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"INTERMEDIATE INEQUALITY AND WELFARE. THE CASE OF SPAIN, 1980-81 TO 1990-91"

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Abstract

We introduce a new centrist or intermediate inequality concept, between the usual relative and absolute notions, which is shown to be a variant of the α-ray invariant inequality measures in Pfingsten and Seidl (1994). We say that distributions x and y have the same (x, π)-inequality if the total income difference between them is allocated among the indivuals as follows: π percent preserving income shares in x, and $(1 - \pi)$ percent in equal absolute amounts. This notion can be made as operational as current standard methods in Shorrocks (1983). The methodology is illustrated, in the first empirical application of centrist concepts, in the comparison of the standard of living in Spain between 1980-81 and 1990-91.

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INTRODUCTION

Suppose we have a population of homogeneous individuals, whose living standards are adecuately represented by a one-dimensional variable we will call income. Traditionally, in welfare economics we are interested in evaluation methods which take into account efficiency preference for larger incomes, and equity preference for less vertical inequality. Moreover, we want our methods to require a minimum of value judgements. In particular, we want unambigous rankings according to which social welfare increases only if efficiency and distribution both improve.

Let us agree that admissible social evaluation functions (SEF for short) must satisfy continuity, population replication invariance, and a preference for equity represented by the S-concavity axiom. Shorrocks (1983) suggests two wide classes of SEFs, depending on an additional monotonicity assumption which captures (i) a preference for higher incomes maintaining constant a relative notion of inequality, so that the proportion of rich and poor individuals does not change; or (ii) a more demanding absolute notion, according to which inequality only remains constant if every household experiences the same absolute income change. Let us denote these two classes by W_R and W_A, respectively. The merit of Shorrocks contribution is that he develops operational methods, based on the usual Lorenz curves, to find out whether one distribution is unambigously better than another according to all SEFs in W_R or W_A[(1)].

But perhaps there is room to improve upon the present methodology. In this paper we side with the minority who argue that there are plenty of "centrist" or intermediate views on inequality which deserve to be explored, between the "rightist" (relative) or "leftist" (absolute) cases in Kolm (1976)'s value ladden terminology. The conceptual interest of such views has been enhanced by recent reports on questionnaires which indicate that people are by no means unanimous in their choice between relative, absolute and other intermediate or centrist notions of inequality[2]. On the other hand, consider a situation in which income distribution y has less relative inequality but more absolute inequality than income distribution x. The following empirical question cannot be answered with present tools: is distribution y "barely better" than x from the relative point of view, and consequently "far away" from it from the absolute one; or is "so much better" from the relative perspective that is "nearly equivalent" to x from the absolute point of view? In other words, present methods only address yes-or-no questions relative to the two polar cases, but in situations like in the above example are silent on whether the improvement in relative inequality (or the worsening of absolute inequality) is "large" or "small".

In order to provide a certain answer to that question, in this paper we introduce a new centrist or intermediate inequality notion, called (x, π)-inequality, depending on an initial income distribution x and a parameter value π in the unit interval. We say that x and y have the same (x, π)-inequality if the total income difference between the two distributions is allocated among the indivuals as follows: π percent preserving income shares in x, and $(1 - \pi)$ percent in equal absolute amounts. Correspondingly, we suggest a monotonicity assumption for SEFs which captures a preference for higher incomes maintaining (x, π)-inequality constant. Let us denote this class of SEFs by $W_{(x, \pi)}$.

It turns out that (x, π)-inequality measures are a variant of the α-ray invariant inequality measures proposed by Pfingsten and Seidl (1994), or PS for short. Our reason for defending the new notion is twofold. In the first place, it has a clearer interpretation than α-ray invariant measures, but retain some of its good properties which distinguish them from the intermediate inequality notions proposed by Kolm (1976) and Bossert and Pfingsten (1992)(3). In the second place, unlike other α-ray invariant measures, it can be made operational. Thus, following up on ideas put forth in Chakravarty (1988) our methods permit to estimate from the data the range of π values for which distribution x has more or less (x, π)-inequality than distribution y(4). Then, taking also into account the mean income change in going from x to y, one can unambigously conclude whether distribution y is superior, inferior or non-comparable to distribution x for all SEFs in the class $W_{(x,\ \pi)}$ for this range of π values. Notice that we do not suggest the "politically correct" (x, π)-inequality concept, but find out from the data for which π values, say (π_1, π_2) in the interval [0, 1], the two income distributions x and y are (x, π)-inequality non comparable. For people with attitudes towards inequality represented by π values in the interval [0, π_1], there would have been a worsening in (x, π)-inequality, while for people with attitudes represented in the interval [π_2, 1], there would have been an improvement.

So far, we have been dealing with the homogeneous case in which all individual incomes are comparable. In practice, we must recognize that individuals come grouped in households with different non-income needs. Our methods can be readily extended to the heterogeneous case. For that purpose, we should first decide which household characteristics ought to be taken as ethically relevant for social evaluation purposes. Then, interhousehold welfare comparisons must be made consistently with the relative, absolute or intermediate inequality concept we care to use.

In Del Río and Ruiz-Castillo (1995) we applied standard procedures to the evolution of the standard of living in Spain between 1980-81 and 1990-91, an interesting period in this country, in which a socialist party occupied power by democratic means for the first time in 40 years.

Household size was taken as the only household characteristic defining ethically relevant non-income needs. To pool all households in a common distribution, we followed Coulter *et al* (1992a, 1992b) and parametrized the role to be given to household size in our definition of adjusted or equivalent income. Finally, following up on recent developments[(5)], when comparing Lorenz curves proper procedures of statistical inference were adscribed to throughout. The main results for the population as a whole, as well as within each homogeneous household type, were as follows. There has been: (i) an important growth in mean household expenditure in real terms; (ii) an statistically significant reduction in relative inequality; (iii) an increase in absolute inequality. These results provide us with a text-book example suggesting an application of a centrist approach.

