INSTITUT D'ECONOMIE INDUSTRIELLE

IDEI Report # 3

Rail Transport

November 2003

Railway (De) Regulation
A European Efficiency Comparison

INSTITUT D'ECONOMIE INDUSTRIELLE

IDEI Report #3 on Passenger Rail Transport

November 2003

Railway (De)Regulation : A European Efficiency Comparison

Railway (De)Regulation : A European Efficiency Comparison

Guido Friebel[1]

Marc Ivaldi[2]

Catherine Vibes[3]

Acknowledgements

We thank Lou Thompson for providing us with the data set and his generous help in using it. We are also grateful for comments of participants at a seminar in Toulouse and at the Eighth European Workshop on Efficiency and Productivity Analysis in Oviedo, and the First Conference on Railroad Industry Structure, Competition and Investment, Toulouse 7th and 8th November 2003. We also thank Tim Coelli, Markus Ksoll, Paul Seabright and Klaus Vestner. The views expressed remain those of the authors alone.

[1] University of Toulouse (EHESS, IDEI and CEPR)
[2] University of Toulouse (EHESS, IDEI and CEPR)
[3] University of Toulouse (GREMAQ and MPSE)

Abstract

Many countries have sought to increase the efficiency of national railroad companies through a range of reforms: separating infrastructure and operations, creating independent regulatory institutions and providing access to the network to third parties. While the European Commission has declared these reforms crucial elements for developing the European railroad industry, little is known about the effects of reforms on railroad efficiency. We investigate a new World Bank panel data set that covers many EU countries over a period of 20 years. We compare the passenger traffic efficiency of national railroad companies by means of a production frontier model and evaluate the effects of reforms on efficiency. We also introduce a new way to control for the effect of freight traffic on efficiency of passenger traffic. We find that reforms have efficiency-increasing effects but that the effect of reforms depends on sequencing: The introduction of multiple reforms in a package has at best neutral effects, but sequential reforms improve efficiency. Using the LISREL technique, we find that our results are robust against potential problems of endogeneity.

Keywords: production frontier, network industries, panel data analysis, passenger and freight traffic.

JEL codes: L51, L92, D24, C23

1. Introduction

By the end of the 20th century, railroads were in dire straits. Although most national railways companies were, and still are, heavily subsidized (Crozet et al., 2000, Friederiszick et al. 2003), the market shares of railways in total (intermodal) transportation were, at best, stable. In many European countries, rail market shares decreased throughout the nineties (European Commission, DG Energy and Transport, 2002). Moreover, surveys show that customer satisfaction with railway services was low in many countries (INRA, 2000).

The European Commission, in its White Paper (EC, 2001), has declared the development of the European railway system one of its priorities in achieving sustainable development in Europe. It is an explicit goal of the Commission to promote railways, increase their market share, and reduce subsidies. Based on the experience in a number of countries that have introduced reforms throughout the 1980s and 1990s, the cornerstones of the EC reform model (EC, Directive 91/440) are: a) to unbundle infrastructure from operations, that is, to them separate them fully or, at least, create separate organizations and accounts within one holding, b) to create independent regulatory institutions for railways, c) to open access to national railway markets for competitors ("third party access").

There is a firm believe among many policy-makers, on both EU and national level, that these reforms ought to increase efficiency. But, while there is a substantial literature on efficiency in the railway industry (Cantos et al. 1999, Cantos et al. 2000, Coelli and Perelman, 1999, Cowie and Riddington, 1996, Gathon and Perelman, 1992, Oum and Yu, 1994), little is known about how regulatory reforms have affected railway efficiency.

We are only aware of two papers. Cantos et al. (1999) analyze the impact of four types of reforms on different dimensions of railway efficiency. They look at separation between infrastructure and operations, changes in the legal constitution of companies, degree of regulation of prices, and degree of government influence over investment. They find that vertical separation appears to have had the strongest impact. However, construction of their regulatory variables does not allow using variations over time, but only across countries. Gathon and Pestieau (1995) cross-sectional study indicates that constraints on managerial autonomy may reduce the efficiency of railway firms.

In this study, we investigate systematically to what extent third-party access, independent regulation, and separation of infrastructure affect railway performance. As different countries have implemented the reforms to different degrees and at different times, we are in position to identify the impact of regulatory regimes on railway performance. To do so, we use the production frontier approach, pioneered by Farrell (1957).

We apply this methodology to a new Worldbank (2001) panel dataset that provides input and output data for 11 European countries, over the period 1980-2000. We match this dataset with information about regulatory reforms in these countries and look at the impact of reforms on the efficiency in passenger traffic.

The paper has four contributions: First, we control for the congestion effects of freight traffic on the efficiency of passenger traffic. Second, we control for the potential endogeneity of explanatory variables and reforms by investigating the structure of the variance-covariance matrix, using the LISREL technique, introduced by Jöreskog (1973) and used in a similar context before by Ivaldi et al (1995). Third, we look at the effects of reforms and fourth, present efficiency measures for the twelve countries in our panel.

The main results are as follows: First, on average, if freight traffic increases by one percent, passenger traffic decreases by two and half tenth of a percent, a value larger than what is usually assumed in comparable studies. Second, there does not seem to be endogeneity of the explanatory variables. Third, reforms increase efficiency, that is, railroad performance would have, *ceteris paribus*, been lower in the absence of reforms. In particular, we find that while it is always efficiency enhancing to implement one reform, the effect of a larger number of reforms depends on sequencing. The introduction of multiple reforms in a package has at best neutral effects, but sequential reforms improve efficiency. It is also noteworthy that our regressions cannot identify that full separation of infrastructure from operations is a *conditio sine qua non* for railroad efficiency. Fourth, the development of efficiency overtime has been quite different across different countries. In general, smaller country railroads have had a more favorable efficiency development than larger countries (measured in terms of network length). Among larger countries, only Sweden and Germany have been able to increase their railroad efficiency, both concerning passenger and total (that is, passenger and freight) traffic, throughout the period of observation (1980-2000).

Section 2 presents the data. Section 3 introduces the econometric model and looks at endogeneity issues. Section 4 presents the results, constructs and compares efficiency measures. Section 5 summarizes and concludes.

2. The Data

The Worldbank (2001) data set comprises coherent and complete input and output information on railway industries of 11 EU countries: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Portugal, Spain, Sweden. Data cover 1980 to 2000, the period in which all reforms in the railroad sector have occurred in Europe.

Unfortunately, data for United Kingdom are not complete. In particular, as a result of the reforms, there is no consistent information about staff of railroad firms for the period from 1995 to 2000. National statistics in UK after the reform of the railways changed: People who formerly were counted as railway staff since then belonged to other industries like the construction industry or consulting. Therefore the official number of employees in the railway industry is much lower than it would be on a comparable basis. This makes it hard to evaluate railroads efficiency in the most interesting period and we will thus have to exclude UK from many of our regressions.

Table 1 provides an overview of the data. In terms of output we will look at *passenger kilometers* and *freight ton kilometers*. Table 1 reports means per country over the investigation period, showing that, in most countries, both outputs have increased in absolute values.

In the case of railroads, it is difficult to identify the correct input measures. Railroads are often integrated firms. The intermediate input "network" is produced by the inputs labor and land. This intermediate input network, additional labor, and rolling stock are then used in the production of the final outputs, *passenger-kilometers* and *freight-ton-kilometers*. The last column represents the measure for labor, *staff*, employed by railway carriers. In the regression this variable will be labelled L_{it}.

To find the right measure of capital input is not so easy a task. There are two problems: First, rolling stock can be interpreted both as input or output. At given labor and finance input, for instance, a company can decide to produce more passenger kilometers with old rolling stocks or higher quality traffic with new rolling stock. Similarly, a company can decide to build new faster or better tracks. In order to avoid this potential confusion between inputs and outputs, we focus on *route kilometers* as the second input besides *staff*. Route kilometers measure the total size of the network without taking into account whether a given connection has single, double or multiple tracks. They thus have the convenient feature to be clearly inputs, not outputs: In the mature networks of European countries, only few new routes are built. Actually, throughout the period we are interested in, route kilometers have decreased rather than increased in most countries. In the regressions, route kilometers are labelled K_{it}.

We have matched these physical data with information about reforms. Table 2 presents these deregulation data. It reports the year in which regulatory reforms were introduced and stems from a variety of documents: Erasmus University (1999), SORT-IT (1999), OECD (1998), Stoffaes et al (1995), Prognos (1998). The data have the advantage to capture the effects of regulatory changes both over time and across countries. They have the disadvantage that they report the state of national laws, and not the implementation of these laws. Moreover, there are many reform specificities across countries. There are thus certain limits concerning the extent to which one can interpret the results. We discuss these issues further in the next section.

3. Econometric specification and endogeneity

The frontier production function specifies what output can be achieved, if all decisions were taken according to “best practice”. As the frontier production function defines a theoretically achievable optimum, all empirical observations must lie below it. Consider the Cobb-Douglas function:

$$y = AK^{\alpha_K} L^{\alpha_L}.$$

In our regressions, output y is either passenger kilometers, or the weighted sum of passenger and freight kilometers. Inputs are route kilometers (*K)* and staff (*L*). The production function is linear in the logs of the inputs. Thus, the log of the output variable can be expressed as the following function:

$$\ln y = A + \alpha_K \ln K + \alpha_L \ln L.$$

As we use a panel data set, we account for individual (country-) fixed effects and time trends through the introduction of effects of railway deregulation. Thus, for country *i* at time *t*, we assume that:

$$A_{it} = \alpha + (\gamma_i + \theta_0 Deregulation_{it})t + \varepsilon_{it}$$

The term $(\gamma_i + \theta_0 Deregulation_{it})$ represents technological progress. It may differ across countries (and is thus indexed by i). Moreover the country specific trend may depend on whether or not a country has reformed its railroad industry. We here introduce a dummy variable $Deregulation_{it}$ that takes value 1 if a country has introduced (and maintained) at least one of three reforms we look at. At this stage we do not distinguish types or intensity of reform, which we will do in other specifications, below.[4] The other components of the expression are an intercept and a normally distributed noise term.

The equation we estimate is then:

$$\ln y_{it} = \alpha + \alpha_K \ln K_{it} + \alpha_L \ln L_{it} + (\gamma_i + \theta_0 Deregulation)t + \varepsilon_{it}. \quad (1)$$

3.1 Output measures: total traffic versus passenger traffic

We are mainly interested the efficiency of passenger transport. Nonetheless, as we have no information about how capital and labor are allocated for the production of passenger and freight traffic, we must control econometrically for the effect of freight transportation on passenger traffic efficiency. In order to do so, we estimate Equation (1), using an aggregate output measure defined as

$$\ln y_{it} = \ln passkm_{it} + \lambda \ln tonkm_{it} \quad (2)$$

In what follows, we use $\hat{\lambda}$, the estimate of λ that provides the best fit of the model, or to be more precise, the $\hat{\lambda}$ that minimizes the fit function for Equation (1). For different specifications, $\hat{\lambda}$ lies between 0.24 and 0.27. That is, on average, if freight traffic increases by one percent, passenger traffic decreases by two and half tenth of a percent. The advantage of our method is that we receive an empirical measure for the congestive effects of freight on passenger traffic, rather than using *ad hoc* measures. Other studies have assumed, for instance, that each passenger equals a certain fixed weight of freight,

[4] One could, alternatively, use a different type of specification in which deregulation would not enter multiplicatively with time, but, rather, a term $\theta_0 Deregulation_{it}$ would be added to the constant. This, however, would not take into consideration that the effects of reforms shift the slope of the trend and not only the level.

specifically 80 kilograms. According to our estimate, the congestion effect owing to freight is higher.[5]

3.2 Endogeneity and LISREL estimates

In our data set and with the specification we use, there is a potential problem of endogeneity: while we control for individual (country-level) effects, we can *a priori*, not exclude correlations of these individual effects with inputs (capital, labor). If there were such correlations, the regression results and the measure for efficiency, which is based on the error terms of the regression, would be biased.

We use the LISREL ("Linear Structural Relations")[6] method to verify whether or not this type of correlation is present in the data. LISREL has the convenient feature of estimating all possible correlations between inputs and individual effects and hence, for our case, between input quantities and individual technical efficiency levels. In Appendix 2, we briefly discuss the method.

Table 6 summarizes the results of the LISREL analysis. By looking at the covariances of different variables and at their associated t-values, it becomes clear that there is no correlation between variables. The results of our regressions are thus unbiased.

---

[5] It should be noted that while our method has advantages over using *ad hoc* aggregation weights, there are some methodological issues associated with aggregating multi-product outputs to a single output (see Alvarez and Orea (2001) for a discussion of the caveats of aggregation to a single output). Note that it is however not our objective to estimate the full production possibility set (PPS), and that equation (2) is not an approximation of this PPS. Rather it is a point on the "true" unobserved PPS at which we investigate inefficiency. By using eq. (2), we are only able to compute inefficiency level of the passenger service conditional on the actual level of freight service. Identifying the full PPS is a research topic beyond the scope of this paper.

[6] See Jöreskog (1973, 1996).

