The Power of Love*

Emmanuel THIBAULT†

GREMAQ, Toulouse University

Abstract: We show that the presence of at least one individual who loves its children, whatever the intensity of this love, allows to avoid a possible global contraction of the economy.

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†GREMAQ - Manufacture des Tabacs, MF 202 - 21, allée de Brienne - 31 000 Toulouse - France. Tel: (+33)-5-61-12-85-57 / Fax: (+33)-5-61-22-55-63 / E-mail: emmanuel.thibault@univ-tlse1.fr

1 Introduction

This note deals with the existence of steady-state equilibria in OLG economies. In this profuse literature the most celebrated result is due to Muller and Woodford (1988): in contrast with to economies constituted exclusively of life-cyclers (Diamond (1965)'s models),[1] economies with at least one infinitely lived agent experience at least one non-trivial steady-state[2] equilibrium.

Since the seminal paper of Barro (1974), infinitely lived agents are often reinterpreted as dynasties of altruists, i.e., finitely lived agents linked though positive bequest. As the result of Muller and Woodford (and the Ricardian Equivalence Theorem) depends on an operative motive for private intergenerational transfers, several papers have studied whether the bequests are positive in a variety of different models.[3]

The purpose of this note is to show that, contrarily to a widespread but erroneous belief, no links exist between the positivity of bequests and the Muller and Woodford's result. Indeed, this result does not hinge upon the presence of a single infinitely lived agent since we shall establish that the presence of an agent with altruistic preferences (but who does not necessarily leave positive bequests) is sufficient to guarantee the existence of at least one non-trivial steady state.

So, one agent who loves his children, whatever the intensity of this love, allows to avoid a possible global contraction of the economy.[4]

To establish this result in a general and realistic setting, we focus on an OLG model

[1] According to Galor and Ryder (1989, 1991) (exogenous labor supply framework) or Nourry (2001) (endogenous labor supply context), the Inada conditions (and well-behaved preferences) are not sufficient to guarantee the existence of a non-trivial steady state in the Diamond model.

[2] A steady state is said to be trivial if the long-run capital /labor ratio is zero and non-trivial if this ratio is positive.

[3] See, e.g., Weil (1987) or Thibault (2000) for explicit conditions under which bequests are positive.

[4] The contraction of the economy is global if and only if whatever the initial capital stock, the long-run capital /labor ratio is zero.

in which the population consists of agents with heterogenous degrees of altruism toward their offspring whose labor supply is endogenous. This framework allows to encompass most of the OLG models in which there is at least one agent *à la Barro*.[5]

Note that such a macroeconomic model acknowledges the great heterogeneity in consumer behavior that is apparent in the data.[6] Indeed, it includes both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption both across years and across generations.

2 The economy

Consider a perfectly competitive economy which extends over infinite discrete time. The economy consists of $N \geq 1$ families denoted with $h \in \{1, ..., N\}$. In each period t, the size of each family h is denoted by N_t^h and grows at rate n. We consider a population of size N_t which consists of a fraction p_t^h of each family h where the proportion p_t^h does not vary through time. Hence:

$$\forall t > 0: \ \frac{N_t^h}{N_t} = p_t^h = p^h \quad \text{and} \quad \sum_{h=1}^{h=N} p^h = 1 \quad \text{and} \quad \frac{N_{t+1}}{N_t} = \frac{N_{t+1}^h}{N_t^h} = 1 + n$$

We assume that $p^h \in (0, 1]$ for $h \in \{1, ..., N\}$.

Consumers

Individuals of a family h are identical within as well as across generations and live for two periods. Hence, a family can be identified with a dynasty. For altruistic agents, we adopt Barro (1974)'s definition of altruism: parents care about their children welfare by weighting their children's utility in their own utility function and possibly leave them a bequest. When young, altruists of dynasty h born at time t receive a bequest x_t^h, work

[5] Our approach embodies a wide class of OLG models with exogenous labor supply: those where agents are only altruists and those where the population is a mix of agents *à la Diamond* and agents *à la Barro* (see, e.g., Michel and Pestieau (1998), Mankiw (2000) or Nourry and Venditti (2001)).

