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Endogenous Bourse Structures[◇]

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Abstract. We propose a novel approach to the market microstructure theory, where a bourse is a club that facilitates asset trading among its members. Under the new perspective of club theory, we provide an equilibrium setting where traders must belong to at least one bourse to trade assets. For this bourse economy, we show that equilibrium exists generically, and give positive predictions regarding the formation of a large unique bourse, and/or a bourse with complete markets. We also give examples that illustrate how traders' attributes and bourse formation costs explain bourse size and composition, market incompleteness, and multiple memberships.

JEL Classification : D52, D53, G12, G14, G15. G18.

Keywords : Bourse structures; Bourse formation cost; Dark liquidity pools; Demutualization; Market incompleteness; Traders' complementarities.

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1 Introduction

Bourses (or stock exchanges) have origins distant in history. Any group of agents who agree to trade their assets in fact constitutes a bourse. Over time bourses evolved and large trading organizations emmerged. By the 20th century, bourses were linked to their respective national countries, where outsiders were charged high commissions and trading fees in order to gain access to their liquidity. But the evolution of bourses has never been as dramatic as in the last decade. In 2007 the European Commission enacted the Market in Financial Instruments Directive (Mifid) to facilitate competition across the region. The competitors are known as MTFs (Multilateral Trading Facilities). The change in regulation and the new electronic trading technologies are driving the old-style stock exchanges out of existence, and they are being progressively replaced by new global trading organizations. Any bourse can now be created, at convenient low formation costs, to trade any assets with self-picked traders and assets.[1] This possibility was a key element in the emergence of "dark pools of liquidity".[2] The following factors inspire the current research.

Fact Set 1 (Toward a unique global bourse?): In October 2006 the Chicago Mercantile Exchange (CME) and the Chicago Board of Trade (CBOT) created the world's largest futures exchange. In May 2006 the New York Stock Exchange (NYSE) (itself a product of the union between the New York Stock Exchange and American Stock Exchange) acquired Euronext (in turn the result of the merger between Paris, Amsterdam, and Brussels stock exchanges). In April 2007 Deutsche Börse acquired the International Securities Exchange (ISE).

Fact Set 2 (Competition by small exchanges): Competition from small exchanges, offshore centers, banks trading consortia, and dark liquidity pools led to an increase in fragmentation in 2008 when the number of trading platforms increased. By 2009 MTFs accounted for about 20% of the trading in the largest shares across European main markets.

In response to the recent wave of demutualization[3], and knowing the importance of such institutions to the market, we provide an equilibrium model of endogenous bourse

[1] See "Derivatives trading platform bypasses intermediary banks", by Jeremy Grant, in Financial Times, January 17, 2011.

[2] Wealthy market participants trading large blocks of shares have smaller costs if the trading occurs in a "dark pool of liquidity" than in standard "lit" exchanges. Those costs are reduced in dark pools since there is no need of a broker (intermediation cost) and also participants are protected against adverse share price movements since the trades are privately negotiated.

[3] Demutualization is the process where traders move freely from their pre-assigned bourses (e.g., national bourses) to their most preferred ones, without no restriction other than paying the corresponding membership fee.

formation. A central concept is our interpretation of a bourse. Precisely, a bourse is a public good that allows traders to diversify risk by trading its assets with the other members of the bourse. The attractiveness of a bourse is thus evaluated in the light of prices for its assets, which ultimately depend on the complementarities in preferences and endowments among the traders that form the bourse.

We propose an economy with three periods. Traders form bourses in period 0 and trade assets in their respective bourses in period 1. In period 2 there is uncertainty over states of nature, when the assets pay returns. Traders evaluate a bourse in period 0 by the risk sharing possibilities associated with periods 1 and 2. Our distinction among periods is also pertinent as it captures the fact that the acquisition of a bourse membership (in period 0) usually involves a commitment for trading in this bourse for a long period of time.[4] Thus, it is different from the short time activity of asset trading in period 1 - trading is usually achieved as a matter of seconds within a day. In such a framework, it is natural to assume that traders are price takers in their decision to enter a bourse - normally these decisions are not driven by a strategic motive of asset price manipulation.

The model proposed here has its foundations in both the cooperative theory of coalition formation with local public goods, and the theory of general equilibrium with incomplete markets (GEI). Our model of bourse formation borrows from the existence result of Allouch and Wooders [2008] (hereinafter AW) as it allows us to incorporate explicitly in our model important characteristics of the bourse industry: large number of traders, price taking behavior, multiple memberships, competition from small dark pools of liquidity, and increasing gains from trade in larger bourse sizes, with bourses being possibly unbounded in size.[5] Other papers in the club / local public goods literature - for instance, Cole and Prescott [1997] and Ellickson, Grodal, Scotchmer, and Zame [2001], and Wooders [1980, 1978] - do not allow clubs to be unbounded in size). There are other papers that do allow unbounded club sizes and also consider price-taking equilibrium – see Wooders [1983, 1997] and, more indirectly, Wooders [1994]. Allouch, Conley, and Wooders [2009] also allow unbounded club sizes, but require that all gains can be realized by coalitions strictly bounded in size.

The application of AW's existence proof to our bourse economy is not immediate,

[4]Usually, the exchange participant stays in the bourse since its year of accession. See, for example, the MICEX list of participants: http://www.micex.com/markets/stock/members/list.

[5]In this setting, ever-increasing gains from trade in larger bourses (the larger the bourse, the smaller are the bourse formation per capita cost) is in fact a possibility, but it is not self-imposed in the model. By considering a large number of traders we can study whether large and small bourses can co-exist in equilibrium.

however, since some of AW's assumptions rely on the individual's utility function being evaluated on the club good. In our paper the utility that the public good (bourse) provides to the traders is endogenous, as it depends on the traders' evaluation of the risk sharing possibilities achieved in equilibrium in the different bourses. Thus, the technical hurdle to reconcile our model of bourse formation with that of AW is to show that there exists an open and dense set of economies where the trading equilibrium for any given bourse structure is continuous in traders' utility and endowment attributes. But our equilibrium model focuses more than equilibrium existence. We provide a sufficient condition for the emergence of both a large unique bourse, and/or a bourse with complete markets. We also give several examples that illustrate how traders' attributes and bourse formation costs explain the different aspects of bourse formation, such as bourse size and composition, market incompleteness, and multiple memberships.

To the best of our knowledge, this paper is the first that seeks to analyze the specific market micro-structure of trading in bourses under the lens of club theory. In particular, we contribute with a different viewpoint to the market microstructure literature that analyzes the issues of concentration and fragmentation of trade across markets - see Pagano [1989][6] and related works - and the impact of trading costs on trading behavior[7]. In our opinion, this club theory approach to finance is powerful for obtaining new insights on the functioning of financial markets, in the same way that the theory of networks and search theory contribute so much to the understanding of different issues in finance. This novel approach provides a useful framework to analyze important issues, such as why do diverse financial market structures exist (included incomplete ones) and what are their welfare implications? More specifically, we provide a micro-founded justification of the emergence of large trading platforms (like the NYSE-Euronext) and also show how, in some cases, small exchanges are ill-suited to certain types of situations.

The outline is as follows. The model is presented in Section 2. Section 3 states the equilibrium concept for our bourses economy, establishes the existence result, and provides an equilibrium characterization. The key technical contribution relative to earlier literature is presented at the end of this section. In Section 4 we provide several tractable numerical examples. Section 5 concludes. The Appendix is devoted for the proofs.

[6] By fully characterizing non-anonymous traders, we depart from Pagano [1989] and related works in that we allow for every possible subset of traders to form a bourse, which contrasts with the two-bourse analysis of Pagano. We also depart from Pagano in that we do not limit our analysis to only one stock (see Pagano's fn. 3). Instead, we consider different asset structures, possibly incomplete, with more than one asset.

[7] See, for example, Lo, Mamaysky, and Wang [2004] for an equilibrium model with fixed transaction costs.

2 Bourse economies

2.1. *Trading*

The economy lasts for three periods, 0, 1, and 2. In period 0 traders form bourses; in period 1 asset trading occurs in each bourse[8]; and in period 2 assets pay returns. The set of states of uncertainty in the last period is $\mathbf{\Xi} \equiv \{1, ..., \Xi\}$, with representative element ξ. In each of the three periods all consumers trade commodities in a common market. Traders are assumed, for all effects, to be price takers in all periods. The set of commodities is $\mathbf{L} \equiv \{1, ..., L\}$, with representative element l. The number of state-time contingent goods is then $(2 + \Xi)\, L$. Commodities are traded at prices $p = (p_0, p_1, p(1), ..., p(\Xi)) \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ \times \mathbb{R}^{L \times \Xi}_+$, where p_0, p_1, and $p(\xi)$ are the price vectors at dates 0, 1, and 2 (state ξ), respectively. Similarly, p_{l0}, p_{l1}, and $p_l(\xi)$ are the good l prices at dates 0, 1, and 2 (state ξ), respectively.

An exchange participant or *trader* is a corporation that may trade on or through the exchange and is licensed under the ordinance of the corresponding exchange financial regulator to carry on asset trading activity. The set of traders is $\mathbf{I} \equiv \{1, ..., n\}$, with n assumed to be large but finite. Let $\mathbf{\Theta}$ be the set of traders' characteristics, endowed with a metric d. An element $\theta \in \mathbf{\Theta}$ describes the endowments and preferences of a trader of this type. We assume that traders' characteristics are observable. Let $\alpha : \mathbf{I} \to \mathbf{\Theta}$ be an attribute function, with $\alpha(i) = \theta$ describing trader i's endowments and preferences.[9] Then, an economy is represented by a pair $(\mathbf{I}, \alpha)$.

Trader i is endowed with a finite positive vector of private commodities $\omega^i = (\omega^i_0, \omega^i_1, (\omega^i(\xi), \xi = 1, ..., \Xi)) \in \mathbb{R}^L_{++} \times \mathbb{R}^L_{++} \times \mathbb{R}^{L\Xi}_{++}$. We assume that the total endowment of commodities is finite, that is, $\sum_{i \in \mathbf{I}} \omega^i < \infty$. We denote by $x^i = (x^i_0, x^i_1, (x^i(\xi), \xi = 1, ..., \Xi)) \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ \times \mathbb{R}^{L\Xi}_+$ trader i's consumption bundle. Then, let $x^I_0 \equiv (x^i_0 \in \mathbb{R}^L_+ : i \in \mathbf{I})$ denote traders' consumption bundles in period 0, and $x^I = (x^i \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ \times \mathbb{R}^{L\Xi}_+ : i \in \mathbf{I})$ denote traders' consumption bundles in the three periods. Let $u^i(x)$ denote trader i's utility function defined on the consumption bundle $x \in \mathbb{R}^{(2+\Xi)L}_+$. In order to introduce a temporal distinction between period 0 (when traders choose bourses) and periods 1 and 2 (when traders already belong to bourses) we assume that the utility function is separable

[8] For simplicity we assume that trading occurs only once in period 1, although period 1 could have been modeled as a period that permits multiple trading rounds.

[9] Our non-anonymous analysis below shows how traders' attributes determine trading behavior. Thus, this paper departs from those models (e.g., Pagano [1989] and related works) that assume traders' decisions to depend on the first and second order moments.

as follows[10]

$$u^i(x_0, x_1, x(1), ..., x(\Xi)) = u_0^i(x_0) u_1^i(x_1, x(1), ..., x(\Xi)). \quad (1)$$

The utility function proposed in (1) incorporates the idea that traders have an economic activity at date 0, and thus some consumption and wealth, before entering in a bourse to exchange its assets. Let us now impose the following assumptions.

(A1.i) For all $i \in \mathbf{I}$ and $l \in \mathbf{L}$, $\omega_{l0}^i > \tau$ with $\tau > 0$, and given $\varepsilon > 0$, there exists $\lambda > 0$ such that for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \lambda$ for any i, then $\omega_0^{\alpha(i)} \leq \omega_0^{\beta(i)} + \varepsilon \bar{1}$, where $\bar{1} = (1, ..., 1) \in \mathbb{R}^L$. u_0^i is continuous, increasing, and strictly quasiconcave.

Assumption (A1.i) requires the endowments to be uniformly bounded away from 0, and for near economies traders' endowments should not differ significantly. The assumptions on trader i's utility at period 0 are standard.

(A1.ii) u_1^i is twice continuous differentiable, increasing, and has the matrix of second derivatives $(D^2 u_1^i)$ negative semidefinite.

Assumption (A1.ii) on trader i's utility evaluated at the consumption in periods 1 and 2 is standard. Notice that we do not need to assume joint concavity of u^i as our equilibrium notion does not require this (see below).

(A1.iii) $u_0^i(x_0) = 0$ if there exists a commodity $l \in \mathbf{L}$ with $x_{l0} = 0$.

Assumption (A1.iii) is merely technical and needed in the proof of Lemma 3 (in the Appendix) to guarantee that the condition "Desirability of endowments" in AW is satisfied.

(A1.iv) Given any attribute θ and any $\varepsilon > 0$, there is $\rho_\varepsilon^\theta > 0$ such that, for all $i \in \mathbf{I}$ with $\alpha(i) = \theta$ and all $x_0^i \in \mathbb{R}_+^L$, $u_0^i(x_0^i)\delta + \rho_\varepsilon^\theta < u_0^i(x_0^i + \varepsilon \bar{1})\delta$ holds, where $\delta = \min_{i \in \mathbf{I}} u_1^i(\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)) > 0$.

Assumption (A1.iv) says that private goods in period 0 are valuable.

(A1.v) Given $\varepsilon > 0$, there exist $\lambda_\varepsilon > 0$ and $\gamma > 0$, such that, for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \gamma$, then $u_0^{\alpha(i)}(x_0^i) + \lambda_\varepsilon < u_0^{\beta(i)}(x_0^i + \varepsilon \bar{1})$, for any i and any $x_0^i \in \mathbb{R}^L$.

Assumption (A1.v) says that private goods in period 0 are valuable for near economies.

[10] This specific functional form is considered only for presentation purposes. We emphasize that other types of functional representations of u_0^i and u_1^i are also admissible. However, we point out that a form $u_0^i + u_1^i$ is not interesting, as the decision of choosing a bourse does not reflect the trading opportunities associated with a bourse.

(A1.vi) Moreover, for $\varepsilon > 0$ and for all $\theta \in \mathbf{\Theta}$ with $\alpha(i) = \theta$, $u_0^{\alpha(i)}(\sum_i \omega_0^{\alpha(i)} + \varepsilon \bar{1})$ is bounded above by some $\bar{u}_0$, and $u_1^{\alpha(i)}(\omega_1^{\alpha(i)}, \omega^{\alpha(i)}(1), ..., \omega^{\alpha(i)}(\Xi))$ is uniformly bounded below from some $\underline{u}_1 > 0$.

