Regulating the Natural Gas Transportation Industry : Optimal Pricing Policy of a Monopolist with Advance-Purchase and Spot Markets

Laurent David* Michel Le Breton† Olivier Merillon‡

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Abstract

Worldwide, natural gas markets have changed drastically after the overall deregulation started in the eighties. In this paper, we introduce an analytical framework to address the supply and pricing policy of a gas supplier that has to reserve transportation capacity in order to deliver natural gas to final users. The main characteristic of our approach lies in the treatment of the demand uncertainty that the supplier faces. Indeed, when it has to book its transportation capacity, the supplier does not know the demand with certainty. This paper defines the optimal policy for the natural gas supplier firstly by the price proposed to final users that are willing to commit themselves in an advanced purchase of gas ; secondly by the price proposed to final users who did not commit themselves in advance and who buy their gas on a spot basis ; thirdly by the optimal capacity reservation made by the gas supplier to the transportation network. Our model is based on two complementary assumptions : we assume that the gas supplier has a market power and that the regulator fixes the network access capacity price on a cost plus basis.

*Gaz De France - R&D Division - Economics and sociology Unit. laurent-m.david@gazdefrance.com
†Université de Toulouse, IDEI. lebreton@cict.fr.
‡Gaz De France - R&D Division - Economics and sociology Unit. olivier.merillon@gazdefrance.com

1 Introduction

Worldwide, the main characteristics of natural gas markets have changed drastically after the overall deregulation started in the eighties. This evolution begun in the United State where Orders 436 and 636 compelled vertically integrated gas companies use their transport facilities. Third Party Access (TPA) to natural gas pipelines is considered as one of the necessary conditions to open natural gas markets to competition. In Europe, the UK has been the first country to go into the gas market opening.

Today, regulated third party access to national pipelines is implemented all over Europe. Every network company, wether it is linked to a supplier or not, has to offer transportation tariffs on a non-discriminatory basis.

Besides, natural gas industry is characterized by uncertainty that, on the demand side comes from weather conditions or economic variables (growth for instance). This uncertainty has a hudge impact on the strategies that can be implemented as far as gas transportation on different networks is concerned. On the other side, natural gas transporters have to take this uncertainty into account while setting their tariffs. Generally, they propose two kinds of service: firm or interruptible. Interruptible capacity is offerred with a discount rate compared to the firm one. For most of the european pipelines, firm capacity can be booked on an annual basis but also on a monthly or a daily basis. Considering the uncertainty that exists on natural gas consumptions, network operators naturally tend to favor long commitment. Thus, the price for monthly capacity is generally set at a higher level than the twelfth of the annual price and for daily capacity, price is superior to the thirtieth of the monthly.

Thus, the reservation strategy that a shipper has to define in order to sell gas to end users is sophisticated because of this demand uncertainty. Precisely, we assume that when it must reach a decision concerning annual firm capacity, it does not know the exact demand that will occur in subsequent periods. To stick with reality, we assume that there is no complete set of markets à la Arrow-Debreu where all the risks could be insured through appropriate insurance contracts or contingent pricing[1]. We assume instead that the firm delivering natural gas for some given period can use two type of simple linear pricing contracts. For any given period of consumption (say a given calendar month), either the client commits in advance to purchase a given quantity of gas (advance purchase) or the client decides upon this quantity at the beginning of the period. The firm offers a menu of two linear prices : one for advance purchases and another one for spot

[1]The analysis of prices and market behavior in environments with a set of incomplete markets raises subtle and sometimes unexpected conclusions (see e.g. Green and Polemarchakis (1976), Green and Sheshinski (1975)). At this stage, we dont offer any insight on the rationale for any particular market architecture. We take the current market environment and the constraints that it imposes on transactions as given.

purchases. The firm faces different risks. First, when fixing the two prices, it cannot predict for sure what will be the volumes demanded for each type of transaction and second, when booking capacity on transportation networks, it cannot assert for sure if the firm annual capacity will be fully utilized. The client who purchases some of its gas in advance also faces a risk as the value (need, utility,..) of gas consumption is not surely known at term. Likely, the second risk faced by the firm price will depend also upon the existence and efficiency of a secondary market for natural gas transportation[2]. In Europe, most of the network operators have settled mechanisms (such as electronic bulletin board) that allow shippers to exchange capacities. Nontheless there have not been so many transactions on these secondary markets. In addition most of the network operators[3] apply the "use-it-or-lose-it" rule in order to prevent forclusion strategy. Indeed, a supplier could be tempted to book capacity far beyond its real needs in order to prevent access to network to other potential suppliers. Currently, the most used allocation rule in Europe is the "first come, first served", the UK and Hungary are exceptions as they use auctions mechanisms to pipeline capacity allocation . This fact can be interpreted as an indicator of sufficient capacity in Europe. That is why, in this paper, we ignore the risk of rationing for capacity reservation.

The structure of the market(s) involving the demand and supply of natural gas is moving rapidly. It is fair to say that the current situation is conveniently described as a market which is imperfectly competitive as only few actors[4] with non negligible market power operate on these markets (and sometimes other energy markets) despite the efforts of the regulator(s). In this paper we will point our attention towards the capacity reservations and pricing decisions of a firm enjoying a monopoly position in the sense that the clients of this firm are captive. This must be viewed as a preliminary step to an assessment of the consequences of the regulatory policies on the functioning of the market and the welfare of the clients. This is part of a sequential game where the regulator moves first and can commit on its decisions. Its move is then followed by moves from the various private actors. The final prediction will depend upon the details of the modelling of the oligopolistic market. Solving backward the game describing the competition, provides the regulator with an exact evaluation of the consequences of the alternative policies that it may consider. Then, we see this manuscript as part of a larger project to understand the main features of the optimal policy of a regulator aiming

[2]They will certainly play an important role when they will be fully implemented. McAfee, Doane, Nayyar and Williams (2006) analyse the implication of pipeline residual rights on the competitiveness of the secundary market for natural gas transportation. They offer a critique of some aspects of the current practices of the US Federal Energy regulatory Commission. Raineri and Kuflik (2003) examine the issue from a dynamic perspective and and apply their methodology to Chile.

[3]Among those network operators, one can find French, Netherland, Italian, Austrian en Denish ones.

[4]On April 1, 2006, only 9% of the population of eligible clients i.e. 63900 clients buy their gas to the market price (there is the possibility to buy gas to a public price totally under the control of the government). Among those, only 18400 lients switch from their historic suppliers to a new entrant. The number of suppliers active on the market has however increased from 10 on January 1, 2005 to 14 on January 1, 2006 (CRE (2006)).

to control a market displaying imperfect competition.

We don't explain in this paper the motivation behind the choices of the regulator. Under the assumption that it is not captured but aiming to maximize some social objective, the access prices imposed to the pipeline networks operators are intended to reflect their costs (capacity and operating). Investment planning in pipeline capacities is a delicate matter as there is no market and these prices can also be used to send signals to firms using the network and determine subsequently possible over or underinvestment[5].

A key ingredient of our approach is the stochastic dimension of the demand for natural gas by either commercial, industrial or residential. We offer a model of the demand of natural gas for some period depending upon the two prices proposed by the provider but also on some other elements which are not modelled explicitely and are uncertain when the client must decide upon its advance purchases. Besides weather conditions, we may think of many other elements like for instance levels of activity or prices of alternative energy inputs in the case or industrial clients. This volatility should be kept distinct from the seasonal "predictable" variations which is well documented problem. Any client is therefore described by a list of parameters including price elasticities and volatilities. In practice, volatility is not easy to quantified unless the client consumption is measured almost in continuous time as done for large clients. On the price elasticities side, we would have in principle two direct elasticities and one cross elasticity. To the best of our knowledge, no empirical analysis of this sophisticated consumer or input choice is available. Past and existing studies concentrate on a single demand for natural gas. While quasi-null in the short run, direct natural gas price elasticities are far from being negligible. In their celebrated paper, Balestra and Nerlove (1966) found for the United States over the period 1950-1962 an elasticity ranging from 0.58 to 0.69. This is confirmed for instance by Beierlein, Dunn and McConnon (1981) for the northeastern United States over the period 1967-1977 who found for industrial clients an elasticity equal to 2.86 with respect to the value added[6].

[5]GDF Réseau-Transport examines the opportunity of an increase of its entry capacities in the northeastern part of France :Le besoin potentiel d'une augmentation des capacités d'acheminement au point d'entrée Obergailach est apparu à l'occasion de différentes demandes non engageantes émanant d'expéditeurs de gaz et de gestionnaires du réseau de transport amont, et de la propre analyse de gaz de france Réseau Transport sur l'évolution des flux de gaz en Europe. Gaz de france Réseau Transport souhaite disposer debons indicateurs pour réaliser les investissements correspondants aux souhaits de ses clients, à des conditions économiques raisonnables, tant pour les bénéficiaires des capacités nouvelles que pour l'ensemble des clients de Gaz de france réseau Transport. A ce titre, des engagements portant sur des capacités et une durée suffisante sont indispensables pour mener à bien le projet". L'opérateur de transport explique que cet investissement est motivé par des demandes reçues par des expéditeurs. L'investissement consiste soit en un doublement de la canalisation jusqu'à Chateau-Salins sur une distance de 37 km avec rajout d'un compresseur à la station de Laneveulotte soit jusqu' à Laneveulotte sur une distance de 54 km avec rajout de deux compresseurs permettant d'augmenter les capacités d'acheminement de 120 à 220 GWh/j suppémentaires. Afin de prévoir les meilleurs choix techniques, l'opérateur demande aux expéditeurs intéressés de bien vouloir se faire connaitre et exprimer leurs besoins de capacités. La durée minimale de demande de réservation non engageante est fixée à 10 ans".

[6]Thse estimations are based on cross section and time series macro-data. there are also a number of studies based

The paper consists then of two parts. In the first part, we develop a model to derive the demand behavior of residential and industrial clients for natural gas. This model ignores income effects and aims to point out some of the key forces behind the decision of a client to buy a large or small quantity of gas and to possibly differ its purchases until uncertainty is resolved. The second part integrates the demand behavior into a multiproduct monopoly problem. We offer a complete solution to the three dimensional problem of such monopoly : how many pipeline capacity to reserve in advance, how to price respectively advance and spot purchases ? This raises a number of questions on top of which : how the two reservation prices imposed by the regulator to the network operator are going to be translated into the prices of the monopoly ? Likely, given its market power, price elasticities and volatilities will play a role and prices will unlikely reflect truly the marginal costs supported by the firm. It is then important to evaluate the gap between the ratio of prices and the ratio of costs. In the process of our analysis, we also derive the optimal reservation rule. Most of the analysis is conducted in the case where the population of clients is composed of two homogeneous groups. Even in that case, the analytical derivation becomes rapidly tricky and we offer a full analysis of the homogeneous case.

Related Literature

While new, the ideas and topics discussed in this paper intersect different branches of the literature on public utility pricing, regulation and industrial organization.

Most of the literature on pricing and regulation with stochastic demand considers as a primitive the aggregate demand. In a pionnering work, Boiteux (1951) has raised the question of optimal pricing of individual stochastic demand under the assumption that the capacity network must meet an exogeneous reliability constraint. In the case where each individual demand was Gaussian, he was able to derive the pair of optimal prices : each client pays according to the mean and the standard deviation of its consumption. His analysis has been continued by Drèze (1964) and Kolm (1970) who point out the necessity of using personalized prices and the role of correlation accross demands. It is very interesting to note that while theoretical, the Hopkinson rate which is a very common method of pricing electricity and gas for industrial use is strongly related to the idea of pricing the standard deviation of consumption as demonstrated by Veall (1983). The Hopkinson rate consists of a demand charge based on the maximum usage during any quarter-hour period during a month. This pricing dimension should be kept distinct from the temporal peak dimension of consumption and the resulting TOU (time-of-use) pricing methods (Seeto, Woo, et Horowitz (1997) and Woo, Horii et Horowitz (2002)). Our model does not take as primitives the individual demands but derives these demands from maximization in a market environment where transactions

on micro-data like for instance Baker, Blundell and Micklewright (1989) and Leth-Petersen.

are however constrained. In contrast to Boiteux who assume inelastic demands, we examine the response of the demands to price changes and point out the role of the derived price elasticities. In our paper, we consider advance purchase pricing and spot pricing but our model could be extended to include more sophisticated pricing options.