4. Regression results and efficiency comparison

In what follows we present and discuss OLS estimates. These are also used to construct efficiency measures.

4.1 Results

Table 3 presents the regression results. We have run the regression both including and excluding United Kingdom. The dependent variable is aggregated output as defined in Equation (2). The parameter estimates for labor and capital are in line with what could be expected for a network industry like railroads. As $\alpha_K + \alpha_L > 1$, there are increasing returns to scale. Note also that in all countries except Finland, the productivity trend is positive. The regression shows that, excluding United Kingdom, deregulation increases the productivity trend of a country at the 7% level of statistical significance.[7] In terms of magnitude of output changes, this corresponds to an additional output of on average 0.4 percent per year after deregulation.[8]

These effects are less significant when one includes United Kingdom. This points to the problem with United Kingdom data. With the beginning of reforms, data quality for United Kingdom has declined, and data for staff since 1995 are missing. In what follows we thus run most regressions without United Kingdom.

---

[7] As an example, total productivity for Austria changes from 0.01 to 0.014, for Germany from 0.02 to 0.024, and for France from 0.05 to 0.054, after introduction of deregulation.

[8] To compute the magnitude, we first write output as $y = AK^{\alpha_K} L^{\alpha_L} e^{\theta Deregulation t} e^{\gamma t} e^{\varepsilon}$. To measure of the effect of deregulation dummy on output, we compute $E(y / Deregulation = 1, t+1) - E(y / Deregulation = 1, t)$. Notice that as $\hat{\theta}_0$ is normally distributed with mean θ_0, $e^{\hat{\theta}_0}$ follows a lognormal distribution with mean: $E(e^{\hat{\theta}_0}) = \exp(\hat{\theta}_0 + \frac{1}{2}\hat{\sigma}_{\hat{\theta}_0}^2)$.

The first regression shows that reforms have affected railroad productivity in a positive way. In order to see whether more reforms are better than one reform, we have constructed a second set of reform variables: $DeregulationOneAspect_{it}$, which takes the value 1 when one and only one aspect of the deregulation is implemented, whatever happens later, and 0 otherwise. $DeregulationTwoAspects_{it}$, which takes the value 1 when exactly two aspects of the deregulation are implemented and 0 otherwise. Both inter multiplicatively with time, as for *Deregulation* before.

The results presented in Table 4 show that the implementation of only one aspect of the deregulation has a positive effect on the productivity trend of a country, whereas the effect of the implementation of two aspects is neutral.

The fact that two reforms do not improve productivity compared to no reform is discomforting. However, it is important to notice that the group of countries with two reforms is very heterogeneous. In some countries (Austria, Finland, Italy, Spain, Sweden), the two reforms (not necessarily the same across countries) were implemented sequentially. In other countries (France, Germany, The Netherlands, Portugal), both were implemented at the same time, as a "package". To get some idea of whether sequencing matters, we define a new set of variables that allows to distinguish the types of reform: $DeregulationPartial_{it}$ takes the value 1 if a reform is implemented, and no further reforms take place, and 0 otherwise. $DeregulationSequential_{it}$ takes the value 1 if a reform is implemented, and it is followed by further reforms, and 0 otherwise. $DeregulationPackage_{it}$, takes the value 1 if more than one reform are implemented by the same time, 0 otherwise.

The results indicate that there is a difference in implementing a given number of reforms in one blow or gradually. Doing too much by the same time appears to be dominated by a more careful gradual strategy.

Two comments arise. First, package reforms have a neutral effect when one excludes United Kingdom, but the effect becomes negative under inclusion of United Kingdom. This does not only stress again that the lacking United Kingdom data are complicating our analysis, it also points to a more important limitation of interpreting the parameter estimates of the reform variables. The results must be taken with a grain of salt, as the variable *DeregulationPackage* entails countries that have quite different models of reforms and different railroad specificities. For instance, while both France and Germany introduced the same reforms into their law books (some unbundling of infrastructure and operations, third party access), the implementation of these reforms have differed largely. In Germany the possibility of third-party access has led to entry of many new competitors, while no new competitors have entered the French market. To a similar extent, the implementation of infrastructure separation has been quite different in Germany from the one in France. While Germany chose an organizational solution in which infrastructure and operations remain in the same holding, France decided to create a separate infrastructure company that is not under the purview of SNCF. However, track allocation and management have been contracted back to SNCF. This example illustrates how difficult it is to operationalize empirically different types of reform implementation.

Second, to investigate the effect of different types of implementation of infrastructure separation, we have regressed output on these two different types of implementation. We use institutional work (Prognos, 1998) that classifies countries according to organizational or institutional types of infrastructure separation (see Table 9). Countries that have opted for organizational separation have created separate bodies and separate accounting, but retain them under the umbrella of one holding infrastructure. Other countries have created two (or more) independent institutions.

Controlling for these two different types of reform and looking at efficiency (results are available on request), we find that there is no significant difference between no separation at all and organizational separation. Full (or institutional separation) has a positive effect on efficiency, but only when one excludes UK. Our model can thus not identify that institutional separation of infrastructure from operations is a *conditio sine qua non* for railroad efficiency.

With the current data, one can unfortunately not go much further in investigating the role of different types of implementation of reforms. The regression results do, however, indicate that there is a need to measure implementation in order to evaluate policy reforms in a comprehensive way.

4.2 Efficiency

Using the regression results, we now investigate the development of railway efficiency in Europe. We construct the efficiency (performance) measure for total (passenger and freight) traffic as follows.[9] Using $\hat{\alpha}, \hat{\alpha}_K, \hat{\alpha}_L, \hat{\lambda}, \hat{\theta}_0, \hat{\gamma}_i$, the parameter estimates, the logarithm of the observed values for capital, labor, and the values for deregulation and time for each country, we compute the estimate of the logarithm of output for country *i* at time *t*. Deducting this estimate from the realized value for the respective country yields

[9] The method is explained in more details in Gathon (1991).

the residual of the regression, ε_{it}. We then rank the residuals and denote as $\varepsilon_t^{\max}$ the highest residual in year *t*.[10] We then express the performance of all other countries compared to the country with the highest performance by the following measure:

$$Eff_{it} \equiv \exp(\varepsilon_{it} - \varepsilon_t^{\max}) \quad (3)$$

The residual here measures the part of the output of a country *i* at time *t* that cannot be captured by the estimates of the productivity parameters of the Cobb-Douglas function. Notice that these productivity parameters do not vary over time. Variations of output that cannot be explained by variations of inputs or productivity trends can in principle be owing to regulatory regimes. We control for these influences. Variations may also be owing to different types of implementation of reforms, and, in particular, to different degrees of managerial efficiency, both of which we do not control for. Notice that the efficiency measure takes the value 1 (or 100%) for the country with the highest performance in the year *t*.

Tables 7a and 7b present the results for smaller and larger countries separately, taking into account total railways transportation, that is, both freight and passengers. Tables 8a and 8b show the efficiency for passenger traffic only, computed as follows:

$$PassEff_{it} \equiv \exp(\varepsilon_{it} - \varepsilon_t^{\max}) + \hat{\lambda}(\ln tonkm_{it} - \ln tonkm^{\max}) \quad (4)$$

The first term on the right hand side represents total traffic efficiency, and the second term represents the impact of freight transportation. As the value in the parenthesis is negative, we thus correct total efficiency by the relative level of freight efficiency of a

[10] The residual here measures the part of the output of a country *i* at time *t* that cannot be captured by the estimates of the productivity parameters of the Cobb-Douglas function. Notice that these productivity parameters do not vary over time. Variations of output that cannot be explained by variations of inputs or productivity trends must thus be either due to regulatory regimes, which we control for, or different degrees of efficiency.

country. Thus, only if a country is both most efficient in terms of freight *and* passenger traffic, it shows a 100% efficiency degree in Tables 7a and 7b.

We have plotted country efficiency levels in different ways. Figures 1 and 2 look at the efficiency development of countries over time, for total and passenger traffic, respectively. The depicted efficiency levels are computed relative to the period in which the country reached its highest efficiency level.

It appears that the development is quite similar for both types of traffic (efficiency levels across the two types of traffics turn out to be highly correlated). While smaller countries, except for the Netherlands, have been able keep or raise their efficiency levels, among larger countries only Sweden and Germany have been able to increase their efficiency, and Spain has been roughly stable.

In Figures 3 and 4 we normalize the sum of all country efficiency levels to 100%. The graph allows comparing the efficiency level of different countries. The component on top of each bar represents the sum of efficiency levels of all small countries. The following components of the bar represent the shares of Sweden, Spain, Italy, Germany and France in the sum of all country efficiency levels. For example, we notice that in 1999, Germany is relatively more efficient than the other countries. Moreover, we see that the relative efficiency of Germany increased from 1993 to 1999, and decreases in 2000. This does not necessarily mean that Germany starts to become “less efficient” in 2000: Germany may continue to be more and more efficient, but other countries efficiency gains may be stronger than the ones of Germany.

Figures 5 and 6 show the relative efficiency levels of large countries only, while figures 7 and 8 present the same information, but by averages of five-year periods.

5. Concluding remarks

This paper has investigated a new panel data set, which we have enriched by information about changes in regulatory regimes over the last twenty years. We find that reforms have had positive impact on output. The efficiency development of European carriers has been quite heterogenous. The LISREL analysis of the variance/covariance structure shows that the results are *not* subject to endogeneity issues. An additional contribution lies in the fact that we have controlled for the effect of freight traffic on passenger traffic efficiency without relying on ad-hoc weights given to freight versus passenger traffic.

An additional important result is that sequencing matters: Introducing a number of reforms by the same time, as a package, does not improve efficiency. Sequential reforms, however, do improve efficiency. The railroad sector seems to be quite sensitive to changes in the regulatory framework and, in particular, to the way reforms are implemented. Better data are needed to come to a conclusion about the most appropriate policy solution for the deregulation of railways.

Some limitations of our study should be noted. First, owing to data problems, we have not been able to include UK data in most of the regressions. Second, we have to date only been able to look at reforms in the law book, and cannot control for different types and intensity of implementation. Third, we have not taken into account that the degree of subsidization is quite different across European countries as Friederiszick et al (2003) have shown. This may have an important impact on our measure of efficiency. Finally, we have only used quantitative measures of output, and not quality, an important issue about which, however, there exist no good data.

The result about positive effects of deregulation corroborates what has been found in studies on other network industries. Ng and Seabright (2001) find that in the period from 1990 to 1995, European airline costs could have dropped by as much as 26%, provided European airlines were privately owned and subject to the same degree of competition as US carriers. Also related is the study of Nicoletti and Scarpetta (2002), who show that product market regulation discouraging entry or distorting competition reduce the efficiency of many industries in the OECD.

The reforms we have looked at are quite similar: Third-party access makes it possible to enter foreclosed markets, and the unbundling of infrastructure and operations makes potential anti-competitive pricing of infrastructure or non-pricing discrimination harder. But, our regressions seem to indicate that much depends on the way reforms are implemented. Other empirical (Ivaldi and McCullough, 2002) and theoretical work, for instance, Vickers (1995) and King (1999) make similar points: Building the reform of network industries on a one-size-fits-all model of separation of infrastructure from operations may not be a fruitful way to enhance efficiency.

REFERENCES

Alvarez, A. and L. Orea (2001), *Different methods of modelling multi-species fishing using a primal approach*, University of Oviedo, Efficiency Series no 4.

Browne, M. W. (1984), "Asymptotically distribution-free methods for the analysis of covariance structures", in: D. J. Aigner and A. Goldberger (eds), *Latent Variables in Socio-Economic Models*, North-Holland, Amsterdam, pp. 205-226.

Cantos Sanchez, P., Pastor Monsalvez, J. M., Serrano Martinez, L, (1999). "Productivity, Efficiency and Technical Change in the European Railways: a Non-Parametric Approach", *Transportation* 26(4), pp 337-357.

Cantos Sanchez, P., Pastor Monsalvez, J. M., Serrano Martinez, L. (2000), "Efficiency measures and output specification: the case of European railways", *Journal of Transportation and Statistics*, vol. 3 (3), pp. 61-68.

Coelli T.J. and S. Perelman (1999), "A comparison of parametric and non-parametric distance functions: with applications to European railways", *European Journal of Operational Research*, pp. 326-339.

Cowie, J and G. Riddington (1996), "Measuring the efficiency of European railways", *Applied Economics*, vol. 28, pp. 1027-1035.

Crozet, Y., L. Guihéry, D. Bouf, P. Péguy (2000), "Markets, hierarchies and tracks: Railway reform in Europe, towards a new lease of life", Paper presented at the 3rd KFB conference, Stockholm.

Erasmus University Rotterdam (1999), "Changing trains - Railway reform and the role of competition: the experience of six countries", Ashgate, Aldershot (*Oxford Studies in Transport Series*), 384 pages (ISBN 1 84014 878 0).

European Commission (1991), *Directive 91/440*, Official Journal 24th August 1991; http://www.rff.fr/biblio_pdf/eur_ref_direct_eu_91_440_CEE.pdf

European Commission (2001), *European Union Energy and Transport in Figures*, Directorate General for Energy and Transport, in cooperation with Eurostat, Brussels.

