AGH University of Science and Technology

FACULTY OF MECHANICAL ENGINEERING AND ROBOTICS

DEPARTMENT OF ROBOTICS AND MECHATRONICS

# FINAL PROJECT

## Analysis of dynamic handling of one motorcycle using simulations tools.

**Autor**

Miguel santana Gallego.

**Tutor**

Michal Manka.

**Cotutor**

José Antonio Calvo Ramos.

**Coordinador académico**

Ana Muñoz Sánchez.

FEBRUARY 2011

# Index of figures and tables.

## Description of the project.

We will study the dynamic handling of one motorcycle using simulations tools through the creation of different models. For this purpose we will use informatics instruments like Matlab/Simulink and the simplification of rigid suspensions in the models.

## Aim of the project.

The main objective is to understand the behavior in rectilinear motion and in steady turning of one vehicle of two wheels and realize the effect of different set up in the motorbike handling.

## Specifics objectives.

- We will study the influence of geometry of motorcycle in its maneuverability.
- We will study the rectilinear motion of motorcycles.
- We will study the form like a vehicle of two wheels take a turn.

## Methodology.

- The first step was to familiarize with the different informatics tools. *Matlab/Simulink*
- The second step was the consulting of different books related with the motorcycle dynamics and different researches about specific points.
- The third step was the consulting of previous works about motorcycle simulations. Simulations that use models of four degrees of freedom and six degrees of freedom.
- The fourth step was the construction of different models that simulate specifics parts of the motorcycle. I studied the geometry of the rear and front frame, the rectilinear motion and the steady turning of one motorcycle, and the way in which changes in the different parameters affect to the maneuverability and handling of the motorbike.

## Expected Results.

The aim of the project is create different models that help to understand the handling of one motorcycle and how the different set up can change the maneuverability of one motorbike.

# 1. Rectilinear Motion of Motorcycles.

The behavior of motorcycles during rectilinear motion depends on the longitudinal forces exchanged between the tires and the road, the aerodynamic forces induced through this motion, and the slope of the road plane if the path is not horizontal. The study of rectilinear motion highlights certain dynamic aspects that are also important for safety, as the motorcycle´s behavior during acceleration with possible wheeling. Also this models are important to study the different set up that the motorcycle can has in the gear box or in the final drive and its effects in the performance of the motorbike.

To explain this model firstly is showed the different components that are involved in the program. These components are the Motorcycle tires, the resistances forces that act in the motorcycle such as Rolling resistance and aerodynamic resistance and the thrust produced by the engine. Secondly we explain the different outputs obtained with the model. The outputs of the model are the dynamic loads in the front and rear tire and the instantaneous speed and acceleration. All of these outputs are obtained using the inputs that are introduced in our model and that represent the real parameters of one specific motorbike. To show the data calculated we use the program graphic.m and finally we finish this chapter with some interesting conclusions.

## 1.1. Motorcycle tires.

The tire is one of the most important components in a motorcycle. Its fundamental characteristic is its deformability, which allows contact between the wheel and the road to be maintained even when small obstacles are encountered.

In the Rectilinear Motion studies we suppose nondeformable tires, and schematized as two toroidal solid bodies with circular sections. Therefore, the parameters that define the real tire in the model, is its radii. The nomenclature used by the factories to describe the size of the tires is this:

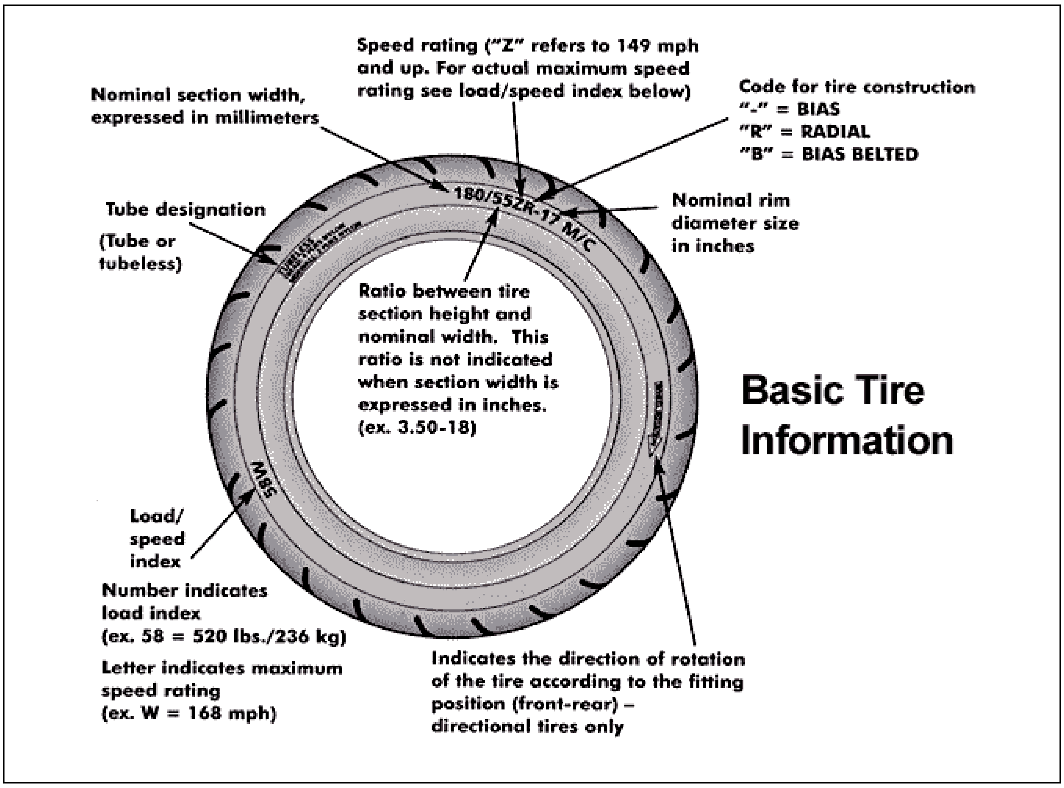


**Figure 1. Basic tire information.**

The parameters that we need in our model are expressed by these three numbers. The meaning of each number is:

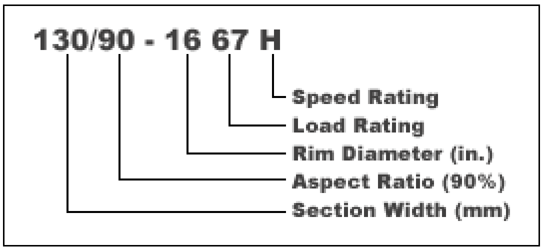


**Figure 2. Parameters of the tire.**

- The section wide of the tire in mm.

$$Wide = 130mm$$

- The high of the tire expressed like a percentage of the section wide in mm:

$$high = 90\%\ of\ 130 = 117mm$$

- The diameter of the rim in inch. To calculate the diameter in mm we have to:

$$Rim = 16in * 25.4\frac{mm}{in} = \phi\ 406.4mm$$

- The diameter of the tire is:

$$Diameter = Rim + 2 * high = 640.4mm$$

- The radii of the tire is:

$$Radii = \frac{Diameter}{2} = 320.2mm$$

In our model, if we have a specific tire and we want to calculate all of this parameters, we use the program *tire.m* the inputs of this program are the three number that describe the size of the tire ***e.g. [180/55-17].*** The outputs of the program are the ***Section wide of the tire, the high of the tire, the diameter of the rim, the diameter of the tire and the radii of it***. They are calculated as explained above. The final data that we need in our model is the radii.

The code of the program *tire.m* is:

```
%                            tire help
%
%    ****************************************************************
%    ***   Analysis of dynamic handling of one motorcycle         ***
%    ***              using simulations tools.                    ***
%    ***                                                          ***
%    ***   Tutor: Michal Manka                                    ***
%    ***   Student: Miguel Santana Gallego                        ***
%    ***                                                          ***
%    ****************************************************************
%
% This program calculates the different parameters that describe the size of
% one motorcycle´s tire.
%
function [wide High Rim Diameter Radii]=tire
disp('                              Tire Measure                             ');
disp('Enter the parameters of the wheel [120/70-17]in this format [120 70 17]');
T=input('Parameters of the tire:    ');
disp('The wide of the tire in mm is:');
Wide=T(1)
disp('The high of the tire in mm is:')
High=(T(2)/100)*T(1)
disp('The diameter of the rim in mm is:')
Rim=T(3)*25.4
disp('The total diameter of the tire is:')
Diameter=Rim+2*High
disp('The total radii of the tire is:')
Radii=Diameter/2
end
```

**Figure 3. Program tire.m**

For example, if we want to identify the different parameters of this tire ***[180/55-17],*** we run the program with these inputs: ***[180 55 17]:***

```
>> tire
                                Tire Measure
Enter the parameters of the wheel [120/70-17]in this format [120 70 17]
Parameters of the tire:    [180 55 17]
The wide of the tire in mm is:

Wide =

   180

The high of the tire in mm is:

High =

   99.0000

The diameter of the rim in mm is:

Rim =

  431.8000

The total diameter of the tire in mm is:

Diameter =

  629.8000

The total radii of the tire in mm is:

Radii =

  314.9000
```

**Figure 4. Tire.m results.**

The final input of our model is the radii of the tire, and it must be expressed in the unit of meter. Hence, the input would be:

| Tire radii 0.3149 m |
|---|

## 1.2. Resistance forces acting in the motorcycles.

During steady state motion, the thrust produced by the engine is equated to the forces that oppose forward motion and depend essentially on three phenomena.

- Resistance to tire rolling.
- Aerodynamic resistance to forward motion.
- The component of the weight force caused by the slope of the road plane.

In our model, we simplify and only are considered the two first forces. Rolling resistance and Aerodynamic resistance.

### 1.2.1. Rolling resistance.

If we consider a completely rigid tire the rolling radius is the same in the load wheel than in the unload wheel. However the effective rolling radius in free motion is smaller than the radius of the unloaded tire because of the deformation of the tire. Its value depends on the type of tire, its radial stiffness, the load, the inflation pressure and the forward velocity.

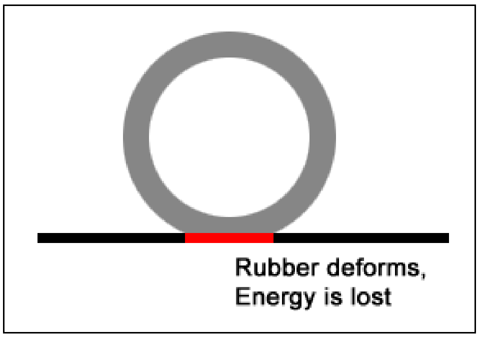


**Figure 5. Energy lost in a tire.**

During the tire´s rolling, the portion of the circumference that passes over the track undergoes a deflection. Due to the hysteresis of the tire material, part of the energy that was spent in deforming the tire carcass is not restored in the following phases of relaxation. This lost of energy is expressed like a force $\boldsymbol{F_w}$ of rolling resistance that opposes the forward motion and whose value is given by the product of the rolling resistance coefficient $\boldsymbol{f_w}$ and the vertical load $\boldsymbol{N}$

The rolling resistance coefficient depends on the type of tire, its dimension, the characteristics of the tire, the temperature, the conditions of use and the inflation pressure. However, we assume the typical value of **2%**

$$\boldsymbol{F_w = N * f_w}$$

The way in which our model computed the rolling resistance force is shown below:

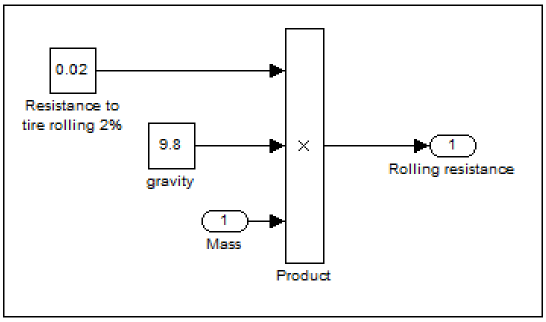


**Figure 6. Simulink, rolling resistance.**

## 1.2.2. Aerodynamic resistance forces.

All the aerodynamics influences that act on the motorcycle can be represented by three forces, which are assumed to be applied on the center of gravity:

- The drag force, in opposition to forward motion.
- The lift force that tends to raise de motorcycle.
- The lateral force that pushes the motorcycle sideways.