The main results obtained from this approach, using our notion of (x, π)-inequality and taking x as the 1980-81 distribution, are as follows. (i) For the population as a whole, when economies of scale in consumption are rather important, the range of π values for which both distributions are (x, π)-inequality equivalent, is (0.75, 0.90) for both the unweighted case and the case in which households are weighted by household size. (ii) Each subgroup in the basic partition by household size is also investigated. In the subgroup of 3 person households, for which the improvement is greatest, the range of π values for which both distributions are (x, π)-inequality equivalent, is (0.52, 0.82)

The rest of the paper is organized in four sections. Section I presents the α-ray invariant inequality measures suggested by PS, and introduces our (x, π)-inequality concept, emphasizing the economic interpretation that singles out our notion within the larger class of α-ray invariant inequality measures. Section II describes how to make operational our inequality concept. Section III contains the empirical results in the Spanish case. Section IV concludes. Proofs are included in an Appendix.

I. RAY INVARIANT INEQUALITY CONCEPTS

I.1. Notation

Let $\mathbf{x} = (x^1,...,x^H) \in R^H_{++}$, $2 \leq H < \infty$, denote an income distribution. Then $D := R^H_{++}$ denotes the set of all possible income distributions, and S the H-dimensional simplex. For any $\mathbf{x} \in D$, let $\mathbf{v_x} = (v^1,...,v^H) \in S$ be the vector of income shares with $v^h = x^h/X$, where $X = \Sigma_h x^h$ is the aggregate income. **1** denotes a row vector whose components are all ones, while **e** denotes the vector (1/H) **1** in S. For any two vectors **x**, **y** $\in D$, let $\mathbf{v_x}$ L $\mathbf{v_y}$ denote weak Lorenz dominance.

Any real valued function I defined on D satisfying continuity, S-convexity and population replication invariance is called an income inequality measure. I(.) satisfies scale invariance when $I(\mathbf{x}) = I(\lambda\mathbf{x})$ for all $\mathbf{x} \in D$ and for all $\lambda > 0$. I(.) satisfies translation invariance when $I(\mathbf{x}) = I(\mathbf{x} + \eta\mathbf{1})$ for all $\mathbf{x} \in D$ and for all $\eta \in R$ such that $(\mathbf{x} + \eta\mathbf{1}) \in D$. If an inequality measure satisfies scale or translation invariance it is called a relative or an absolute inequality measure, respectively.

I.2 Centrist inequality attitudes

It can be argued that, for technical or other reasons, the vast majority of specialists prefer the relative notion. However, Kolm (1976) observed that many people perceive equiproportional increases in all incomes to increase, and equal incremental increases in all incomes to decrease income inequality. He called such an attitude centrist. As indicated in the conclusions to Ballano and Ruiz-Castillo (1993), if because of the influence of political attitudes to redistribution or other unkown concerns people in large numbers declare to favor absolute or intermediate inequality concepts, then perhaps it is time to change the consensus and use more often other types of inequality measures, as Kolm himself and Bossert and Pfingsten (1990), for example, have recommended.

As pointed out in PS, a centrist income inequality attitude can be modelled in various ways. For all $x \in D$, there exists a set of income distributions $E(x)$ such that, first, all $y \in E(x)$ are perceived to be as equally distributed as x, second, for $\lambda x > x$ and $(x + \eta 1) > x$ all $y \in E(x)$ are perceived to be more equally distributed than λx and less equally distributed than $(x + \eta 1)$, and third, for $x > \lambda x$ and $x > (x + \eta 1)$ all $y \in E(x)$ are perceived to be less equally distributed than λx and more equally distributed than $(x + \eta 1)$. Given such a centrist inequality attitude, the question arises whether there are E-invariant income inequality measures, i.e., $I(x) = I(y)$ for all $y \in E(x)$.

As PS indicate, a straightforward case is to assume $E(x)$ to be composed of rays through x. For later reference, the set $E_\alpha(x)$ of α-rays through x is defined by

$$E_\alpha(x) = \{y \in D: y = x + \tau\alpha \text{ , } \tau \in R\}.$$

In accordance with centrist ideas, PS require α-rays to be restricted in two ways: first, they Lorenz dominate the original distribution; and, second, they are more unequally distributed than traslation invariance would require. Thus, given an income distribution $x \in D$, define the set $\Omega(x)$ of value judgements (in income share form) which provide an improvement in relative inequality but a worsening in absolute inequality relative to x:

$$\Omega(x) = \{\alpha \in S: \mathbf{e}\ L\ \alpha\ L\ \mathbf{v_x}\}.$$

In other words, given $x \in D$ and $\alpha \in \Omega(x)$, every $y \in E_\alpha(x)$ is derived from x by superimposing a "more equal" income distribution according to the Lorenz criterion.

To understand in which sense x and α co-determine the domain of α-ray invariant functions, define the set $\Gamma(\alpha)$ of income distributions for which $\alpha \in S$ can represent a centrist inequality attitude:

$$\Gamma(\alpha) = \{x \in D: \alpha \; L \; v_x\}.$$

Clearly, if $x \in D$ and $\alpha \in S$ but $\alpha \notin \Omega(x)$ or $x \notin \Gamma(\alpha)$, then the pair (x, α) does not give rise to a centrist inequality relation. Accordingly, a real valued function $F_\alpha: D \rightarrow R$ is called α-ray invariant in $\Gamma(\alpha)$, if and only if for each $x \in \Gamma(\alpha)$,

$$F_\alpha(x) = F_\alpha(y) \text{ for all } y \in E_\alpha(x).$$

Given an α-ray invariant function $I_\alpha(.)$, we say that it is an α-ray invariant inequality measure if, in addition, it is is continous, S-convex and satisfies the population replication axiom.

As far as existence is concerned, PS show that for any $\alpha \in S$ there exists a non-empty set of income distributions $\Gamma(\alpha)$ on which $I_\alpha(.)$ is an α-ray invariant function; and for any $x \in D$ there exists a non-empty set of value judgements $\Omega(x)$ such that α-ray invariant inequality measures are defined for any $\alpha \in \Omega(x)$.