Assumption (A1.vi) says that utility u_0^i evaluated in a vector above the aggregate endowments does not explode, and that the utility u_1^i evaluated at the trader's endowments is also uniformly bounded but now from below. Assumptions (A1.iv), (A1.v) and (A1.vi) are needed in the proof of Lemma 3 below to prove that Assumption (f) in AW holds, which in our model requires the trader's utility to be continuous in traders' attributes. Notice that Assumption (A1.i), which characterizes endowments and utility in period 0, already appeared in AW. However, the remaining assumptions in (A1) differ slightly. To see this, notice that in our economy the club good associated with each bourse, interpreted here as the assets trading facility, is endogenous. This implies that the trader's utility attained in each bourse structure is endogenous to the model, whereas in AW the trader's utility attained in each coalition structure is pre-defined. Thus, we must impose assumptions on our primitives, given the utility functional form specified in (1), in such a way that AW's assumptions hold (we refer the reader to the proof of our Lemma 3 in the Appendix, for a more explicit relationship between our assumptions and those of AW).

2.2. *Bourse structures*

The set of assets available in the economy is finite and denoted by $\mathbf{J} \equiv \{1, .., J\}$. $\mathcal{J}$ denotes the set of all possible nonempty subsets of $\mathbf{J}$. Each bourse has associated some assets for trading. To each coalition $S \subseteq \mathbf{I}$, we assign a set $J(S) \subseteq \mathcal{J}$, which consists of the different types of assets that are available for trade in bourse S.[11] Then, there is a mapping $S \to A(S)$, where $A(S) = [a_j(\xi)]_{\Xi \times J(S)}$ is the payoff matrix describing the returns at each state of nature of the different assets in $J(S)$. We assume that assets pay in the numeraire commodity, taken here to be good L. The returns matrix $A(S)$ is assumed to have full-column rank for each bourse S. We denote a bourse by the pair $(S, A(S))$. A bourse is thus a club good that allows traders to diversify their risks by offering the specific activity of asset trading. As such, a bourse becomes a source of liquidity.

A *bourse structure* is given by $F(\mathbf{I}) = \{(S_1, A(S_1)), ..., (S_k, A(S_k)), ..., (S_K, A(S_K))\}$. We denote by $\mathbf{F}(\mathbf{I})$ the set of all possible bourse structures. Given two bourse structures $F(\mathbf{I}) = \{(S_k, A(S_k))\}_{k=1}^{K}$ and $F(N) = \{(S'_k, A(S'_k))\}_{k=1}^{K'}$, with $N \subset \mathbf{I}$, let us denote a feasible deviation from $F(\mathbf{I})$, by a set of traders N, by $\tilde{F}_{F(N)}(\mathbf{I}) = F(N) \cup \{(S_k \cap (\mathbf{I} \backslash N)),\ A(S_k \cap (\mathbf{I} \backslash N))\}_{k=1}^{K}$ (i.e., the bourse structure that contains $F(N)$ and also those

[11] This set of assets is exogenously given for bourse S, and can be thought as the assets that traders in bourse S agree to issue for posterior trading.

bourses in $F(\mathbf{I})$ but without those traders belonging to N). In the equilibrium Definition 2 below, we will use the bourse structure $\tilde{F}_{F(N)}(\mathbf{I})$ as the feasible deviation for the set of traders N. Notice that the set of feasible deviations that we consider is large in the sense that we allow the traders in N to form any type of bourse structure among them.

Traders can belong to several bourses, and therefore, a bourse structure is not a partition; it is possible that $S_k \cap S_{k'} \neq \emptyset$. We require the number of bourse memberships of every trader to be bounded (as in AW) - naturally justified if bourse formation is expensive-. In order to guarantee that we can always find a trader for whom the assets are not collinear among different bourses, we require that, for any bourse structure, every bourse has always a trader with no memberships in other bourses. Now, given an economy $(\mathbf{I}, \alpha)$ and a bourse structure $F(\mathbf{I})$, let us denote by $F[i;\mathbf{I}] = \{S_k \in F(\mathbf{I}) : i \in S_k\}$ the set of all bourses in $F(\mathbf{I})$ that contain trader i. Trader i can only trade assets with those traders in $F[i;\mathbf{I}]$. This implies that, for diversifying risks, traders not only choose a bourse because of its assets available for trade, but also because of the other traders' wealth and preferences. We denote by y_j^i the trader i's position on asset $j \in J(S)$ in period 1, with $S \in F[i;\mathbf{I}]$. As usual, $y_j^i > 0$ denotes a purchase of asset j and $y_j^i < 0$ denotes the sale of this asset. Let us denote $y^I = (y^i \in \mathbb{R}^{J(S_k)} : S_k \in F[i;\mathbf{I}], i \in \mathbf{I})$. The prices associated with these assets in S are denoted by $q(S) \in \mathbb{R}_+^{J(S)}$. Notice that our notion of a bourse is substantially different from an over-the-counter (OTC) market. In OTC markets, trading occurs bilaterally (usually between a trader and an intermediary) and an asset can have a different price depending on the bargaining of each couple of traders. Instead, here we model a bourse S as a central market where the asset prices $q(S)$ are common to all traders in that bourse (like the Tradingpoint Stock Exchange, that provides direct access to investors, without the need of intermediaries).

2.3. *Bourse formation costs, communication costs, and transaction fees*

In order to adopt the technology of asset trading and build the trading platform, the bourse S faces (fixed) formation costs $z(S) \in \mathbb{R}_+^L$ (in terms of inputs of the private goods, e.g., hardware costs, software costs, or installation charges)[12]. Asset market characteristics of bourse S are given by the pair $(A, z)(S)$. Let us now impose the following continuity condition on z with respect to attributes, needed for our existence result below. There, S^α and S^β are the bourses comprising the same traders, but characterized by attributes α and β, respectively.

(A2) Given $\varepsilon > 0$, there exists $\lambda > 0$ such that, for any bourse S and any attribute

[12] See the HKEx security trading infrastructure at
http://www.hkex.com.hk/eng/market/sec_tradinfra/CMTradInfra.htm

functions α and β, if $d(\alpha(i), \beta(i)) \leq \lambda$ for every $i \in S$, then $z^{\alpha}_{S^{\alpha}} \leq z^{\beta}_{S^{\beta}} + \varepsilon\bar{1}$.

To fulfill the participantship requirement, all exchange participants are required to hold a bourse membership (or trading right) of the respective exchanges[13]. The membership fee that a trader i pays to participate in a bourse S, denoted by $\pi^i(S)$, is paid in units of account and is dependent on trader's own type $\alpha(i)$ (e.g., broker-dealer and retail investor participants). A participation price system is a set $\Pi = \{\pi^i(S) \in \mathbb{R} : i \in S \text{ and } S \subset \mathbf{I}\}$. According with our interpretation of a bourse as a club that provides the assets trading facility (local public good), we allow membership fees to be non-anonymous, so that these fees reflect the traders' valuation of each bourse.[14]

For simplicity we do not model trading fees, whose importance has declined substantially since the implementation of the Mifid regulation.[15] We consider transaction fees instead, in the form of a transaction levy and a stamp duty, currently at the center of the debate among the leaders in the European Union. We denote the transaction fee for trading an amount y_j of asset j by $g_j(y_j)$. Transaction fees are denominated in units of the numeraire commodity L, and are paid by both sides (buyers and sellers) to a financial regulator.[16] We assume that $g_j : \mathbb{R} \to \mathbb{R}_+$ is twice continuous differentiable (C^2) on y_j, increasing in $|y_j|$, and convex in y_j (i.e., $g'' > 0$). Thus, g has a U-shaped form. Indeed, transaction fees can be non-linear. Typically, most traders pay a constant price per order, but some traders doing a high volume can sign up for a different offer involving a fixed part and a lower unit price. The debate has focused on the linear part, however. But with the increase in competition among exchanges, non-linear transaction fees are more important. From a theoretical point of view, we observe here that the non-linearity of the transaction fee guarantees that the traders' demand functions are smooth in a context with collinear assets among different bourses (for this result, trading fees do not need to be large). Finally, we assume that traders face communication costs in period 0 if they wish to move to different bourses. These communication costs for a bourse of size $|S|$ are given by $c(\varepsilon_0) = \varepsilon_0|S|\bar{1}$, where $\varepsilon_0 > 0$.

[13]As stated in the Hong Kong Exchange (HKEx) rules, any broker-dealer intending to operate a brokerage business for products available on HKEx, using the trading facilities of the Stock Exchange and/or Futures Exchange, must be admitted and registered as an Exchange Participant of that Exchange. This membership fee is HK$500,000 (US$64,100).

[14]Notice that anonymous membership fees can easily be adapted to our theory by making $\pi^i(S) = \pi(S)$ for all $i \in S$, where $\pi(S) = \frac{p_0 z(S)}{|S|}$ is the poll tax in bourse S.

[15]See Colliard [2011] for a survey on this literature and for an analysis of the trading fees on the efficiency of the markets.

[16]For the HKEx the transaction levy is 0.003% of the consideration of the transaction, whereas the stamp duty is 0.001% on the value of stock transactions. These fees are paid to the Securities and Futures Commission and government, respectively.

2.4. *Budget constraints*

Given prices p_0 and π^i, trader i's budget constraint at period 0, when he chooses a set of bourse memberships, is

$$p_0(x_0 - \omega_0^i) + \sum_{S_k \in F[i;\mathbf{I}]} \pi^i(S_k) \leq 0. \tag{2}$$

Observe that the communication costs do not enter in the trader's period 0 budget constraint since these are paid only if the trader decides to move to another bourse (see also AW). The profits of bourse S are given by

$$\sum_{i \in S} \pi^i(S) - p_0 z(S). \tag{3}$$

Given prices p_1 and q, the budget constraint in period 1, once trader i has chosen the bourses he wishes to belong to, is

$$p_1(x_1 - \omega_1^i) + \sum_{S_k \in F[i;\mathbf{I}]} \sum_{j \in J(S_k)} (q_j y_j + p_{L1} g_j(y_j)) \leq 0. \tag{4}$$

Given prices $p(\xi)$, trader i's budget constraint in period 2 and node $\xi \in \mathbf{\Xi}$ is[17]

$$p(\xi)(x(\xi) - \omega^i(\xi)) \leq p_L(\xi) \sum_{S_k \in F[i;\mathbf{I}]} \sum_{j \in J(S_k)} a_j(\xi) y_j. \tag{5}$$

3 Equilibrium

Consistent with our argument that the acquisition of bourse membership (in period 0) usually involves a commitment for trading in that bourse for a long period of time, whereas the asset trading activity occurs constantly (in a matter of seconds) in the bourse that the trader belongs to, we distinguish between the evaluation of bourses (for each bourse structure traders assess the risk sharing attained in their respective bourses) and the formation of bourses (bourses are formed given these evaluations).

3.1. *Bourse evaluation*

Definition 1 (*Asset trading equilibrium for a given bourse structure*): Given the bourse structure $F(\mathbf{I})$, a price taking asset trading equilibrium consists of a system $(x_1^I, x^I(1), ..., x^I(\Xi),\ y^I, p, q)(F(\mathbf{I}))$, such that,

[17] Observe that in period 2 traders' default is not allowed. See Santos and Scheinkman [2001] for a leading model with default and two clearing-houses.

(D1.i) given trader's bourse memberships $F[i;\mathbf{I}]$, the trader chooses optimally his commodities and assets positions, that is, $(x_1^i, x^i(1), ..., x^i(\Xi), y^i)(F(\mathbf{I})) \in \arg\max u_1^i(x_1, x(1), ..., x(\Xi))$, subject to constraints (4) and (5).

(D1.ii) commodity markets clear at periods 1 and 2, i.e.

- $\sum_{i\in\mathbf{I}}(x_{l1}^i - \omega_{l1}^i) = 0$, for all $l \neq L$.
- $\sum_{i\in\mathbf{I}}(x_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j\left(y_j^i\right) - \omega_{L1}^i) = 0$.
- $\sum_{i\in\mathbf{I}}\left(x_l^i(\xi) - \omega_l^i(\xi)\right) = 0$, for all l and $\xi \in \boldsymbol{\Xi}$.

(D1.iii) the asset market clears *for each bourse*, i.e. $\sum_{i\in S_k} y_j^i = 0$, $\forall j \in J(S_k)$, $\forall S_k \in F(\mathbf{I})$.

We denote the set of asset trading equilibria, given a bourse structure $F(\mathbf{I})$, by $E(F(\mathbf{I}))$.

3.2. *Bourse formation*

First, observe that, given the bourse structure $F(\mathbf{I})$, our specification of the utility function (1) allows us to define the trader i's utility in period 0 via an equilibrium point[18] $\tilde{x}(F[i;\mathbf{I}]) \equiv (x_1, x(1), ..., x(\Xi))(F[i;\mathbf{I}])$ as follows

$$V^i(x_0, F[i;\mathbf{I}]) \equiv u_0^i(x_0) U_1^i\left(F[i;\mathbf{I}]\right) \tag{6}$$

where

$$U_1^i\left(F[i;\mathbf{I}]\right) \equiv u_1^i(\tilde{x}(F[i;\mathbf{I}])) \tag{7}$$

denotes the trader i's indirect utility, the utility $u_1^i(\cdot)$ evaluated at the equilibrium point $\tilde{x}(F[i;\mathbf{I}])$. Observe that the evaluation of trader's bourse memberships enters indirectly into his utility u^i through the access to income and the risk sharing that he gains from trading the securities offered in those bourses he belongs to. The following proposition asserts that the utility evaluated at the assets trading equilibrium $\tilde{x}\left(F[i;\mathbf{I}]\right)$ is continuous in trader's attributes. There, we write $\mathbf{I}^\alpha$ to refer to an economy $(\mathbf{I}, \alpha)$, and call an open and dense set with null complement a generic set.

Proposition 1: *There exists a generic set of economies for which, given* $\lambda > 0$, *there is a* $\gamma > 0$ *such that, for any pair of economies* $(\mathbf{I}, \alpha)$ *and* $(\mathbf{I}, \beta)$, *if* $d(\alpha(i), \beta(i)) \leq \gamma$ *for any* i, *then* $|U_1^{\alpha(i)}\left(F[i;\mathbf{I}^\alpha]\right) - U_1^{\beta(i)}\left(F[i;\mathbf{I}^\beta]\right)| < \lambda$.