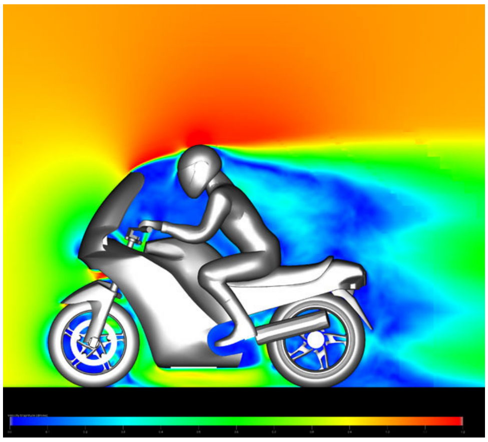

**Figure 7. Aerodynamic influences.**

The most important component is the drag forces and it is applied at a point, called the pressure center, which does not coincide with the center of gravity, but rather is generally located above it. In our model, we assume that there is only one aerodynamic resistance force and it acts in the center of gravity.

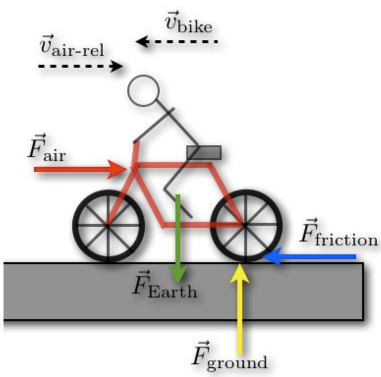


**Figure 8. Forces acting the motorcycle.**

The drag force influences both the maximum attainable velocity and performance in acceleration. The drag force $F_D$ is approximately proportional to the square of the motorcycle´s forward velocity and depends on the density of the air $\rho$, the frontal area of the motorcycle $A$ and the coefficient of aerodynamic resistance $C_D$

$$F_D = \frac{1}{2} \rho \cdot C_D \cdot A \cdot V^2$$

The way in which our model computed the aerodynamic resistance force is shown below:

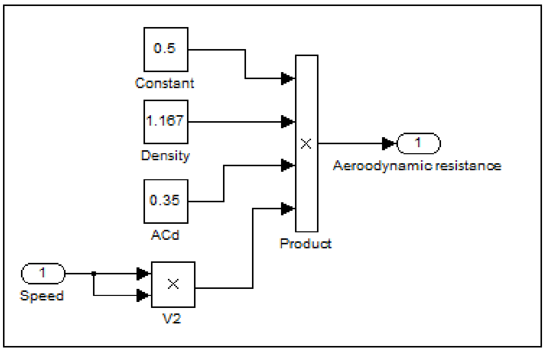

**Figure 9. Simulink, aerodynamic resistance.**

The value of the coefficient $C_D$ is strongly influenced by the shape of the motorcycle. The aerodynamic characteristics of motorcycle is given by the drag area $C_D \cdot A$ the typical values of this coefficient vary between **0.18**$m^2$ for speed record motorbikes to **0.7**$m^2$ for motorcycles with no fairing and the rider in and erected position. The typical value for a super bike motorcycle is **0.35**$m^2$**.**

## 1.3. Engine Power.

During steady state motion, the thrust produced by the engine is equated to the forces that oppose forward motion. Therefore we have to calculate this thrust based on the real power curve of the motorcycle´s engine. The engine that we will use in our model is the engine used in the **ducati 848.**

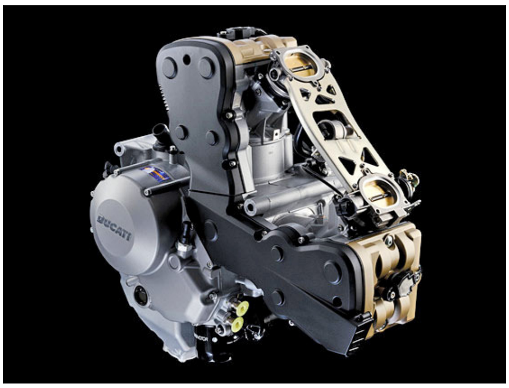

**Figure 10. Ducati 848 engine.**

The real power curve of the engine is simplified by three linear curves that we can see below in the next figure.

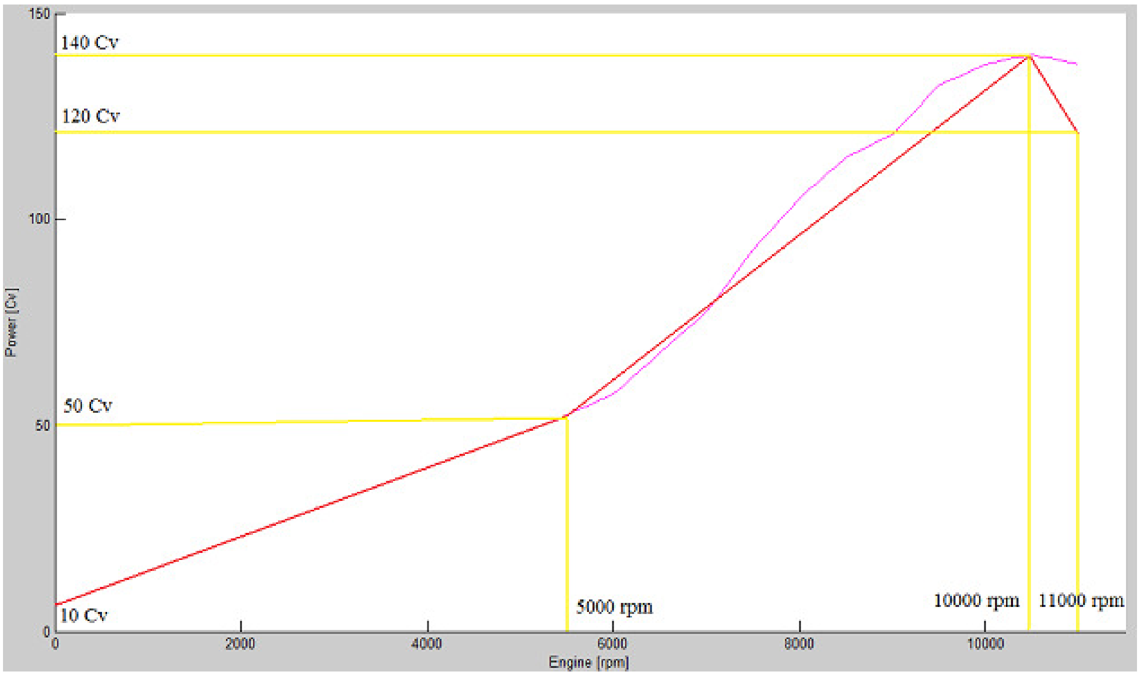


**Figure 11. Curve power of the engine.**

The purple curve represents the real power curve, while the three red curves represent the simplification that we use in our model.

To obtain these curves, we interpolate three linear curves using the real power curve in the intervals of [0-5000],[5000-10000],[10000-11000] (rpm). The three curves obtained are:

| | |
|---|---|
| **If the rpm of the engine are: Engine<5000 rpm** | $\boldsymbol{Power = 10 + 0.008 \cdot rpm}$ |
| **If the rpm of the engine are: 5000<Engine<10000 rpm** | $\boldsymbol{Power = -40 + 0.018 \cdot rpm}$ |
| **If the rpm of the engine are: 10000<Engine<11000 rpm** | $\boldsymbol{Power = 340 - 0.02 \cdot rpm}$ |

**Table 1. Interpolation curves.**

At this moment we only know the power in ***Cv***. as a function of the revolution per minute of the engine. However the output is the force in ***Newton*** and the input of the model is the instantaneous speed of the motorcycle in $(\frac{m}{s})$**,** so we need to transform this linear speed of the motorcycle in the instantaneous ***rpm*** of the engine and then calculate the force using the interpolation curves. The procedure is as follow.

| | |
|---|---|
| Instantaneous linear speed of the motorbike | $V(\frac{m}{s})$ |
| Angular speed of the tire in $\frac{rad}{s}$ | $w = \frac{V(\frac{m}{s})}{Radii_{tire}} \; (\frac{rad}{s})$ |
| Angular speed if the tire in rpm | $w = w(\frac{rad}{s}) \cdot \frac{60}{2\pi}(rpm)$ |
| Revolution per minute of the engine through the gearbox rpm | $w = w \cdot (\frac{39}{15})_{\substack{Final \\ drive}} \cdot (\frac{1.84}{1})_{\substack{Primary \\ drive}} \cdot (\frac{39}{15})_{Gear}$ |
| After know the rpm of the engine, we interpolate the power in this moment. | ***Power (Cv)*** |
| Power in (W) | ***Power (W)=Power(Cv)***$\cdot\, 735.5(\frac{w}{Cv})$ |
| Instantaneous thrust force | $F = \frac{Power(w)}{V(\frac{m}{s})} \; (N)$ |

**Table 2. Program flow.**

The information relative to the gearbox, primary drive and final drive of the ducati 848 is obtained directly of the catalogue of Ducati's factory.

| Transmission | |
|---|---|
| GEARBOX | 6 speed |
| RATIO | 1=37/15 2=30/17 3=28/20 4=26/22 5=24/23 6=23/24 |
| PRIMARY DRIVE | Straight cut gears, Ratio 1.84:1 |
| FINAL DRIVE | Chain; Front sprocket 15; Rear sprocket 39 |
| CLUTCH | Wet multiplate with hydraulic control |

**Figure 12. Ducati 848 Transmission parameters.**

The way in which our model computed the instantaneous thrust force is shown below:

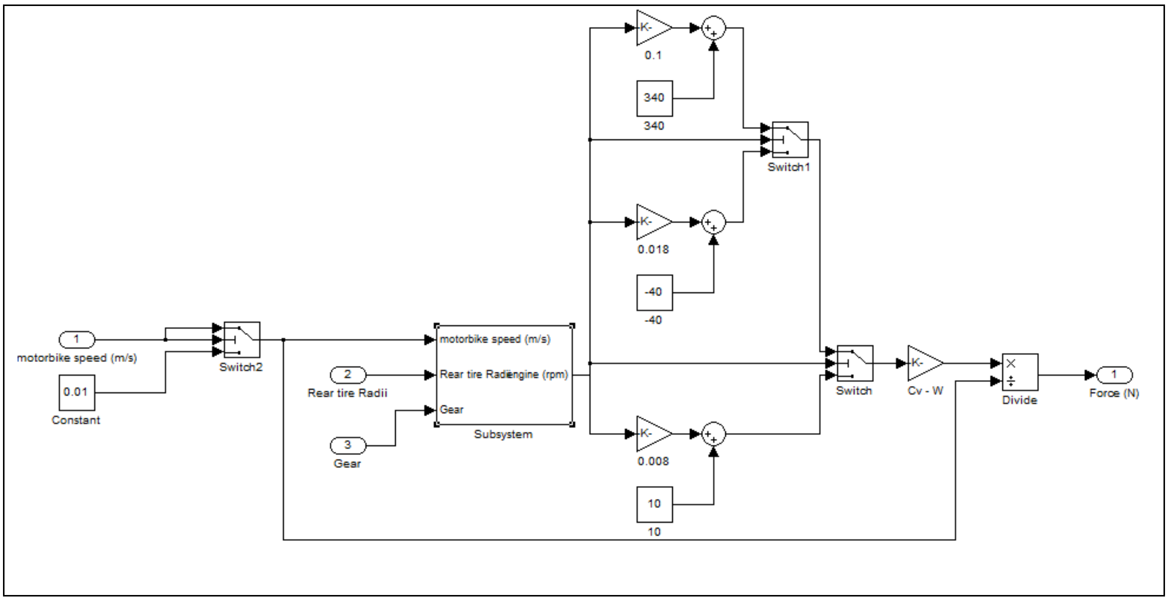


**Figure 13. Simulink, thrust force.**

The subsystem calculate the rpm of the engine based on the Rear tire radii, gear selected, primary and final drive and instantaneous motorbike speed.