Our paper is also very related to the literature on equilibria in markets where both individual and aggregate demand is uncertain and firms set prices before demand is known (Carlton (1978)(1979)(1991), Dana (1998)(1999), Eden (1990)). In the competitive case, small firms sell in a spot market but must set their spot market prices and quantities before demand is known. As first shown by Prescott (1975) with such a context of uncertain aggregate demand, goods may be offered and sold at different prices in equilibrium. Firms must decide how much to supply at each price. Since spot prices do not adjust to clear the market, rationing occurs in equilibrium and capacity may not be fully utilized. Dana (1998) generalizes the basic Prescott model to allow firms to make advance-purchase sales. Like in our paper, these contracts are restricted to standard sales contracts, contingent contracts or contracts that specify probabilistic delivery of the good (e.g. priority service pricing) [7]. Gales and Holmes ((1992),(1993)) examines the rationale of an advance-purchase discount policy from the perspective of a monopoly facing demand uncertainty.

Finally, our paper is also a contribution to the analysis of multiproduct monopolies. In our model, there is a single physical product but the timing of purchase leads to two different products from the perspective of the clients. The optimal pricing policy of a multiproduct monopoly has been examined in many papers with focus on questions like bundling or cross subsidization[8]. Our paper adds demand uncertainty and price rigidities to the standard model of monopoly pricing. By setting with those features, it has been demonstrated that posting several prices may be optimal (Dana (2001), Wilson (1988)[9]). There are very few models of imperfect competition in market environments like those considered here. A notable exception is Deneckere and Peck (1995) who consider a game where firms set prices and capacities and then a random number of consumers attend the market and select a firm to visit[10].

The rest of the paper is organized as follows. In section 2, we develop our model of residential and industrial stochastic demand for natural gas. Then in section 3, we derive and interpret the optimal policy of the monopoly in terms of pipeline capacity reservations and advance-purchase and spot pricing. We conclude with a description of a number of directions of extension of the

[7]See, for instance, Spulber (1992a,b) and Wilson (1989) for the examination of alternative policies.

[8]See Tirole (1988) for an overview.

[9]In Wilson (1988) there is no aggregate uncertainty but it is assumed that a large number of consumers visit the firm in random order.

[10]There is also a related literature on the implications of future markets on the equilibria of oligopolistic spot markets (see e.g. Allaz (1992), Allaz and Vila (1993), Mahenc and salanié (2004) and Murphy and Smeers (2005)). Note however that advance-pruchase discounts is something different from forward trading.

current work.

2 The Demand for Natural Gas

We consider a monopolist distributing gas to a "captive" population of heterogeneous consumers. We are interested in explaining the volumes of trade and the prices paid by these customers at time t or more precisely during the period $[t, t+\Delta]$. The firm holding this monopoly position has two important technological decisions : it can reserve some transportation capacity one period ahead i.e. at time $t-\Delta$ for the unitary price of $\underline{c}$. It has however the possibility to proceed to some extra reservation of transportation capacity at time t for the unitary price $\overline{c}$. We assume here that there is no secondary market for unused capacity which implies that the monopoly may well be in a situation where it has to incur the full cost of a transportation capacity which is under utilized at time t. We also ignore here the event that the monopoly could be unable to reserve the "missing" capacity at time t[11]. The monopoly must precommit to two prices : a unitary price $\underline{p}$ for any volume of gas that a client may ordered at time $t-\Delta$ for a delivery/consumption at time t and a unitary price $\overline{p}$ for any volume of gas ordered at time t for an immediate delivery/consumption. The contractual universe consists simply of these two linear prices.

The specific feature of our model is the stochastic character of the demand from the perspective of the firm. The firm "investment" (here reservations) decisions cannot be delayed until the resolution of uncertainty. The analysis is conducted within a reference time period and we will not pay any attention to the temporal dimension of gas consumption[12]. If it was possible to design a complete set of contingent markets a la Arrow-Debreu, we would have a price for each possible contingency and each consumer would have to select a contingent consumption plan. In such market environment, uncertainty would disappear for the local gas supplier without implying, of course, that there is no excess capacity in some contingency. We also know that such organization of the transactions leads to an efficient allocation of resources and in particular optimal risk sharing. We assume that this market organization is not possible or, more precisely, we assume that it is not possible to offer all the contingent delivery contracts associated to these markets. In this second best setting, the gas supplier cannot accomodate all contingencies without investing in capacity up to the point that even the worst case could be handled without risk of default. In this section, we first model the demand behavior of household and firms when a single price is posted for the period which is considered. Then, we describe the costs of the firm. Finally, we describe the market

[11] This sounds at first glance inconsistent with our assumption that the reservation price is regulated and therefore not flexible to accomodate an excess demand of transportation capacity at time t.

[12] The explicit introduction of time does not raise any specific conceptual difficulty. However, this aspect, on top of which peak load pricing, has been widely discussed in the literature and is not the main focus of our manuscript which deals with other sources of the variability in demand.

environment i.e. the type of transactions that are possible.

2.1 The Demand for Natural Gas at Time t from residential Clients

The main purpose of this section is to offer a simple model[13] explaining how a client of the monopoly reacts to the pair of prices $(\underline{p}, \overline{p})$ i.e. plans its gas consumption for the period. This simple model aims not only to explain what will be the volume of gas consumed by this client in reaction to the menu of prices but also how it will share this total consumption between an advance-purchase and the spot market. This will depends of course upon its need/preference/value for gas consumption in contrast to other commodities. The key assumption is that the valuation of this client depends upon informations which are not all disclosed at time $t-\Delta$. Precisely, we assume that the preference of a generic household for gas consumption at time $t-\Delta$ is described by the quasi-linear utility function

$$V(x, \omega) + M \tag{1}$$

where x denotes its consumption for the period $[t, t+\Delta]$, ω is a real number and M denotes the other consumption expenditures. We denote by R the consumption budget of this household. As explained above, at time $t-\Delta$, this household is offered the possibility of ordering some gas at a unitary price $\underline{p}$. Then at time t, he can always proceeds to some ultimate arrangements (if needed) once the all relevant information will have been conciled. It is assumed here that the real parameter ω is random and that its realization will take place at time t. This implies that any household planning to order some gas at time $t-\Delta$ faces some uncertainty. The expression in (1) describes in monetary units the value of the consumption plan (x, M) when the realization of the random variable $\widetilde{\omega}$ is ω. To evaluate ex ante, i.e. at time $t-\Delta$, the value of this plan, we need to introduce the von Neuman-Morgenstern utility U of this household which reflects its attitude towards risk. The value of the consumption plan (x, M) is then :

$$U\left(V(x, \omega) + M\right)$$

In the contractual environment considered here, a consumption plan for the period $[t, t+\Delta]$ is a vector $(\underline{x}, \overline{x}(\omega), M)$ where $\underline{x}$ represents its advance purchase at time $t-\Delta$ and $\overline{x}(\omega)$ represents its spot purchase at time t when uncertainty has been resolved : $\overline{x}(\omega)$ denotes its purchase when the realization of the random variable is ω. When the range of the random variable $\widetilde{\omega}$ consists of

[13] This model is of course simplistic in many respects as it wants to focus on a specific feature, namely the volatility in gas consumption and its consequences on pricing. A complete model of gas consumption should recognize the time dimension of the problem and the impossibility to separate the investment decision in some durable equipments for energy consumption and the subsequent decisions on input consumptions. This

a finite set Ω and $\pi(\omega)$ is the probability of the event $\{\widetilde{\omega}=\omega\}$, the expected utility derived from the purchase plan $(\underline{x}, \overline{x}(\omega), M)$ is :

$$\sum_{\omega\in\Omega}\pi(\omega)U(V(\underline{x}+\overline{x}(\omega),\omega)-\underline{p}\underline{x}-\overline{p}\overline{x}(\omega)+R) \tag{2}$$

The first order conditions are :

$$\sum_{\omega\in\Omega}\pi(\omega)U'(V(\underline{x}+\overline{x}(\omega),\omega)-\underline{p}\underline{x}-\overline{p}\overline{x}(\omega)+R)\left[\frac{\partial V}{\partial x}(\underline{x}+\overline{x}(\omega),\omega)-\underline{p}\right]=0 \text{ si } \underline{x}>0 \tag{3}$$

and

$$\frac{\partial V}{\partial x}(\underline{x}+\overline{x}(\omega),\omega)-\overline{p}=0 \text{ si } \overline{x}(\omega)>0 \tag{4}$$

Without any further assumption on the primitives, equations (3) and (4) are not easy to solve in full generality. For instance, when U is of the CARA type i.e. $U(z)=-e^{-\lambda z}$ where λ is a positive parameter, equation (3) simplifies to :

$$\sum_{\omega\in\Omega}\pi(\omega)e^{-\lambda(V(\underline{x}+\overline{x}(\omega),\omega)-\overline{p}\overline{x}(\omega))}\left[\frac{\partial V}{\partial x}(\underline{x}+\overline{x}(\omega),\omega)-\underline{p}\right]=0$$

Note however that U does not play any role in equations (4) which is fairly natural as they describe optimal supplementary purchase of gas once uncertainty has totally disappeared.

To handle these equations, we introduce a number of simplifications. First, we suppose that each household is risk neutral. In this case, equation (3) simplifies to :

$$\sum_{\omega\in\Omega}\pi(\omega)\left[\frac{\partial V}{\partial x}(\underline{x}+\overline{x}(\omega),\omega)-\underline{p}\right]=0 \text{ if } \underline{x}>0 \tag{5}$$

When we will consider a population of households, each of them will be identified by an index $i=1,.....,I$[1]. The parameters and variables will be subsequently indexed with i : $V_i, U_i, \Omega_i, \pi_i$ and R_i. Heterogeneity across households dervies from multidimensional spanning income, preferences for gas and risk attitudes. Under risk neutrality and quasi linearity, income and risk effects are eliminated. We are left with intrinsic preference heterogeneity with two channels : the impact of x for a given ω and the impact of ω for a given x.

2.2 The Binomial Specification

We now specify further the framework by describing the influence of uncertainty on the value of gas consumption. The key feature is the binomial character[14] of the stochastic influence of the state of the world ω: for each client i either the state of the world is favorable to gas consumption x_i or it is not. Moreover, the states of the world favorable to gas consumption may differ sharply accross clients. A state of the world is a vector describing the subpopulation of clients receiving a favorable signal. Precisely :

$$\Omega = \prod_{i=1}^{I^1} \{\underline{\omega}_i, \overline{\omega}_i\}$$

where for all $i = 1, 2, ..., I^1$, $\underline{\omega}_i$ et $\overline{\omega}_i$ are two real numbers such that : $\underline{\omega}_i < \overline{\omega}_i$. Without loss of generality, we suppose hereafter that $\underline{\omega}_i = 0$; $\overline{\omega}_i$ refers to circumstances unfavorable to gas consumption from the perspective of client i. We denote π_i the probability of the event $\{\omega_i(t) = 0\}$.