Condition (f) in AW requires that traders who are similar in attribute space are near-substitutes in the economy. For our bourse economy this is not a trivial issue, as the public good in our paper is endogenous to the model, and this makes the utility $u_1^i(\tilde{x}(F[i;\mathbf{I}])$

[18] We consider a measurable selector from the equilibrium correspondence $E(F(\mathbf{I}))$ (see the Appendix).

dependent on the assets trading equilibrium. In Proposition 1 we demonstrate that there exists an open and dense set of traders' attributes (endowments and utilities) for which the assets trading equilibrium is continuous in traders' attributes. This proof is the most critical point for the application of AW's existence result to our economy. The proof of Proposition 1, in the Appendix, makes use of the Transversality theorem. For simplicity, we consider a finite dimensional manifold of utility functions.[19]

It is also important to note that $V^i(x_0, F[i;\mathbf{I}])$ may represent ever-increasing gains from trade in larger bourse sizes, depending on how $u_0^i(x_0)$ and $u_1^i\left(\tilde{x}(F[i;\mathbf{I}])\right)$ interrelate in the functional form dictated by $u^i(x_0, \tilde{x}(F[i;\mathbf{I}]))$. In order for this possibility to be compatible with equilibrium existence, we need to impose the following assumption:

(A3) There is a bundle of goods $x_0^* \in \mathbb{R}_+^L$ such that for any economy $(\mathbf{I}, \alpha)$, any consumer $i \in \mathbf{I}$, and any $x^i \in \mathbb{R}_+^L \times \mathbb{R}_{++}^L \times \mathbb{R}_{++}^{L\Xi}$, we have

$$u_0^i(x_0^i + x_0^*)u_1^i(x_1^i, x^i(1), ..., x^i(\Xi)) \geq u_0^i(x_0^i)u_1^i\left(\sum\nolimits_i \left(\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)\right)\right).$$

Assumption A3 permits ever-increasing gains from larger and larger "bourses" while, at the same time, allows for small "bourses". The assumption says that, even in the worst scenario where trader i cannot diversify risk in any bourse and, as a consequence, may consume an inferior bundle $(x_1^i, x^i(1), ..., x^i(\Xi))$, the trader prefers to consume a very large amount of private goods in period 0, $x_0^i + x_0^*$, rather than consuming the total endowments in periods 1 and 2. Notice that the consumption $x_0^i + x_0^*$ can be very large and even unfeasible for the trader - we require only the existence of such a large bundle x_0^*. Assumption A3 assures that the "Desirability of wealth" assumption of AW holds for our economy.

Definition 2 ($c(\varepsilon_0)$-*equilibrium of the bourse formation*): A price taking $c(\varepsilon_0)$-bourse structure equilibrium for period 0 is an ordered triple $((x_0^I, F(\mathbf{I})), p_0, \Pi)$ that consists of an allocation of commodities x_0^I, a bourse structure $F(\mathbf{I})$, a commodity price vector p_0, and a participation price system Π such that,

(D2.i) $\sum_{i\in\mathbf{I}}(x_0^i - \omega_0^i) + \sum_{S_k\in F(\mathbf{I})} z(S_k) \leq 0$.

(D2.ii) For each $S \subset \mathbf{I}$, profits are non positive, i.e., $\sum\limits_{i\in S} \pi^i(S) - p_0 z(S) \leq 0$.

(D2.iii) for each $i \in \mathbf{I}$, any $N \subset \mathbf{I}$ with $i \in N$, and any bourse structure deviation $\tilde{F}_{F(N)}[i;\mathbf{I}]$, if $V^i(y_0^i, \tilde{F}_{F(N)}[i;\mathbf{I}]) > V^i(x_0^i, F[i;\mathbf{I}])$, then $p_0(y_0^i - \omega_0^i) + \sum\limits_{S_k\in\tilde{F}_{F(N)}[i;\mathbf{I}]} \pi^i(S_k) > -\varepsilon_0 p_0 \bar{1}$.

[19]Extending our framework to consider instead an infinite dimensional manifold of utility functions would complicate substantially the proofs, and thus is left for some future research.

The equilibrium condition (D2.iii) says that, if a set of traders N deviates and forms another bourse structure with the remaining traders of $\mathbf{I}$ staying in the same bourses as in $F(\mathbf{I})$ but without the traders of N, then the budget constraints of those traders in N are violated in excess of the communication costs. These communication costs consist of small frictions in the economy, which affect the opportunities to change bourse memberships. Also, notice that $V^i(y_0^i, \tilde{F}_{F(N)}[i;\mathbf{I}])$ is well defined since it is the indirect utility evaluated at the trading equilibrium associated with the bourse structure $\tilde{F}_{F(N)}(\mathbf{I})$. Since the equilibrium selection $\tilde{x}(\tilde{F}_{F(N)}[i;\mathbf{I}])$ is defined for any bourse structure that contains all traders in $\mathbf{I}$ we have that it is uniquely defined. Finally, the off-equilibrium deviating bourses that we consider belong to the bourse structure $\tilde{F}_{F(N)}(\mathbf{I})$. Such deviating bourses have always associated a trading equilibrium, and therefore are feasible.

An additional equilibrium condition in Definition 2 would be to require that most consumers cannot be very far outside their budget constraints. As AW remark, this condition can be derived from conditions of the model and other parts of the definition of equilibrium for the bourse formation process. Therefore, we omit such condition in Definition 2. It may also occur that, depending on the composition of the set of traders, some traders cannot be accommodated in their preferred bourses. AW show that if the economy is large, then these traders constitute only a small proportion of the total population. For that, AW accommodate the equilibrium notion by taking into account these reminders. We prefer to avoid further notation and refer to the original paper for such refinement.

3.3. *Equilibrium for the bourse economy*

Finally, we introduce the equilibrium concept associated with the bourse economy proposed here.

Definition 3 ($c(\varepsilon_0)$-*equilibrium of the bourse economy*): We say that the vector $(((x_0^I, F(\mathbf{I}), p_0, \Pi), (x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)(F(\mathbf{I})))$ constitutes a price taking $c(\varepsilon_0)$-equilibrium for our bourse economy if

(D3.i) $(x_0^I, F(\mathbf{I}), p_0, \Pi)$ is a $c(\varepsilon_0)$-equilibrium for the bourse formation.

(D3.ii) $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)(F(\mathbf{I}))$ is an asset trading equilibrium for $F(\mathbf{I})$.

Observe that for a given bourse structure $F(\mathbf{I})$ there can be more than one asset trading equilibrium. It is well known that different beliefs among traders on the equilibrium realizations may lead to a problem of non-existence. To avoid this possibility we impose the standard "rational expectations hypothesis" (see Dutta and Morris [1997]); that is, traders agree on the realization of prices at each state (consensus) and simultaneously believe that there is a single possible price in each state (degenerate beliefs). No information problems are considered here. Thus, traders' beliefs about the realization of prices in each state are self-fulfilling.

3.4. *Existence*

Theorem 1: *Let us assume A1, A2, and A3. If there are sufficiently many traders with attributes represented in the economy, there exists a generic set of bourse economies for which there is a price taking $c(\varepsilon_0)$-equilibrium with possibly ever-increasing gains from larger bourses.*

The proof of Theorem 1 is left for the Appendix. In this proof we first show that a trading equilibrium exists given a bourse structure. Notice that we do not require short sales to be bounded. A subtlety in this part of the proof is that market clearing no longer occurs for the whole economy (as in previous general equilibrium models), but occurs in each bourse.[20] In the second part of the proof we show that there exists a measurable selector of the trading equilibria (recall that there can exist more than one), at which traders evaluate their bourse memberships. Finally, we show that a bourse structure equilibrium exists, given the bourse evaluation at the selected trading equilibrium.

The existence result of a $c(\varepsilon_0)$-*equilibrium of the bourse formation* relies on AW [2008, Theorem 2], which says that: *under appropriate assumptions* (namely, (a)-(h) and "desirability of wealth" - see our proof of Lemma 3 below)*, there exists a* ε_0*-price taking equilibrium with communication costs if there exist sufficiently many players with attributes represented in the economy* (*that is, with attributes in the range of* $\alpha(\cdot)$).

Technical contribution: The application of AW's result to our economy is not immediate. We need to assure that all assumptions required in AW [2008, Theorem 2] are satisfied. The trickiest one is their assumption (f) ("utility of a bourse structure is continuous on the attributes"; see proof of Lemma 3 in the Appendix). This cannot be an assumption in our model because the utility that a trader obtains in a bourse (public good) is endogenous in our setup. Proposition 1 proves continuity on traders' attributes for a generic set of economies. In this proof we follow the lines of Geanakoplos and Polemarchakis [1986, Section 6]. There are three difficult steps in this proof. The first is that we have to show that the assets trading equilibrium is a continuous differentiable function of commodity and asset prices. The second difficulty has to do with equilibrium regularity. Precisely, we have to show that, there exists a generic set of endowments and utilities,

[20] Neither does the related field of security design properly model bourses as the place where traders issue and trade securities.

such that the set of asset trading equilibria is a continuously differentiable function of the endowment and utility assignment. Our framework is however different than Geanakoplos and Polemarchakis [1986], as we must adapt their proof to our economy where: 1) the asset market clearing occurs in each bourse; 2) the trading period accounts for assets and commodities; and 3) trading of assets involve paying a non-linear transaction fee (in order to come across the collinearity of asset returns among different bourses), which is itself a function of the assets trading. The third difficulty is that, for each economy in a generic set of endowments and utilities, we have to find a compact subset (this set exists because the generic set is open), and use this set as the compact set used by AW to extend replica economies.[21]

3.5. *Positive predictions*

In this paper we are also able to provide a sufficient condition for the formation of a unique large bourse, and also for the formation of a bourse with complete markets. For this, we consider a framework with one commodity and single membership. We shall take as given the traders' indirect utilities derived from trading in their respective bourses.[22]

Sufficient condition (SC): *For any S and S^*, and $\alpha \in \mathbb{R}$, the following inequality holds:*

$$z(S^*)\sum_{i\in S}\left(\frac{U_1^i(S^*)}{U_1^i(S)}\right)^{\alpha} - z(S) \leq \sum_{i\in S}\left(\left(\frac{U_1^i(S^*)}{U_1^i(S)}\right)^{\alpha} - 1\right)\omega_0^i$$

Condition (SC) becomes less binding the higher is the relative value of bourse S^* with respect to bourse S $(U_1^i(S^*)/U_1^i(S))$, and the lower is difference between bourse S^* formation cost $(z(S^*))$ and bourse S formation cost of $(z(S))$. By bourse S^* we refer either to a large unique bourse or to a bourse with complete markets, whereas by bourse S we refer either to a small bourse or a bourse with incomplete markets, respectively.

Proposition 2: *Let us assume that, for all $i \in \mathbf{I}$, the inverse of u_0^i is homogeneous of degree α, and also that, for any bourse $\tilde{S} \subseteq \mathbf{I}$, each trader' endowment does not exceed the cost of forming a bourse (i.e., for all i, $\omega_0^i \leq z(\tilde{S})$). Then,*

(2.1) *a large unique bourse S^*, with $|S^*| = |\mathbf{I}|$, forms in equilibrium if condition (SC) holds, for any bourse S with $|S| < |\mathbf{I}|$.*

(2.2) *a bourse with complete markets S^* forms in equilibrium if condition (SC) holds, for any bourse S with incomplete markets.*

[21] AW's result relies on the use of a compact set of attributes where the continuity property on traders' attributes for a generic set of economies holds.

[22] In the examples below, we show how bourse members' attributes and bourse structure determine the indirect utility $U_1^i\left(F[i;\mathbf{I}]\right)$.

On the one hand, Result (2.1) in Proposition 2 is interesting as it gives a prediction - based on the relationship among the technology of bourse formation, traders' endowments, and traders' valuation of bourses - for the formation of a unique global bourse. Thus, this result helps to clarify which conditions will make trading in a single global exchange a natural monopoly[23]. On the other hand, Result (2.2) in Proposition 2 gives a sufficient condition for markets to be complete, and thus for the *asset trading equilibrium* to be efficient.[24] Moreover, we can also assert that *equilibrium of the bourse formation process* is also efficient, following the result of AW, who show that a ε_0-equilibrium is in the core for an economy with sufficiently many traders of each attribute. Therefore, an $c(\varepsilon_0)$-*equilibrium of the bourse economy* is efficient if condition (SC) holds. This result contributes to the recent literature on endogenous market completeness.[25]

4 Complementarities, technology, and optimal bourse structures: Examples

Next, we provide several examples that illustrate the various trade-offs among the bourse' formation costs, trading complementarities, and the inherent asset structure of the bourses. Example 1 illustrates how the bourse structure affects welfare and trading volume through varying complementarities among different sets of traders. Motivated by Fact Set 1, Example 2 shows that large bourses form in equilibrium when bourse formation costs are proportional in size, as long as there exist good complementarities among traders. Example 3 provides the opposite case, pointed out in Fact Set 2, where small bourses provide more welfare given the bad complementarities in a larger bourse, with formation cost again proportional in size. Example 4 provides a case with multiple memberships, and compares different bourse structures for different technology scenarios. Finally, Example 5 focuses on the endogeneity of the market incompleteness and the related inefficiency.

All examples below are valid for any N-replication of these economies (see explanations in Example 3). In these examples, for simplicity the transaction fee is made equal to zero, as the main insights of the examples remain valid. Our terminology should give

[23] "*Liquidity and technology will inevitably make trading a natural monopoly*". See The Economist, March 25, 2006 (http://www.economist.com/node/6978712).

[24] The issue of the optimality of the equilibrium is related to previous results in the literature of general equilibrium with incomplete markets. A well known argument is that equilibrium efficiency is "constrained" to the market incompleteness. This result naturally extends to our asset trading equilibrium.

[25] See Carvajal, Rostek, and Weretka (2012) for a model of competition in financial innovation and a result on the endogeneity of market completeness, and references therein.

no space to confusion since we always give a name to each set of numbers, indicating its nature. For example, the set $S^1 = (1, 2)$ indicates a bourse with traders 1 and 2, the set $\Xi = \{1, 2\}$ indicates that the states of nature under consideration are 1 and 2, and the set $A(S) = \{(1, 1), (1, 2)\}$ indicates that there are two assets, the first paying one unit in each of the two states of nature, whereas the second asset is paying one unit in the first state of nature and two units in the second state of nature.

