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On the influence of extreme parties in electoral competition with policy-motivated candidates

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Abstract We study and compare equilibrium platforms in models of unidimensional electoral competition with two and four policy motivated parties. We first analyze the plurality game, where the party getting the most votes is elected and implements its proposed platform. Restrictions on the set of credible announcements are needed to get existence of equilibria. Comparing equilibria with two and four parties, we obtain that moderate parties react to the introduction of extreme parties by proposing the same or more extreme equilibrium platforms. We then study the proportional system, where the policy implemented is a weighted sum of the proposals, with the voting shares as weights. Here, the existence of extreme parties leads moderate ones to choose more centrist platforms. We finally test the robustness of our results with respect to, first, the enlargement of the strategy space to entry decisions and, second, to asymmetric distributions of voters' blisspoints.

1 Introduction

The traditional model of electoral competition, known as the Downsian model, is characterized by two key features. First, the political parties are able to commit to the policy announced during the electoral campaign. Second, they do not care about the policy implemented, their only goal being to get elected. When there are two parties, the policy space is unidimensional and voters' preferences satisfy some conditions (single-peakedness, single-crossing), we end up with the *median voter theorem*: the parties adopt the same position at equilibrium, this position corresponding to the median voter's ideal policy. Considering multidimensional

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voting or more than two parties leads to much less optimistic and clear-cut results. In the first case, there is generically no equilibrium as shown for example by Plott (1967). When there are more than two candidates and voting is sincere, Osborne (1993) argues that an equilibrium is unlikely to exist.[1]

Although the Downsian approach has been the dominant model in political economy for a long time, its basic premises are questionable. Parties do have policy preferences and they happen to renege on their campaign promises. Roemer (2001), building on work by Wittman (1983) and keeping the assumption of commitment, develops a theory of political competition with parties having policy preferences. In the one dimensional case, he shows that parties propose the median ideal policy if there is no uncertainty, because parties must propose this policy in order to win the election and to influence the policy implemented. However, introducing uncertainty on the voting behavior of the citizens leads the parties to propose differentiated policies at equilibrium.

If one acknowledges that parties attach some importance to the policy implemented then commitment becomes an issue.[2] Indeed, in a one shot game, they should implement their preferred policy once they have been elected. Rational voters anticipate this and do not believe any promise different from the ideal policies of the parties. Matters are clearly different when the game is repeated. Alesina (1988) shows that parties credibly converge to the center of the political spectrum in a repeated elections setting. The basic idea supporting this result is that the parties, who are risk averse, prefer to have a moderate policy for sure than different extreme policies at random. In this model, voters are not strategic and the result only stems from the interaction between the parties. Aragones and Postlewaite (2000) consider also a repeated elections model, but with strategic voters, who punish the parties implementing a policy different from the one announced during the campaign. They obtain that the parties can credibly commit to policies belonging to an interval centered on their ideal policies. If they announce a platform too far away from their preferred one, the voters recognize that their incentives to renege on their promise is too high and they punish the party by not reelecting it in the future.

In this paper, we consider an electoral competition game with *policy motivated candidates*. Mainly, two questions are addressed. First, we are interested in knowing under which circumstances an equilibrium with *more than two parties* exists. Second, we want to determine the impact on the political equilibrium, starting from the standard two party case, of introducing extreme parties in the analysis. Our interest for this question has been aroused by the development of extreme political parties in many European countries, such as Italy, Belgium, the Netherlands, Austria or France. We study a setting where parties and voters have symmetric, single-peaked utility functions and where voting is sincere. The distribution of the voters' and parties' preferred policies is also symmetric.[3] Two electoral rules are considered: *plurality rule* and *proportional representation*. In the

[1] The issue of sincere vs. strategic voting does not arise in the model with two candidates where the winner implements its announced policy. Indeed, deleting weakly dominated strategies ensures that individuals vote for the party proposing the best platform from their point of view.
[2] If parties are indifferent with respect to the policy implemented, as in the Downsian framework, there is nothing outrageous in assuming that they stick to the policy announced during the campaign.
[3] We consider later on asymmetric distributions of voters' blisspoints.

first case, the party obtaining the largest number of votes is elected and chooses a policy. In the second case, the implemented policy is a weighted sum of the platforms of the parties, where the weights correspond to the number of votes received by the parties. This second framework has been adopted by Ortuño-Ortin (1997) in the two party case.

In this setting, we first show that there is generally no equilibrium under plurality rule with four parties, two moderates and two extremes, when any announcement is credible. From the discussion above, we know that the assumption of perfect commitment is not reasonable when parties are policy motivated. We thus follow Aragones and Postlewaite (2000) and assume that there exist credibility sets containing the credible announcements of a party. Under this assumption, equilibria with four parties possibly exist. Compared to the two party case, moderate parties *propose either the same or more extreme policies*. The intuition is the following. We start with the equilibrium proposals with only two parties, where each party proposes the credible policy that is closest to the median. Faced with the introduction of two extreme parties, moderate parties get more extreme if, by deviating, they attract more votes from extreme voters than they lose from centrists, i.e. if there are not too many centrists in the voters population. On the other hand, they cannot move closer to the center even if they wished since they already are on the boundary of their credibility set. In other words, the centripetal forces embedded in the plurality system are maximum with only two parties, and the introduction of extreme parties can only dampen these forces and lead to more extreme platforms from moderate parties.

Considering proportional representation, we find that an equilibrium with four parties generally exists and that moderate parties choose more moderate platforms than in the two party case. The intuition for this result goes as follows. Observe first that, with a symmetrical equilibrium, the policy implemented is the one preferred by the median voter. The left (moderate) party would thus like to decrease the implemented policy. Compared with the two party case, the introduction of extreme parties adds a new incentive for the left party to move closer to the median. Such a move will increase the extreme left party share of the vote at the expense of the left party, which is a very effective way to decrease the implemented policy since the extreme left party platform is the most leftist among the policies proposed. With an added incentive to move closer to the median, moderate parties end up with more moderate platforms than without extreme parties.

The paper is organized as follows. Section 2 presents the model. Section 3 studies plurality rule while Section 4 analyzes proportional representation. Section 5 tests the robustness of our results with respect to, first, the enlargement of the strategy space to entry decisions and, second, to asymmetric distributions of voters' blisspoints. Section 6 discusses our results and concludes. The proofs of the propositions are in the Appendix.

2 The model

The set of possible policies is $P=[-1,1]$. There are four political parties: extreme left (EL), left (L), right (R) and extreme right (ER). Their ideal policies are respectively denoted $\widehat{\alpha}_{EL}$, $\widehat{\alpha}_{L}$, $\widehat{\alpha}_{R}$ and $\widehat{\alpha}_{ER}$, with $-1<\widehat{\alpha}_{EL}=-\widehat{\alpha}_{ER}<\widehat{\alpha}_{L}=-\widehat{\alpha}_{R}<0$.

Preferences of party i, $i = EL, L, R, ER$, are given by a utility function $v_i : P \rightarrow \mathbb{R}$, assumed to be single-peaked and symmetric around its mode, $\widehat{\alpha}_i$.

Each voter has a single-peaked, symmetric utility function. The distribution of voters' ideal policies is given by a cumulative distribution function F over P. The density function, f, is continuous everywhere. We assume in what follows that f is symmetric around 0, except in Section 5.2. where we drop this assumption. Voting is assumed to be sincere, that is an individual votes for the party whose platform is closest to her ideal point. We denote by α_i the platform proposed by party $i = EL, L, R, ER$.

We study both the two party (L and R) and the four party (EL, L, R, ER) cases. The number of votes received by each party is

$$n_L = F\left(\frac{\alpha_L + \alpha_R}{2}\right) = 1 - n_R \tag{1}$$

in the two party case where $\alpha_L < \alpha_R$ and

$$n_{EL} = F\left(\frac{\alpha_{EL} + \alpha_L}{2}\right), \tag{2a}$$

$$n_L = F\left(\frac{\alpha_L + \alpha_R}{2}\right) - F\left(\frac{\alpha_{EL} + \alpha_L}{2}\right), \tag{2b}$$

$$n_R = F\left(\frac{\alpha_R + \alpha_{ER}}{2}\right) - F\left(\frac{\alpha_L + \alpha_R}{2}\right), \tag{2c}$$

$$n_{ER} = 1 - F\left(\frac{\alpha_R + \alpha_{ER}}{2}\right) \tag{2d}$$

in the four party case where $\alpha_{EL} < \alpha_L < \alpha_R < \alpha_{ER}$.

We first consider plurality rule before turning to the proportional case.

3 Plurality rule

In this section, elections are held according to plurality rule. The party with the largest number of votes is elected and implements its announced platform. In case of a tie, each party is elected with equal probability. Each party's goal is to maximize its expected utility. Party j ($j = EL, L, R, ER$)'s expected utility is given by:

$$\prod\nolimits_j (\alpha_{EL}, \ \alpha_L, \ \alpha_R, \ \alpha_{ER}) = \sum_{i=EL,L,R,ER} \pi_i(\alpha_{EL}, \ \alpha_L, \ \alpha_R, \ \alpha_{ER}) v_j(\alpha_i),$$

where π_i is the probability that party i gets elected.

