Gamson's Law and Hedonic Games

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Abstract

This note adds one celebrated coalition formation game due to Gamson (1961) in the list of applications of the theory of hedonic games explored by Banerjee, Konishi and Sömnez (2001) and Bogomolnaia and Jackson (2002). We apply their results to study the original Gamson game and offer extensions both to a multi-dimensional characteristics space and to an infinite number of players.

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1 Motivation

In his seminal work Gamson (1961a) examined a coalition formation game where each player in the "winning" coalition receives the share of the total coalition resources proportional to her own contribution. This celebrated coalition formation game, often referred as the Gamson's game or Gamson's law, has generated a tremendous interest in various areas of social sciences. Gamson's research was motivated by early work of Mills (1953) and Caplow (1956) who analyzed the structure of coalitions formed in three-person families and pointed out the presence of the weakest player in the equilibrium coalition. These theoretical considerations have produced an important volume of research in *social psychology*, which made coalitional experiments[1] an important part of the standard repertory in this field (Bonacich, Grusky and Peyrot (1985), Chertkoff (1967,1971), Gamson (1964), Miller and Crandall (1980), Murnighan (1978), Nail and Cole (1985). A great deal has been learned about choices of coalition partners and the way a division of coalition winnings is affected by variations in game rules and players' attributes.

As noted and illustrated by Caplow (1968), the theory of coalitions in triads could be used to analyze conflict and cooperation in many social and political environments, including *international relations*. Hsiung (1987) offers coalitional analysis of the strategic triangle of the United States, China and the Soviet Union during the Cold War era of 1950-1985. Along similar lines, Caplow (1989) discusses the failure of peace planning in 1815, 1919 and 1945, while Zagare (1984) examines the outcome of the Geneva conference on Vietnam in 1954.

Together with the minimal size principle introduced by Riker (1962), Gamson's law is also among the most popular hypothesis in *political science* and is one of the prominent landmarks in empirical models of the allocation of cabinet portfolios in coalition governments.[2] Laver (1998) points out that "Gamson's law boasts one of the highest non-trivial R-squared figures in political science". One of the great appeals of Gamson's law is its intuitive nature and the parsimony it offers, as it is independent of the game form underlying the legislative bargaining process. The latter feature of Gamson's continues to generate the interest in this field of political science as evidenced by recent contributions of Carroll, Cox and Pachon (2004) and Fréchette, Kagel and Morelli (2004).

The purpose of this brief note is to place the game-theoretical interpretation of Gamson's law in the context of hedonic games developed by Banerjee, Konishi and Sönmez (2001) (BKS - henceforth) (see also Bogomolnaia and Jackson (2002)). In this framework the payoff of

[1]Gamson (1961b) describes the results of an experiment designed to test his theory.

[2]See Brown and Franklin (1973), Brown and Freindreis (1980) and Warwick and Druckman (2001), Acemoglu, Egorov and Sonin (2006) on modified versions of Gamson's law in this context.

every player payoff depends only on the composition of the coalition she belongs to. BKS show that if players share common preferences over possible coalitions, there is a stable partition of of all players into disjoint coalitions. This so-called "top-coalition property" allows us to guarantee the existence and to characterize the equilibria of the Gamson game. We then generalize the original Gamson game and examine the case where each player is characterized by a multiple array of characteristics. We consider two different types of a generalized Gamson game and show that under the single division method, the BKS weak top-coalition property still applies, thus, yielding the existence of a stable coalition. However, under the double division method, a stable coalition may fail to exist. Incidentally, we offer a concept of "congruent" game and demonstrate that a game admits a stable coalition if and only if it is congruent[3]. Finally, we consider the Gamson game with a continuum of players and derive a necessary and sufficient condition for the existence of a stable coalition and explore the political environment where one of the players' characteristics is determined by their ideological preferences.

2 Model

Let us now introduce the Gamson game. Consider a finite set of players $N = \{1, \dots, n\}$, where each player $i \in N$ is described by a positive parameter θ_i, interpreted as her endowment. Without loss of generality, we assume that the players are ordered according to the value of this parameter, i.e., $\theta_1 \geq \theta_2 \geq \dots \geq \theta_n$. For every $S \subset N$ denote by

$$\Theta(S) = \sum_{i \in S} \theta_i$$

the total endowment of coalition S and put $\Theta(N) = 1$.

