WORKING PAPERS

N°TSE- 378

February 2013

Allocating fixed costs in the postal sector in the presence of changing letter and parcel volumes: applied in outdoor delivery

P.De Donder, H.Cremer, P.Dudley and F.Rodriguez

INSTITUT D'ECONOMIE INDUSTRIELLE

Toulouse School of Economics

Allocating fixed costs in the postal sector in the presence of changing letter and parcel volumes: applied in outdoor delivery[1]

P De Donder[2]**, H Cremer**[3]**, P Dudley**[4]**, F Rodriguez**[5]

1. Introduction

The mix of volumes handled by the Universal Service Provider (USP) is changing. Letter mail volumes are in decline and parcel volumes are growing. These trends are expected to continue with the e-substitution of letter mail and the e-commerce of parcels. Current costing methods are based on a letter mails market and not set up to take account of this change. With the current method, the market trends lead to the transfer of common and joint costs to parcels, and consequently an ever increasing allocation of fully allocated costs per item within parcels. The purpose of this paper is to look at the properties of the cost function that would provide a more intuitive and, potentially more appropriate, cost allocation in the changing circumstances.

Cost allocation is included as part of the European Postal Services Directive, Article 14.3. This sets out that "*costs which can be directly assigned to a particular service or product shall be so assigned*"; and "*common costs, that is costs which cannot be directly assigned to a particular service or product, shall be allocated in the following manner (i) whenever possible, common costs shall be allocated on the basis of direct analysis of the origin of the costs themselves; (ii) when direct analysis is not possible, common cost categories shall be allocated on the basis of an indirect linkage to another cost category or group of cost categories for which a direct assignment or allocation is possible; the indirect linkage shall be based on comparable cost structures; (iii) when neither direct nor indirect measures of cost allocation can be found, the cost category shall be allocated on the basis of a general allocator computed by using the ratio of all expenses directly or indirectly assigned or allocated, on the one hand, to each of the universal services and, on the other hand, to the other services; (iv) common costs, which are necessary for the provision of both universal services and non-universal services, shall be allocated appropriately; the same cost drivers must be applied to both universal services and non-universal services*."

Within the postal sector, Robinson and McMurdie (2009) developed an operational model and method for the allocation of cost using the drivers of class and products within the universal service; their approach allocated proportionately more cost to higher class and Universal Service Obligation (USO) products. De Donder et al (2002) modelled parcels within the USP in the context of competition within the delivery to rural areas; in subsequent papers by De Donder et al (2008), the USP is characterised as having fixed costs and marginal costs for the provision of its services. Mautino et al (2013) developed an approach to estimate long run average incremental costs for the postal sector. This included an explanation of the economies of scale and scope found within outdoor collection and delivery; there are joint costs in outdoor delivery which mean

[1] The analysis contained in this paper reflects the view of the authors and may not necessarily be those of the institutions that employ them.
[2] Corresponding author, Toulouse School of Economics (GREMAQ-CNRS and IDEI), philippe.dedonder@tse-fr.eu
[3] Toulouse School of Economics (GREMAQ, Institut universitaire de France and IDEI).
[4] Head of Economics, Royal Mail Group.

[5] Associate, Oxera.

that the incremental cost of an additional item is often associated with the likelihood of having to deliver, and therefore walk up the path, to an address; the incremental cost of delivering an item reduces with increased volume, as the proportion of addresses visited increases at a decreasing rate.

This paper develops a model with welfare maximisation by the USP with separate goods, subject to a budget constraint where there are marginal, joint and fixed costs which take account of the economies of scale and scope in delivery. Under the Postal Services Directive, first direct costs are identified for goods, second indirect attributable costs are allocated and finally indirect non-attributable costs are to be apportioned using EPMU. In the context of the model, the direct costs are the marginal costs, the indirect attributable costs are the joint costs and additional unit costs and the indirect non-attributable costs are the residual fixed costs (i.e. the fixed costs less the marginal and joint (unit) costs).