Finally, we assume that for all $i = 1, ..., I^1$:

$$V_i(x_i, \omega) = v_i(x_i + \omega_i)$$

where v_i is an increasing and strictly concave continuously differentiable function. In this simplified setting, a consumption purchase plan is a three dimensional vector $(\underline{x}_i, \overline{x}_i(\underline{\omega}_i), \overline{x}_i(\overline{\omega}_i)) = (\underline{x}_i, \overline{x}_i(0), \overline{x}_i(\overline{\omega}_i))$. The expected utility of client i for such plan becomes :

$$\pi_i v_i(\underline{x}_i + \overline{x}_i(\underline{\omega}_i)) + (1 - \pi_i) v_i(\underline{x}_i + \overline{x}_i(\overline{\omega}_i) + \overline{\omega}_i) - \underline{p}\underline{x}_i - \overline{p}(\pi_i \overline{x}_i(0) + (1 - \pi_i)\overline{x}_i(\overline{\omega}_i))$$

Equations (4) simplifies to :

$$\begin{cases} v_i'(\underline{x}_i + \overline{x}_i(0)) = \overline{p} \text{ iff } v_i'(\underline{x}_i) \geq \overline{p} \\ v_i'(\underline{x}_i + \overline{x}_i(\overline{\omega}_i) + \overline{\omega}_i) = \overline{p} \text{ iff } v_i'(\underline{x}_i + \overline{\omega}_i) \geq \overline{p} \end{cases} \tag{6}$$

If $v_i'(\underline{x}_i) < \overline{p}$ (respectively $v_i'(\underline{x}_i + \overline{\omega}_i) < \overline{p}$), then $\overline{x}_i(0) = 0$ (respectively $\overline{x}_i(\overline{\omega}_i) = 0$). Since v_i has been assumed to be strictly concave, we deduce that if $v_i'(\underline{x}_i) < \overline{p}$ then $v_i'(\underline{x}_i - \overline{\omega}_i) < \overline{p}$. Therefore, from (6), if $\overline{x}_i(\overline{\omega}_i) > 0$, then $\overline{x}_i(0) > 0$. On the other hand, equation (5) simplifies to :

$$\pi_i v_i'(\underline{x}_i + \overline{x}_i(0)) + (1 - \pi_i)\, v_i'(\underline{x}_i + \overline{x}_i(\overline{\omega}_i) + \overline{\omega}_i) = \underline{p} \text{ si } \underline{x}_i > 0 \tag{7}$$

We deduce from equations (6) et (7) that necessarily :

[14] We could opt for a continuous state space. Our modelling choice is just driven by convenience.

$$\overline{x}_i(\overline{\omega}_i) = 0$$

Indeed, if on the contrary $\overline{x}_i(\overline{\omega}_i) > 0$, since $\overline{x}_i(0) > 0$, we deduce from (6) :

$$\pi_i v_i'(\underline{x}_i + \overline{x}_i(0)) + (1 - \pi_i)\, v_i'(\underline{x}_i + \overline{x}_i(\overline{\omega}_i) + \overline{\omega}_i) = \overline{p}$$

which contradicts (7) since $\overline{p} > \underline{p}$. The intuition driving this result is fairly simple. Here, a circumstance which is adverse to gas consumption leads to a decrease of the marginal utility of gas with respect to a reference consumption. In our binary setting, this happens when $\omega_i = \overline{\omega}_i$ and in such case, it is optimal to purchase the contingent optimal quantity of gas at the lowest possible price i.e. in advance. If in contrast, circumstances turn to be favorable, then the spot market will be (likely) used to proceed to some additional purchases. An immediate implication of this observation is that a purchase plan of client i reduces to a two dimensional vector $(\underline{x}_i, \overline{x}_i(0))$, that we will denote simply $(\underline{x}_i, \overline{x}_i)$. The first order conditions become :

$$\pi_i v_i'(\underline{x}_i + \overline{x}_i) + (1 - \pi_i)\, v_i'(\underline{x}_i + \overline{\omega}_i) = \underline{p} \text{ if } \underline{x}_i > 0 \tag{8}$$

and

$$v_i'(\underline{x}_i + \overline{x}_i) = \overline{p} \text{ si } \overline{x}_i > 0$$

Client i finds optimal to purchase its gas in advance if the unique solution $\underline{x}_i$ of the following equation :

$$\pi_i v_i'(\underline{x}_i) + (1 - \pi_i)\, v_i'(\underline{x}_i + \overline{\omega}_i) = \underline{p} \tag{9}$$

satisfies

$$v_i'(\underline{x}_i) \leq \overline{p} \tag{10}$$

Similarly, client i finds optimal to purchase all its gas on the spot market if :

$$\pi_i v_i'(\overline{x}_i) + (1 - \pi_i)\, v_i'(\overline{\omega}_i) \leq \underline{p} \tag{11}$$

where $\overline{x}_i$ is the unique solution of the equation:

$$v_i'(\overline{x}_i) = \overline{p} \tag{12}$$

This happens if and only if the following inequality holds true :

$$v_i'(\overline{\omega}_i) \leq \frac{\underline{p} - \pi_i \overline{p}}{1 - \pi_i}$$

For instance when $\underline{p}$ is smaller than $\pi_i \overline{p}$, we conclude that this cannot happen. The inequality is less likely to hold true when $\underline{p}$ is small, $\overline{p}$ is large and $v_i'(\overline{\omega}_i)$ is large. In contrast, the effect of π_i is ambiguous.

Finally, client i will not purchase gas (at all) if :

$$\pi_i v_i'(0) + (1 - \pi_i)\, v_i'(\overline{\omega}_i) \leq \underline{p} \text{ et } v_i'(0) \leq \overline{p}$$

Let $\underline{x}_i(\underline{p})$ be the unique solution to equation (8). Inequality (9) becomes :

$$\overline{p} \geq \frac{\underline{p} - (1 - \pi_i)\, v_i'(\underline{x}_i(\underline{p}) + \overline{\omega}_i)}{\pi_i} \equiv \varphi\left(\underline{p}\right)$$

From the implicit function theorem, we deduce :

$$\underline{x}_i'(\underline{p}) = \frac{1}{\pi_i v_i''(\underline{x}_i) + (1 - \pi_i)\, v_i''(\underline{x}_i + \overline{\omega}_i)}$$

and then :

$$\varphi'\left(\underline{p}\right) = \frac{1 - \frac{(1-\pi_i) v_i'"(\underline{x}_i + \overline{\omega}_i)}{\pi_i v_i''(\underline{x}_i) + (1-\pi_i) v_i'"(\underline{x}_i + \overline{\omega}_i)}}{\pi_i} = \frac{v_i''(\underline{x}_i)}{\pi_i v_i''(\underline{x}_i) + (1 - \pi_i)\, v_i''(\underline{x}_i + \overline{\omega}_i)}$$

It should be noted that as soon as $v_i'(x)$ tends to 0 when x tends to $+\infty$, $\varphi\left(\underline{p}\right)$ tends to 0 when $\underline{p}$ tends to 0. Moreover, combining (10) and (11) lead to the inequality :

$$\overline{p} \leq \frac{\underline{p} - (1 - \pi_i)\, v_i'(\overline{\omega}_i)}{\pi_i} \equiv \psi(\underline{p})$$

The functions φ et ψ make the identification of the four potential groups of households in the population easier : those who do consume gas, those who purchase their gas exclusively in advance, those who purchase their gas exclusively on the spot market and those who mix with the both. We note first that the functions intersect at $\underline{p} = \pi_i v_i'(0) + (1 - \pi_i)\, v_i'(\overline{\omega}_i)$ and $\varphi\left(\underline{p}\right) = \psi\left(\underline{p}\right) = v_i'(0)$. The curvature of the function φ depends upon the monotonicity of the coefficient $-\frac{v_i''(x)}{v_i''(x)}$. For the sake of illustration, we consider the case where $v_i(x) = -e^{-\lambda_i x}$ where λ_i is a positive parameter. In such a case :

$$\varphi\left(\underline{p}\right) = \frac{\underline{p}}{\pi_i + (1 - \pi_i) e^{-\lambda_i x}} \text{ and } \psi(\underline{p}) = \frac{\underline{p} - (1 - \pi_i)\, \lambda_i e^{-\lambda_i x}}{\pi_i}$$

At time $t - \Delta$, the total gas consumption of household i is a Bernouilli random variable with mean $\underline{x}_i\left(\underline{p}, \overline{p}\right) + \pi_i \overline{x}_i\left(\underline{p}, \overline{p}\right)$ and standard deviation $\sqrt{\pi_i(1 - \pi_i)}\overline{x}_i\left(\underline{p}, \overline{p}\right)$..

2.3 The Demand for Natural Gas at Time t from Industrial Clients

Our preceding analysis of the demand has focused exclusively on households. In this subsection, we sketch a Bernouilli model of demand for natural gas from firms. Here, gas is an energy input used by firms in their production process and it is implicitely assumed that this input is in competition with others.

Consider the case of a monoproduct firm described by the cost function $C(y,x)+px$ conditional upon the purchase of a quantity of gas equal to x at a unitary price of p. Let q denote the unitary price of the product. When there is no uncertainty on q, the determination of the optimal quantity of gas to buy is a conventional exercice.

Suppose now that instead, there is a binomial uncertainty on the sale price q : the price q takes the values $\overline{q}$ et $\underline{q}$ with probabilities π and $1-\pi$. The desired quantity of gas does not need to be the same for the two states of the world. Assume that $\overline{q} > \underline{q}$. In such case and under the assumption that the inputs are normal, then $y(\overline{q}) > y(\underline{q})$. As in the case of households, we distinguish favorable and unfavorable circumstances for gas consumption. This channel of influence privileges the impact of a change in the activity level of the firm following a decline in rentability. Other channels of influence could consist for instance in modifications of the price of inputs which are complements or subsitutes to gas.

The analysis of the optimal choices of the firm is very similar to the analysis conducted for the household. First, there will be no purchase on the spot market when $q = \underline{q}$. Indeed, given the normality assumption, the demand of all the inputs in the favorable case is larger than in the unfavorable case. The lowest demand of gas is attached to the sale price $\underline{q}$ and it is better to purchase gas in advance at the lower price $\underline{p}$. This means that the firm will buy at least the quantity $x(\underline{p},\underline{q})$. We are left with the question : is it profitable to buy in advance a quantity larger than $x(\underline{p},\underline{q})$? Let $\underline{x}$ be the quantity of gas purchased in advance by the firm. The expected profit is then :

$$\pi \underset{y,x}{Max} \left[\overline{q}y - C(y, x+\underline{x}) - \overline{p}x - \underline{p}\underline{x}\right] + (1-\pi) \underset{y,x}{Max} \left[\underline{q}y - C(y, x+\underline{x}) - \overline{p}x - \underline{p}\underline{x}\right]$$

The important observation is that, at time t, the cost $\underline{p}\underline{x}$ is sunk. Note also that ouput plan of the firm and demand factors will depend upon the ex ante decision $\underline{x}$. On the spot market, it faces the unit price $\overline{p}$. When circumstances are unfavorable, the constraint $x \geq 0$ will be active. In such a case, the manager of the firm optimizes with respect to the short run cost function : everything is as if, with respect to the prices $\underline{p}$ et $\underline{q}$, his gas input was in excess. Given $\underline{p}$, this quantity of gas would then be optimal for some sale price $q > \underline{q}$. We deduce then from the Viner-Wong envelope's principle that the production y is in between $y(\underline{p},\underline{q})$ and $y(\underline{p},q)$.

