bing Shopping, Google Shopping, Yahoo Shopping)	Products/information user is looking for	Other products advertised on the page	>0 (referral fees paid by 3 rd party merchants of products listed)	Advertising fees	No
Content portals (e.g. Forbes, MSN, New York Times, Yahoo)	Editorial content	Display advertising	=0	Advertising fees (CPM and CPC)	Yes (NYT) No (Forbes, MSN, Yahoo)
E-commerce sites (e.g. Fandango, Zappos)	Products/information user is looking for (e.g. movie schedules; shoe descriptions)	Display advertising	>0 (booking fees paid by users buying movie tickets or margins made on shoes sold)	Advertising fees	No

Table 1

All of these platforms provide users with first-party content (cf. Hagiu and Spulber 2012), such as organic search results, information, editorial stories or products sold in their own name. All of them make positive revenues from user exposure to advertising or products users were

¹CPM is the advertising industry term for cost per impression (literally, "cost per mille", i.e. a thousand impressions), while CPC stands for "cost per click".

not necessarily looking for ($\pi_2 > 0$). Some of them (search engines, content portals) make no revenues from first-party content, while others (shopping portals, e-commerce sites) derive positive revenues from exposing consumers to first-party content. For shopping portals, π_1 is equal to the click-through rate of listed products multiplied by the referral fees charged to the third-party merchants who sell those products. For e-commerce sites, π_1 is the conversion rate multiplied by the booking fees charged to users (Fandango) or margins made on shoes sold (Zappos).

The extent of search diversion varies across these platforms from minimal (small and unintrusive ads on Fandango.com, sponsored search results at the *bottom* of Google Shopping pages) to moderate (sponsored search results at the *top and right-hand side* of Google's search engine pages) to very high (in addition to showing several large ads on every content page, Forbes.com *requires* users to view a video ad prior to watching every piece of video content and oftentimes to click through a full-page display ad before reaching the desired content page).

1.2 Related literature

Our paper builds upon the model of search diversion introduced by Hagiu and Jullien (2011). That paper established that search diversion allows platforms to: (i) trade off higher total consumer traffic for higher revenues per consumer visit; and (ii) influence independent product sellers' choices of strategic variables (e.g., pricing). It also showed that search diversion is a strategic instrument that cannot be easily replaced by contractual extensions and that it can be socially desirable because consumers do not internalize the benefits of their search activities for product sellers. We extend Hagiu and Jullien (2011)'s analysis in two important and novel directions: competition among platforms and endogenous product and consumer affiliation (Hagiu and Jullien 2011 focus exclusively on a monopoly platform with exogenously given product and consumer affiliation).

We contribute to the economics and strategy literature on two-sided platforms by introducing a key design decision (search diversion) that many platforms have to make, but has not been formally studied. Indeed, most of the existing work on two-sided platforms focuses on pricing strategies (Armstrong 2006, Parker and Van Alstyne 2005, Rochet and Tirole 2006, Weyl 2010)

and market outcomes (Caillaud and Jullien 2003, Hossain et al. 2011) in the presence of indirect network effects. Our paper is aligned with an emerging body of work aiming to expand the formal study of platforms to design decisions (e.g. Parker and Van Alstyne 2008, Boudreau 2010, Hagiu and Spulber 2012, Veiga and Weyl 2012).

At a broader level, several articles have pointed out that platforms have to make design compromises between the interests of their two sides (e.g. Kaplan and Sawhney 2000, Evans and Schmalensee 2007). To the best of our knowledge however, this issue has received limited formal modelling treatment.

Finally, our research is also connected to the literature on advertising-supported platforms: Anderson and Coate (2005), Casadesus-Masanell and Zhu (2010), Crampes et al. (2009), Gabszewicz et al. (2006), Peitz and Valletti (2008). These papers study product positioning and product quality choices by ad-sponsored platforms. In contrast, our focus is on the design of platforms' product exposure mechanism. A recent strand of this literature studies the effect of consumers' and advertisers' "multihoming" behavior (i.e. their presence on multiple online outlets) on firm profits. Athey Calvano and Gans (2012) show that the value of advertising in one outlet depends on consumer multihoming and focus on tracking technologies, while Athey and Gans (2010) study targeted advertising. In our competition sections, we analyze the impact of consumer and advertiser multihoming on the endogenous rate of exposure to advertising (search diversion in our model). Ambrus Calvano and Reisinger (2012) discuss the nature of price competition when all agents can multi-home. We share with them the conclusion that competition does not restore efficiency, but their paper focuses on the volume of advertising, whereas we focus on the design of the exposure service. Broadly, targeting, tracking and diversion can all be viewed as various instances of platform service design, thus our work is complementary to this stream of literature.

The remainder of the paper is organized as follows. Section 2 lays out the modeling set-up and analyzes the monopoly platform case, with endogenous consumer and advertising affiliation. Section 3 introduces competition between platforms and analyzes three scenarios: a) platforms compete for the exclusive affiliation of consumers, whereas the advertiser multihomes; b) platforms compete for the exclusive affiliation of the advertiser, whereas consumers multihome; c)

platforms compete for the exclusive affiliation of both consumers and the advertiser. In section 4 we introduce the possibility for platforms to charge consumers access fees. Section 5 introduces incomplete information regarding the advertiser's payoff in the context of a monopoly platform. Section 6 concludes.

2 Monopoly platform set-up

In this section we lay out the foundation for our analysis using a variant of the model in Hagiu and Jullien (2011). We present the model here but postpone its discussion until after proposition 1, where it will be more transparent. There is a monopoly platform which allows a unit mass of consumers to access two products, 1 and 2. Product 1 is *already* available through (or affiliated with) the platform, while product 2 must be attracted by the platform (its affiliation is endogenous). To access either product, a consumer must first affiliate with (i.e. visit) the platform and then be exposed to the product through a search process described below. Consumers are interested in product 1 only, which can be interpreted as content, e.g. editorial stories, videos, organic search results. They are not interested in product 2, which can be interpreted as advertising.

2.1 Consumers

All consumers derive *net* expected utility $u_1 = u$ from being exposed to product 1 and $u_2 = 0$ from being exposed to product 2, where $0 < u < 1$ is exogenously given. These utilities should be interpreted as encompassing the utility of just viewing the product plus the expected utility of actually consuming it, net of the price paid (we do not model product pricing decisions).

Ex-ante, i.e. before affiliating with the platform, consumers only differ in their location x , uniformly distributed on $[0, 1]$. The monopoly platform is located at 0. When consumer x affiliates with the platform, she incurs transportation costs tx , where $t > 0$.

Ex-post, i.e. after deciding whether or not to affiliate with the platform, consumers learn their unitary cost of exposure (or cost of search) c , which they incur whenever they are exposed

to a product. When both products are affiliated with the platform, consumers can only view them *sequentially* and therefore are subject to one or two product exposures. The search cost c can be interpreted as the cost of consumer attention; it is distributed on $[0, 1]$ according to a twice continuously differentiable cumulative distribution function F . From an *ex-ante* perspective, a consumer located at any position x perceives the same *ex-post* distribution of search costs $F(\cdot)$.

Thus, consumers make two decisions: whether or not to visit the platform, and whether or not to engage in product search if they decided to visit the platform.

2.2 The platform

For conciseness, we assume that the monopoly platform is vertically integrated into product 1. The platform derives expected profits $\pi_1 \geq 0$ for each consumer exposure to product 1, where π_1 is known by all players and exogenously fixed.

Product 2 is supplied by an independent seller (advertiser), who must be induced to affiliate. The advertiser derives profit π_2 for every consumer exposure to its product, which is also publicly known (we study the case in which π_2 is unobservable in section 5). In order to affiliate with the platform and gain access to its consumers, the advertiser must pay the platform a per-impression (per-click) fee r . We assume the platform has all the bargaining power when setting r .

Throughout the paper, only the ratio π_2/π_1 affects the level of search diversion. For this reason, we normalize our model by setting

$$\pi_1 = 1 \text{ and } \pi_2 = \pi.$$

Note that a platform with no revenue from first-party content ($\pi_1 = 0$) corresponds to $\pi = +\infty$, which can be accommodated by the analysis.

For now, we assume the platform does not charge any access fees to consumers. We study the effect of allowing access fees in section 4.

2.3 Search diversion

The platform has a design technology that allows it to choose a probability $s \in [0, 1]$ with which it first exposes *any* given consumer to product 2 before showing her product 1. The probability s represents the level of *search diversion* induced by the platform. Once a consumer has been exposed to product 2, she knows that she will next be exposed for sure to product 1, but she will then need to incur her search cost c again. The focus of our paper is on platforms' choice of s . We assume that s can be costlessly set to any value between 0 and 1.

One can think of $(1 - s)$ as a measure of how efficient the design of the platform is for consumers. Does the platform provide quick and clear access to the products or content that consumers are searching for (low s)? Or does it try to expose consumers to various forms of unsolicited content before providing the service they came for in the first place (high s)?

2.4 Timing

The timing of the game we consider in this section is as follows:

1. The platform commits to s publicly and credibly
2. The platform sets the fee r to be paid by the advertiser
3. The advertiser decides whether or not to affiliate with the platform
4. Consumers observe s and the advertiser's affiliation decision and decide whether or not to affiliate with the platform
5. Affiliated consumers learn their individual cost c , then engage in product search.

Three aspects of this set-up deserve mention. First, separating the choices of s and r has no effect on the solution of the monopoly game.

Second, the design parameter s is observed by consumers before deciding whether or not to visit the platform (for instance, through reviews or word of mouth) and by the advertiser before deciding to affiliate or not, and we assume it is not subject to *ex-post* opportunism, i.e.

cannot be adjusted once affiliation decisions have been made. In fact, it is in the platform's best interest to credibly announce s upfront because s affects not only consumer utility, but also expected payoffs for the advertiser that the platform is courting.²

Third, affiliation decisions by consumers typically involve longer time horizons than activity (search) decisions. Moreover, affiliation is usually based on less information than activity, as consumers learn about the platform gradually. This is captured by our assumption that affiliation is based on the level of search diversion and *expectation* of search costs, while activity (search) is based on the *realized* individual search cost. This assumption simplifies the analysis without loss of substance: the key feature that we need is that *total* consumer demand is decreasing in s (as is realistic).

2.5 The consumer search process and affiliation decision

In stage 4, consumers affiliated with the platform must decide whether to search or not. When the platform has chosen $s > 0$ and product 2 is affiliated, consumers know that they may be diverted. A consumer with search cost $c \leq u$ who is first diverted to product 2 will still proceed to product 1, because she knows with certainty that she will obtain net utility $u - c \geq 0$. If the consumer is not diverted, i.e. if she is directly exposed to product 1, then she stops searching immediately and will not be exposed to product 2 (which would yield negative net utility $-c$). To fix ideas, it is useful to think of an advertising-supported news website. If a user is first shown an ad, she will still click or scroll to find the news content. If she is shown the content right away, she will never go on searching for ads.

The consumer's net expected utility from searching is thus $u - (1 + s)c$ and is positive for $c \leq u/(1 + s)$. Consumers with search cost above $u/(1 + s)$ do not engage in search at all. Using the news website example, the expected utility provided by the site to such consumers is not sufficient to justify the time wasted clicking through or scrolling over ads.

Working backwards to stage 4, a consumer located at x affiliates with the platform if and

²If ex-post opportunism was possible, then the unique equilibrium with consistent expectations would be $s^* = 1$. Indeed, once consumers and the advertisers are affiliated, the platform can unambiguously increase revenues by increasing s . Thus, it is in the platform's interest to credibly commit to (or develop a reputation for) a level of search diversion $s^* < 1$ ex-ante ($s^* = 1$ is almost never optimal).

only if $V(s) - tx \geq 0$, where:

$$V(s) \equiv \int_0^{u/(1+s)} (u - (1+s)c) f(c) dc \quad (1)$$

is the expected consumer utility from the perspective of stage 4, gross of access price and transportation costs. Note that $V(s)$ is decreasing.

If only one product is affiliated with the platform then consumers who visit the platform find the affiliated product with probability 1 in just one round of search. If only product 1 is affiliated then expected consumer utility from the perspective of stage 4 is $V(0)$. If only product 2 is affiliated then expected consumer utility is 0.

2.6 Optimal search diversion

Since the advertiser's per impression profit π is common knowledge, the platform sets $r = \pi$ (slightly below) in stage 2, which ensures the advertiser affiliates and extracts its entire profit.³ The incentives to divert search are thus the same as if the platform were also vertically integrated with the product 2 seller, i.e. if it owned both products.

Total consumer demand for (or traffic to) the platform is then $\min(V(s)/t, 1)$, weakly decreasing in the level of search diversion s . The platform's profits as a function of s are

$$X(s, \pi) \min(V(s)/t, 1), \quad (2)$$

where we have denoted⁴

$$X(s, \pi) \equiv (1 + s\pi) F\left(\frac{u}{1+s}\right)$$

the revenues derived by the platform from the product exposures of each affiliated consumer.