To focus on the principles of cost allocation the model has two goods of letters and parcels in outdoor delivery; the same model could be extended to include more dimensions but the outdoor delivery is a significant component of the USP and is known to have significant economies of scale and scope. In the first two stages of analysis, the cost function includes the marginal and fixed costs for letters and parcels and, in the final stage, additionally a joint cost component. The marginal costs increase at a decreasing rate and the joint cost is such that an increase in volume of one good increases the cost of the other good, but at a decreasing rate. Hence both marginal and joint costs reflect the presence of economies of scale.

More specifically, the initial analysis assumes that the marginal costs for letters and parcels are separate and decreasing in volume, with the same mode of delivery method. This is sufficient to obtain an outcome whereby the marginal cost of letters increases and that for parcels reduces as letter volumes reduce and parcels increase, such that the fixed cost (allocated under constant demand elasticity assumptions or equi-proportional mark up (EPMU)), leads to the unit cost of letters rising more than parcels. This is illustrated by a numerical example. The illustration is then extended to consider the case where parcels are more demand elastic than letters, such that, under Ramsey rules, the movements in unit costs for letters and parcels are further enhanced.

The second analysis takes account of two delivery methods for letters and parcels, one of fixed capacity trolleys, known as High Capacity Trolleys (HCTs), that are used on walks to carry letters and some parcels, and one of a higher capacity vans which lend themselves more to parcels delivery. The letters are assumed to have priority use of the HCTs and the residual space on a HCT is used for parcels, with residual parcels then delivered by van. When the cost functional form is Cobb Douglas for a single route only, the results have similar properties to those in the initial analysis.

The third analysis develops the second analysis further to include multiple routes where there is a distribution for the portion of HCT fill by letters by route. When the cost functional form is Cobb Douglas assuming multiple routes, the results again can have similar properties to those in the initial analysis, but in this instance there are not only marginal and fixed costs, but also joint costs with the cost function having cross elasticities of its costs.

The analytical model is set out and developed in Section 2. The illustrations are included in Section 3. Section 4 concludes.

2. Models

2.1 First model

For the model there is one firm and two products, 1 and 2. One could be "packets" and the other one "letters". The demands for the two products are independent of each other and given by $x_1(p_1)$ and $x_2(p_2)$ where x_i denotes the quantity and p_i the price of good i. These demands are obtained when consumers maximize their utility

$$U_1(x_1)+U_2(x_2)-p_1x_1-p_2x_2,$$

for given p_1 and p_2, with respect to x_1 and x_2. The cost function of the firm has both variable and fixed components, and components that can be attributed to one good versus overheads.

With no joint cost and decreasing marginal costs the total cost function of the firm is given by:

$$C(x_1,x_2)=c_1(x_1)x_1+F_1+c_2(x_2)x_2+F_2+F,$$

with decreasing marginal costs: $c_i^{'}(x_i)<0$ for i = 1, 2.

The firm maximises welfare subject to some profit constraint, where profit $\prod$, given by

$$\prod = p_1x_1+p_2x_2-C(x_1,x_2),$$

has to be larger than some minimum amount $\overline{\prod}\geq 0$.

The Lagrangian expression of this problem is

$$L=U_1(x_1)+U_2(x_2)-p_1x_1-p_2x_2+(1+\lambda)\prod-\lambda\overline{\prod}.$$

When maximizing L with respect to p_i, the following Ramsey expression for good i is obtained

$$\frac{p_i-\partial C(x_1,x_2)/\partial x_i}{p_i}=\frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_i},$$

where ε_i is the (absolute value of the) own-price elasticity of the demand for good i, and where

$$\frac{\partial C(x_1,x_2)}{\partial x_i}=c_i^{'}(x_i)x_i+c_i(x_i)<c_i(x_i)$$

since marginal cost is decreasing.

Observe that, since marginal cost is decreasing, the Ramsey formula with $\lambda=0$ generates prices (equal to marginal costs) that do not even cover variable costs.

If the volume of good 1 decreases for an exogenous reason (meaning, for any given price the demand is now lower) while the demand for good 2 increases, than marginal costs will increase for good 1 while they will decrease for good 2, meaning a larger price of good 1 and a smaller price for good 2.

