AUTOMATIC MODELLING OF DAILY SERIES OF ECONOMIC ACTIVITY

Antoni Espasa, J. Manuel Revuelta
and J. Ramón Cancelo

96-26

WORKING PAPERS

Universidad Carlos III de Madrid

Working Paper 96-26
Statistics and Econometrics Series 07
April 1996

Departamento de Estadística y Econometría
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-9849

AUTOMATIC MODELLING OF DAILY SERIES OF ECONOMIC ACTIVITY

Antoni Espasa, J. Manuel Revuelta and J. Ramón Cancelo*

Abstract

Daily series of economic activity have not been the object of as a rigorous study as financial series. Nevertheless, the possibility of having adequate models available at a reasonable cost would give companies and institutions powerful management tools. On the other hand, the peculiarities that these series show advise specific treatment, differentiated from that of the series which show a higher level of time aggregation. In this article the previous problem is illustrated and an automatic methodology for the analysis of such series is proposed.

Key Words

Multiple seasonality; Varying seasonality; Calendar effect; Meteorological variables; Threshold variables; Intervention analysis; Deterministic seasonality

* Espasa, A., and Revuelta, J.M., Department of Statistics and Econometrics, University Carlos III of Madrid. E-mail: revuelta@est-econ.uc3m.es
Cancelo, J.R., Department of Applied Economics II, University of La Coruña.
This research has been supported by DGICYT under grant PB93-0236. Paper prepared for the *XII Symposium on Computational Statistics* to be held in Barcelona, August 1996. For a translation in Spanish see "Documento de Trabajo" 96-07(03), Statistics and Econometric Department, University Carlos III, Madrid.

1. Introduction

The object of our study are those daily series related directly or indirectly to economic activity and which from here on we shall refer to as *economic activity series*. In them the quantitative problem of interest is the modelling of the conditional means. This counterposes them against financial yield series in which the fundamental objective is the modelling of magnitudes related to second moments and for which a wholly different methodology has been developed, based on the ARCH and GARCH models or the stochastic volatility models (Ruiz, 1993; Taylor, 1994; etc.).

Some examples of the series that interest us are: consumption of energy products, aggregate monetary variables, pollution levels, sales in large companies, traffic series, occupation of means of transport, etcetera.

Our efforts to find an adequate model are justified by the great importance that these daily series have for the day-to-day activity of many enterprises and institutions. One example of how this may be important is in aspects such as the reduction of costs in the production of goods or in the provision of services. Here, the key to an improvement in competitivity would be to meet demand without incurring excessive costs owing to, for example, the maintenance of idle resources. This requires a precise quantification of the impact on consumption caused by various social, institutional, meteorological and other factors, in order to achieve predictions which are as accurate as possible. Apart from the predictive element, these models allow us to characterise the variables in question by setting parameters, for example: (a) the change of some short seasonal cycles (weekly) in terms of other longer ones (monthly or yearly) or in terms of meteorological variables; (b) the effect of a public holiday in terms of the day of the week, the season of the year, its position in the month and the values of meteorological variables in the days inmediately previous to it; (c) the non-linear effect of temperature, etc. All of these parameters are useful tools for management, control and diagnosing.

On the basis of the above we can justify efforts which tend towards a greater knowledge of the essential characteristics of these series and towards the development of modelling techniques which are as systematised as possible and allow a simple and general treatment.

The rest of the paper is organised in the following way: in section 2 a descriptive analysis is made of this type of time series; we then go on to look more deeply into basic schemes for dealing with them (section 3) and in the application to the simultaneous modelling of various seasonalities (section 4); the correction of the calendar effect is dealt with in section 5; frecuently these series are very sensitive to exogenous variables such as meteorological ones which are analysed in section 6. With all the previous developments the paper lays out in section 7 a basic automatic modelling scheme and ends with a concluding section 8.

2. General Characteristics

One first question which ought to be asked is if the usual techniques for time series which are applied to monthly, quarterly, etc. series, which from here on we shall call *low frecuency series*, may be directly applied to daily series of economic activity or if, on the contrary, it is convenient to develop specific techniques for these series which take into account the particular set of problems that they pose.

In general terms it can be said that the low-frequency series are characterised by the following aspects: (a) the existence of one seasonal cycle, (b) whose period seems to be perfectly defined and (c) for which simple schemes of a stochastic or deterministic nature seem adequate.

The above facilitates the development of systematised treatment, such as the ARIMA methodology developed by Box and Jenkins (1970).