The optimization of (2) over s involves a trade-off between *total* consumer traffic and participation in the search process on the one hand, and the average number of product exposures

³Note that the platform always (weakly) prefers to attract the advertiser. Indeed, the platform can always replicate the scenario with no advertiser affiliation by choosing $s = 0$.

⁴The argument π is included for consistency with the competition section, where the price per exposure charged to the advertiser can be lower than π .

per consumer on the other hand. Indeed, an increase in s induces a consumer to see two products with probability s , which yields revenues $1 + s\pi$ to the platform, but it also reduces the proportion of consumers who engage in search, $F(u/(1+s))$, as well as total consumer traffic to the platform, $V(s)/t$. Variations of this tradeoff are analyzed at length in Hagiu and Jullien (2011). The key novelty here is the term $V(s)/t$: indeed, Hagiu and Jullien (2011) treat total consumer affiliation with the platform as exogenously given, equal to 1.

We also denote:

$$\begin{aligned} s_{XV}(\pi) &\equiv \arg \max_s \{X(s, \pi) V(s)\} \\ s_X(\pi) &\equiv \arg \max_s \{X(s, \pi)\}, \end{aligned} \tag{3}$$

so that $s_X(\pi) > s_{XV}(\pi)$ because $V(\cdot)$ is decreasing. With this notation, we obtain:

Proposition 1 *The optimal level of search diversion for a monopoly platform is:*

$$\begin{aligned} s^M(\pi, t) &= \arg \max_s \{X(s, \pi) \min(V(s)/t, 1)\} \\ &= \begin{cases} s_X(\pi) & \text{if } t \leq V(s_X(\pi)) \\ V^{-1}(t) & \text{if } t \in [V(s_X(\pi)), V(s_{XV}(\pi))] \\ s_{XV}(\pi) & \text{if } t \geq V(s_{XV}(\pi)) \end{cases} \end{aligned} \tag{4}$$

It is (weakly) increasing in π and (weakly) decreasing in t .

The second part of the proposition (proven in the appendix) states that the platform diverts search more when it derives higher revenues from the product that consumers are *not* interested in (advertising) relative to the product that they are interested in (content). The reason is straightforward: when the platform derives more revenues from content (advertising), its interests are more (less) aligned with those of consumers, therefore the optimal level of search diversion is lower (higher). Recall indeed that consumers always prefer less search diversion. The comparative static in t is easily understood: the platform diverts search more when consumer traffic is less elastic in (i.e. less responsive to) search diversion.

This result is consistent with the examples discussed in the introduction. Restricting attention to platforms that do not charge access fees, those platforms that derive no revenues from content, i.e. with $\pi = +\infty$ (e.g. search engines, Forbes.com), clearly engage in more search diversion relative to platforms with content revenue, i.e. with π finite (e.g. Google Shopping, Fandango). Note in particular the contrast between Google search, on which sponsored search results are at the top and on the right hand side of the page, and Google Shopping, where sponsored search results only appear at the bottom of the page (much less intrusive).

Thus, although highly stylized, our model contains the two ingredients necessary to capture the key tradeoffs associated with search diversion. First, platforms' profit incentives are imperfectly aligned with consumer preferences: platforms derive positive revenues from exposing consumers to products that they do not care about ($\pi > 0$ and possibly $\pi > 1$). Second, exposure to individual products is costly for consumers and the platform can make design decisions (captured by s) that influence the degree to which consumers are exposed to one product relative to the other.

Of course, in most real-world settings there are more than two products, multiple sellers or advertisers per product and perhaps even complementarity or substitutability across products. Introducing any of these aspects would unnecessarily complicate our analysis, since the fundamental mechanics of search diversion remain unchanged. For the same reason, we treat π as exogenously given in our model, i.e. we do not endogenize price-setting by independent sellers. Some of these extensions are treated by Hagiu and Jullien (2011) in the context of a monopoly platform choosing search diversion. Finally, while the assumption $u_2 = 0$ best fits contexts in which product 2 is advertising (as in the examples listed in Table 1 above), the general implications we derive hold for any platforms that have incentives to divert consumers away from the products that best suit their preferences and towards products they are *less* - but still positively - interested in ($u_2 > 0$). For instance, Netflix uses its recommender system in an attempt to steer users towards less popular movies, which entail lower licensing costs and are less likely to run out of stock, which in turn means they generate higher margins for Netflix (Shih et al. 2007). Similarly for Amazon.com. Indeed, the difference between diverting consumers to advertising and diverting them to products that they find less desirable is simply

one of degree. Consumers may derive 0 expected utility from being exposed to advertisements, whereas they might perceive a (small) positive expected utility from being exposed to products other than the ones that they initially came to the platform for. The only thing that matters is that the platform derives positive margins from such unsolicited products. In a previous draft version, we worked with $u_2 > 0$: the analysis turned out to be more complex than the one presented here but the main results were the same. This is why we have opted to work with $u_2 = 0$.

3 Competition

In this section we analyse how competition affects search diversion incentives. We maintain the same structure of consumer preferences, except that there are now two competing platforms, A and B, one at each end of the Hotelling $[0, 1]$ segment. Each platform is vertically integrated into product 1. Although we use the same label, product 1 on platform A may differ from the product 1 on platform B. We consider three competition scenarios:

- i) Competition for consumers: the advertiser can multihome and the platforms compete solely for the exclusive affiliation of consumers
- ii) Competition for advertising: consumers can multihome (at no charge) and the platforms compete for the exclusive affiliation of a unique advertiser
- iii) Two-sided competition: the platforms compete for the exclusive affiliation of both consumers and the independent advertiser.

In all three scenarios, the timing is the same as in the monopoly case. Platforms A and B commit simultaneously to s_A and s_B respectively in stage 1 (publicly and credibly), then simultaneously set fees r_A and r_B to be paid by the advertiser in stage 2. The other stages are similar, with agents (advertiser and consumers) choosing to affiliate exclusively or not, depending on the competition scenario. When a consumer is affiliated with both platforms, her stage 5 utility from engaging in product search on one platform is independent of what she does on the other platform.

We have separated the choices of s_i and r_i between the first two stages of the game in order to better reflect reality: fees charged to sellers are typically set after committing to platform design. Our equilibrium characterization below would be the same if we worked with the entire space of (s_i, r_i) deviations. The difference is that the set of equilibrium conditions to satisfy would be significantly more complicated: this is another reason for adopting our simpler set-up.

3.1 Competition for consumers

In this scenario, the advertiser multi-homes whereas consumers single-home. In stage 2, each platform sets $r_i = \pi$ and the advertiser affiliates with both platforms.⁵

If t is not too large so that the two platforms actually compete against each other, then platform i 's profits from the perspective of stage 1 are:

$$\Pi_i = X(s_i, \pi) \left(\frac{1}{2} + \frac{V(s_i) - V(s_j)}{2t} \right) \quad (5)$$

In this case, the equilibrium level of search diversion solves:

$$s_{XV2}(\pi, t) = \arg \max_s \left\{ X(s, \pi) \left(\frac{1}{2} + \frac{V(s) - V(s_{XV2}(\pi, t))}{2t} \right) \right\} \quad (6)$$

In contrast, if t is large, then each platform acts as a local monopolist and chooses the monopoly level of search diversion. Relegating the remaining details to the appendix, we obtain:

Lemma 1 *There exists $\bar{t}_{cons}^C > V(s_{XV}(\pi))$ such that the symmetric equilibrium level of search diversion when platforms compete for consumers only is:*

$$s_{cons}^C(\pi, t) = \begin{cases} s_{XV2}(\pi, t) & \text{if } 0 \leq t \leq \bar{t}_{cons}^C \\ V^{-1}\left(\frac{t}{2}\right) & \text{if } \bar{t}_{cons}^C \leq t \leq 2V(s_{XV}(\pi)) \\ s_{XV}(\pi) & \text{if } t \geq 2V(s_{XV}(\pi)) \end{cases}$$

⁵This scenario is equivalent to assuming that each platform is vertically integrated into products 1 and 2.

It is increasing in t for $t \in [0, \bar{t}_{cons}^C]$, decreasing in t for $t \in [\bar{t}_{cons}^C, 2V(s_{XV}(\pi))]$, and everywhere (weakly) increasing in π .

On the interval $t \in [0, \bar{t}_{cons}^C]$, i.e. when platforms compete, t has the opposite effect on s_{cons}^C relative to s^M . To explain this, recall that the level of search diversion results from a trade-off between revenue per user ($1 + sr$) and total participation by consumers. The latter becomes less elastic when competition for consumers is less intense (larger t), which shifts the tradeoff towards extracting more revenues per user, i.e. towards more diversion. On the other hand, if t is above the \bar{t}_{cons}^C threshold, then platforms no longer compete against each other, therefore s_{cons}^C is decreasing in t , just like s^M . The comparative statics in π is the same as for the monopoly platform and the same interpretation applies.

We can now compare s_{cons}^C with s^M :

Proposition 2 *Relative to the level of search diversion chosen by a monopoly platform, the equilibrium level of search diversion when platforms compete for consumers only is strictly lower for low t , strictly higher for intermediate t , and equal for large t . Specifically:*

- $s_{cons}^C(\pi, t) < s^M(\pi, t)$ for $0 \leq t < V(s_{XV}(\pi))$
- $s_{cons}^C(\pi, t) > s^M(\pi, t)$ for $V(s_{XV}(\pi)) < t < 2V(s_{XV}(\pi))$
- $s_{cons}^C(\pi, t) = s^M(\pi, t)$ for $t \geq 2V(s_{XV}(\pi))$

Restricting attention to the region of interest $t \in [0, \bar{t}_{cons}^C]$ (on which platforms actually compete), our model predicts that the equilibrium level of search diversion with competing platforms is lower than the one chosen by a monopolist when competition is intense (low t) and higher when competition is not too intense (high t). Figure 1 represents s_{cons}^C and s^M as functions of t .

To see why competing platforms may end up diverting search more than a monopolist, consider the equilibrium elasticity of consumers' affiliation with respect to search diversion.

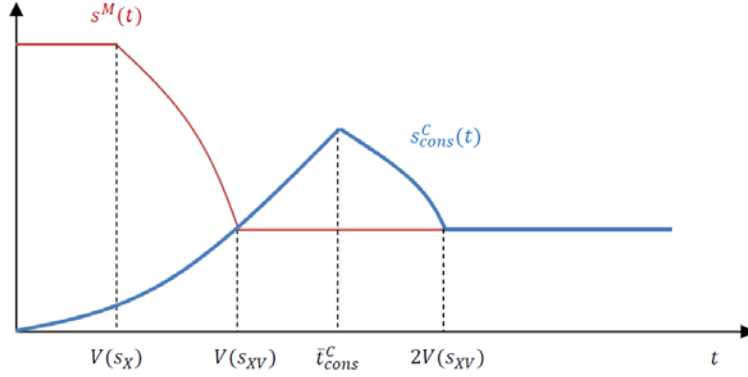


Figure 1:

This elasticity is $-sV'(s)/V(s)$ in the case of a monopoly that does not cover the entire consumer market (i.e. $t > V(s)$), whereas it is equal to $-sV'(s)/t$ for a duopoly with equal market shares. Thus, if $t > V(s)$ then the former elasticity is larger, which leads to less search diversion under monopoly. On the other hand, the opposite holds if the monopolist covers the entire consumer market ($t < V(s_{XV}(\pi))$): the monopolist's choice of search diversion ignores the impact on consumer affiliation decisions, which competing platforms can never ignore.

3.2 Competition for unsolicited content

Consider now the opposite scenario relative to the previous subsection: consumers can costlessly multihome and the platforms compete for the exclusive affiliation of an advertiser. Specifically, we assume that the platforms offer non-substitutable versions of product 1 (content), so that a consumer who affiliates with both intermediaries derives utility $2u$ from consuming the two versions of product 1 (gross of search and transportation costs). We assume the advertiser affiliates exclusively with one platform.⁶ This scenario occurs in mature markets, where consumers have long-standing and stable affiliation decisions, but platforms must attract new advertisers.

If the advertiser affiliates with platform A, then platform A profits are $X(s_A, r_A) \min(V(s_A)/t, 1)$,

⁶Exclusive affiliation by the advertiser could be obtained endogenously by assuming that, for each consumer, only the first product exposure matters and that there is a sufficiently large probability that the same consumers are exposed to advertising on both platforms (see Athey Calvano and Gans 2012).

whereas platform B profits are $F(u) \min(V(0)/t, 1)$. The advertiser's payoff is

$$(\pi - r_A) s_A F\left(\frac{u}{1 + s_A}\right) \min\left(\frac{V(s_A)}{t}, 1\right) = (X(s_A, \pi) - X(s_A, r_A)) \min\left(\frac{V(s_A)}{t}, 1\right).$$

Relegating the remaining details to the appendix, we obtain:

Proposition 3 *When consumers multihome and platforms compete for the exclusive affiliation of an advertiser, the equilibrium level of search diversion is the same as that chosen by a monopoly platform: $s_{adv}^C(\pi, t) = s^M(\pi, t)$.*

Thus, competition for product 2 (advertising) does not affect search diversion relative to a monopolist. This result is driven by Bertrand competition for the advertiser. Each platform i sets its search diversion level s_i to maximize its joint profits with the advertiser when the latter affiliates with i exclusively, i.e. $X(s_i, \pi) \min(V(s_i)/t, 1)$. Then both platforms compete in fees r_i so that the advertiser ends up capturing all of the joint profits in excess of each platform's outside option, $F(u) \min(V(0)/t, 1)$, which does not depend on search diversion levels. Consequently, although competition shifts the repartition of the joint vertical profit in favor of the advertiser, it is still optimal for platforms to choose the design that maximizes this joint profit. The difference is that a monopoly platform maximizes the value that can be extracted from the advertiser, whereas competing platforms seek to maximize the chance to attract the advertiser.