2.2 Second model

The first model is developed to account for two modes of delivery. Parcels are delivered jointly with letters using High Capacity Trolleys (HCT). When the capacity constraint of the HCT is binding, the parcels that cannot fit are delivered with a van.

For simplicity it is assumed that there is a single, “aggregate” route, that the HCT capacity is large enough to deliver all letters but not large enough to accommodate all parcels.

The volume of parcels delivered with HCT is denoted by x_2^T and the volume of parcels delivered by van is denoted by x_2^V , with

$$x_2^T + x_2^V = x_2 .$$

The capacity per item taken by a letter in the HCT by a and the capacity per item taken by a parcel by b , with $b > a$. The capacity constraint of the HCT is

$$ax_1 + bx_2^T \le K ,$$

and it is assumed that this constraint is binding (i.e., holds with equality).

The following expression is then obtained

$$x_2^T = \frac{K - ax_1}{b} , \tag{1}$$

and

$$x_2^V = x_2 - x_2^T = x_2 - \frac{K - ax_1}{b} , \tag{2}$$

The cost function for vans is denoted by

$$C^V\left(x_2^V\right) = \theta\, c_2^V\left(x_2^V\right) x_2^V + F^V$$

and it is assumed that there are increasing returns to scale, so that marginal costs are decreasing:

$$\frac{\partial^2 c_2^V\left(x_2^V\right)}{\partial^2 x_2^V} < 0.$$

The cost function for HCTs is

$$C^T\left(x_1, x_2^T\right) = \left(x_1\right)^{\gamma} \left(x_2^T\right)^{1-\gamma} + F^T$$

When combined with (1), this yields

$$C^T\left(x_1, x_2^T\right) = \left(x_1\right)^{\gamma}\left(\left(K - ax_1\right)/b\right)^{1-\gamma} + F^T$$

Observe that this cost is only a function of x_1, with

$$\frac{\partial C^T\left(x_1, x_2^T\right)}{\partial x_1} = \frac{\left(x_1\right)^{\gamma-1}\left(\left(K - ax_1\right)/b\right)^{-\gamma}\left(K\gamma - ax_1\right)}{b}$$

and

$$\frac{\partial^2 C^T\left(x_1, x_2^T\right)}{\partial x_1^2} = \frac{\gamma(\gamma-1)K^2\left(x_1\right)^{\gamma-2}\left(\left(K - ax_1\right)/b\right)^{-1-\gamma}}{b} < 0,$$

so that marginal costs are decreasing.

Hence, this has similar properties to the first model with no joint costs but decreasing marginal costs. The role played by the number of letters, x_1, is more complex because having more letters shifts parcels from HTC to vans. The parameters used include: K, a and b (the capacity of a HCT in terms of both parcels and letters), the shape of c_2^V (the variable cost function for vans); and F^V and F^T (the fixed costs of vans and trolleys).

2.3 Third model

In a further development of the second model, it is assumed that some HCTs on some routes are not full so that the total volume of parcels delivered by HCT increases with the total amount of parcels delivered. As volumes increase, the capacity constraint on HCTs becomes binding for an increasing fraction of routes, so that the relationship between x_2^T and x_2 is concave and given by the function

$$x_2^T = f\left(x_2\right)$$

with $0 < f'\left(x_2\right) < 1$ and $f''\left(x_2\right) < 0$.

The intuition for this assumption runs as follows. When x_2 is very small, there is enough space on HCTs on every route to accommodate parcels, so that $f\left(x_2\right) = x_2$ and $f'\left(x_2\right) = 1$. As parcels volumes increase, the capacity constraint for HCTs becomes binding on an increasing number of routes, so that in the aggregate $f\left(x_2\right) < x_2$ and $0 < f'\left(x_2\right) < 1$. At the limit, when volumes are very large, HCTs are saturated on all routes and $f\left(x_2\right) = nK$ (where K is the capacity constraint of an individual HCT and where n is the number of routes), so that $f'\left(x_2\right) = 0$. It is assumed that the operation of volumes is such that the capacity constraint of the HCTs is binding on some but not all routes, so that $0 < f'\left(x_2\right) < 1$ and $f''\left(x_2\right) < 0$.