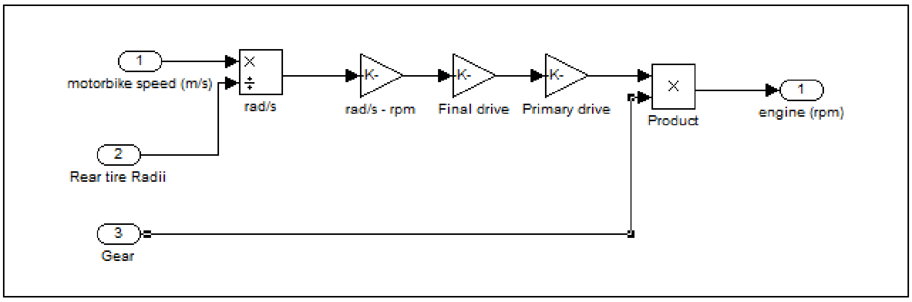


**Figure 14. Simulink, transmission.**

If we analyze the Simulink model, we can see first how the model solve the initial condition using a non zero initial speed, setting this initial speed in 0.01 $(\frac{m}{s})$ and secondly how the model choose the different curves of power in function of the instantaneous speed to calculate the power. The output of this block is the instantaneous thrust force.

## 1.4. Outputs of the model.

The outputs of the model are the instantaneous speed in $\left(\frac{Km}{h}\right)$, the acceleration in $\left(\frac{m}{s^2}\right)$, the aerodynamic force in (N), the engine force in (N), the dynamic load in the front wheel in (N) and the dynamic load in the rear wheel in (N).

### 1.4.1. The dynamic load in the front and rear wheel.

The equations of equilibrium of a motorcycle enable us to determinate the unknown values of the reaction forces $\boldsymbol{N_f}$ and $\boldsymbol{N_r}$ (Front and Rear Load), once the weight force ***mg***, driving force ***S*** and drag force $\boldsymbol{F_D}$ are known.

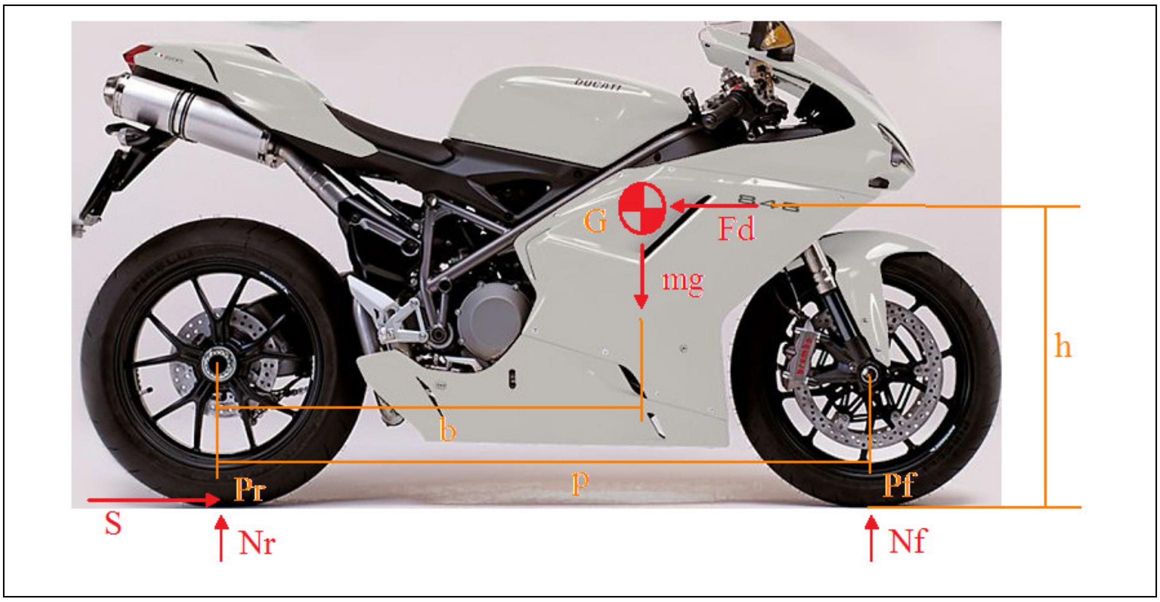


**Figure 15. Ducati 848 Equilibrium forces.**

To calculate the reaction forces in the front and in the rear wheel, we introduce the following three hypotheses:

- The rolling resistances forces are the same in the two wheels, $\mathbf{0.01}\boldsymbol{N_r}$ *and* $\mathbf{0.01}\boldsymbol{N_f}$ assuming that the rolling resistance coefficient is **0.02** and it is equally distributed in the two wheels.
- The aerodynamic drag force is applied in the center of gravity.
- The road surface is flat.

| | |
|---|---|
| **Equilibrium of horizontal forces** | $S - F_d - 0.01N_r - 0.01N_f = 0$ |
| **Equilibrium of vertical forces** | $mg - N_r - N_f = 0$ |
| **Equilibrium of moments with respect to the center of gravity** | $Sh - 0.01N_r h - N_r b + N_f(p - b) - 0.01N_f h = 0$ |

**Table 3. Equilibrium equations.**

Therefore, the vertical forces exchanged between the tires and the road plane is:

| | |
|---|---|
| **Dynamic load on the front wheel** | $N_f = \frac{mg}{p} \cdot (0.01h + b) - \frac{sh}{p}$ |
| **Dynamic load on the rear wheel** | $N_r = \frac{mg}{p} \cdot (P - b - 0.01h) + \frac{sh}{p}$ |

**Table 4. Dynamic loads.**

The way in which our model computed the loads in the front and rear wheels are:

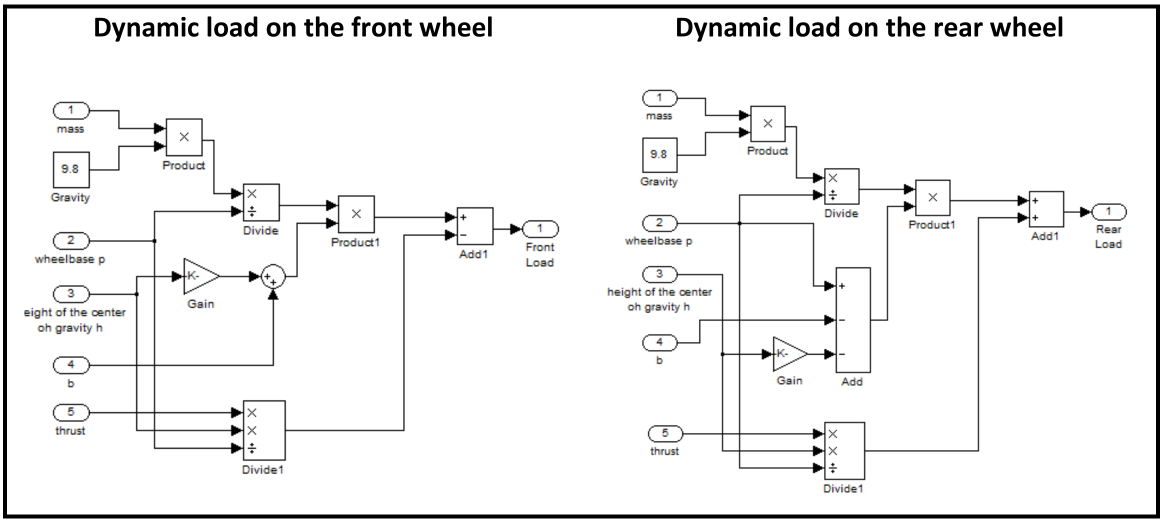


**Figure 16. Simulink, dynamic loads.**

## 1.4.2. The instantaneous speed and acceleration.

The thrust produced by the engine is equated to the forces that oppose forward motion therefore if we want to calculate the instantaneous speed and acceleration of the motorcycle, we only need to use the equation of equilibrium. The forces that are involved in the rectilinear motion are: The thrust of the engine (+), the rolling resistance force (-) and the aerodynamic resistance force (-). Using the Newton´s law of motion, we can calculate the instantaneous acceleration and if we integrate this value, we obtain the instantaneous speed.

The way in which our model computed these values is:

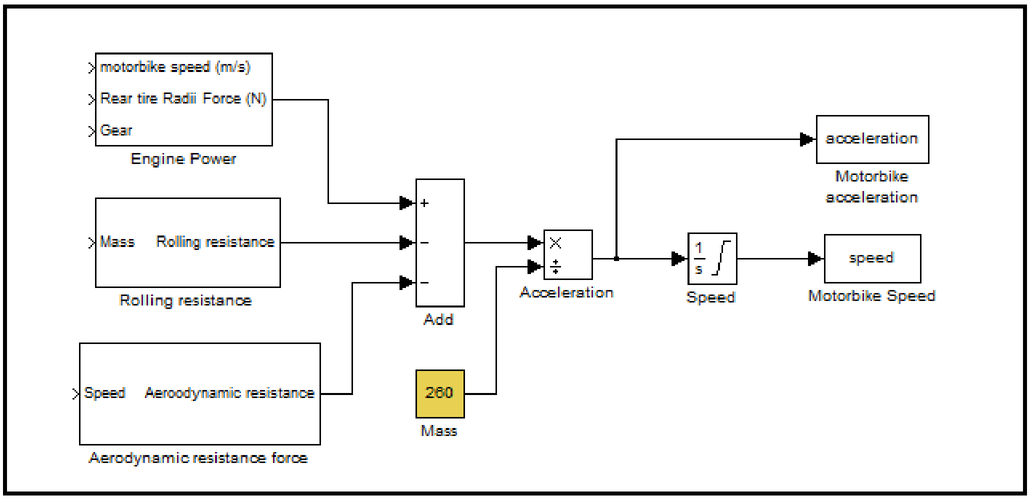


**Figure 17. Simulink, Acceleration and speed.**

To see the data we use in our model the block parameters: ***To workspace***. With this block we created a variable in the workspace of Matlab which name could be chosen by us.

## 1.5. Inputs of the model

The most important part of our model is the inputs of the program. To run the model we only need a few numbers of data. These values are: The radii of the tire which has been calculated by the program *tire.m* and the gear selected in the gear box, used to calculate the rolling resistance and the instantaneous thrust of the engine using the power curve of the engine. To calculate the aerodynamic resistance force we need the density of the air at normal temperature (20 ºC), the Drag area of the motorcycle which is the product between the Drag coefficient and the frontal area ($C_D \cdot A$). Beside to calculate the rolling resistance and the dynamic loads in the tires of the motorcycle, we need the mass of it. To compute the amount of each dynamics loads we need also geometric data. These parameters are the wheel base of the motorcycle, the height of the center of gravity, and the longitudinal position of it.