In general, α-ray invariance requires an inequality measure not to change provided any income change is distributed according to the value judgement represented by the relative pattern α. Thus, let **x** = (200, 800), so that v_x = (0.2, 0.8), and, for example, let α = (0.4, 0.6) so that **e** L α L v_x. Then

$$E_\alpha(x) = \{y \in R^2_{++}: y = (200, 800) + \tau(0.4, 0.6), \tau \in R\}.$$

Therefore, if we have 100 units of extra income to allocate, to preserve such α-ray invariance we must add up the vector (40, 60) to x to reach (240, 860).

I.3. Social Evaluation Functions

A Social Evaluation Function (SEF for short) is a real valued function W defined on D, with the interpretation that for each income distribution x, W(x) provides the "social" or, simply, the aggregate welfare from a normative point of view. In the area of income distribution analysis, it is generally agreed upon that SEFs must express, at a minimum, social preference for a more equitable profile and higher incomes, *ceteris paribus*. These value judgements are refered to as "equity preference" and "efficiency preference" respectively.

In I.2. we have presented the PS notion of centrist or intermediate inequality. We must now incorporate a preference for efficiency. As pointed out in the Introduction, Shorrocks (1983) made two suggestions in this respect. The first condition is that

$$W(\lambda x) \geq W(x) \text{ for all scalars } \lambda \geq 1,$$

that is, welfare improves if all incomes are increased proportionately. This corresponds to a preference for higher incomes keeping relative inequality constant. The second condition is that

$$W(x + \eta 1) \geq W(x) \text{ for all scalars } \eta \geq 0,$$

so that welfare improves if all incomes are increasing by the same amount. This corresponds to a preference for higher incomes keeping absolute inequality constant.

The natural extension in our context is as follows. A SEF W: D $\rightarrow$ R is called monotonic along α-rays in $\Gamma(\alpha)$, if and only if for each $x \in \Gamma(\alpha)$

$$W(x + \tau\alpha) \geq W(x) \text{ for all scalars } \tau \geq 0.$$

This property of monotonicity along α-rays corresponds for a preference for higher incomes keeping α-ray invariant inequality constant. For any

$\alpha \in S$, let W_α be the class of SEF satisfying continuity, population replication invariance, S-concavity and monotonicity along α-rays.

I.4. A new concept of intermediate inequality

In principle, given two distributions **t**, **u**$\in$D, we could search for τ^* and α^* so that **u** is α^*-ray invariant inequality equivalent to **t**, that is, **u** = **t** + $\tau^* \alpha^*$. Assume, without loss of generality, that $\tau^* \geq 0$. Then people with equal or less demanding inequality views than α^* would say that society is better off in **u** relative to **t**, while people with more demanding inequality views than α^* would say that **u** and **t** are non-comparable. We do not follow that path for the following two reasons. First, in practice τ^* is given by the total income difference between the two distributions under comparison, but we do not know how to find the vector α^* for which **u** is statistically α^*-ray invariant inequality equivalent to **t**. Second, even if such value judgement α^* were to be found, it is not obvious how to interpret it.

These problems do not affect our own inequality concept which will be presently introduced. We concentrate our attention on α-ray invariant inequality measures which can receive a clear economic interpretation. For that purpose, we start from an initial income distribution $x_0 \in D$, and a value of $\pi \in [0, 1]$. Then we consider only rays through $x \in D$ constructed so that π per cent of any extra income is allocated to individuals according to income shares in x_0, and $(1 - \pi)$ per cent in equal absolute amounts. That is, we define

$$P_{(x_0, \pi)}(x) = \{y \in D: y = x + \tau(\pi v_{x_0} + (1 - \pi)e), \tau \in R\}.$$

Clearly, if we let $\alpha_0 = \pi v_{x_0} + (1 - \pi)e$, then

$$P_{(x_0, \pi)}(x) = E_{\alpha_0}(x).$$

Correspondingly, we define the subset $\Gamma'(\alpha_0)$ of $\Gamma(\alpha_0)$ of income distributions along a $P_{(x_0,\ \pi)}(.)$ ray, for which α_0 can represent a centrist inequality attitude:

$$\Gamma'(\alpha_0) = \{x \in D: \pi' v_x + (1 - \pi')e = \alpha_0 \text{ for some } \pi' \in [0, 1]\}.$$

Clearly, for any $x \in \Gamma'(\alpha_0)$, $\alpha_0 \, L \, v_x$. Then we say that a real valued function $I_{(x_0,\ \pi)}: D \rightarrow R$ is a (x_0, π)-inequality measure in $\Gamma'(\alpha_0)$, if and only if it is the restriction to $\Gamma'(\alpha_0)$ of the I_{α_0}-ray invariant inequality measure. In this case, of course,

$$I_{(x_0,\ \pi)}(x) = I_{(x_0,\ \pi)}(y) \text{ for all } y \in P_{(x_0,\ \pi)}(x)$$

or, what is the same,

$$I_{\alpha_0}(x) = I_{\alpha_0}(y) \text{ for all } y \in E_{\alpha_0}(x).$$

In general, the set $\Gamma'(\alpha_0)$ is clearly non-empty(6), so that the (x_0, π)-inequality measures are well defined. This means that they enjoy all the properties discussed by PS for α-ray invariant inequality measures. Given $x_0 \in D$, changes in income in the scale τ are allocated as a linear combination of the vectors e and v_{x_0} in S. If we let x_0 = (200, 800) as before and π = 0.5, then 50 per cent of all income differences are allocated according to the income shares vector (1/5, 4/5), and 50 percent in equal absolute amounts according to the proportions (1/2, 1/2). Thus, the (x_0, π)-ray of income distributions through x_0 is given by

$$P_{(x_0,\ \pi)}(x_0) = \{y \in R^2_{++}: y = x_0 + \tau(7/20, 13/20), \tau \in R\},$$

so that 100 extra units of income are allocated as (35, 65) to reach the new distribution (235, 865). On the other hand, notice that if π = 1, (x_0, π)-inequality becomes the relative view, whereas π = 0 leads to the absolute view.