3.3 Competition for both consumers and advertising

Let us now turn to the case in which the two platforms compete for exclusive affiliation on both sides of the market. The expressions of platforms' profits and of the advertiser's payoff when the latter affiliates exclusively with platform A are:

$$\Pi_A = X(s_A, r_A) \left(\frac{1}{2} + \frac{V(s_A) - V(0)}{2t}\right) \quad \text{and} \quad \Pi_B = F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_A)}{2t}\right)$$

$$\Pi^{adv}(A) = (X(s_A, \pi) - X(s_A, r_A)) \left(\frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right)$$

The key difference with the previous scenarios is that here it is no longer clear whether both platforms wish to compete for the advertiser. This is because the "losing" platform B obtains higher consumer demand, which may compensate for its lower revenues per consumer (no advertising). Thus, given the levels of search diversion set in stage 1, it is possible that B obtains a larger profit without the advertiser than the maximum profit it could expect to achieve if it were to attract the advertiser. When this is the case, platform B prefers not to make an offer to the advertiser in stage 2 and platform A is a de facto monopoly on the advertiser side of the market. Consequently, according to the same logic as before, platform A's optimal choice of search diversion maximizes its joint profits with the advertiser. Let then:

$$s_{VI}^C(\pi, t) \equiv \arg \max_{s \in [0,1]} \left\{ X(s, \pi) \left(\frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) \right\} \quad (7)$$

be the level of search diversion that maximizes the joint profit of the advertiser and the platform it affiliates with, when facing a platform that offers product 1 (content) only.

Of course, the alternative scenario is that the two platforms actually compete to attract the advertiser. For this case, we define

$$s_T^C(\pi, t) \equiv \arg \max_{s \in [0,1]} \left\{ X(s, \pi) \left(\frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s)}{2t} \right) \right\} \quad (8)$$

as the level of search diversion that maximizes total industry profit, i.e. the joint profits of the advertiser and both platforms. Since $V(s)$ is decreasing, we have:

$$s_{VI}^C(\pi, t) < s_T^C(\pi, t)$$

Whether or not the two platforms effectively compete for the advertiser depends on $z^C(\pi, t) \in [0, 1]$, which solves:

$$X(s_{VI}^C(\pi, t), \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C(\pi, t)) - V(0)}{2t} \right) = F(u) \left(\frac{1}{2} + \frac{V(0) - V(z^C(\pi, t))}{2t} \right),$$

with the convention $z^C(\pi, t) \equiv +\infty$ if:

$$X(s_{VI}^C(\pi, t), \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C(\pi, t)) - V(0)}{2t} \right) > F(u) \left(\frac{1}{2} + \frac{V(0) - V(1)}{2t} \right)$$

When $z^C \in [0, 1]$, it represents the level of search diversion that platform A (who obtains the advertiser) would have to choose in order to render platform B indifferent between extracting maximum joint profits with the advertiser and ceding the advertiser to platform A. If $z^C = +\infty$ then platform B always prefers competing for the advertiser (no matter the level of search diversion chosen by A). Furthermore, if $z^C \in [0, 1]$ then, by choosing $s_A > z^C$, platform A can secure the affiliation of the advertiser (i.e. platform B prefers not to compete for it).

Focusing on the range of t such that the two platforms are actually competing⁷ and relegating the proof to the appendix, we obtain:

Lemma 2 *There exists $\bar{t}^C > V(0)$ such that, for all $t \in [0, \bar{t}^C]$, the maximum level of search diversion that can be sustained in equilibrium is:*

$$s^C(\pi, t) = \begin{cases} s_{VI}^C(\pi, t) & \text{if } z^C(\pi, t) \leq s_{VI}^C(\pi, t) \\ z^C(\pi, t) & \text{if } s_{VI}^C(\pi, t) \leq z^C(\pi, t) \leq s_T^C(\pi, t) \\ s_T^C(\pi, t) & \text{if } z^C(\pi, t) \geq s_T^C(\pi, t) \end{cases}$$

The maximum level of search diversion $s^C(\pi, t)$ is continuous in its 2 arguments, (weakly) increasing in t for $t \in [0, \bar{t}^C]$ and (weakly) increasing in π .

Note that the equilibrium is not unique when $z^C(\pi, t) \geq s_{VI}^C(\pi, t)$ (in that case, there is a continuum of equilibria), which is why we have chosen to focus on the equilibrium that involves the highest level of search diversion. Fundamentally, equilibrium multiplicity stems from the fact that search diversion has opposite effects on the two sides of the market. It arises when the platform that wins the advertiser is in a situation where raising s would strongly depress

⁷Once again, if t is large enough, then the two platforms behave as local monopolists. Platform A then chooses $s_A = s_{XV} = s^M$ for this range of t .

consumer demand, whereas reducing s would lead the advertiser to jump ship to the other platform.

According to the most basic intuition, the equilibrium level of search diversion ought to maximize joint profits of the "winning" platform and the advertiser, i.e. $s^C = s_{VI}^C(\pi, t)$. Perhaps surprisingly, this is not always the case here. It is important to understand how the other two outcomes may arise.

First, when $z^C \geq s_T^C$, the equilibrium level of search diversion s_T^C maximizes joint profits of the advertiser, the winning platform (say, A) and the losing platform (B). The fact that platform B's profit is taken into account (note that it depends on the search diversion level chosen by A) might seem odd at first glance. The interpretation is as follows: A must offer the advertiser a payoff just above the largest payoff that can be offered by B, who in turn can only offer the advertiser the difference between joint profits and B's outside option, i.e. what B gets when the advertiser affiliates with A. This term is *increasing* in the search diversion level chosen by A: a higher s_A increases B's outside option (more consumers go to B instead of A) and thereby decreases the hurdle that A needs to overcome in order to attract the advertiser. In other words, when it expects platform B to compete for the advertiser, the winning platform A maximizes total industry profit because it internalizes the fact that yielding more consumer demand to platform B (through more search diversion) reduces the cost of attracting the advertiser, by reducing platform B's willingness to compete.

On the contrary, when $z^C \leq s_{VI}^C$, the winning platform can, with the same level of search diversion, obtain maximum joint profits and secure the affiliation of the advertiser without having to give up additional consumer demand. The equilibrium is then the same as if one platform were vertically integrated with product 2.

The intermediate range $s_{VI}^C \leq z^C \leq s_T^C$ arises because the total industry profit is maximized at a higher level of search diversion (s_T^C) than the vertically integrated profit (s_{VI}^C). In this range, when neither s_T^C nor s_{VI}^C are sustainable, the winning platform is cornered in an equilibrium where the losing platform is just indifferent between competing and not competing for the advertiser (as discussed above).

Finally, the comparative statics in (t, π) are the same as in the case of competition for

consumers and interpreted in the same way. Comparing with the level of search diversion chosen by a monopolist, we obtain (the proof is in the appendix):

Proposition 4 *Relative to the level of search diversion chosen by a monopoly platform, the maximum equilibrium level of search diversion when platforms compete for both consumers and the advertiser is strictly lower for low t and strictly higher for large t . Specifically, there exists $t^C \in [0, V(0)]$ such that:*

- $s^C(\pi, t) \leq s^M(\pi, t)$ for $0 \leq t \leq t^C$
- $s^C(\pi, t) > s^M(\pi, t)$ for $t^C < t \leq \bar{t}^C$

This result confirms the one obtained under competition for consumers only (despite a significantly more complex analysis): once again, the equilibrium level of search diversion with competing platforms is lower than the one chosen by a monopolist when competition is intense (low t) and higher when competition is not too intense (high t). The explanation is the same.

4 Access fees

In this section we introduce the possibility that platforms can charge access fees to consumers, denoted by P and paid before search costs c are observed. A priori, P can be positive or negative. A negative access fee can be interpreted as a monetary subsidy (e.g. cash or redeemable points) or first-party content (beyond product 1) that consumers value at more than the price being charged.

The timing we use throughout this section (monopoly as well as competing platforms) is:

1. Platforms choose s_A and s_B simultaneously
2. Platforms choose r_A and r_B simultaneously
3. The advertiser decides which platform(s) to affiliate with

4. Platforms choose consumer access fees P_A and P_B simultaneously
5. Consumers decide which platform(s) to affiliate with
6. Affiliated consumers learn c and engage in product search.

Our timing ensures that decisions regarding consumer access fees do not interfere with decisions affecting the quality of the services offered to consumers (search diversion). The value expected by consumers from each platform results from the combination of search diversion and the advertiser's affiliation decisions, and it is known at the time price competition for consumers occurs.⁸ As in the previous sections, we have separated the choices of s_i and r_i between the first two stages of the game.

4.1 Monopoly

The monopoly platform's profits are now:

$$\max_{P,s} \left\{ (P + X(s, \pi)) \min \left(\frac{V(s) - P}{t}, 1 \right) \right\}. \quad (9)$$

It is straightforward to obtain (details are in the appendix):

Proposition 5 *The optimal level of search diversion for a monopoly platform that can charge access fees is:*

$$\widehat{s}^M(\pi) = s_{X+V}(\pi) \equiv \arg \max_s \{X(s, \pi) + V(s)\} \quad (10)$$

Allowing the monopoly platform to charge access fees results in less search diversion ($\widehat{s}^M \leq s^M$) if and only if the profit-maximizing access fee is non-negative ($\widehat{P}^M \geq 0$)

The first part of the proposition says that when the platform can monetize consumer participation, it chooses search diversion to maximize the joint surplus (platform + consumers)

⁸This allows us to avoid coordination issues in affiliation decisions (see Caillaud and Jullien 2003), which are not the focus of this paper.

from participation. Also, note that just like $s^M(\pi)$, the level of search diversion with access fees $\widehat{s}^M(\pi)$ is weakly increasing in π .

The second part of the proposition says that the ability to monetize consumer participation reduces search diversion incentives. This result is intuitive: if the platform wishes to charge consumers a positive access fee, it must increase their willingness-to-pay, which means it needs to *reduce* search diversion. Conversely, a platform wishes to subsidize consumers when this sacrifice allows it to mitigate the effect of increasing search diversion on consumers' participation.

This result is consistent with some of the examples discussed in the introduction. Compare the websites of Forbes and The New York Times. Both rely on advertising, but The New York Times charges users a subscription fee, whereas access to Forbes is entirely free for users. Advertising on the New York Times web page is moderate; in contrast, advertising is highly intrusive on Forbes.com.

4.2 Competition

Consider the first competition scenario, in which the advertiser multihomes and consumers singlehome. In stage 2 the platforms set $r_A = r_B = \pi$, thus fully extracting the advertiser's surplus. As a result, stage 4 profits for platform i are:

$$\Pi_i = (P_i + X(s_i, \pi)) \left(\frac{1}{2} + \frac{V(s_i) - P_i - V(s_j) + P_j}{2t} \right)$$

Given (s_i, s_j) chosen in stage 1, the stage 2 pricing equilibrium is easily verified to be $P_i = t + (V(s_i) - V(s_j) - X(s_j, \pi) - 2X(s_i, \pi)) / 3$, leading to stage 1 platform profits:

$$\Pi_i(s_i, s_j) = \frac{t}{2} \left(1 + \frac{V(s_i) + X(s_i, \pi) - V(s_j) - X(s_j, \pi)}{3t} \right)^2 \quad (11)$$

Going backwards to stage 1, the symmetric equilibrium level of search diversion is:

$$\widehat{s}_{cons}^C(\pi) = \arg \max_s \{X(s, \pi) + V(s)\} = \widehat{s}^M(\pi). \quad (12)$$

In other words, we obtain the same level of search diversion as the one chosen by a monopoly platform. The following proposition (proven in the appendix) confirms that this is also true under the other two competition scenarios (their analysis is more complicated):

Proposition 6 *When platforms can charge access fees, the equilibrium levels of search diversion under all three competition scenarios are the same and equal to the level chosen by a platform monopolist:*

$$\widehat{s}_{cons}^C(\pi) = \widehat{s}_{adv}^C(\pi) = \widehat{s}^C(\pi) = s_{X+V}(\pi) = \widehat{s}^M(\pi)$$

Furthermore, in the two competition scenarios in which one side multihomes, the equilibrium level of search diversion with access fees is lower relative to the case with no access fees if and only if the equilibrium access fee is positive.