By contrast, in daily, or *high frecuency series*, the following aspects, among others, stand out:

a) The existence of various seasonal cycles, the most common being weekly, monthly and annual ones.
b) The appearance of cycles of variable periods owing to irregularities in the calendar, such as leap years, the different length of months or the effect of there being a different number of weekends within the different months.
c) The need for the combination of deterministic and stochastic schemes to capture seasonalities.
d) Deterministic schemes, when they are necessary for a specific cyclical effect, are usually variable in terms of another cycle or meteorological variables.
e) An important dependency, and frecuently of a non-linear nature, on exogenous variables such as the meteorological ones and, specially, on the temperature, rain, light and wind.
f) The complex calendar effect in terms of public holidays, holiday periods etc.

Together with these characteristics, daily series, as happens with low-frequency series, usually show a non-stationary level which in many cases tend to grow.

As a consequence of the above the systematic search for models, taking as a base the approach commonly used is difficult and the setting out of specific modelling strategies for this type of series becomes of great interest. That is the fundamental objective of the present study.

In the charts 1, 2 and 3 are some of the series on which we shall focus our attention. In them, it can be appreciated how, even though they are of a different nature, all of them fit the above-mentioned aspects. In chart 4(A), in greater detail, are the cyclical patterns for the case of spanish electricity consumption series, proving how in this example the weekly pattern is dominant. In 4(B) we illustrate how this pattern varies according to the time of year. The calendar effect and the temperature effect can be appreciated in charts 4(C) y 4(D) respectively.

3. Modelling Schemes

For the modelling of any characteristic to be found in a time series it is possible to fall back on two basic types of schemes which may be purely deterministic or purely stochastic. The peculiar characteristics of some series also oblige us to use mixed schemes.