When the circumstances are favorable, the constraint will be likely inactive and the manager will buy some additional quantity of gas on the spot market. To examine, the validity of these claims in full generality, we write down the first oder conditions of the firm. A production plan is a five dimensional vector $\left(y(\underline{q}), y(\overline{q}), \overline{x}(\underline{q}), \overline{x}\left(\overline{q}\right), \underline{x}\right)$. In the interior case, we obtain :

$$\overline{q} - \frac{\partial C}{\partial y}(y(\overline{q}), \overline{x}\left(\overline{q}\right) + \underline{x}) = 0$$

$$\underline{q} - \frac{\partial C}{\partial y}(y(\underline{q}), \overline{x}\left(\underline{q}\right) + \underline{x}) = 0$$

$$-\frac{\partial C}{\partial x}(y(\overline{q}), \overline{x}\left(\overline{q}\right) + \underline{x}) = \overline{p}$$

$$-\frac{\partial C}{\partial x}(y(\underline{q}), \overline{x}\left(\underline{q}\right) + \underline{x}) = \overline{p}$$

$$\pi\left[-\frac{\partial C}{\partial x}(y(\overline{q}), \overline{x}\left(\overline{q}\right) + \underline{x}) - \underline{p}\right] + (1-\pi)\left[-\frac{\partial C}{\partial x}(y(\underline{q}), \overline{x}\left(\underline{q}\right) + \underline{x}) - \underline{p}\right]$$

Assume that $\frac{\partial^2 C}{\partial x \partial y}(y, x) < 0$. From the implicit function theorem and the second order condition $\frac{\partial^2 C}{\partial x^2}(y, x) > 0$, we deduce that x is an increasing function of y. Further, the supply curve is increasing in q and therefore $y(\underline{q}) < y(\overline{q})$. We deduce that if $\overline{x}\left(\underline{q}\right) > 0$, then $\overline{x}\left(\overline{q}\right) > 0$. Indeed, if instead $\overline{x}\left(\overline{q}\right) = 0$, we would obtain

$$-\frac{\partial C}{\partial x}(y(\overline{q}), \underline{x}) \leq \overline{p}$$

But since $\frac{\partial^2 C}{\partial x \partial y}(y, x) < 0$ and $y(\underline{q}) < y(\overline{q})$, we would have :

$$-\frac{\partial C}{\partial x}(y(\underline{q}), \underline{x}) \leq \overline{p}$$

and then, since $\frac{\partial^2 C}{\partial x^2}(y, x) > 0$:

$$-\frac{\partial C}{\partial x}(y(\underline{q}), \overline{x}\left(\underline{q}\right) + \underline{x}) < \overline{p}$$

But this contradicts the fourth first order condition. To conclude, we can now observe that it is impossible to have $\overline{x}\left(\underline{q}\right) > 0$ because then we would have $\overline{x}\left(\overline{q}\right) > 0$ and after substitution in the fifth equation, we would deduce $\overline{p} = \underline{p}$ in contradiction to our assumption.

Like for households, we will consider a population of firms, each of them will be identified by an index $j = 1, ..., I^2$ and the parameters and variables will be subsequently indexed with j : C_i and π_j. Heterogeneity across firms spans differences in technology and risk.

2.4 Aggregate Demand Uncertainty

To conclude the modeling of the demand side, it remains to agregate the individual demand behaviors. To do so, we must describe the structure of the uncertainty. We have assumed that each client i is described by a Bernouilli model totally summarized by a single number π_i representing the marginal distribution attached to this client. We now introduce the joint distribution accross clients. In the case of two clients i.e. $I = 2$, a state of the world is described by a vector $\omega \in \{0, \overline{\omega}_1\} \times \{0, \overline{\omega}_2\}$. the joint distribution is defined by the following contingency table :

	0	$\overline{\omega}_2$	
0	$\rho\pi_1\pi_2$	$\pi_1 (1 - \rho\pi_2)$	π_1
$\overline{\omega}_1$	$\pi_2 (1 - \rho\pi_1)$	$1 - \pi_1 - \pi_2 + \rho\pi_1\pi_2$	$1 - \pi_1$
	π_2	$1 - \pi_2$	

Table 1

The last row and last colum correspond to the marginals. All the information about the correlation accross states is contained in the coefficient ρ. The circumstances infuencing the gas demand of the two clients are independent when $\rho = 1$. In contrats, they are perfectly correlated when $\rho = \frac{1}{\pi_1} = \frac{1}{\pi_2}$.

An alternative way to model simply the correlation would consist in adding an extra component in the product space $\prod_{i=1}^{I} \{\underline{\omega}_i, \overline{\omega}_i\}$, say $\Omega = \left\{\underline{\theta}, \overline{\theta}\right\} \times \prod_{i=1}^{I} \{\underline{\omega}_i, \overline{\omega}_i\}$ and assuming the joint distribution as the product of the marginals. In such setting, the uncertainty affecting client i would consist of two terms : a macroeconomic or climatic term θ together with an idiosyncratic term ω_i. The analysis of the demand of gas by households and firms could be conducted as before, under the asssumption that $V_i(x_i, \omega) = v_i(x_i + \theta + \omega_i)$. However, there are four states of the world at the level of each client and the analytics become more tedious.

Hereafter , we focus on the case where only the idiosyncratic risk is taken into consideration i.e. we assume that the individual demands are independent. From the perspective of the firm supplying gas to this population of clients, the stochastic demand at time $t - \Delta$ is therefore a sum of independent (but not identically distributed) Bernouilli random variables $\widetilde{x}_i$ where :

$$\widetilde{x}_i = \begin{cases} \underline{x}_i \left(\underline{p}, \overline{p}\right) + \overline{x}_i \left(\underline{p}, \overline{p}\right) & \text{with probability } \pi_i \\ \underline{x}_i \left(\underline{p}, \overline{p}\right) & \text{with probability } 1 - \pi_i \end{cases}$$

The aggregate demand consists of a deterministic term $\sum_{i=1}^{I} \underline{x}_i \left(\underline{p}, \overline{p}\right)$ and a random term $\sum_{i=1}^{I} \overline{x}_i \left(\underline{p}, \overline{p}\right)$. The first term is the aggregate advance purchase while the second terme is the aggregate purchase on the spot market. Both are influenced by the two dimensional price policy $\left(\underline{p}, \overline{p}\right)$.

In some cases, it will be useful to replace the exact aggregate demand by its Gaussian approximation[15]. We do it in the case of the residential aggregate demand but the same argument applies in the case of the aggregate industrial demand. If :

$$\frac{\sum_{i=1}^{I^1} \pi_i \left(1-\pi_i\right) \left(\pi_i^{1+\delta}+(1-\pi_i)^{1+\delta}\right)(\overline{x}_i(\overline{\omega}_i))^{2+\delta}}{(\sqrt{\sum_{i=1}^{I} \sigma_i^2})^{2+\delta}} \underset{I\to\infty}{\to} 0$$

we deduce from the Lyapounov 's central limit theorem that if I is large enough, $X\left(\underline{p}, \overline{p}\right) \equiv \sum_{i=1}^{I^1} \left(\underline{x}_i \left(\underline{p}, \overline{p}\right) + \overline{x}_i \left(\underline{p}, \overline{p}\right)\right)$ behaves approximatively as a Gaussian random variable $N(\mu, \sigma)$ where

$$\mu = \sum_{i=1}^{I^1} \mu_i \text{ with } \mu_i = \underline{x}_i \left(\underline{p}, \overline{p}\right) + \pi_i \overline{x}_i \left(\underline{p}, \overline{p}\right) \text{ for all } i = 1, ..., I^1$$

$$\sigma = \sqrt{\sum_{i=1}^{I^1} \sigma_i^2} \text{ with } \sigma_i = \sqrt{\pi_i(1-\pi_i)}\overline{x}_i \left(\underline{p}, \overline{p}\right) \text{ for all } i = 1, ..., I^1$$

3 The Monopoly Optimal Policy

The main contribution of this section is to derive the optimal capacity reservation and pricing policy of the monopoly. On the one hand, the monopoly has to decide how much capacity to reserve in advance. Two events may occur. If the capacity can match the realized demand, then there is no need to buy extra capacity on the spot market. Otherwise, the monopoly will be obliged to buy on the spot market the missing capacity. The unitary prices of these two markets are regulated and denoted respectively $\underline{c}$ and $\overline{c}$ for advance and spot capacity reservations. We assume that there is no possible resale for capacities i.e. that there is no secundary market, any unused capacity is worthless. On the pricing side, the monopoly offer two contractual options to its clients : any unitary quantity of gas purchased firmly in advance is sold at the price $\underline{p}$ while any unitary quantity purchased a day in advance is sold at the price $\overline{p}$. We assume that the monopoly can credibly commit on its prices.

The aggregate demand is stochastic. This implies two sources of randomness : on the sales side, the revenues are uncertain and on the costs side, the final cost is also dependent upon the amount of extra capacity that the operator may have to buy on the spot market. As it will bve transparent below, we will separate the input decision (advance capacity reservations) from the

[15] Any Gaussian variable takes negative values with positive probability. Since demand is non negative, we will have to be careful with that implication of the approximation.

pricing decisions. we ignore here the other part of the cost incurred by the monopoly, namely the purchase of gas at some entry point. Under the assumption that this cost is constant and governed by long term contracts (say with importers), the extension of our results to this setting is straightforward : the optimal relative markups are the same as without this additional cost. This assumption is realistic for the case where the monopoly is the the part of the historical monopoly after unbundling. It is less realistic in the case where the monopoly is a new entrant which does not access to these long term arrangements and can be obliged (besides gas release arrangements) in some circumstances to buy gas on the "marché de gros".

Since the profit of the monopoly is random, it is important to know how it evaluates risky strategies. Denoting by $\widetilde{\pi}$ the random profit, a good and simple utility function f to describe that feature is given by the CARA class. Precisely :

$$f(\pi) = -e^{-\rho\pi}$$

where ρ is a positive parameter representing its absolute aversion towards risk. A computatinal advantage offered by this class is that we can demonstrate that if $\widetilde{\pi}$ is a Gaussian random variable $N(\mu, \sigma)$, then the certainty equivalent objective is simply :

$$\mu - \frac{\rho\sigma^2}{2}$$

The solution of the problem is already complicated in that case[16]. This is why, we will focus for the moment on the case where the monopoly is risk neutral

3.1 The General Formulation

We denote by $\widetilde{\pi}\left(\underline{p}, \overline{p}\right)$ the monopoly profit. It is the random variable defined as follows :

$$\begin{array}{ll} \underline{p} & \left[\sum_{i=1}^{I^1} \underline{x}_i(\underline{p}, \overline{p}) + \sum_{j=1}^{I^2} \underline{x}_j\left(\underline{p}, \overline{p}\right)\right] + \overline{p}\left[\sum_{i=1}^{I^1} \widetilde{\overline{x}}_i\left(\underline{p}, \overline{p}\right) + \sum_{j=1}^{I^2} \widetilde{\overline{x}_j}\left(\underline{p}, \overline{p}\right)\right] \\ & -\underline{c}z - \overline{c}\widetilde{z} \end{array}$$

where z is the capacity reserved in advanced and :

$$\widetilde{z} = \begin{cases} 0 \text{ if } \sum_{i=1}^{I^1} \widetilde{x}_i(\underline{p}, \overline{p}) + \sum_{j=1}^{I^2} \widetilde{x_j}\left(\underline{p}, \overline{p}\right) \leq z \\ \sum_{i=1}^{I^1} \widetilde{x}_i(\underline{p}, \overline{p}) + \sum_{j=1}^{I^2} \widetilde{x_j}\left(\underline{p}, \overline{p}\right) - z \text{ if } \sum_{i=1}^{I^1} \widetilde{x}_i(\underline{p}, \overline{p}) + \sum_{j=1}^{I^2} \widetilde{x_j}\left(\underline{p}, \overline{p}\right) > z \end{cases}$$

We deduce that the expected profit of the monopoly is equal to :

[16]Preliminary results are available upon request from the authors.

$$
\begin{aligned}
&\underline{p}\left[\sum_{i=1}^{I^1}\underline{x}_i(\underline{p},\overline{p})+\sum_{j=1}^{I^2}\underline{x}_j\left(\underline{p},\overline{p}\right)\right]+\overline{p}\left[\sum_{i=1}^{I^1}\pi_i\overline{x}_i\left(\underline{p},\overline{p}\right)+\sum_{j=1}^{I^2}\pi_j\overline{x}_j\left(\underline{p},\overline{p}\right)\right]\\
&-\underline{c}z-\overline{c}\int_z^{+\infty}\left(\sum_{i=1}^{I^1}\widetilde{x}_i(\underline{p},\overline{p})+\sum_{j=1}^{I^2}\widetilde{x_j}\left(\underline{p},\overline{p}\right)-z\right)P(d\widetilde{x})
\end{aligned}
$$

where P denotes the distribution of the aggregate random demand $\widetilde{x}$ where

$$
\widetilde{x}\equiv\sum_{i=1}^{I^1}\widetilde{x}_i(\underline{p},\overline{p})+\sum_{j=1}^{I^2}\widetilde{x_j}\left(\underline{p},\overline{p}\right)
$$