In the rest of the section, with the exception of Proposition 1, we restrict the set of platforms that can be credibly made by parties. We adopt a simple formulation, which can be rationalized using the reputation model of Aragones and Postlewaite (2000). The set of credible promises of party i is comprised in the interval

$[\widehat{\alpha}_i - d_i,\ \widehat{\alpha}_i + d_i]$, with $i \in \{EL,\ L,\ R,\ ER\}$. We assume in the following that $d_i =$ d for all i. Moreover, the credibility intervals do not overlap: $\widehat{\alpha}_{EL} + d \leq \widehat{\alpha}_L - d$, $\widehat{\alpha}_L + d \leq \widehat{\alpha}_R - d$ and $\widehat{\alpha}_R + d \leq \widehat{\alpha}_{ER} - d$. Observe in particular that this implies that credible announcements of L (resp. R) are to the left (resp. right) of 0. If a party makes an announcement outside the credibility set, then the voters believe it would implement its ideal policy if elected.

We first study the equilibria with two political parties (L and R) and then with four parties.

3.1 Two-party competition

In the standard Wittman model without uncertainty, the two parties announce the ideal position of the median voter, namely 0. Here the only modification is that the parties will not choose a policy outside the credible set. The equilibrium platforms are then $\alpha_L = \widehat{\alpha}_L + d,\ \alpha_R = \widehat{\alpha}_R - d$. There is still “maximal” convergence.

3.2 Four party competition

We first show that there are no equilibria with four parties when any announcement is credible.

Proposition 1 Non existence of equilibria when any platform is credible
Assume that all platforms are credible and that, at equilibrium, policy announcements follow the same ranking as parties’ ideal policies: $\widehat{\alpha}_i < \widehat{\alpha}_j \Rightarrow \alpha_i \leq \alpha_j$. Then,

(i) There is no pure strategy symmetric equilibrium.
(ii) There is no asymmetric equilibrium if $\widehat{\alpha}_{EL} = -\widehat{\alpha}_{ER} \leq -1/2$ and

$$F(\widehat{\alpha}_{EL}) = 1 - F(\widehat{\alpha}_{ER}) \leq 1/4. \tag{3}$$

To prove part (i), we show that for any configuration of platforms whose ranking is the same as the ideal points ranking, at least one party has an interest to deviate marginally.[4] To prove part (ii), we have to use non-marginal deviations, for which the ranking of the proposed platforms differ from the ranking of ideal points. The meaning of the two assumptions exposed in part (ii) is clear. The assumption that $\widehat{\alpha}_{EL} = -\widehat{\alpha}_{ER} \leq -1/2$ makes sure that any extreme party prefers any policy on its side of the median voter’s blisspoint to any policy on the other side. Moreover, the EL (resp. ER) party prefers any policy to the left (resp. right) of $x > 0$ (resp. $x <$ 0) to policy x. Both consequences seem very much in accord with our intuition of what preferences of extreme parties should be. The second assumption guarantees that an extreme party cannot win the elections outright by proposing its most preferred policy. This puts an upperbound of one fourth on the proportion of voters with more extreme views than that of any extreme party.

[4] Except in the (symmetric) case where the two extreme parties are located at the boundaries of the political spectrum and the two moderate parties tie for winning. In that case, extreme parties resort to large deviations.

Note also that we have not shown in Proposition 1 that equilibria where the ranking of announcements is *not* the same as the ranking of parties' ideal points fail to exist. However, it seems to us highly improbable that voters would consider any announced platform as credible. We thus restrict in the following the set of platforms that can be credibly made by parties and give existence results when credibility sets are imposed.

Proposition 2 Symmetric equilibria where central parties tie for victory

Assume that only platforms in the interval $[\widehat{\alpha}_i - d,\ \widehat{\alpha}_i + d]$ are credible for party $i = EL,\ L,\ R,\ ER$.

(i) For all $d > 0$ such that credibility intervals do not overlap, the platforms $\alpha_{EL} = \widehat{\alpha}_{EL} - d,\ \alpha_L = \widehat{\alpha}_L - d,\ \alpha_R = \widehat{\alpha}_R + d$ and $\alpha_{ER} = -\widehat{\alpha}_{ER} + d$ constitute an equilibrium if

$$F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2}\right) \leq \frac{1}{4} \tag{4}$$

and

$$2f\left(\frac{\alpha_L + \widehat{\alpha}_R + d}{2}\right) < f\left(\frac{\alpha_L + \widehat{\alpha}_{EL} - d}{2}\right) \tag{5}$$

for all $\alpha_L \in [\widehat{\alpha}_L - d,\ \widehat{\alpha}_L + d]$.

(ii) For all $d > 0$ such that credibility intervals do not overlap, the platforms $\alpha_{EL} = \widehat{\alpha}_{EL} - d,\ \alpha_L = \widehat{\alpha}_L + d,\ \alpha_R = \widehat{\alpha}_R - d$ and $\alpha_{ER} = \widehat{\alpha}_{ER} + d$ constitute an equilibrium if

$$F(\widehat{\alpha}_L) \leq \frac{1}{2} - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2}\right) \tag{6}$$

and

$$2f\left(\frac{\alpha_L + \widehat{\alpha}_R - d}{2}\right) > f\left(\frac{\alpha_L + \widehat{\alpha}_{EL} - d}{2}\right) \tag{7}$$

for all $\alpha_L \in [\widehat{\alpha}_L - d,\ \widehat{\alpha}_L + d]$.

In the two kinds of equilibria described in this proposition, extreme parties are sure losers whereas L and R tie for winning. Extreme parties would like to deviate toward the extreme in order to induce the victory of the closest moderate party. They cannot credibly make such a deviation because their platform lies on the boundary of their credibility set. Condition (4), or in the second case, the more restrictive condition (6) guarantees that they do not wish to deviate towards the center. These conditions imply that the maximal number of votes obtained by an extreme party is always lower than the number of votes obtained by at least one moderate party, and therefore that extreme parties cannot win the election.

In the first equilibrium, moderate parties propose their most extreme credible platform. They do not deviate toward the center. Consider for example party L. If it deviates to the right, it attracts votes from centrist voters, who were previously supporting party R. However, it also loses votes to the benefit of EL. Condition (5) ensures that the net effect is negative and moreover that L loses more votes than R.

As a consequence, R is elected for sure following such a move and L should not deviate. Note that condition (5) can hold only if the density function $f(.)$ has at least two modes, so that such an equilibrium is excluded with a unimodal density function.

On the other hand, the second equilibrium described in the proposition is compatible with a unimodal density function. In this equilibrium, moderate parties would like to deviate towards the center, for the reason that a lot of people are concentrated there. Credibility prevents them from doing so.

In Proposition 2, extreme parties are sure losers.[5] We investigate in the following proposition whether there exist (symmetric) equilibria in which they have some chance of winning.

Proposition 3 Symmetric equilibria where extreme parties tie for victory
Assume that only platforms in the interval $[\widehat{\alpha}_i - d,\ \widehat{\alpha}_i + d]$ are credible for party $i = EL,\ L,\ R,\ ER$.

(i) At an equilibrium in which EL and ER tie for victory and L and R are sure losers, we necessarily have $\alpha_{EL} = \widehat{\alpha}_{EL} + d,\ \alpha_L = \widehat{\alpha}_L + d,\ \alpha_R = \widehat{\alpha}_R - d$ and $\alpha_{ER} = \widehat{\alpha}_{ER} - d$.

(ii) A sufficient condition for the existence of such an equilibrium for d small enough is given by

$$F\left(\frac{\widehat{\alpha}_{EL} + \widehat{\alpha}_L}{2}\right) > \frac{1}{4}. \tag{8}$$

(iii) A sufficient condition for the non-existence of such an equilibrium is given by

$$F\left(\frac{\widehat{\alpha}_{EL} + \widehat{\alpha}_L}{2}\right) < \frac{1}{2} - F\left(\min\left\{\frac{\widehat{\alpha}_{EL} - \widehat{\alpha}_L}{2},\ \widehat{\alpha}_L\right\}\right) \tag{9}$$

and

$$v_{EL}(\widehat{\alpha}_L + d) > \frac{1}{2} v_{EL}(\widehat{\alpha}_{EL} + d) + \frac{1}{2} v_{EL}(\widehat{\alpha}_{ER} - d). \tag{10}$$

The first part of the proposition identifies the only configuration of platforms that could constitute an equilibrium where extreme parties tie for victory. Since, at such an equilibrium, all parties have an incentive to deviate slightly towards the

[5] In addition to Proposition 2, there may also exist equilibria where moderate parties tie for winning by proposing their most preferred policy. First note that no other policy in the interior of the moderate parties' credibility intervals will be proposed at equilibrium, since moderates would rather deviate either to win outright or to tie with a policy closer to their blisspoint. Moderates do not deviate from their blisspoint if they cannot win by deviating. Locally, this requires that $2f(0) = f((\alpha_L + \alpha_{EL} - d)/2)$, which is an extremely improbable event. Moreover, looking at finite deviations would add so many demanding conditions that we just mention these potential equilibria for the sake of completeness. Finally, note that such equilibria do not contradict our main point that moderate platforms are more extreme with four than with two parties.

center of the policy spectrum,[6] they all propose their most moderate but still credible platform.

The second part of the proposition identifies a sufficient condition for the existence of such an equilibrium: condition (8), which says that people who prefer the ideal policy of party *EL* (resp. *ER*) outnumber those who prefer the ideal policy of *L* (resp. *R*). It does not then come as a surprise that an equilibrium with extreme parties winning exist. We know from the discussion in the previous paragraph that moderate parties are not tempted to make small deviations towards the extreme as it would induce the sure victory of the furthest extreme party. The requirement that d be small enough guarantees that they do not want to make large deviations either.