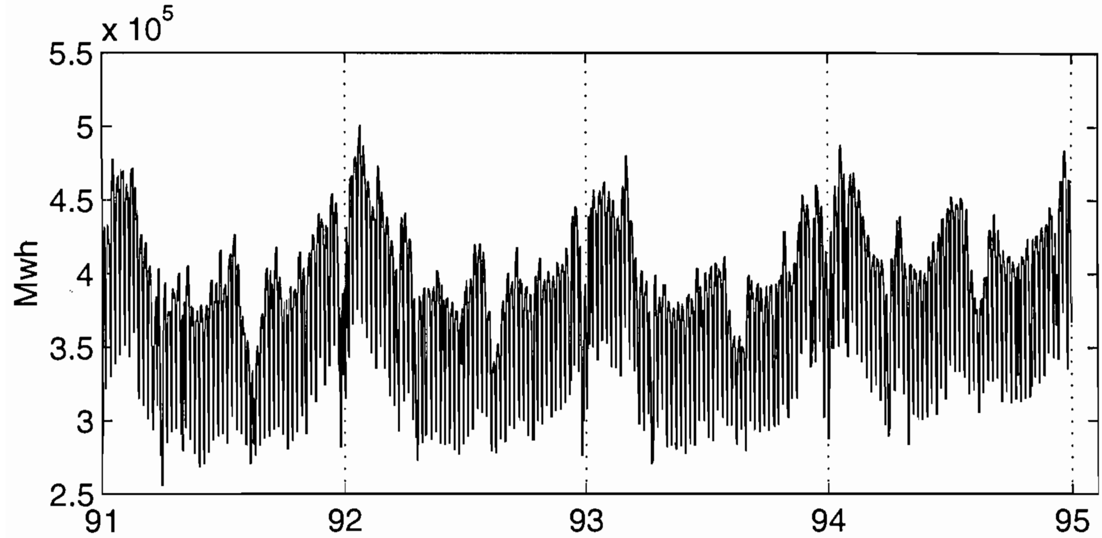

Fig. 1. Demand for electricity in Spain

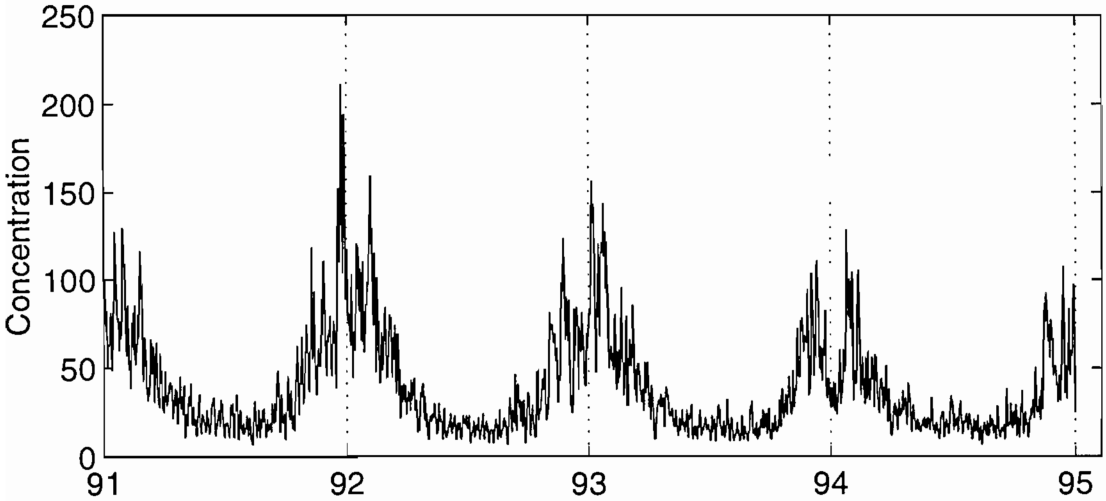

Fig. 2. Contamination of SO2 in Madrid

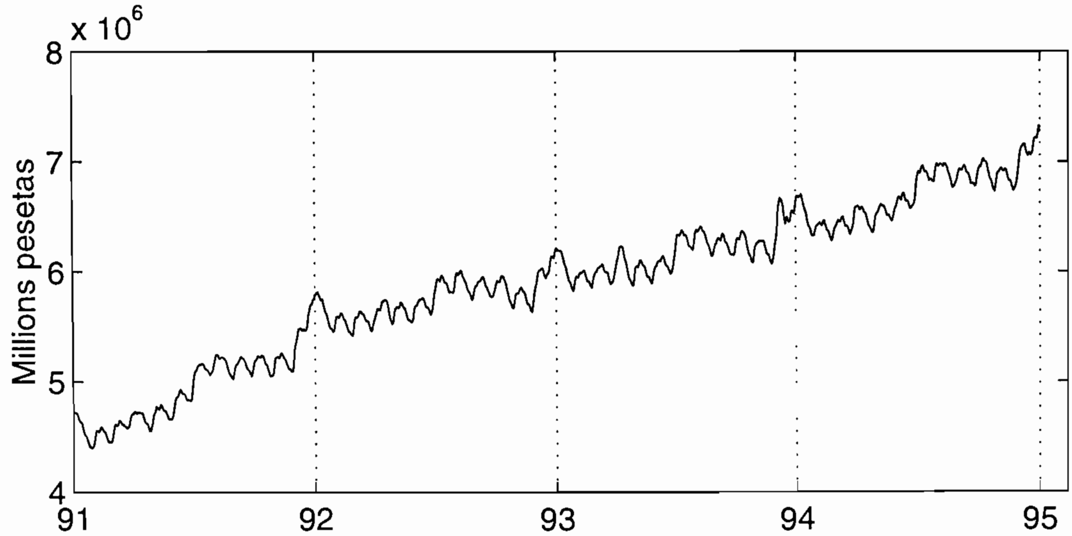

Fig. 3. Notes and coin in circulation in Spain

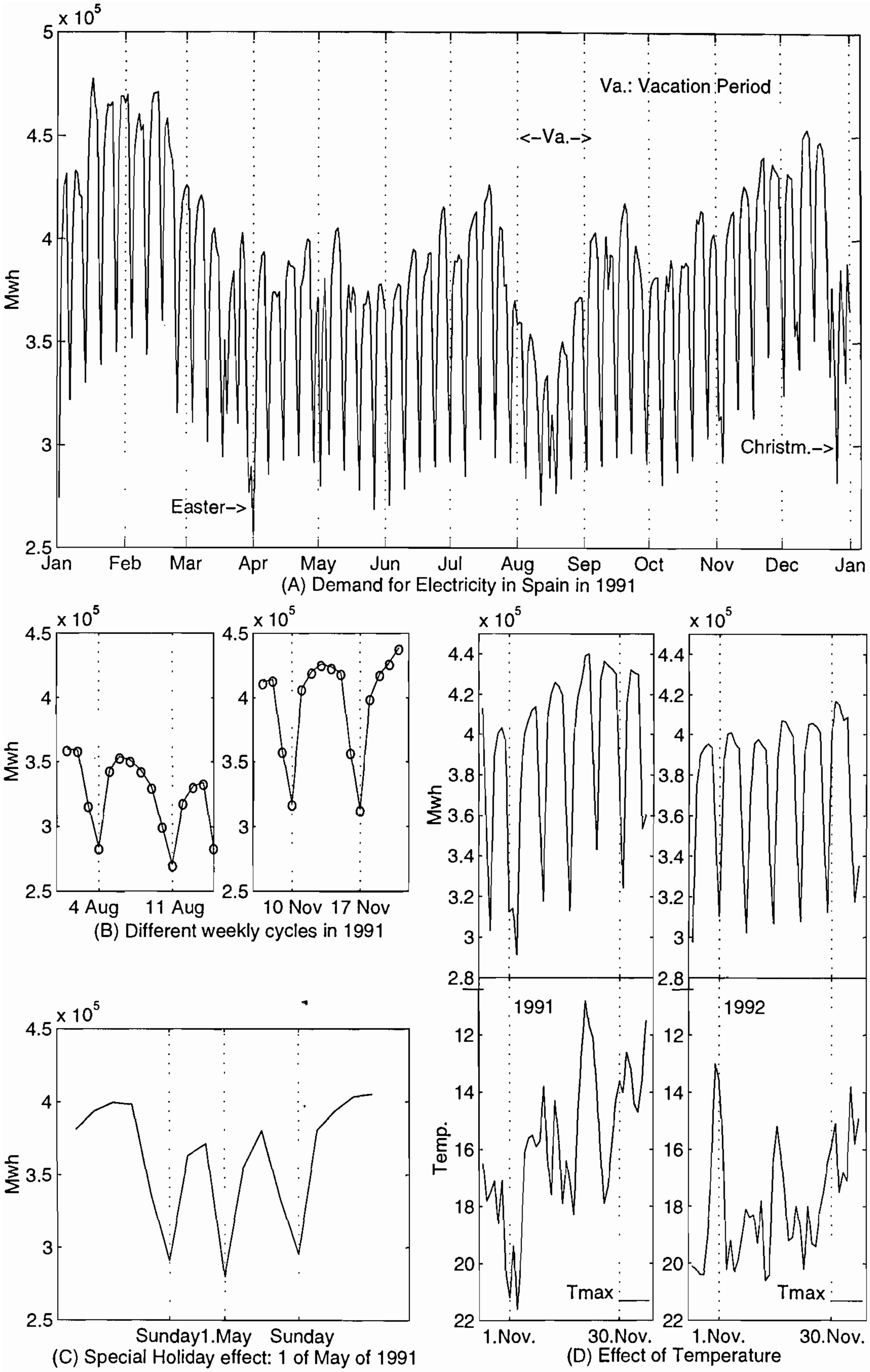

Fig. 4. Study of the Demand for Electricity in Spain

3.1. Deterministic Schemes

3.1.1. Trend

This is modelled by means of time polynomials. These schemes usually turn out to be excessively rigid for which reason the stochastic, or mixed, schemes are, in general, preferable.

3.1.2. Seasonal Cycles

Seasonality can follow a stable or variable pattern in time. The modelling of a cycle with a period length s which shows a constant pattern in time may be done by means of s dummy variables, according to the formula

$$Y_t = w_1\delta_{1t} + w_2\delta_{2t} + \ldots + w_s\delta_{st},$$

where the w_i's are constants and the δ_{it}'s are dummy variables which take the value 1 when t corresponds to the seasonal moment i and 0 in the rest of the cases. Given that the trend is modelled with an additional structure, the whole set of seasonal variables used must make the sum of the coefficients w_i to be worth zero.

The treatment of cycles with time periods of varying length, as the monthly one may be, (see section 3.2.4), by means of this type of scheme requires certain approximations (Espasa, 1993). In general, days of homogenous behaviour are usually grouped after the same dummy variable. One example of this would be what happens with many series of monetary aggregates, sales, etc., in which we can appreciate a monthly seasonality restricted to an effect of the beginning (evolutive effect), the middle (fixed effect) and the end of month (evolutive effect). In this case three dummy variables would be enough even though that of the beginning and that of the end of the month would be affected by a dynamic filter. In the case of annual deterministic seasonality a greater number of restrictions, evidently, are required and the different schemes to be contemplated are extremely varied depending on each particular series. Nevertheless an automatic procedure to identify these restrictions can be developed and it has been implemented in the program described in section 7.