Thus, when access fees are feasible, the equilibrium level of search diversion is identical to the one chosen by a monopoly platform. In this case, platforms maximize the joint surplus of the relationship with consumers and use the access fee to share this surplus with consumers. Competition only affects the level of the access fee.

In all three competition cases, the central part of the proof is showing that in the equilibrium of the game starting at stage 2 the advertiser affiliates with the platform i that creates the highest joint surplus $X(s_i, \pi) + V(s_i)$. This was straightforward for the scenario with competition for consumers only, but turns out to be more complicated for the other two scenarios (cf. appendix).

The fundamental reason we obtain the equilibrium level of search diversion that maximizes $X(s, \pi) + V(s)$ for all monopoly and competition cases is that the access fee allows platforms to transfer surplus so all that matters beyond stage 1 of the game is the total surplus per consumer. This is clear in the scenario when the advertiser multihomes, because each platform extracts the entire profit from the vertical structure (platform plus advertiser) and this profit is increasing in the total surplus per consumer. The two scenarios with advertiser singlehoming

are more complex because competition for the advertiser reduces the profit that platforms can extract per consumer. Still, we show that the total profit that can be promised by each platform to the advertiser remains increasing in the total surplus per consumer. Thus, in equilibrium, competition for the advertiser still leads the platform winning the advertiser to maximize the total surplus per consumer. Indeed, the platform generating the highest total surplus is able to offer better terms to the advertiser, holding constant the profit on the consumer side. This logic applies both when consumers multihome and when they singlehome.

For the second part of the proposition, the result and interpretation is the same as in the monopoly case above. If platforms charge positive access fees to consumers, they must offer them more value, i.e. less diversion. Conversely, if platforms subsidize the participation of consumers, they need to make up for the loss by increasing advertising revenues, which they can achieve by increasing diversion.

5 Incomplete information and vertical integration (monopoly)

Another interesting extension of our baseline model is to introduce incomplete information regarding the advertiser's profit π (up to now, we have assumed all players observe π). Accordingly, in this section we revert to the monopoly platform scenario and we assume that π is unobservable to the platform. In particular, we assume that π is drawn from a cumulative distribution function $G(\cdot)$, which is common knowledge. The timing is exactly as in the previous section: we cover the possibility that the platform may charge an access fee P .⁹

⁹Here, allowing the platform to choose P *after* the advertiser's affiliation decision creates a form of *ex-post* opportunism by the platform. Once the advertiser decides to affiliate in stage 3, the platform sets P too high from a joint perspective in stage 4, because it does not fully internalize the advertiser's profits. This hold-up issue arises more generally in two-sided settings whenever one side makes participation decisions before the other and the platform cannot commit to the price it will charge to the second side *ex-ante*. The implications of commitment in such settings are analyzed in depth in Hagiu and Spulber (2012). The timing we have chosen to work with is arguably the most realistic: the advertiser's participation determines to a large extent the platform's business model (in the sense of Casadesus-Masanell and Zhu 2010). It seems unrealistic for the platform to commit to a specific consumer access fee *before* knowing whether it will derive advertising revenues. Furthermore, none of our generalizable results below would change if we allowed the platform to commit to P prior to the advertiser's affiliation decision. In fact, results would be identical if we allowed the platform to commit to two different P 's, contingent on whether the advertiser affiliates or not.

The advertiser affiliates with the platform in stage 3 if and only if $\pi \geq r$. When the platform does not charge any access fees, its expected profits from the perspective of stage 1 are:

$$\max_{s,r} \left\{ (1 - G(r)) \cdot X(s, r) \min\left(\frac{V(s)}{t}, 1\right) + G(r) \cdot F(u) \min\left(\frac{V(0)}{t}, 1\right) \right\} \quad (13)$$

When the platform can charge access fees, its stage 1 expected profits are:

$$\max_{s,r} \left\{ \begin{array}{l} (1 - G(r)) \max_P \left\{ (P + X(s, r)) \min\left(\frac{V(s)-P}{t}, 1\right) \right\} \\ + G(r) \max_P \left\{ (P + F(u)) \min\left(\frac{V(0)-P}{t}, 1\right) \right\} \end{array} \right\} \quad (14)$$

In both cases, with probability $G(r)$ the platform only has product 1 to offer, hence there is no search diversion and the platform's revenues come from consumer exposure to product 1 and access fees. With probability $(1 - G(r))$ the independent advertiser affiliates and the platform derives higher revenues, which come from consumer exposures to both products and access fees.

For the sake of concision, we omit the first-order conditions of (13) and (14) with respect to the advertiser fee r . In both cases, the optimal fee (r^M and \hat{r}^M respectively) is set such that the marginal net gain from a small reduction in r (higher probability of advertiser affiliation multiplied by the revenue differential) is equal to the marginal net loss (lower advertising revenues whenever the advertiser affiliates). We focus directly on the optimal levels of search diversion, s_{end}^M and \hat{s}_{end}^M respectively, which are given by:

$$\begin{aligned} s_{end}^M &= \arg \max_s \left\{ X(s, r^M) \min\left(\frac{V(s)}{t}, 1\right) \right\} = s^M(r^M, t) \\ \hat{s}_{end}^M &= \arg \max_s \left\{ X(s, \hat{r}^M) + V(s) \right\} = \hat{s}^M(\hat{r}^M) \end{aligned}$$

Thus, in both cases, the optimal level of search diversion under incomplete information is equal to the monopoly level under exogenous affiliation and complete information (expression (4) without access fees and (10) with access fees), when profit per exposure to advertising is equal to the profit of the *marginal advertiser*, i.e. $\pi = r^M$ or $\pi = \hat{r}^M$ respectively. Of course, the marginal advertisers are different in the two cases, but the general principle is the same.

It is interesting to compare this outcome to the corresponding benchmark with exogenous

affiliation (or vertical integration) under incomplete information. Specifically, assume product 2 is exogenously affiliated with the platform (stages 2 and 3 are eliminated), but we maintain the incomplete information setting by assuming the platform does not know π prior to setting s and P (when access fees are feasible). Alternatively, one could assume that, when access fees are feasible, the platform observes π prior to setting P : the analysis that follows would then produce very similar results.

When access fees are not feasible, the platform's expected profits in stage 1 are:

$$\max_s \left\{ \int_{\pi} X(s, \pi) \min\left(\frac{V(s)}{t}, 1\right) dG(\pi) \right\} = \max_s \left\{ X(s, E\pi) \min\left(\frac{V(s)}{t}, 1\right) \right\},$$

where $E\pi \equiv \int_{\pi} \pi dG(\pi)$ is the expected value of π and the equality above follows from the linearity of $X(s, \pi)$ in π .

When access fees are feasible, the platform's expected profits in stage 1 are:

$$\begin{aligned} & \max_{s,P} \left\{ \int_{\pi} (P + X(s, \pi)) \min\left(\frac{V(s) - P}{t}, 1\right) dG(\pi) \right\} \\ &= \max_{s,P} \left\{ (P + X(s, E\pi)) \min\left(\frac{V(s) - P}{t}, 1\right) \right\} \end{aligned}$$

The optimal level of search diversion is therefore $s^M(E\pi, t)$ without access fees and $\widehat{s}^M(E\pi)$ with access fees (recall expressions (4) and (10) above). In other words, the optimal level of search diversion in the benchmark exogenous affiliation case is determined by the *average* advertiser $E\pi$, both with and without access fees.

Combining the previous results, we have:

Proposition 7 *With incomplete information about the advertiser's payoff, the platform's optimal level of search diversion is determined by the marginal advertiser who affiliates (corresponding to $\pi = r^M$ or $\pi = \widehat{r}^M$). Endogenous advertiser affiliation results in more search diversion relative to the exogenous affiliation benchmark if and only if the marginal advertiser under endogenous affiliation derives higher profits per consumer exposure relative to the average*

advertiser, i.e. if and only if $r^M > E\pi$ (respectively, $\hat{r}^M > E\pi$) without (respectively, with) access fees.

This result is reminiscent of Spence (1975)'s insight that a monopolist's incentives to invest in product quality are driven by the marginal customer. In our model, things are more complicated because there are two sides (or two customer groups): consumers and the advertiser. Since consumers all have the same marginal disutility from search diversion, the relevant "customer" for determining search diversion (the inverse of "quality") is the advertiser.

An important implication of this result is that endogenous product affiliation and incomplete information create a novel potential source of search diversion incentives, relative to the two basic sources of incentives identified in Hagiu and Jullien (2011). Specifically, endogenous affiliation of the advertiser leads to more search diversion relative to exogenous affiliation if and only if the marginal advertiser derives higher profits relative to the average advertiser. This is because consumers are *not* interested in advertising, therefore higher advertising revenues make the platform's profit incentives run counter to consumer preferences.

If we endogenized the affiliation of product 1 (content) instead of that of product 2 (advertising), we would obtain a very similar result, except that the inequalities in Proposition 7 would be reversed. In other words, endogenous affiliation of product 1 would lead to *more* search diversion if and only if the marginal seller of product 1 derived *lower* profits per consumer exposure than the average seller of product 1.

6 Conclusion

Our study of search diversion by competing platforms has yielded several important and novel insights (relative to Hagiu and Jullien 2011). First and most importantly, the effect of competition between platforms on the equilibrium level of search diversion is determined by the nature of competition. When horizontal differentiation between competing platforms in the eyes of consumers is reduced, the equilibrium level of search diversion decreases (i.e. search quality increases), as expected. On the other hand, we have shown that entry of a competitor

may lead to more or at least as much search diversion as under monopoly. Specifically, when consumers singlehome, entry of a competitor leads to more (less) search diversion relative to monopoly when the degree of horizontal differentiation is intermediate (low). When the degree of horizontal differentiation is large, "competing" platforms behave as local monopolies and therefore divert search to the exact same extent as a monopolist would. An important result is that effective competition for exclusive affiliation on both sides of the market sometimes leads to a level of search diversion that maximizes total industry profit.

Second, allowing platforms to charge unrestricted access fees to consumers leads to the striking result that competing platforms choose the exact same level of search diversion as a monopoly platform, irrespective of the nature of competition and of the degree of horizontal differentiation. Furthermore, under monopoly and competition with at least one multihoming side, platforms that charge positive access fees to consumers have weaker incentives to divert search relative to platforms that cannot (or choose not to) charge such fees. On the other hand, platforms that subsidize consumer participation have stronger incentives to divert search.

Third and finally, we have shown that endogenous affiliation of both sides (consumers and advertisers or content sellers) combined with incomplete information regarding the advertisers' (or content sellers') payoff per consumer can create an additional reason to divert search. Specifically, a monopoly platform's incentives to divert consumer search are stronger relative to the exogenous affiliation (or vertical integration) benchmark if the marginal independent advertiser (respectively, content seller) derives higher (respectively, lower) profits per consumer exposure than the average advertiser (respectively, content seller). This result suggests that the extent of vertical integration should be an important predictor of the magnitude of search diversion.

A broader implication of our analysis is that design decisions by two-sided platforms (search diversion being but one specific example) must balance the potentially conflicting interests of the two sides of the market. One should then not be surprised that the entry of a competitor may lead to design decisions which go against consumers' preferences, but instead favor third-party sellers, advertisers, or whatever the relevant second side might be. A recent example was the privacy compromise made by Microsoft with the design of its Internet Explorer 8 web browser

in favor of advertisers and against users' interests. The episode was reported by the Wall Street Journal: "*As the leading maker of Web browsers, the gateway software to the Internet, Microsoft must balance conflicting interests: helping people surf the Web with its browser to keep their mouse clicks private, and helping advertisers who want to see those clicks. In the end, the product planners lost a key part of the debate. The winners: executives who argued that giving automatic privacy to consumers would make it tougher for Microsoft to profit from selling online ads. Microsoft built its browser so that users must deliberately turn on privacy settings every time they start up the software.*" (Wingfield (2010))

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7 Appendix

In order to reduce clutter, throughout the appendix we will drop the (π, t) arguments of the various levels of s , unless they are needed for clarity.

7.1 Proof of Proposition 1

If $t \in [V(s_X), V(s_{XV})]$ then $s^M(\pi, t) = V^{-1}(t)$, decreasing in t and constant in π .

If $t \leq V(s_X)$ then $s^M(\pi, t) = \arg \max_s \{X(s, \pi)\}$, so the F.O.C. determining s^M can be written:

$$F\left(\frac{u}{1+s}\right) - \left(\frac{1}{\pi} + s\right) \frac{u}{(1+s)^2} F'\left(\frac{u}{1+s}\right) = 0$$

The left-hand side is increasing in π , which implies (assuming second-order conditions are satisfied) that $s^M(\pi, t)$ is increasing in π . It is obviously constant in t .

If $t \geq V(s_{XV})$ then $s^M(\pi, t) = \arg \max_s \{X(s, \pi) V(s)\}$, so the F.O.C. determining s^M is:

$$\left(\frac{1}{\pi} + s\right) \cdot \left[\frac{-u}{(1+s)^2} F'\left(\frac{u}{1+s}\right) V(s) + F\left(\frac{u}{1+s}\right) V'(s) \right] + F\left(\frac{u}{1+s}\right) V(s) = 0$$

Since $V'(s) < 0$, the left-hand side is increasing in π , which implies (assuming second-order conditions are satisfied) that $s^M(\pi, t)$ is increasing in π and constant in t .