A coalition $S \subset N$ is *winning* if the total endowment of S exceeds the total endowment of its complement $N \backslash S$. We denote by $\mathcal{W}$ the set of winning coalitions, i.e.,

$$\mathcal{W} = \{S \subset N | \Theta(S) > \frac{1}{2}\}.$$

If a winning coalition S forms, every member of S derives the payoff $U_i(S)$, which is a share of her endowment in S. A coalition which is not winning, has nothing to offer to its members. That is, for every $i \in S$

$$U_i(S) = \begin{cases} \frac{\theta_i}{\Theta(S)} & \text{if } S \in \mathcal{W} \\ 0 & \text{otherwise.} \end{cases}$$

[3]We also point out that the notions of congruency and top-coalition property in general do not coincide.

The Gamson game is hedonic in the sense that each player's strategic considerations take into account the payoff distribution after the coalition she belongs is formed, and thus, the payoff is the function of the composition of the formed coalition only. We now turn to the notion of stability:

Definition: A partition of the set N in two coalitions S and $N\backslash S$ is called *stable* if there is no coalition T such that

$$U_i(T) > U_i(S) \text{ for all } i \in T \cap S \text{ and } U_i(T) > U_i(N\backslash S) \text{ for all } i \in T \cap (N\backslash S)\,.$$

Obviously, if a partition is stable, one of its two elements, say S, is a winning coalition. For the above inequality to hold true, it must be the case that T is winning. Therefore, we deduce that $T \cap S \neq \varnothing$ and that the second part part of the test can be deleted. Thus, a partition is stable if and only if there is no winning coalition T with $U_i(T) > U_i(S)$ for all $i \in T \cap S$. We have the following observation:

Result 2.1: A winning coalition S is stable if and only if it is weakest (in terms of endowment) among all winning coalitions:

$$\Theta(S) = \min_{T \in \mathcal{W}} \Theta(T).$$

A simple direct proof of this result can be provided. However, it immediately follows from Corollary 2 in BKS.

It is worth pointing out that the hedonic framework of BKS allows for an extension of the Gamson's game. Consider a positive-valued function $H : \Re_+ \times \Re_+ \times 2^N \rightarrow \Re_{++}$, which is increasing in the first argument and is decreasing in the second. Put, for all $i \in S$

$$U_i(S) = \begin{cases} H(\theta_i, \Theta(S), S) & \text{if } S \in \mathcal{W} \\ 0 & \text{otherwise} \end{cases}$$

Since this generalized game satisfies the *weak top-coalition property*, the main theorem in BKS yields the existence of a stable coalition. Note, however, that this formulation allows for increasing returns to scale in terms of coalition size. Thus, a stable winning coalition is not necessarily minimal. Note that this framework covers the case where coalitions are not necessarily either winning or losing. In the case where a surplus is also distributed among small coalitions, a core stable partition may contain more than two elements.

In the case of a triad, an equilibrium coalition contains the weakest player:

Result 2.2: Let $n = 3$ and $\frac{1}{2} > \theta_1 > \theta_2 > \theta_3$, i.e., no singleton is a winning coalition. Then, the unique stable coalition is $\{2, 3\}$.

However, if $n \geq 4$, the stable coalition does not necessarily contain the weakest player. Consider the following example:

Example 2.3: Let $n = 4$ and $\theta_1 = 0.40$, $\theta_2 = 0.27$, $\theta_3 = 0.25$ and $\theta_4 = 0.08$. Then the unique stable coalition is $\{2, 3\}$, which does not contain player 4.

Finally, one can raise the question whether stable coalitions are *connected* or *consecutive* (Greenberg and Weber (1986)), where, to recall, a coalition S is connected if for every three different players, $i < j < k$, $i, k \in S$ implies that j belongs to S as well. Assume now that game is *strict*, i.e., there is no coalition S with $\Theta(S) = \frac{1}{2}$.We have

Result 2.4: Let $n = 4$ and $\frac{1}{2} > \theta_1 > \theta_2 > \theta_3 > \theta_4$. If a stable coalition S does not contain the weakest player, it is connected.