[6] See, e.g., Mankiw (2000).

a portion l_t^h of their first period, receive the market wage $w_t l_t^h$, consume c_t^h and save s_t^h. When old, they consume part of the proceeds of their savings and bequeath the remainder $(1+n)x_{t+1}$ to their $(1+n)$ children. Agents perfectly foresee[7] the interest factor R_{t+1}. Importantly, the bequest is restricted to be non-negative. We denote by V_t^h the utility of an altruist of dynasty h:

$$
\begin{aligned}
V_t^h(x_t^h) = \max_{c_t^h,\ell_t^h,s_t^h,d_{t+1}^h,x_{t+1}^h} \quad & U(c_t^h,\ell_t^h,d_{t+1}^h) + \beta^h\, V_{t+1}^h(x_{t+1}^h) \\
s.t \quad & w_t(1-\ell_t^h) + x_t^h = c_t^h + s_t^h && (1) \\
& R_{t+1}s_t^h = d_{t+1}^h + (1+n)x_{t+1}^h && (2) \\
& x_{t+1}^h \geq 0 && (3) \\
& \ell_t^h \in [0,1] && (4)
\end{aligned}
$$

where $V_{t+1}^h(x_{t+1}^h)$ denotes the utility of a representative descendant who inherits x_{t+1}^h, $U(c^h,\ell^h,d^h)$ his life cycle utility[8] which depends on consumptions (c^h,d^h) and leisure $\ell^h = 1 - l^h$ and β^h the intergenerational degree of altruism of the dynasty h.

We assume that $\beta^N \in (0,1)$ and (if $N > 1$) $\beta^h \in [0,\beta^N)$ for $h \in \{0,...,N-1\}$. Therefore, N is the most altruistic dynasty and, when $N > 1$, this dynasty can coexist with agents *à la Diamond* (i.e., with a dynasty h with $\beta^h = 0$).

Solving $V_t(x_t)$ gives the following optimality conditions:

$$U_c(c_t^h,\ell_t^h,d_{t+1}^h) = R_{t+1}U_d(c_t^h,\ell_t^h,d_{t+1}^h) \tag{5}$$

$$w_tU_c(c_t^h,\ell_t^h,d_{t+1}^h) - U_\ell(c_t^h,\ell_t^h,d_{t+1}^h) \begin{cases} = 0 & \text{if } l_t^h > 0 \quad (6a) \\ \leq 0 & \text{if } l_t^h = 0 \quad (6b) \end{cases}$$

$$-(1+n)U_d(c_t^h,\ell_t^h,d_{t+1}^h) + \beta U_c(c_{t+1}^h,\ell_{t+1}^h,d_{t+2}^h) \leq 0 \quad (= \text{if } x_{t+1}^h > 0) \tag{7}$$

[7] de la Croix and Michel (2000) compare the dynamics under myopic foresight and under perfect foresight in an OLG model with capital accumulation and two-period lived individuals.

[8] $U(c^h,\ell^h,d^h)$ is strictly concave, twice continuously differentiable over $\mathbb{R}_+^\star \times (0,1) \times \mathbb{R}_+^\star$ and $U_c(.) > 0$, $U_\ell(.) > 0$, $U_d(.) > 0$, and $\lim_{\varrho\to 0} U_c(\varrho,\ell^h,d^h) = \lim_{\varrho\to 0} U_\ell(c^h,\varrho,d^h) = \lim_{\varrho\to 0} U_d(c^h,\ell^h,\varrho) = +\infty$. The Hessian of U is negative definite. Moreover, c_t^h, ℓ_t^h and d_{t+1}^h are normal goods.

and the transversality condition (see Michel (1990)): $\lim\limits_{t\to+\infty}\beta^{t+1}U_d(c_{t-1}^h,\ell_{t-1}^h,d_t^h)x_t^h=0$.