European Commission (2001), *Whitepaper: European transport policy for 2010: time to decide*, Brussels.

European Commission (2003), *Rail transport and interoperability*, Directorate-General for Energy and Transport http://europa.eu.int/comm/transport/rail/overview/infrastructure_en.htm

European Conference of Ministers of Transports, http://www1.oecd.org/cem/

Farrell M. (1957), "The measurement of productive efficiency", *Journal of the Royal Statistical Society, Series A, General*, 120, part 3, pp. 253-281.

Friederiszick, H., L.-H. Röller, C. Schulz (2003), Evaluation of the effectiveness of state aid as a policy instrument: The Railway Sector, mimeo.

Gathon, H.J. (1991), *La performance des chemins de fer européens: gestion et autonomie*, PhD dissertation, University of Liege, Belgium.

Gathon, H.J and S. Perelman (1992), "Measuring technical efficiency in European railways: a panel data approach", *The Journal of Productivity Analysis*, vol. 3, pp. 135-151.

Gathon, H. and P. Pestieau (1995), "Decomposing efficiency into its managerial and its regulatory components: the case of European railways", *European Journal of Operational Research*, pp. 500-507.

INRA (2000), *Les Européens et les services d'intérêts généraux* (The Europeans and their services of public interest), Report for *Direction générale santé et protection des consommateurs*, http://europa.eu.int/comm/dgs/health\consumer/library/surveys/sur15_fr.pdf

Ivaldi, M. and G. Mc Cullough (2002), "Subadditivity tests for network separation", mimeo Toulouse and Northwestern University.

Ivaldi, M., S. Monier-Dilhan, M. Simioni (1995), "Stochastic production frontiers and panel data: a latent variable framework", *European Journal of Operational Research*, pp. 534-547.

Jöreskog, K. G. (1973), "Analysis of covariance structures", in: P.R. Krishnaiah (ed.), *Multivariate analysis III*, Academic Press, New York, 263-285.

Jöreskog, K.G. and G. Sörbom (1996), *LISREL 8 User's Reference Guide*, Chicago: Scientific Software International, (378 pages).

King, S. (1999), "Price discrimination, separation and access", *Australian Journal of Management*, vol. 24, pp. 21-36.

Ng, C. and P. Seabright (2001), "Competition, privatisation and productive efficiency: Evidence from the airline industry", *The Economic Journal*, vol. 111, pp. 591-619.

Nicoletti, G. and S. Scarpetta (2002), "Regulation, productivity and growth", forthcoming, *Economic Policy*.

Obermauer, A. (2001), "National Railway Reform in Japan and the EU: Evaluation of Institutional Changes", *Japan Railway & Transport Review*, No. 29, pp.24-31.

OECD (1998), *Railways: Structure, regulation and competition policy*, Competition Policy Roundtables, No 15
http://www.oecd.org/pdf/M000015000/M00015190.pdf

Oum, T.H. and C. Yu (1994), "Economic efficiency of railways and implications for public policy", *Journal of Transport Economics and Policy*, vol. 28, pp. 121-138.

Prognos, European Centre for Economic Research and Strategy Consulting, (1998), *Examination of the implementation and impact of directive 91/440/EC on the development of the community's railways*, Report to the Union Internationale des Chemins de fer, Basel.

SORT-IT, Strategic Organization and Regulation in Transports (1999), *Final Report to the European Commission*.

Stoffaes, C., J. Berthod, M. Feve (1995), *L'Europe: avenir du ferroviaire* (The future of European railways), Report to the French Ministry of Transportation.

UIC, Union Internationale des Chemins de Fer (2001), *Railways time-series data*, 1970-1999.

Vickers, J. (1995), "Competition and regulation in vertically regulated markets", R*eview of Economic Studies*, vol. 62, pp. 1-17.

The Worldbank (2001), *Railway Performance Database*, The Worldbank Transportation, Water and Urban Development Department.

http://www.worldbank.org/transport/rail/rdb.htm

APPENDIX 1: Tables

Table 1: Summary statistics

	period	*pass-km (millions)*	*ton-km (millions)*	*route (km)*	*nbber of employees*
Austria	1980-1989	7410	10770	5751	68976
	1990-2000	8856	13558	5625	59414
Belgium	1980-1989	6565	7709	3742	58017
	1990-2000	7008	7948	3428	42272
Denmark	1980-1989	4489	1656	2318	21908
	1990-2000	4906	1825	2293	15785
Finland	1980-1989	3185	7899	5972	25470
	1990-2000	3243	9182	5867	16253
France	1980-1989	59507	57274	34559	234564
	1990-2000	62058	50858	32378	186959
Germany	1980-1989	63368	115886	41933	534424
	1990-2000	61714	74304	39828	314748
Italy	1980-1989	40148	17649	16225	218337
	1990-2000	47078	21336	16086	142302
Netherlands	1980-1989	9293	3194	2776	27188
	1990-2000	13970	3062	2773	26417
Portugal	1980-1989	5738	1253	3555	22598
	1990-2000	4906	1908	2113	14831
Spain	1980-1989	15061	10893	13094	65184
	1990-2000	16398	10294	12473	40514
Sweden	1980-1989	6385	16413	11367	32577
	1990-2000	6363	18801	10145	20316

1980-19894(-19)-5.4(89)-557(63)-5.4(85 164)-5.4(13 63)--2-2-9.5(a)-9

[11] ÙK data are missing from 1995 to 2000.

Table 2: Deregulation events (three main aspects)[12]

	separation infra-structure, operations	*third party access*	*independent regulatory entity*
Austria	1997	1995	
Belgium	1998		
Denmark	1997	1999	
Finland	1995	1999	
France	1997	1997	
Germany	1994	1994	
Italy	1998	1999	
The Netherlands	1995	1995	
Portugal	1997		1997
Spain	1996	1995	
Sweden	1988	1989	
United Kingdom	1993	1993	1993

Table 3: OLS regression estimates: global deregulation

	Without United Kingdom		*With United Kingdom*	
Variables	***Parameter estimate***	***t-value***	***Parameter estimate***	***t-value***
Intercept	-1.327***	-4.92	-1.271***	-4.92
Logarithm (Capital)	0.526***	9.77	0.520***	9.89
Logarithm (Labor)	0.737***	15.53	0.728***	15.91
Deregulation Productivity trend	0.004*	1.76	0.003	1.52
Productivity trend Austria	0.009**	2.03	0.010**	2.16
Productivity trend Belgium	0.020***	4.40	0.020***	4.55
Productivity trend Denmark	0.038***	7.75	0.038***	7.98
Productivity trend Finland	-0.002	-0.39	-0.002	-0.43
Productivity trend France	0.049***	9.62	0.050***	10.09
Productivity trend Germany	0.024***	4.42	0.025***	4.79
Productivity trend Italy	0.050***	11.57	0.051***	12.23
Productivity trend The Netherlands	0.081***	16.47	0.082***	16.99
Productivity trend Portugal	0.032***	6.28	0.032***	6.52
Productivity trend Spain	0.039***	7.15	0.040***	7.43
Productivity trend Sweden	0.026***	3.85	0.026***	3.92
Productivity trend United Kingdom	-	-	0.031***	4.98

$\lambda = 0.26$
$R^2 = 0.9798$
Number of observations: 231.
*: significant at a 10% level, **: significant at a 5% level, ***: significant at a 1% level.

[12] Defined according to discussion p 1.

Table 4: OLS regression estimates, intensity of reforms

Variables	*Parameter estimate*	*t-value*
Intercept	-1.251***	-4.63
Logarithm (Capital)	0.505***	9.34
Logarithm (Labor)	0.739***	15.57
DeregulationOneAspect Productivity trend	0.008**	2.33
DeregulationTwoAspect Productivity trend	0.002	0.83
Productivity trend Austria	0.009**	2.00
Productivity trend Belgium	0.018***	3.86
Productivity trend Denmark	0.036***	7.34
Productivity trend Finland	-0.003	-0.55
Productivity trend France	0.051***	9.75
Productivity trend Germany	0.026***	4.68
Productivity trend Italy	0.051***	11.73
Productivity trend The Netherlands	0.082***	16.53
Productivity trend Portugal	0.033***	6.46
Productivity trend Spain	0.040***	7.36
Productivity trend Sweden	0.028***	4.04

$\lambda = 0.25$
$R^2 = 0.9797$
Number of observations: 231.
*: significant at a 10% level, **: significant at a 5% level, ***: significant at a 1% level.

Table 5: OLS regression estimates, sequencing of reforms

Variables	*Parameter estimate*	*t-value*
Intercept	-1.133***	-4.23
Logarithm (Capital)	0.518***	9.53
Logarithm (Labor)	0.711***	14.93
DeregulationPartial Productivity trend	0.008*	1.59
DeregulationSequential Productivity trend	0.011**	3.05
DeregulationPackage Productivity trend	-0.005	-1.28
Productivity trend Austria	0.004	0.89
Productivity trend Belgium	0.018***	3.63
Productivity trend Denmark	0.035***	6.98
Productivity trend Finland	-0.009	-1.48
Productivity trend France	0.054***	10.15
Productivity trend Germany	0.031***	5.31
Productivity trend Italy	0.049***	11.04
Productivity trend The Netherlands	0.085***	16.98
Productivity trend Portugal	0.034***	6.87
Productivity trend Spain	0.034***	5.80
Productivity trend Sweden	0.017**	2.25

$\lambda = 0.24$
$R^2 = 0.9801$
Number of observations: 231.
*: significant at a 10% level, **: significant at a 5% level, ***: significant at a 1% level.

Table 6: LISREL estimates

	Estimates	***t-values***
Logarithm (Capital)	0.4790***	8.967
Logarithm (Labor)	0.7840***	16.552
Cov(*γ_i*,LogCapital)	-0.0010	-0.438
Cov(*γ_i*,LogLabor)	-0.0005	-0.156
Var(*γ_i*)	0.0005***	2.18
σ^2	0.0370***	10.00
Ridge constant	0.001	

*: significant at a 10% level, **: significant at a 5% level, ***: significant at a 1% level.

Remarks:

The endogenous variable here is aggregate output. We report here an experiment with global individual effects, not controlling for deregulation. Covariances between capital and individual effects and between labor and individual effects are constrained to be constant over time. To avoid "near multi-collinearity" among predictors, LISREL automatically applies a ridge estimation (Jöreskog, 1996).

We have also run a model with $\delta_{it} = \gamma_i + \theta_0 Deregulation_{it}$. Because of their time pattern and structure, the deregulation dummies create a problem of non-invertibelity. This imposes to restrict the panel to the period 1995 to 1999. In this regression we find similar results: covariances between individual effects and inputs are not significant.

Table 7: Relative efficiency measures, total traffic

a) Smaller countries

	Austria	*Belgium*	*Denmark*	*Finland*	*Netherlands*	*Portugal*
1980	0.43	0.49	0.54	0.37	1.00	0.51
1981	0.41	0.48	0.56	0.39	1.00	0.51
1982	0.45	0.49	0.66	0.39	1.00	0.51
1983	0.49	0.54	0.63	0.45	1.00	0.54
1984	0.53	0.60	0.64	0.47	1.00	0.61
1985	0.58	0.64	0.68	0.50	1.00	0.66
1986	0.66	0.68	0.80	0.47	1.00	0.78
1987	0.69	0.74	0.79	0.59	1.00	0.82
1988	0.74	0.77	0.74	0.64	0.97	0.80
1989	0.78	0.81	0.73	0.68	0.94	0.82
1990	0.87	0.88	0.80	0.79	0.99	0.78
1991	0.87	0.87	0.75	0.74	1.00	0.76
1992	0.86	0.86	0.68	0.70	1.00	0.77
1993	0.93	0.90	0.71	0.81	1.00	0.89
1994	0.96	0.91	0.76	0.90	0.96	0.99
1995	0.93	0.86	0.82	0.92	0.79	0.95
1996	0.98	0.87	0.77	0.94	0.71	0.85
1997	0.80	0.85	0.90	1.00	0.68	0.79
1998	0.80	0.88	0.92	1.00	0.64	0.85
1999	0.72	0.67	0.79	0.90	0.50	0.61
2000	0.61	0.57	0.82	0.80	0.42	0.68
Mean	0.72	0.73	0.74	0.69	0.89	0.74

b) Larger countries

	France	*Germany*	*Italy*	*Spain*	*Sweden*
1980	0.76	0.50	0.62	0.50	0.52
1981	0.75	0.52	0.61	0.50	0.52
1982	0.78	0.54	0.62	0.53	0.50
1983	0.86	0.58	0.60	0.60	0.57
1984	0.89	0.61	0.67	0.63	0.61
1985	0.90	0.68	0.63	0.72	0.66
1986	0.96	0.76	0.77	0.78	0.72
1987	0.97	0.75	0.79	0.82	0.63
1988	1.00	0.76	0.77	0.88	0.61
1989	1.00	0.77	0.77	0.82	0.61
1990	1.00	0.72	0.82	0.87	0.73
1991	0.91	0.62	0.84	0.78	0.62
1992	0.85	0.60	0.86	0.76	0.61
1993	0.82	0.67	0.89	0.84	0.75
1994	0.86	0.78	1.00	0.81	0.82
1995	0.76	0.77	1.00	0.79	0.81
1996	0.79	0.84	1.00	0.80	0.87
1997	0.70	0.87	0.92	0.88	0.89
1998	0.69	0.90	0.78	0.85	0.97
1999	0.61	1.00	0.57	0.75	0.92
2000	0.50	0.90	0.49	0.68	1.00
Mean	0.83	0.72	0.76	0.74	0.71