7.2 Proof of Lemma 1

Given s_j , platform i 's profits are:

$$\Pi_i(s_i) = \begin{cases} X(s_i, \pi) \left[\frac{1}{2} + \frac{V(s_i) - V(s_j)}{2t} \right] & \text{if } V(s_i) + V(s_j) \geq t \\ X(s_i, \pi) \frac{V(s_i)}{t} & \text{if } V(s_i) + V(s_j) \leq t \end{cases}$$

There are therefore 3 possible equilibria:

1) $s_{cons}^C = s_{XV2}$, where s_{XV2} solves equation (6), so that s_{XV2} is determined by the following F.O.C.:

$$X(s, \pi) V'(s) + t X_s(s, \pi) = 0 \quad (15)$$

From this, it is easily seen that s_{XV2} is increasing in t . It is also increasing in π by a very similar argument to that employed in the proof of proposition 1. This is an equilibrium if and only if $V(s_{XV2}) \geq t/2$ (the consumer in the middle of the Hotelling segment obtains non-negative utility), i.e. only if $t \leq \bar{t}_{cons}^C$, where \bar{t}_{cons}^C is uniquely defined by:

$$V\left(s_{XV2}\left(\pi, \bar{t}_{cons}^C\right)\right) = \bar{t}_{cons}^C/2$$

Furthermore, comparing (15) with (3) we have:

$$s_{XV2}(\pi, V(s_{XV})) = s_{XV}, \quad (16)$$

so that $V(s_{XV2}(\pi, V(s_{XV}))) > V(s_{XV})/2$, which implies $\bar{t}_{cons}^C > V(s_{XV})$.

2) $s_{cons}^C = s_{XV}$. This is an equilibrium if and only if $V(s_{XV}) \leq t/2$, i.e. if and only if $t \geq 2V(s_{XV})$. Let us show that $2V(s_{XV}) > \bar{t}_{cons}^C$. The first-order conditions that determine $s_{XV2}(\pi, \bar{t}_{cons}^C)$ and $s_{XV}(\pi)$ are, respectively:

$$\begin{aligned} X(s, \pi) V'(s) + 2V(s) X_s(s, \pi) &= 0 \text{ for } s_{XV2}\left(\pi, \bar{t}_{cons}^C\right) \\ X(s, \pi) V'(s) + V(s) X_s(s, \pi) &= 0 \text{ for } s_{XV}(\pi) \end{aligned}$$

Comparing the two, it is clear that $s_{XV2}(\pi, \bar{t}_{cons}^C) > s_{XV}(\pi)$, which implies:

$$\bar{t}_{cons}^C = 2V\left(s_{XV2}(\pi, \bar{t}_{cons}^C)\right) < 2V(s_{XV})$$

3) $s_{cons}^C = V^{-1}(t/2)$. Suppose $s_2 = V^{-1}(t/2)$. If $s_1 \geq V^{-1}(t/2)$ then platform 1's profits are $X(s_1, \pi)V(s_1)/t$. Thus, for $s_1 = V^{-1}(t/2)$ to be a best response to $s_2 = V^{-1}(t/2)$, it must be that $s_{XV} \leq V^{-1}(t/2)$, i.e. if $t \leq 2V(s_{XV})$.

If $s_1 \leq V^{-1}(t/2)$ then platform 1's profits are:

$$X(s_1, \pi) \left(\frac{1}{2} + \frac{V(s_1) - t/2}{2t} \right)$$

The maximizer s^* of this profit expression is defined by the first-order condition:

$$X(s^*, \pi)V'(s^*) + (t/2 + V(s^*))X_s(s^*, \pi) = 0$$

Thus, for $s_1 = V^{-1}(t/2)$ to be a best response to $s_2 = V^{-1}(t/2)$, it must be that $s^* \geq V^{-1}(t/2)$, i.e. $V(s^*) \leq t/2$, which implies:

$$X(s^*, \pi)V'(s^*) + 2V(s^*)X_s(s^*, \pi) \leq 0$$

Comparing this with the first-order condition determining $s_{XV2}(\pi, \bar{t}_{cons}^C)$,

$$X(s, \pi)V'(s) + 2V(s)X_s(s, \pi) = 0,$$

we have $s^* \leq s_{XV2}(\pi, \bar{t}_{cons}^C)$, which is equivalent to $V(s^*) \geq V(s_{XV2}(\pi, \bar{t}_{cons}^C)) = \bar{t}_{cons}^C/2$. Consequently, we must have $t \geq \bar{t}_{cons}^C$.

Thus, $s_{cons}^C = V^{-1}(t/2)$ is an equilibrium if and only if $t \in [\bar{t}_{cons}^C, 2V(s_{XV})]$. Note that $V^{-1}(t/2)$ is decreasing in t .

7.3 Proof of Proposition 2

On the interval $t \in [0, \bar{t}_{cons}^C]$, we know from proposition 1 and lemma 1 that s^M is decreasing in t , whereas $s_{cons}^C = s_{XV2}$ is increasing in t . Furthermore, from (16), we have $s_{XV2} = s_{XV} = s^M$ at $t = V(s_{XV})$. We can conclude that $s_{cons}^C < s^M$ for $t < V(s_{XV})$ and $s_{cons}^C > s^M$ for $V(s_{XV}) < t <$

\bar{t}_{cons}^C .

7.4 Proof of Proposition 3

Consider stage 3. The advertiser's payoff from affiliating exclusively with platform i is:

$$(\pi - r_i) s_i F\left(\frac{u}{1 + s_i}\right) \min\left(\frac{V(s_i)}{t}, 1\right) = (X(s_i, \pi) - X(s_i, r_i)) \min\left(\frac{V(s_i)}{t}, 1\right),$$

while platform i 's payoff is $X(s_i, r_i) \min(V(s_i)/t, 1)$. In stage 2, platform i is prepared to lower its fee r_i until its payoff is equal to its outside option, $F(u) \min(V(0)/t, 1)$. Consequently, in the equilibrium of the game starting at stage 2, the advertiser affiliates with the platform that has the highest $X(s_i, r_i) \min(V(s_i)/t, 1)$. The fees in the stage 2 equilibrium are determined by:

$$(X(s_i, \pi) - X(s_i, r_i)) \min\left(\frac{V(s_i)}{t}, 1\right) = X(s_j, \pi) \min\left(\frac{V(s_j)}{t}, 1\right) - F(u) \min\left(\frac{V(0)}{t}, 1\right)$$

for the "winning" platform i and:

$$X(s_j, r_j) \min\left(\frac{V(s_j)}{t}, 1\right) = F(u) \min\left(\frac{V(0)}{t}, 1\right)$$

for the "losing" platform j .

Consider now stage 1. If $X(s_i, \pi) \min(V(s_i)/t, 1) > X(s_j, \pi) \min(V(s_j)/t, 1)$ then platform i attracts the advertiser with probability 1 and obtains profits

$$F(u) \min\left(\frac{V(0)}{t}, 1\right) + X(s_i, \pi) \min\left(\frac{V(s_i)}{t}, 1\right) - X(s_j, \pi) \min\left(\frac{V(s_j)}{t}, 1\right).$$

This is an equilibrium if and only if $s_i = \arg \max_s \{X(s, \pi) \min(V(s)/t, 1)\}$. If $X(s_i, \pi) \min(V(s_i)/t, 1) = X(s_j, \pi) \min(V(s_j)/t, 1)$ then the advertiser is indifferent between affiliating with either platform and both platforms' profits are equal to $F(u) \min(V(0)/t, 1)$. This is an equilibrium if and only if $s_i = s_j = \arg \max_s \{X(s, \pi) \min(V(s)/t, 1)\}$. Thus, we have proven that the equilibrium level of search diversion in all cases is $\arg \max_s \{X(s, \pi) \min(V(s)/t, 1)\} = s^M(\pi, t)$.

7.5 Proof of Lemma 2

We assume that $X(s, \pi) \left(\frac{1}{2} + \frac{V(s) - V(0)}{2t}\right)$ is quasi-concave and its maximizer s_{VI}^C is interior to $[0, 1]$.

For any s , denote then by $z(s) \in [0, 1]$ the solution to

$$X(s, \pi) \left(\frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) = F(u) \left(\frac{1}{2} + \frac{V(0) - V(z)}{2t} \right)$$

with the convention $z(s) = +\infty$ for all s such that:

$$X(s, \pi) \left(\frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) > F(u) \left(\frac{1}{2} + \frac{V(0) - V(1)}{2t} \right)$$

Since $V(z)$ is decreasing, the function $z(s)$ is inverted U-shaped in s , with maximal value

$$z_{VI}^C \equiv \max z(s) = z(s_{VI}^C).$$

We restrict attention to levels of search diversion (s_A, s_B) such that:

$$X(s_i, \pi) \left(\frac{1}{2} + \frac{V(s_i) - V(0)}{2t} \right) \geq F(u) \frac{1}{2} \text{ for } i = A, B$$

Fix (s_A, s_B) chosen in the first stage of the game. Denote by $\Pi^{adv}(i)$ the payoffs obtained by the advertiser when it affiliates exclusively with platform i and by Π^i the profits derived by platform i , where $i \in \{A, B\}$. If the advertiser affiliates exclusively with platform $i \in \{A, B\}$ in the equilibrium of the game beginning in the second stage then the equilibrium fees (r_A, r_B) chosen in stage 2 must be such that (assuming interior demand¹⁰) the following conditions hold:

$$\begin{aligned} \Pi^{adv}(i) &= \max \{ \Pi^{adv}(j), 0 \} \\ \Pi^{adv}(j) &= X(s_j, \pi) \left(\frac{1}{2} + \frac{V(s_j) - V(0)}{2t} \right) - F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_i)}{2t} \right) \\ \Pi^i &= X(s_i, \pi) \left(\frac{1}{2} + \frac{V(s_i) - V(0)}{2t} \right) - \Pi^{adv}(i) \\ \Pi^i &\geq F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_j)}{2t} \right) \text{ if } \Pi^{adv}(j) \geq 0 \\ \Pi^i &\geq F(u) \frac{1}{2} \text{ if } \Pi^{adv}(j) < 0 \end{aligned} \tag{17}$$

The first condition ensures that the advertiser weakly prefers to affiliate with platform i and that platform i cannot increase its profits by raising r_i . The second condition requires platform j to offer the advertiser all the surplus in excess of j 's outside option that would be created if the advertiser were

¹⁰Choosing s_i that induces zero demand for platform $i \in \{A, B\}$ is weakly dominated by a small s_i , so we rule it out.

to affiliate with j instead of i . If j was offering any less in equilibrium, it could then profitably deviate by slightly decreasing r_j and getting the advertiser to affiliate with it (because of the first condition). The third condition is a simple accounting equality. The fourth and fifth conditions ensure that platform i prefers the equilibrium to a deviation in which it would let the advertiser affiliate with platform j or no platform at all (depending on whether platform j offers the advertiser a positive payoff).

With the notation above, we have $\Pi^{adv}(j) \geq 0$ if and only if $s_i \leq z(s_j)$.

Furthermore, if $\Pi^{adv}(j) \geq 0$ then the conditions above imply:

$$\begin{aligned} & X(s_i, \pi) \left(\frac{1}{2} + \frac{V(s_i) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_i)}{2t} \right) \\ \geq & X(s_j, \pi) \left(\frac{1}{2} + \frac{V(s_j) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_j)}{2t} \right) \end{aligned}$$

Suppose that the equilibrium (s_A^*, s_B^*) is such that the advertiser affiliates with platform A. There are three possibilities:

Case I: $s_A^* < z(s_B^*)$

In this case, we must have:

$$\begin{aligned} \Pi^A &= X(s_A^*, \pi) \left[\frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right] \\ &\quad - X(s_B^*, \pi) \left[\frac{1}{2} + \frac{V(s_B^*) - V(0)}{2t} \right] + F(u) \left[\frac{1}{2} + \frac{V(0) - V(s_A^*)}{2t} \right] \\ &\geq F(u) \left[\frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right]. \end{aligned}$$

Local optimality of s_A^* implies that we must have $s_A^* = s_T^C$. For this to be an equilibrium, we first must be able to find s_B^* such that $z(s_B^*) \geq s_T^C$, which is possible if and only if $z_{VI}^C \geq s_T^C$.

Second, neither A nor B can be able to profitably deviate. Consider any deviation s_A by platform A. For the deviation to be profitable, A must continue to win the advertiser, otherwise it would obtain $F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right)$, which is (weakly) dominated by the pre-deviation profits. Since $s_A^* = s_T^C$, we also know that there is no profitable deviation $s_A \leq z(s_B^*)$, so the only possibility would be $s_A \geq z(s_B^*)$, in which case platform A's deviation profits would be:

$$\Pi^A = X(s_A, \pi) \left(\frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right)$$

But $s_{VI}^C < s_T^C < z(s_B^*)$, so the expression above is decreasing for $s_A \geq z(s_B^*)$. Thus, there is no profitable deviation for A.