Under the Gaussian approximation, the expected profit simplifies to :

$$
\begin{aligned}
&\underline{p}\left[\sum_{i=1}^{I^1}\underline{x}_i(\underline{p},\overline{p})+\sum_{j=1}^{I^2}\underline{x}_j\left(\underline{p},\overline{p}\right)\right]+\overline{p}\left[\sum_{i=1}^{I^1}\pi_i\overline{x}_i\left(\underline{p},\overline{p}\right)+\sum_{j=1}^{I^2}\pi_j\overline{x}_j\left(\underline{p},\overline{p}\right)\right]\\
&-\underline{c}z-\overline{c}\int_z^{+\infty}\left(\ \widetilde{x}-z\ \right)\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\widetilde{x}-\mu)^2}{2\sigma^2}}d\widetilde{x}
\end{aligned}
$$

$\int_2^{+\infty}(x-2)^2\,\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-1)^2}{2\sigma^2}}dx$

where $\widetilde{x}$ is the Gaussian random variable $N(\mu,\sigma)$ with :

$$
\mu\equiv\mu^1+\ \mu^2 \text{ et } \sigma=\sqrt{\sum_{i=1}^{I^1}\sigma_i^2+\sum_{j=1}^{I^2}\sigma_j^2}
$$

Under the assumption that the monopoly is risk neutral, then its optimal policy $\left(\underline{p}^*,\overline{p}^*,z^*\right)$ is solution of the program :

$$
\begin{aligned}
&\underset{\left(\underline{p},\overline{p},z\right)}{Max}\ \underline{p}\left[\sum_{i=1}^{I^1}\underline{x}_i(\underline{p},\overline{p})+\sum_{j=1}^{I^2}\underline{x}_j\left(\underline{p},\overline{p}\right)\right]+\overline{p}\left[\sum_{i=1}^{I^1}\pi_i\overline{x}_i\left(\underline{p},\overline{p}\right)+\sum_{j=1}^{I^2}\pi_j\overline{x}_j\left(\underline{p},\overline{p}\right)\right]\\
&-\underline{c}z-\overline{c}\int_z^{+\infty}\left(\widetilde{x}-z\right)\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\widetilde{x}-\mu)^2}{2\sigma^2}}d\widetilde{x}
\end{aligned}
$$

The first order condition with respect to z leads to the fondamental equation :

$$
\int_z^{+\infty}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\widetilde{x}-\mu)^2}{2\sigma^2}}d\widetilde{x}=\frac{\underline{c}}{\overline{c}}
$$

i.e. the optimal reservation of transport capacity $z^*(\underline{p},\overline{p})$ at time $t-\Delta$ satisfies :

$$
\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{z-\mu}{\sigma}}e^{-\frac{t^2}{2}}dt=1-\frac{\underline{c}}{\overline{c}}
$$

or equivalently :

$$z^* = \mu + \sigma\Phi(1 - \frac{\underline{c}}{\overline{c}})$$

where $\Phi(\kappa)$ is the unique solution to the equation :

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\Phi(\kappa)} e^{-\frac{t^2}{2}}dt = \kappa$$

For the record, table 2 below provides a sample of values of $\Phi(\kappa)$.

κ	0.8	0.9	0.95	0.99
$\Phi(\kappa)$	0.845	1.285	1.645	2.325

Table 2

Then after solving for z, the monopoly choice problem simplifies to the following two dimensional problem :

$$\underset{(\underline{p},\overline{p})}{Max}\,\underline{p}\left[\sum_{i=1}^{I^1}\underline{x}_i(\underline{p},\overline{p}) + \sum_{j=1}^{I^2}\underline{x}_j\left(\underline{p},\overline{p}\right)\right] + \overline{p}\left[\sum_{i=1}^{I^1}\pi_i\overline{x}_i\left(\underline{p},\overline{p}\right) + \sum_{j=1}^{I^2}\pi_j\overline{x}_j\left(\underline{p},\overline{p}\right)\right]$$
$$-\underline{c}\left(\mu + \sigma\Phi(1 - \frac{\underline{c}}{\overline{c}})\right) - \overline{c}\int_{\mu+\sigma\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}\left(\widetilde{x} - \mu - \sigma\Phi(1 - \frac{\underline{c}}{\overline{c}})\right)\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\widetilde{x-\mu})^2}{2\sigma^2}}d\widetilde{x}$$

where :

$$\mu = \sum_{i=1}^{I^1}\left(\underline{x}_i\left(\underline{p},\overline{p}\right) + \pi_i\overline{x}_i\left(\underline{p},\overline{p}\right)\right) + \sum_{j=1}^{I^2}\left(\underline{x}_j\left(\underline{p},\overline{p}\right) + \pi_j\overline{x}_j\left(\underline{p},\overline{p}\right)\right)$$

and

$$\sigma = \sqrt{\sum_{i=1}^{I^1}\pi_i(1-\pi_i)\overline{x}_i^2\left(\underline{p},\overline{p}\right) + \sum_{j=1}^{I^2}\pi_j(1-\pi_j)\overline{x}_j^2\left(\underline{p},\overline{p}\right)}$$

For the sake of calculus, it is useful to observe[17] that :

$$\int_z^{+\infty}(\widetilde{x} - z)\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\widetilde{x-\mu})^2}{2\sigma^2}}d\widetilde{x} = \sigma\int_{\frac{z-\mu}{\sigma}}^{+\infty}(1 - F(t))dt$$

where :

$$F(x) \equiv \int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$$

[17]The equality follows from a straightforward integration by parts.

This reduced form indicates that the vector of prices $\left(\underline{p}, \overline{p}\right)$ influences the objective through direct channels like the classical demand and cost components. The analysis here is complicated by the fact that the optimal technological choice which consists here in a mix of advance transportation capacity reservations and spot transportation capacity reservations cannot be separated from the pricing decisions. Under suitable second order conditions, the determination of the optimal policy $\left(\underline{p}^*, \overline{p}^*\right)$ is solution of the two first order conditions attached to these variables. The analytical derivation as well as the interpretation of these equations is quite tricky in the general case. In the next section, we offer a complete analysis in the case where the total population of clients consists of two homogenous groups..

3.2 Two Homogeneous Classes of Clients

Hereafter, we assume that the populations of residential and industrial clients of this monopoly are homogeneous : all the residential clients on the one hand and all the industrial clients on the other hand display the same characteristics concerning their needs/tastes for gas consumption and the volatility of their demand. As it should be clear, this binary assumption accomodates of course many alternative illustrations like for instance the case of a monopoly whose clients are all either residential or industrial but partitioned into two groups. Formally, we assume that :

$$\pi_i \equiv \pi^1,\ v_i \equiv v^1,\ \overline{\omega}_i \equiv \overline{\omega}^1 \text{ for all } i = 1, ..., I^1$$

$$\pi_j \equiv \pi^2,\ v_j \equiv v^2,\ \overline{\omega}_j \equiv \overline{\omega}^2 \text{ for all } j = 1, ..., I^2$$

The problem of the monopoly simplifies to :

$$\underset{\left(\underline{p},\overline{p}\right)}{Max} \left[\begin{array}{c} \underline{p}\left(I^1\underline{x}^1\left(\underline{p},\overline{p}\right) + I^2\underline{x}^2\left(\underline{p},\overline{p}\right)\right) + \overline{p}\left(I^1\pi^1\overline{x}^1\left(\underline{p},\overline{p}\right) + I^2\pi^2\overline{x}^2\left(\underline{p},\overline{p}\right)\right) \\ -\underline{c}z^*\left(\underline{p},\overline{p}\right) - \sigma\overline{c}\int_{\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}}^{+\infty}\left(1-F(t)\right)dt \end{array} \right]$$

The first order conditions are as follows :

$$
\begin{aligned}
& \underline{p}\left[I^1\frac{\partial\underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial\underline{p}}+I^2\frac{\partial\underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial\underline{p}}\right]+\left[I^1\underline{x}^1\left(\underline{p},\overline{p}\right)+I^2\underline{x}^2\left(\underline{p},\overline{p}\right)\right] \\
& +\overline{p}\left[I^1\pi^1\frac{\partial\overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial\underline{p}}+I^2\pi^2\frac{\partial\overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial\underline{p}}\right] \\
& -\underline{c}\left[\frac{\partial z^*\left(\underline{p},\overline{p}\right)}{\partial\underline{p}}\right]-\overline{c}\left[\left(1-F(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma})\right)\frac{\partial z^*\left(\underline{p},\overline{p}\right)}{\partial\underline{p}}\right] \\
& -\overline{c}\left(1-F(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma})\right)\frac{\partial\mu}{\partial\underline{p}} \\
& -\overline{c}\left[\int_{\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}}^{+\infty}\left(1-F(t)\right)dt+\left(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}\right)\left(1-F(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma})\right)\right]\frac{\partial\sigma}{\partial\underline{p}} \\
= \; & 0
\end{aligned}
$$

and

$$
\begin{aligned}
& \left[I^1\frac{\partial\underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial\overline{p}}+I^2\frac{\partial\underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial\overline{p}}\right]+\overline{p}\left[I^1\pi^1\frac{\partial\overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial\overline{p}}+I^2\pi^2\frac{\partial\overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial\overline{p}}\right] \\
& +\left[I^1\pi^1\overline{x}^1\left(\underline{p},\overline{p}\right)+I^2\pi^2\overline{x}^2\left(\underline{p},\overline{p}\right)\right] \\
& -\underline{c}\left[\frac{\partial z^*\left(\underline{p},\overline{p}\right)}{\partial\overline{p}}\right]-\overline{c}\left[\left(1-F(z^*\left(\underline{p},\overline{p}\right)\right)\frac{\partial z^*\left(\underline{p},\overline{p}\right)}{\partial\overline{p}}\right] \\
& -\overline{c}\left(1-F(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma})\right)\frac{\partial\mu}{\partial\overline{p}} \\
& -\overline{c}\left[\int_{\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}}^{+\infty}\left(1-F(t)\right)dt+\left(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}\right)\left(1-F(\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma})\right)\right]\frac{\partial\sigma}{\partial\overline{p}} \\
= \; & 0
\end{aligned}
$$

Remembering that :

$$z^*\left(\underline{p},\overline{p}\right)=\mu+\sigma\Phi(1-\frac{\underline{c}}{\overline{c}})$$

where here :

$$
\begin{aligned}
\mu &= I^1\left(\underline{x}^1\left(\underline{p},\overline{p}\right)+\pi^1\overline{x}^1\left(\underline{p},\overline{p}\right)\right)+I^2\left(\underline{x}^2\left(\underline{p},\overline{p}\right)+\pi^2\overline{x}^2\left(\underline{p},\overline{p}\right)\right) \\
\sigma &= \sqrt{I^1\pi^1(1-\pi^1)\left(\overline{x}^1\left(\underline{p},\overline{p}\right)\right)^2+I^2\pi^2(1-\pi^2)\left(\overline{x}^2\left(\underline{p},\overline{p}\right)\right)^2}
\end{aligned}
$$

we obtain after simplifications and substitutions :

$$\begin{aligned}
& \underline{p}\left[I^1\frac{\partial \underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\frac{\partial \underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right]+\left[I^1\underline{x}^1\left(\underline{p},\overline{p}\right)+I^2\underline{x}^2\left(\underline{p},\overline{p}\right)\right] \\
& +\overline{p}\left[I^1\pi^1\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\pi^2\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right] \\
& -\underline{c}\frac{\partial \mu}{\partial \underline{p}}-\overline{c}\left[\int_{\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}}^{+\infty}\left(1-F(t)\right)dt+\frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\frac{\partial \sigma}{\partial \underline{p}} \\
= \; & 0
\end{aligned}$$

and

$$\begin{aligned}
& \underline{p}\left[I^1\frac{\partial \underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\frac{\partial \underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\right]+\overline{p}\left[I^1\pi^1\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\pi^2\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\right] \\
& +\left[I^1\pi^1\overline{x}^1\left(\underline{p},\overline{p}\right)+I^2\pi^2\overline{x}^2\left(\underline{p},\overline{p}\right)\right] \\
& -\underline{c}\frac{\partial \mu}{\partial \overline{p}}-\overline{c}\left[\int_{\frac{z^*\left(\underline{p},\overline{p}\right)-\mu}{\sigma}}^{+\infty}\left(1-F(t)\right)dt+\frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\frac{\partial \sigma}{\partial \overline{p}} \\
= \; & 0
\end{aligned}$$