Using the same formulation as in the Section 2.2 for the cost functions, the following is obtained:

$$C^T\left(x_1, x_2\right) = \left(x_1\right)^{\gamma}\left(f\left(x_2\right)\right)^{1-\gamma} + F^T,$$

While we simplify the van delivery cost to have

$$C^V(x_2) = (x_2 - f(x_2))c_2^V + F^V,$$

Where c_2^V is the constant marginal delivery cost with vans. Denoting total cost by

$$C(x_1, x_2) = C^T(x_1, x_2) + C^V(x_2),$$

the following is obtained

$$\frac{\partial C(x_1, x_2)}{\partial x_1} = \gamma \left(f(x_2)/x_1 \right)^{1-\gamma} > 0,$$

$$\frac{\partial^2 C(x_1, x_2)}{\partial x_1 \partial x_2} = \gamma (1-\gamma) \left(f(x_2) \right)^{-\gamma} (x_1)^{\gamma-1} f'(x_2) > 0,$$

so that a joint cost function is obtained with positive cross derivative.

In addition,

$$\frac{\partial^2 C(x_1, x_2)}{\partial x_1^2} = \gamma (\gamma - 1) \left(f(x_2) \right)^{1-\gamma} (x_1)^{\gamma-2} < 0,$$

so that the total cost is increasing and concave in the number of letters.

As for the impact of the number of parcels, we obtain

$$\frac{\partial C(x_1, x_2)}{\partial x_2} = (1-\gamma) x_1^\gamma \left(f(x_2) \right)^{-\gamma} f'(x_2) + c_2^V (1 - f'(x_2)) > 0,$$

where the first term is the marginal HCT cost and the second term the marginal van cost, and where both terms are positive since having more parcels increases the number of parcels delivered both with HCTs and with vans.

Consequently,

$$\frac{\partial^2 C(x_1, x_2)}{\partial x_2^2} = (\gamma - 1) \left(f(x_2) \right)^{-1-\gamma} x_1^\gamma \left[\gamma \left(f'(x_2) \right)^2 - f'(x_2) f''(x_2) \right] - c_2^V \left(f''(x_2) \right),$$

where the first term pertains to the HCT cost function and is negative (so that this part of the function is concave) while the second term pertains to the van cost function and is positive (so that this part of the cost function is convex). The aggregate cost function is then concave in the number of parcels provided that the second term is not too positive (i.e., that c_2^V is not too large). In that case, the aggregate cost function is increasing and concave separately in both the number of parcels and of letters, and exhibits a positive first order cross-derivative. These are the same properties as those of a simple Cobb-Douglas function.

$$C(x_1, x_2) = x_1^\gamma x_2^{1-\gamma}.$$

3. Illustration

For the first model analysis in Section 2.1 a numerical example is developed assuming an iso-elastic demand function,

$$x_i(p_i) = k_i p_i^{-r_i},$$

and where costs are given by

$$c(x_1) = x_1^{-1/2},$$

$$\text{and } c(x_2) = 2x_2^{1/2},$$

so that

$$C(x_1, x_2) = x_1^{1/2} + 2x_2^{1/2},$$

Good 2 is twice more expensive than good 1 for any $x_1 = x_2 = x,$ marginal costs are decreasing but total costs are increasing. The same elasticity is assumed for both goods: $r_1 = r_2 = 0.3.$

The parameters k_i in the demand functions are trend parameters. These affect the size of the demand without changing the sensitivity of the demand functions to price (the demand elasticities).

For simplicity it is assumed that $F_1 = F_2 = F = 0.$

Table 1 shows the optimal values of p_1 and p_2 when $k_1 = k_2 = 1,$ when $\lambda = 0$ (so that $\prod < 0$) and when λ is obtained endogenously so that $\prod = 0.$

Table 1: optimal values of p_1 and p_2 when $k_1 = k_2 = 1,$ when $\lambda = 0$ (so that $\prod < 0$) and when λ is obtained endogenously so that $\prod = 0.$

	$\lambda = 0$	$\prod = 0$
p_1	0.44	1
p_2	1	2.26
λ	0	0.18
$\prod$	-1.565	0

As explained above, $\prod < 0$ even when there is no fixed costs when λ is set exogenously at 0. When λ is set endogenously so that $\prod = 0$, prices increase and we have $p_2 > p_1$ since good 2 is more expensive. Observe that p_2 is more than twice p_1 (even though it costs exactly twice as much for the same quantity), because higher prices translate into lower quantities and thus into larger marginal costs.