Example 1 (*Bourse structure affects welfare*): Our objective in this first example is to compare three different economies, one with trader set $S^1 = (1, 2)$, another with $S^2 = (1, 2, 3)$, and another with $S^3 = (1, 2, 4)$. In this simple example, we shall focus on the bourse evaluation process, which is necessary for the analysis of period 0 in the following examples. We consider just one good for consumption at each node. Trading of assets and commodities occurs as described in the model above. The set of states of nature in period 2 is $\Xi = \{1, 2\}$. The asset structure is assumed to be complete, with $A(S) = \{(1, 1), (1, 2)\}$, for all $S = S^1, S^2, S^3$. Traders' attributes for the different economies are defined as follows. Trader i's utility is $u_1^i(x_1, x(1), x(2)) = \alpha_1^i \ln x_1 + \alpha^i(1) \ln x(1) + \alpha^i(2) \ln x(2)$. Traders 1 and 2's endowments and preference parameters are $(\omega_1^1, \omega^1(1), \omega^1(2)) = (2, 2, 6)$, $(\alpha_1^1, \alpha^1(1), \alpha^1(2)) = (1, 1, 0)$, $(\omega_1^2, \omega^2(1), \omega^2(2)) = (2, 6, 2)$ and $(\alpha_1^2, \alpha^2(1), \alpha^2(2)) = (1, 0, 1)$, respectively. We perform comparative statics with the three economies. Trader 3 is rich today and prefers to consume today, i.e., $(\omega_1^3, \omega^3(1), \omega^3(2)) = (6, 2, 2)$ and $(\alpha_1^3, \alpha^3(1), \alpha^3(2)) = (1, 1/2, 1/2)$. Trader 4 is rich today and prefers to consume tomorrow, i.e., $(\omega_1^4, \omega^4(1), \omega^4(2)) = (6, 2, 2)$ and $(\alpha_1^4, \alpha^4(1), \alpha^4(2)) = (1/2, 1, 1)$. Observe that trader 4 allows traders 1 and 2 to transfer wealth to those nodes where consumption is more valued to them. Trader 3 is not bound to make such transfers, as trader 3 has high endowments in the node where he most values consumption (period 1). Thus, trader 4 has better complementarities (in endowments and preferences) with traders 1 and 2 than trader 3 has. Moreover, trader 4 allows traders 1 and 2 to better diversify their risk in bourse S^3 than if they were alone in bourse S^1. Let us abbreviate notation and redefine $U_1^i(F[i; \mathbf{I}]) \equiv U_1^i(S)$, with $i \in S$ (as in this example each trader belongs to just one bourse). The following tables give the traders' indirect utilities and portfolios for different bourses.[26]

$\mathbf{S}^1 = (1, 2)$	$\mathbf{S}^2 = (1, 2, 3)$	$\mathbf{S}^3 = (1, 2, 4)$
$U_1^1(S^1) = 2.7726$	$U_1^1(S^2) = 2.8904$	$U_1^1(S^3) = 3.0754$
$U_1^2(S^1) = 2.7726$	$U_1^2(S^2) = 2.8904$	$U_1^2(S^3) = 3.0754$
n.a.	$U_1^3(S^2) = 2.7726$	$U_1^4(S^3) = 3.3965$

[26] The numerical computation procedure for this example and the following ones can be found in the working paper Faias and Luque [2011].

Table 1: Indirect utilities for a given bourse structure

	$\mathbf{S}^1 = (1,2)$	$\mathbf{S}^2 = (1,2,3)$	$\mathbf{S}^3 = (1,2,4)$
$(y_1^1(S), y_2^1(S))$	$(18, -12)$	$(14, -10)$	$(12.5882, -9.2941)$
$(y_1^2(S), y_2^2(S))$	$(-18, 12)$	$(-16, 10)$	$(-15.2941, 9.2941)$
$(y_1^3(S), y_2^3(S))$	n.a.	$(2, 0)$	n.a.
$(y_1^4(S), y_2^4(S))$	n.a.	n.a.	$(2.7058, 0)$

Table 2: Asset trading

In Table 1 we can see that the indirect utilities of traders 1 and 2 are substantially greater when they trade in the bourse with trader 4 than when they trade in the bourse with trader 3. Table 2 shows that trading in different bourses results in different trading volumes.[27] These observations indicate that trading complementarities are, indeed, an important determinant of traders' welfare and asset trading. ♣

Example 2: (*Large bourses are optimal if trading complementarities are good*): Being inspired by Fact Set 1, we illustrate here how good trading complemetarities are enough for a large bourse to emerge. Let us add to the set-up presented in Example 1 an initial period 0 where bourses form. We assume no uncertainty between periods 0 and 1. Our framework is again characterized by non-anonymity and market completeness. We restrict our attention to the set of traders $\mathbf{I} = \{1, 2, 4\}$. The possible bourses are $S^1 = (1,2)$, $S^3 = (1,2,4)$, $S^4 = (1,4)$, and $S^5 = (2,4)$. Let $u_0^i(x_0) = (1/2) \ln x_0$, where x_0 is the good consumption in period 0. The modified utility function is then $V^i(x_0, S) = u_0^i(x_0) U_1^i(S) = (1/2) \ln x_0 [\alpha_1^i \ln x_1^i + \alpha^i(1) \ln x^i(1) + \alpha^i(2) \ln x^i(2)]$. Good endowments in period 0 are $\omega_0^1 = 7$, $\omega_0^2 = 7$ and $\omega_0^4 = 5.9$. In period 0 traders must pay for the membership fee to have access to the bourse trading facility. Membership fees cover the bourse formation cost. Trader i's membership fee in bourse S is denoted by $\pi^i(S)$. We compute the non-anonymous membership fees by considering a welfarist agent that maximizes the weighted sum of indirect utilities subject to individual budget constraints in period 0. It can be shown that the optimal membership fee for a trader i in a two-traders bourse $S = (i,k)$ is given by

$$\pi^i(S) = \frac{U_1^k(S)\omega_0^i - U_1^i(S)\omega_0^k}{U_1^i(S) + U_1^k(S)} + \frac{U_1^i(S)}{U_1^i(S) + U_1^k(S)} z(S) \tag{8}$$

[27] We can define trading volume of an asset j in a bourse S by $\eta_j(S) \equiv \frac{1}{2}\sum_{i \in S} |y_j^i(S)|$, where the coefficient 1/2 corrects for the double counting when summing the trades over all traders. Then, using the values in Table 2, we can order trading volumes as $\eta_j(S^1) > \eta_j(S^2) > \eta_j(S^3)$, for $j = 1, 2$. Observe that a higher volume of trade does not necessarily correlate with traders' welfare. In fact, in this example, trading volume is lower when traders' welfare is higher.

whereas if it is a three-traders bourse $S = (i, j, k)$ we would have

$$\pi^i(S) = \frac{\omega_0^i \left[U_1^j(S) + U_1^k(S)\right] - U_1^i(S)(\omega_0^j + \omega_0^k)}{U_1^i(S) + U_1^j(S) + U_1^k(S)} + \frac{U_1^i(S)}{U_1^i(S) + U_1^j(S) + U_1^k(S)} z(S) \qquad (9)$$

These formulas give an efficient characterization of the non-anonymous bourse membership pricing.[28] Notice that in both cases the equilibrium membership fee equations consist of the sum of a pure transfer (first term on the right hand side) and a poll tax (second term). The poll tax is such that all traders share the bourse S's formation cost $z(S)$. The pure transfer reflects the trader's valuation of the trading opportunities in bourse S. Then, $\sum_{i \in S} \pi^i(S) = z(S)$. We consider bourse formation costs of the form $z(S) = 3|S|$. These costs are proportional to the bourse size[29] because we seek to emphasize the role of complementarities in traders' attributes (we wish to see the complementarities as the driving force that determine the bourse composition). The membership fee values are $\pi^1(S^1) = \pi^2(S^1) = 3$, $\pi^1(S^4) = \pi^2(S^5) = 4.0022$, $\pi^4(S^4) = \pi^4(S^5) = 1.9978$, $\pi^1(S^3) = \pi^2(S^3) = 3.4889$ and $\pi^4(S^3) = 2.0222$. The indirect utilities are[30]

	$\mathbf{S}^1 = (1, 2)$	$S^4 = (1, 4)$	$S^5 = (2, 4)$	$\mathbf{S}^3 = (1, 2, 4)$
Trader 1	$V^1(S^1) = 1.9218$	$V^1(S^4) = 1.3418$	$u^1(\omega^1) = 1.3488$	$V^1(S^3) = 1.9312$
Trader 2	$V^2(S^1) = 1.9218$	$u^2(\omega^2) = 1.3488$	$V^2(S^5) = 1.3418$	$V^2(S^3) = 1.9312$
Trader 4	$u^4(\omega^4) = 2.0253$	$V^4(S^4) = 2.1662$	$V^4(S^5) = 2.1662$	$V^4(S^3) = 2.3016$

Table 3: Indirect utilities in period 0

In this example with complete markets and proportional bourse formation costs, we find that the three traders prefer to sort in the largest possible bourse, $S^3 = (1, 2, 4)$. This happens even if traders 1 and 2 have to pay a higher membership fee in the three-trader bourse, since the gain in utility of the bourse evaluation process more than offsets this extra cost. Thus, good complementarities in preferences and endowments are sufficient here to obtain a large bourse in equilibrium. ♣

[28] An anonymous pricing context would make better (worse) off those traders who value more (less) the bourse than if the context were non-anonymous (not all surplus can be subtracted when pricing is anonymous).

[29] Denoting by $|S|$ the cardinality of S, we say that two bourses S^1 and S^2 with $|S^1| < |S^2|$ have bourse formation costs: proportional in size if $\frac{z(S^1)}{|S^1|} = \frac{z(S^2)}{|S^2|}$, more than proportional in size if $\frac{z(S^1)}{|S^1|} < \frac{z(S^2)}{|S^2|}$, and less than proportional in size if $\frac{z(S^1)}{|S^1|} > \frac{z(S^2)}{|S^2|}$.

[30] The indirect utility of a trader who does not belong to a bourse is given by his utility u^i evaluated in his good endowments (recall that there is only one good and u^i is strictly increasing in the consumption of the good).

Example 3: (*Size versus tailored efficiency*[31]) This example illustrates the case opposite to Example 2, that poor complementarities alone can lead a small bourse to form in equilibrium. We assume that markets are complete. For this example it is worth considering larger replica bourses with N traders of each type. For example, in the three-traders bourse case, the bourse $S_N^2 = (1,2,3)^N$ will denote a bourse composed by N traders of each type. Observe that for any given bourse S_N with m types of traders and N traders of each type, the indirect utility (given in Example 1) of a given trader i is such that $U^i(S) = U^i(S_N)$, for any $N \in \mathbb{N}$.[32] This remains true for all the examples in this paper. Again, we assume that bourse formation costs are proportional to bourse size in order to isolate the role of complementarities on the equilibrium outcome. In particular, we assume $z(S_N) = 3N|S_N|$, for any bourse S_N. Trader 3's endowment in period 0 is $\omega_0^3 = 5.9$. Our objective here is to compare the small bourse $S^1 = (1,2)$ with the large bourse $S_N^2 = (1,2,3)^N$, with N large, where the third trader has poor complementarities with traders 1 and 2. The indirect utilities in these bourses are

	$\mathbf{S^1} = (1,2)$	$\mathbf{S_N^2} = (1,2,3)^N$
Trader 1	$V^1(S^1) = 1.9218$	$V^1(S_N^2) = 1.8843$
Trader 2	$V^2(S^1) = 1.9218$	$V^2(S_N^2) = 1.8843$
Trader 3	$u^3(\omega^3) = 2.0253$	$V^3(S_N^2) = 1.7498$

Table 4: Indirect utilities at period 0

In Table 4 we see that all traders have a greater indirect utility in bourse $S^1 = (1,2)$ than in the larger bourse $S_N^2 = (1,2,3)^N$, for any N (possibly large). This result contrasts with the values given in Table 1, where traders 1 and 2 prefer to be in a larger bourse with trader 3 (there, only the bourse evaluation process was under consideration). However, in the bourse formation process, the benefit for traders 1 and 2 of enlarging the bourse with a third trader with poor complementarities (like trader 3) is not enough to compensate the cost of paying a higher membership fee - in bourse S^1 the membership fee for both traders 1 and 2 is 3, while in bourse S_N^2 the membership fee is $\pi^1(S^2) = \pi^2(S^2) = 3.3166$ (and $\pi^3(S^2) = 2.3667$ for trader 3). We conclude that poor complementarities between

[31] The properties of self-picked traders and assets motivate the expression *tailored-efficiency*.

[32] To see this, notice that commodities and assets market clearing equations in S_N are the same as in S. Also, the membership pricing expressions (8) and (9) hold true for any N-replica bourse with formation cost $z(S_N) = Nz(S)$. For example, in an N-replica two-traders bourse, the objective function would be $N\left(\ln x_0^i\right) U^i(S_N) + N(\ln x_0^j)U^j(S_N)$, given the restriction $N\pi^i(S_N) + N\pi^j(S_N) = Nz(S)$. This maximization problem is equivalent to the same problem when $N = 1$. Thus, this result relates to Allouch, Conley, and Wooders [2009], where a group strictly bounded in size can achieve all gains to club formation.

trader 3 and traders 1 and 2 leads to a situation where the small bourse S^1 is preferred in equilibrium by traders of types 1 and 2 to the larger bourse S^2_N, for any N-replica. This result identifies poor traders' complementarities (in preferences and endowments) as an important force against the tendency toward a unique bourse. This example is in accordance with Fact Set 2, which suggests that traditional large exchanges are ill-suited to certain types of institutions (e.g., dark liquidity pools). ♣

Remark 1: *What is the effect of the implementation of a Tobin tax on certain (but not all) bourses?* It can easily be shown that for two possible bourses - subject to legislation by regulators - with the same traders and asset structure, traders will participate in the bourse with lower execution rates for their trades. Higher execution rates in a bourse, due to a Tobin tax (see Tobin [1984]), can be accommodated in the form of higher formation cost, which penalizes traders in that bourse through higher memberships. As traders are free to move to their most preferred bourse, we can infer that the bourse with such high execution rates (or equivalently, high formation costs) may not be formed. In other words, if traders are free to choose their preferred bourses, then a Tobin tax on the financial transactions in some but not all bourses may not be effective. This result adds to the current debate on international financial transactions (Stiglitz [1989]).