Proof: Suppose there is a stable coalition S that does not contain player 4 and is not connected. Then $S = \{1, 3\}$. Since S is stable and the game is strict, $\theta_2 + \theta_3 < \frac{1}{2}$. But this implies that $\theta_1 + \theta_4 > \frac{1}{2}$ and $\{1, 4\}$ is a stable coalition, a contradiction.□

However, Result 2.4 cannot be extended to a larger number of players:

Example 2.5: Let $n = 5$ and $\theta_1 = 0.40$, $\theta_2 = 0.30$, $\theta_3 = 0.12$, $\theta_4 = 0.11$ and $\theta_5 = 0.07$. Then the unique stable coalition is $\{1, 4\}$.

We leave open an interesting problem of the characterization of vectors $\theta \equiv (\theta_1, \theta_2, \ldots, \theta_n)$ for which stable coalitions exhibit connectedness or include the weakest player.

In the next section we examine a multi-dimensional variant of the Gamson game, where each player is characterized by a multiple array of characteristics.

3 Multi-characteristic extension

Note taht Gamson's law has been formulated for the weighted majority games and we could conceivably consider an extension to the case where, unlike in the previous section, each player $i \in N$ is identified by $K > 1$ characteristics, specifically, by the vector $\theta_i = \left(\theta_i^1, \theta_i^2, \ldots, \theta_i^K\right) \in \Re_+^K$. A winning coalition S must satisfy

$$\Theta^k(S) = \sum_{i \in S} \theta_i^k > \frac{\Theta^k(N)}{2} \text{ for every } k = 1, 2, \ldots, K.$$

This extension of weighted majority games is relevant to describe various voting environments. An important illustration is provided by the qualified majority provisions in the treaty of Nice for which $K = 3$ and every country i is described by the vector $\Theta_i = (1, \theta_i^2, \theta_i^3)$, where θ_i^2 is the country i's population, and θ_i^3 is an assigned voting weight. Thus, in forming a winning coalition one takes into account the number of countries it contains as well as their total population and aggregated voting weight (Felsenthal and Machover (2001)). The more recent qualified majority decision rules for the Council of Ministers of the EU that were included in the draft European Constitution proposed by the 2003 European Convention reduced the number of parameters to $K = 2$ (Felsenthal and Machover (2004)). Another example is provided by the amendment of Canada's Constitution Act of 1982 (Kilgour (1983)).

There are several ways to extend the division of surplus in the Gamson game. Let $\alpha = \left(\alpha^1, \alpha^2, \ldots, \alpha^K\right)$, be the vector of positive weights assigned to each of K characteristics. Assume that $\sum_{k=1}^{k=K} \alpha^k = 1$.

First, consider a *single division method*, where the payoff $U_i(S)$ of every member i of the winning coalition S is the share of her "weighted" endowment in S. As before, a coalition which is not winning, has nothing to offer to its members. Thus, for every $i \in S$ we have

$$U_i(S) = \begin{cases} \frac{\sum_{k=1}^{k=K} \alpha^k \theta_i^k}{\sum_{k=1}^{k=K} \alpha^k \Theta^k(S)} & \text{if } S \in \mathcal{W} \\ 0 & \text{otherwise.} \end{cases}$$

The extension of Result 2.1 to this setting is straightforward.

Result 3.1: Under the single division method, a winning coalition S is stable if it has the smallest total endowment among all winning coalitions:

$$\Theta(S) = \min_{T \in \mathcal{W}} \sum_{k=1}^{k=K} \alpha^k \Theta^k(S).$$

The situation is quite different if instead of weighting the different characteristics, the pie is divided according to the *double division* method. Namely, for each characteristic k the share of each player is determined according to the unidimensional Gamson rule. The total share is then calculated as a weighted average by utilizing the weights α^k for each characteristic k. Thus, if a winning coalition S forms, every member i of S derives the payoff $V_i(S)$, defined as follows:

$$V_i(S) = \begin{cases} \sum_{k=1}^{k=K} \alpha^k \frac{\theta_i^k}{\Theta^k(S)} & \text{if } S \in \mathcal{W} \\ 0 & \text{otherwise} \end{cases}$$

Interestingly enough, this game does not always admit an equilibrium.