Consider now a deviation s_B by platform B. The only way it can be profitable is if B wins the advertiser and makes profits larger than $F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_T^C)}{2t} \right)$. This requires $s_B > z(s_T^C)$ (otherwise either A wins the advertiser or B wins it but its resulting profits are still $F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_T^C)}{2t} \right)$), which yields deviation profits (recall conditions (17)):

$$X(s_B, \pi) \left(\frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right)$$

But $s_B > z(s_T^C)$ is equivalent to:

$$F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) > X(s_T^C, \pi) \left(\frac{1}{2} + \frac{V(s_T^C) - V(0)}{2t} \right)$$

and we have:

$$\begin{aligned} & X(s_B, \pi) \left(\frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) \\ \leq & X(s_T^C, \pi) \left(\frac{1}{2} + \frac{V(s_T^C) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_T^C)}{2t} \right) \end{aligned}$$

Combining the last two inequalities, we obtain:

$$X(s_B, \pi) \left(\frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) < F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_T^C)}{2t} \right),$$

which means B's deviation is not profitable.

We have thus proven that if $z_{VI}^C > s_T^C$ then there exists an equilibrium (s_A^*, s_B^*) with $s_A^* = s_T^C$ and $s_B^* = s_{VI}^C$.

Case II: $s_A^* > z(s_B^*)$

In this case, we must have:

$$\Pi^A = X(s_A^*, \pi) \left(\frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) \geq F(u) \frac{1}{2}$$

Local optimality of s_A^* implies that we must have $s_A^* = s_{VI}^C$. For this to be an equilibrium, we first must be able to find s_B^* such that $z(s_B^*) < s_{VI}^C$, which is always possible since $z(0) = 0$.

Second, neither A nor B can be able to profitably deviate. Any deviation s_A by platform A such that $s_A \geq z(s_B^*)$ cannot be profitable, so we must have $s_A \leq z(s_B^*)$. If the advertiser affiliates with platform A after such a deviation then A's deviation profits are:

$$X(s_A, \pi) \left(\frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right) - X(s_B^*, \pi) \left(\frac{1}{2} + \frac{V(s_B^*) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_A)}{2t} \right)$$

But $z(s_B^*) < s_{VI}^C < s_T^C$, so these profits are increasing in s_A for $s_A \leq z(s_B^*)$, which means this deviation is not profitable. The remaining possibility is that $s_A \leq z(s_B^*)$ and the advertiser affiliates with platform B, which platform A can always induce by setting $s_A = 0$. In this case, A's deviation profits are $F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right)$. This deviation is not profitable if:

$$X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) \geq F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right),$$

i.e. if $s_B^* \leq z_{VI}^C$. Note that it is always possible to find s_B^* that satisfies both this condition and $z(s_B^*) < s_{VI}^C$ above: for example, $s_B^* = 0$ works.

Consider now a deviation s_B by platform B: it can be profitable only if it leads to the advertiser affiliating with B. In such a deviation, platform B's profit would be:

$$X(s_B, \pi) \left(\frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) - \max \left\{ 0, X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) - F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) \right\} \quad (18)$$

Suppose that $s_{VI}^C \geq z_{VI}^C$, i.e.

$$X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) \leq F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right)$$

Then platform B's best deviation is $s_B = s_{VI}^C$, which yields profits:

$$X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right),$$

less than the pre-deviation profits $F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right)$. Thus, B has no profitable deviation.

Now suppose that $s_{VI}^C < z_{VI}^C$. There are two possibilities:

- If $z_{VI}^C \geq s_T^C$ then B's best deviation (from (18)) is achieved for $s_B = s_T^C$, yielding profits:

$$\begin{aligned} & X(s_T^C, \pi) \left(\frac{1}{2} + \frac{V(s_T^C) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_T^C)}{2t} \right) \\ & - X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) \\ & > F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right) \end{aligned}$$

- If $s_{VI}^C < z_{VI}^C < s_T^C$ then B's best deviation (from (18)) is achieved for $s_B = z_{VI}^C$, yielding profits:

$$X(z_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(z_{VI}^C) - V(0)}{2t} \right)$$

But $s_{VI}^C < z_{VI}^C < s_T^C$ implies that:

$$\begin{aligned} & X(z_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(z_{VI}^C) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(z_{VI}^C)}{2t} \right) \\ & > X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right), \end{aligned}$$

which is equivalent to (recall the definition of z_{VI}^C):

$$X(z_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(z_{VI}^C) - V(0)}{2t} \right) > F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right)$$

Thus, in both cases B has a profitable deviation so $(s_A^* = s_{VI}^C, s_B^*)$ cannot be an equilibrium for any s_B^* .

We have thus proven that there exists an equilibrium $(s_A^* = s_{VI}^C, s_B^* = 0)$ if and only if $s_{VI}^C \geq z_{VI}^C$.

Case III: $s_A^* = z(s_B^*)$

In this case, we must have:

$$\Pi^A = X(s_A^*, \pi) \left(\frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) \geq F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right),$$

i.e. $s_B^* \leq z(s_A^*)$ (this ensures A does not want to deviate by ceding the advertiser to B). Furthermore, any deviation $s_A > z(s_B^*)$ such that the advertiser stays with A cannot be profitable, so we must have $s_A^* = z(s_B^*) \geq s_{VI}^C$. Similarly, any deviation s_A by platform A such that the advertiser stays with A and $s_A < z(s_B^*)$ cannot be profitable, so we must also have $s_A^* = z(s_B^*) \leq s_T^C$.

Platform B's profit is

$$F(u) \left(\frac{1}{2} + \frac{V(0) - V(z(s_B^*))}{2t} \right) = X(s_B^*, \pi) \left(\frac{1}{2} + \frac{V(s_B^*) - V(0)}{2t} \right).$$

For B to profitably deviate, it must obtain the advertiser. In such a deviation (s_B), platform B's profit would be:

$$X(s_B, \pi) \left(\frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) - \max \left\{ 0, X(s_A^*, \pi) \left(\frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) - F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) \right\}$$

There are three possibilities:

a) If $z(s_A^*) < s_{VI}^C$ then B's optimal deviation is $s_B = s_{VI}^C$, which yields $X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right)$.

This deviation is strictly profitable unless $s_B^* = s_{VI}^C$, which means $s_A^* = z(s_{VI}^C)$.

b) If $z(s_A^*) > s_T^C$ then B's optimal deviation is $s_B = s_T^C$, which yields:

$$\begin{aligned} & X(s_T^C, \pi) \left(\frac{1}{2} + \frac{V(s_T^C) - V(0)}{2t} \right) \\ & - X(s_A^*, \pi) \left(\frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_T^C)}{2t} \right) \\ & \geq F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_A^*)}{2t} \right) \end{aligned}$$

This deviation is strictly profitable unless $s_A^* = s_T^C$, which is possible only if $z(s_{VI}^C) \geq s_T^C$.

c) The remaining case is $s_{VI}^C \leq z(s_A^*) \leq s_T^C$. Here, B's optimal deviation is $s_B = z(s_A^*)$, which yields $X(z(s_A^*), \pi) \left(\frac{1}{2} + \frac{V(z(s_A^*)) - V(0)}{2t} \right)$. For this deviation not to be profitable we must have $z(z(s_A^*)) \leq s_A^*$.

Using the analysis of the 3 cases above, we can infer the following:

- In all cases, the equilibrium level of search diversion s_A^* verifies $s_{VI}^C \leq s_A^* \leq s_T^C$

- If $z_{VI}^C > s_T^C$ then the maximum level of search diversion that can be supported in equilibrium is $s_A^* = s_T^C$ (case I).
- If $z_{VI}^C < s_{VI}^C$ then the only possible equilibrium level of search diversion is $s_A^* = s_{VI}^C$ (case II).
- If $s_{VI}^C \leq z_{VI}^C \leq s_T^C$ then the case I equilibrium cannot be sustained since $z(s_B^*) \leq z_{VI}^C \leq s_T^C$ for all s_B^* . Consequently, the maximum level of search diversion that could potentially be sustained in equilibrium is $s_A^* = z_{VI}^C$ (case 3). Let us make sure that this equilibrium does indeed work. Let $s_B^* = s_{VI}^C$ so that $s_A^* = z(s_B^*) = z_{VI}^C$. To ensure that A has no profitable deviation, we must show that $s_{VI}^C \leq z(s_A^*) = z(z_{VI}^C)$. Recalling the definition of s_T^C and $s_{VI}^C \leq z_{VI}^C \leq s_T^C$, we have:

$$\begin{aligned} & X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right) \\ & \leq X(z_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(z_{VI}^C) - V(0)}{2t} \right) + F(u) \left(\frac{1}{2} + \frac{V(0) - V(z_{VI}^C)}{2t} \right) \end{aligned}$$

By the definition of z_{VI}^C we also have:

$$X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) = F(u) \left(\frac{1}{2} + \frac{V(0) - V(z_{VI}^C)}{2t} \right)$$

We can then derive:

$$F(u) \left(\frac{1}{2} + \frac{V(0) - V(s_{VI}^C)}{2t} \right) \leq X(z_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(z_{VI}^C) - V(0)}{2t} \right),$$

i.e. $s_{VI}^C \leq z(z_{VI}^C)$, as desired. Finally, $s_{VI}^C \leq z_{VI}^C \leq s_T^C$ also implies $z(z_{VI}^C) \leq z_{VI}^C \leq s_T^C$ (recall $z(s)$ is weakly decreasing for $s \geq s_{VI}^C$). To ensure that B does not have a profitable deviation we must then verify that $z(z(z_{VI}^C)) \leq z_{VI}^C$, which follows directly from $s_{VI}^C \leq z(z_{VI}^C)$. We can then conclude that the maximum level of search diversion that can be sustained is indeed $s_A^* = z_{VI}^C$.

This concludes the proof of the first part of the proposition.

Let us now turn to the second part. $s^C(\pi, t)$ is continuous because $s_{VI}^C(\pi, t)$, $z^C(\pi, t)$ and $s_T^C(\pi, t)$ are continuous in both of their arguments. The fact that s_{VI}^C and s_T^C are increasing in t is

seen from the respective first-order conditions that determine them:

$$\begin{aligned} X_s(s_{VI}^C, \pi) (t + V(s_{VI}^C) - V(0)) + X(s_{VI}^C, \pi) V'(s_{VI}^C) &= 0 \\ X_s(s_T^C, \pi) (t + V(s_T^C) - V(0)) + (X(s_T^C, \pi) - F(u)) V'(s_T^C) &= 0 \end{aligned}$$

Both $X_s(s_{VI}^C, \pi)$ and $X_s(s_T^C, \pi)$ are positive, so both left-hand sides above are increasing in t .

The proof that s_{VI}^C and s_T^C are increasing in π is very similar to the proof that s^M is increasing in π (cf. proof of Proposition 1 above), therefore omitted.

Let us now turn to $z^C(\pi, t)$. From the definition of s_{VI}^C we have:

$$X(s_{VI}^C, \pi) \left(\frac{1}{2} + \frac{V(s_{VI}^C) - V(0)}{2t} \right) \geq X(0, \pi) \frac{1}{2} = F(u) \frac{1}{2},$$

which implies $X(s_{VI}^C, \pi) > F(u)$. We can then write the equation determining $z^C(\pi, t)$ as:

$$\begin{aligned} V(0) - V(z^C) &= \frac{X(s_{VI}^C, \pi) (t + V(s_{VI}^C) - V(0))}{F(u)} - t \\ &= \max_s \left\{ \frac{X(s, \pi) (t + V(s) - V(0))}{F(u)} - t \right\} \equiv W(\pi, t), \end{aligned}$$

where the last equality follows from the definition of s_{VI}^C . Using the envelope theorem, we have:

$$\begin{aligned} W_t(\pi, t) &= \frac{X(s_{VI}^C, \pi)}{F(u)} - 1 > 0 \\ W_\pi(\pi, t) &= s_{VI}^C F \left(\frac{u}{1 + s_{VI}^C} \right) \frac{t + V(s_{VI}^C) - V(0)}{F(u)} > 0 \end{aligned}$$

Since $V(0) - V(z^C)$ is increasing in z^C , we can therefore conclude that $z^C(\pi, t)$ is increasing in t and increasing in π , just like s_{VI}^C and s_T^C .

Note that $s^C(\pi, t)$ is always equal to the middle value among s_{VI}^C , s_T^C and $z^C(\pi, t)$. As a result, $s^C(\pi, t)$ inherits the same properties: increasing in t and decreasing in π .

Furthermore, note that $s^C(\pi, t)$ is a well-defined equilibrium only if t is such that:

$$V(s^C) + V(0) > t,$$

i.e. such that the marginal consumer on the Hotelling segment derives positive net utility when the platform that attracts the advertiser chooses s^C in equilibrium. Since s^C is increasing in t , the above

inequality holds for all $t \in [0, \bar{t}^C]$, where $\bar{t}^C > V(0)$.