One very useful way of giving flexibility to the above-mentioned deterministic schemes is by allowing them to vary. So, for example, weekly seasonality can be made to vary in terms of the position of the week within the month, the period of the year in which it is situated and of the value which certain meteorological variables take.

3.2. Stochastic Schemes

3.2.1. Trend

The characteristics of economic series usually make a stochastic modelling of the trend desirable. This is implemented by means of the difference operator Δ^d. Following Espasa and Peña (1995) the trend of a process can be characterised by means of the binomial $I(d,m)$, where the first figure indicates the number of unit roots and the values one or zero in the second show the presence, or not, of a non-zero mean in the stationary process. The behaviour most commonly found in these series of economic activity is quasilinear - polynomial trend in the forecasting function of order *(d+m-1)* - which requires, in terms of the previous reference, schemes of the type *I(1,1)* or *I(2,0)*.

3.2.2. Seasonal Cycles

The modelling of a seasonality of stable period *s* by means of stochastic schemes is typically done via the application to the original series of the sum operator

$$U_{s-1}=(1+L+L^2+\ldots+L^{s-1}),$$

where *L* is the lag operator.

In the case of the series also needing a regular difference, both schemes combined give us a seasonal difference operator

$$(1-L)U_{s-1}=(1-L^s).$$

The modelling of a varying seasonality through stochastic schemes is complex and requires many degrees of freedom. On the other hand, their use is normally rendered unnecessary owing to the good adaptation to the data which the varying deterministic schemes show. When it is not like this, it may be due to a strong dependency of the seasonality on some meteorological variables. In this case a good adjustment could be achieved by falling back once again on a varying seasonal deterministic scheme but one which also is a direct function of these variables.

3.2.3. Decomposition of Sum Operators

Every sum operator U_{s-1} can be broken down in terms of its harmonics, each one being associated to a determined frequency according to the expression $f_{i,s}=2\pi i/s$, with i=1...[s/2]. Thus, for example the weekly operator (U_6), very common in these daily series, will have the following breakdown

$$U_6=(1-2\cos(2\pi/7)L+L^2)(1-2\cos(2\pi/3.5)L+L^2)(1-2\cos(2\pi/2.3)L+L^2),$$

with each one of the terms picking up time periods of 7, 7/2 and 7/3 days respectively. In this case there are only three frequencies affected, but for the case of sum operators of greater order such as the annual one (U_{364}) the number of frequencies filtered will be much more, 182 in this example. This suggests the possibility of working directly with the really necessary harmonics instead of with the complete sum operator. In addition, the possible excess of unit roots in determined narrow frequency bands, if various sum operators corresponding to different seasonalities are employed, becomes evident. For example, if we take the operators $(1-L)^{30}$ y $(1-L)^{365}$, for any harmonic of the first there exists a *j* so that $f_{i,30}\simeq(2\pi j/365)$, for which reason it is not recommendable to apply the first filter in presence of the second.

3.2.4. Heterogeneity in Seasonal Periods

In daily series the periods of some cycles present in the series are not generally constant owing to the different length of months and the existence of leap years. This is even more relevant if the series we are dealing with are series lacking in data for one or other day of the week. The typical case are what we call the *weekday series*, characterised by the lack of information concerning the weekend.

This would be the case, for example, of the series for notes and coin in circulation denoted in pesetas (fig. 3). In this case there may be months of between 20 and 23 working days and years of 260, 261 or 262 working days.

In this case, the sum operators do not, in general, compare homogenous moments in the cycle. This leaves a residual seasonality which may be compensated by deterministic schemes, as is done in Espasa and Cancelo (1987).

This effect does not occur in the weekly cycle and its corresponding stochastic scheme U_6, but becomes specially appreciable in the annual and monthly seasonalities. The presence of months of different lengths even makes us doubt the adequate order of the monthly sum operator. This would make the use of specific stochastic schemes for this seasonality inadvisable.

4. Modelling of Different Simultaneous Seasonalities

4.1. Problems Posed

As has already been commented, a quite common factor in daily series is the presence of more than one seasonality, for which reason various modelling schemes have to be combined.

The approach of various stochastic schemes based on sum operators would pose two fundamental problems:

a) Overlapping of harmonics of similar frequencies which come from two different sum operators. This, as we have seen already, leads to an excessive number of unit roots.
b) The application of harmonics which correspond to periodicities not present in the series. This is specially relevant the greater the order of the sum operator.