In our program, all of these data refer to a specific model of motorcycle. How we said above, this model is the **Ducati 848.** Using the parameters that describe this model we can obtain the following inputs.

| | |
|---|---|
| Rear tire radii | 0.316 (m) |
| Gear box ratio | 37/15 ; 30/17 ; 28/20<br>26/22 ; 24/23 ; 23/24 |
| Final drive | 39/15 |
| Primary drive | 1.84/1 |
| Density of the air | 1.167 (kg/m3) |
| Drag area | 0.35 |
| mass | 260 (kg) |
| Wheel base | 1.414 (m) |
| Eight of C.G. | 0.461 (m) |
| Longitudinal position of C.G. | 0.480 (m) |

**Table 5. Inputs of rectilinear_motion.mdl**

## 1.6. Rectilinear motion of motorcycle model

Once, we have all the inputs that our model need, we can run the program rectilinear_motion.mdl

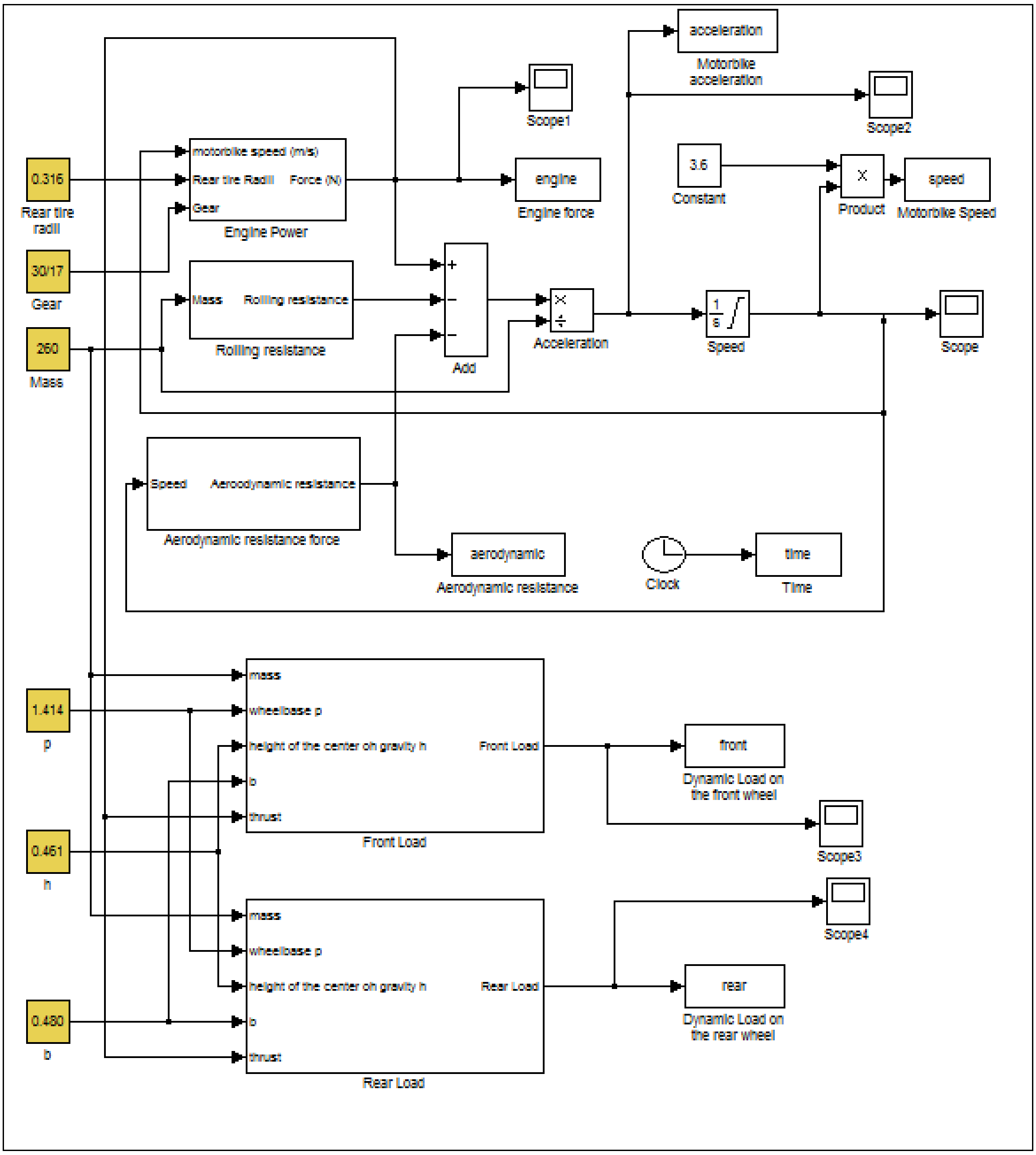


**Figure 18. Simulink, rectilinear_motion.mdl.**

## 1.7. The function graphic.m

To run the problem, firstly we need to fix the simulation stop time. After a number of tests, we decided that with 30 seconds is enough time. It means that in 30 seconds, the motorbike reach a constant speed with non-acceleration.

After the simulation time, the program generates six variables in the work space of Matlab. These variables are: ***acceleration, aerodynamic, engine, front, rear, speed and time***. All of these data are an array with the instantaneous values of each parameter in the interval of time of ***0.01*** second.

To make easier the understanding of these values we represented each of them in the interval of 30 seconds. To do it, we use the program graphic.m

```
%                            graphic help
%
%    ******************************************************************
%    ***   Analysis of dynamic handling of one motorcycle          ***
%    ***               using simulations tools.                    ***
%    ***                                                           ***
%    ***   Tutor: Michal Manka                                     ***
%    ***   Student: Miguel Santana Gallego                         ***
%    ***                                                           ***
%    ******************************************************************
%
% This program represented the outputs of the program rectilinear_motion.mdl
%

function graphic(time,speed,acceleration,aerodynamic,engine,front,rear)
hold on
subplot(2,3,1),plot(time,speed,'b')
title('speed (Km/h)')
grid
axis([0,30,0,300])
subplot(2,3,2),plot(time,acceleration,'k')
title('acceleration (m/s2)')
grid
axis([0,30,0,15])
subplot(2,3,3),plot(time,aerodynamic,'r')
title('aerodynamic (N)')
grid
axis([0,30,0,1000])
subplot(2,3,4),plot(time,engine,'g')
title('engine force (N)')
grid
axis([0,30,0,3000])
subplot(2,3,5),plot(time,front,'b')
title('Dynamic load on the front wheel (N)')
grid
axis([0,30,-300,700])
subplot(2,3,6),plot(time,rear,'b')
title('Dynamic load on the rear wheel (N)')
grid
axis([0,30,1000,3000])
end
```

**Figure 19. Graphic.m**

After run the program, we obtain the six values that the function graphic.m need. Therefore, if we put in the command line:

*>> graphic(time,speed,acceleration,aerodynamic,engine,front,rear)*

We graphic the different values of the outputs of our model.

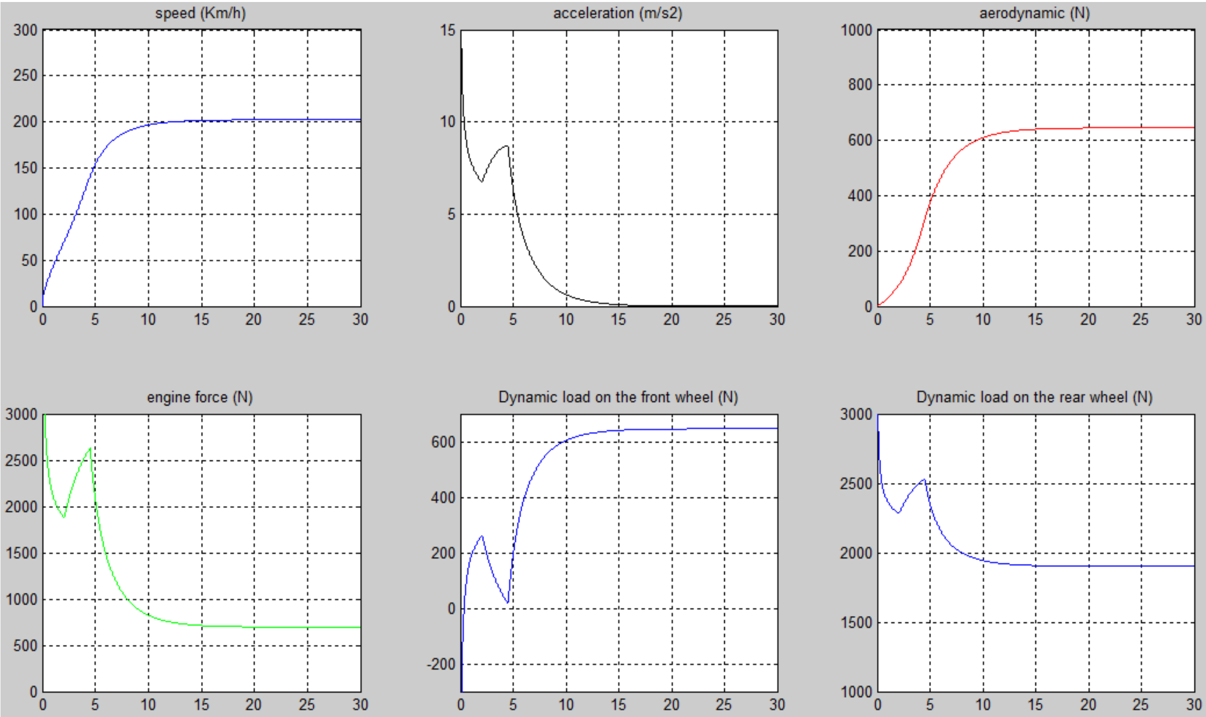

**Figure 20. Outputs of rectilinear_motion.mdl**

## 1.8. Analysis of results.

In this section I will explain and analyze how the different factors affect the rectilinear motion of one motorbike. Scilicet, why we obtain these different graphic and how affect the changes of the parameters in the behavior of our motorcycle.

- The simulation stop time is fixed in 30 seconds, however we can see in the graphic of acceleration, that the motorbike after ***20 seconds***, its acceleration tends to zero. It means that the thrust of the engine is equal to the resistance forces.
- If we compare the graphic of ***engine force*** and ***aerodynamic force***, we can see how the two values in the equilibrium state are very similar but not equal. It is caused by the fact that we must sum the action of the rolling resistance to the aerodynamic force.

- The ***engine force*** has three parts that correspond with the three interpolation curves. At low speed, the engine thrust is very high because in the equation $\boldsymbol{F} = \frac{\boldsymbol{Power(w)}}{\boldsymbol{V(\frac{m}{s})}}\ (\boldsymbol{N})$ the value of the denominator is very small. When the speed start to increase, the rpm of the engine are increasing also, however the power that gives the engine increase but in a smaller ratio than the angular speed of the rear tire, so the thrust of the engine start to decrease until it reaches the equilibrium with the resistance forces. In this point, the acceleration of the motorcycle is zero.

- The aerodynamic force, increases very quickly because it value is proportional to the square of the instantaneous velocity.

- Analyzing the dynamic load in the front and in the rear tire, we can see how in the first seconds, we have a negative force in the front wheel. It means that the motorbike is doing a wheeling. This phenomenon is very common in the high motorbike acceleration like race starts. It could be a dangerous phenomenon if the rider don´t control the torque of the engine with the throttle, because it could finish in the overturn of the motorbike. However, the rear suspension of the motorbike helps to solve this problem.

- In the equilibrium state, the sum of front load and rear load is equal to the total weight of the motorcycle and rider.

- The curves of loads are very similar to the thrust of the engine, because it depends highly of this value.

- Let´s now to compare the different between thrust in second gear or in a higher gear like fourth gear. Firstly we must obtain the two different graphics.