7.6 Proof of Proposition 4

From lemma 2, s^C is continuous and increasing in t for $t \in [0, \bar{t}^C]$. We also know that s^M is continuous and decreasing in t for all $t \geq 0$. Furthermore, $\lim_{t \rightarrow 0} s^C = 0 < s_X = \lim_{t \rightarrow 0} s^M$, whereas if $t \in (V(0), \bar{t}^C)$ then $s^M = s_{XV} < s_{VI}^C < s_T^C$, which implies $s^M < s^C$. Consequently, there exists $t^C \in [0, V(0)]$ such that $s^C \leq s^M$ for $t \in [0, t^C]$ and $s^C > s^M$ for $t \in (t^C, \bar{t}^C)$.

7.7 Proof of Proposition 5

Denote by $(\widehat{P}^M, \widehat{s}^M)$ the solutions to the optimization program:

$$\max_{P,s} \left\{ (P + X(s, \pi)) \min \left(\frac{V(s) - P}{t}, 1 \right) \right\}$$

There are only two possibilities:

- if $V(\widehat{s}^M) - \widehat{P}^M \geq t$ then it must be that $\widehat{P}^M = V(\widehat{s}^M) - t$ and profits are equal to $V(\widehat{s}^M) + X(\widehat{s}^M, \pi) - t$, which means we must have $\widehat{s}^M = s_{X+V}$
- if $V(\widehat{s}^M) - \widehat{P}^M < t$ then it must be that $\widehat{P}^M = (V(\widehat{s}^M) - X(\widehat{s}^M, \pi)) / 2$ and profits are equal to $(V(\widehat{s}^M) + X(\widehat{s}^M, \pi))^2 / 4t$, so that we must have $\widehat{s}^M = s_{X+V}$

Thus, $\widehat{s}^M = s_{X+V}$ in all cases. The first-order condition determining s_{X+V} can then be written:

$$\frac{-u}{(1+s)^2} F' \left(\frac{u}{1+s} \right) + F \left(\frac{u}{1+s} \right) + V'(s) / \pi = 0$$

The left-hand side is increasing in π , which implies (assuming the second-order condition is satisfied) that s_{X+V} is increasing in π .

The optimal access fee is:

$$\widehat{P}^M = \begin{cases} V(s_{X+V}) - t & \text{if } t \leq (V(s_{X+V}) + X(s_{X+V}, \pi)) / 2 \\ (V(s_{X+V}) - X(s_{X+V}, \pi)) / 2 & \text{if } t \geq (V(s_{X+V}) + X(s_{X+V}, \pi)) / 2 \end{cases}$$

Now compare s_{X+V} and s_{XV} by looking at the first-order conditions that determine them:

$$\begin{aligned} X_s(s_{X+V}, \pi) + V'(s_{X+V}) &= 0 \\ X_s(s_{XV}, \pi) V(s_{XV}) + X(s_{XV}, \pi) V'(s_{XV}) &= 0 \end{aligned}$$

It is thus apparent that $\max\{s_{X+V}, s_{XV}\} \leq s_X$ and $s_{X+V} \leq s_{XV}$ if and only if $V(s_{X+V}) \geq X(s_{X+V}, \pi)$. Consider then the two possible cases:

- If $V(s_{X+V}) \geq X(s_{X+V}, \pi)$ then $\widehat{P}^M \geq 0$ for all t and $s_{X+V} \leq s_{XV}$. Recalling the expression of $s^M(t, \pi)$ from 4, this implies that $\widehat{s}^M(\pi) \leq s^M(t, \pi)$ for all t
- If $V(s_{X+V}) < X(s_{X+V}, \pi)$ then $\widehat{P}^M \geq 0$ if and only if $t \leq V(s_{X+V})$. But in this case we also have $s_X > s_{X+V} > s_{XV}$, i.e. $V(s_X) < V(s_{X+V}) < V(s_{XV})$, which implies that $s^M(V(s_{X+V}), \pi) = s_{X+V} = \widehat{s}^M(\pi)$. Since $s^M(t, \pi)$ is decreasing in t , we therefore conclude that $\widehat{s}^M(\pi) \leq s^M(t, \pi)$ if and only if $t \leq V(s_{X+V})$.

Thus, we have shown that in all cases, $\widehat{s}^M \leq s^M$ if and only if $\widehat{P}^M \geq 0$.

7.8 Proof of Proposition 6

7.8.1 Consumers singlehome and the advertiser multihomes

The determination of the equilibrium level of search diversion $\widehat{s}_{cons}^C(\pi)$ is in the main text. For the second part of the proposition, the equilibrium access fee charged by the two platforms is $P_{cons}^C = t - X(\widehat{s}_{cons}^C(\pi), \pi)$. The two first order conditions that determine $\widehat{s}_{cons}^C(\pi)$ and $s_{cons}^C(\pi) = s_{XV2}$ (focusing on the case in which platforms actually compete) are, respectively:

$$\begin{aligned} X'(\widehat{s}_{cons}^C, \pi) + V'(\widehat{s}_{cons}^C) &= 0 \\ X'(s_{cons}^C, \pi) + X(s_{cons}^C, \pi) \frac{V'(s_{cons}^C)}{t} &= 0 \end{aligned}$$

Comparing, it is easily seen that $\widehat{s}_{cons}^C \leq s_{cons}^C$ if and only if $t \geq X(\widehat{s}_{cons}^C(\pi), \pi)$, i.e. if and only if $P_{cons}^C \geq 0$.

7.8.2 Consumers multihome and the advertiser singlehomes

Suppose that in stage 3 the advertiser affiliates with platform $i \in \{A, B\}$. Then, in stage 4, platform i 's profits are $(P_i + X(s_i, r_i)) \min\{(V(s_i) - P_i)/t, 1\}$, which it optimizes over P_i to obtain profits equal to $\Pi_P(V_i(s_i) + X(s_i, r_i))$, where:

$$\Pi_P(z) \equiv \begin{cases} \frac{z^2}{4t} & \text{if } z \leq 2t \\ z - t & \text{if } z \geq 2t \end{cases}$$

Clearly, $\Pi_P(\cdot)$ is increasing. In turn, platform j 's stage 4 profits are $\Pi_P(V(0) + F(u)) \equiv \Pi_0$.

The advertiser's payoff from affiliation with platform i is then:

$$\Pi_{adv}^i = (\pi - r_i) s_i F\left(\frac{u}{1 + s_i}\right) \min\left(\frac{V(s_i) + X(s_i, r_i)}{2t}, 1\right)$$

In Stage 2, platforms choose (r_A, r_B) taking (s_A, s_B) as given, which is equivalent to choosing (Z_A, Z_B) , where:

$$Z_A \equiv V(s_A) + X(s_A, r_A) \text{ and } Z_B \equiv V(s_B) + X(s_B, r_B)$$

Indeed, recall that $X(s_i, r_i)$ is increasing in r_i so there is a one-to-one relationship between r_i and Z_i for each $i \in \{A, B\}$.

To simplify notation, we also denote:

$$W_i \equiv V(s_i) + X(s_i, \pi) \text{ and } V_i \equiv V(s_i) \text{ for } i \in \{A, B\}$$

which are fixed from the perspective of stage 2.

The advertiser's payoff from affiliation with platform i is then:

$$(W_i - Z_i) \min\left(\frac{Z_i}{2t}, 1\right) \equiv \Pi_{adv}(Z_i, W_i)$$

It is easily seen that $\Pi_{adv}(Z_i, W_i)$ is single-peaked in Z_i and increasing in W_i . Let also:

$$\widehat{Z}(W) \equiv \arg \max_Z \{\Pi_{adv}(Z, W)\} = \begin{cases} \frac{W}{2} & \text{if } \frac{W}{4t} \leq 1 \\ 2t & \text{if } \frac{W}{4t} \geq 1 \end{cases} \quad (19)$$

We first prove the following lemma:

Lemma 3 *In the stage 2 equilibrium, if the advertiser affiliates with platform i then $W_i \geq W_j$.*

Proof. If the advertiser affiliates with platform i in stage 3, then in the stage 2 equilibrium (choices of Z_i and Z_j) we must have:

$$\Pi_i = \max_{Y_i} \{\Pi_P(Z_i)\} \quad \text{such that } \Pi_{adv}(Z_i, W_i) \geq \Pi_{adv}^j \quad (20)$$

$$\Pi_{adv}^j = \max_{Z_j} \{\Pi_{adv}(Z_j, W_j)\} \quad \text{s.t. } \Pi_P(W_j) \geq \Pi_0 \quad (21)$$

Denote by (Z_i^*, Z_j^*) the resulting equilibrium choices.

Since $\Pi_P(Z_i)$ is increasing in Z_i , whereas $\Pi_{adv}(Z_i, W_i)$ is single-peaked in Z_i and zero for $Z_i = W_i$, in equilibrium the constraint in the program (20) must be binding with the highest possible value of Z_i , so that

$$\Pi_{adv}(Z_i^*, W_i) = \Pi_{adv}^j$$

There are two possibilities. First, if the constraint in program (21) is not binding in equilibrium then:

$$\Pi_{adv}^j = \max_{Z_j} \{\Pi_{adv}(Z_j, W_j)\} = \max_{Z_j} \left\{ (W_j - Y_j) \min\left(\frac{Z_j}{2t}, 1\right) \right\}$$

Combined with (20), this implies:

$$\begin{aligned} \max_{Z_i} \left\{ (W_i - Y_i) \min\left(\frac{Z_i}{2t}, 1\right) \right\} &\geq (W_i - Z_i^*) \min\left(\frac{Z_i^*}{2t}, 1\right) \\ &= \max_{Z_j} \left\{ (W_j - Y_j) \min\left(\frac{Z_j}{2t}, 1\right) \right\}, \end{aligned}$$

It is easily verified that this implies $W_i \geq W_j$.

Second, suppose instead the constraint in (21) is binding in equilibrium:

$$\Pi_P(Z_j^*) = \Pi_0 \leq \Pi_P(Z_i^*),$$

where the last inequality is required in equilibrium (otherwise platform i would prefer to not attract the advertiser). Since $\Pi_P(\cdot)$ is increasing, this is equivalent to:

$$Z_j^* = V(0) + F(u) \leq Z_i^* \quad (22)$$

Furthermore, if the constraint in (21) binds then $Z_j = \widehat{Z}(W_j)$ violates the constraint, i.e.

$$\widehat{Z}(W_j) < V(0) + F(u) \quad (23)$$

Once again, there are two possibilities:

- If $\widehat{Z}(W_i) \geq V(0) + F(u)$ then

$$\widehat{Z}(W_i) > \widehat{Z}(W_j) \quad (24)$$

From (19), this is only possible if $W_i > W_j$ (and $W_j < 4t$).

- If $\widehat{Z}(W_i) < V(0) + F(u)$ then (22) and (20) imply

$$\begin{aligned} \Pi_{adv}(Z_i^*, W_i) &\leq \Pi_{adv}(V(0) + F(u), W_i) \\ \Pi_{adv}(Z_i^*, W_i) &= \Pi_{adv}(Z_j^*, W_j) = \Pi_{adv}(F(u) + V(0), W_j), \end{aligned}$$

which implies

$$\Pi_{adv}(V(0) + F(u), W_i) \geq \Pi_{adv}(F(u) + V(0), W_j),$$

i.e. $W_i \geq W_j$.

■

Suppose that in equilibrium platform A wins the advertiser. Then $W_A \geq W_B$ and platform A's profits can be rewritten:

$$\begin{aligned} \Pi_A &= \Pi_w(s_A, s_B) \equiv \max_{Z_A} \{\Pi_P(Z_A)\} \\ \text{s.t. } (W_A - Z_A) \min\left(\frac{Z_A}{2t}, 1\right) &\geq \Pi_{adv}(s_B) \end{aligned} \quad (25)$$

where:

$$\Pi_{adv}(s_B) \equiv \max_{Z_B} \left\{ \frac{(W_B - Z_B) Z_B}{2t} \right\} \quad \text{s.t. } \Pi_P(Z_B) \geq \Pi_0 \quad (26)$$

Suppose there exists s'_A such that $W'_A = X(s'_A, \pi) + V(s'_A) > W_A = X(s_A, \pi) + V(s_A)$, which implies $W'_A > W_B$. Thus, if platform A deviates to s'_A in stage 1 then it wins the advertiser in stage 2 with probability 1. Furthermore, since $\Pi_{adv}(s_B)$ remains unchanged, the optimization problem above

immediately implies $\Pi_w(s'_A, s_B) > \Pi_w(s_A, s_B)$. Therefore s'_A is a profitable deviation. Thus, it must be that in the stage 1 equilibrium $s_A = s_{X+V} \equiv \arg \max_s \{X(s, \pi) + V(s)\}$.

If $s_B \neq s_{X+V}$ then platform B makes profits Π_0 with probability 1 and Π_0 does not depend on s_B . If $s_B = s_A = s_{X+V}$ then we may assume that A wins the advertiser with probability 1. Thus, in all cases it is an equilibrium that platform A chooses $s_A = s_{X+V}$ and wins the advertiser.