Using the implicit function theorem, we can prove that the solution s_t^h of (5) can be expressed by a differentiable function $\breve{s}^h(l_t^h,w_t,R_{t+1},x_t^h,x_{t+1}^h)$. After substitution of c_t^h and d_{t+1}^h in (5) and ($6a$), the solutions l_t^h and s_t^h of these equations can be expressed by differentiable functions $s^h(.)$ and $l^h(.)$ of w_t, R_{t+1}, x_t^h and x_{t+1}^h. Since c and d are normal goods, $s^h(.)$ is increasing with respect to (w.r.t.) x_t^h and x_{t+1}^h and $l^h(.)$ is decreasing w.r.t. x_t^h, increasing w.r.t. x_{t+1}^h. The higher is the inheritance, the higher are savings and leisure. The more an altruist wants to leave a bequest, the more he works and saves. An increase in w_t can induce two opposite effects: it can increase labor supply because a higher wage incites to work more but it may decrease labor supply because, to keep his income constant, an agent can work less. Hence, $s^h(.)$ is not necessarily increasing w.r.t. its first argument. Concerning the second argument, things are more complex and the sign of s_2^h and l_2^h are indeterminate. Taking into account the constraint $l_t^h\geq 0$, the labor supply and the saving levels of an altruist who inherits x_t^h and wants to bequeath x_{t+1}^h to each of his children may be locally expressed by some continuous functions $\tilde{l}^h(.)$ and $\tilde{s}^h(.)$ of $(w_t,R_{t+1},x_t^h,x_{t+1}^h)$:[9]

$$l_t^h=\tilde{l}^h(w_t,R_{t+1},x_t^h,x_{t+1}^h)\equiv\max[0,l^h(w_t,R_{t+1},x_t^h,x_{t+1}^h)]$$

$$s_t^h=\tilde{s}^h(w_t,R_{t+1},x_t^h,x_{t+1}^h)\equiv\breve{s}^h(\tilde{l}^h(w_t,R_{t+1},x_t^h,x_{t+1}^h),w_t,R_{t+1},x_t^h,x_{t+1}^h)$$

These functions allow to characterize the bequest and labor supply of an agent of dynasty h. Note that if an altruist chooses to work, his savings function $\tilde{s}^h(.)$ corresponds to the function $s^h(.)$. Since $l^h(.)$ is increasing with respect to x_t^h, the higher is the inheritance of an altruist, the lower is his labor supply.

It is also important to note that when x_t^h and x_{t+1}^h are zero, then the functions $s^h(.)$ and $l^h(.)$ give the savings and the labor supply of each selfish agents as functions of

[9] All the details of these tedious computations are in Appendix 1 of Thibault (2003). In that companion paper we focus on the existence and specific characters of rentiers. Indeed, since altruists can inherit, they can behave as rentiers, i.e., as individuals who can choose not to work.

the wage rate and the interest rate. As the life cycle utility function U is identical for the N dynasties,[10] there exist some differentiable functions $s^{De}(.)$ and $l^{De}(.)$ such that for all h: $s^{De}(w_t, R_t) \equiv s^h(w_t, R_t, 0, 0)$ and $l^{De}(w_t, R_t) \equiv l^h(w_t, R_t, 0, 0)$. From these function, we also define the function $\vartheta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that:

$$\vartheta(z) = \frac{s^{De}(f(z) - zf'(z), f'(z))}{(1+n)l^{De}(f(z) - zf'(z), f'(z))}$$

This function will be useful to analyze the bequest motive of an altruistic dynasty.