However, condition (8) is not necessary and one can find examples in which it is violated and an equilibrium nevertheless exists. Therefore, even if *L*'s ideal policy is more popular than *EL*'s, *L* may still lose the election at equilibrium. It will be the case if, by deviating to the left from $\widehat{\alpha}_L + d$, *L* obtains more votes than *EL* but less votes than *ER* which becomes the only winner. Condition (9) gives a lower bound on the popularity of *EL*'s ideal policy for an equilibrium with winning extreme parties to exist. If *EL*'s popularity is less than this threshold, *L* becomes the winner of the election when *EL* deviates to $\widehat{\alpha}_{EL} - d$. For this deviation to be in the interest of *EL*, (10) must hold, that is *EL* must prefer that *L* be the winner alone rather than a tie between *EL* and *ER*. This is a mild condition as it is satisfied as soon as the utility function of *EL* is concave on $[\widehat{\alpha}_{EL} + d,\ \widehat{\alpha}_{ER} - d]$. Together, conditions (9) and (10) then constitute sufficient conditions for the non-existence of any equilibrium where extreme parties tie for winning.

4 Proportional representation

We now turn to the case of proportional representation. In this setting, the policy implemented is not the announced policy of the party with the highest voting score but a weighted average of the policies announced by the four parties, where the weights correspond to the proportion of votes obtained by each party. This corresponds to a special case of the analysis in Ortuño-Ortin (1997) for two party competition. The policy implemented is given by

$$\alpha = n_L \alpha_L + n_R \alpha_R \tag{11}$$

and

$$\alpha = n_{EL} \alpha_{EL} + n_L \alpha_L + n_R \alpha_R + n_{ER} \alpha_{ER}, \tag{12}$$

in the cases of two party and four party competition respectively. The payoff function of party i is $\prod_i (\alpha_{EL},\ \alpha_L,\ \alpha_R,\ \alpha_{ER}) = v_i(\alpha)$.

As before we proceed in two steps, describing first the equilibrium of the two party game and then the four party game.

[6] the extreme parties in order to win, the moderates to make the closest extreme party win.

4.1 Two party competition

We describe in the following proposition the symmetric equilibrium when there are only two parties.

Proposition 4 A sufficient condition for the existence of a symmetric equilibrium of the two party competition game is that $F(x)$ be log concave ($f(x)/F(x)$ decreasing with x). The platforms proposed by the parties at equilibrium are:

$$\alpha_L^{2P} = -\alpha_R^{2P} = \begin{cases} -\frac{1}{2f(0)} \ if \ f(0) \geq \frac{1}{2} \\ -1 \ if \ f(0) < \frac{1}{2} \end{cases}.$$

The logic of the proof is the following. From (1) and (11), the implemented policy for given platforms α_L and α_R is

$$\alpha = F\left(\frac{\alpha_L + \alpha_R}{2}\right)\alpha_L + \left(1 - F\left(\frac{\alpha_L + \alpha_R}{2}\right)\right)\alpha_R.$$

At a symmetric equilibrium, $\alpha = 0$. Necessary conditions for an interior equilibrium are then that $d\alpha/d\alpha_L = 0$ and $d\alpha/d\alpha_R = 0$, which are given by, respectively, Eqs. (26) and (27) in the Appendix. Consider for example the case of party L. On the one hand, increasing α_L allows to increase the number of votes and therefore the weight received by α_L. It also decreases the weight received by the platform proposed by party R. These two effects correspond to the first and third term in (26). As long as $\alpha_L < \alpha_R$, the net effect is to decrease α. On the other hand, increasing α_L leads to a mechanical increase in α, for given weights. This effect is represented by the second term in (26). At the interior solution, both effects cancel out. Moreover, the condition that $f(x)/F(x)$ be decreasing with x is sufficient to guarantee that the second-order conditions are satisfied. As for the corner equilibrium ($\alpha_L = -1$, $\alpha_R = 1$), we check that α is increasing in both α_L and α_R at the equilibrium, meaning that no party has an incentive to deviate towards the center.

The condition we obtain on the hazard rate is mirrored in Ortuño-Ortin (1997, Proposition 4). Our restricting to symmetric equilibria allows us to go further than Ortuño-Ortin and to obtain information on the platforms proposed at equilibrium. Their most striking characteristic is that they are independent of the parties' most preferred policies (as long as $\widehat{\alpha}_L < 0$ and thus that $\widehat{\alpha}_R > 0$). One can also show that the interior equilibrium will emerge with all unimodal distributions of voters' ideal points. Moreover, a non-unimodal distribution is a necessary but not sufficient condition for the appearance of a corner equilibrium. Finally, note that in all cases the policy that is implemented is that most preferred by the median voter.

A last remark concerns the existence of asymmetric equilibria. Indeed, equilibria such that $\alpha = \widehat{\alpha}_R$ (resp. $\alpha = \widehat{\alpha}_L$) and $d\alpha/d\alpha_L = 0$ (resp. $d\alpha/d\alpha_R = 0$) cannot be a priori ruled out.

4.2 Four-party competition

Equilibria with four parties are analyzed in the following proposition.

Proposition 5 Any completely interior symmetric equilibrium ($-1 < \alpha_{EL} < \alpha_L$) of the four party competition game satisfies

$$\frac{d\alpha}{d\alpha_{EL}} = \frac{1}{2} f\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)(\alpha_{EL} - \alpha_L) + F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right) = 0,$$

$$\alpha_L = -\frac{1-4F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)}{2f(0)}.$$

A sufficient condition for the second-order condition to be satisfied is that $f(x)/F(x)$ be decreasing in x.

If $f(0) > 1/2$, any symmetric equilibrium is either completely or partially $(-1 = \alpha_{EL} < \alpha_L)$ interior. In all cases, the platforms chosen by L and R are closer to 0 than in the two party case.

The main difference between Propositions 4 and 5 is that moderate parties platforms are closer to the median with four than with just two parties. This is true whether the equilibrium is interior or not. Consider the case $f(0) \leq 1/2$. In the two party competition, the parties locate at the boundaries of the political space. Moderate parties are then necessarily closer to the center when there are four parties. Consider now the converse case $f(0) > 1/2$. This leads to an interior equilibrium in the two party case. We show that whether the equilibrium with four parties is completely or partially interior (a full corner solution is impossible in this case), the moderate parties choose more centrist policies.

The intuition for this result runs as follows. Consider the consequences of an increase in party L's platform. The effects described after the statement of Proposition 4 are still at work. On the one hand, the left party gains votes at the expense of the right party, the electoral constitution then putting more weight on the left platform and less on the right. On the other hand, for given weights the implemented policy increases following the increase in the left's platform. Observe first that this second effect is smaller with four parties since the vote share of the left is smaller $(F((\alpha_L + \alpha_R)/2) - F((\alpha_{EL} + \alpha_L)/2))$ than with two parties $(F((\alpha_L + \alpha_R)/2))$. Furthermore, another effect appears, since the left party now loses votes to the extreme left, which in turn lowers the value of the implemented policy. The net effect of the introduction of extreme parties is thus to make an increase of the left party platform more appealing, resulting in a more moderate platform at equilibrium. Notwithstanding this effect, the implemented policy is unaffected by the number of parties since we concentrate on symmetric equilibria.

5 Robustness of the results

In this section, we test the robustness of our results with respect to, first, the enlargement of the strategy space to entry decisions and, second, to asymmetric distributions of voters' blisspoints.

5.1 Entry in the political competition

In the previous section, we have restricted the strategy space of political parties to (credible) policy platforms. This implicitly assumes that parties always find in their interest to run for elections. A first look at the results obtained suggests that this is not always the case: in the equilibria identified by Proposition 2, an extreme party would be better off by dropping out since it would allow the closest moderate party to win *provided all running parties keep the same platforms*.

This argument implicitly assumes that whether to run and which platform to propose are simultaneous decisions. We think that a better way to model these choices is to assume that the decision to run is taken first and that the precise platform is chosen later. A first justification for this sequence can be found in the fact that conveying information on the platform proposed is a costly and time-consuming process which requires the existence of a political machine. Such a machine also takes time to build, and has to pre-exist at the time the platform is chosen. In our model, we equate the decision to build such a machine with the decision to run. A second justification results from the observation that the only activity of a party in our model is to run for elections: a party which does not run is equivalent to having no such party. One can then take a long run view and equate the decision to run and the decision to form a party in order to influence elections results. Founding a party implies sizeable stranded costs (in time, money and energy), so that parties' lifespan usually cover several rounds of elections. It then makes sense to model first the decision to found a party and then after the selection of an electoral platform. This is the strategy adopted by the related industrial organization literature: "Sunk costs are, by definition, a multiperiod phenomenon, as is entry deterrence. For the modeling, we will need an explicit dynamic model" (Tirole, 1992, p. 315).

We look for subgame perfect Nash equilibria (SPNE) of this sequential game. Our objective is to assess which of the four party equilibria described in the preceding sections survive the introduction of a prior stage where parties decide whether to enter or not the political arena. For the plurality case, we obtain that the equilibrium identified in Proposition 2 (ii) does not constitute a SPNE while those of Propositions 2 (i) and 3 may still arise. In other words, taking into account the decision to enter reinforces our point that the introduction of extreme parties induces moderate parties to choose more extreme platforms. As for the proportional case, we first obtain that all parties are indifferent whether to run for elections or not if the three-party electoral competition results in a symmetric equilibrium. On the other hand, we show by means of numerical simulations that there exist three party asymmetric equilibria resulting in a worse outcome for the party which does not run than the four-party (symmetric) equilibrium. In other terms, the equilibrium depicted in Proposition 5 is a SPNE of the two-stage game at least for certain configurations of the parameters.