For the second part of the proposition corresponding to this scenario, simply note that $\widehat{s}_{adv}^C = \widehat{s}^M$ (which we have just proven) and $s_{adv}^C = s^M$ (from proposition 3). And we have already proven that $\widehat{s}^M \leq s^M$ if and only if $P^M \geq 0$ (proposition 5). We can therefore directly conclude that $\widehat{s}_{adv}^C \leq s_{adv}^C$ if and only if $P_{adv}^C \geq 0$ (since $P_{adv}^C = P^M$).

7.8.3 Both sides singlehome

Suppose that in stage 3 the advertiser affiliates with platform $i \in \{A, B\}$. Then, in stage 4, platform i and platform j 's profits are, respectively:

$$\begin{aligned} & (P_i + X(s_i, r_i)) \frac{1}{2t} (t + V(s_i) - V(0) - P_i + P_j) \\ & (P_j + F(u)) \frac{1}{2t} (t + V(0) - V(s_i) - P_j + P_i) \end{aligned}$$

Calculating the Nash equilibrium in prices, we obtain that stage 4 equilibrium profits are, respectively:

$$\begin{aligned} \Pi_i &= \frac{1}{2t} \left(t + \frac{V(s_i) + X(s_i, r_i) - V(0) - F(u)}{3} \right)^2 \\ \Pi_j &= \frac{1}{2t} \left(t + \frac{V(0) + F(u) - V(s_i) - X(s_i, r_i)}{3} \right)^2 \end{aligned}$$

The advertiser's payoff from affiliation with platform i is then:

$$\Pi_{adv}^i = (X(s_i, \pi) - X(s_i, r_i)) \frac{1}{2t} \left(t + \frac{V(s_i) + X(s_i, r_i) - V(0) - F(u)}{3} \right)$$

In Stage 2, platforms choose (r_A, r_B) taking (s_A, s_B) as given, which is equivalent to choosing (Z_A, Z_B) , where:

$$Z_i \equiv X(s_i, r_i) + V(s_i)$$

Indeed, recall that $X(s_i, r_i)$ is increasing in r_i so there is a one-to-one relationship between r_i and Z_i for each $i \in \{A, B\}$.

Denote also:

$$\begin{aligned} W_i &\equiv X(s_i, \pi) + V(s_i) \text{ for } i \in \{1, 2\} \\ W_0 &\equiv V(0) + F(u) \end{aligned}$$

which are fixed from the perspective of stage 2.

Suppose that platform i wins the advertiser. Then platform profits in stage 4 can be written:

$$\begin{aligned} \Pi_i &= \Pi_w(Z_i) \equiv \frac{1}{2t} \left(t + \frac{Z_i - W_0}{3} \right)^2 \\ \Pi_j &= \Pi_l(Z_i) \equiv \frac{1}{2t} \left(t + \frac{W_0 - Z_i}{3} \right)^2 \end{aligned}$$

Meanwhile, the advertiser's payoff from affiliation with platform i is:

$$\Pi_{adv}(Z_i, W_i) \equiv (W_i - Z_i) \frac{1}{2t} \left(t + \frac{Z_i - W_0}{3} \right)$$

First, we prove the following lemma:

Lemma 4 *In the stage 3 equilibrium, if the advertiser affiliates with platform i then $W_i \geq W_j$.*

Proof. Suppose platform i wins the advertiser in the equilibrium of the stage 2 game and denote by (Z_i^*, Z_j^*) the equilibrium choices in stage 2. We must then have:

$$Z_i^* = \arg \max_{Z_i} \{\Pi_w(Z_i)\} \text{ s.t. } \Pi_{adv}(Z_i, W_i) \geq \widehat{\Pi}_{adv}(W_j, \Pi_l(Z_i^*)) \quad (27)$$

$$\widehat{\Pi}_{adv}(W_j, \Pi_l(Z_i^*)) \equiv \max_{Z_j} \{\Pi_{adv}(Z_j, W_j)\} \text{ s.t. } \Pi_w(Z_j) \geq \Pi_l(Z_i^*) \quad (28)$$

Since $\Pi_w(Z_i)$ is increasing in Z_i , whereas $\Pi_{adv}(Z_i, W_i)$ is concave in Z_i and equals 0 at $Z_i = W_i$, in equilibrium the constraint in (27) must be binding and $\Pi_{adv}(Z_i, W_i)$ must be decreasing in Z_i at the point $Z_i = Z_i^*$ where it intersects $\widehat{\Pi}_{adv}(W_j, \Pi_l(Z_i^*))$. This is equivalent to:

$$W_i - 3t + W_0 \leq 2Z_i^* \quad (29)$$

Suppose the constraint in (28) is not binding. Then in equilibrium:

$$\begin{aligned} Z_j^* &= \frac{W_j - 3t + W_0}{2} \\ \widehat{\Pi}_{adv}(W_j, \Pi_l(Z_i^*)) &= \frac{1}{24t} (3t + W_j - W_0)^2 \end{aligned}$$

so that:

$$\frac{1}{24t} (3t + W_i - W_0)^2 \geq \frac{(W_i - Z_i^*)}{2t} \left(t + \frac{Z_i^* - W_0}{3} \right) = \frac{1}{24t} (3t + W_j - W_0)^2,$$

which directly implies $W_i \geq W_j$.

Suppose now the constraint in (28) is binding, which means:

$$Z_j^* - W_0 = W_0 - Z_i^* \tag{30}$$

Furthermore, this requires that the peak of $\Pi_{adv}(Z_j, W_j)$ in Z_j violates the constraint, i.e.:

$$\frac{W_j - 3t + W_0}{2} - W_0 < W_0 - Z_i^*$$

which can be rewritten:

$$W_j < 3t + 3W_0 - 2Z_i^* \tag{31}$$

Taking the sum of inequalities (29) and (31) above, we obtain:

$$W_i + W_j < 6t + 2W_0 \tag{32}$$

We can then write the fact that the constraint in (27) is binding, $\Pi_{adv}(Z_i^*, W_i) = \widehat{\Pi}_{adv}(W_j, \Pi_l(Z_i^*))$, as:

$$\frac{(W_i - Z_i^*)}{2t} \left(t + \frac{Z_i^* - W_0}{3} \right) = \frac{(W_j - Z_j^*)}{2t} \left(t + \frac{Z_j^* - W_0}{3} \right),$$

which, after using (30) and re-arranging, is equivalent to:

$$t(W_i - W_j - Z_i^* + Z_j^*) + \frac{Z_i^* - W_0}{3} (W_i + W_j - Z_i^* - Z_j^*) = 0$$

Using (30) again, this is equivalent to:

$$t(W_i - W_j) + \frac{Z_i^* - W_0}{3} (W_i + W_j - 2W_0 - 6t) = 0$$

But (32) implies $W_i + W_j - 2W_0 - 6t < 0$ and we must have $Z_i^* - W_0 \geq 0$ (otherwise platform i would prefer to not provide any advertising in stage 4). Thus, we conclude that $W_i - W_j \geq 0$. ■

Suppose platform A wins the advertiser in equilibrium. Then the lemma implies $X(s_A, \pi) + V(s_A) \geq X(s_B, \pi) + V(s_B)$ and platform A's profit in stage 2 is $\Pi_A(s_A, s_B) = \Pi_w(Z_A^*)$, where Z_A^* solves:

$$Z_A^* = \arg \max_{Z_A} \left\{ \frac{1}{2t} \left(t + \frac{Z_A - W_0}{3} \right)^2 \right\} \text{ s.t. } \frac{(W_A - Z_A)}{2t} \left(t + \frac{Z_A - W_0}{3} \right) \geq \widehat{\Pi}_{adv}(W_B, Z_A^*)$$

$$\widehat{\Pi}_{adv}(W_B, Z_A^*) \equiv \max_{Z_B} \left\{ \frac{(W_B - Z_B)}{2t} \left(t + \frac{Z_B - W_0}{3} \right) \right\} \text{ s.t. } \frac{1}{2t} \left(t + \frac{Z_B - W_0}{3} \right)^2 \geq \frac{1}{2t} \left(t + \frac{W_0 - Z_A^*}{3} \right)^2$$

Suppose there exists s'_A such that $X(s'_A, \pi) + V(s'_A) > X(s_A, \pi) + V(s_A)$, which implies $X(s'_A, \pi) + V(s'_A) > X(s_B, \pi) + V(s_B)$. Thus, if platform A deviates to s'_A in stage 1 then the lemma above implies that platform A wins the advertiser in stage 2 with probability 1. Let then $\Pi_A(s'_A, s_B) = \Pi_w(Z_A^{*'})$, where $Z_A^{*'}$ is defined similarly to Z_A^* , except that W_A is replaced by $W'_A \equiv X(s'_A, \pi) + V(s'_A)$.

There are several possibilities. First suppose the constraint in the definition of $\widehat{\Pi}_{adv}(W_B, Z_A^*)$ is not binding, i.e.

$$\widehat{\Pi}_{adv}(W_B, Z_A^*) = \frac{1}{24t} (3t + W_B - W_0)^2,$$

which does not depend on Z_A^* . Pick then any ε such that $0 < \varepsilon < W'_A - W_A$. We have $Z_A^{*' } \geq Z_A^* + \varepsilon > Z_A^*$. To see this, note that $\frac{1}{2t} \left(t + \frac{Z_A - W_0}{3} \right)^2$ is increasing in Z_A and:

$$\begin{aligned} \frac{(W'_A - Z_A^* - \varepsilon)}{2t} \left(t + \frac{Z_A^* + \varepsilon - W_0}{3} \right) &> \frac{(W_A - Z_A^*)}{2t} \left(t + \frac{Z_A^* - W_0}{3} \right) \\ &= \widehat{\Pi}_{adv}(W_B, Z_A^*) = \widehat{\Pi}_{adv}(W_B, Z_A^* + \varepsilon) \end{aligned}$$

where the last equality follows from the fact that $\frac{1}{2t} \left(t + \frac{W_0 - Z_A^*}{3} \right)^2$ is decreasing in Z_A^* .

Second, suppose the constraint in the definition of $\widehat{\Pi}_A(W_B, Z_A^*)$ is binding, which implies

$$Z_B^* - W_0 = W_0 - Z_A^*$$

and

$$\begin{aligned}
Z_A^* &= \arg \max_{Z_A} \left\{ \frac{1}{2t} \left(t + \frac{Z_A - W_0}{3} \right)^2 \right\} \\
\text{s.t. } \frac{(W_A - Z_A)}{2t} \left(t + \frac{Z_A - W_0}{3} \right) &\geq \frac{(W_B - 2W_0 + Z_A^*)}{2t} \left(t + \frac{W_0 - Z_A^*}{3} \right)
\end{aligned} \tag{33}$$

There are now two possibilities regarding Z_A^* :

- If the constraint in the definition of $\widehat{\Pi}_{adv}(W_B, Z_A^*)$ is also binding then we have:

$$\begin{aligned}
Z_A^{*'} &= \arg \max_{Z_A} \left\{ \frac{1}{2t} \left(t + \frac{Z_A - W_0}{3} \right)^2 \right\} \\
\text{s.t. } \frac{(W_A' - Z_A)}{2t} \left(t + \frac{Z_A - W_0}{3} \right) &\geq \frac{(W_B - 2W_0 + Z_A^{*'})}{2t} \left(t + \frac{W_0 - Z_A^{*'}}{3} \right)
\end{aligned} \tag{34}$$

Comparing (33) and (34), the only difference is $W_A' > W_A$, so $Z_A^{*'} > Z_A^*$.

- If the constraint in the definition of $\widehat{\Pi}_{adv}(W_B, Z_A^{*'})$ is not binding then:

$$\frac{W_B - 3t + W_0}{2} - W_0 \geq W_0 - Z_A^{*'}.$$

and, since the constraint in the definition of $\widehat{\Pi}_{adv}(W_B, Z_A^*)$ is binding, we also have:

$$\frac{W_B - 3t + W_0}{2} - W_0 < W_0 - Z_A^*.$$

The last two inequalities imply $Z_A^{*'} > Z_A^*$.

Thus, in all possible cases, we have $Z_A^{*'} > Z_A^*$, which means that s_A' is a profitable deviation for platform A: $\Pi_A(s_A', s_B) > \Pi_A(s_A^*, s_B)$. Therefore, it must be that in the stage 1 equilibrium $s_A = s_{X+V} \equiv \arg \max_s \{X(s, \pi) + V(s)\}$.

If $s_B \neq s_{X+V}$ then platform B makes profits $\frac{1}{2t} \left(t + \frac{W_0 - Z_A^*}{3} \right)^2$ with probability 1.

If $s_{2B} = s_A = s_{X+V}$ then:

- with probability 1/2 platform 1 wins the advertiser so platform B's profits are still Π_0
- with probability 1/2 platform B wins the advertiser so its profits are $\Pi_w(s_{X+V}, s_{X+V}) \geq \Pi_0$ (the inequality is strict if the constraint in program 26 is not binding).

Thus, in all cases it is an equilibrium for platform B to also choose $s_B = s_{X+V}$ (this is the only equilibrium if the constraint in program 26 is not binding).