Firms

Production occurs according to a constant returns to scale technology $F(.)$ using two inputs, capital K_t and labor L_t.[11] Homogeneity of degree one allows us to write output per young as a function of the capital/labor ratio per young $f(z_t) = F(z_t, 1)$ where $z_t = K_t/L_t$. Markets are perfectly competitive. Each factor is paid its marginal product. Assuming that capital fully depreciates after one period we obtain:

$$w_t = F_L(z_t, 1) = f(z_t) - z_t f'(z_t) \quad \text{and} \quad R_t = F_K(z_t, 1) = f'(z_t) \tag{8}$$

In each period, the labor market clears, i.e., $L_t = N_t l_t$ with $l_t = 1 - \sum_{h=1}^{h=N} p^h \ell_t^h$. The capital stock at time $t+1$ is financed by the savings of the young generation born in t. Hence, $K_{t+1} = N_t s_t$ with $s_t = \sum_{h=1}^{h=N} p^h s_t^h$. Therefore, in intensive form:

$$k_{t+1} = s_t/(1+n) \quad \text{with} \quad k_{t+1} = K_t/N_t \tag{9}$$

[10]For simplicity, the utility U is the same for all dynasties. Using Cobb-Douglas specifications, it is possible to analyze equilibria of an economy in which this is not the case (see, e.g., Thibault (2001)).

[11]$F(K, L)$ is twice continuously differentiable, homogeneous of degree 1 with respect to capital and labor over the set $\mathbb{R}_+^\star \times \mathbb{R}_+^\star$ and satisfies: $\forall L > 0$ $F_K(., L) > 0$, $F_{KK}(., L) < 0$ and $\lim_{L \rightarrow 0} F(K, L) = 0$.

3 When bequest motive avoids global contraction

We now confine our analysis to steady states. According to equations (5) and (7), the long-term behavior of each dynasty must satisfy:

$$\beta^h R \leq 1 + n \quad (= \text{if } x^h > 0) \tag{10}$$

Hence, only agents of dynasty N i.e., the dynasty endowed with the highest degree of altruism, have the possibility to leave a bequest to their children.[12]

Note that it is sufficient to have some unconstrained altruistic agents to reach the modified golden rule, and this result holds regardless of the proportion p^N.[13] Indeed, when x^N is positive, according to (8) and (10) the steady state capital/labor ratio z is equal to $f'^{-1}((1+n)/\beta^N)$. We denote this capital/labor ratio by $\hat{z}$.

First, using methodology developed in Thibault (2000), we derive a general condition under which the most altruist agents leave a positive bequests.[14]

Lemma 1 *The most altruist agents leave positive bequests if and only if $\vartheta(\hat{z})$ is lower than the modified golden rule capital/labor ratio $\hat{z}$.*

A strength of this condition is that it is valid whatever the number and stability properties of steady states of the subjacent Diamond model with endogenous labor supply. This model is defined as the economy described in section 2 with only life-cyclers (i.e., with $\beta^N = 0$). The steady states with no bequests of our economy are linked with the steady states of the subjacent Diamond model in the following way:

[12] Indeed, if there exists $i \in \{1, ..., N-1\}$ such that $x^i > 0$ then (10) is not satisfied for dynasties j where $j \in \{i+1, ..., N\}$.

[13] Whatever their size, as well-known since Becker (1980)'s model with heterogenous infinitely lived agents, the most patient (or altruistic) impose their view on the long-run capital accumulation.

[14] This lemma is a simple extension of Proposition 1 of Thibault (2000). Details of the proof of this lemma are available at: `http://durandal.cnrs-mrs.fr/GREQAM/dt/wp-pdf00/00A32.pdf`

Lemma 2 *A steady state z^{De} of the subjacent Diamond model is a steady state with zero bequests of our economy if and only if z^{De} is greater or equal to $\hat{z}$.*

Proof. The steady states z^{De} of this subjacent Diamond model are solutions to $z = \vartheta(z)$. According to (8) and (10), it is straightforward that a steady state z^{De} of the subjacent Diamond model is a steady state with zero bequests of our economy if and only if $\beta^h f'(z^{De}) \leq 1 + n$, i.e., if and only if $z^{De} \geq \hat{z}$. **QED**

Before focusing on the non-trivial steady states of our model, we establish a useful and important property of the function ϑ.