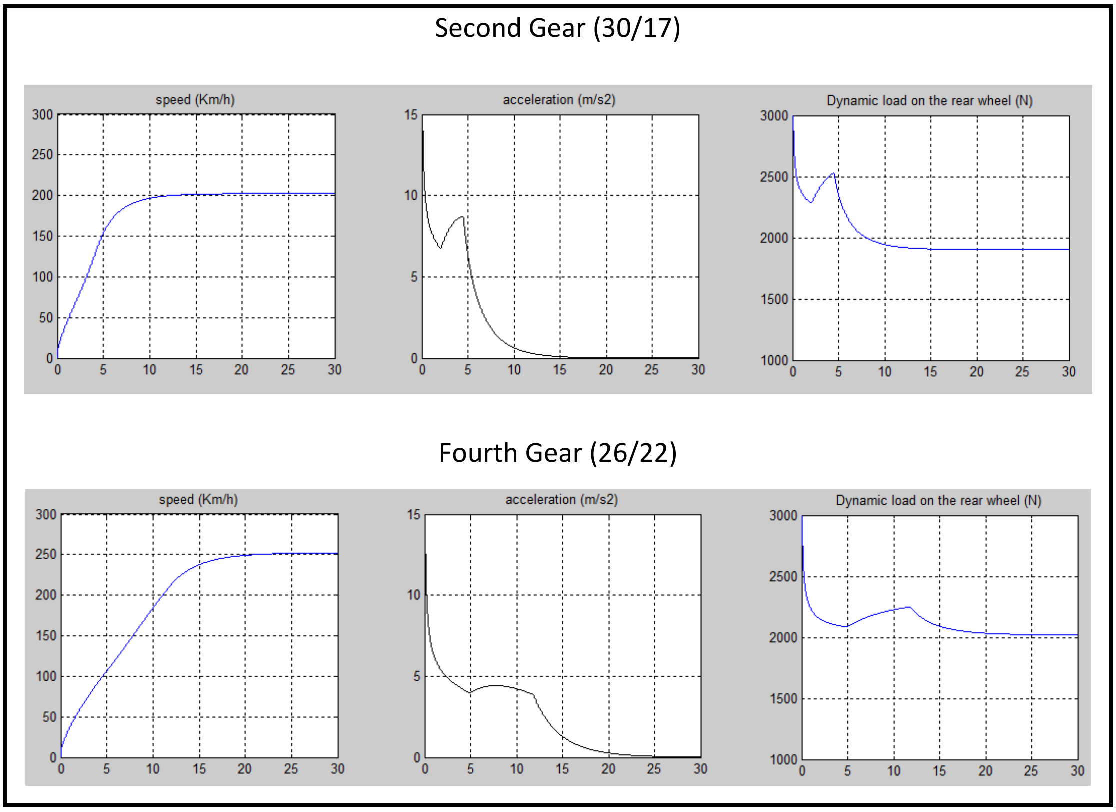


**Figure 21. Acceleration in 2º or 4º gear.**

- Firstly, in the second gear the speed reach by the motorbike is smaller than with the fourth gear, however with the second gear we need half time than with the forth gear to reach it.
- Secondly, the acceleration is softer with the higher gear.
- If we compare the load in the rear wheel, we can see how the load is bigger in the second case. It is cause of the equilibrium speed is higher and therefore the aerodynamic force effect is bigger in this case. We need to highlight the important of the speed rating of our motorbike`s tires. Because, if the top speed of our tire is not too big, we could have big problem at high speed. Firstly by the angular forces in the wheel and secondly by this effect that could destroy our tire causing a very huge accident.
- When rider drive their motorbikes, is very common that they want a big acceleration capacity at the exit of one curve. Looking these two graphics, we can study what happen when we accelerate at the exit of a curve. In the second gear, we have a big acceleration capacity at low rpm of the engine (Typical situation at the exit of a curve). This capacity is bigger with the lower gear. For this reason, is very typical that the rider change their final drive kit by one with a lower ratio, and so have greater acceleration capacity.

## 2. Steady turning motion.

A motorcycle`s dynamic properties are described using terms like maneuverability, handling and stability. Maneuverability and handling describe the motorcycle´s ability to execute complicated maneuvers, and how difficult it is for the rider to perform them. Stability, on the other hand, means a motorcycle´s ability to maintain equilibrium in response to outside disturbance like an uneven road surface or gust of wind.

During steady turning motion the motorcycle can have neutral, under or over steering behavior. To maintain equilibrium the rider applies a torque to the handlebar that can be zero, positive, (in the same direction to the handlebar rotation) or negative (applied in the direction opposite to the rotation of the handlebar). These characteristics are important and concur to define the sensation of the motorcycle´s handling.

**Figure 22. Steady turning Vs Over steering behavior.**

Motorcycles in motion need to be controlled by the rider at all the time. Rider input affects the motorcycle´s equilibrium and direction of forward motion. In rectilinear motion, a motorcycle is called "directional stable" if it easy to control or naturally tends to maintain its equilibrium and follow a rectilinear path. However, that a large tendency towards directional stability makes a motorcycle hard to handle. The best example of it phenomena are the custom motorbike that are cumbersome to turn and control through twists and turns.

This section discusses the directional stability of motorcycles, which is determined by a number of factors:

- Inertial properties of the motorcycle.
- Forward speed.
- Geometric properties of the steering head.
- Tire properties.

## 2.1. The function steadyturning.m.

A motorcycle can be described as a system of six rigid bodies: Sprung steering components, unsprung steering components, rear frame (Including frame, engine, tank and driver), rear swinging arm and the two wheels. The driver is considered to be rigid body firmly attached to the frame.

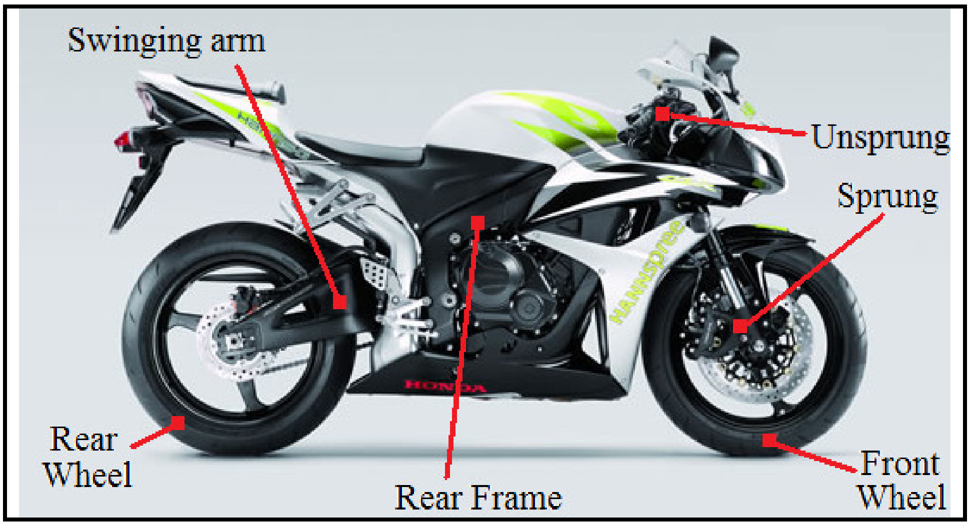


**Figure 23. Model of six rigid bodies.**

Nevertheless, our model refers to a rigid motorcycle, that is to say one without suspensions with the wheels fitted to nonderformable tires, and schematized as two toroidal solid bodies with circular sections.

The equations of motion of the motorcycle in steady turning motion are described below.

The equilibrium conditions give six equations:

- Three force equilibrium equations.
- Three moment equilibrium: Equilibrium around the X-axis (roll), Y-axis (pitch) and the Z-axis (yaw).

In addition we have two equations that give the lateral forces as function of sideslip and camber angles.

Once the roll angle and the steering angle are assigned, the eight equations allow us to obtain the eight unknowns:

- Forward velocity. $V$
- Vertical forces applied respectively to the front and rear wheels. $N_f$ , $N_r$
- Lateral forces applied respectively to the front and rear wheels. $F_{sf}, F_{sr}$
- Sideslip angles $\lambda_f$, $\lambda_r$
- The driving force. $S$

Finally, the equilibrium of the front and the rear frame around the steering axis gives the torque exerted by the rider and applied to the handlebar, which provides and equal and opposite reaction on the rear frame.

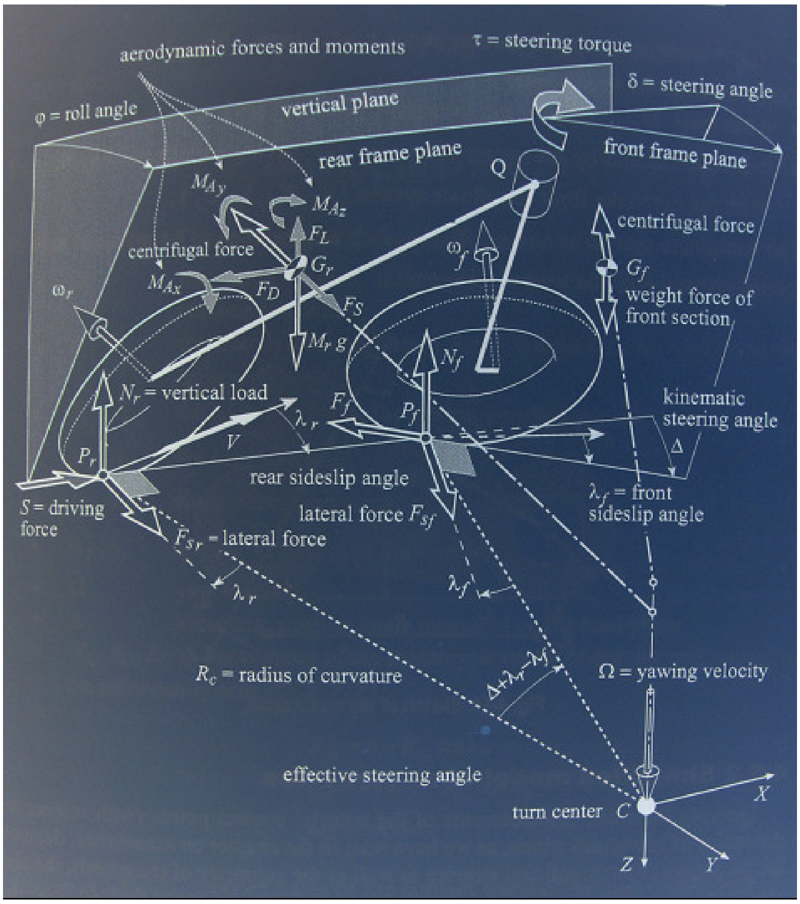


**Figure 24. Forces and moments acting on the motorcycle.**

The inertial and geometrical properties are defined with respect to the coordinate system represented below.

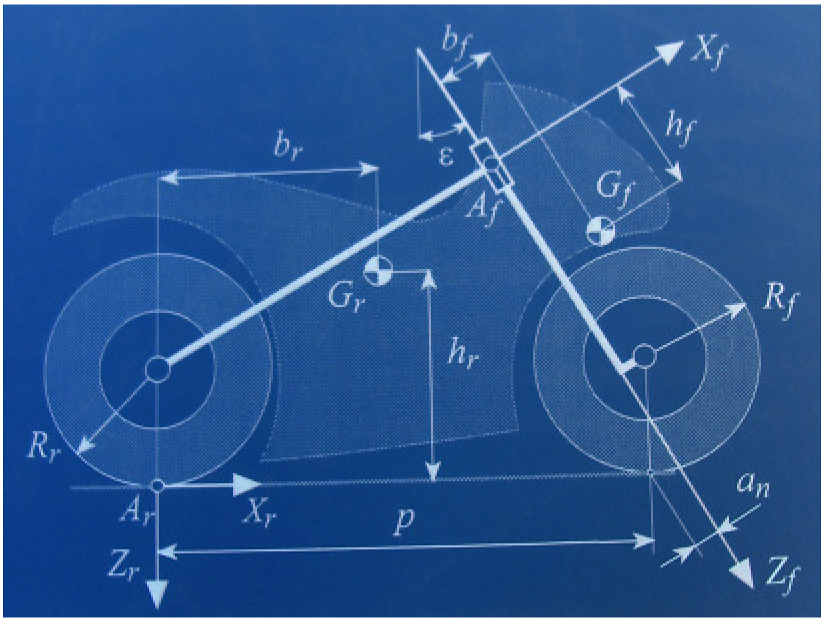


**Figure 25. Sketch of the motorcycle.**

The rear frame is described by the following parameters:

- It has mass $M_r$
- It is characterized by the center of gravity $G_r$ having coordinates $(b_r, 0, -h_r)$ with respect to the rear coordinate system $(A_r, X_r, Y_r, Z_r)$.
- It is considered symmetrical with respect to the *X-Z* plane, hence its inertial characteristics are represented by the following four terms:
  - $I_{xr}$= mass center moment of inertia about Xr axis (roll moment of inertia).
  - $I_{yr}$= mass center moment of inertia about Yr axis (pitch moment of inertia).
  - $I_{zr}$= mass center moment of inertia about Zr axis (yaw moment of inertia).
  - $I_{xzr}$= mass center inertia product about Xr-Zr axes.