Using the fact that :

$$\begin{aligned}
\frac{\partial \mu}{\partial \underline{p}} &= I^1\frac{\partial \underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\frac{\partial \underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^1\pi^1\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\pi^2\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}} \\
\frac{\partial \mu}{\partial \overline{p}} &= I^1\frac{\partial \underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\frac{\partial \underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^1\pi^1\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\pi^2\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}} \\
\frac{\partial \sigma}{\partial \underline{p}} &= \frac{\left[I^1\pi^1(1-\pi^1)\overline{x}^1\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\pi^2(1-\pi^2)\overline{x}^2\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right]}{\sqrt{\left[I^1\pi^1(1-\pi^1)\left(\overline{x}^1\right)^2\left(\underline{p},\overline{p}\right)+I^2\pi^2(1-\pi^2)\left(\overline{x}^2\right)^2\left(\underline{p},\overline{p}\right)\right]}} \\
\frac{\partial \sigma}{\partial \overline{p}} &= \frac{\left[I^1\pi^1(1-\pi^1)\overline{x}^1\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\pi^2(1-\pi^2)\overline{x}^2\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\right]}{\sqrt{\left[I^1\pi^1(1-\pi^1)\left(\overline{x}^1\right)^2\left(\underline{p},\overline{p}\right)+I^2\pi^2(1-\pi^2)\left(\overline{x}^2\right)^2\left(\underline{p},\overline{p}\right)\right]}}
\end{aligned}$$

we obtain after some rearrangements :

$$
\begin{aligned}
& \left(\underline{p}-\underline{c}\right)\left[I^1\frac{\partial \underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\frac{\partial \underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right]+\left(\overline{p}-\underline{c}\right)\left[I^1\pi^1\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\pi^2\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right] \\
= \quad & -\left[I^1\underline{x}^1\left(\underline{p},\overline{p}\right)+I^2\underline{x}^2\left(\underline{p},\overline{p}\right)\right] \\
& +\overline{c}\left[\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}\left(1-F(t)\right)dt+\frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\frac{\left[I^1\pi^1(1-\pi^1)\overline{x}^1\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\pi^2(1-\pi^2)\overline{x}^2\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right]}{\sqrt{\left[I^1\pi^1(1-\pi^1)\left(\overline{x}^1\right)^2\left(\underline{p},\overline{p}\right)+I^2\pi^2(1-\pi^2)\left(\overline{x}^2\right)^2\left(\underline{p},\overline{p}\right)\right]}}
\end{aligned}
$$

and

$$
\begin{aligned}
& \left(\underline{p}-\underline{c}\right)\left[I^1\frac{\partial \underline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\frac{\partial \underline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\right]+\left(\overline{p}-\underline{c}\right)\left[I^1\pi^1\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}+I^2\pi^2\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\right] \\
= \quad & -\left[I^1\pi^1\overline{x}^1\left(\underline{p},\overline{p}\right)+I^2\pi^2\overline{x}^2\left(\underline{p},\overline{p}\right)\right] \\
& +\overline{c}\left[\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}\left(1-F(t)\right)dt+\frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\frac{\left[I^1\pi^1(1-\pi^1)\overline{x}^1\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^1\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}+I^2\pi^2(1-\pi^2)\overline{x}^2\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}^2\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\right]}{\sqrt{\left[I^1\pi^1(1-\pi^1)\left(\overline{x}^1\right)^2\left(\underline{p},\overline{p}\right)+I^2\pi^2(1-\pi^2)\left(\overline{x}^2\right)^2\left(\underline{p},\overline{p}\right)\right]}}
\end{aligned}
$$

or equivalently :

$$
\begin{aligned}
& \left(\underline{p}-\underline{c}\right)\left[\alpha_1 e^1+\alpha_2 e^2\right]+\left(\overline{p}-\underline{c}\right)\left[\beta_1\alpha_1\pi^1 f^1+\beta_2\alpha_2\pi^2 f^2\right] \\
= \quad & -1+\overline{c}\left[\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}\left(1-F(t)\right)dt+\frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\left[\frac{I^2\alpha_1\left[\pi^1(1-\pi^1)\left(\beta_1\right)^2 f^1+\pi^2\left(\beta_2\right)^2(1-\pi^2)\gamma^2 f^2\right]}{\sqrt{I^1\left(I^2\right)^2\pi^1(1-\pi^1)\left(\beta_1\right)^2+I^2\left(I^1\right)^2\pi^2\left(\beta_2\right)^2(1-\pi^2)\gamma^2}}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
& \left(\underline{p}-\underline{c}\right)\left[\alpha_1 g^1+\alpha_2 g^2\right]+\left(\overline{p}-\underline{c}\right)\left[\beta_1\alpha_1\pi^1 h^1+\beta_2\alpha_2\pi^2 h^2\right] \\
= \quad & -\frac{1+\gamma}{\beta^1\pi^1+\gamma\beta^2\pi^2}+ \\
& \overline{c}\left[\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}\left(1-F(t)\right)dt+\frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\left[\frac{I^2\alpha_1\left[\pi^1(1-\pi^1)\left(\beta_1\right)^2 h^1+\pi^2\left(\beta_2\right)^2(1-\pi^2)\gamma^2 h^2\right]}{\sqrt{I^1\left(I^2\right)^2\pi^1(1-\pi^1)\left(\beta_1\right)^2+I^2\left(I^1\right)^2\pi^2\left(\beta_2\right)^2(1-\pi^2)\gamma^2}}\right]
\end{aligned}
$$

where for $i=1,2$:

$$
\begin{aligned}
\alpha_i &\equiv \frac{I^i \underline{x}^i\left(\underline{p},\overline{p}\right)}{I^1\underline{x}^1\left(\underline{p},\overline{p}\right)+I^2\underline{x}^2\left(\underline{p},\overline{p}\right)},\ \gamma \equiv \frac{\alpha_2}{\alpha_1}\ ,\ \beta_i \equiv \frac{\overline{x}^i\left(\underline{p},\overline{p}\right)}{\underline{x}^i\left(\underline{p},\overline{p}\right)},\ e^i \equiv \frac{\partial \underline{x}^i\left(\underline{p},\overline{p}\right)}{\partial \underline{p}}\frac{\underline{p}}{\underline{x}^i\left(\underline{p},\overline{p}\right)} \\
f^i &\equiv \frac{\partial \overline{x}^i\left(\underline{p},\overline{p}\right)}{\underline{p}}\frac{\underline{p}}{\overline{x}^i\left(\underline{p},\overline{p}\right)}\partial\overline{p},\ g^i \equiv \frac{\partial \underline{x}^i\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\frac{\overline{p}}{\underline{x}^i\left(\underline{p},\overline{p}\right)} \text{and } h^i \equiv \frac{\partial \overline{x}^i\left(\underline{p},\overline{p}\right)}{\partial \overline{p}}\frac{\overline{p}}{\overline{x}^i\left(\underline{p},\overline{p}\right)}
\end{aligned}
$$

These two fundamental equations describe in reduced form the optimal pricing policy of the monopoly. They express the differential between the two prices and the unit cost $\underline{c}$ for reserving in advance i.e. at time $t-\Delta$ some transportation capacity. In matrix form, they are as follows :

$$
A\begin{pmatrix} \underline{p}-\underline{c} \\ \overline{p}-\underline{c} \end{pmatrix} = \begin{pmatrix} \underline{B} \\ \overline{B} \end{pmatrix}
$$

where :

$$
A \equiv \begin{pmatrix} \alpha_1 e^1 + \alpha_2 e^2 & \beta_1\alpha_1\pi^1 f^1 + \beta_2\alpha_2\pi^2 f^2 \\ \alpha_1 g^1 + \alpha_2 g^2 & \beta_1\alpha_1\pi^1 h^1 + \beta_2\alpha_2\pi^2 h^2 \end{pmatrix}
$$

and :

$$
\underline{B} = \left(-1 + \overline{c}\left[\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))\,dt + \frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\left[\frac{I^2\alpha_1\left[\pi^1(1-\pi^1)(\beta_1)^2 f^1+\pi^2(\beta_2)^2(1-\pi^2)\gamma^2 f^2\right]}{\sqrt{I^1(I^2)^2\pi^1(1-\pi^1)(\beta_1)^2+I^2(I^1)^2\pi^2(\beta_2)^2(1-\pi^2)\gamma^2}}\right]\ \right)
$$

$$
\begin{aligned}
\overline{B} &= -\frac{1+\gamma}{\beta^1\pi^1+\gamma\beta^2\pi^2} \\
&\quad +\overline{c}\left[\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))\,dt + \frac{\underline{c}}{\overline{c}}\Phi(1-\frac{\underline{c}}{\overline{c}})\right]\left[\frac{I^2\alpha_1\left[\pi^1(1-\pi^1)\left(\beta_1\right)^2 h^1 + \pi^2\left(\beta_2\right)^2(1-\pi^2)\gamma^2 h^2\right]}{\sqrt{I^1\left(I^2\right)^2\pi^1(1-\pi^1)\left(\beta_1\right)^2 + I^2\left(I^1\right)^2\pi^2\left(\beta_2\right)^2(1-\pi^2)\gamma^2}}\right]
\end{aligned}
$$

A closed form solution as well as an interpretation of these equations is not immediate in this general two class case. Note however that as formulated, the system is ready for calibration as the market shares and elasticity parameters playing a major role in the determination of the prices are identified.

· The parameters α^1 and α^2 measure the respective *market shares* of the two groups of clients on the advance-purchase markets.

· The parameters $\beta^1$and β^2 measure the *respective magnitude* of the advance-purchase and spot sales for the two groups of clients.

· The different *direct and cross price elasticities* : e^1, e^2, f^1, f^2, g^1, g^2, $h^1$and h^2.

· The *respective volatilities* of the two groups of clients measured by the parameters π^1 et π^2. The volatility is measured here through the quantity $\mid \pi^i - \frac{1}{2} \mid$.

It would be interesting to look at a calibrated model based on a reasonable set of values for the parameters in order to evaluate differences in prices proposed by european pipelin operatord for different terms (month, year) of transportation capacity. Note however that these parameters are not independent as we should anticipate for basic consumer or producer theory. Further, as in the case of Ramsey-Boiteux prices, they don't represent, strictly speaking, a solution as the parameters depend themselves upon the pair of prices. In the next section, we continue this examination in the simplest one class case i.e. with an homogeneous population of clients.