We first address the plurality rule case and then turn to the proportional system.

5.1.1 Plurality rule

From the symmetry of our model, we only need to consider the entry decision of two parties, for example L and EL, the problem being identical for R and ER. As usual, this two-stage game is solved backward. We thus start by solving the electoral competition stage. Since we concentrate on the four party equilibria, we assume that parties other than the one under consideration run for elections. We then compare, from this party's viewpoint, whether its payoff is increased by running for government when all other parties do so.

Three party equilibria

a) Equilibrium if the extreme left party does not run
   In this case, the only possible equilibrium is for L to win the elections by proposing its most favored policy, $\widehat{\alpha}_L$. First note that L can always guarantee its sure victory by moving close enough to $\widehat{\alpha}_L + d$. Such a result is preferred by L to any credible policy offered by a right party. Further observe that if L wins with proposal $\alpha_L > \widehat{\alpha}_L$, it has an incentive to deviate to the left. Hence, the only possible equilibrium is for L to win the elections by proposing its most favored policy, $\widehat{\alpha}_L$.
b) Equilibrium if the left party does not run
   First, *ER* never wins at equilibrium since *EL* can always propose the platform symmetrical to that chosen by *ER* (which it prefers to *ER*'s platform) and gather more vote than *ER*. There are then three possible equilibria: *EL* wins for sure, *R* wins for sure or they tie. By the same argument as presented above, when a party i is a sure winner, its platform is necessarily $\widehat{\alpha}_i$, otherwise it should deviate slightly towards its ideal policy.

Entry decision We now turn to the entry decision. We first argue that the situation depicted in Proposition 2 (ii) is not a SPNE of the two-stage game, since any extreme party would prefer not to run for elections in this case. Under Proposition 2 (ii), moderate parties tie for winning and the result is a lottery between policies $\widehat{\alpha}_L + d$ and $\widehat{\alpha}_R - d$. Since both realizations are worse from *EL*'s viewpoint than the sure $\widehat{\alpha}_L$ it would get by not entering, *EL* prefers to stay out of the contest.

We then show that there exist configurations of parameters such that the situations depicted in Propositions 2 (i) and 3 are SPNE, i.e. that both moderate and extreme parties fare better when running for elections. We concentrate on the three party equilibrium in which *EL* is elected for sure when *L* stays out of the competition. This requires that

$$\begin{aligned} n_{EL} &= F\left(\frac{\widehat{\alpha}_{EL} + \alpha_R}{2}\right) \\ &> \max\left\{n_R = F\left(\tfrac{\alpha_R+\alpha_{ER}}{2}\right) - F\left(\tfrac{\widehat{\alpha}_{EL}+\alpha_R}{2}\right);\ n_{ER} = 1 - F\left(\tfrac{\alpha_R+\alpha_{ER}}{2}\right)\right\}. \end{aligned}$$

We start with Proposition 2 (i), where the two moderate parties adopt extreme platforms and tie for victory. The conditions for *L* and *EL* to be better off by entering are then respectively

$$\frac{1}{2} v_L(\widehat{\alpha}_L - d) + \frac{1}{2} v_L(\widehat{\alpha}_R + d) > v_L(\widehat{\alpha}_{EL}) \tag{14}$$

and

$$\frac{1}{2} v_L(\widehat{\alpha}_{EL} - d) + \frac{1}{2} v_{EL}(\widehat{\alpha}_R - d) > v_{EL}(\widehat{\alpha}_L). \tag{15}$$

Resorting to numerical simulations,[7] we show that conditions (13), (14) and (15) can be satisfied simultaneously. In words, L prefers to enter in order to avoid the implementation of an extreme policy by EL; EL wants to enter because it leads party L to propose a more extreme policy.

We now show that Proposition 3 can be a SPNE too. In this equilibrium, the two extreme parties are tying for victory when the four parties run for election. The conditions for L and EL to be better off by entering are then respectively

$$\frac{1}{2} v_L(\widehat{\alpha}_{EL} + d) + \frac{1}{2} v_L(\widehat{\alpha}_{ER} - d) > v_L(\widehat{\alpha}_{EL}) \tag{16}$$

and

$$\frac{1}{2} v_{EL}(\widehat{\alpha}_{EL} + d) + \frac{1}{2} v_{EL}(\widehat{\alpha}_{ER} - d) > v_{EL}(\widehat{\alpha}_L). \tag{17}$$

Once again, we resort to a numerical simulation to show that conditions (13), (16) and (17) can be satisfied simultaneously.[8] In words, L wants to enter because it induces a more moderate choice by EL; EL wants to enter because it has a one half chance of winning the elections.

We now turn to the proportional system.

5.1.2 Proportional system

As before, we examine the incentives to enter for the two leftist parties, L and EL. We first compute the first-order conditions of any (interior) equilibrium where the left party is the only one not to enter the race. In the case where this equilibrium is symmetric, we obtain that the right party proposes the central policy and gathers half of the votes while the other two parties each gather one fourth of the votes.[9] As in any symmetric equilibrium under proportional voting, the implemented policy is 0, and party L is indifferent between entering or not. Moreover, we obtain the same equilibrium whatever the identity of the party which does not run for elections, i.e. as long as two of the three parties are located on one side of the median voter's blisspoint.

More interesting is the possibility of a non-symmetric equilibrium. We obtain that in any asymmetric equilibrium, the central party still gathers one half of the votes while extreme parties always share unequally the remaining half. Comparing with the four party (symmetric) equilibrium, we obtain that a party from, say, the left has no incentive to drop out if not entering results in an asymmetric equilibrium with an implemented policy greater than zero. This is the case with the following cali-

[7] In this simulation, we consider a bimodal distribution for the ideal points of the voters. The preferred policies of the parties are $\widehat{\alpha}_{EL} = -0.8$, $\widehat{\alpha}_L = -0.4$, $\widehat{\alpha}_R = +0.4$, $\widehat{\alpha}_{ER} = +0.8$ and $d = 0.1$.

[8] The parameter values that we use are the same as in the previous footnote. The only modification concerns the ideal policies of the parties that become $\widehat{\alpha}_{EL} = -0.7$, $\widehat{\alpha}_L = -0.3$, $\widehat{\alpha}_R = +0.3$, $\widehat{\alpha}_{ER} = +0.7$.

[9] The second order conditions are satisfied if $f(x)/F(x)$ is decreasing in x, as in Propositions 4 and 5.

bration: the distribution of the citizens' preferred policies is a Beta distribution[10] with $a=b=2$. At equilibrium, party L proposes $\alpha_L=-0.67$ whereas R proposes $\alpha_R=0.44$ and ER proposes $\alpha_{ER}=0.83$, resulting in an implemented policy of $\alpha=0.014$.

[11] In this numerical example, all the parties enter the political competition in the first stage. The (symmetric) SPNE of the two-stage game is then described by Proposition 5.

5.2 Asymmetric distribution of blisspoints

The preceding sections study political competition in a perfectly symmetric environment: the distribution of voters' blisspoints, political parties' blisspoints, voters' and parties' utility functions are all symmetric. It is important to know to what extent our results depend on these symmetry assumptions. To test the robustness of our results, we relax the one symmetry assumption that seems the most contestable, i.e. the voters' blisspoints distribution. As suggested by an associate editor, if one views these blisspoints as reflecting the voters' income, a positively skewed distribution would be more relevant. We now look at how an asymmetric distribution of voters' blisspoints affects our results, first under proportional rule and then in the plurality case.

5.2.1 Proportional Rule

In this section, we show that the results obtained under a symmetric distribution of voters' blisspoints can easily be generalized to the case of an asymmetric distribution, provided that left parties' blisspoints are to the left of the median while right parties' blisspoints are to the right. Roughly speaking, the role played by policy zero in the symmetric case is now played by the median policy, that we denote by *med*. We still obtain that moderate parties locate closer to the median when extreme parties are added.

In the two party case, we obtain the following result, which is a generalization of Proposition 4.

Proposition 6 Keep all the assumptions described in Section 2 with two parties, except that the distribution function f does not have to be symmetric and that we do not impose the symmetry of parties' ideal policies but simply require that $\widehat{\alpha}_L < med < \widehat{\alpha}_R$. Sufficient conditions for the existence of asymmetric equilibrium of the two party proportional game are that $f(x)/F(x)$ be decreasing with x and $f(x)/(1-F(x))$ be increasing with x. The platforms proposed by the parties at equilibrium are:

[10] The density function of the Beta distribution is
$f(x) = \frac{1}{B(a,b)}(1+x)^{a-1}(1-x)^{b-1}$,
where $B(a,b) = \int_{-1}^{+1}(1+s)^{a-1}(1-s)^{b-1}ds$.

[11] This equilibrium holds for any $\widehat{\alpha}_L < 0.014 < \widehat{\alpha}_R < \widehat{\alpha}_{ER}$.

$$\alpha_L = med - \frac{1}{2f(med)} \tag{18}$$

$$\alpha_R = med + \frac{1}{2f(med)} \tag{19}$$

in case of an interior equilibrium, i.e. if $f(med) > max\left\{\frac{1}{2(1+med)}, \frac{1}{2(1-med)}\right\}$. The implemented policy is the median one and each party receives one half of the votes.