Result 3.2: Under the double division method, a stable coalition may fail to exist.

Proof: Consider the following example with three players and three characteristics. Let $\theta_1 = (0.40, 0.35, 0.25)$, $\theta_2 = (0.25, 0.40, 0.35)$, $\theta_3 = (0.35, 0.25, 0.40)$ and $\alpha = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. The winning coalitions consists of all groups with at least two players. Straightforward calculations lead to :

$$\begin{aligned} V_1(\{1,2\}) &= \frac{1}{3}\left(\frac{40}{75}+\frac{25}{65}+\frac{35}{60}\right) \simeq 0.500\,43 > 0.499\,57 \simeq V_1(\{1,3\}) = \frac{1}{3}\left(\frac{40}{65}+\frac{25}{60}+\frac{35}{75}\right) \\ V_2(\{2,3\}) &= \frac{1}{3}\left(\frac{40}{75}+\frac{25}{65}+\frac{35}{60}\right) \simeq 0.500\,43 > 0.499\,57 \simeq V_2(\{1,2\}) = \frac{1}{3}\left(\frac{40}{65}+\frac{25}{60}+\frac{35}{75}\right) \\ V_3(\{1,3\}) &= \frac{1}{3}\left(\frac{40}{75}+\frac{25}{65}+\frac{35}{60}\right) \simeq 0.500\,43 > 0.499\,57 \simeq V_3(\{2,3\}) = \frac{1}{3}\left(\frac{40}{65}+\frac{25}{60}+\frac{35}{75}\right) \end{aligned}$$

A stable coalition does not exist due to the emergence of the Condorcet cycle among winning coalitions.□

The last example suggests that the existence of stable coalitions calls for some degree of congruence among a subset of players. The following notion of congruence which applies to any game where the economies of scale are fully described by a proper[4] set of winning coalitions $\mathcal{W}$ (i.e. at equilibrium, if any, a single coalition forms) slightly generalizes the notion of weak-top coalition property introduced by BKS. In fact, congruence is necessary and sufficient for stability.

Definition: A coalition $S \subseteq N$ is *congruent* if for all $i \in S$ and all $T \subseteq N$ the inequality $U_i(T) > U_i(S)$ implies that there exists $j \in T \cap S$ such that $U_j(S) \geq U_j(T)$. A coalition formation game is called *congruent* if there exists a congruent coalition.

Since we consider proper simple games, there is no loss of generality in considering partitions consisting of two coalitions,[5] S and $N \backslash S$, where one, say S, is winning. Like for the Gamson's game, we will refer to such coalition as a stable coalition. We have the following result:

Result 3.3: Let $\mathcal{W}$ be a proper family of winning coalitions. A coalition formation game is stable (i.e., admits a stable coalition) if and only if it is congruent.

[4]A simple game $(N, \mathcal{W})$ is proper if $S, T \in \mathcal{W}$ implies $S \cap T \neq \emptyset$.

[5]If the simple game is not proper, then a stable coalition structure may consists of more than two coalitions. A standard example of such assertion is provided by hedonic matching games when the population of at least six players is divided into two types.

Proof: Let the game be congruent. We show that the partition of the set N into coalitions S and $N\backslash S$, where S is congruent, is stable. Indeed, if not, then there is a winning coalition T such that $U_i(T) > U_i(S)$ for all $i \in T \cap S$, a contradiction to the congruency of S.

Now, let S be a stable coalition. We show that S is congruent. Indeed, let $i \in S$ and $T \subseteq N$ be such that $U_i(T) > U_i(S)$. Since $U_j(T) > U_j(N\backslash S) = 0$ for all $j \in T \cap (N\backslash S)$, we deduce that $U_j(S) \geq U_j(T)$ for some $j \in T \cap S$. Otherwise, we would contradict our assumption that S is stable.□

The notion of congruence generalizes the notion of weak top-coalition property in BKS as it does not imply that a congruent coalition S would be ranked at the top by all members of S. Consider the following example:

Example 3.4: Let $n = 4$ and

$$\begin{aligned}
V_1(\{1,2,4\}) &> V_1(\{1,2,3\}) > V_1(T) \text{ for every other } T \text{ that includes } 1, \\
V_2(\{2,3,4\}) &> V_2(\{1,2,3\}) > V_2(T) \text{ for every other } T \text{ that includes } 2, \\
V_3(\{1,3,4\}) &> V_3(\{1,2,3\}) > V_3(T) \text{ for every other } T \text{ that includes } 3, \\
V_4(\{1,2,4\}) &> V_4(\{1,2,3\}) > V_4(T) \text{ for every other } T \text{ that includes } 4.
\end{aligned}$$

Let $\mathcal{W}$ to be the set of majority coalitions. It is easy to see that $S = \{1,2,3\}$ is a congruent coalition. However, S does not satisfy the weak top-coalition property, as it represents only the second best choice for the members of S.

We conclude this section by the examination of another extension of the Gamson game framework which could also accommodate a wide spectrum of applications. In the current formulation, the outcome of the game is determined by the division of a surplus among a subset of players. As we already pointed out, in politics it may represent a cabinet portfolio allocation in coalition governments. However, governments must also choose their policy. Assuming that the policy space is represented by the M-dimensional Euclidean space and that each player i, is identified by her her weight θ^i as well as her ideal point $x^i \in \Re^M$. We then consider a game where a winning coalition does not only distribute a surplus of size B but also decides upon a policy. A natural outcome function for this second dimension is given by the weighted average of the ideal points of players (political parties) in the government coalition S:

$$x(S) = \frac{\sum_{i \in S} \theta^i x^i}{\sum_{i \in S} \theta^i}.$$

We assume that if a winning coalition S forms, every member i of S derives a payoff $U_i(S)$, which is a convex combination of the share of B she receives and the ideological cost represented by the distance $\| x^i - x(S) \|$ between her preferred ideology x^i and $x(S)$. As before, a coalition which is not winning, is powerless:

$$U_i(S) = \delta \frac{\theta_i B}{\theta(S)} - (1-\delta) \| x^i - x(S) \|,$$

where $\delta \in [0,1]$ is an exogenous parameter describing the respective importance of the two motives.[6] When player i is not a member of a winning coalition, it seems natural to assume that she incurs only ideological cost:

$$U_i(S) = -(1-\delta) \| x^i - x(S) \| .$$

The properties of this hedonic game where players are described by two parameters are yet unknown. Does there always exist an equilibrium? If so, what can we say about the features of an equilibrium coalition? We conclude the paper by stating that in the special case where $M = 1$, $n = 3$ and players care exclusively about ideology, there always exist an equilibrium.

Result 4.3: Let $M = 1$, $n = 3$, $\delta = 0$ and $0 < \theta^i < \frac{1}{2}$ for all $i = 1, 2, 3$. Then, there exists an equilibrium coalition.

Proof: Let the vector of ideologies be such that $x^1 < x^2 < x^3$ and $a \equiv x(\{1,2\})$, $b \equiv x(\{2,3\})$ and $c \equiv x(\{1,3\})$. Then we have $x^1 < a < x^2$, $x^2 < b < x^3$.

Suppose that $x^2 - a \leq b - x^2$. We argue then that $\{1,2\}$ is an equilibrium coalition. (If the opposite inequality $x^2 - a > b - x^2$ holds, we can show in a similar way that $\{2,3\}$ is an equilibrium coalition.)

If $c \geq a$, it is obvious that neither player 1 nor player 2 would deviate to form a coalition with player 3. If player 1 forms a coalition with 3 the outcome would be c, which is further away from x^1 than a. If player 2 forms a coalition with 3, the outcome would be b, which, by assumption, is more distant from x^2 than a.

If $c < a$, player 1 would deviate and form a coalition with player 3. However, player 3 does not want to form a coalition with player 1 since she prefers outcome a over c.$\square$

In the next section we discuss an extension of the Gamson game to environments that consist of infinite number of players.

[6] This is a particular case of the setting considered by Jackson and Moselle (2002).