Example 4: (*Multiple memberships*): Let us now consider an example with multiple memberships. Traders' names are now 5, 6, and 7. Traders' endowments and preferences parameters are $(\omega^5_0, \omega^5_1, \omega^5(1),\ \omega^5(2)) = (15, 6, 2, 2)$, $(\alpha^5_0, \alpha^5_1, \alpha^5(1),\ \alpha^5(2)) = (1/2, 1/2, 1, 0)$, $(\omega^6_0, \omega^6_1, \omega^6(1), \omega^6(2)) = (9, 2, 6, 6)$, $(\alpha^6_0, \alpha^6_1,\ \alpha^6(1), \alpha^6(2)) = (1/2,\ 1,\ 0, 1/2)$, $(\omega^7_0, \omega^7_1, \omega^7(1), \omega^7(2)) = (9, 2, 6, 6)$, and $(\alpha^7_0,\ \alpha^7_1,\ \alpha^7(1), \alpha^7(2)) = (1/2,\ 1, 1/2,\ 0)$. We assume that every possible bourse has an incomplete asset structure with only one asset with payoffs $(1, 1)$. Traders 6 and 7 are similar in preferences and endowments and, therefore, the two interesting bourse structures to compare are $\{S^6, S^7\} = \{(5, 6), (5, 7)\}$ and $\{S^8\} = \{(5, 6, 7)\}$, one with multiple memberships where trader 5 is common to both bourses, and the other with all traders in a single bourse. We consider that bourse formation costs are $z(S) = 3|S|$ if $S = S^6, S^7$, and $z(S) = 7|S|$ if $S = S^8$. One possible justification of these bourse formation costs follows a location argument. Trader 5 can be thought of as in between traders 6 and 7, and this location causes the cost of creating the bourse S^6 or S^7 to be smaller than $z(S^8)$.[33] Indirect utilities are $V^5(\{S^6, S^7\}) = 2.1239$ and $V^6(S^6) = V^7(S^7) = 1.9406$ for structure $\{S^6, S^7\}$, while $V^5(S^8) = 1.9512$ and $V^6(S^8) = V^7(S^8) = 1.2171$ for the unique bourse. For these values,

[33] For these costs, memberships are $\pi^5(S^6) = \pi^5(S^7) = 4.7619$ and $\pi^6(S^6) = \pi^7(S^7) = 1.2380$ for the multiple membership scenario, whereas $\pi^5(S^8) = 10.2310$ and $\pi^6(S^8) = \pi^7(S^8) = 5.3846$ for the unique bourse.

all traders prefer the bourse structure $\{(5,6),(5,7)\}$. Therefore, one insight of this example is that low bourse formation costs for bourses with traders located relatively "close" can explain the existence of multiple memberships in equilibrium. Another insight is that it is not necessarily true that a multiple membership scenario is Pareto superior to a unique bourse scenario, or vice versa. To see this, we can consider instead a smaller $z(S^8)$, for instance, if $z(S) = 3|S|$ for $S = S^6, S^7, S^8$. Then traders 6 and 7 can be shown to be better off with multiple memberships, but trader 5 would be better off in the unique bourse.[34] ♣

Example 5 (*Market incompleteness as a consequence of bourse formation costs and complementarities*): We consider an economy with new traders, named traders 8, 9, and 10. Traders' endowments and preferences parameters are $(\omega_0^8, \omega_1^8, \omega^8(1), \omega^8(2)) = (10,6,2,1)$, $(\alpha_0^8, \alpha_1^8, \alpha^8(1), \alpha^8(2)) = (1/2, 1/2, 1, 0)$, $(\omega_0^9, \omega_1^9, \omega^9(1), \omega^9(2)) = (6,2,6,1)$, $(\alpha_0^9, \alpha_1^9, \alpha^9(1), \alpha^9(2)) = (1/2, 1, 1/2, 0)$, $(\omega_0^{10}, \omega_1^{10}, \omega^{10}(1), \omega^{10}(2)) = (6,2,6,1)$ and $(\alpha_0^{10}, \alpha_1^{10}, \alpha^{10}(1), \alpha^{10}(2)) = (1/2, 1/2, 1, 1/2)$.

The possible bourses are $S^9 = (8,9)$, $S^{10} = (8,10)$, $S^{11} = (9,10)$, and $S^{12} = (8,9,10)$. The asset structure is complete, with $A(S) = \{(1,1),(1,2)\}$, for bourses $S = S^{10}, S^{11}, S^{12}$, but incomplete for bourse S^9, $A(S^9) = \{(1,1)\}$. We consider that bourses S^{10}, S^{11}, and S^{12} have formation costs proportional to their sizes, with $z(S) = 3|S|$, for $S = S^{10}, S^{11}, S^{12}$, but assume that bourse S^9 has less than proportional formation cost of the form $z(S^9) = 2|S^9|$ to illustrate that it is cheaper to provide an incomplete asset structure than a complete one.

$S^9 = (8,9)$	$S^{10} = (8,10)$	$S^{11} = (9,10)$	$S^{12} = (8,9,10)$
$V^8(S^9) = 1.7416$	$V^8(S^{10}) = 1.3581$	$V^8(\{8\}) = 1.546$	$V^8(S^{12}) = 1.6734$
$V^9(S^9) = 1.7416$	$V^9(\{9\}) = 1.546$	$V^9(S^{11}) = 1.1638$	$V^9(S^{12}) = 1.4728$
$V^{10}(\{10\}) = 2.0805$	$V^{10}(S^{10}) = 1.7262$	$V^{10}(S^{11}) = 1.7741$	$V^{10}(S^{12}) = 1.6973$

Table 5: Indirect utilities at period 0

From the results in Table 5, we conclude that, if bourse S^9 formation costs are (appropriately) less than proportional to bourse size, then traders 8 and 9 end up strictly preferring the incomplete asset structure associated with bourse S^9, *even if* a complete asset structure is available in another bourse. In Example 5 we can see how technology, in the form of bourse formation costs, plays a crucial role in determining the incompleteness of the markets. Thus, we conclude that the incompleteness of the asset structure is

[34] For these proportional bourse formation costs, indirect utilities for the unique bourse are $V^5(S^8) = 2.5456$ and $V^6(S^8) = V^7(S^8) = 1.8735$.

endogenous to the model, and crucially depends on the technology associated with the bourse formation costs.[35]♣

5 Final remarks

This paper is pioneering in examining financial market structures and their welfare properties under the new perspective of club theory. The paper provides the first equilibrium model of bourse formation, where a bourse is a financial platform that allows traders to share risks by exchanging assets. In our view this is an important conceptualization of an exchange, and opens a new way for analyzing competition among exchanges using the powerful theory of group formation. Our hope is to bring to the field of financial economics, and to the market microstructure in particular, the powerful tool of club theory. Our approach revealed interesting insights on the optimal size of a bourse. The paper also addresses the question of what forces drive the efficiency of equilibrium. We relate the endogeneity of complete markets to the existing technology of bourse formation, traders' valuation of bourses, and their endowments. We also think that the paper paves the way to study other interesting questions, like default, formation of clearing houses, and the contagion effects associated with multiple bourse memberships.

6 Appendix

Proof of Theorem 1: This Theorem follows by Lemmas 1, 2, and 3 below. The proof of existence of equilibrium is not trivial anymore. It requires elaboration. Lemma 1 constructs a sequence of truncated economies and shows that equilibrium exists for this truncation. Then, we show that there exists equilibrium for the limit of this sequence of truncations. It is important to notice that for this proof we do not need to assume bounded short sales (securities pay in the numeraire commodity). Lemma 2 proves that there exists a measurable selector of the asset trading equilibria in which a trader evaluates his utility in period 0 (to acquire bourse memberships). In Lemma 3 we show that there exists a generic set of bourse economies for which there is a$c(\varepsilon)$-equilibrium with possibly ever-increasing gains from larger bourses. The most difficult and subtle part of the proof is to guarantee that assumption (f) in AW(2008) holds. For this proof we need

[35] Other approaches to obtain market incompleteness consider issuing costs faced by intermediaries who offer securities in a context of imperfect competition, or some type of collateral requirements for the household borrowers.

to use Proposition 1. For convenience of exposition we present the proof of Proposition 1 immediately after the proof of Lemma 1.

Lemma 1: *Let us assume (A1.ii). Then, for a fixed bourse structure, there exists an asset trading equilibrium.*

Proof of Lemma 1: Let us consider a generalized game where allocated consumption and portfolios are restricted to a closed cube $\mathbf{K} \subseteq \mathbb{R}^{(\Xi+1)L+\sum_{S\in F(\mathbf{I})} J(S)}$ with center at the origin and large enough to contain the double of the aggregate endowment. Since a separate budget constraint must be satisfied at every state ξ and the demand is homogenous of degree zero in spot prices, the prices can be chosen in the simplex. In period 1 the simplex is such that $\sum_{l\in\mathbf{L}} p_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} q_j(S) = 1$, whereas in state ξ of period 2 it is such that $\sum_{l\in\mathbf{L}} p_l(\xi) = 1$.

The players of the generalized game are the traders and $(1+\Xi)$ additional auctioneers.

Given a bourse structure $F(\mathbf{I})$, each trader chooses a vector $(x_1^i, x^i(1), ..., x^i(\Xi), y^i)(F(\mathbf{I}))$ on $\mathbf{K}$ to maximize $u_1^i(x_1, x(1), ..., x(\Xi))$, subject to constraints (4) and (5).

The first auctioneer chooses period 1 commodities prices $p_1 \in \mathbb{R}_+^L$ and assets prices $q \in \mathbb{R}_+^{J(S_1)} \times ... \times \mathbb{R}_+^{J(S_k)}$ in order to maximize

$$B_1 \equiv \sum_{l\neq L} p_{l1} \sum_{i\in\mathbf{I}} (x_{l1}^i - \omega_{l1}^i) + p_{L1} \sum_{i\in\mathbf{I}} (x_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j(y_j^i) - \omega_{L1}^i) + \sum_{S_k\in F(\mathbf{I})} \sum_{j\in J(S_k)} q_j \sum_{i\in S_k} y_j^i. \quad (10)$$

The last part of B_1 accounts for the fact that the asset market clearing is achieved for each bourse. In period 2 there is an auctioneer for each node $\xi \in \Xi$ that chooses the commodity prices $p(\xi) \in \mathbb{R}_+^L$ in order to maximize

$$B(\xi) \equiv p(\xi) \sum_{i\in\mathbf{I}} \left(x^i(\xi) - \omega^i(\xi)\right). \quad (11\text{-}\xi)$$

An equilibrium for this generalized game, parameterized in the bourse structure $F(\mathbf{I})$, is a vector $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)(F(\mathbf{I}))$ such that, for each player (the n traders and the $(1+\Xi)$ auctioneers), the respective action solves his optimization problem parameterized by the other players' actions. We have that the generalized game has an equilibrium since it satisfies all the assumptions of Debreu's [1952] theorem. In fact, the auctioneers' objective functions (10) and $(11\text{-}\xi)_{\xi\in\boldsymbol{\Xi}}$ are linear in their respective price variables, and for each period and state, prices are in the simplex. Traders' utilities are continuous and strictly concave by (A1.ii), and their choice variables $(x_1, x(1), ..., x(\Xi), y)$ belong to non-empty, convex and compact sets. We can show that for this equilibrium there is no excess of demand of commodities, that is, $\sum_{i\in\mathbf{I}}(x_1^i - \omega_1^i) \leq 0$ and $\sum_{i\in\mathbf{I}}(x^i(\xi) - \omega^i(\xi)) \leq 0$, for every state ξ. First, observe that $B_1 \leq 0$ and $B(\xi) \leq 0$ as B_1 and $B(\xi)$ represent the aggregation of their respective traders' budget constraints. Now we argue that $\sum_{i\in\mathbf{I}}(x_{l1}^i - \omega_{l1}^i) \leq 0$, $l \neq L$. Otherwise, there exists a commodity $l' \neq L$ at period 1 with $\sum_{i\in\mathbf{I}}(x_{l'1}^i - \omega_{l'1}^i) > 0$. But, then the auctioneer would choose $p_{l'1} = 1$, a contradiction with $B_1 \leq 0$. Analogously, for good L, $\sum_{i\in\mathbf{I}}(x_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]}\sum_{j\in J(S_k)} g_j(y_j^i) - \omega_{L1}^i) \leq 0$, which implies $\sum_{i\in\mathbf{I}}(x_{L1}^i - \omega_{L1}^i) \leq 0$ since $g_j(y_j^i) \geq 0$, for all y_j^i. By the same arguments, it also holds that $\sum_{i\in S_k} y_j^i \leq 0$, $\forall j \in J(S_k), \forall S_k$. The proof that there is no excess of supply of commodities in each node ξ of period 2 follows by similar arguments, taking into account the aggregation of budget constraints in every state and the fact that the aggregated portfolios are non-positive.

We now consider a sequence of increasing closed cubes, $\mathbf{K}^n$, with center at the origin. Then, for each cube in the sequence, fix an equilibrium of the generalized game, $(x_1^{In}, x^{In}(1), ..., x^{In}(\Xi), y^{In})(F(\mathbf{I}))$. Notice that, as shown above, for each n and every i, $(x_1^{in}, x^{in}(1), ..., x^{in}(\Xi))(F(\mathbf{I})) \in [0, \sum_{i\in\mathbf{I}}\omega_1^i] \times \prod_{\xi\in\Xi}[0, \sum_{i\in\mathbf{I}}\omega^i(\xi)]$. Moreover, the corresponding sequence of equilibrium prices belongs to the simplex. Thus, there exists a converging subsequence. Let $(\breve{x}_1^I, \breve{x}^I(1), ..., \breve{x}^I(\Xi), \breve{p}, \breve{q})$ denote the limit of this subsequence.

Let us assume that $\breve{p}_{L1} > 0$ (we will prove this later). We now show that the corresponding subsequence of portfolios also converges. Indeed, the first order necessary and sufficient condition for trader i relative to an asset j in $J(S_k)$, with $S_k \in F[i, \mathbf{I}]$, is $-\tilde{\beta}_1^{n_q}[q_j^{n_q} + p_{L1}^{n_q} D_{y_j^{n_q}} g_j] + \sum_{\xi\in\Xi}\tilde{\beta}^{n_q}(\xi)p_L^{n_q}(\xi)a_j(\xi) = 0$, that is, $\tilde{\beta}_1^{n_q} p_{L1}^{n_q} D_{y_j^{n_q}} g_j = \sum_{\xi\in\Xi}\tilde{\beta}^{n_q}(\xi)p_L^{n_q}(\xi)a_j(\xi) - \tilde{\beta}_1^{n_q} q_j^{n_q}$. Since the function g_j is nonlinear, $D_{y_j} g_j$ is continuous and $D_{y_j}^2 g_j > 0$ we conclude that $y_j^{n_q}$ converges, provided that $\tilde{\beta}_1^{n_q}$ and $\tilde{\beta}^{n_q}(\xi)$ also converge and $\lim_{n_q}\tilde{\beta}_1^{n_q} > 0$. We prove that the sequence of Lagrange multipliers is bounded and, therefore, converges. For each truncated economy, we have $u_1^i(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi)) \leq u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$. Actually, there are non-negative multipliers $(\tilde{\beta}_1^{n_q}, \tilde{\beta}^{n_q}(1), ..., \tilde{\beta}^{n_q}(\Xi))$ such that, for each nonnegative bundle, the following saddle point property is satisfied (see Rockafellar [21, Theorem 28.3])

$$\mathcal{L}^i(x_1, x(1), ..., x(\Xi), y, \tilde{\beta}_1^{n_q}, \tilde{\beta}^{n_q}(1), ..., \tilde{\beta}^{n_q}(\Xi)) \leq u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi)),$$

which turns into $\tilde{\beta}_1^{n_q} p_1^{n_q}\omega_1^i + \sum_{\xi\in\Xi}\tilde{\beta}^{n_q}(\xi)p^{n_q}(\xi)\omega^i(\xi) \leq u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$ if we choose $(x_1, x(1), ..., x(\Xi), y) = 0$. This inequality implies that the sequence of multipliers is bounded since $\omega_1^i \gg 0$, $\omega^i(\xi) \gg 0$ for each state ξ, $p^{n_q}(\xi)$ belongs to the simplex for each state ξ and $\breve{p}_{L1} > 0$.