With a positively skewed blisspoints' distribution, we can have a full corner equilibrium ($\alpha_L = -1$, $\alpha_R = 1$) or a partial corner equilibrium with ($\alpha_L = -1$, $\alpha_R < 1$). In both cases, the left party has more than half of the votes and the implemented policy is negative.

The four party case is given by the following proposition, which generalizes Proposition 5.

Proposition 7 Keep all the assumptions described in Section 2 with four parties, except that the distribution function f does not have to be symmetric and that we do not impose the symmetry of parties platforms but simply require that $\alpha_{EL} < \alpha_L < med < \alpha_R < \alpha_{ER}$. Any completely interior symmetric equilibrium $(-1 < \alpha_{EL} < \alpha_L)$ of the four party competition game satisfies

$$\frac{d\alpha}{d\alpha_{EL}} = \frac{1}{2} f\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)(\alpha_{EL} - \alpha_L) + F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right) = 0$$
$$\alpha_L = med - \frac{1-4F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)}{2f(med)}.$$

A sufficient condition for the second-order condition to be satisfied is that $f(x)/F(x)$ be decreasing in x and $f(x)/(1-F(x))$ be increasing in x.

In symmetric completely interior as well as (partially) corner equilibria, the platforms chosen by L and R are closer to 0 than in the two party case.

We now turn to the plurality rule.

5.2.2 Plurality Rule

We assume wlog that the blisspoints' distribution is positively skewed, so that $med < 0$, and that $\widehat{\alpha}_L + d < med < \widehat{\alpha}_R - d$. We first look at the equilibrium platforms with two parties and plurality voting. It is easy to see that both parties will converge as close to the median policy as credibility allows them to. Under the assumption that $\widehat{\alpha}_L = -\widehat{\alpha}_R$ and that the credibility intervals have the same length for all parties, party L would then win the election for sure. We thus see a need to reformulate the assumption of symmetry of parties' blisspoints by assuming that they are symmetrical with respect to the median blisspoint, i.e.

$$F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_R}{2}\right) = \frac{1}{2} = n_L(\widehat{\alpha}_L,\ \widehat{\alpha}_R). \tag{20}$$

Our first conclusion is then that the symmetry of the parties' blisspoints has to be redefined with respect to the median voter's blisspoint. We do not try in this paper to endogeneize the parties' blisspoints location and leave this topic for future research. One can nevertheless conjecture that endogenous parties' blisspoints would satisfy condition (20) since it is a necessary condition for parties to influence the implemented policy. If this condition is satisfied, we still obtain maximum convergence towards the median.

Turning to the four party case, we need to impose that the two parties tying for victory have the same number of votes at equilibrium. On the other hand, we do not need to impose symmetry assumptions for the platforms of the loosing parties. In other terms, loosing parties (the two extreme parties in Proposition 2 and the two moderate in Proposition 3) need not have the same number of votes at equilibrium.

Moreover, the asymmetry of the distribution of voters' blisspoints implies that deviations incentives are typically different for leftist and rightist parties, and that one cannot concentrate, as in the symmetric case, on one extreme and one moderate party. More precisely, the conditions listed in Propositions 2 and 3 make sure that parties L and EL have no interest in deviating from the equilibrium locations. For these equilibria to carry through to the asymmetric distribution case, one must explicitly impose similar conditions on parties R and ER.

To illustrate this, we present the following result, which is a generalization of Proposition 2 (ii). Propositions 2 (i) and 3 are generalized in the same way and are not reported here for the sake of space.[12] The reader can check that conditions (22) and (23) are equivalent to condition (6) when f is symmetric, while conditions (24) and (25) collapse to condition (7). Condition (21) is necessarily satisfied under the assumptions of Section 2.

Proposition 8 Asymmetric voters' blisspoints distribution: Symmetric equilibria where central parties tie for victory.

Keep all the assumptions described in Section 2 with four parties, except that the distribution function f need not be symmetric. Assume that only platforms in the interval $[\widehat{\alpha}_i - d,\ \widehat{\alpha}_i + d]$ are credible for party $i = EL,\ L,\ R,\ ER$.

For all $d > 0$ such that credibility intervals do not overlap, the platforms $\alpha_{EL} = \widehat{\alpha}_{EL} - d,\ \alpha_L = \widehat{\alpha}_L + d,\ \alpha_R = \widehat{\alpha}_R - d$ and $\alpha_{ER} = \widehat{\alpha}_{ER} + d$ constitute an equilibrium if

$$F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_R}{2}\right) - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2}\right) = F\left(\frac{\widehat{\alpha}_R + \widehat{\alpha}_{ER}}{2}\right) - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_R}{2}\right), \tag{21}$$

$$F(\widehat{\alpha}_L) \leq F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_R}{2}\right) - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2}\right) \tag{22}$$

$$1 - F(\widehat{\alpha}_R)F\left(\frac{\widehat{\alpha}_R + \widehat{\alpha}_{ER}}{2}\right) - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_R}{2}\right) \tag{23}$$

$$2f\left(\frac{\alpha_L + \widehat{\alpha}_R - d}{2}\right) > f\left(\frac{\alpha_L + \widehat{\alpha}_{EL} - d}{2}\right) \tag{24}$$

[12] Statements of results and proofs are available upon request.

for all $\alpha_L \in [\widehat{\alpha}_L - d,\ \widehat{\alpha}_L + d]$ and

$$2f\left(\frac{\widehat{\alpha}_L + \alpha_R + d}{2}\right) > f\left(\frac{\alpha_R + \widehat{\alpha}_{ER} + d}{2}\right) \tag{25}$$

for all $\alpha_R \in [\widehat{\alpha}_R - d,\ \widehat{\alpha}_R + d]$.

The conditions imposed on the rightists parties are very likely to remain satisfied when one starts with an equilibrium under a symmetric distribution and perturbs a little this distribution. In this sense, the results depicted in Section 3.2. are robust to the introduction of a small asymmetry in the voters' blisspoints distribution. This reasoning is upheld by the results of the following simulation. We start with a symmetrical Beta(5,5) distribution function f with $\widehat{\alpha}_{EL} = -0.8 = -\widehat{\alpha}_{ER},\ \widehat{\alpha}_L = -0.4 = -\widehat{\alpha}_R$ and $d = 0.1$. We check that the conditions listed in Proposition 2 (ii) are satisfied so that there exists a symmetrical equilibrium with platforms $\alpha_{EL} = -0.9 = -\alpha_{ER},\ \alpha_L = -0.3 = -\alpha_R$ for which each extreme party garners 1.96% of the votes while moderate parties each have 48.04%. We then modify the distribution function to introduce some positive skewness by modeling f as a Beta(4.5, 5) distribution. We have to modify the parties' blisspoints so that condition (21) holds. In absence of explicit modeling of the parties' blisspoint determination, we choose a configuration of parties' blisspoints where all blisspoints slide slightly to the left when the distribution becomes positively skewed. More precisely, we assume that $\widehat{\alpha}_{EL} = -0.85,\ \widehat{\alpha}_L = -0.468,\ \widehat{\alpha}_R = 0.35$ and $\widehat{\alpha}_{EL} = 0.75$. We then obtain that the conditions listed in Proposition 8 are satisfied so that there exists an equilibrium with platforms $\alpha_{EL} = -0.95,\ \alpha_L = -0.368,\ \alpha_R = 0.25$ and $\alpha_{EL} = 0.85$ for which moderate parties tie for winning with 47.89% of the votes each. Note also that, as stated above, the extreme parties do not have the same number of votes at equilibrium, with 1.82% for party *EL* and 2.4% for party *ER*.

6 Conclusion

This paper constitutes an attempt to formalize electoral competition with more than two parties. Two main results emerge. First, equilibrium existence with four parties competing for election is unlikely under plurality unless one places bounds on the promises that parties can credibly make. Second, the introduction of extreme parties induces moderate parties to propose more extreme policies under plurality whereas we obtain the converse effect in a proportional system.

We have tested the robustness of our results with respect to two departures from our basic assumptions. First, we add a first stage where parties decide whether to run for elections or not before choosing their electoral platform. We show that most of our results still hold, in the sense that there exist parameters values for which the Nash equilibria of Sections 3 and 4 also constitute subgame perfect Nash equilibria of this sequential game.

We also generalize our results to the case of an asymmetric distribution of voters' blisspoints. Results for the proportional case are basically unaffected, with the median policy appearing explicitly in the formulas. As for the plurality rule, for the results to carry through requires two modifications. First, unlike in the pure symmetric case, it is now necessary to check the incentives to deviate of all the parties, and not only of the, say, leftist parties. This requirement does not threaten the robustness of our results with respect to small departures from a symmetric distribution function. With a symmetric distribution function, incentives to deviate for parties L and R are identical. By continuity, they remain very close when considering small departures from the symmetry of the distribution function. We illustrate this reasoning by mean of a numerical example.

The second modification required by the introduction of an asymmetric distribution function is to modify the winning parties' blisspoints in order to keep the symmetry of these platforms. In absence of such a modification, one party would win the election for sure. It is difficult to think of this situation as a long term equilibrium. Opposing parties would then have a strong incentive to move to increase their chance of getting elected. Since credibility prevents parties from moving further from their blisspoint, it seems reasonable to think that the blisspoints themselves will move with time. We do not explicitly model the process of parties' blisspoints determination in this paper, but simply state the symmetry condition that is needed for our results to generalize to asymmetric distribution functions. We plan to endogeneize this determination in future work.