The dependence of centrist or intermediate inequality measures on an initial situation deserves to be emphasized. In our case, given $x_0 \in D$ and $\pi \in [0, 1]$, $\alpha_0 = \pi v_{x_0} + (1 - \pi)e$ is determined. Then, for all $y \in \Gamma'(\alpha_0)$ there exists some $\pi' \in [0, 1]$ such that $\alpha_0 = \pi' v_y + (1 - \pi')e$. Thus, (y, π')-inequality coincides with (x_0, π)-inequality for all such $y \in \Gamma'(\alpha_0)$. The interpretation is clear. Suppose first that $y \in \Gamma'(\alpha_0)$ and x_0 have the same (x_0, π)-inequality. Assume that $Y' - X_0 > 0$. Then, as we show in Proposition 1 in the Appendix, $\pi' \geq \pi$. This means that the same centrist attitude is captured when, starting from x_0, π per cent of all income exceeding X_0 is allocated according to v_{x_0} and $(1 - \pi)$ per cent in equal absolute amounts, as when, starting from y, π' percent of the income difference $Y - X_0$ is substracted from the individuals according to v_y and $(1 - \pi')$ in equal absolute amounts. This is undestandable, since y has a greater mean but the same centrist inequality as x_0 and, therefore, less relative inequality. Thus, to get down to x_0 from y so as to preserve intermediate inequality, we can follow the pattern v_y more closely than the pattern v_{x_0} from x_0. On the other hand, suppose that $y \in \Gamma'(\alpha_0)$ and x_0 have the same mean, but y, for instance, has greater or equal (x_0, π)-inequality than x_0. Then, as we show in Proposition 1 in the Appendix, $\pi' \leq \pi$. Now that y has greater relative inequality than x, to maintain the same centrist inequality from y, a smaller π' per cent of all income exceeding Y must be allocated according to v_y along the relative ray through y.

In the 2-dimensional case, given any $x_0 \in D$ and $\pi \in [0, 1]$, all distributions y in $\Gamma(\alpha_0)$, where $\alpha_0 = \pi v_{x_0} + (1 - \pi)e$, have the property that $\alpha_0 = \pi' v_y + (1 - \pi')e$ for some $\pi' \in [0, 1]$. This means that $\Gamma'(\alpha_0)$ and $\Gamma(\alpha_0)$ coincide, in which case the (x_0, π)-inequality and the α_0-ray invariant inequality concepts also coincide. In general, of course, the set $\Gamma(\alpha_0)$ is much richer than $\Gamma'(\alpha_0)$. However, as we will see in the next section, the structure possesed by $\Gamma'(\alpha_0)$ permits the new concept to be made operational.

Finally, given $x_0 \in D$ and $\pi \in [0, 1]$, so that $\alpha_0 = \pi v_{x_0} + (1 - \pi)e$, a SEF W: D → R is called monotonic along (x_0, π)-rays in $\Gamma'(\alpha_0)$, if and only if

$$W(x + \tau\alpha_0) \geq W(x) \text{ for all scalars } \tau \geq 0 \text{ and all } x \in \Gamma'(\alpha_0).$$

This property of monotonicity along along (x_0, π)-rays corresponds to a preference for higher incomes keeping (x_0, π)-inequality constant. For any $x_0 \in D$ and $\pi \in [0, 1]$, let $W_{(x_0, \pi)}$ be the class of SEF satisfying continuity, population replication invariance, S-concavity and monotonicity along (x_0, π)-rays.

II. OPERATIONAL METHODS

II. 1. The homogeneous case

An empirical situation in which intermediate inequality concepts might prove useful, arises in the presence of two income distributions **t**, **u**∈D such that **u** dominates in the relative Lorenz sense to **t**, but **t** dominates **u** in the absolute Lorenz sense. Define the absolute and the relative rays through **t**, A(**t**) and R(**t**), by

$$A(\mathbf{t}) = \{x \in D:\ \mathbf{x} = \mathbf{t} + \tau \mathbf{e}, \tau \in R\},\ R(\mathbf{t}) = \{x \in D:\ \mathbf{x} = \mathbf{t} + \tau \mathbf{v}_t, \tau \in R\},$$

respectively. Let m(.) denote the function giving the income distribution mean, and let us call **a** and **r** the income distributions in A(t) and R(t), respectively, with mean m(**u**). Then, the starting situation will be described by the fact that $\mathbf{v_a}\ L\ \mathbf{v_u}\ L\ \mathbf{v_r}$. The following theorem, inspired in Chakravarty (1988), summarizes the connection between Lorenz dominance and SEFs in the class $W_{(t,\ \pi)}$.

<u>Theorem 1</u>. Let **t**, **u** ∈D such that $\mathbf{v_a}\ L\ \mathbf{v_u}\ L\ \mathbf{v_r}$. Then the following statements are equivalent:

(1.i) m(**u**) ≥ m(**t**), and

(1.ii) there exists some $\pi^{\#} \in [0, 1]$ such that, when we define

$$\mathbf{z} = \mathbf{t} + \tau\,(\pi^{\#}\mathbf{v}_t + (1 - \pi^{\#})\mathbf{e}),\ \tau = U - T,$$

we have $\mathbf{v_u}\ L\ \mathbf{v_z}$.

(2) W(**u**) ≥ W(**t**) for all $W \in W_{(t,\ \pi^{\#})}$.

<u>Corollary</u>. Under the conditions of the above Theorem,

$$W(\mathbf{u}) > W(\mathbf{t}) \text{ for all } W \in W_{(t,\ \pi)} \text{ with } \pi \in (\pi^{\#}, 1].$$

Assume without loss of generality that $\tau^* = U - T > 0$, so that $\mathbf{r} = \mathbf{t} + \tau^* \mathbf{v}_t$ and $\mathbf{a} = \mathbf{t} + \tau^* \mathbf{e}$. Define the line segment {**r**, **a**} in H-dimensional space by

$$\{r, a\} = \{z \in D: z = t + \tau^*(\pi v_t + (1 - \pi)e) \text{ for some } \pi \in [0,1]\}.$$