3.3 The Optimal Prices in the Homogeneous Case

In order to highlight the main ingredients shaping the derivation of the optimal pricing policy $\left(\underline{p}^*, \overline{p}^*\right)$, we examine the fundamental equations in the case where the population of clients consists of one, instead of two, homogeneous group of I clients. Each client is characterized by the triple $\left(\pi, \underline{x}\left(\underline{p}, \overline{p}\right), \overline{x}\left(\underline{p}, \overline{p}\right)\right)$ and we denote simply by e, f, g, h the four direct elasticities introduced in the preceding subsection. To derive the optimal prices in a more transparent way, we first look back to the total cost function of the monopoly. Remember that it results from an optimization process through the adequate choice of the capacity reservation z. If the monopoly knows that the aggregate stochastic demand $\widetilde{x}$ is described by a Gaussian random variable $N(\mu, \sigma)$, the optimal choice of z results from the following minimization problem :

$$\underset{z}{Min}\underline{c}z + \overline{c}\left[\int_z^{+\infty} (\widetilde{x} - z)\, \frac{e^{-\frac{\left(\widetilde{x}-\mu\right)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} d\widetilde{x}\right]$$

As already demonstrated, the total cost can be expressed alternatively as :

$$\underline{c}z + \overline{c}\sigma \int_{\frac{z-\mu}{\sigma}}^{+\infty} (1 - F(t))dt$$

The optimal capacity reservation satisfies :

$$\underline{c} = \overline{c}(1 - F(\frac{z-\mu}{\sigma})) \text{ i.e. } z^* = \mu + \sigma\Phi(1 - \frac{\underline{c}}{\overline{c}})$$

The cost function of the monopoly is therefore :

$$C(\mu, \sigma) = \underline{c}\mu + \left(\underline{c}\Phi(1 - \frac{\underline{c}}{\overline{c}}) + \overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty} (1 - F(t))dt\right)\sigma$$

This formulation of the cost function emphasizes the contribution of the aggregate first and second moments of the total gas consumption at time t. We note that it is linear with respect to both μ and σ. The constant marginal costs of both variables are respectively :

$$\frac{\partial C(\mu,\sigma)}{\partial \mu}=\underline{c} \text{ and } \frac{\partial C(\mu,\sigma)}{\partial \sigma}=\underline{c}\Phi(1-\frac{\underline{c}}{\overline{c}})+\overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))dt$$

To trace back the marginal cost of the individual demands $\left(\underline{x}\left(\underline{p},\overline{p}\right),\overline{x}\left(\underline{p},\overline{p}\right)\right)$, we will substitute the following expressions of μ and σ :

$$\mu=I\left(\underline{x}\left(\underline{p},\overline{p}\right)+\pi\overline{x}\left(\underline{p},\overline{p}\right)\right)$$

and

$$\sigma=\overline{x}\left(\underline{p},\overline{p}\right)\sqrt{I\pi\left(1-\pi\right)}$$

The first order conditions describing the optimal policy $\left(\underline{p}^{*},\overline{p}^{*}\right)$ are :

$$I\underline{x}\left(\underline{p},\overline{p}\right)+I\underline{p}\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)+I\pi\overline{p}\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)=\frac{\partial C(\mu,\sigma)}{\partial \mu}\frac{\partial \mu}{\partial \underline{p}}+\frac{\partial C(\mu,\sigma)}{\partial \sigma}\frac{\partial \sigma}{\partial \underline{p}}$$

$$I\underline{p}\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)+I\pi\overline{x}\left(\underline{p},\overline{p}\right)+I\pi\overline{p}\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)=\frac{\partial C(\mu,\sigma)}{\partial \mu}\frac{\partial \mu}{\partial \overline{p}}+\frac{\partial C(\mu,\sigma)}{\partial \sigma}\frac{\partial \sigma}{\partial \overline{p}}$$

After substitution, we obtain :

$$\begin{aligned}&\left(\underline{p}-\underline{c}\right)\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)+\pi\left(\overline{p}-\underline{c}-\left(\underline{c}\Phi(1-\frac{\underline{c}}{\overline{c}})+\overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))dt\right)\sqrt{\frac{(1-\pi)}{\pi I}}\right)\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\\ =\;& -\underline{x}\left(\underline{p},\overline{p}\right)\end{aligned}$$

$$\begin{aligned}&\left(\underline{p}-\underline{c}\right)\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)+\pi\left(\overline{p}-\underline{c}-\left(\underline{c}\Phi(1-\frac{\underline{c}}{\overline{c}})+\overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))dt\right)\sqrt{\frac{(1-\pi)}{\pi I}}\right)\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)\\ =\;& -\pi\overline{x}\left(\underline{p},\overline{p}\right)\end{aligned}$$

For the sake of notational simplification, let us denote respectively by $\underline{q}$ and $\overline{q}$ the expressions $\underline{p}-\underline{c}$ and $\overline{p}-\left(\underline{c}+\left(\underline{c}\Phi(1-\frac{\underline{c}}{\overline{c}})+\overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))dt\right)\sqrt{\frac{(1-\pi)}{\pi I}}\right)$. In matrix form, the two first order conditions simplify to :

$$\begin{pmatrix}\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right) & \pi\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\\ \frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right) & \pi\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)\end{pmatrix}\begin{pmatrix}\underline{q}\\ \overline{q}\end{pmatrix}=\begin{pmatrix}-\underline{x}\left(\underline{p},\overline{p}\right)\\ -\pi\overline{x}\left(\underline{p},\overline{p}\right)\end{pmatrix}$$

We obtain :

$$\underline{q} = \frac{-\pi\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)\underline{x}\left(\underline{p},\overline{p}\right)+\pi^2\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\overline{x}\left(\underline{p},\overline{p}\right)}{\pi\left(\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)-\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\right)}$$

and

$$\overline{q} = \frac{\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)\underline{x}\left(\underline{p},\overline{p}\right)-\pi\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\overline{x}\left(\underline{p},\overline{p}\right)}{\pi\left(\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)-\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p},\overline{p}\right)\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p},\overline{p}\right)\right)}$$

or more compactly, in terms of direct and cross elasticities and market shares :

$$\frac{\underline{p}-\underline{c}}{\underline{p}} = \frac{-h+\pi g\frac{\overline{R}}{\underline{R}}}{eh-fg}$$

and

$$\frac{\overline{p}-\left(\underline{c}+\left(\underline{c}\Phi(1-\frac{\underline{c}}{\overline{c}})+\overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty}(1-F(t))dt\right)\sqrt{\frac{(1-\pi)}{\pi I}}\right)}{\overline{p}} = \frac{f-\pi e\frac{\overline{R}}{\underline{R}}}{\pi\left(eh-fg\right)}$$

where :

$$\overline{R} \equiv \overline{p}\overline{x}\left(\underline{p},\overline{p}\right) \text{ and } \underline{R} \equiv \underline{p}\underline{x}\left(\underline{p},\overline{p}\right)$$

These two formulas provide the optimal relative markups of the monopoly on the two markets. They point out immediately the respective roles of the four elasticities e, f, g and h as well as the relative value of transactions $\frac{\overline{R}}{\underline{R}}$ on both markets. The knowledge of the parameters could follow from an econometric analysis of the demand for natural gas. While doing so, it should be mentionned that there are some structural implications of the demand theory that we are using that should be pointed out. In the homogeneous case considered in this subsection, these implications are easy to derive. We limit ourselves to the case where all the clients are households purchasing gas on both markets. The two first order conditions describing the optimal interior purchase plan $\left(\underline{x}^*\left(\underline{p},\overline{p}\right),\overline{x}^*\left(\underline{p},\overline{p}\right)\right)$ of a generic household are :

$$\pi v'(\underline{x}+\overline{x})+(1-\pi)v'(\underline{x}+\overline{\omega})=\underline{p}$$

$$v'(\underline{x}+\overline{x})=\overline{p}$$

Denoting by ϕ the inverse of v', we obtain :

$$\underline{x}=\phi\left(\frac{\underline{p}-\pi\overline{p}}{1-\pi}\right)-\overline{\omega}$$

$$\overline{x} = \phi\left(\overline{p}\right) - \phi\left(\frac{\underline{p} - \pi\overline{p}}{1-\pi}\right) + \overline{\omega}$$

from which we deduce :

$$\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right) = \frac{1}{1-\pi}\phi'\left(\frac{\underline{p} - \pi\overline{p}}{1-\pi}\right) < 0$$

$$\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right) = \frac{-\pi}{1-\pi}\phi'\left(\frac{\underline{p} - \pi\overline{p}}{1-\pi}\right) > 0$$

$$\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right) = \frac{-1}{1-\pi}\phi'\left(\frac{\underline{p} - \pi\overline{p}}{1-\pi}\right) > 0$$

$$\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right) = \frac{\pi}{1-\pi}\phi'\left(\frac{\underline{p} - \pi\overline{p}}{1-\pi}\right) + \phi'\left(\overline{p}\right) < 0$$

The sign inequalities follow from the fact that ϕ is decreasing. We note that the quantities of gas bought on the advance-purchase and spot markets are substitutes[18]. Moreover, up to the normalization by π, we obtain the Slutsky 's symmetry conditions :

$$\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right) = \pi\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right)$$

and

$$\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right)\frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right) - \frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right)\frac{\partial \overline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right) = \frac{1}{1-\pi}\phi'\left(\frac{\underline{p} - \pi\overline{p}}{1-\pi}\right)\phi'\left(\overline{p}\right) > 0$$

Finally, we also obtain[19] :

$$\frac{\partial \underline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right) = -\pi\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right)$$

$$\frac{\partial \underline{x}}{\partial \underline{p}}\left(\underline{p}, \overline{p}\right) < \frac{\partial \overline{x}}{\partial \overline{p}}\left(\underline{p}, \overline{p}\right)$$

In terms of elasticities, these restrictions translate as follow :

$$f = \pi g\frac{\overline{R}}{\underline{R}}$$

[18]This, together with the inequality below, implies that both markups are positive : the monopoly does not underprice strategically one of the product to gain more on the other.

[19]The inequality does not hold in general and is a peculiar implication of our specification of utility.

$$f = -\pi e \frac{\overline{p}}{\underline{p}}$$

and

$$e < h \frac{\overline{R}}{\underline{R}} \left(\frac{\underline{p}}{\overline{p}}\right)^2$$

These behavioral restrictions should be of course tested against empirical data. For the time being, we just notice that irrespective of whether they are true or not, the optimal pricing policy is characterized by the pair of equations derived above. The fundamental question to be answered now is to understand under which circumstances the ratio $\frac{\overline{p}}{\underline{p}}$ of prices is larger or smaller than the ratio of access costs $\frac{\overline{c}}{\underline{c}}$. While, as already noted, there is a positive markup on both markets, an inequality like $\frac{\overline{p}}{\underline{p}} > \frac{\overline{c}}{\underline{c}}$ could be interpreted as a biais towards the spot market. Finally, when the right hand side of the above sytem of equations is equal to 0, the corresponding prices

$$\underline{p} = \underline{c} \text{ and } \overline{p} = \underline{c} + \left(\underline{c}\Phi(1 - \frac{\underline{c}}{\overline{c}}) + \overline{c}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty} (1 - F(t))dt\right)\sqrt{\frac{(1-\pi)}{\pi I}}$$

are the optimal prices from the perspective of a regulator maximizing social surplus. If the unit costs were themselves true marginal costs derived from technological constraints, then the comparison between $\frac{\overline{p}}{\underline{p}}$ and $\frac{\overline{c}}{\underline{c}}$ would also be of some interest. In the case where $\pi = \frac{1}{2}$ and $I = 1$, we obtain :

$$\frac{\overline{p}}{\underline{p}} = 1 + \Phi(1 - \frac{\underline{c}}{\overline{c}}) + \frac{\overline{c}}{\underline{c}}\int_{\Phi(1-\frac{\underline{c}}{\overline{c}})}^{+\infty} (1 - F(t))dt$$

Table 3 below reproduces a sample of ratios.