We believe that these results are of interest, allowing to explain some of the features of the political systems in European countries, in which many political parties are active. As one referee pointed out, many democracies exhibit three parties, one at the center of the policy spectrum and one on each side. Although the main focus of this paper is on the impact of the introduction of extreme parties on the platforms proposed by moderates, our model enables us to shed some light on the effect of a centrist party on the moderates' decisions. We consider three symmetric parties and a symmetric distribution function f, so that the center party C's ideal policy is at 0. We have already seen in Section 5.1.2 that the symmetric equilibrium under proportional representation is such that the center party proposes zero and gathers one half of the votes while the L and R parties each get one fourth. The implemented policy is zero, the same as when C does not run. Party C is then indifferent whether to run or not.

We now turn to the plurality case. It is easily seen that at equilibrium, the center party proposes its ideal policy while the other two parties locate as close to the center as is credible.[13] The incentives faced by the two moderate parties are basically the same as in the two party case. The implemented policy is either zero (sure victory of C, in which case the moderate parties are in fact indifferent as to their location) or a lottery between either the two moderate platforms or the three platforms with in both cases an expected policy of zero.

We now check whether this three party equilibrium is a SPNE of the sequential game where parties choose first to enter and then their platform. A moderate party, by not entering, guarantees the sure victory of party C with a platform of zero. It is then indifferent as to whether to enter if the three party equilibrium also results in the sure victory of party C. On the other hand, if the three party equilibrium is a lottery (whose expected result is still zero), a risk averse moderate party will not run for elections. This suggests that the three-party configuration is not very stable.[14]

[13] It is well known that no equilibrium exists in absence of credibility sets.

[14] It is reminiscent of the results obtained by Palfrey (1984) in a three party model where two parties (simultaneously) choose their platform before the third one. In this model where parties maximize their plurality, the third party locates in between the other two but never wins the elections.

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1 Appendix

Before proving Proposition 1, we state the following useful lemma whose proof is immediate and left to the reader.

Lemma 1 if $\widehat{\alpha}_{EL} = -\widehat{\alpha}_{ER} \leq -1/2$ then

1) *EL* (resp. *ER*) prefers any policy $x < 0$ (resp. $x > 0$) to any policy $y > 0$ (resp. $y < 0$)
2) Fix any policy $x > 0$ (resp. $x < 0$). Then, *EL* (resp. *ER*) prefers any policy smaller (resp. larger) than x to policy x.

Proof of Proposition 1 (i) Symmetric equilibrium.
Four cases must be distinguished:

1) $\alpha_{EL} < \alpha_L < \alpha_R = -\alpha_L < \alpha_{ER} = -\alpha_{EL}$
In such a symmetric equilibrium, either all parties tie for victory or only two (the two moderate or the two extreme ones) do. If only two tie for victory, one sure loser can always ensure the victory of the party whose platform it prefers (i.e. the party on the same side of the political spectrum) by deviating.[15] If all parties tie for victory, each extreme party has an incentive to adopt a less extreme position in order to attract votes from moderate individuals.
2) $\alpha_{EL} = \alpha_L < \alpha_R = \alpha_{ER} = -\alpha_L$
Platforms α_L and α_R are implemented with probability 1/2. If *EL* deviates slightly to the left, then either *L* or *EL* is elected for sure and *EL* is better off. Therefore *EL* should deviate.
3) $\alpha_{EL} < \alpha_L = \alpha_R = 0 < \alpha_{ER} = -\alpha_{EL}$
As in case 1, extreme parties should deviate (marginally) towards the center when their probability of winning is not 0. In the converse case, a sufficiently large deviation towards the center allows an extreme party to be elected for sure with a preferred policy (without deviating, the policy implemented is 0).
4) $\alpha_{EL} = \alpha_L = \alpha_R = \alpha_{ER} = 0$
A left (resp. right) party deviates to the left (resp. right) and wins for sure with a policy it prefers to *0*.

(ii) Asymmetric equilibrium.
We first study the case where two or more candidates tie for victory. We treat separately the cases where winners offer different platforms and the case where they make the same offer. We first claim that, if parties tying for victory offer different platforms, extreme parties always have an incentive to deviate. A first

[15] If only extremes tie for winning, a moderate party should deviate towards the center. If only moderates tie, an extreme should propose a more extreme platform if feasible or should leapfrog the closest moderate to locate just next to the other moderate but closer to the center.

subcase arises when (at least) one of the tied winners is an extreme party. Three possibilities open up:

- the extreme party prefers its own platform to the platform proposed by any other tied winner. The extreme party then has an incentive to deviate towards the center, which ensures it to win the election.
- the extreme party is indifferent between its platform and that proposed by another tied winner. Its most preferred platform then lies in between these two platforms, and deviating towards the center allows it to win with a platform closer to its most preferred one.
- the extreme party prefers another tied winner's platform to its own. W.l.og., assume that this extreme party is *EL*. We first prove that the only platform that could be preferred by *EL* to its own is that proposed by *L*. The argument consists in two steps. First, it is impossible to have an equilibrium situation with tied winners and three proposed platforms on the same side of 0. In such a case, the extreme party with the only platform on a given side of 0 would win for sure by moving sufficiently closed to 0, and Lemma 1 guarantees that it would benefit from this move. Second, since two platforms must be located on each side of 0, Lemma 1 ensures that the only policy who might be preferred by an extreme party to its own is that proposed by its neighbor. Finally, if *EL* prefers the policy proposed by *L* to its own, and if both tie for winning, than *EL* can guarantee that *L* win for sure by deviating to -1.

The second subcase of tied winners with different platforms that we consider is that where no extreme party ties for winning, i.e. when the two moderate parties tie for winning with different platforms. In this case, it is impossible for both extreme parties to be indifferent between the policies proposed by *L* and *R*, and any non-indifferent extreme party can guarantee the victory of the moderate policy it prefers by moving to a slightly less extreme platform than the moderate party it does not like.

We now show that the situation where tied winners offer the same platform cannot be an equilibrium either. The reasoning above shows that no equilibrium can have three parties proposing the same policy (and thus ending up on the same side of 0) and tying for winning. We then have to treat the two following cases: one extreme and one moderate parties tying, and two moderates tying. Consider the first case, where *EL* and *L* tie for winning. They must propose a policy $x < 0$ (or else *EL* would be better off by deviating to the left and win the elections). Moreover, to tie for winning, *EL* and *L* must together win more than half of the votes, which means that the policy proposed by *R* (and a fortiori by *ER*) must be greater than $|x|$. But then, *ER* has an incentive to move to the immediate left of $|x|$ in order to win the election or make *R* win with a policy that it prefers to $x < 0$ according to Lemma 1. The other case with tied winners offering the same platform would involve both *L* and *R* proposing the same policy x. If x differs from 0, the extreme party located on the other side of zero would win outright by moving sufficiently close to zero (once more, lemma 1 ensures that this party benefits from this move). If $x = 0$, any extreme party would win by moving sufficiently close to zero and would benefit from this move.

We consider next the possibility of an equilibrium with one party being the sure winner. We first treat the case where the winner is a moderate party. Suppose w.l.o.g.

that this is party R. Its location must then be $\widehat{\alpha}_R$, otherwise R would have an incentive to move closer to its ideal point.

The number of votes obtained by each party when they propose different policies is

$$
\begin{aligned}
n_{EL} &= F\left(\tfrac{\alpha_{EL}+\alpha_L}{2}\right) = A \\
n_L &= F\left(\tfrac{\alpha_L+\widehat{\alpha}_R}{2}\right) - F\left(\tfrac{\alpha_L+\alpha_{EL}}{2}\right) = B \\
n_R &= F\left(\tfrac{\widehat{\alpha}_R+\alpha_{ER}}{2}\right) - F(\widehat{\alpha}_R) + F(\widehat{\alpha}_R) - F\left(\tfrac{\alpha_L+\widehat{\alpha}_R}{2}\right) = D + C \\
n_{ER} &= 1 - F\left(\tfrac{\widehat{\alpha}_R+\alpha_{ER}}{2}\right) = E.
\end{aligned}
$$

If EL deviates to $\widehat{\alpha}_R - \varepsilon$, with ε small, we have

$$
\begin{aligned}
n_{EL} &= F(\widehat{\alpha}_R) - F\left(\tfrac{\alpha_L+\widehat{\alpha}_R}{2}\right) = C \\
n_L &= F\left(\tfrac{\alpha_L+\widehat{\alpha}_R}{2}\right) = A + B \\
n_R &= F\left(\tfrac{\widehat{\alpha}_R+\alpha_{ER}}{2}\right) - F(\widehat{\alpha}_R) = D \\
n_{ER} &= 1 - F\left(\tfrac{\widehat{\alpha}_R+\alpha_{ER}}{2}\right) = E.
\end{aligned}
$$

If $A + B > \max\{C,\ D,\ E\}$, L is elected and implements α_L. Since $\widehat{\alpha}_{EL} \leq 1/2$, EL prefers any policy to the left of $\widehat{\alpha}_R$ to $\widehat{\alpha}_R$ and has therefore an incentive to deviate. If $C > \max\{A + B,\ D,\ E\}$, EL is elected with a more favorable policy than $\widehat{\alpha}_R$. On the other hand, if $\max\{D,\ E\} > \max\{A + B,\ C\}$, R or ER are elected and EL should not deviate. However, if ER deviates to $\widehat{\alpha}_R + \varepsilon$, with ε small, then $n_{EL} = A$, $n_L = B$, $n_R = C$, $n_{ER} = D + E$. Therefore, in the case $\max\{D,\ E\}$ $\max\{A + B,\ C\}$, ER is elected if it deviates and obtains a better policy.