The front frame is described by the following parameters:

- It has mass $M_f$
- It is characterized by the center of gravity $G_f$ having coordinates $(b_f, 0, -h_f)$ with respect to the rear coordinate system $(A_f, X_f, Y_f, Z_f)$.

The coordinate system axes $(A_f, X_f, Y_f, Z_f)$ are assumed to be principal axes of inertia so that the inertia tensor is diagonal. i.e. $I_{xzf} = 0$

### 2.1.1 Simplified model of motorcycles.

Ignoring the small displacements of the wheel`s contact point which depend on the angle of pitch, roll and steering as well as the geometry of the wheels, the six equations of equilibrium for the motorcycle in a turn can be easily derived.

| | |
|---|---|
| **Equilibrium of the forces along the X axis** | $S - F_{sf}sin\Delta + mX_G\Omega^2 - F_A = 0$ |
| **Equilibrium of the forces along the Y axis** | $F_{sf}\,cos\Delta + F_{sr} + mY_G\Omega^2 = 0$ |
| **Equilibrium of the forces along the Z axis** | $-N_f - N_r + mg = 0$ |
| **Equilibrium of the moment around the X axis** | $I_{YZ}\Omega^2 - (N_f + N_r)Y_r + mgY_G + (I_{wf}w_f + I_{wr}w_r)\Omega cos\varphi = 0$ |
| **Equilibrium of the moment around the Y axis** | $-I_{XZ}\Omega^2 - N_f(p + X_r) + mgX_G + F_AZ_G - N_rX_r = 0$ |
| **Equilibrium of the moment around the Z axis** | $(p + X_r)F_{sf}\,cos\Delta + Y_rF_{sf}sin\Delta + F_{sr}X_r - SY_r + F_AY_G = 0$ |

**Table 6. Equations of equilibrium.**

The meaning of the symbols as the follow:

| | |
|---|---|
| **S** | The thrust which is necessary for holding the motorcycle stationary in a turn. |
| $F_A$ | The aerodynamic resistance force assumed to be applied to the center of gravity. |
| $F_{sf}, F_{sr}$ | The lateral force applied to the tires by the road. |
| $N_fN_r$ | The vertical loads. |
| $I_{wf}, I_{wr}$ | Spin moments of inertia of the wheels |
| $w_f, w_r$ | Angular velocities of the wheels |
| $\Omega$ | Yaw velocity |
| $\Delta$ | Kinematic steering angle measured on the road plane. |
| $X_G, Y_G, Z_G$ | Coordinates of the motorcycle center of gravity with respect to the reference system (C,X,Y,Z) |
| $X_R, Y_R$ | Coordinates of the contact point of the rear wheel with respect to the reference system (C,X,Y,Z) |

**Table 7. Meaning of the symbols.**

| | |
|---|---|
| $X_G = b - R_{cr}\sin\lambda_r$ | $X_R$= -$R_{cr}$ $\sin\lambda_r$ |
| $Y_G = h\sin\varphi - R_{cr}\cos\lambda_r$ | $Y_R = -R_{cr}\cos\lambda_r$ |
| $Z_G$= -hcos$\varphi$ | |

**Table 8. Coordinate systems.**

### 2.1.2 Solving the model of motorcycle.

The six equations constitute a non-linear system. Expressing the roll angle $\varphi$ as a function of the yaw velocity $\Omega$ and of the radius $R_{cr}$ , and expressing the lateral forces of the tires as linear functions of the sideslip angles $\lambda_f$, $\lambda_r$ and roll angle $\varphi$, we can calculate the six unknowns.

In our model we set the steering angle δ and the roll angle and the six unknowns are:

- The sideslip angles $\lambda_f$, $\lambda_r$
- The radius $R_{cr}$
- The vertical loads $N_f$ , $N_r$
- The thrust S necessary for assuring motion at a constant velocity.

The first step to solve our problem is to introduce the different parameters needed by the model. These parameters are summary in the next table.

| | | | |
|---|---|---|---|
| GEOMETRY | | Cornering stiffness | $K_{\lambda f}$ = 11.2 kN/rad |
| Wheel base | P=1.414 m | Camber stiffness | $K_{\varphi f}$ = 0.94 kN/rad |
| Trial | a=0.116 m | REAR ASSEMBLY | |
| Caster angle | E=27º | Mass | Mr=217.5 kg |
| FRONT ASSEMBLY | | Longitudinal position Gr | br=0.480 m |
| Mass | Mf=30.7 Kg | Height of Gr | hr=0.616 m |
| X-position of Gf | bf=0.024 m | Moments of Inertia | |
| Z-position of Gf | hf=0.461 | Around x-axis | Ixr=31.20 kgm2 |
| Moments of inertia | | Around z-Axis | Izr=21.08 kgm2 |
| Around x-axis | Ixf=1.23 kgm2 | Product Inertia x-z | Ixz=1.74 kgm2 |
| Around z-axis | Izf=0.44 kgm2 | REAR WHEEL | |
| Steering damper | c=6.8 Nm/rad/s | Wheel radius | Rr=0.305 m |
| FRONT WHEEL | | Axial Inertia | Iwr=1.05 kgm2 |
| Wheel radius | Rf=0.305 m | Cornering stiffness | $K_{\lambda f}$ = 15.8 kN/rad |
| Axial inertia | Iwf=0.72 kgm2 | Camber stiffness | $K_{\lambda f}$ = 1.32 kN/rad |

**Table 9. Inputs of steadyturnig.m.**

This part in the function steadyturning.m is:

```
%GEOMETRY
p=1.414; %Wheel base (m)
a=0.116; %Trial (m)
E=27; %caster angle (°)

%FRONT ASSEMBLY
Mf=30.7; %Mass (Kg)
bf=0.024; %X position of Gf (m)
hf=0.0461; %Y position of Gf (m)
Ixf=1.23; %Moments of inertia (Kgm2)
Izf=0.44;
c=6.8; %Steering damper (Nm/rad/s)

%FRONT WHEEL
Rf=0.305; %Wheel radius (m)
Iwf=0.72; %Axial Inertia (Kgm2)
Klf=11200; %Cornering stiffness (N/rad)
Kfif=940; %Camber Stiffness (N/rad)

%REAR ASSEMBLY
Mr=217.5; %Mass (Kg)
br=0.480; %X position of Gr (m)
hr=0.616; %Y position of Gr (m)
Ixr=31.20; %Moments of inertia (Kgm2)
Izr=21.08;
Ixzr=1.74;

%REAR WHEEL
Rr=0.305; %Wheel radius (m)
Iwr=1.05; %Axial Inertia (Kgm2)
Klr=15800; %Cornering stiffness (N/rad)
Kfir=1320; %Camber Stiffness (N/rad)
```

The second part of the program has assigned the value of the roll and the steering angle. This is the typical situation that a rider must decide when take twists and turns. Depending of the speed and the experience of the rider, he will fix these two parameters.

```
%MOTORCYCLE IN A TURN WITH ASSIGNED ROLL AND STEERING ANGLE

                        %%%%%%%%%%%%%
                        %%%%KNOWS%%%%
                        %%%%%%%%%%%%%

 Fi=30; %Roll angle (°)

             %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
             %%%%CHANGE OF UNITS (°)->(rad)%%%%
             %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Fi=Fi*pi/180; %Roll angle (rad)
E=E*pi/180; %caster angle (rad)
L=i*pi/180; %Steering angle (rad)
```

Once knows factors are fixed, is time to calculate the unknowns. The ordered followed is show below:

1. The sideslips angles as function of the roll angle and the cornering and camber stiffness on both wheels.

```
%The sideslip angles

Lf=(1-Kfif)*Fi/Klf; %Front (rad)
Lr=(1-Kfir)*Fi/Klr; %Rear (rad)
```

2. The effective steering angle as function of the caster angle, steering angle, roll angle and the two sideslips angles.

```
%The effective steering angles

A=cos(E)*L/cos(Fi)+Lr-Lf; %(rad)
```

3. The turning radius of the trajectory describes by the rear wheel as function of the wheel base, effective steering angle and both sideslips angles.

```
%The turning radius of the trajectory described by the rear wheel

Rcr=p/(tan(A-Lf)*cos(Lr)+sin(Lr)); %(m)
```

4. The forward velocity as function of the roll angle and the turning radius.

```
%Forward velocity

V=sqrt(tan(Fi)*9.8*Rcr); %(m/s)
```

5. The angular velocity as function of the forward velocity and the turning radius.

```
Omega=V/Rcr; %(rad/s)
```

After we have calculated these values is time to express the different coordinate system with respect to the reference system [C]:

```
%Coordinate systems

    %Coordinates of the motorcycle center of gravity
Xg=br-Rcr*sin(Lr); %(m)
Yg=hr*sin(Fi)-Rcr*cos(Lr); %(m)
Zg=-hr*cos(Fi); %(m)

    %Coordinates of the contact point of the rear wheel
Xr=-Rcr*sin(Lr);
Yr=-Rcr*cos(Lr);
```

Before to calculate the loads in the front and rear wheels, we need to estimate first the values of the drag force, acting in the center of gravity and product of inertia of the front assembly.

```
Ro=1.167; %Density of the air (Kg/m3)
CdA=0.35; %Drag Area
Fa=0.5*Ro*CdA*V^2; %Drag force
Ixz=Mr*Xg*Zg+Ixzr*cos(Fi); %Product of inertia
```

Once we have all the unknowns' necessaries to know the loads in the wheels, we can compute their values.

```
Nf=(-Ixz*Omega^2+Mr*9.8*Xg-Mr*9.8*Xr+Fa.*Zg)/(p); %The vertical load in the front wheel
Nr=Mr*9.8-Nf; %The vertical load in the rear wheel

%Lateral force in the front wheel
Fsf=(-Mr*Yg*Omega^2*Xr+Fa*Yg+Mr*Xg*Omega^2*Yr-Fa*Yr)/((p+Xr)*cos(A)+Yr*sin(A)-Xr*cos(A)-Yr*sin(A)); %(N)

%Lateral force in the rear wheel
Fsr=-Mr*Yg*Omega^2-Fsf*cos(A); %(N)

%Driving force
S=Fsf*sin(A)-Mr*Xg*Omega^2+Fa; %(N)
```

The final step of our program consists in compute the value of the torque necessary to apply in the handlebar to maintain the desired path.

```
%THE TORQUE APPLIED TO STEERING

%With a non-zero steering angle the normal trail is

d=0; %offset

an=Rf*cos(L)*sin(E)/sqrt(1-(sin(L)*sin(E))^2)-d; %(m)

%If the pitch is ignored with respect to the caster angle, the front
% wheel camber angle is:

B=asin(cos(L)*sin(Fi)+cos(Fi)*sin(L)*sin(E)); %(°)

%The torque applied to steering
M=-(Fsf*cos(B)-Nf*sin(B))*an; %(Nm)
```

## 2.2. Outputs.

With this program, is easy to calculate the different parameters that describe the behavior of the motorcycle, such as the turning radius, the forward velocity or the torque necessary to be applied in the handlebar to maintain the desired path. In addition we can compute the values of the different loads that act in the tires of our motorcycle. These values are very important for safety, because we can understand the performance limit of our vehicle trough the grip limit of the tires.

To study the different unknowns, we will graphic each of them. To make it we will set firstly the roll angle and then, programming a ***loop for*** with which we will increment step by step the value of the steering angle. In this way we can study and understand the way in which all of these parameters depend on each other.

## 2.3. Analysis of results.

### 2.3.1. Roll angle.

The motorcycle, in steady turning, is subject to both a restoring moment, generated by the centrifugal force that tends to return the motorcycle to a vertical position, and to a tilting moment, generated by the weight force, that tends to increase the motorcycle´s inclination, or roll angle.