Table 8: Relative efficiency measures, passenger traffic

a) Smaller countries

	Austria	*Belgium*	*Denmark*	*Finland*	*Netherlands*	*Portugal*
1980	0.23	0.24	0.18	0.18	0.40	0.15
1981	0.22	0.24	0.18	0.20	0.40	0.15
1982	0.24	0.24	0.22	0.20	0.39	0.15
1983	0.26	0.26	0.21	0.23	0.39	0.16
1984	0.29	0.31	0.21	0.23	0.39	0.19
1985	0.31	0.32	0.23	0.25	0.39	0.20
1986	0.36	0.33	0.27	0.23	0.39	0.24
1987	0.37	0.36	0.26	0.29	0.39	0.26
1988	0.39	0.38	0.25	0.32	0.38	0.26
1989	0.42	0.40	0.24	0.34	0.36	0.27
1990	0.50	0.46	0.28	0.42	0.40	0.26
1991	0.54	0.48	0.28	0.40	0.43	0.28
1992	0.55	0.51	0.26	0.39	0.43	0.30
1993	0.60	0.53	0.27	0.49	0.44	0.35
1994	0.62	0.52	0.30	0.54	0.42	0.37
1995	0.61	0.48	0.32	0.55	0.35	0.38
1996	0.65	0.48	0.30	0.55	0.32	0.33
1997	0.52	0.47	0.33	0.60	0.31	0.32
1998	0.52	0.49	0.34	0.59	0.30	0.34
1999	0.48	0.37	0.31	0.54	0.23	0.25
2000	0.41	0.32	0.32	0.47	0.19	0.27
Mean	0.43	0.39	0.27	0.38	0.37	0.26

b) Larger countries

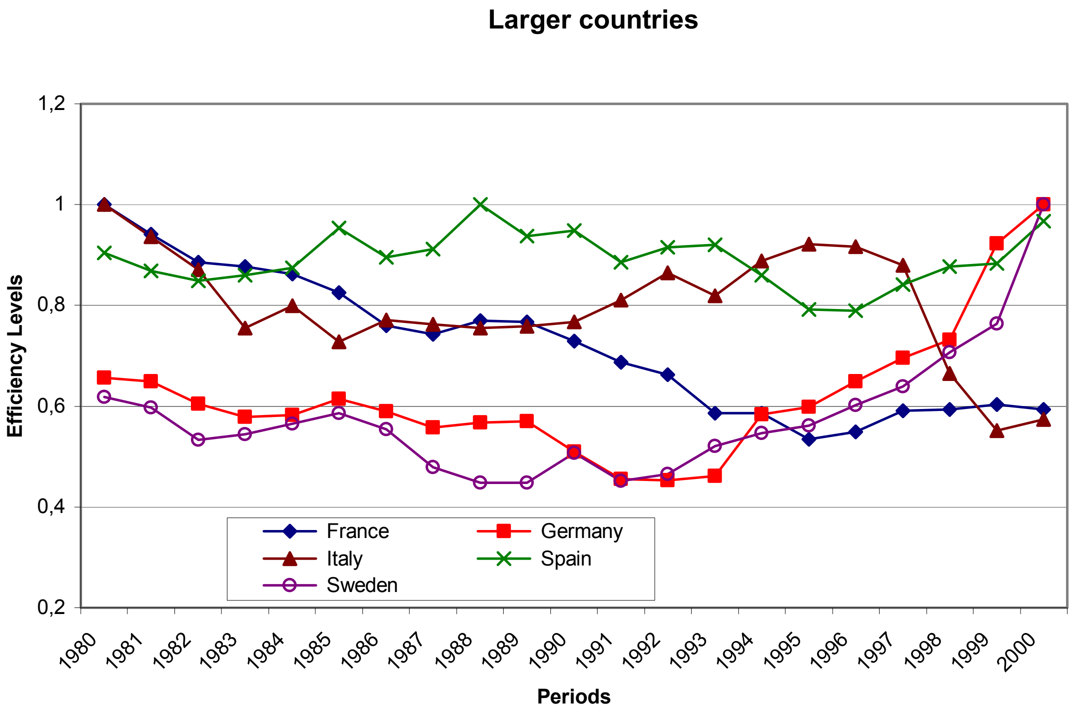
	France	*Germany*	*Italy*	*Spain*	*Sweden*
1980	0.66	0.50	0.38	0.26	0.31
1981	0.65	0.52	0.37	0.26	0.30
1982	0.67	0.54	0.38	0.29	0.29
1983	0.74	0.58	0.37	0.32	0.34
1984	0.75	0.61	0.41	0.35	0.37
1985	0.73	0.68	0.38	0.39	0.40
1986	0.77	0.76	0.47	0.42	0.44
1987	0.78	0.75	0.49	0.44	0.39
1988	0.81	0.76	0.47	0.48	0.38
1989	0.81	0.77	0.48	0.45	0.38
1990	0.84	0.72	0.53	0.48	0.47
1991	0.81	0.62	0.58	0.45	0.42
1992	0.77	0.60	0.63	0.45	0.44
1993	0.75	0.67	0.66	0.50	0.55
1994	0.78	0.78	0.72	0.47	0.58
1995	0.69	0.77	0.74	0.48	0.58
1996	0.72	0.84	0.74	0.48	0.62
1997	0.65	0.87	0.68	0.50	0.63
1998	0.64	0.91	0.57	0.52	0.69
1999	0.57	1.00	0.41	0.46	0.65
2000	0.46	0.80	0.36	0.42	0.70
Mean	0.72	0.72	0.52	0.42	0.47

Table 9: Types of separation of infrastructure from operations

	Organisational	***Institutional or Full***
Austria	From 1997	
Belgium	From 1998	
Denmark		From 1997
Finland		From 1995
France		From 1997
Germany	From 1994	
Italy	From 1998	
The Netherlands	From 1995	
Portugal		From 1997
Spain	From 1996	
Sweden		From 1988
United Kingdom		From 1993

APPENDIX 2: Figures

Figure 1: Efficiency development over time by country, total traffic



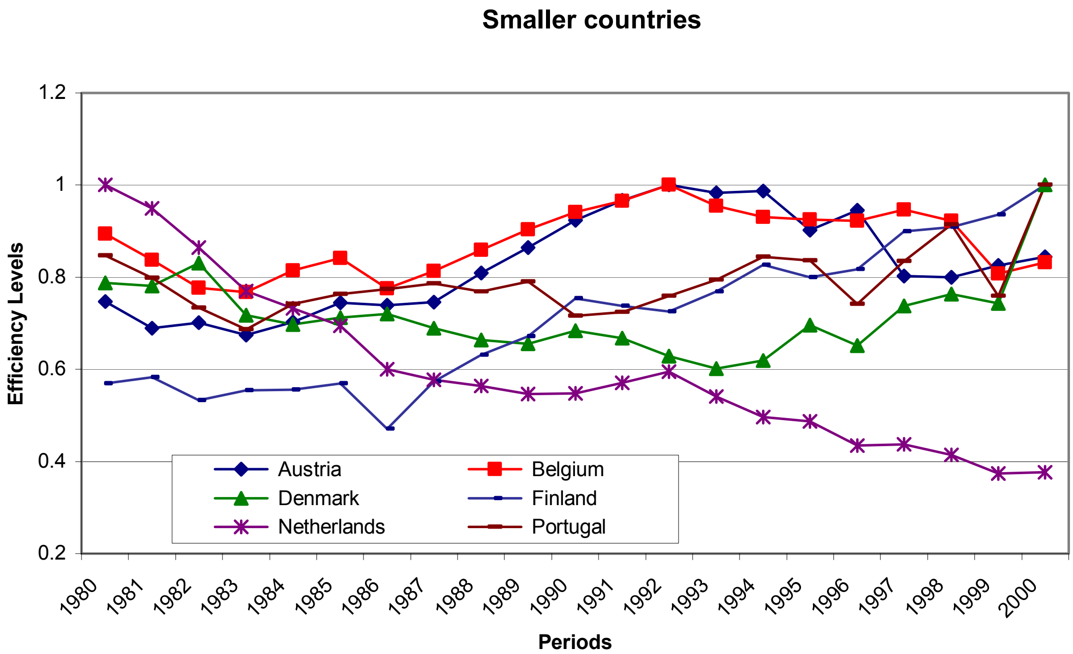
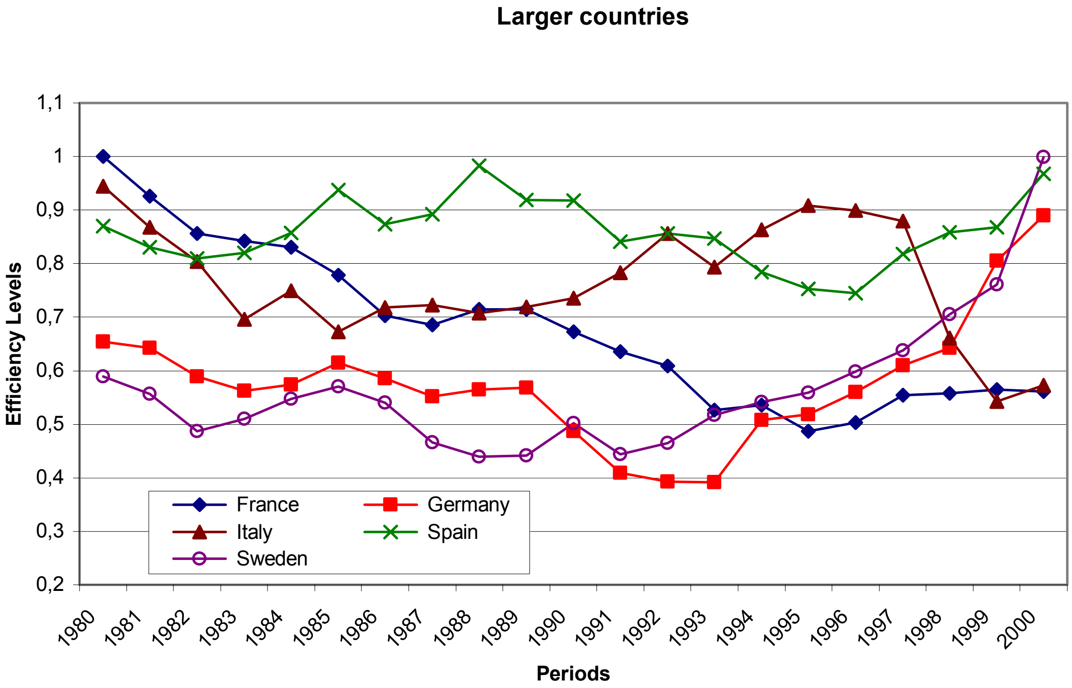