When $k_1 = 0.7$ and $k_2 = 1.2$, the demand elasticities are unchanged but the increase in the quantity of good 2 is smaller than the decrease in the quantity demanded of good 1, for any price $p_1 = p_2 = p$. The results are reported in Table 2.

Table 2: optimal values of p_1 and p_2 when $k_1 = 0.7$ and $k_2 = 1.2$, when $\lambda = 0$ (so that $\prod < 0$) and when λ is obtained endogenously so that $\prod = 0$.

	$\lambda = 0$	$\prod = 0$
p_1	0.55	1.23
p_2	0.89	2.03
λ	0	0.18
$\prod$	-1.571	0

Compared to Table 1, p_1 increases while p_2 decreases (both with $\lambda = 0$ and with endogenous λ), because lower quantities increase marginal costs for good 1, while the opposite occurs for good 2.

In contrast if it is assumed that parcels are more price elastic than letters such that $r_1 = 0.3$ and $r_2 = 1$, the equivalent results to Tables 1 and 2 are shown in Tables 3 and 4 respectively.

Table 3: optimal values of p_1 and p_2 when $k_1 = k_2 = 1$, when $\lambda = 0$ (so that $\prod < 0$) and when λ is obtained endogenously so that $\prod = 0$ with more elastic demand for parcels.

	$\lambda = 0$	$\prod = 0$
p_1	0.44	1.78
p_2	1	1.59
λ	0	0.26
$\prod$	-1.565	0

Table 4: optimal values of p_1 and p_2 when $k_1 = 0.7$ and $k_2 = 1.2$, when $\lambda = 0$ (so that $\prod < 0$) and when λ is obtained endogenously so that $\prod = 0$ with more elastic demand for parcels.

	$\lambda = 0$	$\prod = 0$
p_1	0.55	2.66
p_2	0.83	1.38
λ	0	0.29
$\prod$	-1.658	0

Increasing r_2 does not change the equilibrium prices with $\prod = 0$ when $k_1 = k_2 = 1$, but slightly decreases p_2 when $k_1 < k_2$. Observe that there is marginal cost pricing when $\prod = 0$. The reason why the equilibrium price changes only marginally is because the marginal cost is a function of quantity, and the quantity demanded is affected by a variation of r_2, except in the special case where $p_2 = 1$, which happens when $k_1 = k_2 = 1$.

When $\lambda > 0$, increasing r_2 increases the price of the price-inelastic good (letter) and decreases the price of the price-elastic good (parcel). Hence $p_1 > p_2$ even though good 2 is twice more costly to deliver than good 1 when $x_1 = x_2$

The variation of k_i's has the same qualitative impact on equilibrium prices for the two configurations of demand elasticities.

Observe that the Ramsey prices with endogenous λ reported in Tables 1 and 2 are also EPMU prices here, because the mark-up over marginal costs is proportional to the same elasticity (with $r_1 = r_2$). Table 5 compares the EPMU prices with the Ramsey prices when the elasticities differ. With EPMU, the mark-up over marginal cost is exactly 100% for both goods. Good 1 is twice as cheap to deliver as good 2 (when $x_1 = x_2$). With $p_1 < p_2$, this cost difference is further magnified because the marginal costs decrease with volume (when $x_1 > x_2$), such that $p_2 = 4p_1$ under EPMU. Hence an EPMU rule for the recovery and allocation of fixed costs leads to a significantly higher cost allocation into good 2 in Table 5, when compared to the results shown in the previous tables.

Table 5: recovery of fixed costs under EMPU, optimal values of p_1 and p_2, with more elastic demand for parcels.