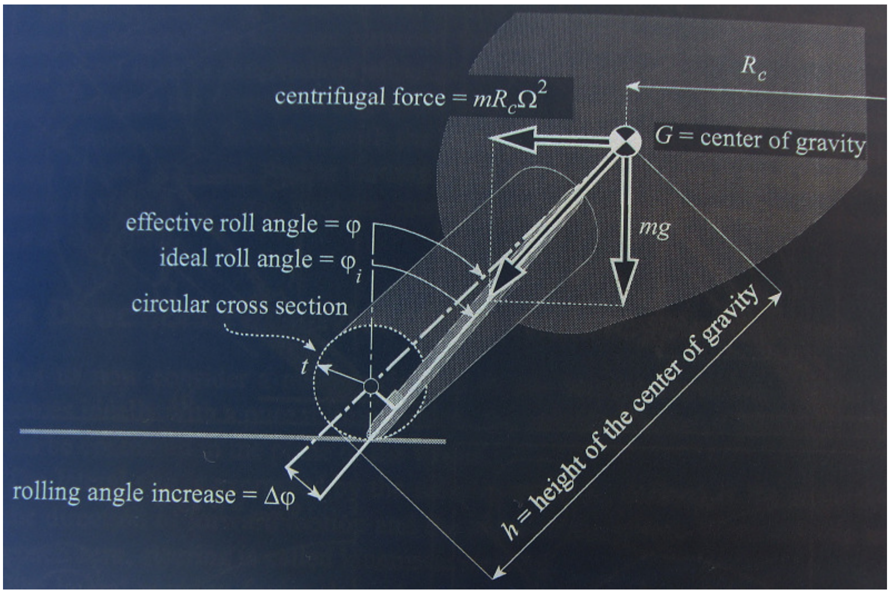


**Figure. 26. Roll angle.**

As shown in the graph above, the effective roll angle is the sum between the ideal roll angle and the roll angle increase.

The equilibrium of the moments allows us to derive the ideal roll angle in terms of the forward velocity V and the radius of the turn Rc, which is measured from the center of gravity to the turning axis.

$$\varphi = \arctan\frac{V^2}{gR_c}$$

Looking the previous figure, we can see how the roll angle increase is function of the cross section radius of the tire and the eight of the center of gravity. Therefore, the equation which describes the real roll angle is:

$$\varphi = \arctan\frac{V^2}{gR_c} + \arcsin\frac{t \cdot \sin\left(\arctan\frac{V^2}{gR_c}\right)}{h-t}$$

Looking the previous equation shows us that the real roll angle increase with the cross section radius and decrease with the height of the center of gravity. Therefore, the use of wide tires forces the rider to use greater roll angles with respect to the angle necessary with a motorcycle equipped with tires that have smaller cross sections. Furthermore, with equal cross section of the tires, to describe the same turn with the same forward velocity, a motorcycle with a low center of gravity needs to be tilted more than a motorcycle with a higher center of gravity. The grip that have the tires in a motorcycle, are greater if the roll angle is low. For this reason, the handling in a supermotard is better than the maneuverability of for example, super sport motorbikes.

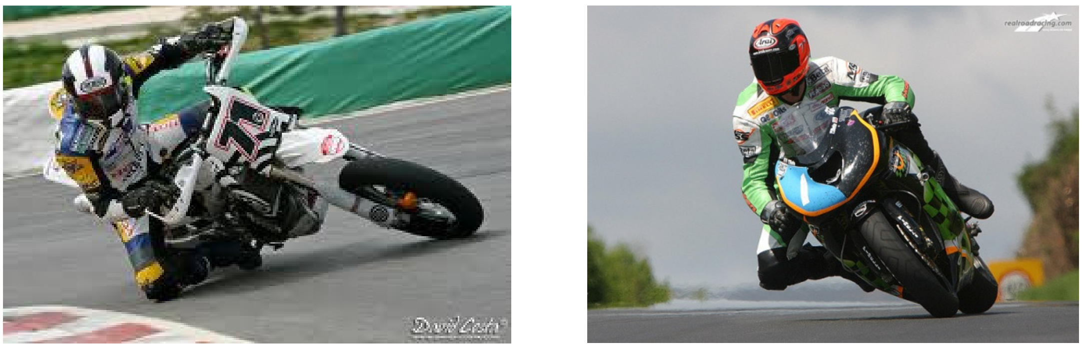

**Figure 27. Supermotard Vs Super sport.**

### 2.3.2. Driving style.

The motorcycle roll angle on a turn is influenced, in a significant way, by the rider´s driving style. By the leaning with respect to the vehicle, the rider changes the position of his center of gravity with respect to the motorcycle. The four following pictures illustrate the possible situations.

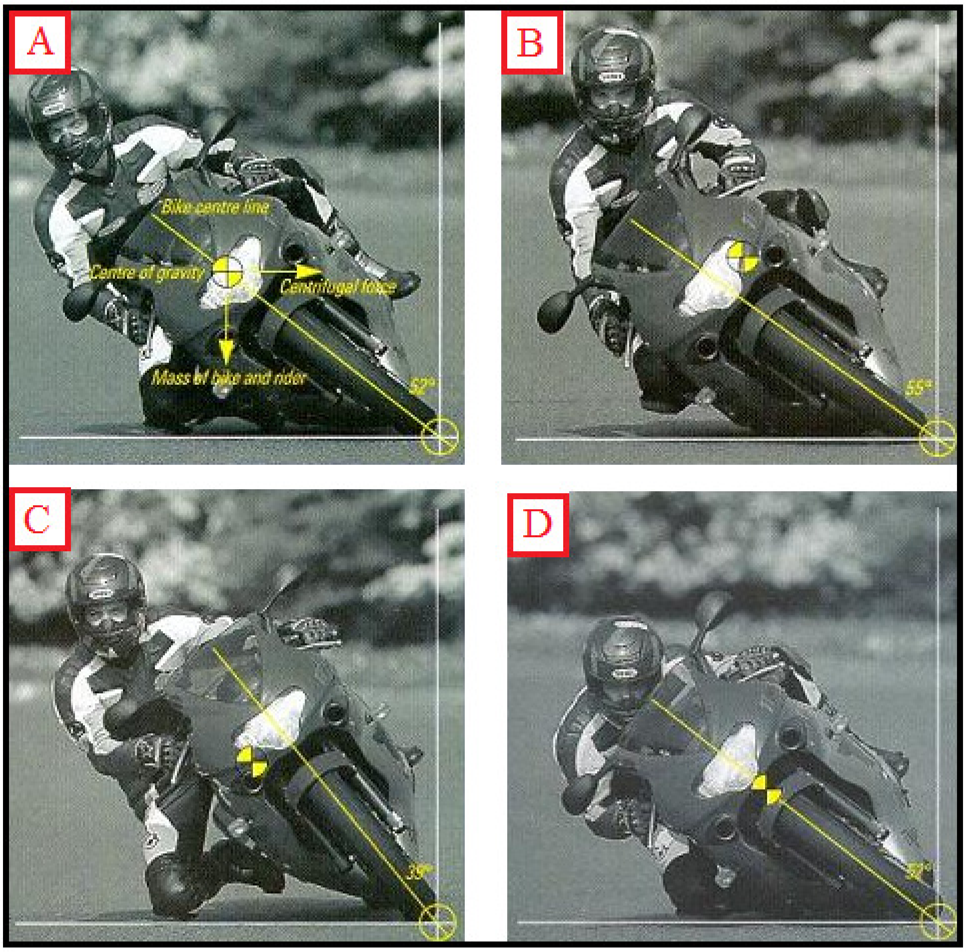


**Figure 28. Driving style.**

- ***Figure 28, A:*** If the rider remains immobile with respect to the chassis, the center of gravity of the motorcycle-rider system remains in the motorcycle plane. This is an elegant way of handling the turns, but not the best. In this case, the actual roll angle is the same that was previously calculated.

- ***Figure 28, B:*** If the rider leans towards the exterior of the turn, the center of gravity is also moved to the exterior of the turn with respect to the motorcycle. As a result, he needs to incline the motorcycle further so that the tire, being more inclined than necessary, operate under less favorable conditions.

- ***Figure 28, C:*** If the rider leans his torso towards the interior of the turn and at the same time rotates his leg so as to nearly touch the ground with his knee, he manage to reduce the roll angle of the motorcycle plane.

- ***Figure 28, D:*** When racing, the riders move their entire bodies to the interior of the turn, both to reduce the roll angle of the motorcycle and to better control the vehicle on the turn. The displacement of the motorcycle-rider system´s center of gravity towards the interior is carried out both by moving the leg and by the movement of the body in the saddle.

### 2.3.3. Directional behavior of the motorcycle in a turn.

Let us consider a motorcycle in a steady turning condition. Using our program we can calculate the radius of the curvature path of the motorcycle, which depend on three independent factors, the roll angle, the steering angle and the forward velocity.

Like we saw above, the roll angle of the motorcycle is function of the speed and turning radius of the path curvature. So, if we know the roll angle and the radius of the curvature path, we can compute the value of the motorcycle`s speed.

$$V = \sqrt{\tan(\varphi) \cdot 9.8 \cdot R_c}$$

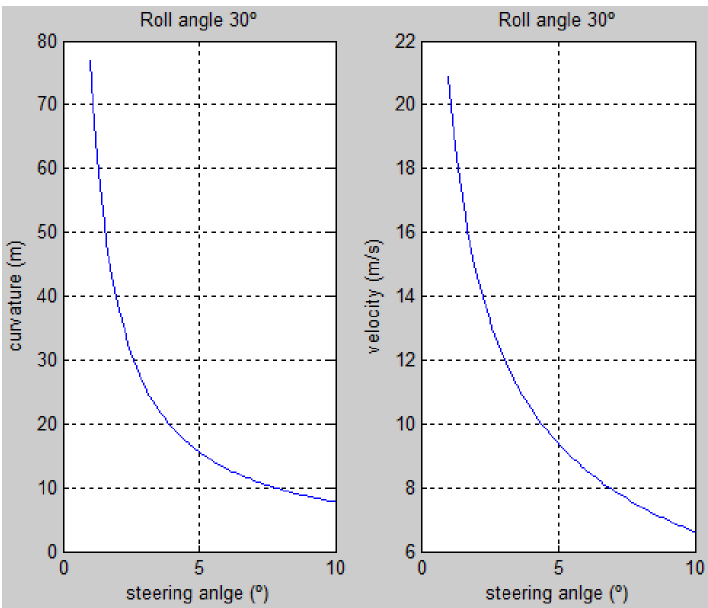


**Figure 29. Curvature and velocity**

In steady condition the driver set the roll angle and the steering angle, tilting the motorcycle and applying a torque in the handlebar. As we can see in the previous graph, if we set the roll angle, the radius of the curvature path decrease with the increase of the steering angle. It means that the motorcycle trajectory is in a closed curve. If we see the previous equation, we can understand that in closer curve with the same roll angle, the forward velocity decrease with the decrease of path radius. If we want to improve the velocity of the motorcycle, we need to increase the roll angle of our motorcycle. However, an increase in the roll angle of our motorcycle brings an increase in the loads of the tires like we will see below. It is a problem because we could reach the limit grip between the tires and the road. So, one way to improve this forward velocity is with higher center of gravity that allows us to reduce the roll angle of the motorcycle.

This theory was applied by the Spanish great engineer Antonio Cobas in his famous motorbikes´ frames. One of his most important works was the deltabox frame. Design used by most of all sport bikes today.