$\frac{\overline{c}}{\underline{c}}$	$\frac{3}{2}$	2	5	10
$\frac{\overline{p}}{\underline{p}}$	1.5454	1.7979	2.3998	2.7550

Table 3

We may wonder for which value(s) of $\frac{\overline{c}}{\underline{c}}$, the ratio $\frac{\overline{p}}{\underline{p}}/\frac{\overline{c}}{\underline{c}}$ reaches its highest value. This amounts to look at the maximum of the function φ defined as follows :

$$\varphi(x) \equiv \frac{1}{x} + \frac{1}{x}\Phi(1 - \frac{1}{x}) + \int_{\Phi(1-\frac{1}{x})}^{+\infty} (1 - F(t))dt$$

Since :

$$\begin{aligned}
\varphi'(x) &= -\frac{1}{x^2} - \frac{1}{x^2}\Phi(1 - \frac{1}{x}) + \frac{1}{x^3}\Phi'(1 - \frac{1}{x}) + \frac{1}{x^3}(1 - F(\Phi(1 - \frac{1}{x}))) \\
&= -\frac{1}{x^2} - \frac{1}{x^2}\Phi(1 - \frac{1}{x})
\end{aligned}$$

we deduce that :

$$\varphi'(x) = 0 \text{ iff } \Phi(1 - \frac{1}{x}) = -1 \text{ which implies } x = 1.19$$

It can be verified that in such case, the ratio $\frac{\overline{p}}{\underline{p}}$ exceeds the ratio $\frac{\overline{c}}{\underline{c}}$ by 8.4%.

4 Concluding Remarks

In this paper, we have developed a simple model to predict the pricing behavior of a firm selling gas in a regulated market environment. To reflect the current situation, as experimented in France but also in some other countries, we have assumed that the access prices to the pipelines network were decided by the regulator. The term network should be interpreted in a broad sense including : entry/exit capacities and intermediate transportation ones. Of course, the effectiveness of the access to these facilities is essential in an opening market situation. At a given point in time, these facilities cannot be adjusted and, therefore, there is a maximal supply which acts as a constraint. European gas pipeline companies offer a wide variety of contractual arrangements to sell these capacities. Operators have the possibility to make reservation for a year, a month or a day. In each case, the contract specifies the daily amount of gas which is permitted to enter, circulate or exit. Some variability is allowed but it is rather limited. For each of them, the allocation rules describe how many days ahead these reservations must be introduced and, at any time t, to whom the available residual capacity is assigned during the all period preceding the termination of the process. Besides offering firm capacities, most of European gas pipeline companies also offer interruptible capacities and sometimes impose some clauses of restoration when the capacity share of an operator exceeds 20% of the total. Any operator is constrained to balance its gas flows; any major departure from that balancing constraint results in penalties. Finally, a kind of secondary market to exchange capacities is organized but, while transparent in terms of posting all the relevant informations, it is not yet very effective. In practice the service consists only in an electronic bulletin board that provide anonymous listing of supplies and demands for transport capacity.

In this paper, we have concentrated our attention on the case of a shipper selling gas to a captive population of clients. To handle the supply service, the shipper has to subscribe to the network operator a transportation capacity adapted to the global need of its portfolio. Therefore, the determining of the relevant booking strategy becomes crucial. We have assumed that any client does not know in advance for sure what will be its gas consumption: many uncertain events may increase or decrease its consumption at any time during the contractual period. Some clients may have very predictable consumption while others may display a significant volatility. This second class of clients is really problematic from the perspective of the shipper trying to plan its optimal capacity reservations. To simplify the analysis, we have assumed that the shipper could either make annual reservations or daily reservations. He will propose to its clients two kinds of linear tariffs : a first tariff applies to the gas purchased in advance and a second tariff applies to the gas which is purchased during the considered period.

This model paves the way for a better understanding of the reactions of some of the economic agents acting on the market for natural gas. This little step has concentrated on the relationship between the prices to access to the transportation network (under the control of the regulator) and the prices by a monopoly delivering natural to a (captive) population of residential and (or) industrial customers. Obviously, it can be generalized in many directions.

One first direction could consist in removing the monopoly assumption and replace it by a true model of imperfect competition between a limited number of firms. This game theoretical setting raises a number of new and challenging questions as the oligopolists compete on several markets and may face problems of capacities. From that perspective, we can see the contribution of this paper as the determination of the best response of an agent in this competitive world.

A second direction of research would consist in an examination of a problem analogous to the one examined in this paper but where the two contractual instruments would be replaced by two other ones. A possible suggestion along these lines could be to consider two two-part tariffs: one with a large fixed component and a small variable component and another one with a smaller fixed part but a larger variable one. This would be consistent with the current practice on the retail market : three different two-part tariffs are offered together (for clients with large consumption) with a three-part tariff which includes a third component based on an annual capacity reservation. The per unit price of gas of a client varies with the contractual arrangement that he has selected and its real consumption. In the context of uncertainty considered here, the clients will partition themselvelves into several groups depending upon the expected volumes of consumption but also volatility parameters. It seems worthwhile to look at the optimal pricing behavior of a monopoly in this environment.

A third promising direction of research would consist in reintroducing the gas markets at the entry and exchange points. We could assume that any firm selling gas can either buy gas through long term contracts with gas producers or buy (or sell) on some "marché de gros". There are only three such markets in Europe with limited scope in terms of volumes but also financial instruments but this picture may change rapidly. In such richer environment, the joint impact of the cost of gas and the cost of capacity reservation would become more intricated.

5 References

Allaz, B. (1992) "Oligopoly, Uncertainty and Strategic Forward markets and Efficiency", *International Journal of Industrial Organization*, 10, 297-308.

Allaz, B. and J.L. Vila (1993) "Cournot Competition, Forward markets and Efficiency", *Journal of Economic Theory*, 59, 1-16.

Baker, P., Bluendell, R. and J. Micklewright (1989) "Modelling Household Energy Expenditures Using Micro-Data", *Economic Journal*, 99, 720-738.

Balestra,P. and M. Nerlove (1966) "Pooling Cross Section and Time Series data in the Estimation of a Dynamic Model : The Demand for natural Gas", *Econometrica*, 34, 585-612.

Baron, D. (1971) "Demand Uncertainty and Imperfect Competition", *International Economic Review*, 12, 196-208.

Beierlein, J.G., Dunn, J.W. and J.C. McConnon, Jr (1981) "The Demand for Electricity and Natural gas in the Northeastern United States" *Review of Economics and Statistics*, 63, 403-408.

Boiteux, M. (1951) "La Tarification au Coût Marginal et les Demandes Aléatoires", *Cahiers du Séminaire d'Econométrie*, 1, 56-69.

Carlton, D.W. (1978) "Market Behavior with Demand Uncertainty and Price Inflexibility", *American Economic Review*, 68, 571-587.

Carlton, D.W. (1979) "Contracts, Price Rigidity and Market Equilibrium", *Journal of Political Economy*, 87, 1034-1062.

Carlton, D.W. (1991) "The Theory of Market Allocation and its Implications for Marketing and Industrial Organization", *Journal of Law and Economics*, 34, 231-262.

Commission de Régulation de l'Energie (2006) *La Régulation du Marché du Gaz Naturel*, Rapport d'Activité.

Dana, J.D. (1998) "Advance-Purchase Discounts and Price Descrimination in Competitive Markets", *Journal of Political Economy*, 106, 395-422.

Dana, J.D. (1999) "Equilibrium Price Dispersion under Demand Uncertainty : the Roles of Costly Capacity and Market Structure", *Rand Journal of Economics*, 30, 632-660.

Dana, J.D. (2001) "Monopoly Price Dispersion under Demand Uncertainty", *International Economic Review*, 42, .

Deneckere, R. and J. Peck (1995) "Competition over Price and Service Rate when Demand is Stochastic : a Strategic Analysis", *Rand Journal of Economics*, 26, 148-162.

Doane, M.J., McAfee, P.R. and M.A. Williams (2004) "Evaluating and Enhancing Competition in the Interstate Natural Gas Transportation Industry", *Natural Resources Journal*, 44,761-808

Drèze, J.H. (1964) "Some Postwar Contributions of French Economists to Theory and Public

Policy", *American Economic Review*, 54, 1-64.

Eden, B. (1990) "Marginal Cost pricing when Spot Markets are Complete", *Journal of Political Economy*, 1293-1306.

Gale, I.L. and T.J. Holmes (1992) "The Efficiency of Advance Purchase Discounts in the Presence of Aggregate Demand Uncertainty", *International Journal of Industrial Organization*, 10, 413-437.

Gale, I.L. and T.J. Holmes (1993) "Advance Purchase Discounts and Monopoly Allocation of Capacity", *American Economic Review*, 83, 135-146.

Green, J. and H. Polemarchakis (1976) "A Brief Note on the Efficiency of Equilibria with Costly Transactions", *Review of Economic Studies*, 43, 537-542.

Green, J. and E. Sheshinski (1975) "Competitive Inefficiencies in the Presence of Constrained Transactions", *Journal of Economic Theory*, 10, 343-357 .

Kolm, S.C. (1970) "Service Optimal d'une Demande Variable et Prix de l'Incertitude", *Revue Economique*, 21, 1970, 243-271.

Leth-Petersen, S. (2002) "Micro Econometric Modelling of Household Energy Use : Testing for Dependence between Demand for Electricity and Natural Gas", *Energy Journal*, 23, 57-84.

Mahenc, P. and F. Salanié (2004) "Softening Competition through Forward Trading", *Journal of Economic Theory*, 116, 282-293.

McAfee, P.R. and P.J. Reny (2006) "The Role of Excess Capacity in Determining Market Power in Natural Gas Transportation Markets", Caltech, Mimeo.

McAfee, P.R., Doane, M.J., Nayyar, A. and M.A. Williams " (2006) "Interpretating Concentration Indices in the secundary Market for natural gas Transporation : The Implication of Pipeline Residual Rights", Caltech, Mimeo.

Murphy, F. and Y. Smeers (2005) "Forward Markets may not Decrease Market Power when Capacities are Endogenous", CORE, Mimeo.

Prescott, E.C. (1975) "Efficiency and the Natural rate", *Journal of Political Economy*, 83, 1229-1236.

Raineri, R.B. and A.T. Kuflik (2003) "Secondary Market and Future Market for the Problem of Provision of Gas Pipeline Transportation Capacity", *Energy Journal*, 24, 23-47.

Seeto, D.Q., Woo, C.K. and I. Horowitz (1997) "Time-of Use rates vs Hopkinson Tariffs Redux: An analysis of the Choice of Rate Structures in a Regulated Electricity Distribution Company", *Energy Economics*, 19, 169-185.

Spulber, D.F. (1992a) "Capacity-Contingent Nonlinear Pricing by Regulated Firms", *Journal of Regulatory Economics*, 4, 299-319.

Spulber, D.F. (1992b) "Optimal Nonlinear Pricing and Contingent Contracts", *International*

Economic Review, 33, 747-772.

Talluri, K.T. and G.J. Van Ryzin (2004) *The Theory and Practice of Revenue Management*, Kluwer Academic Publishers, London.

Tirole, J. (1988) *The Theory of Industrial Organization*, The MIT Press, Cambridge.

Veall, M.R. (1983) "Industrial Electricity Demand and the Hopkinson rate : An Application of the Extreme value Distribution", *Bell Journal of Economics*, 14, 427-440.

Wilson, C.A. (1988) "On the Optimal Pricing of a Monopolist", Journal of Political Economy, 96, 164-176.

Wilson, R. (1989) "Ramsey pricing of Priority Service", *Journal of Regulatory Economics*, 1, 189-202.

Woo, C.K., Horii, B. and I. Horowitz (2002) "The Hopkinson Tariff Alternative to TOU Rates in the Israel Electric Corporation", *Managerial and Decision Economics*, 23, 9-19.

www.gazdefrance-reseau-transport.com (Web site of "Gaz de France Reseau-Transport")

www.cre.fr (Web site of the "Commission de régulation de l'énergie")

www.ingaa.org (Web site of the "Interstate Natural Gas Association of America")