4 Atomless environments

The analysis in previous sections has been conducted under the assumption that the set of players was finite. Consider instead an environment with a continuum of players given by the unit interval $[0,1]$ and a vector $\theta = \left(\theta^1, \theta^2, \ldots, \theta^K\right)$ of K positive random variables, where $\theta(t) = \left(\theta^1(t), \theta^2(t), \ldots, \theta^K(t)\right)$ denotes the characteristics of a player of type $t \in [0,1]$. Without loss of generality we assume that $\int_0^1 \theta^k(t)dt = 1$ for all $k = 1, \ldots, K$. Given a small positive number ε,[7] consider a proper simple game $\mathcal{W}_\varepsilon$ defined as follows:

$$S \in \mathcal{W}_\varepsilon \text{ if and only if } \theta^k(S) = \int_S \theta^k(t)dt \geq \frac{1}{2} + \varepsilon.$$

We assume that the fraction $\frac{1}{K}$ of the pie is divided among the members of the coalition on the basis of their k^{th} characteristic by using the double division method. That is, if a coalition S forms, every player of type t of S derives the payoff $V_t(S)$, defined as follows:

$$V_t(S) = \begin{cases} \frac{1}{K}\sum_{k=1}^{k=K} \frac{\theta^k(t)}{\theta^k(S)} & \text{if } S \in \mathcal{W}_\varepsilon \\ 0 & \text{otherwise.} \end{cases}$$

An obvious sufficient condition for a coalition S to be stable is

$$\theta^k(S) = \frac{1}{2} + \varepsilon \tag{1}$$

for all $k = 1, \ldots, K$.

In the finite case, condition (1) is not necessary for stability as illustrated by the following example:

Example 4.1: Let $n = 4$, $K = 2$, $\theta^1 = (0.40, 0.32, 0.17, 0.11)$ and $\theta^2 = (0.17, 0.10, 0.31, 0.42)$. Then there is a unique stable coalition $\{1,4\}$. is the only stable coalition. However $\theta^2(\{2,4\}) = 0.59 > \theta^2(\{2,4\}) = 0.52$.

The situation is however different when the set of players is atomless:

Result 4.2: There exists a stable coalition. Moreover, S is stable if and only if it satisfies (1).

Proof: The fact that condition (1) is necessary and sufficient condition for stability is immediate. For the existence of a stable coalition, consider the K-dimensional valued

[7] In the case of the classical majority simple game, existence of a stable coalition fails as the game is discontinuous if $\varepsilon = 0$.

measure μ defined as follows:

$$\mu(S) = \begin{pmatrix} \theta^1(S) \\ \theta^2(S) \\ \dots \\ \dots \\ \theta^K(S) \end{pmatrix}$$

Since μ is atomless, $\mu(\varnothing) = (0, 0, .., 0)$ and $\mu([0, 1]) = (1, 1, .., 1)$. Thus, by the Lyapunov's theorem[8] that there exits $S \subseteq [0, 1]$ such that

$$\mu(S) = \left(\frac{1}{2} + \varepsilon, \frac{1}{2} + \varepsilon, \dots, \frac{1}{2} + \varepsilon\right).$$

□

Result 4.2 indicates that the existence of an equilibrium coalition is a relatively simple question in the case where we have a continuum of players. In fact, the argument above applies for any hedonic game of the following form:

the payoff of a player of type $t \in [0, 1]$ is defined by

$$V_t(S) = \begin{cases} U(\theta^1(S), \theta^2(S), \dots, \theta^K(S)) & \text{if } S \in \mathcal{W} \\ 0 & \text{otherwise,} \end{cases}$$

where U is a decreasing function and there exist the values $\gamma^k \in \left(\frac{1}{2}, 1\right], k = 1, \dots, K$ such that the set of winning coalitions $\mathcal{W}$ is determined by

$$\mathcal{W} = \{S \subset [0, 1] | \theta^k(S) \geq \gamma^k \text{ for all } k = 1, \dots, K\}.$$

5 References

Acemoglu, D. Egorov, G. and K. Sonin (2006) "Coalition Formation in Political Games", mimeo.

Banerjee, S., Konishi, H. and T. Sömnez (2001) "Core in a Simple Coalition Formation Game", *Social Choice and Welfare* 18, 135-153.

Bogomolnaia and M.O. Jackson (2002) "The Stability of Hedonic Coalition Structures", *Games and Economic Behavior* 38, 201-230.

Bonacich, P., Grusky, O. and M. Peyrot (1985) "Family Coalitions: A New Approach and Method", *Social Psychology Quarterly* 44, 42-50.