Let $\breve{y}^i$ be the portfolio limit. We prove now that $(\breve{x}_1^I, \breve{x}^I(1), ..., \breve{x}^I(\Xi), \breve{y}^I, \breve{p}, \breve{q})(F(\mathbf{I}))$ is an asset trading equilibrium. All budget constraints and inequalities $\sum_{i\in\mathbf{I}}(\breve{x}_1^i - \omega_1^i) \leq 0$, $\sum_{i\in\mathbf{I}}(\breve{x}^i(\xi) - \omega^i(\xi)) \leq 0$, $\sum_{i\in S_k} \breve{y}_j^i \leq 0$, $\forall j \in J(S_k)$ and $\forall S_k$, are satisfied, since all of them hold in each truncated economy and, therefore, still hold in the limit.

To obtain market clearing in all markets we look at the first order conditions of the optimization problem of the first period auctioneer, who chooses p_1 and q. Let $\breve{\mu}$ denote the Lagrange multiplier for the constraint $\sum_{l\in\mathbf{L}} \breve{p}_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} \breve{q}_j(S) = 1$. Then, $\sum_{i\in\mathbf{I}}(\breve{x}_{l1}^i - \omega_{l1}^i) = \breve{\mu}$ for $l \neq L$, $\sum_{i\in\mathbf{I}}(\breve{x}_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j(\breve{y}_j^i) - \omega_{L1}^i) = \breve{\mu}$, and $\sum_{i\in S_k} \breve{y}_j^i = \breve{\mu}$, $\forall j \in J(S_k), \forall S_k$. But by the Walras' law we can write, $\sum_{l\neq L} \breve{p}_{l1} \sum_{i\in\mathbf{I}}(\breve{x}_{l1}^i - \omega_{l1}^i) + \breve{p}_{L1} \sum_{i\in\mathbf{I}}(\breve{x}_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j(\breve{y}_j^i) - \omega_{L1}^i) + \sum_{S_k\in F(\mathbf{I})} \sum_{j\in J(S_k)} \breve{q}_j \sum_{i\in S_k} \breve{y}_j^i = \breve{\mu} \sum_{l\neq L} \breve{p}_{l1} + \breve{\mu}\breve{p}_{L1} + \breve{\mu} \sum_{S_k\in F(\mathbf{I})} \sum_{j\in J(S_k)} \breve{q}_j = \breve{\mu}\ (\sum_{l\in\mathbf{L}} \breve{p}_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} \breve{q}_j(S)) = 0$. Since $\sum_{l\in\mathbf{L}} \breve{p}_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} \breve{q}_j(S) = 1$ then $\breve{\mu} = 0$ which implies market clearing in all markets of period 1. The proof of market clearing in each state of nature for period 2 follows the same argument. Notice that market clearing in the asset market simplifies the Walras' law in each state of nature to $\breve{p}(\xi) \sum_{i\in\mathbf{I}}(\breve{x}^i(\xi) - \omega^i(\xi)) = 0$.

Now, we show that, given the structure $F(\mathbf{I})$ and prices $(\breve{p}, \breve{q})(F(\mathbf{I}))$, the vector $(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi), \breve{y}^i)$ is an optimal solution for consumer i with utility $u_1^i(x_1, x(1), ..., x(\Xi))$. Suppose it were not, say $(\hat{x}_1^i, \hat{x}^i(1), ..., \hat{x}^i(\Xi), \hat{y}^i)$ is budget feasible at $(\breve{p}, \breve{q})(F(\mathbf{I}))$, and $u_1^i(\hat{x}_1^i, \hat{x}^i(1), ..., \hat{x}^i(\Xi)) > u_1^i\ (\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$. For n large enough $(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi), \breve{y}^i)$ belongs to the interior of the cube $\mathbf{K}^n$, and for λ small enough, $\lambda\,(\hat{x}_1^i, \hat{x}^i(1), ..., \hat{x}^i(\Xi), \hat{y}^i) + (1 - \lambda)\,(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi), \breve{y}^i)$ belongs to $\mathbf{K}^n$, is budget feasible at prices $(\breve{p}, \breve{q})(F(\mathbf{I}))$ and $u_1^i(\lambda(\hat{x}_1^i, \hat{x}^i(1), ..., \hat{x}^i(\Xi)) + (1 - \lambda)(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))) > u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$ by strict concavity of the utility function. By continuity of preferences, $\lambda\,(\hat{x}_1^i, \hat{x}^i(1), ..., \hat{x}^i(\Xi), \hat{y}^i) + (1 - \lambda)(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi), y^{in_q})$ would be chosen instead of $(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi), y^{in_q})$ in the truncated economy associated to $\mathbf{K}^{n_q}$ at prices $(p^{n_q}, q^{n_q})(F(\mathbf{I}))$, a contradiction.

Finally, let us prove that $\breve{p}_{L1} \neq 0$. Suppose $\breve{p}_{L1} = 0$. Let e_{L1} be the canonical vector in the direction of this commodity. Monotonicity allows to conclude that $u_1^i((\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi)) + ke_{L1}) > u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$. Let $k = \min_{l\in\mathbf{L}} \omega_{l1}^i$, then for n large enough, $p_{L1}^{n_q} < 1$ and $u_1^i((1 - p_{L1}^{n_q})(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi)) + ke_{L1}) > u_1^i(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi))$. But the bundle $((1-p_{L1}^{n_q})(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi))+ke_{L1}, (1-p_{L1}^{n_q})y^{in_q})$ would be affordable at prices (p^{n_q}, q^{n_q}) which contradicts the fact that $(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi), y^{in_q})$ is an equilibrium for the truncated economy $\mathbf{K}^{n_q}$. ■

Proof of Proposition 1: This proof is a consequence of the following four steps.

Step 1: $\tilde{x}^i(F[i;I])$ *is a* C^1 *function in prices* p *and* q.

For this, notice that this proof does not follow straightforward the well known proof of Geanakoplos and Polemarchakis [1986, Section 3] since in our context with multiple bourse membership it is possible that there exists the same asset in two bourses to which a trader might belong. This introduces a linear dependence in the asset structure for this trader. That is the asset structure may not be full column rank. The non-linear function g allows us to solve this problem, in the same way as the bid-ask spread solves an analogous problem in Faias [2008].

Let us fix $p_{L1} = 1$ and $p_L(\xi) = 1$, for every ξ. The first order necessary and sufficient conditions for trader i's problem are:

$$\begin{aligned}
&D_1 u_1^i - \tilde{\beta}_1 p_1 = 0\\
&D_\xi u_1^i - \tilde{\beta}(\xi) p(\xi) = 0, \xi = 1, ..., \Xi\\
&-p(\xi)(x(\xi) - \omega^i(\xi)) + A_\xi^i y = 0, \xi = 1, ..., \Xi\\
&\tilde{\beta}^T A^i - \tilde{\beta}_1 [q + D_y g] = 0\\
&-p_1(x_1 - \omega_1^i) - qy - g(y) = 0
\end{aligned}$$

where T refers to the transpose of a matrix and $g(y) = \sum_j g_j(y_j)$. The columns of the return matrix $A^i = |\cdots A(S) \cdots|$ are those $A(S)$ with $S \in F[i, \mathbf{I}]$. Then, the element A_ξ^i denotes the line ξ of the return matrix A^i.

The Jacobian matrix with the second order derivatives with respect to $(x_1, x(\xi), \tilde{\beta}(\xi), y, \tilde{\beta}_1)$, with $x(\xi)$ and $\tilde{\beta}(\xi)$ generic elements of the corresponding Ξ-vector, is:

$$\mathbf{J} = \begin{bmatrix} D_1^2 u_1^i & 0 & 0 & 0 & -p_1 \\ 0 & D_\xi^2 u_1^i & -p(\xi) & 0 & 0 \\ 0 & -p(\xi)^T & 0 & A^i & 0 \\ 0 & 0 & A^{iT} & -\tilde{\beta}_1 D_y^2 g & -q - D_y g \\ -p_1 & 0 & 0 & -q^T - D_y g^T & 0 \end{bmatrix}$$

It is easy to see that the matrix $\mathbf{J}$ is non-singular. In fact, let $z = (\check{x}_1, \check{x}(\xi), \check{\beta}(\xi), \check{y}, \check{\beta}_1)$ such that $\mathbf{J}z = 0$, then $z^T \mathbf{J} z = 0$, and using $\mathbf{J}z = 0$, it reduces to $\check{x}_1^T(D_1^2 u_1^i)\check{x}_1 + \check{x}(\xi)^T(D_\xi^2 u_1^i)\check{x}(\xi) - \check{y}^T(v_1 D_y^2 g)\check{y}$. Notice that this last equality can be written as

$$\begin{bmatrix} \check{x}_1^T & \check{x}(\xi)^T & \check{y}^T \end{bmatrix} \begin{bmatrix} D_1^2 u_1^i & 0 & 0 \\ 0 & D_\xi^2 u_1^i & 0 \\ 0 & 0 & -\check{\beta}_1 D_y^2 g \end{bmatrix} \begin{bmatrix} \check{x}_1 \\ \check{x}(\xi) \\ \check{y} \end{bmatrix}$$

which implies $\check{x}_1 = 0$, $\check{x}(\xi) = 0$, and $\check{y} = 0$ by negative definiteness of $D^2 u_1^i$ and $\check{\beta}_1 D_y^2 g$. Then, back to $\mathbf{J}z = 0$, we obtain $\check{\beta}(\xi) = 0$. Finally, again with $\mathbf{J}z = 0$, $\check{\beta}_1 = 0$. Therefore, by the implicit function theorem, we conclude that individual excess demand is a C^1 function.

Step 2: *For any choice of utilities $U \equiv (u_1^i)_{i\in\mathbf{I}}$ in a given utility space $\mathbf{U}$, there exists a generic set $\mathbf{W}(U)$ of endowments, such that for every economy $(\mathbf{I},\alpha)$, $\alpha(i)$ $(i\in\mathbf{I})$ characterized by u_1^i and $(\omega^i)_{i\in\mathbf{I}}\in\mathbf{W}(U)$, the set of asset trading equilibria is a continuously differentiable function of the endowments.*

This proof follows the lines of Geanakoplos and Polemarchakis' [1986] proof of generic regularity. Our framework is different however, as we must adapt their proof to an economy where: 1) the asset market clearing occurs in each bourse; 2) the trading period accounts for assets *and* commodities; and 3) trading of assets involve paying g, which is itself a function of the asset trading.

Denote the price domain in period 1 by $\mathbf{M}_1 = \mathbb{R}_{++}^{L-1}\times\mathbb{R}_{+}^{\sum_{S\in F(\mathbf{I})}J(S)}$. In state ξ of period 2 the price domain is $\mathbf{M}(\xi)=\mathbb{R}_{++}^{L-1}$. Then, let $\mathbf{M}=\mathbf{M}_1\times\mathbf{M}(1)\times...\times\mathbf{M}(\Xi)$. In every node we can normalize the price of the numeraire commodity to be 1. Denote by $f:\mathbf{U}\times\mathbf{W}\times\mathbf{M}_+\rightarrow\mathbb{R}^{(L-1)(\Xi+1)}\times\mathbb{R}^{\sum_{S\in F(\mathbf{I})}J(S)}$ the aggregate excess demand function of commodities (other than the numeraire) and assets, given utilities, endowments, commodity prices, and asset prices. Let us fix the utilities to $\mathcal{U}$ and show that f restricted to $\mathcal{U}$, denoted by $f_{|\mathcal{U}}$, is transverse to 0 (see Geanakoplos and Polemarchakis [1986, Section 5]). That is, if for all $(\omega,p,q)\in\mathbf{W}\times\mathbf{M}_+$ with $f_{|\mathcal{U}}(\omega,p,q)=0$, the Jacobian matrix $D_{(\omega,p,q)}f_{|\mathcal{U}}$ has full rank. This amounts to showing that there exists a set of independent vectors of directional derivatives that has dimension $(L-1)(\Xi+1)+\sum_{S\in F(\mathbf{I})}J(S)$.

Let us fix an element $(\omega,p,q)\in f_{|\mathcal{U}}^{-1}(0)$ and a given trader i. Now, consider an increase of one unit in $\omega_l^i(\xi)$ with $l\in\mathbf{L}\backslash L$, and a decrease in the endowment of the numeraire good, $\omega_L^i(\xi)$, in $p_l(\xi)$ units. Trader i's demand of good l remains unchanged in ξ, but the total supply in the l-commodity market in state ξ has increased one unit. Thus, there is a net effect of aggregate excess demand of $(0,...,-1,...,0)$. This same argument also holds in period 1.

Consider a bourse S and let trader k be the trader with the only membership in this bourse S (guaranteed by assumption). Now, for each asset $j(S)=1,...,J(S)$ in this bourse, we can increase $\omega_L^k(\xi)$ by $a_{j(S)}(\xi)$, for all ξ, and decrease ω_{L1}^k by $q_{j(S)}+(g_{j(S)}(y_{j(S)}^k)-g_{j(S)}(y_{j(S)}^k-1))$. The *only* effect on trader k's demand is a decrease in asset $j(S)$ by one unit. As a consequence, the aggregate excess supply of asset $j(S)$ is now $(0,...,-1,...,0)$. This argument holds for any asset $j(S)$ in each bourse S. This proves Step 2.

Let us conclude this Step 2 with two observations. First, notice that in each bourse we need to work with the trader k chosen above, since in an environment of multiplicity of bourse memberships, a trader who belongs to two different bourses could have available the same asset in two different bourses. If this occurs, nothing guarantees that the previous argument holds. Second, the assumption of matrix $A(S)$ having full rank is needed in this proof to guarantee that the increase of $\omega_L^k(\xi)$ by $a_{j(S)}(\xi)$ is offset by a change in trader k's demand of asset $j(S)$, and is not offset by an equivalent change in the demand of the co-linear assets.

Step 3: There exists a generic set $\mathbf{U}' \times \mathbf{W}'$, *such that for every economy* $(\mathbf{I}, \alpha)$, $\alpha(i)$ $(i \in \mathbf{I})$ *with* $(u_1^i)_{i \in \mathbf{I}} \in \mathbf{U}'$ *and* $(\omega^i)_{i \in \mathbf{I}} \in \mathbf{W}'$, *the set of asset trading equilibria is a continuously differentiable function of both the endowment and the utility assignment.*[36]

The proof mimics Geanakoplos and Polemarchakis' [1986] proof of generic strong regularity, since for that proof the asset positions are fixed, and hence our bourse economy does not pose any further complication to their arguments (observe that for this proof we need to show that $f_{|\mathcal{U}}$ is transverse to 0, which has already been proven in the previous Step 2).