For all these reasons, our approach starts from the selection of one of the seasonalities as a principal one, giving it priority when it comes to choosing the scheme it will be treated with, although most of the time it will be stochastic. As well as this, it allows for the breakdown of the sum operator in terms of the harmonics that make it up. Later, our experience shows that residual seasonalities from other periods that still remain are modelled in a more precise manner by means of combinations of fixed and varying deterministic schemes or via some frequency harmonics relevant to the said seasonalities. The clearest case of this is monthly seasonality with respect to annual seasonality.

On the other hand, the possibility that certain seasonalities are picked up by exogenous variables must always be born in mind. For example, in the case of annual seasonality, on occasions it can be modelled through meteorological variables and binary variables which pick up the effect of holiday periods.

It still remains to determine a discrimination criterion in order to use it for the selection amongst the multiple alternative schemes which could be considered, in such a way that a list of provisional starting models for more in-depth studies can be determined. We are inclined to choose the *criterion of reduction of the residual variance.*

4.2. The Criterion of Reduction of the Residual Variance

There is no theoretical base which makes this method unquestionable, rather it is intuitive grounds and experience which have demonstrated its good results. To this can be added its computational simplicity and speed, a decisive factor in series as ours, which consist of many thousands of observations. This is illustrated in references such as Cancelo and Espasa (1991a) and Espasa (1993).

5. Calendar Effect

The calendar effects takes on a special importance in our application owing to the great sensitivity that is shown by these series towards the presence of public holidays, holiday periods, and special events such as the celebration of elections, general strikes, timetable changes, etc. One example can be seen in chart 4(C). For that reason, a rigorous intervention analysis previous to any study seems compulsory. We incorporate it into the model through a complex system of dummy variables accompanied by their corresponding dynamic filters. A minucious treatment of this problem is usually indispensable. Here we give a brief outline of the principal problems. For applications of the calendar effect on daily series see Cancelo and Espasa (1991a) and Espasa (1993).

5.1. Public Holidays

It is proved that the effect of a public holiday is not the same depending on various factors. This impels us to group the public holidays into categories whose effect can be considered similar with the aim being to preserve the principle of parsimony and at the same time explain the data adequately. Fundamentally, we have used the following criteria for grouping: (a) day of the week, (b) period of year, (c) position in the month and (d) temperature abnormally low or high.

The public holidays which cannot be grouped with others, frequently the first of May and Christmas Day, receive specific treatment. As well as this, the existence of a great number of public holidays of a local or regional character obliges us to affect the analysis with corrective coefficients in terms of the percentage of the population that enjoy the public holiday.

5.2. Holiday Periods

The most relevant holiday periods are Holy Week, Christmas and Summer Holidays (July or August in the majority of the Européan countries).

The fundamental effects to reflect in holiday periods of long duration are:

a) Changes in trend (chart 4(A)).
b) Changes in the structure of the weekly cycle (chart 4(B)).
c) Special effects of determined days (Holy Friday, Christmas Eve, Christmas Day, Easter Monday, etc., chart 4(A) and 4(C)).

To model these effects we have used combinations of truncated step dummies onto which we have superimposed dynamic filters appropiate for each case.

5.3. Special Events

Events such as strikes, election days, etc., also have to be taken into account. In some series it is even recommendable to incorporate the change of the hour as happens on the last Sunday in March when the clock goes forward (a day of 23 hours) or as happens in September when the clock goes backwards (a day of 25 hours).

6. Exogenous Variables

The use of indicators or other types of exogenous variables in the modelling of daily series depends in great measure on the sector to which the series belongs. Nevertheless, it has been proved that a great many of them are very much affected, appart from by the possible previously mentioned seasonal considerations, by meteorological variables. This relationship becomes essential in series relating to the electrical sector, pollution or transport. It is for this reason that in the proposed modelling strategy these mentioned variables and, very specially temperature are taken into account.

The effect of these variables on our series is, in a good many cases, non-linear and different schemes for their formulation and estimation can be conceived. Engle *et al.* (1986) propose a semiparametric method; Engle *et al.* (1992) use first and seconds powers of the meteorological variable; Cancelo and Espasa (1991b) look for significant thresholds with which to define different segments in the rank of variation of the meteorological variable, approaching in each one the relationship by a linear function.

These variables may also show a dynamic effect. This means that the same temperature can cause different effects in terms of the values that have been registered in previous days. In many cases, the scheme can be even more complex owing to the fact that it may be different depending on the season of the year, wether it is a working day, or a weekend etc.