Brown, E.C. and M.N. Franklin (1973) "Aspects of Coalition Payoffs in European Parliamentary Democracies", *American Political Science Review* 67, 453-469.

[8]See Halmos (1950).

Brown, E.C. and J.P. Frendreis (1980) “Allocating Coalition Payoffs by Conventional Norm: An Assessment of the Evidence from Cabinet Coalition Situations”, *American Journal of Political Science* 24, 753-768.

Caplow, T. (1956) “A Theory of Coalitions in the Triad”, *American Sociological Review* 21, 489-493.

Caplow, T. (1968) *Two against One: Coalitions in Triads*, Prentice-Hall, Englewood Cliffs, N.J.

Caplow, T. (1989) *Peace Games*, Wesley University Press, Middletown.

Caplow, T. (1995) “Coalitions” in *Encyclopedia of Sociology*, MacMillan, New York, 208-212.

Carroll, R., Cox, G.W. and M. Pochon (2004) “Gamson’s Law: How Governments Allocate Offices”, Mimeo, UC San Diego.

Chertkoff, J.M. (1967) “A Revision of Caplow’s Coalition Theory”, *Journal of Experimental Social Psychology* 3, 172-177.

Chertkoff, J.M. (1971) “Sociopsychological Theories and Research on Coalition formation” in *The Study of Coalition Behavior*, Groennings, S., Kelley, E.W. and M. Leiserson (Eds), Holt, Rinehart and Winston, New York, N.Y.

Felsenthal, D.S. and M. Machover (2001) ”The Treaty of Nice and Qualified Majority Voting”, *Social Choice and Welfare* 18, 431-464.

Felsenthal, D.S. and M. Machover (2004) “Analysis of QM Rules in the Draft Constitution for Europe Proposed by the European Convention, 2003”, *Social Choice and Welfare* 23, 1-20.

Fréchette, G.R., Kagel, J.H. and M. Morelli (2004) “Gamson’s Law versus Non-Cooperative Bargaining Theory”, Mimeo, Ohio State University.

Gamson, W.A. (1961a) “A Theory of Coalition Formation”, *American Sociological Review* 26, 373-382.

Gamson, W.A. (1961b) “An Experimental Test of a Theory of Coalition Formation”, *American Sociological Review* 26, 565-573.

Gamson, W.A. (1964) “Experimental Studies of Coalition Formation”, in *Advances in Experimental Social Psychology*, L. Berkowitz, ed., Academic Press, New York, N.Y.

Greenberg, J. and S. Weber (1986) “Strong Tiebout Equilibrium under Restricted Preferences Domain” *Journal of Economic Theory* 38, 101-117.

Hsiung, J.C. (1987) “Internal Dynamics in the Sino-Soviet-US Triad”, in *The Strategic Triangle*, Kim, I.J., ed., Paragon, New York, N.Y.

Kilgour, D.M. (1983) “A Formal Analysis of the Amending Formula of Canada’s Constitution Act, 1982", *Canadian Journal of Political Science* 16, 771-777.

Laver, M. (1998) "Models of Government Formation", *Annual Review of Political Science* 1, 1-25.

Miller, C.E. and R. Crandall (1980) "Experimental Research on the Social Psychology of Bargaining and Coalition Formation", in *Psychology of Group Influence*, R. Paulus, ed., Lawrence Erlbaum Associates, Hillsdale, N.J.

Murninghan, J.K. (1978) "Models of Coalition Behavior: Social Psychology and Political Perspectives", *Psychological Bulletin* 85, 1130-1153.

Nail, P. and S.G. Cole (1985) "Three Theories of Coalition Behavior: A Probabilistic Extension", *British Journal of Social Psychology* 24, 181-190.

Mills, T.M. (1954) "Coalition Patterns in Three Person Groups", *American Sociological Review* 19, 657-667.

Riker, W.H. (1962) *The Theory of Political Coalitions*, Yale University Press, New Haven, CN.

Warwick, P.V. and J.N. Druckman (2001) "Portfolio Salience and the Proportionality of Payoffs in Coalition Governments", *British Journal of Political Science* 31, 627-649.

Zagare, F.C. (1984) *Game Theory: Concepts and Applications*, Sage publications, Beverly Hills, CA.