This is the subset of $\cup_{\alpha \in \Omega(t)} E_\alpha(t)$ with the following structure: it consists of all income distributions with mean equal to m(u) which can be reached by (t, π)-rays through t. Assume first that the Lorenz dominance relation $v_a \; L \; v_u \; L \; v_r$ is strict. Then there must exist two values $\pi_1^* \in [0, 1)$ and $\pi_2^* \in [\pi_1^*, 1]$ which induce the following partition of {r, a}:

$$\{a, z_1^*\} = \{z \in \{r, a\}: z = t + \tau^* (\pi \, v_t + (1 - \pi)e), \pi \in [0, \pi_1^*]\};$$

$$\{z_1^*, z_2^*\} = \{z \in \{r, a\}: z = t + \tau^* (\pi \, v_t + (1 - \pi)e), \pi \in (\pi_1^*, \pi_2^*)\};$$

$$\{z_2^*, r\} = \{z \in \{r, a\}: z = t + \tau^* (\pi \, v_t + (1 - \pi)e), \pi \in [\pi_2^*, 1]\}.$$

The partition has the following property:

$v_z \; L \; v_u$ para todo $z \in \{a, z_1^*\}$;
$v_u \; L \; v_z$ para todo $z \in \{z_2^*, r\}$;
v_u is non-comparable to v_z in the Lorenz sense for all $z \in \{z_1^*, z_2^*\}$.

Since, for instance,

$$\{a, z_1^*\} = \cup_{\pi \in [0, \pi_1^*]} P_{(t, \pi)}(t) \cap \{z \in D: m(z) = m(u)\},$$

for every $z \in \{a, z_1^*\}$, $I_{(t, \pi)}(z) = I_{(t, \pi)}(t)$ for some $\pi \in [0, \pi_1^*]$. Therefore,

$$I_{(t, \pi)}(u) \geq I_{(t, \pi)}(t) \text{ for all } \pi \in [0, \pi_1^*].$$

Similarly,

$$I_{(t, \pi)}(u) \leq I_{(t, \pi)}(t) \text{ for all } \pi \in [\pi_2^*, 1],$$

while for any $\pi \in (\pi_1^*, \pi_2^*)$, u and t are non comparable from the point of view of (t, π)-inequality. Notice that if v_u is statiistically equivalent to v_z in the Lorenz sense, then $\pi_1^* = \pi_2^* = \pi^*$ with $z = t + \tau^* (\pi^* \, v_t + (1 - \pi^*)e)$. In this case

$$I_{(t, \pi^*)}(u) = I_{(t, \pi^*)}(t).$$

Finally, if v_a is Lorenz equivalent to v_u, then $\pi_1^* = \pi_2^* = 0$; but if v_a is non comparable to v_u, then there exists no $\pi_1^* \in [0, 1]$. Similarly, if v_u is Lorenz equivalent to v_t, then $\pi_1^* = \pi_2^* = 1$, while if v_u is non comparable to v_t, then there exists no $\pi_2^* \in [0, 1]$.

II.2. The heterogeneous case

Let us now admit that we have a population of h = 1,...,H households which can differ in income, x^h, and/or a vector of household characteristics. In this paper, households of the same size are assumed to have the same needs and, therefore, their incomes will be directly comparable. Larger households have greater needs, but also greater opportunities to achieve economies of scale in consumption. Assume that there are $\kappa = 1,\ldots,K$ household sizes. Following Coulter et al (1992a, 1992b), for each household h of size κ define adjusted income in the relative case by

$$z^h(\Theta) = x^h/\kappa^\Theta, \ \Theta \in [0,1].$$

When $\Theta = 0$, adjusted income coincides with unadjusted household income, while if $\Theta = 1$, it equals *per capita* household income. Taking a single adult as the reference type, the expression κ^Θ can be interpreted as the number of equivalent adults in a household of size κ. Thus, the greater is Θ, the smaller are the economies of scale within the household or the larger is the number of equivalent adults. Notice that, given Θ, the number of equivalent adults is a non linear increasing function of κ.

In the absolute case, given Θ, for each household h of size κ define adjusted income by

$$z^h(\lambda^\kappa) = x^h - \lambda^\kappa(\kappa - 1),$$

where λ^κ is such

$$m(z^\kappa(\lambda^\kappa)) = m(z^\kappa(\Theta)).$$

It is easy to see that

$$\lambda^\kappa = [m(z^\kappa(\Theta))(\kappa^\Theta - 1)]/(\kappa - 1) = [m(x^\kappa)(\kappa^\Theta - 1)]/[(\kappa - 1)\kappa^\Theta].$$

The parameter λ^κ can be interpreted as the cost of an adult when household size is κ. Of course, the greater is Θ, the greater is λ^κ and the smaller are the economies of scale within the household.

Notice that, if I is any index of relative inequality, for each κ

$$I(z^\kappa(\Theta)) = I(x^\kappa/(\kappa^\Theta) = I(x^\kappa).$$

Similarly, if A is any index of absolute inequality

$$A(z^\kappa(\lambda)) = A(x^\kappa - \lambda^\kappa(\kappa - 1)) = A(x^\kappa).$$

Thus, in both cases, within each ethically homogeneous subgroup, the inequality of adjusted income is equal to the inequality of original income, independently of individual incomes and prices.

We now extend this adjustment procedure to the (x^κ, π)-inequality case. Let X^κ and H^κ be the total income and the number of households of size κ. Given π and Θ, for each household h of size κ define adjusted income by

$$z^h(\tau^\kappa) = x^h - \tau^\kappa[\pi(x^h/X^\kappa) + (1 - \pi)/H^\kappa],$$

where τ^κ is such

$$m(z^\kappa(\tau^\kappa)) = m(z^\kappa(\Theta)).$$

Equivalently,

$$z^h(\tau^\kappa) = \pi(z^h(\Theta)) + (1 - \pi)(z^h(\lambda^\kappa)).$$

It can be shown that

$$\tau^\kappa = [(\kappa^\Theta - 1)X^\kappa]/\kappa^\Theta.$$

Again, the greater is Θ, the greater is τ^κ and the smaller are the economies of scale within the household. Finally, if for every π and every x^κ, $I_{(x^\kappa, \pi)}$ is any index of (x^κ, π)-inequality, we have

$$I_{(x^\kappa, \pi)}(z^\kappa(\tau^\kappa)) = I_{(x^\kappa, \pi)}(x^\kappa).$$

III. EMPIRICAL RESULTS

III.1. The data

Our data come from two household budget surveys, the *Encuestas de Presupuestos Familiares* (EPF for short), collected by the *Instituto Nacional de Estadística* (INE for short) in 1980-81 and 1990-91. The EPFs are large, comparable surveys of 23.972 and 21.155 observations, respectively, for a population of approximately 10 or 11 million households. The basic demographic information is in Table 1. Household and personal distributions have been estimated taking into account the blowing up factors provided by the surveys.