Figure 2: Efficiency development over time by country, passenger traffic



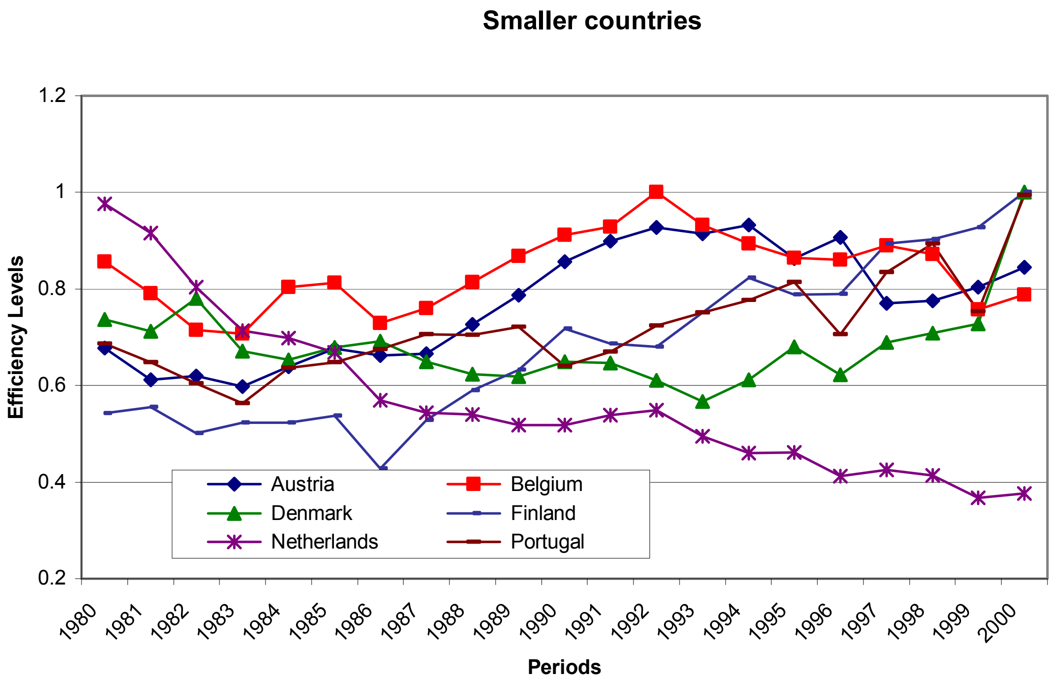


Figure 3: Normalized relative efficiency comparisons, total traffic

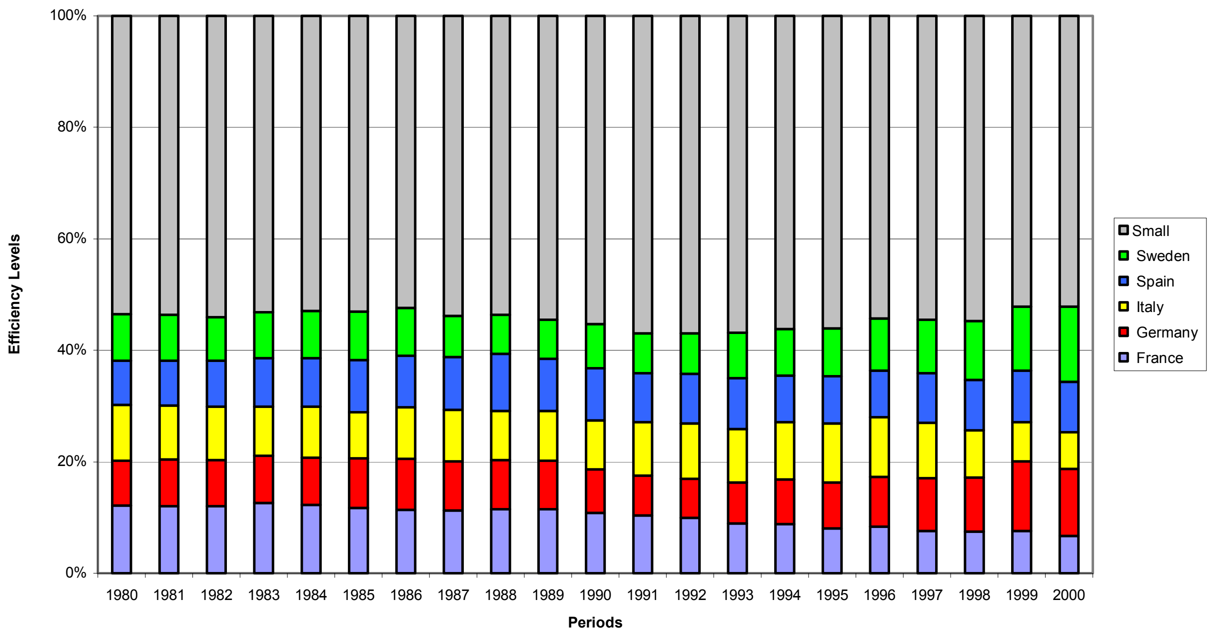


Figure 4: Normalized relative efficiency comparisons, passenger traffic

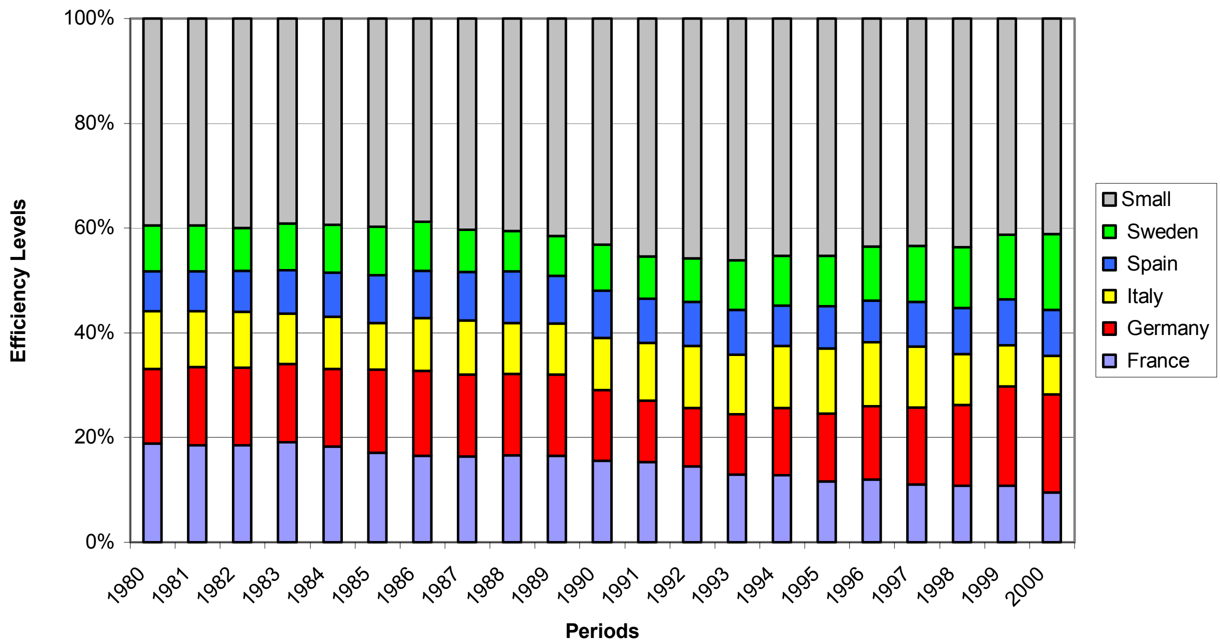


Figure 5: Normalized relative efficiency comparisons, larger countries, total traffic

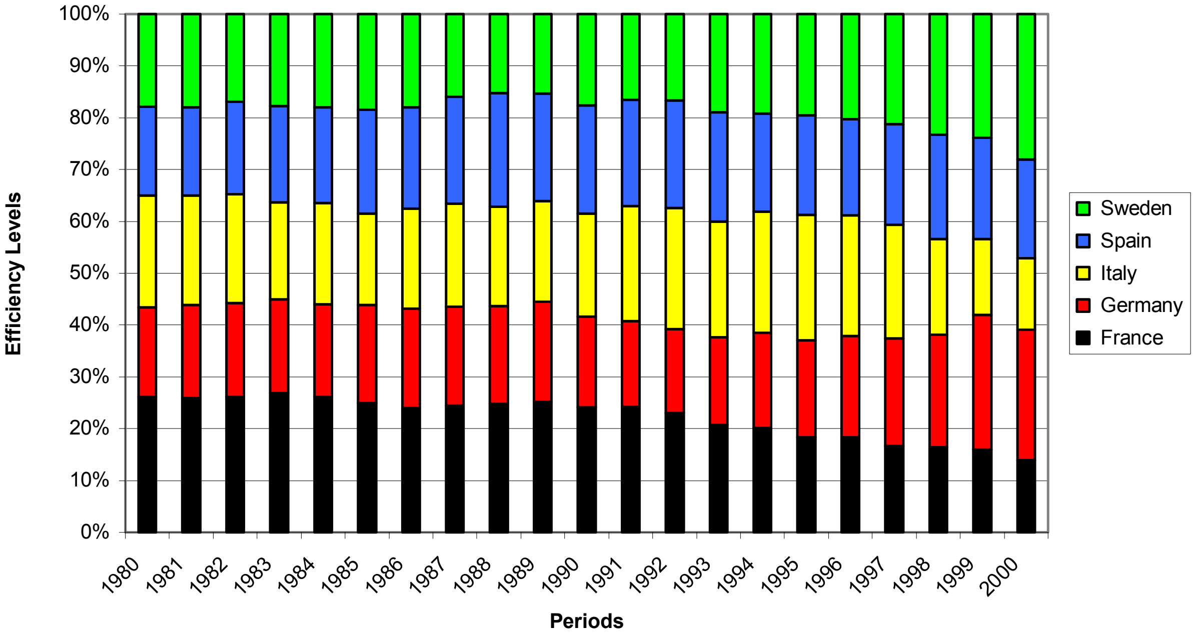


Figure 6: Normalized relative efficiency comparisons, larger countries, passenger traffic

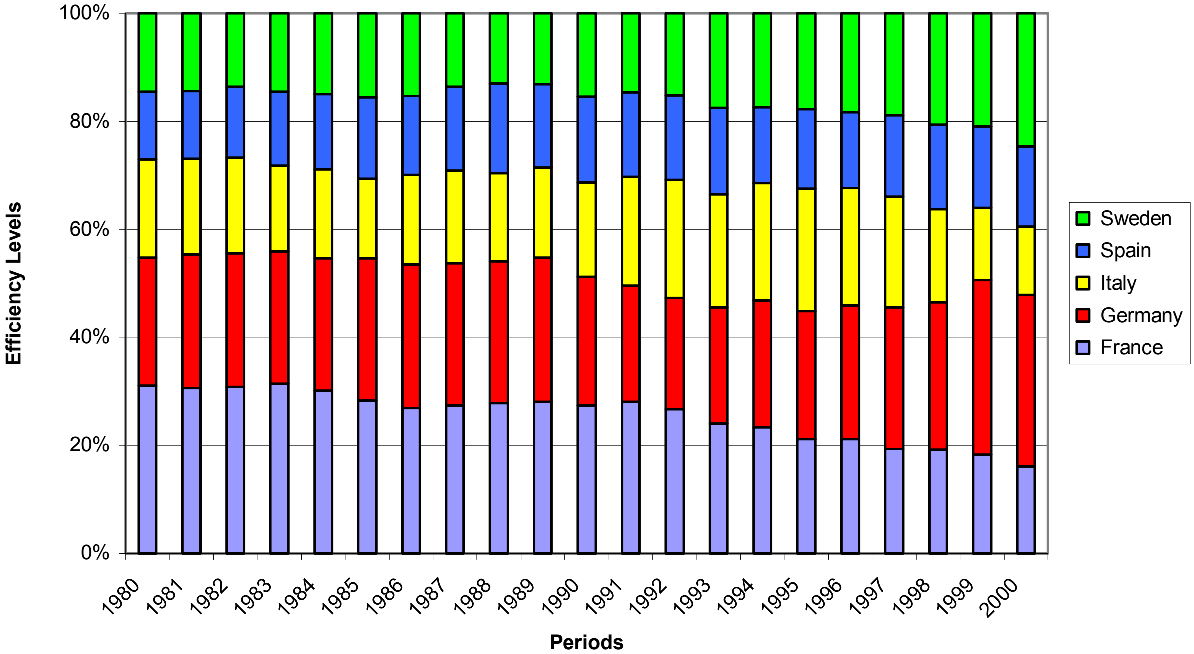


Figure 7: Relative efficiency comparisons, larger countries, by average of five-year periods, total traffic

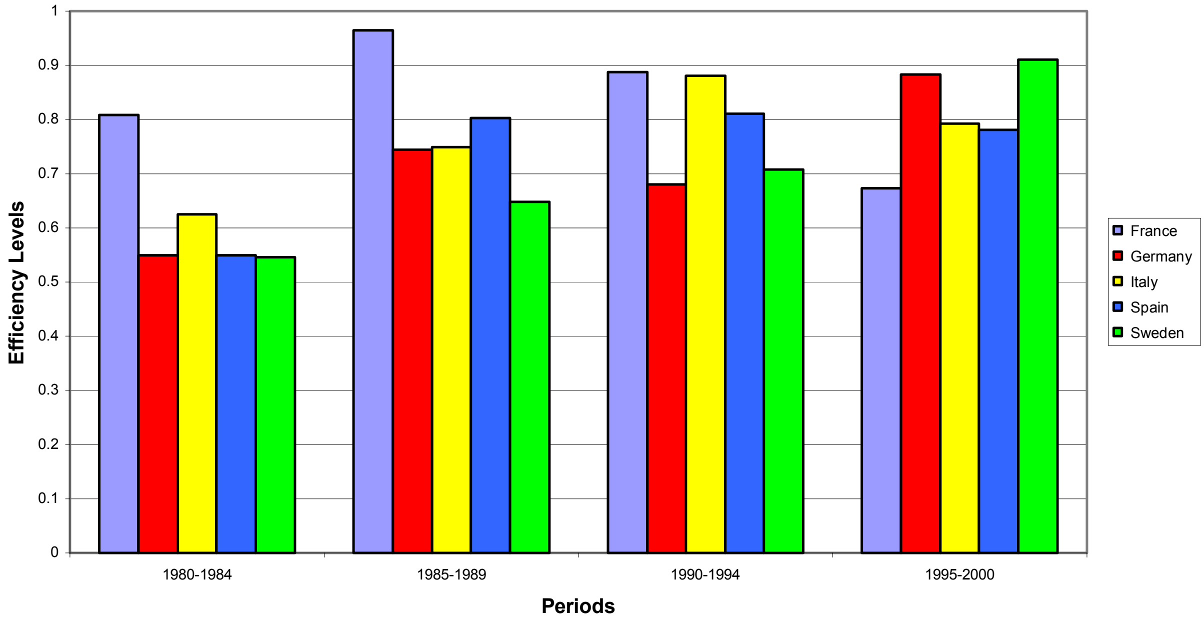


Figure 8: Relative efficiency comparisons, larger countries by average of five-year periods, passenger traffic