Step 4: There exists a generic set of economies for which, given $\lambda > 0$, *there is a* $\gamma > 0$ *such that for any set* I *and pair of economies* (I, α) *and* (I, β), *if* $d(\alpha(i), \beta(i)) \leq \gamma$ *for any* i, *then* $|U_1^{\alpha(i)}(F[i; \mathbf{I}^\alpha]) - U_1^{\beta(i)}(F[i; \mathbf{I}^\beta])| < \lambda$.

The proof of this last step follows by the continuity of u_1^i (by A1.ii) and the continuity of asset trading equilibria (Step 3). ■

Now, at this part of the proof of Theorem 1, we must observe that, given a bourse structure, the asset trading equilibrium may not be unique. This would imply that there is an indirect utility $U_1^i(F[i; \mathbf{I}])$ for each equilibrium solution, and thus more than one function $V^i(x_0^i, F[i; \mathbf{I}])$. Existence of an equilibrium for the bourse economy would require choosing a measurable selector of the equilibrium correspondence $E(\cdot)$. Then, we have to consider that for each bourse structure $F(\mathbf{I})$, the utility U_1^i is evaluated at $\tilde{x}^i(F[i; \mathbf{I}])$, the respective equilibrium consumption bundle of trader i at the equilibrium selection. The next proposition asserts the existence of a measurable selection.

Lemma 2: *There exists a measurable selection* $\tilde{x}^I(F(\mathbf{I})) = (\tilde{x}^i(F[i; \mathbf{I}]) : i \in \mathbf{I})$ *for the equilibrium correspondence* $E(F(\mathbf{I}))$.

Proof of Lemma 2: The proof follows by the Kuratowski-Ryll-Nardzewski measura-

[36] A set is a continuously differentiable function if all its elements are continuously differentiable functions.

ble selection theorem (a weak measurable correspondence with non-empty closed values into a separable metrizable space admits a measurable selection)[37]. In fact, we have that $\mathbf{F}(\mathbf{I})$ is a finite set, and therefore the equilibrium correspondence $E(\cdot)$ defined in $\mathbf{F}(\mathbf{I})$ is trivially a weak measurable correspondence (see Aliprantis and Border [2006, p. 600]). The correspondence takes values in the positive coordinate subset of a finite dimensional space and therefore it follows immediately that it is a separable metrizable space.

The correspondence $E(\cdot)$ takes closed values, i.e., if $(x_1^{I,s}, x^{I,s}(1), ..., x^{I,s}(\Xi), y^{I,s}, p^s, q^s)$ is a sequence in $E(F(\mathbf{I}))$ that converges to $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)$, then $(x_1^I, x^I(1), ...,$ $x^I(\Xi), y^I, p, q)$ also belongs to $E(F(\mathbf{I}))$. Given an equilibrium sequence, if we consider the budget constraints of each trader and pass to the limit, we obtain that in the limit the budget constraint of each trader is satisfied. The same reasoning allows us to prove that the market clearing also holds in the limit.

Finally, it remains to show that in the limit each trader is maximizing his utility. Suppose not, so for a trader i there exists another bundle $(\breve{x}^i, \breve{y}^i)$ which is budget feasible and such that $u_1^i(\breve{x}^i) > u_1^i(\tilde{x}^i)$. Now, let $(\check{x}^i, \check{y}^i) = (\lambda\tilde{x}^{i,s} + (1-\lambda)\breve{x}^i, \lambda y^{i,s} + (1-\lambda)\breve{y}^i)$ with $\lambda \in [0,1]$. Observe that $(\check{x}^i, \check{y}^i)$ is budget feasible for s large enough and for λ close to one. Moreover, by continuity we have that $u_1^i(\breve{x}^i) > u_1^i(\tilde{x}^{i,s})$, for s large enough. Then, the strict quasiconcavity implies that $u_1^i(\check{x}^i) = u_1^i(\lambda\tilde{x}^{i,s} + (1-\lambda)\breve{x}^i) > u_1^i(\tilde{x}^{i,s})$. This is a contradiction because $((\tilde{x}^{i,s})_{i\in\mathbf{I}}, y^{I,s}, p^s, q^s)$ was an equilibrium for the given bourse structure $F(\mathbf{I})$.■

An immediate consequence of Lemma 2 is that $V^i(x_0, F[i;\mathbf{I}])$ is well defined.

Lemma 3: *Let us assume that A1, A2, and A3 hold. Then there exists a generic set of bourse economies for which there is a $c(\varepsilon_0)$-equilibrium with possibly ever-increasing gains from larger bourses.*

Proof of Lemma 3: To prove Lemma 3 we need to assure that all assumptions required in AW [2008, Theorem 2] are satisfied. In the next items we rewrite AW's assumption in our notation, to make clear how our steps proceed in the proofs.

First we indicate that our assumption (A1.i) over utility $u_0^i(x_0)$ implies AW's assumptions (a) monotonicity, (b) continuity, and (c) convexity on $V^i(\cdot, F[i;\mathbf{I}])$.

AW's condition (d) "Desirability of endowment" can be rewritten with our notation as

[37] We remark that the equilibrium correspondence is defined in the finite set of bourse structures, and therefore, a continuous measurable selector is not needed. Continuous selectors are in general used to construct continuous objective functions. Thus, they only fit if the correspondence is defined in a continuum set.

follows:

$$\text{if } u_0^i(\omega_0^i - \tau\bar{1})U_1^i(\{i\}) < u_0^i(x_0^i)U_1^i(F[i;\mathbf{I}]), \text{ then } x_0 \gg 0.$$

We show this by contradiction. Assume $x_{l0} = 0$ for some l. Then, $u_0^i(x_0^i)U_1^i(F[i;\mathbf{I}]) = 0$ by (A1.iii). Now, $u_0^i(\omega_0^i - \tau\bar{1})U_1^i(\{i\}) > 0$ since $\omega_0^i - \tau\bar{1} \gg 0$ and $u_0^i(x_0)$ is increasing (by (A1.i)). Finally, $U_1^i(\{i\}) \geq 0$ since $U_1^i(\{i\}) \geq u_1^i(\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)) > 0$ by (A1.vi). Thus, we obtain an impossibility.

AW's condition (e) "Private goods are valuable" says that, given any attribute θ and any $\varepsilon > 0$, there is $\rho_\varepsilon^\theta > 0$ such that, for all $i \in \mathbf{I}$ with $\alpha(i) = \theta$ and all $x_0^i \in \mathbb{R}_+^L$, $V^i(x_0^i, F[i;\mathbf{I}]) + \rho_\varepsilon^\theta < V^i(x_0^i + \varepsilon\bar{1}, F[i;\mathbf{I}])$ holds. This assumption is implied by our assumption (A1.iv). Actually, given $\varepsilon > 0$ and given $\rho_\varepsilon^\theta > 0$ satisfying assumption (A1.iv), and for any x_0^i, we have $(u_0^i(x_0^i + \varepsilon\bar{1}) - u_0^i(x_0^i))\, U_1^i(F[i;\mathbf{I}]) \geq (u_0^i(x_0^i + \varepsilon\bar{1}) - u_0^i(x_0^i))\, \delta > \rho_\varepsilon^\theta$, where the first inequality follows by the optimality of the equilibrium $\tilde{x}(F[i;\mathbf{I}])$ and the definition of δ, whereas the last inequality follows by assumption (A1.iv).

AW's condition (g), "Continuity with respect to attributes 2", says that, given $\varepsilon > 0$, there exists $\lambda > 0$ such that for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \lambda$ for any i, then $\omega_0^{\alpha(i)} \leq \omega_0^{\beta(i)} + \varepsilon\bar{1}$, where $\bar{1} = (1, ..., 1) \in \mathbb{R}^L$. This is precisely what was assumed in (A1.i).

AW's assumption (h), "Continuity with respect to attributes 3" is precisely our assumption (A2).

Let us prove AW's assumption (f), which says that given $\varepsilon > 0$, there exists $\gamma > 0$, such that, for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \gamma$, then $V^{\alpha(i)}(x_0^i, F[i;\mathbf{I}^\alpha]) < V^{\beta(i)}(x_0^i + \varepsilon\bar{1}, F[i;\mathbf{I}^\beta])$, for any i and any $x_0^i \in \mathbb{R}^L$. Proposition 1 asserts that, for each bourse structure $F(\mathbf{I})$, there is a generic set $\mathbf{U}' \times \mathbf{W}'(F(\mathbf{I}))$ where the assets trading equilibrium is continuous in traders' attributes. Now, the finite intersection $\mathbf{U}'' \times \mathbf{W}'' \equiv \bigcap_{F(\mathbf{I}) \in \mathbf{F}(\mathbf{I})} (\mathbf{U}' \times \mathbf{W}'(F(\mathbf{I})))$ is a generic set, where the assets trading equilibrium is continuous in traders' attributes for every bourse structure. Given an economy $(\mathbf{I}, \alpha)$ belonging to the generic set $\mathbf{U}'' \times \mathbf{W}''$, we can find a compact subset of economies containing $(\mathbf{I}, \alpha)$ (since $\mathbf{U}'' \times \mathbf{W}''$ is open) where Proposition 1 holds. We now prove that assumption (f) of AW holds in this compact set of economies for $x_0^i \leq \sum \omega_0^i + \varepsilon$, with $\varepsilon > 0$.[38]. Now, given $\varepsilon > 0$, by assumption (A1.v), there exists $\lambda_\varepsilon > 0$ and $\gamma_1 > 0$ such that if $d(\alpha(i), \beta(i)) \leq \gamma_1$, then $u_0^{\beta(i)}(x_0^i + \varepsilon\bar{1}) - u_0^{\alpha(i)}(x_0^i) > \lambda_\varepsilon$. By Proposition 1, there exists $\gamma_2 > 0$ such that, if $d(\alpha(i), \beta(i)) \leq \gamma_2$, then $|U_1^{\beta(i)}(F[i;\mathbf{I}]) - U_1^{\alpha(i)}(F[i;\mathbf{I}])| < \frac{\underline{u}_1}{\bar{u}_0}\lambda_\varepsilon$, where $\bar{u}_0$ and $\underline{u}_1$ are the upper and lower bounds in assumption (A1.vi). Now, we prove

[38] Observe that AW [2008, p. 271-272] only require assumption f) to be satisfied for a consumption x_0^i bounded above by the aggregate endowments plus some $\varepsilon > 0$.

that there is a $\gamma > 0$ such that, if $d(\alpha(i), \beta(i)) \leq \gamma$ for all i, the following inequality holds $V^{\alpha(i)}(x_0^i, F[i; \mathbf{I}^\alpha]) < V^{\beta(i)}(x_0^i + \varepsilon\bar{1}, F[i; \mathbf{I}^\beta])$, which can be written as

$$u^{\alpha(i)}(x_0^i)U_1^{\alpha(i)}(F[i; \mathbf{I}]) < u^{\beta(i)}(x_0^i + \varepsilon\bar{1})U_1^{\beta(i)}(F[i; \mathbf{I}]).$$

This inequality is equivalent to

$$u^{\beta(i)}(x_0^i + \varepsilon\bar{1}) - u^{\alpha(i)}(x_0^i) > \frac{u^{\alpha(i)}(x_0^i)}{U_1^{\beta(i)}(F[i; \mathbf{I}])}(U_1^{\alpha(i)}(F[i; \mathbf{I}]) - U_1^{\beta(i)}(F[i; \mathbf{I}])).$$

If $\gamma = \min\{\gamma_1, \gamma_2\}$, then the previous inequality holds. To see this, notice that if $d(\alpha(i), \beta(i)) \leq \gamma$ for all i, then

$$u^{\beta(i)}(x_0^i + \varepsilon\bar{1}) - u^{\alpha(i)}(x_0^i) > \lambda_\varepsilon > (\bar{u}_0/\underline{u}_1)(U_1^{\alpha(i)}(F[i; \mathbf{I}]) - U_1^{\beta(i)}(F[i; \mathbf{I}])) \geq$$
$$\geq (u^{\alpha(i)}(x_0^i)/U_1^{\alpha(i)}(F[i; \mathbf{I}]))(U_1^{\alpha(i)}(F[i; \mathbf{I}]) - U_1^{\beta(i)}(F[i; \mathbf{I}])).$$

It remains to show that our economy satisfies AW's assumption "Desirability of wealth", which says that there is $x_0^* \in \mathbb{R}_+^L$ and an integer η such that for any economy $(\mathbf{I}, \alpha)$ and any $i \in \mathbf{I}$, there is a coalition $S \in \mathbf{I}$ with $|S| \leq \eta$ and a club structure $F(S)$ satisfying $V^i(x_0^i + x_0^*, F[i; S]) \geq V^i(x_0^i, F[i; \mathbf{I}])$, for any $F(\mathbf{I})$ and any $x_0^i \in \mathbb{R}_+^L$. Notice that by (A3), $\tilde{x}(\{i\})$ satisfies $u_0^i(x_0^i + x_0^*)u_1^i(\tilde{x}(\{i\})) \geq u_0^i(x_0^i)\, u_1^i(\sum_i (\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)))$, and also observe that $u_0^i(x_0^i)u_1^i(\sum_i (\omega_1^i, \omega^i(1), ..., \omega^i(\Xi))) \geq u_0^i(x_0^i)\, u_1^i(F[i; \mathbf{I}])$. Then, transitivity implies that $u_0^i(x_0^i + x_0^*)u_1^i(\tilde{x}(\{i\})) \geq u_0^i(x_0^i)u_1^i(F[i; \mathbf{I}])$, which satisfies the "Desirability of wealth" assumption for a bourse $S = \{i\}$ (and therefore, for $\eta = 1$ in AW's terminology).■
■

Proof of Proposition 2: The proof is by contradiction. Assume that condition (SC) holds, and also that there is a bourse S whose traders improve upon bourse S^* (that is, for all $i \in S$, $u_0^i(\omega_0^i - \pi^i(S))U_1^i(S) \geq u_0^i(\omega_0^i - \pi^i(S^*))U_1^i(S^*)$, with strict inequality for some $i \in S$). The latter is equivalent to $\omega_0^i - \pi^i(S) \geq (\omega_0^i - \pi^i(S^*)) \left(\frac{U_1^i(S^*)}{U_1^i(S)}\right)^\alpha$, since the inverse of u_0^i is strictly increasing and homogeneous of degree α. Aggregating over all traders in S, we get $\sum_{i \in S}(\omega_0^i - \pi^i(S)) > \sum_{i \in S}(\omega_0^i - \pi^i(S^*)) \left(\frac{U_1^i(S^*)}{U_1^i(S)}\right)^\alpha$. Since $\frac{U_1^i(S^*)}{U_1^i(S)} > 0$ and $\pi^i(S^*) \leq z(S^*)$ (as $\omega_0^i \leq z(S^*)$ and $\omega_0^i \geq \pi^i(S^*)$), we get $z(S^*) \sum_{i \in S} \left(\frac{U_1^i(S^*)}{U_1^i(S)}\right)^\alpha - z(S) > \sum_{i \in S} \left(\left(\frac{U_1^i(S^*)}{U_1^i(S)}\right)^\alpha - 1\right) \omega_0^i$, a contradiction with (SC).