TABLE 1. Household and personal population by household size in 1980-81 and 1990-91

	1980-81				1990-91			
Household size	Households	%	Persons	%	Households	%	Persons	%
1 person	779.135	7.8	779.135	2.1	1.128.990	10.0	1.128.990	2.9
2 persons	2.116.476	21.1	4.232.951	11.4	2.519.291	22.3	5.038.581	13.1
3 persons	1.866.104	18.6	5.598.312	15.1	2.347.041	20.8	7.041.124	18.3
4 persons	2.364.574	23.6	9.458.297	25.5	2.821.017	25.0	11.284.067	29.3
5 persons	1.490.503	14.9	7.452.513	20.1	1.493.602	13.2	7.468.011	19.4
6 persons	774.309	7.7	4.645.852	12.5	614.983	5.4	3.689.897	9.7
7 persons	359.818	3.6	2.518.725	6.8	245.154	2.2	1.716.075	4.5
8 or more	271.414	2.7	2.383.123	6.4	128.432	1.1	1.127.260	2.9
All	10.022.332	100.0	37.068.908	100.0	11.298.509	100.0	38.494.006	100.0

Smaller households, consisting of 1 to 4 persons, are more important at the end of the decade, and the opposite is the case for larger households.Thus, whereas the household population grows by more than 10 per cent, the number of person increases only by approximately 4 per cent. Correspondingly, household size decreases from 3.7 in 1980-81 to 3.41 in 1990-91.

For reasons spelled out elsewhere(7), we believe that household welfare is best approximated by a measure of current consumption, namely, household total current expenditure on private goods and services, net of expenditures on the acquisition of certain durables, but inclusive of imputations for self-consumption, wages in kind, meals subsidised at work, and the rental value for owner-occupied and other non rental housing. We express total hoshold expenditure at constant prices of the Winter of 1991 by means of household specific statistical price indices.

III.2. Previous results in Del Río y Ruiz-Castillo (1995)

Table 2 contains the percentage change in real terms of household expenditures for households of different size. Table 3 presents the corresponding information for two distributions: the distribution of household expenditure, adjusted for household size, and the distribution in which each person is assigned the adjusted expenditure of the household to which she belongs. In both cases, the change in the mean is given as a function of the parameter Θ, which determines the weight we give to household size in the definition of household adjusted expenditure: $z^h(\Theta) = x^h/\kappa^\Theta$, $\Theta \in [0,1]$, where x^h is original household expenditure and κ is household size.

TABLE 2. Percentage change in the mean of household expenditure in real terms, by household size

Number of persons:	1	2	3	4	5	6	7	8 or more
In %:	37.8	27.3	28.3	32.5	28.8	29.2	17.8	25.1

TABLE 3. Percentage change in the mean of adjusted household expenditure in real terms as a function of Θ (the greater Θ is, the smaller the economies of scale in consumption)

Θ =	0.0	0.2	0.4	0.7	1.0
Households	24.2	26.2	28.2	31.2	34.3
Persons	23.8	26.0	28.3	31.6	34.8

From the point of view of efficiency, it is quite clear that there has been an important improvement over the decade for all household types. Single person households and the large group of 4-person households, experiment an increase above 30 per cent. At the opposite side, large households of 7 or more persons grow only between 17 and 25 per cent. The increase for all other households is in the 27-29 range. For the population as a whole, the rates of growth are quite similar for households and persons. In both cases, the smaller the economies of scale, the greater the growth in mean adjusted expenditure, which varies between 24 and 34 percentage change.

As far as inequality is concerned, the main findings are as follows: (i) There has been an statistically significant reduction in relative inequality at constant prices. Therefore, real aggregate welfare has improved for all SEFs in the class W_R . This result is robust to the parametrization of the weight to be given to economies of scale, the unit of analysis -the household or the person- and the scale variable used to approximate the household standard of living. (ii) Although these results are also obtained for every subgroup in the partition by household size, smaller households (up to 3 persons) show greater improvements. (iii) There has been an increase in absolute inequality, both for the population as a whole and for every subgroup within the partition by household size.

III.3. Results on intermediate inequality: the homogeneous case

The results just summarized provide us with a text-book example demanding for an application of a centrist approach. We start with the analysis of each subgroup in the partition by household size. Let us denote by t and u the 1980-81 and 1990-91 distributions, respectively. We have just seen that **u** has a greater mean than **t** for all subgroups. In terms of the notation introduced in Section II, we must search for a pair of values 0 ≤

$\pi_1^* \le \pi_2^* \le 1$, where at least the first or the last inequality is strict. The purpose is to establish that:

- **t** y **u** are statistically non comparable from the point of view of (t, π)-inequality for all $\pi \in (\pi_1^*, \pi_2^*)$;

- **t** has less or equal (t, π)-inequality than **u** for all $\pi \in [0, \pi_1^*]$

- **t** has more or equal (t, π)-inequality than **u** for all $\pi \in [\pi_2^*, 1]$.

Then we may conclude that

(i) **t** has less welfare than **u** for all SEF in the class $W_{(t,\ \pi)}$ for all $\pi \in [\pi_1^*, 1]$;

(ii) **t** is non comparable to **u** for all SEF in the class $W_{(t,\ \pi)}$ for all $\pi \in [0, \pi_1^*]$.

The results are in Table 4. Household sizes are ordered, first, by the minimum π_2^* value, then by the minimum π_1^* value.