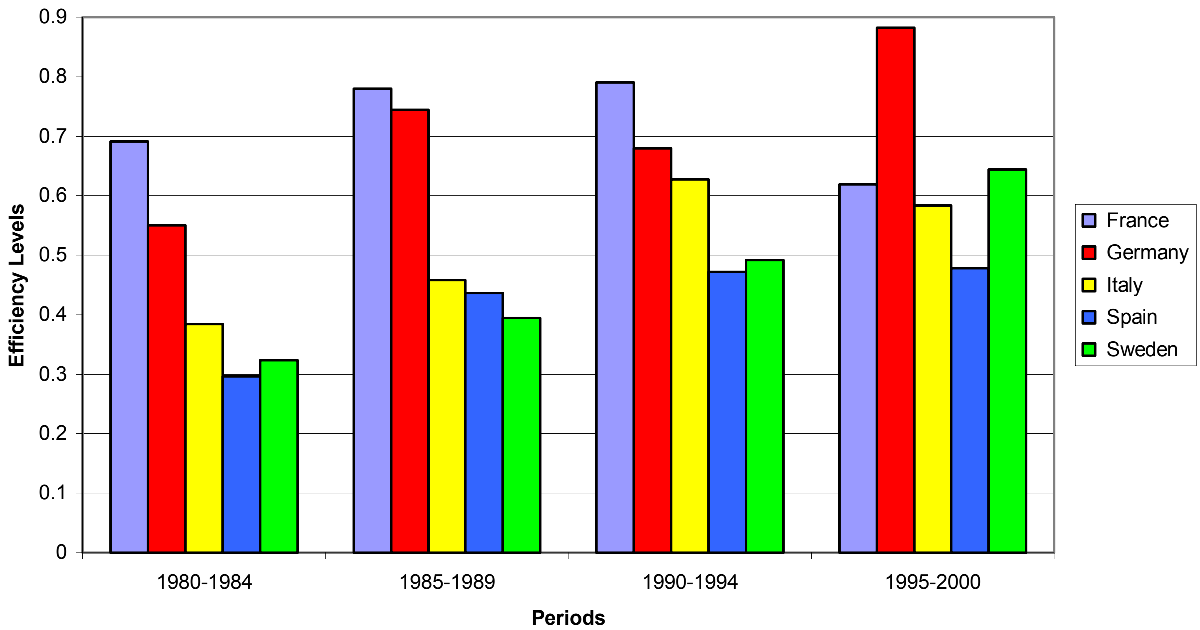
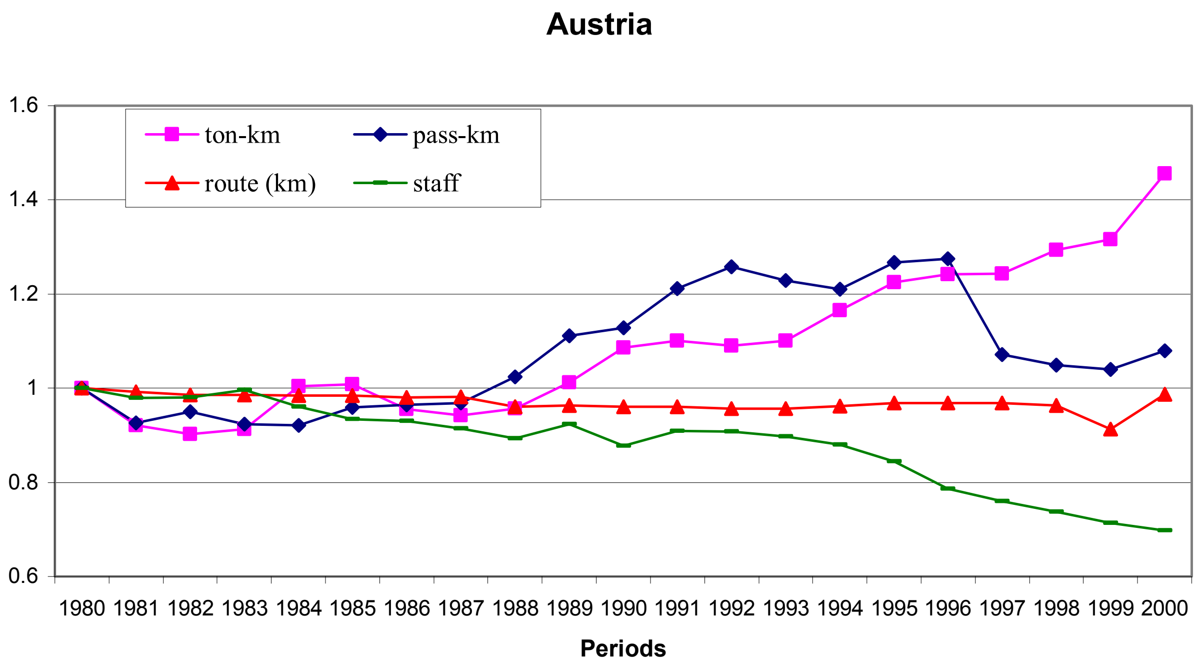
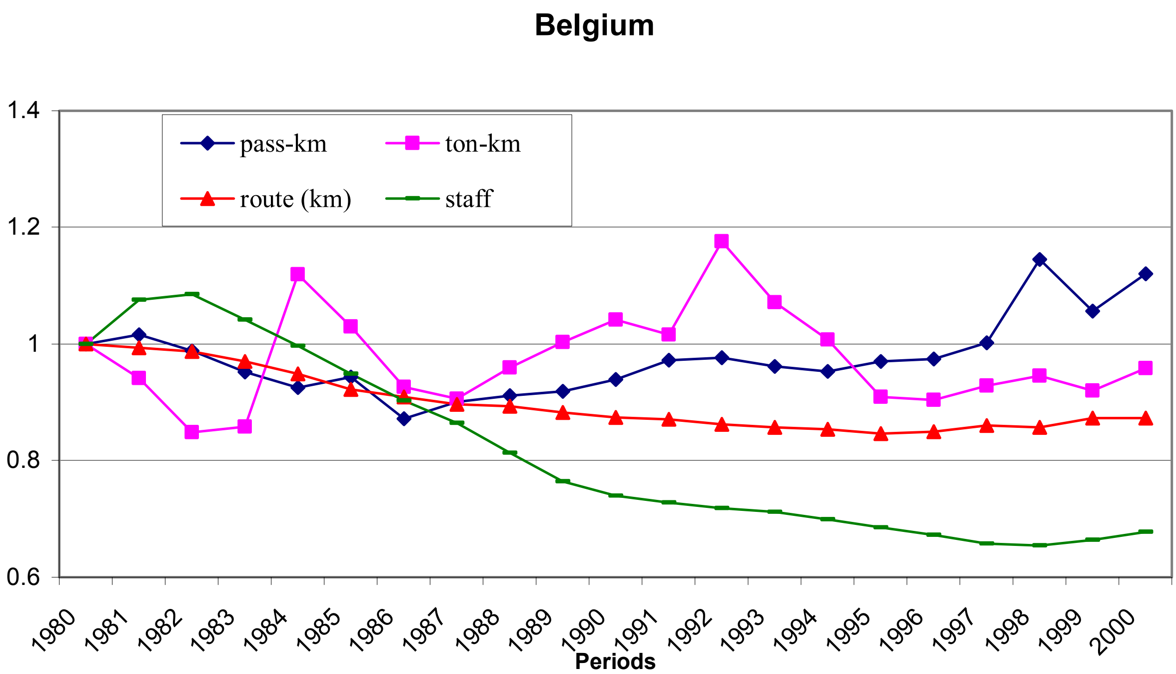
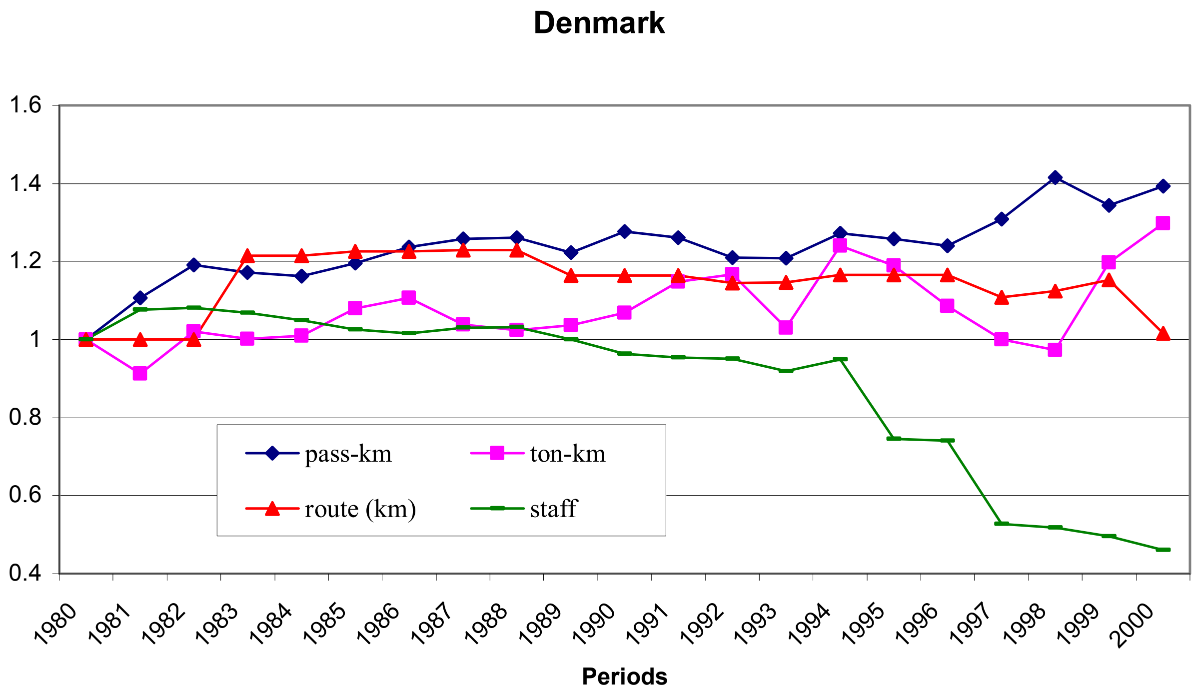
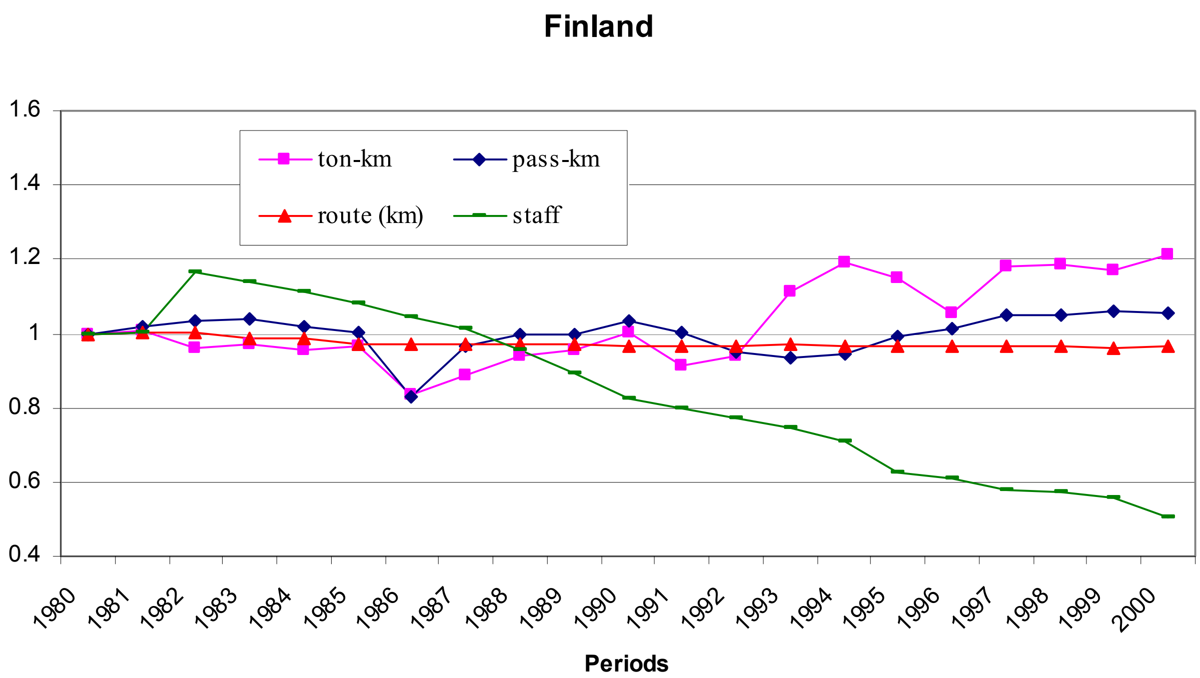


Figure 9: Input and output development of twelve countries over time.