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Supplementary Material: Computation procedures to the examples

Abstract. In this Supplementary Material we provide the computation procedures to solve the examples of the paper "Endogenous bourse structures". This material is not for publication.

Procedure to compute $U_1^i(S)$ for examples 1, 2, and 3: With out loss of generality, consider any bourse $S = S^1$, S^2, S^3, S^4, S^5. Given that markets are complete, we find the Radner equilibrium by solving the Arrow-Debreu equilibrium, where each consumer i maximizes $u^i(x_1, x_2, x_3)$ subject to his budget constraint $\sum_\xi p_\xi(x_\xi - \omega_\xi^i) = 0$, and markets clear, $\sum_i x_\xi^i = \sum_i \omega_\xi^i$, $\forall \xi \epsilon \mathbf{\Xi}$. The procedure is standard. The steps are:

1) Take the first order condition of the Lagrangian function $\mathcal{L}^i$ with respect to the consumption variables and shadow price λ^i (one Arrow-Debreu restriction). We then obtain $\lambda^i(\mathbf{p}\boldsymbol{\omega}^i)$ and $x_\xi^i(\mathbf{p}\boldsymbol{\omega}^i)$, for all ξ, where $\mathbf{p}\boldsymbol{\omega}^i \equiv p_1\omega_1^i + \sum_{\xi=1,2} p(\xi)\omega^i(\xi)$. In general, we obtain the following expressions: $x_1^i = \frac{\alpha_1^i \cdot \mathbf{p}\boldsymbol{\omega}^i}{p_1 \cdot \boldsymbol{\alpha}^i}$ and $x^i(\xi) = \frac{\alpha^i(\xi) \cdot \mathbf{p}\boldsymbol{\omega}^i}{p(\xi) \cdot \boldsymbol{\alpha}^i}$ for $\xi = 1, 2$,where $\boldsymbol{\alpha}^i = \alpha_1^i + \sum_{\xi=1,2} \alpha^i(\xi)$.

2) Substitute these values in the Arrow-Debreu market clearing equilibrium conditions (see condition (D1.ii)) for bourse S and obtain the commodity prices that clear the markets. We obtain:

	$\mathbf{S}^1$	$\mathbf{S}^2$	$\mathbf{S}^3$	$\mathbf{S}^4$	$\mathbf{S}^5$
p_1	4	2	1.2941	8	8
$p(1)$	1	1	1	21	5
$p(2)$	1	1	1	5	21

Table SP-1

3) Find the equilibrium consumption values using these prices. We obtain

	x_1^1	$x^1(1)$	$x^1(2)$	x_1^2	$x^2(1)$	$x^2(2)$
$\mathbf{S}^1$	2	8	0	2	0	8
$\mathbf{S}^2$	3	6	0	3	0	6
$\mathbf{S}^3$	4.0909	5.2941	0	4.0909	0	5.2941
$\mathbf{S}^4$	5.5	2.0952	0	n.a.	n.a.	n.a.
$\mathbf{S}^5$	n.a.	n.a.	n.a.	5.5	0	2.0952

Table SM-2

	x_1^3	$x^3(1)$	$x^3(2)$	x_1^4	$x^4(1)$	$x^4(2)$
$\mathbf{S}^1$	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
$\mathbf{S}^2$	4	4	4	n.a.	n.a.	n.a.
$\mathbf{S}^3$	n.a.	n.a.	n.a.	1.8182	4.7059	4.7059
$\mathbf{S}^4$	n.a.	n.a.	n.a.	2.5	1.9048	8
$\mathbf{S}^5$	n.a.	n.a.	n.a.	2.5	8	1.9048

Table SM-3

4) Substitute the equilibrium consumption values in the utility function to obtain the trader's indirect utility in the second stage when he belongs to that bourse S. These values for bourses S^1, S^2 and S^3 are given in Example 1. The indirect utility values for bourses S^4 and S^5, needed in Example 2, are given in the following table. Notice that the indirect utilities of the traders that do not belong to any bourse are obtained by evaluating the trader's utility u_1^i in his good endowments (there is only one good and all traders have their utilities strictly increasing in the consumption of this good).

$\mathbf{S}^4 = (1,4)$	$\mathbf{S}^5 = (2,4)$
$U_1^1(S^4) = 2.4444$	$u_1^1(\omega^1) = 1.3862$
$u_1^2(\omega^2) = 1.3862$	$U_1^2(S^5) = 2.4444$
$u_1^3(\omega^3) = 2.4849$	$u_1^3(\omega^3) = 2.4849$
$U_1^4(S^4) = 3.1819$	$U_1^4(S^5) = 3.1819$

Table SM-4

The following step is only needed for Example 1.

5) Given that we are in a complete markets framework, the Arrow-Debreu equilibrium coincides with the Radner equilibrium. Therefore, to compute traders' portfolios we substitute the above Arrow-Debreu equilibrium consumption values in the corresponding Radner restrictions:

$$x^i(\xi) - \omega^i(\xi) - \sum_{j=1,2} a_j(\xi) y_j^i = 0, \text{ for } \xi = 1, 2.$$

Solving for each trader's system of equations and unknowns, we get the asset trades of a trader in the bourse S. The values are given in Table 2 of the paper.

Procedure to compute examples 2 and 3: The non-anonymous membership fees are obtained by considering a welfarist agent that maximizes the weighted sum of indirect utilities subject to individual budget constraints in period 0. That is, the maximization problem is $\max_{(\pi^i)_{i\in S}} \left[\sum_{i\in S}(1/2)\,(\ln x_0^i)\,U_1^i(S)\right]$ subject to $x_0^i = \omega_0^i - \pi^i(S)$, $\forall i \in S$, and $\sum_{i\in S} \pi^i(S) = z(s)$. Budget constraint in period 0 can be written with equality, as utility is increasing in the consumption of the single good. The membership fee formulas are obtained by taking the first order condition with respect to $(\pi^i(S))_{i\in S}$. The memberships fees for bourses S^1, S^3, S^4 and S^5 are given in the text of Example 2, whereas the membership fees for bourse S^2 are given in the text of Example 3.

Given the good endowments at period 0 and the membership fees values, we obtain the consumption values and indirect utilities at period 0 for bourses S^1, S^3, S^4 and S^5 (the values for bourse S^2 are not needed in our examples). These are[39]

	x_0^1	x_0^2	x_0^4
$\mathbf{S}^1$	4	4	*5.9*
$\mathbf{S}^3$	3.5111	3.5111	3.8778
$\mathbf{S}^4$	2.9978	*7*	3.9022
$\mathbf{S}^5$	*7*	2.9978	3.9022

Table SM-5

	V^1	V^2	V^4
$\mathbf{S}^1$	1.9218	1.9218	*2.0253*
$\mathbf{S}^3$	1.9312	1.9312	2.3016
$\mathbf{S}^4$	1.3418	*1.3488*	2.1662
$\mathbf{S}^5$	*1.3488*	1.3418	2.1662

Table SM-6

Procedure to compute example 4: The equilibrium of the second stage is computed by solving a Radner type economy (one budget constraint for each node). By writing each trader's budget constraints for states 1 and 2 in period 2 in equality form, and substituting the state-consumption expressions in the utility function of the second stage, we get the following objective functions: for trader 5, $(1/2)\ln(4 - q(S^6)y^5(S^6) - q(S^7)y^5(S^7))+$ $\ln(2 + y^5(S^6) + y^5(S^7))$; for trader 6, $\ln(2 - q(S^6)y^6(S^6))+$ $(1/2)\ln(6 + y^6(S^6))$; for trader 7, $\ln(2 - q(S^7)y^7(S^7))+$ $(1/2)\ln(6 + y^7(S^7))$. Portfolios parametrized in asset prices are then obtained by solving the system of first order conditions on these objective functions. For traders 6 and 7, these are $y^6(S^6) = (2 - 12q(S^6))/3q(S^6)$ and $y^7(S^7) = (2 - 12q(S^7))/3q(S^7)$. The first order conditions for trader 5 determine a system of two equations and two unknowns: $\frac{-0.5q(S^6)}{6-q(S^6)y^5(S^6)-q(S^7)y^5(S^7)} + \frac{1}{2+y^5(S^6)+y^5(S^7)} = 0$ and $\frac{-0.5q(S^7)}{6-q(S^6)y^5(S^6)-q(S^7)y^5(S^7)} + \frac{1}{2+y^5(S^6)+y^5(S^7)} = 0$. These two equations imply that $q(S^6) = q(S^7)$. Denoting this price by q, and solving for $y^5(S^6) + y^5(S^7)$ in one of these two equations, we get $y^5(S^6)+y^5(S^7) = (12-2q)/3q$. Now, using the asset market clearing equations for the multiple memberships bourse structure, $y^5(S^6)+y^6(S^6) = 0$ and $y^5(S^7)+y^7(S^7) = 0$,

[39] Numbers in italics indicate that trader i's consumption x_0^i and utility u^i are evaluated at the trader's good endowments (as the trader does not belong to any bourse).

we get $q(S^6) = q(S^7) = 8/13$ (notice that there is only one good and also that traders 6 and 7 are symmetric in preferences and endowments in the second stage). Portfolios are $(y^5(S^6), y^5(S^7)) = (35/12, 35/12)$, $y^6(S^6) = -35/12$ and $y^7(S^7) = -35/12$. On the other hand, it can be shown that for the bourse structure with a unique bourse $S^8 = (5, 6, 7)$, with market clearing equation $y^5(S^8)+ y^6(S^8)+ y^7(S^8) = 0$, portfolios are $y^5(S^8) = 70/12$, $y^6(S^8) = -35/12$ and $y^7(S^8) = -35/12$. Indirect utilities in the second stage are $U_1^5 = 2.4982$, $U_1^6 = 1.8940$ and $U_1^7 = 1.8940$, for either the bourse structure with multiple memberships or the bourse structure with a unique bourse. However, membership fees are different in the two types of bourse structures, which will determine different traders' indirect utilities in period 0. These are: $\pi^5(S^6) = \pi^5(S^7) = 4.7619$, $\pi^6(S^6) = \pi^7(S^7) = 1.2380$, $\pi^5(S^8) = 10.2310$, $\pi^6(S^8) = \pi^7(S^8) = 5.3846$. Then, consumptions at period 0 are obtained using the respective Radner restriction $x_0^i = \omega_0^i - \sum_{S:i\in S} \pi^i(S)$. These are $x^5(\{S^6, S^7\}) = 5.4762$, $x^6(S^6) = x^7(S^7) = 7.7620$, $x^5(S^8) = 4.769$, $x^6(S^8) = x^7(S^8) = 3.6154$. The indirect utilities in period 0 are given in the text of Example 4.

Procedure to compute Example 5: Bourse S^9 is characterized by incomplete markets and therefore the procedure to compute equilibrium is different that the one described above when markets are complete. For bourse S^9 the equilibrium for the second stage is obtained by solving a Radner type economy (one budget constraint for each node). The steps are:

1) Since there is only one good, we can make the price of the good equal to 1 in every node. Then, we write the Radner budget constraints in equality form and obtain the equilibrium consumption ($x_1^i = \omega_1^i - q_1 y_1^i$ and $x^i(\xi) = \omega^i(\xi) + a_{1\xi} y^i(\xi)$, for $\xi = 1, 2$). Consumption in period 1 is then parametrized by the asset trades and good endowment, while consumption at node ξ of period 2 is parametrized by the asset returns and good endowment.

2) Substitute these parametrized consumption functions in the utility function $u_1^i(x_1^i, x^i(1), x^i(2))$ and take the first order conditions with respect to $y^i(1)$ and $y^i(2)$ to obtain the asset trades as a function of asset prices.

3) Apply asset market clearing equations ($\sum_{i\in S^9} y^i = 0$) to obtain the asset price: $q = 1$. Then, substitute this price in the in the previous expressions to obtain asset trades: $y^8(S^9) = 10/3$ and $y^9(S^9) = -10/3$.

In what follows we also indicate the values for the other bourses (with complete markets) S^{10}, S^{11} and S^{12}.

4) Substitute the values of $(y^i)_{i\in S}$ into period 1 and period 2 budget constraints to calculate the equilibrium consumption values $(x_1^i, x^i(1), x^i(2))$.

	S^9	S^{10}	S^{11}	S^{12}
$(x_1^8, x^8(1), x^8(2))$	(2.6666,5.3333,0)	(4,4,0)	n.a.	(2.7964,6.4489,0)
$(x_1^9, x^9(1), x^9(2))$	(5.3333,2.6666,0)	n.a.	(2.909,4.8,0)	(5.2389,3.0204,0)
$(x_1^{10}, x^{10}(1), x^{10}(2))$	n.a.	(4,4,2)	(1.0909 ,7.2,2)	(1.9646,4.5306,3)

Table SM-7

These consumption values will determine the value of the indirect utility function:

	S^9	S^{10}	S^{11}	S^{12}
$U_1^8(\cdot)$	2.1643	2.0794	n.a.	2.378
$U_1^9(\cdot)$	2.1643	n.a.	1.8521	2.2088
$U_1^{10}(\cdot)$	n.a.	2.426	2.3641	2.3978

Table SM-8

5) Membership fees are obtained using the formulas (8) and (9) given in the paper. The values are:

	S^9	S^{10}	S^{11}	S^{12}
$\pi^8(\cdot)$	3	3.3077	n.a.	2.9144
$\pi^9(\cdot)$	3	n.a.	3.4857	3.2051
$\pi^{10}(\cdot)$	n.a.	2.6922	2.5142	2.8804

Table SM-9

6) Substitute the values of the membership fees and good endowments in budget constraint of period 0 and calculate consumption in period 0, x_0^i. Then, substitute the value of x_0^i in u_0^i, and obtain $V^i(S) = (1/2)\cdot \ln x_0 \cdot U_1^i(x_1^i, x^i(1), x^i(2))$. Table 5 in the paper gives the values of traders' indirect utilities $V^i(S)$ at S^9, S^{10}, S^{11} and S^{12}.