Observe that this proof relies on the assumption that EL can credibly commit to the policy $\widehat{\alpha}_{EL} = \widehat{\alpha}_R - \varepsilon$. Moreover the reasoning extends to the case where $\alpha_{EL} = \alpha_L$.

Suppose now that party ER is the winner. By the same argument presented above, the platform chosen by this party must be $\widehat{\alpha}_{ER}$. Assumed for the moment that all parties propose different platforms. In this case, the number of votes for each party is given by:

$$
\begin{aligned}
n_{ER} &= 1 - F(\widehat{\alpha}_{ER}) + F(\widehat{\alpha}_{ER}) - F\left(\tfrac{\alpha_R+\widehat{\alpha}_{ER}}{2}\right) = A + B \\
n_R &= F\left(\tfrac{\alpha_R+\widehat{\alpha}_{ER}}{2}\right) - F\left(\tfrac{\alpha_L+\alpha_R}{2}\right) = C \\
n_L &= F\left(\tfrac{\alpha_L+\alpha_R}{2}\right) - F\left(\tfrac{\alpha_{EL}+\alpha_L}{2}\right) = D \\
n_{EL} &= F\left(\tfrac{\alpha_{EL}+\alpha_L}{2}\right) = E.
\end{aligned}
$$

If EL deviates to $\widehat{\alpha}_{ER} - \varepsilon$, we get $n_{ER} = A$, $n_R = C$, $n_L = D + E$ and $n_{EL} = B$.

Three cases must be distinguished:

– $D+E \in \max\{A,\ B,\ C,\ D+E\}$: L wins or ties;
– $C \in \max\{A,\ B,\ C,\ D+E\}$: R wins or ties;
– $B \in \max\{A,\ B,\ C,\ D+E\}$: EL wins or ties.

In all these three cases, *EL* is better off following the deviation since it prefers any policy to the left of $\widehat{\alpha}_{ER}$ to $\widehat{\alpha}_{ER}$. Finally, note that, under (3), A cannot be the (strict) maximum of A, B, C and $D+E$, since $A \leq 1/4$ means that one of the three other parties must receive at least 1/4 of the votes. Finally, the reader can check that the logic of this proof (*EL* deviating to $\widehat{\alpha}_{ER}-\varepsilon$ and ending up with a preferred policy) extends to the cases where the platforms of some parties (except of course *ER*) coincide.

Proof of Proposition 2 (*i*) We examine the deviations available to parties *EL* and *L*, the cases of parties *R* and *ER* being symmetric.

Deviations by EL. We first show that condition (4) implies that extreme parties are sure losers and that party *EL* does not gain by deviating. Party *EL* maximizes the number of votes it obtains by moving to $\widehat{\alpha}_{EL}+d$ in which case it gets $F((\widehat{\alpha}_L+\widehat{\alpha}_{EL})/2)$. On the other hand, the number of votes received by *R* is

$$F\left(\frac{\widehat{\alpha}_R+\widehat{\alpha}_{ER}}{2}+d\right)-\frac{1}{2}=\frac{1}{2}-F\left(\frac{\widehat{\alpha}_L+\widehat{\alpha}_{EL}}{2}-d\right).$$

Then *R* receives more votes than *EL* for any $d \geq 0$ if

$$F\left(\frac{\widehat{\alpha}_L+\widehat{\alpha}_{EL}}{2}\right)\leq\frac{1}{4}$$

which is condition (4). Notice further that this condition, together with the symmetry of the equilibrium, ensures that both intermediate parties receive more than one fourth of the vote and thus tie for victory. It follows from this analysis that *EL* should not deviate to the right, since it would not win and would cause party *R* to win for sure. Therefore condition (4) is sufficient (but not necessary) to ensure that party *EL* does not want to deviate to the right.

A deviation to the left by party *EL* is not profitable either. The policy announced would not be believed by the voters who would anticipate the choice of $\widehat{\alpha}_{EL}$ by *EL* if elected. This would then be equivalent to a deviation to the right by *EL*.

Deviations by L. As before a deviation to the left by *L* is equivalent to a deviation to $\widehat{\alpha}_L$. We then look at deviations to the right by party *L*. We write the number of votes received by parties *L* and *R* as a function of the location of party *L*, α_L:

$$\begin{aligned} n_L(\alpha_L) &= F\left(\tfrac{\alpha_L+\alpha_R}{2}\right)-F\left(\tfrac{\alpha_L+\alpha_{EL}}{2}\right)\\ &= F\left(\tfrac{\alpha_L+\widehat{\alpha}_R+d}{2}\right)-F\left(\tfrac{\alpha_L+\widehat{\alpha}_{EL}-d}{2}\right)\end{aligned}$$

and

$$\begin{aligned} n_R(\alpha_L) &= F\left(\tfrac{\alpha_R+\alpha_{ER}}{2}\right)-F\left(\tfrac{\alpha_L+\alpha_R}{2}\right)\\ &= F\left(\tfrac{\widehat{\alpha}_R+\widehat{\alpha}_{ER}}{2}+d\right)-F\left(\tfrac{\alpha_L+\widehat{\alpha}_L+d}{2}\right)\end{aligned}$$

The effect of a marginal deviation, $d\alpha_L$, on the number of votes is given by:

$$\frac{dn_L}{d\alpha_L} = \frac{1}{2}f\left(\frac{\alpha_L + \widehat{\alpha}_R + d}{2}\right) - \frac{1}{2}f\left(\frac{\alpha_L + \widehat{\alpha}_{EL} + -d}{2}\right)$$

and

$$\frac{dn_R}{d\alpha_L} = -\frac{1}{2}f\left(\frac{\alpha_L + \widehat{\alpha}_R + d}{2}\right).$$

When moving to the right, L receives additional votes from people previously supporting R. However it also loses votes to EL. A sufficient condition for any deviation to the right from $\alpha_L = \widehat{\alpha}_L - d$ not to be profitable for party L is that $n_L - n_R$ be strictly decreasing on the whole credibility interval, i.e. to have

$$2f\left(\frac{\alpha_L + \widehat{\alpha}_R + d}{2}\right) < f\left(\frac{\alpha_L + \widehat{\alpha}_{EL} - d}{2}\right)$$

for all $\alpha_L \in [\widehat{\alpha}_L - d,\ \widehat{\alpha}_L + d]$. Then R is elected for sure and L is strictly worse off. Note that this condition is satisfied only if the distribution of ideal points has at least two modes, since for $\alpha_L = \widehat{\alpha}_L - d$ the condition reduces to

$$2f(0) < f\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2} - d\right).$$

(*ii*) Party EL maximizes the number of votes it obtains by moving to $\widehat{\alpha}_{EL} + d$ in which case it gets a number of votes equal to $F\left(\frac{\widehat{\alpha}_{EL}+\widehat{\alpha}_L}{2} + d\right)$. This is less than $F(\widehat{\alpha}_L)$ since $\widehat{\alpha}_{EL} + d \leq \widehat{\alpha}_L - d$. Condition (6) then ensures that EL does not deviate towards the center and that moderate parties tie for winning. The proof that L does not wish to deviate follows the same line as above, using condition (7) instead of (5).

Proof of Proposition 3 (*i*) *Position of EL*. At equilibrium, we must necessarily have $\alpha_{EL} = \widehat{\alpha}_{EL} + d$, otherwise EL would have an incentive to make a small deviation to the right and become the winner for sure. A marginal move to the left would give rise to the victory of party ER and is therefore not desirable from the point of view of party EL.

Position of L. A marginal deviation to the left by L would lead to the election of ER. Because $\alpha_{EL} = -\alpha_{ER}$ and $\widehat{\alpha}_L < 0$, L prefers that EL be elected rather than ER. At equilibrium, we cannot have $\alpha_L < \widehat{\alpha}_L + d$, or L would prefer to deviate toward the center in order to ensure the election of EL.

(*ii*) We have to show that, for the configuration of platforms derived in (*i*), the extreme parties garner more votes than the center ones. The number of votes received by EL and L at equilibrium are given by

$$n_{EL} + F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2} + d\right)$$

and

$$n_L = \frac{1}{2} - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2} + d\right).$$

Consequently,

$$n_{EL} > n_L \Leftrightarrow F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2} + d\right) > \frac{1}{4}.$$

This is always true if (8) is satisfied.

The analysis in (i) has shown that no party has an interest to slightly deviate from its position. By continuity, no party has an incentive to deviate at all provided that d is low enough.

(iii) The sufficient conditions for an equilibrium *not* to exist are derived from the following observations. First, we write the number of votes obtained by party L when EL deviates to $\alpha_{EL} = \widehat{\alpha}_{EL} - d$:

$$n_L\big|_{\alpha_{EL}=\widehat{\alpha}_{EL}-d} = \frac{1}{2} - F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2}\right).$$

Moreover, we have that the number of votes received by ER is not affected by this deviation:

$$n_{ER} = F\left(\frac{\widehat{\alpha}_L + \widehat{\alpha}_{EL}}{2} + d\right) \leq \min\left\{F\left(\frac{\widehat{\alpha}_{EL} - \widehat{\alpha}_L}{2}\right),\ F(\widehat{\alpha}_L)\right\},$$

where the inequality follows from the fact that $\widehat{\alpha}_L + d \leq 0$ and $\widehat{\alpha}_{EL} + d \leq \widehat{\alpha}_L - d$ (following the assumption that the credibility sets do not overlap). Then if (9) is satisfied, n_L is larger than n_{ER} (which is itself larger than n_R and n_{EL}) and L is the winner of the election. Condition (10) simply states that EL prefers that L be the winner alone rather than a tie between EL and ER.