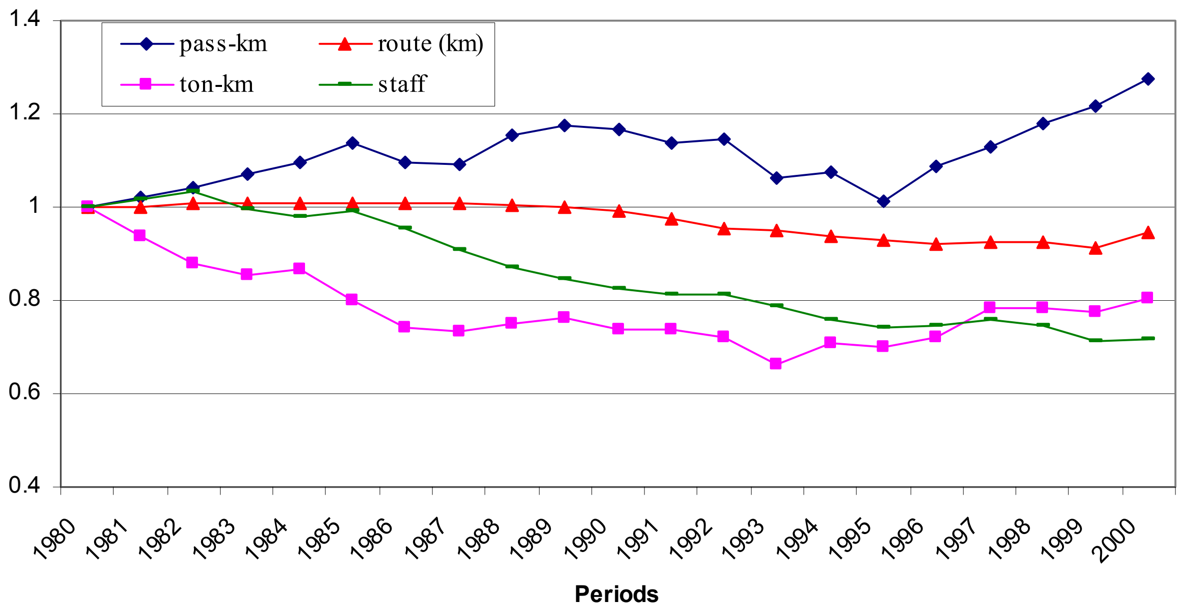




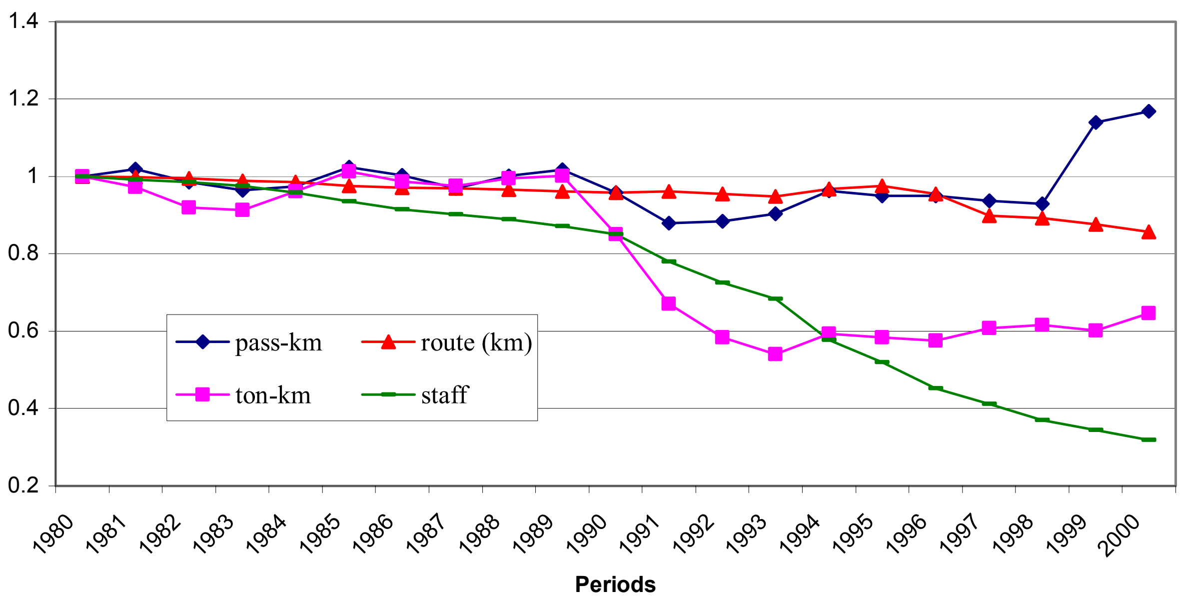

Denmark
1.6
1.4
1.2
1
0.8
0.6
0.4
pass-km
ton-km
route (km)
staff
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Periods


Finland
1.6
1.4
1.2
1
0.8
0.6
0.4
ton-km
pass-km
route (km)
staff
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Periods