TABLE 4. Intermediate inequality within the basic partition:1980-81 vs. 1990-91

Household size	π_2^*	π_1^*
3 persons	0.82	0.52
1 person	0.85	0.75
2 persons	0.89	0.67
5 persons	0.99	0.64
7 persons	1.00	0.18
6 persons	1.00	0.66
4 persons	1.00	0.85

Let us comment on 3 person households. For the range [0.82, 1] of π values, the (t, π)-inequality at **t** is greater than at **u**. Since the mean is greater at **u**, by 28.33 to be exact, the social welfare of 3 person households has increased unambigously for that range of centrist attitudes to inequality. For the range [0, 0.52] of π values, the (t, π)-inequality at **t** is smaller than at u. Therefore, there is nothing we can say about social welfare for this range.

III.4. Results on intermediate inequality: the heterogeneous case

Naturally, all results presented in Table 5 are in terms of of Θ parameter values, corresponding to different assumptions on the importance of economies of scale in the definition of adjusted household expenditure.

TABLE 5. Intermediate inequality for the population as a whole as a function of Θ

	Households		Persons	
	π_2^*	π_1^*	π_2^*	π_1^*
Θ = 0.0	0.92	0.81	0.91	0.78
Θ = 0.2	0.90	0.78	0.91	0.75
Θ = 0.4	0.89	0.75	0.89	0.74
Θ = 0.7	0.88	0.76	0.89	0.75
Θ = 1.0	0.91	0.79	0.90	0.78

The two main conclusions are clear. In the first place, as Θ grows and we diminish the importance of economies of scale, there is an improvement in intermediate inequality, until we reach the last interval when Θ goes from 0.7 to 1, in which case there is a slight deterioration in inequality. In the second place, the differences obtained with the household or the personal distributions are practically negligible.

IV. CONCLUSIONS

Present empirical methods, pioneered by Shorrocks (1983), allow us to test whether an income distribution y unambigously provides greater social welfare than distribution x according to all members of two wide classes of SEFs. The two classes differ in the way a preference for higher incomes is made compatible with the invariance of either relative or absolute inequality.

Assume that relative to x, we have found with this methodology that income distribution y has greater mean income, less relative inequality, but more absolute inequality. This is exactly the case when x and y are taken to be, respectively, the 1980-81 and 1990-91 household expenditure distributions, adjusted for household size, after a decade of socialist governments in Spain. What present methods cannot say is whether the improvement in relative inequality (or the worsening of absolute inequality) is "large" or "small".

We believe the previous example provides good reasons to inmerse oneself in the continuum of centrist or intermediate inequality notions. The question can now be rephrased: for what type of centrist attitudes towards inequality the situation in 1990-91 in Spain is statistically equivalent or non comparable to the situation in 1980-81? An answer will tell us also for what type of centrist attitudes there has been an improvement, and for what type a worsening in inequality.

To provide such an answer, in this paper we introduce a centrist or intermediate (x, π)-inequality concept, where x is an initial income distribution and π a parameter which takes values in the unit interval. Technically, it is seen to be a variant of the α-ray invariant inequality concept proposed by Pfingsten and Seidl (1994). In practice, it has two advantages. (i) The first is that it has a clear economic interpretation. We say that x and y have the same (x, π)-inequality if the total income difference between the two distributions is allocated among the indivuals as follows: π percent preserving income shares in x, and (1 - π) percent in equal absolute amounts. (ii) The second advantage is that it can be made operational in the Shorrocks way. Specifically, given a situation like the Spanish one, the data reveal the range of π values for which the 1990-91 household expenditure distribution provides a greater social welfare, in terms of both the mean and (x, π)-inequality, than the 1980-81 distribution.

Whether social welfare went unambigously up or down according to measurement instruments consistent with a relative or an absolute inequality notion, is a very important piece of knowledge to have. However, in situations like the Spanish one, to know precisely under which set of centrist value judgements inequality was unchanged, increased or reduced, provides some value added worth having. In our opinion, the methodology presented in this paper goes one step in the direction pointed out by Atkinson (1989), when he indicates that we ought to follow procedures and, above all, report empirical estimates, making clear their dependence on the various axioms and value judgements involved.

For the record, according to the EPFs, during the 80's (i) Spain has experienced an important increase in mean household expenditure, ranging from 24 to 34 per cent in real terms, depending on our hypothesis about the economies of scale in consumption due to household size. (ii) For the population as a whole, when economies of scale in consumption are rather important, the range of π values for which the 1990-91 distribution has a smaller (x, π)-inequality is (0.90, 1) for both the unweighted case and the case in which households are weighted by household size. (iii) Each subgroup in the basic partition has also smaller

(x, π)-inequality in 1990-91. In the subgroup of 3 person households, for which the improvement is greatest, the range of π values for which this is the case is (0.82, 1).

NOTES

(1) Moyes (1987) develops analogous criteria, based on absolute Lorenz curves, to establish whether one distribution is unambigously better than another according to all SEFs in W_A.

(2) For example, see Amiel and Cowell (1992) and Harrison and Seidl (1990). In the Spanish case, Ballano and Ruiz-Castillo (1993) found that, for the subsample that showed an acceptable degree of consistency over the questionnaire, only 31 percent supported a relative view of inequality, 24 percent supported an absolute view, and 27 percent an intermediate notion (the rest supported other extreme views).

(3) For the shortcomings of Bossert and Pfingsten μ-inequality concept and Kolm's centrist γ-inequality measure, we refer to PS's discussion.

(4) Alternatively, the data reveal the range of π values for which the two income distribution x has less or more (y, π)-inequality than distribution y. As we will see, both inequality concepts represent exactly the same centrist attitude toward inequality.

(5) See Beach and Davidson (1983) and, for applications, Bishop et al (1989).

(6) Similarly, the subset $\Omega'(x_0)$ of $\Omega(x_0)$, defined by $\Omega'(x_0) = \{\alpha \in S: \alpha = \pi' v_{x_0} + (1 - \pi')e$ for some $\pi' \in [0, 1]\}$, is also non-empty.

(7) See for instance Del Río and Ruiz-Castillo (1995) and references quoted there.