Cancelo and Espasa's previously mentioned method (1991b) has shown itself to give good results as much for the modelling of the non-linear effect as for picking up the dynamic effect.

7. Automatic Modelling Methodology

All of the above can be materialised in a methodology for automatic modelling of daily series. In its formulation we have limited ourselves to a description of the essential and most frequent aspects of daily series of economic activity.

It is possible to find various frameworks for the automatic programming of time series. Worthy of mention here is the SCA-Expert program (Long-Mu, 1993), the STAMP program in the context of structural models (developed by professor Harvey and associates) or the TRAMO program (Gomez and Maravall, 1994a and 1994b). All are designed for series with, generally, only one seasonality, which is in any case stable. This generally serves reasonably well in low-frequency series but not for the daily ones. For that reason a more specific treatment is necessary for these series. From here on, we shall suppose that two important seasonalities exists, *s1* and *s2*, which in the majority of cases will be weekly and annual.

The fundamental steps that our organigram follows are:

A.- Starting from the general scheme for modelling the calendar effect described in section 5, a first approximate estimation is made by OLS to remove from the series, Z_t, such effect ($Z_{At}=Z_t-Y_c\cdot\beta_{OLS}$).

B.- We calculate the dominant seasonal effect by comparing, according to the criterion of minimal variance, the schemes Δ, Δ^2, Δ_{s1}, Δ_{s2}, $\Delta\Delta_{s1}$, $\Delta\Delta_{s2}$ over the series Z_{At} and the Z_{At} series itself. With that an operator, say $\Delta^d U_{s1-1}$, is chosen and the correspondng *s1* will be considered the dominant seasonality. If the chosen scheme were Δ^d, then the principal seasonality is selected by calculating which scheme of the type $\Delta^d U_{sj-1}$ applied to Z_{At} gives minimal variance.

C.- We rank, according to the criterion of reduction of the residual variance, the schemes $\Delta^d\cdot ESQ_{s1}\cdot Z_{At}$, so that ESQ_{s1} may be:

i) Adequate deterministic schemes related to *s1*
ii) Combinations of harmonics, components of U_{s1-1}
iii) Related exogenous variables
iv) Certain combinations of i, ii and iii

The estimates are obtained by OLS. From such estimates the best *n* schemes are chosen: $\Delta^d\cdot ESQ1_{s1}$, $\Delta^d\cdot ESQ2_{s1}$,...and $\Delta^d\cdot ESQn_{s1}$. As a value of *n* in general 2 will be taken, although it could be higher if there are models very close in residual variance and if it is computationally admissible.

D.- Over the resultant series from $\Delta^d\cdot ESQ1_{s1}\cdot Z_{At}$,..., $\Delta^d\cdot ESQn_{s1}\cdot Z_{At}$ we apply again all possible combinations of i, ii, iii and iv, but this time related to seasonality *s2*. Again we keep the *n* best models $\Delta^d\cdot ESQ1_{s1s2}$, $\Delta^d\cdot ESQ2_{s1s2}$,... and $\Delta^d\cdot ESQn_{s1s2}$.

E.- To the previous models we apply other possible schemes which pick up residual effects of those seen in previous sections - including a third seasonality - and without the possibility of being filtered in previous stages.

E1.- From here onwards we have a list of possible models ($\Delta^d\cdot ESQi_{s1s2*}$), generally 2, of which we focus our attention on the first for consequent stages ($\Delta^d\cdot ESQ1_{s1s2*}\cdot Z_{At}=Z_{Et}$).

F.- In a similar way to the programms TRAMO or SCA-Expert, and on the basis of the results obtained by Tiao and Tsay (1983,1984) and Tsay (1984), a checking stage is established to see whether any of the non-stationary seasonalities present in the series has not been reflected adequately. In the case of this problem occuring we pass on to the next following model on the list from the paragraph *E1*. If all the pre-selected models are rejected *F* is inmediately applied to Z_{At} and the purely non-stationary stochastic scheme is taken as given by the unit roots that may appear here.

G.- We specify an ARMA(p,q) model to the series Z_{Ft}, the stationary series which comes out of the previous point. In the specification process the procedure of Revilla *et al.*(1991) is used.

H.- We estimate the complete model by maximum likehood, including all the deterministic elements that we have incorporated throughout the process and the meteorological variables.