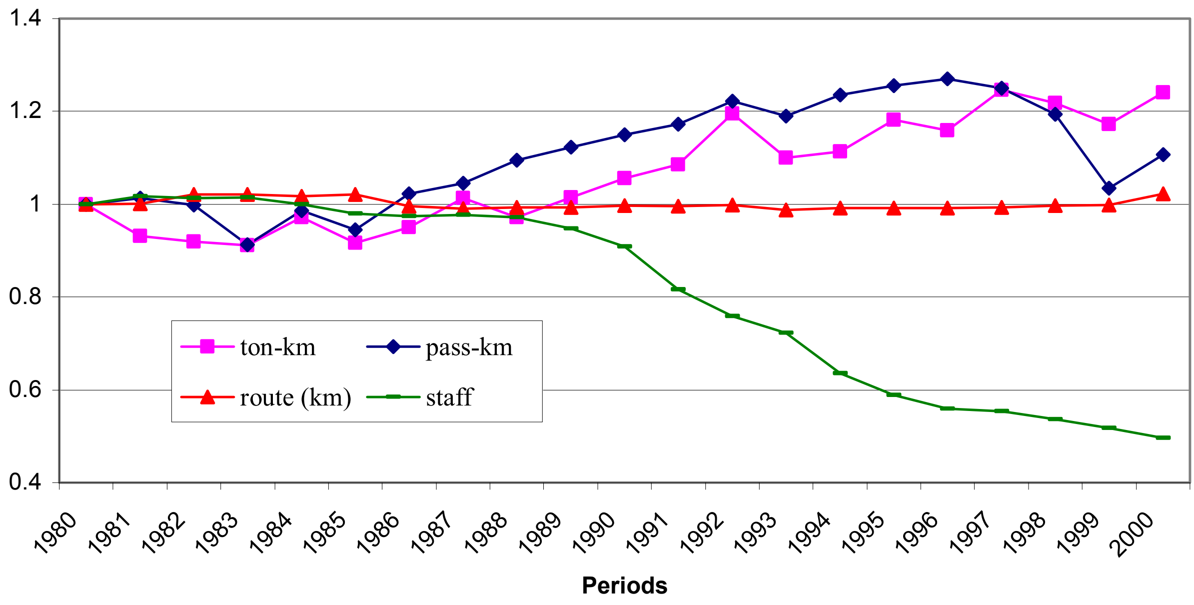
France



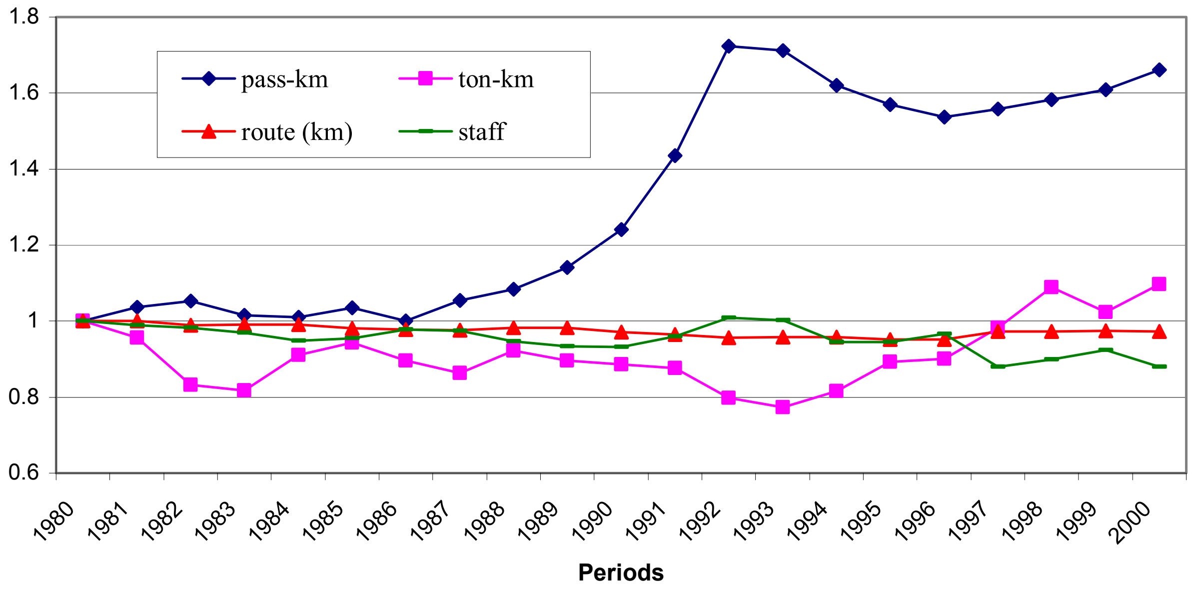
Germany

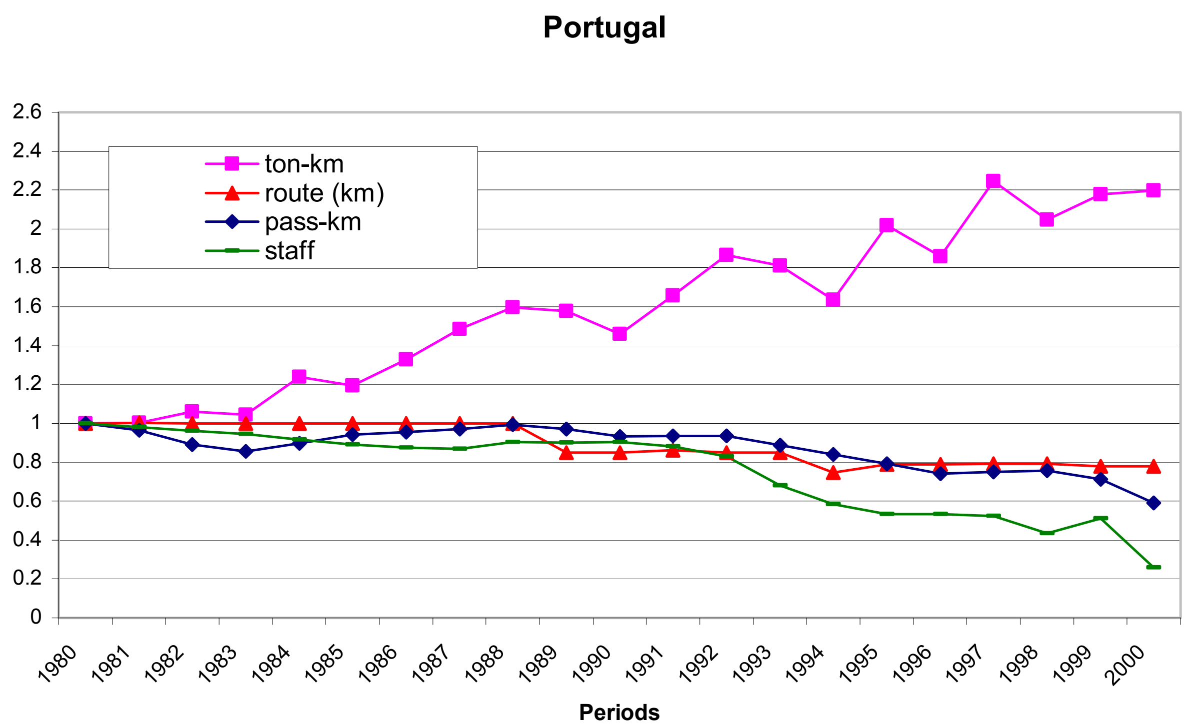


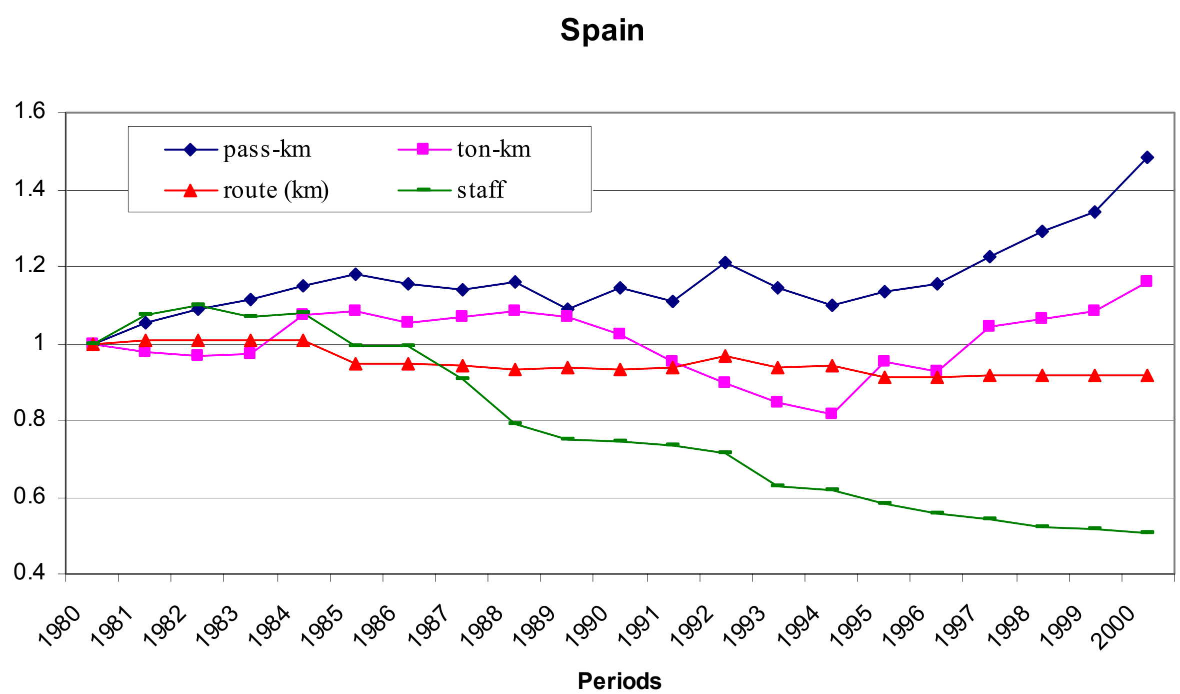
Italy



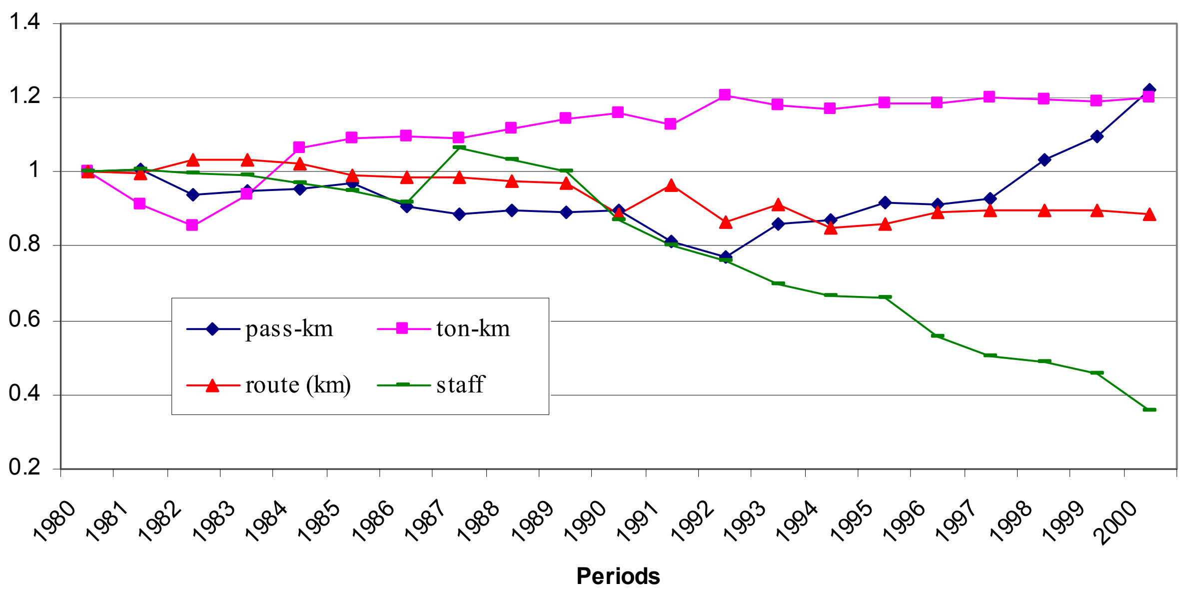
The Netherlands



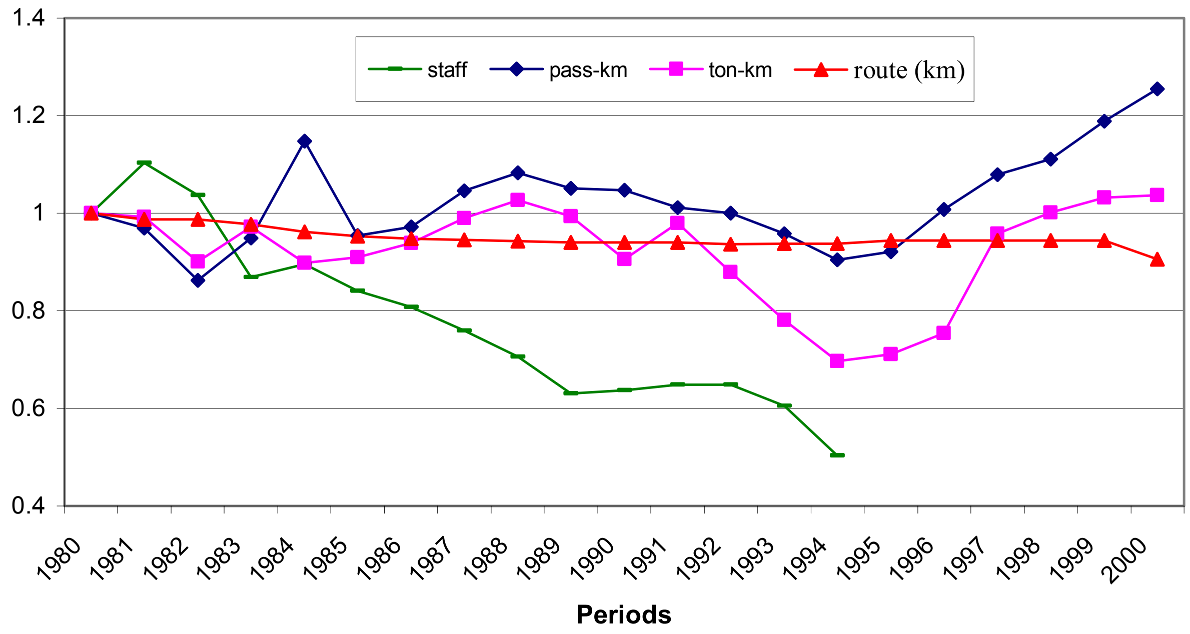

Portugal
ton-km
route (km)
pass-km
staff
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Periods


Spain
pass-km
ton-km
route (km)
staff
1.6
1.4
1.2
1
0.8
0.6
0.4
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Periods

Sweden



United Kingdom



APPENDIX 3: The mean-and-covariance structure analysis

Estimation of our model is complicated by the fact that individual effects and inputs are often correlated in empirical studies. The method proposed here has the convenient feature of estimating all possible correlations between inputs and individual effects, and hence between input quantities and individual technical efficiencies. The correlation problem is directly addressed by treating individual effects as latent variables. Another advantage of this method is that contrary to the usual instrumental variable estimation, it does not require additional information.

Definition of the LISREL method (Jöreskog (1973), "Analysis of covariances structures"):

There are:

A structural equation system of the form:

$$\eta = B\eta + \varsigma \qquad \text{(a)}$$

Here η is the vector of latent variables (individual effects), B the coefficient matrix of the regression of the latent variables on themselves, and ς are disturbances with zero means.

Measurement equations which link the latent variables η to the observed variables z according to:

$$z = \Lambda\,\eta + \varepsilon \qquad \text{(b)}$$

Λ is the coefficient matrix of the regression of the observed variables z on the latent variables η, and ε disturbances with zero means. It is assumed that: $E(\varsigma\varepsilon') = E(\eta\varepsilon') = 0$.
Let the variables z be measured as deviations from their respective means. The moment matrix of z can be expressed as:

$$\Sigma = \Lambda(I - B)^{-1}\Psi(I - B')^{-1}\Lambda' + \Theta$$

where $\Psi = E(\varsigma\varsigma')$ and $\Theta = E(\varepsilon\varepsilon')$.

The elements of Σ are functions of the elements of B, Λ, Ψ, and Θ, which can be referred to as fixed parameters or free parameters.

Estimation of the model entails choosing values for the free parameters so that the predicted moment matrix Σ fits the empirical moment matrix, denoted by S, in different ways depending on the estimation method used. All common fit functions are special cases of a general family (see Browne (1984)):

$$F(\Sigma, S) = (s - \sigma)'\Omega^{-1}(s - \sigma),$$

where $s = vec_*(S)$, $\sigma = vec_*(\Sigma)$.
F(.,.) is a twice differentiable continuous function which maps S and Σ into a non-negative scalar. It requires positive definitiveness of the weight matrix Ω.

The estimation we use here consists in maximizing the likelihood of the parameters. Under the assumption of a multivariate normal distribution of the observed variables, the fit function becomes:

$$F(\Sigma, S) = \log\det(\Sigma) + trace(S\Sigma^{-1}) - \log\det(S) - p$$

Minimizing this function is equivalent to maximizing the log-likelihood of the sample.

Application to our framework:

The system of equations defining a production function with individual effects can be equivalently formulated as:

$$\begin{cases} y_{it} = \beta x_{it} + \varepsilon_{it} \\ \varepsilon_{it} = w_t \gamma_i + v_{it} \end{cases} \quad i=1, \ldots, N,\ t=1, \ldots, T$$

Here, w_t represents our previous t and γ_i is the individual random parameter, called individual effects. v_{it} is an error-term which is identically and independently distributed with variance σ^2.

This system can be reconstructed as a multivariate system by stacking the *T* equations for each individual. The system becomes:

$$\begin{cases} y_i = (I_T \otimes \beta) x_i + \varepsilon_i \\ \varepsilon_i = W \gamma_i + v_i \end{cases} \quad i=1, \ldots, N$$

y_i, ε_i, and v_i are $(T \times 1)$ vectors, x_i is a $(KT \times 1)$ vector and W is the $(T \times 1)$ matrix resulting from stacking the *T* vectors w_t.

Our model in matrix form becomes the following structural equation system (page 37 Equation (a)):

$$\begin{bmatrix} 1 \\ y_i \\ x_i \\ \gamma_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ e_T\alpha & 0 & I_T \otimes \beta & W \\ \bar{x} & 0 & 0 & 0 \\ \bar{\gamma} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ y_i \\ x_i \\ \gamma_i \end{bmatrix} + \begin{bmatrix} 0 \\ \xi_i \\ x_i - \bar{x} \\ \gamma_i - \bar{\gamma} \end{bmatrix}$$

$$with \quad \eta \equiv \begin{bmatrix} 1 \\ y_i \\ x_i \\ \gamma_i \end{bmatrix}, \quad B \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ e_T\alpha & 0 & I_T \otimes \beta & W \\ \bar{x} & 0 & 0 & 0 \\ \bar{\gamma} & 0 & 0 & 0 \end{bmatrix}, \quad \varsigma \equiv \begin{bmatrix} 0 \\ \xi_i \\ x_i - \bar{x} \\ \gamma_i - \bar{\gamma} \end{bmatrix}$$

We can now interpret the stochastic production frontier model through the mean-and-covariance structure analysis.

The vector γ_i is treated as a vector of latent variables, and the moment matrix of ζ has the following form:

$$\Psi = E(\varsigma\varsigma') = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma^2 I_T & 0 & 0 \\ 0 & 0 & \Sigma_{x,x} & \Psi'_{\theta,x} \\ 0 & 0 & \Psi_{\theta,x} & \Delta \end{bmatrix}$$

where $\Sigma_{x,x}$ is the $KT \times KT$ covariance matrix of vector x_i, $\Psi_{\theta,x}$ is the $1 \times KT$ covariance matrix between the vector γ and the vector x_i. Δ is the covariance matrix (here a scalar) of the vector γ_i.

The measurement equations which link the observable variables y_i and x_i to the latent variable η take the following form:

$$z \equiv \begin{bmatrix} 1 \\ y_i \\ x_i \end{bmatrix} = \Lambda \begin{bmatrix} 1 \\ y_i \\ x_i \\ \gamma_i \end{bmatrix} = \Lambda\eta \quad \text{with} \quad \Lambda \equiv \begin{bmatrix} I_{T(K+1)+1} & 0 \end{bmatrix}$$