	EPMU
p_1	1
p_2	4
λ	-
Π	0

$k_1 = k_2 = 1, r_1 = 0.3, r_2 = 1$

4. Conclusion

The postal market is changing such that letter volumes are in decline and parcel volumes increasing. Current costing methods may not take account of this change and lead unit costs for parcels increasing disproportionately. As parcel volume increases and letter volume declines it is possible that the portion of fixed cost attributed to parcels increases, stays the same or decreases depending on the specific cost function and its calibration. The analysis shows how this may be addressed through the identification and application of appropriate cost functions.

The first analysis assumes that the marginal costs of letters and packets are separate and decreasing in volume. With letter volumes decreasing and parcel volumes increasing this functionality would increase the unit cost for letters relative to parcels. However, this is a simple analysis. The second analysis introduces two delivery methods (e.g. HCTs and vans) for letters and parcels and a Cobb Douglas functional form, with priority for HCTs given to letters. For a single route, the results have similar properties whereby with letter volumes decreasing and parcel volumes increasing the same functional form for costs would increase the unit cost for letters relatively more than parcels. However, this analysis assumes a single route. Hence, the third analysis, introduces multiple routes and variation in the share of HCT capacity dedicated to letters within a Cobb Douglas functional form. While the analytic calculations are more complex and introduce cross cost elasticities and therefore joint costs, the result again can be obtained whereby with letter volumes decreasing and parcel volumes increasing the functional form would increase the unit cost for letters relatively more than parcels.

Hence, in summary, our analysis identifies the functional form for supply side cost functions that has the property of allocating proportionally more of the fixed cost to letters and less to parcels if market trends continue. This outcome occurs simply through the supply side cost function, that is when the demand elasticities for letters and parcels are assumed to be the same or there is an EPMU mark up on marginal & joint costs. The direction of the cost allocation could be further strengthened from the demand side under Ramsey rules, where the demand elasticity for parcels is greater than that for letters as illustrated in this paper.

A Cobb Douglas cost function is a particular form that assumes constant cost elasticities for the two goods and a particular relationship between those two elasticities. It may be appropriate through empirical analysis to assess whether the cost functions have a Cobb Douglas functional form and what its parameter values look like in practice. Published econometric models in delivery to-date have tended to focus on forms with constant cost elasticities in volume, but have not considered this specific point. Rather than provide numerical estimates this paper assumes values to illustrate through simple examples for the first analysis, and identifies the parameters that would need to be valued from empirical analysis. That is to say, it assists in identifying what to look for within the empirical analysis for the cost function to have the said properties.

In terms of the future development and application of this type of model, its focus here has been on two goods and two delivery methods in outdoor delivery. Outdoor delivery comprises a significant portion of overall USP costs, its daily provision is a significant component within the universal service provision and there is known to be considerable economies of scale and scope. The model could be expanded to consider more goods and delivery methods. Further, consideration could also be given to consider whether Cobb Douglas cost functions apply in other parts of the pipeline. Moreover, consideration also could be given as to whether the USP's goods continue to recover their marginal and joint costs in the presence of competition to the USP from alternative suppliers.

References

De Donder P., Cremer H., and Rodriguez F., (2002), "*Access Pricing and Parcels Delivery*" in Postal and Delivery Services: delivering on competition, ed. Crew and Kleindorfer, Kluwer Academic Publishers.

De Donder P., Cremer H., Dudley P., Rodriguez F., (2008), "*Pricing, welfare and organizational constraints for postal operators*" in Competition and Regulation in the Postal and Delivery Sector, ed. Crew and Kleindorfer, Edward Edgar.

Mautino L., Dudley P., Prettyman J., and Heagney F., (2013), " *Estimating long run incremental costs in the postal sector: A UK perspective*" in Reforming the Postal Sector in the Face of Electronic Competition, ed. Crew and Kleindorfer, Edward Edgar.

Robinson R., and McMurdie J., (2009) , "*Postal costing beyond ABC: estimating the economic cost of mail services*"in Progress in the Competitive Agenda in the Postal and Delivery Sector, ed. M Crew and P R Kleindorfer, Edward Edgar.