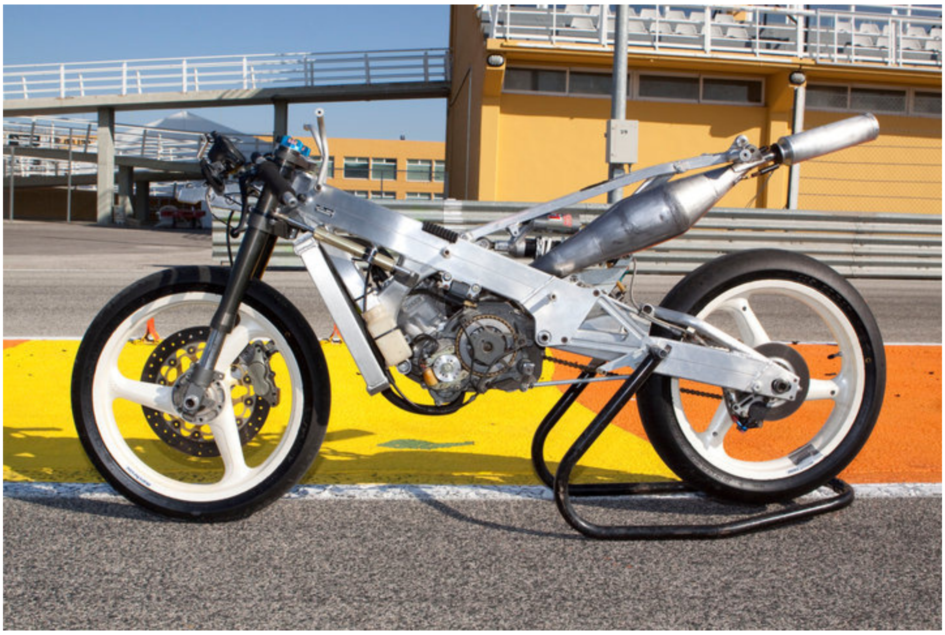

**Figure 30. Deltabox frame.**

## 2.3.4. Cornering forces.

Using the same hypothesis, we will use our program to see how the cornering forces in the tires vary. Setting the roll angle and increasing step by step the steering angle we can study this phenomenon. Like we saw above, if we maintain the roll angle and increase the steering angle, the radius of the curve decrease. The curve is more closed. If we observe the following graphs we can see how in this case, the vertical load in the front wheel increase while the vertical force in the rear tire decrease. In steady condition, the sum of these forces is equal to the mass of the motorcycle. The front load increases because the relative angle between the plane of the motorcycle and the road is grater, so the sideslip in the front tire increase and the resistance force become greater.

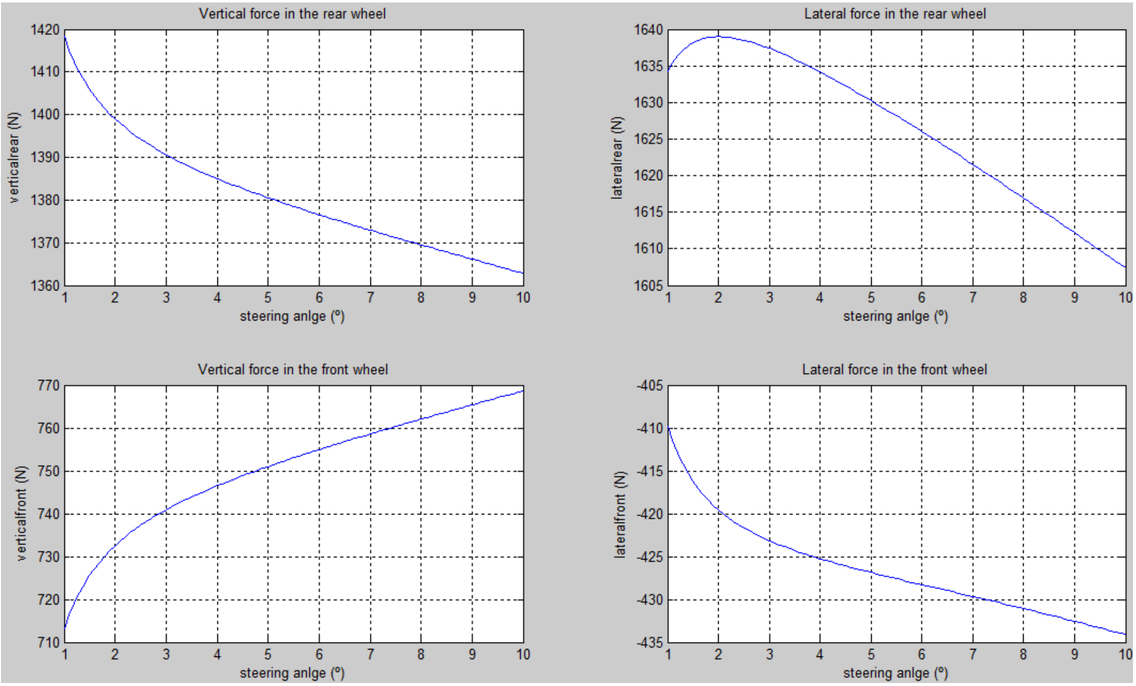


**Figure 31. Forces acting in the tires.**

If we study the lateral force in the tires, we can prove the above theory. Looking the graph of the "Lateral force in the front wheel", we can see how the absolute value of the lateral force in the front wheel increase with the increase of the steering angle, like we explain above. This theory can be proved too, looking the graph of the thrust. If we ignore the effect of the aerodynamic force, the thrust necessary to maintain a constant forward velocity in the steady turning, should increase in smaller curvature radius path.

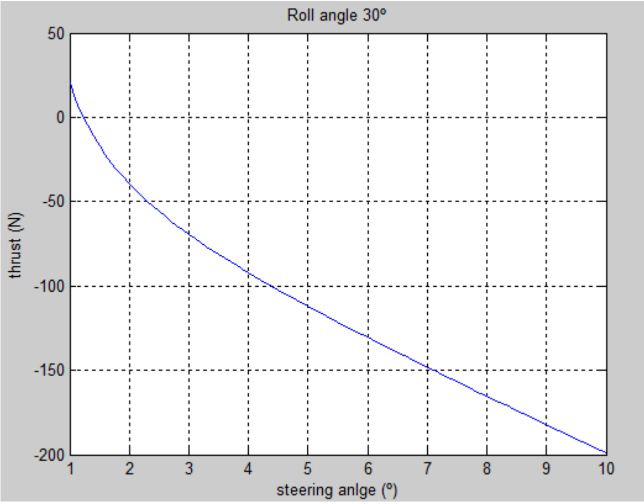


**Figure 32. Motorcycle thrust.**

The absolute value of the thrust, increase with the steering angle like we assumed above.

Is interesting to observe, how the lateral force in the rear tire decrease with the increase of the steering angle. Like we explained, with the same roll angle, the speed of the motorcycle is higher with small angles in the steering. So, the load in the rear wheel increases with the speed in steady turning. It is also interesting to observe the values of the loads in the front and in the rear tires. We can see how the forces in the rear tire are two or three times higher than in the front tire. For this reason, the rear tires are always bigger than the front tires.

## 2.3.5. Torque in the handlebar.

Using our program is easy to compute the value of the torque necessary to be applied for the rider in the handlebar to maintain the desired path. Like we can see in the following graph the value of the torque is between 100 and 108 (N), it means that this value is rather in the same range. To have a motorcycle more maneuverability, is needed that the rider feels the lowest possible force in the handlebar. So, if the torque is always the same, we need to increase the distance between the axis of the steering system and the point in where the rider apply the force with his arms. It is very typical in sport bikes, that the riders can set the angle of the handlebar. So is normal that they increase this value when they have a nervous motorcycle. The torque is the same, but the force that they need to apply is lees and they get a more easy motorcycle, with softer reaction.

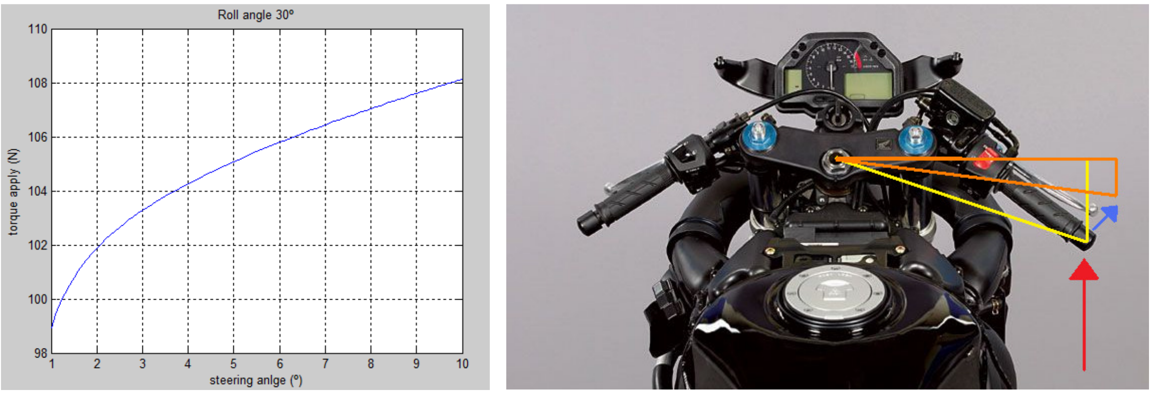


**Figure 33. Torque in the handlebar.**

## 3. Conclusion.

With these models we can develop a tool which permits to us to understand the behavior of the motorcycle and how affect the different parameters in its stability and maneuverability.

The model of the motorcycle in rectilinear motion can compute how the motorcycle performance the acceleration. At this phase we can obtain the values of the forces in the tires and the resistances forces that act in the motorcycle. Therefore we can change different parameters in our model to see how affect these to the performance of the motorcycle. For example, we could see the influence to use one final drive kit or other, or how affect accelerated in one or other gear after the exit of a curve path. We can study the operation of the engine and try to optimize its performance. Also we can understand the limit of acceleration performance of the motorcycle and avoid the overturning of our vehicle. However, the most important feature of our model is that is too easy to change the different parameters which define the model of the motorcycle. Therefore we can compare two different configurations and make decision based on the results.

The model of the steady turning motion can compute the different forces that take part in a motorbike when it takes twist and turns. With this model we can study two different groups of forces. Firstly we can compute the forces that act in the tires of the motorcycle. So we can compare how affect different configuration in the geometry of the motorcycle in these forces. It study is important to discover the limit grip of the tires with the road and hence find the best configuration to take curve path in the faster and safer way.

Secondly, the feeling of the rider on the motorcycle depends on the forces that the rider must apply in the handlebar to take the desire path. The better situation is when the rider has to apply the lees force possible in the handlebar. Hence, a motorcycle will have a better handling and maneuverability if the torque necessary to be applies in the handlebar is the lees possible. With our model we can compute these values and understand how the geometry of the motorcycle´s frame and suspension and how the features of the tires affect to it torque and hence in the forces that the rider must apply in the handlebar.

Finally, we can say that with these models we can understand the behavior of the motorcycle and how affect each of parameters in it. Therefore with these models we can study and compare two different configuration or motorcycles and make decision taking in account the values computed by the models.

# 4. Future Working.

The study of the rectilinear motion and the steady turning of a motorcycle is the first step in the research of the study of the motorcycle dynamics. After make these two models we should obtain the 3D model of a real motorcycle. Using software of virtual design like CATIA or Solid Edge we could obtain all the mechanical properties of each element such as mass, inertial matrix or center of gravity. Once we have this data we could develop a more complex model of a motorcycle and study the dynamic of a motorcycle in transitory state. For example, we could study how the motorcycle enters in a curve path and exits of it. After make this model we could study the dynamic of the motorcycle in plane, through the study of the front and rear suspension. With these studies we could discover the correct setting of the front and rear suspension to have a good handling in the motorcycle, looking for the more safety situation in the driving of vehicles of two wheels. After understand the dynamic in plane we could include these two parameters in our model and do a more complex model. With this model we could study the vibration model of the motorcycle and understand how affect the different setting in the suspension, how affect the geometry of the frame and how affect the features of the tires in the maneuverability and stability of our motorcycle.

To make all of this research, we should firstly develop the 3D model of the motorcycle with the program of 3D design and secondly take all the data and make a numerical model of the motorcycle. With this numerical model should be very easy to change the different parameters, which define the behavior of the motorcycle and therefore, we will have a tool with which to understand the stability and maneuverability of the motorcycle.

# 5. Bibliography.


- Numeric tools: [1] Matlab 7.0 and Simulink tutorial.
- Technical bibliography: [2] Vittore Cossalter – Motorcycle Dynamics.
- Technical papers:
  - [3] Model simulation of tires TYRES TECHONOLOGY.
  - [4] A Motorcycle Model for Stability and Control Analysis.
  - [5] Motorcycle Modeling for High-Performance Maneuvering.
  - [6] Motorcycle modeling and roll motion stabilization by rider leaning and steering torque.
- Technical information: [7] Manufacture catalogue of Ducati 848.