Lemma 3 *There exists a ratio $\tilde{z}$ above which the curve $\vartheta(z)$ is flatter than the $45°$ line.*

Proof. Since agents cannot save more than their income: $0 \leq s^{De}(w, R) \leq wl^{De}(w, R)$. Hence $0 \leq \vartheta(z) \leq (f(z) - zf'(z))/(1 + n)$ *i.e.,* $0 \leq \vartheta(z)/z \leq [f(z)/z - f'(z)]/(1 + n)$. As $f > 0$, $f' > 0$ and $f'' < 0$ we have $\lim\limits_{z \to +\infty} (f(z)/z - f'(z)) = 0$. Then, $\lim\limits_{z \to +\infty} \vartheta(z)/z = 0$. This infinite limit implies the lemma. **QED**

From lemma 1, 2, and 3 we now can establish the existence of a non-trivial steady state.

The existence result.

There exists at least one non-trivial steady-state equilibrium.

Proof. We can distinguish two cases according the number of equilibria of the subjacent Diamond model. First, assume that the subjacent Diamond model has no positive steady state. Then, the function $\vartheta(z)$ has non-trivial fixed point. According to lemma 3, its form is necessarily as indicated in Figure 1 (left side). Then, we have $\vartheta(\hat{z}) < \hat{z}$. Therefore, according to lemma 1, our model has a non-trivial steady state: the modified golden rule.

Second, assume that the subjacent Diamond model has positive steady states. If the highest z_{max}^{De} of these equilibria is larger than, or equal to, $\hat{z}$ then, according to lemma 2, it is a steady state with zero bequests of our economy. If z_{max}^{De} is smaller

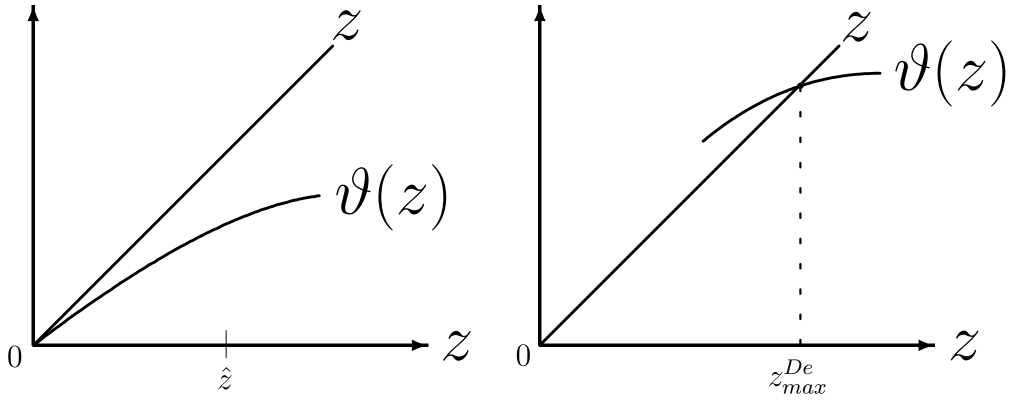

Figure 1: First Case (left side) vs Second Case (right side)

than $\hat{z}$ then the dynastic model has a steady state with operative bequests, because we necessarily have $\vartheta(\hat{z}) < \hat{z}$. Indeed, according to lemma 3, in this second case the function $\vartheta(z)$ has the form depicted Figure 1 (right side). **QED**

To summarize, an agent with altruistic preferences (but who does not necessarily leave positive bequests) is sufficient to guarantee the existence of at least one non-trivial steady state. So, one agent who loves its children, whatever the intensity of this love, allows to avoid a possible global contraction of the economy: it is the power of love...

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