APPENDIX

A. Proposition 1.

Let $\mathbf{x},\mathbf{y}\in\Gamma'(\alpha_0)$ *and* $\alpha_0\in S$, *where* $\alpha_0=\pi v_x+(1-\pi)e$ *for some* $\pi\in[0,1]$. *If* xLy (yLx) *then the value of* π' *which satisfies* $\alpha_0=\pi' v_y+(1-\pi')e$ *is such that* $\pi'\leq\pi$ $(\pi'\geq\pi)$.

Proof:

By contradiction, suppose that $\pi'>\pi$. This means $\pi'=\pi+\varepsilon$, $\varepsilon>0$ being. Consider $\mathbf{x},\mathbf{y}\in\Gamma'(\alpha_0)$, therefore, we can write

$$\boldsymbol{\alpha}_0 = \pi'\boldsymbol{v}_y+(1-\pi')\boldsymbol{e} = \pi\boldsymbol{v}_x+(1-\pi)\boldsymbol{e} \ .$$

By substituting π' in this expression we obtain

$$\pi\boldsymbol{v}_x+(1-\pi)\boldsymbol{e} = \pi\boldsymbol{v}_y+(1-\pi)\boldsymbol{e}+(\boldsymbol{v}_y-\boldsymbol{e})\epsilon \ .$$

This implies that $v_x^h>v_y^h$ for the rich $(v_y^h>(1/H))$ and that $v_x^h<v_y^h$ for the poor $(v_y^h<(1/H))$, in the **y** distribution. We can conclude that **x** can be obtained from **y** by transferring income from the poor to the rich. And thus gives us **yLx**, a contradiction.

Q.E.D.

B. Proof of Theorem 1.

1) $\Rightarrow$ 2):

As m(**u**)≥m(**t**), for any SEF, $W\in W_{(t,\pi\#)}$ it must be verified:

$$W(\boldsymbol{z}) = W(\boldsymbol{t}+(U-T)(\pi^{\#}\boldsymbol{v}_t+(1-\pi^{\#})\boldsymbol{e})) \geq W(\boldsymbol{t}) \ .$$

Moreover, as **u** Lorenz-dominates **z** and both distributions have the same mean, m(**u**), by Dasgupta-Sen-Starret (1973) we know that for any S-Concave function, W

$$W(\mathbf{u}) \geq W(\mathbf{z}) \ .$$

By combining these two expressions we conclude that

$$W(\mathbf{u}) \geq W(\mathbf{t}) \ .$$

2) $\Rightarrow$ 1):

Let us suppose that

$$W(\mathbf{x}) = (m(\mathbf{x}))^n f[\mathbf{z}'] \quad (**)$$

where $\mathbf{z}'=\mathbf{x}+(U-X)[\pi^{\#}\mathbf{v}_t+(1-\pi^{\#})\mathbf{e}]$, $n\geq 0$, and $f(.)$ is a S-concave function. It can be prove that for any function W verifying (**):

$$W(\mathbf{x}+\tau'(\pi^{\#}\mathbf{V}_t+(1-\pi^{\#})\mathbf{e})) = \left(m(\mathbf{x})+\frac{\tau'}{H}\right)^n f[\mathbf{z}'] \ ,$$

holds for any $\tau' \in \mathbb{R}$. In fact, for $\tau'>0$, this means $m(\mathbf{x})+(\tau'/H) \geq m(\mathbf{x})$, it can be shown

$$W(\mathbf{x}) \leq W(\mathbf{x}+\tau'(\pi^{\#}\mathbf{V}_t+(1-\pi^{\#})\mathbf{e})) \ .$$

Notice that S-concavity of f also implies S-concavity of W. Therefore, expression (**) warranties that function W(.) satisfies the assumptions of the theorem. Now then, knowing that $W(\mathbf{t}) \leq W(\mathbf{u})$, and choosing $f(.)=1$ we obtain condition (1.i):

$$W(\mathbf{t}) = (m(\mathbf{t}))^n \leq (m(\mathbf{u}))^n = W(\mathbf{u}) \ .$$

On the other hand, if n=0 we get

$$W(\boldsymbol{t}) = f[\boldsymbol{z}'] = f[\boldsymbol{z}] \leq f[\boldsymbol{u}] = W(\boldsymbol{u}) \ .$$

As **z** and **u** have the same mean, m(**u**) > 0, and *f*(.) being any arbitrary S-concave function, by Dasgupta-Sen-Starret (1973), this means that **u** Lorenz-dominates **z**.

Q.E.D.

Proof of Corollary:

If $\pi \in (\pi^{\#}, 1]$ we can write $\pi^{\#} = \pi - \beta$, for some $\beta \in \mathbb{R}$. Then,

$$\pi^{\#}\boldsymbol{v}_t + (1-\pi^{\#})\,\boldsymbol{e} = \pi\boldsymbol{v}_t + (1-\pi)\,\boldsymbol{e} - \beta\,(\boldsymbol{v}_t - \boldsymbol{e}) \ .$$

It can be shown that $\pi^{\#}\mathbf{v}_t + (1-\pi^{\#})\mathbf{e}$ is obtained from $\pi\mathbf{v}_t + (1-\pi)\mathbf{e}$ by using a sequence of rank preserving transformations transferring income from the rich to the poor, in a proportionally way: $(\mathbf{v}_t - \mathbf{e})$. Then, $\pi^{\#}\mathbf{v}_t + (1-\pi^{\#})\mathbf{e}$ strictly dominates $\pi\mathbf{v}_t + (1-\pi)\mathbf{e}$ in the Lorenz sense. And therefore, using that

$$\boldsymbol{z}' = t + \tau\,[\pi\boldsymbol{v}_t + (1-\pi)\,\boldsymbol{e}] \ , \quad \tau = U - T \ ,$$

this demonstrates that $\mathbf{v}_z$ strictly dominates $\mathbf{v}_{z'}$ in the Lorenz sense. Therefore, under the assumptions of Theorem 1:

$$W(\boldsymbol{t}) = W(\boldsymbol{z}') < W(\boldsymbol{z}) \leq W(\boldsymbol{u}) \ ,$$

must hold for any function $W \in W_{(t,\pi)}$, with $\pi \in (\pi^{\#}, 1]$.

Q.E.D.

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