I.- We analyse the t-statistics from the previous estimation to see whether the model can be simplified. Also F-statistics are considered for each one of the groups of seasonal dummy variables. Finally, an analysis of residuals is made in order to simplify the model or detect the need for alternative ARMA specifications. The analysis of residuals considers also the contrasts over the presence of conditional variance structure. If all of these contrasts do not reject the model, this is chosen as the final model. In the contrary case, the model is reformulated and we return to paragraph *I*. If the reformulation of the model turns out to be confusing we move on to the following model in the *E1* list.

The enlargement of this automatic modelling process to include effects mentioned in section 6 is quite direct if we apply Cancelo and Espasa's procedure (1991b).

8. Conclusions

In this study we have explained the great use that low cost models of daily series for the most sensitive variables could be for enterprises and institutions. On the other hand we have discussed the principal characteristics of these series and the general inadequacy of usual treatment methods for dealing with them. We have seen, case by case, not in an exhaustive manner, alternative methods of modelling which may be of great interest for our application, as well as their associated problems, which motivates a modelling strategy. The success of this strategy is based on the adequate design of alternative schemes in each of the sub-headings i) to iv) in paragraph *C* of the previous section. These have to be able to incorporate varying multiple seasonalities, with a non-linear structure and of a mixed nature, stochastic and deterministic, when the series thus require it.

Lastly, all the above takes the form of an automatic scheme of modelling. With similar procedures, but without the structuring formulated here, Cancelo and Espasa have obtained highly satisfactory results with series relating to notes and coin in circulation and electricity consumption.

References

Box, G.E. and Jenkins, G.M. (1970). *Time Series Analysis: Forecasting and Control.* San Francisco, Holden-Day.

Cancelo, J.R. and Espasa, A. (1987). "Un nuevo modelo diario para la predicción de la circulación fiduciaria". Unpublished work. *Servicio de estudios del Banco de España*, Madrid.

Cancelo, J.R. and Espasa, A. (1991a). "Forecasting Daily Demand for Electricity with Multiple-Input Nonlinear Transfer Function Models: A case Study". Working Paper 91-05. Universidad Carlos III de Madrid.

Cancelo, J.R. and Espasa, A. (1991b). "Threshold Modelling of Nonlinear Dynamic Relationships: An application to a Daily Series of Economic Activity". Working Paper 91-05. Universidad Carlos III de Madrid.

Cancelo, J.R. and Espasa, A. (1996). "Using high-frecuency data and time series models to improve yield management". Working Paper. Universidad Carlos III de Madrid.

Engle, R.F., Granger, C.W.J., Rice, J. and Weiss, H. (1986). "Semiparametric Estimates of the relation between weather and electricity sales". *Journal of the American Statistic Association*, 81:310-320.

Engle, R.F., Granger, C.W.J., Romanathan, R. and Valid Araghi, F. (1992). "Probabilistic Methods in Peak Forecasting". *Quantitative Economic Research Inc.* San Diego, California.

Espasa, A. (1993). "Modelling Daily Series of Economic Activity". *Proceedings of the BES section of the Amer. Stat. Assoc.*

Espasa, A. and Peña, D. (1995). "The Decomposition of Forecast in Seasonal ARIMA Models". *Journal of Forecasting*, 14: 565-583.

Gomez, V. and Maravall, A. (1994a). "Program TRAMO. Time Series Regression with ARIMA Noise Missing Observations and Outliers". EUI Working Paper ECO No. 94/31, Department of Economics, European University Institute, Florence.

Gomez, V. and Maravall, A. (1994b). "Estimation, Prediction, and Interpolation for Nonstationary Series with the Kalman Filter". *Journal of the American Statistical Association*, 89: 611-624.

Lon-Mu, L. (1993). "Modeling and Forecasting Time Series Using an Expert System Approach". Working Paper No. 127, University of Illinois at Chicago.

Revilla, P., Rey, P. and Espasa A. (1991). "Characterization of Production in Different Branches of Spanish Industrial Activity, by Means of Time Series Analysis". Working Paper 91-28. Universidad Carlos III de Madrid.

Ruiz, E. (1993). "Modelos para Series Temporales Heterocedásticas". *Cuadernos Económicos del ICE*, 56: 73-108.

Taylor, S. (1994). "Modelling Stochastic Volatility". *Mathematical Finance*, 4: 183-204.

Tiao, G.C. and Tsay, R.S. (1983). "Consistency Properties of Least Squares Estimates of Autoregressive Parameters in ARMA Models". *The Annals of Statistics*, 11: 856-871.

Tsay, R.S. (1984). "Regression Models with Time Series Errors". *Journal of the American Statistical Association*, 79: 118-124.