Proof of Proposition 4 The first-order conditions on α_L and α_R are:

$$\frac{d\Pi_L}{d\alpha_L} = 0 \Leftrightarrow \frac{d\alpha}{d\alpha_L} v_L'(\alpha) = 0$$

$$\frac{d\Pi_R}{d\alpha_R} = 0 \Leftrightarrow \frac{d\alpha}{d\alpha_R} v_R'(\alpha) = 0.$$

In a symmetric equilibrium, $\alpha = 0$. Therefore $\widehat{\alpha}_L < \alpha$ (resp. $\widehat{\alpha}_R > \alpha$) and $v_L'(\alpha) < 0$ (resp. $v_R'(\alpha) > 0$). It follows that the first-order conditions for an interior solution are satisfied if and only if

$$\frac{d\alpha}{d\alpha_L} = \frac{1}{2} f\left(\frac{\alpha_L + \alpha_R}{2}\right)\alpha_L + F\left(\frac{\alpha_L + \alpha_R}{2}\right) - \frac{1}{2} f\left(\frac{\alpha_L + \alpha_R}{2}\right)\alpha_R = 0 \tag{26}$$

$$\frac{d\alpha}{d\alpha_R} = \frac{1}{2} f\left(\frac{\alpha_L + \alpha_R}{2}\right)\alpha_L + 1 - F\left(\frac{\alpha_L + \alpha_R}{2}\right) - \frac{1}{2} f\left(\frac{\alpha_L + \alpha_L}{2}\right)\alpha_R = 0. \tag{27}$$

This yields

$$\alpha_L = -\alpha_R = -\frac{1}{2f(0)}$$

if $f(0) > 1/2$, where we have used the fact that $\alpha_L = -\alpha_R$ and $F(0) = 1/2$. If $f(0) \leq 1/2$,

$$\left.\frac{d\alpha}{d\alpha_L}\right|_{\alpha_L=-1,\alpha_R=1} = -f(0) + \tfrac{1}{2} \geq 0$$
$$\left.\frac{d\alpha}{d\alpha_R}\right|_{\alpha_L=-1,\alpha_R=1} = -f(0) + \tfrac{1}{2} \geq 0.$$

Therefore $\alpha_L = -1$ and $\alpha_R = 1$ is a local equilibrium when $f(0) \leq 1/2$.

It remains to be shown that the second-order conditions are satisfied for the interior solution. The second-order condition for party L is given by

$$\frac{d^2\Pi_L}{d\alpha_L^2} = \frac{d\alpha}{d\alpha_L} v_L''(\alpha) + \frac{d^2\alpha}{d\alpha_L^2} v_L'(\alpha) < 0.$$

If $f(0) > 1/2$, we have $d\alpha/d\alpha_L = 0$ and $v_L'(\alpha) < 0$ when $\alpha_L = -\alpha_R = -1/2f(0)$. A sufficient condition for $d^2\Pi_L/d\alpha_L^2 < 0$ is then $d^2\alpha/d\alpha_L^2 > 0$. Dividing the left hand side of (26) by $F((\alpha_L + \alpha_R)/2)$, we obtain:

$$A = \frac{\frac{d\alpha}{d\alpha_L}}{F\left(\frac{\alpha_L+\alpha_R}{2}\right)} = \frac{1}{2}(\alpha_L - \alpha_R)\frac{f\left(\frac{\alpha_L+\alpha_R}{2}\right)}{F\left(\frac{\alpha_L+\alpha_R}{2}\right)} + 1$$

and

$$\frac{dA}{d\alpha_L} = \frac{1}{2}\frac{f}{F} + \frac{1}{4}(\alpha_L - \alpha_R)\left(\frac{f}{F}\right)'.$$

At equilibrium, recalling that $\alpha_R = 1/2f(0) > 0$, party L should never play $\alpha_L \geq \alpha_R$. Therefore $\alpha_L - \alpha_R < 0$. If $(f/F)' \leq 0$, $dA/d\alpha_L$ is positive. Knowing that $F((\alpha_L + \alpha_R)/2)$ is increasing with α_L, necessarily $d\alpha/d\alpha_L$ must be increasing with α_L, that is $d^2\alpha/d\alpha_L^2 > 0$.

The second-order condition for party R is given by

$$\frac{d^2\Pi_R}{d\alpha_R^2} = \frac{d\alpha}{d\alpha_R} v_R''(\alpha) + \frac{d^2\alpha}{d\alpha_R^2} v_R'(\alpha) < 0.$$

If $f(0) > 1/2$, we have $d\alpha/d\alpha_R = 0$ and $v_R'(\alpha) > 0$ when $\alpha_L = -\alpha_R = -1/2f(0)$. A sufficient condition for $d^2\Pi_L/d\alpha_L^2 < 0$ is then $d^2\alpha/d\alpha_L^2 < 0$. Dividing the left hand side of (27) by $1 - F((\alpha_L + \alpha_R)/2)$, we obtain:

$$B = \frac{\frac{d\alpha}{d\alpha_R}}{1 - F\left(\frac{\alpha_L+\alpha_R}{2}\right)} = \frac{1}{2}(\alpha_L - \alpha_R)\frac{f\left(\frac{\alpha_L+\alpha_R}{2}\right)}{1 - F\left(\frac{\alpha_L+\alpha_R}{2}\right)} + 1$$

and

$$\frac{dB}{d\alpha_R} = -\frac{1}{2}\frac{f}{1-F} + \frac{1}{4}(\alpha_L - \alpha_R)\left(\frac{f}{1-F}\right)'.$$

This expression is negative if $(f/(1-F))' \geq 0$. One can show that, when the distribution function is symmetric, the two conditions, $(f/F)' \leq 0$ and $(f/(1-F))' \geq 0$, are equivalent.

Proof of Proposition 5 Following (2*a*)–(2*d*) and (12), we have:

$$\alpha = F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)\alpha_{EL} + \left(F\left(\frac{\alpha_L+\alpha_R}{2}\right) - F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)\right)\alpha_L$$
$$+\left(F\left(\frac{\alpha_R+\alpha_{ER}}{2}\right) - F\left(\frac{\alpha_L+\alpha_R}{2}\right)\right)\alpha_R + \left(1 - F\left(\frac{\alpha_R+\alpha_{ER}}{2}\right)\right)\alpha_{ER}.$$

First-order conditions for parties *EL* and *L* at a symmetric interior equilibrium are

$$\frac{d\alpha}{d\alpha_{EL}} = \frac{1}{2}f\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)(\alpha_{EL}-\alpha_L) + F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right) = 0 \tag{28}$$

$$\begin{aligned}\frac{d\alpha}{d\alpha_L} = \frac{1}{2}f\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)(\alpha_{EL}-\alpha_L) - F\left(\frac{\alpha_{EL}-\alpha_L}{2}\right)\\ +\frac{1}{2}f\left(\frac{\alpha_L+\alpha_R}{2}\right)\times(\alpha_L-\alpha_R) + F\left(\frac{\alpha_L+\alpha_R}{2}\right) = 0.\end{aligned} \tag{29}$$

From the first-order condition on α_{EL}, one immediately sees that $\alpha_{EL} < \alpha_L$ at an interior equilibrium.

Using (28) together with the symmetry of the solution, (29) becomes

$$-2F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right) + \alpha_L f(0) + \frac{1}{2} = 0$$

which means that

$$\alpha_L = -\frac{1 - 4F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)}{2f(0)}$$

Comparing this with the result for two party competition, one immediately sees that moderate platforms are closer to 0 with four parties than with 2. Furthermore, we obtain that $F((\alpha_{EL}+\alpha_L)/2) < 1/4$ at an interior equilibrium.

We next verify that the above result (moderate platforms closer to the center with four parties) holds true when some parties propose corner platforms. When $f(0) \leq 1/2$, the two parties competition leads to corner solutions. The outcome with four parties cannot then be more extreme in such a case. We now prove that the moderate platform is closer to the center with four parties when $f(0) > 1/2$:

$$\left.\frac{d\alpha}{d\alpha_L}\right|_{\alpha_{EL}=-1,\alpha_L=-1/2f(0)} = \frac{1}{2}f\left(\frac{-1-\frac{1}{2f(0)}}{2}\right)\left(-1+\frac{1}{2f(0)}\right) - F\left(\frac{-1-\frac{1}{2f(0)}}{2}\right).$$

As soon as $f(0) > 1/2$, this expression is negative. Hence the conclusion. Observe that this implies that a full corner solution is impossible when $f(0) > 1/2$.

We now turn to the second-order conditions. The proof that $d^2\alpha/d\alpha_{EL}^2 > 0$ is similar to the proof for party L in the two party case. As for party L, making use of (28), (29) becomes

$$\frac{d\alpha}{d\alpha_L} = f\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)(\alpha_{EL}-\alpha_L) + \frac{1}{2}f\left(\frac{\alpha_L+\alpha_R}{2}\right)(\alpha_L-\alpha_R) + F\left(\frac{\alpha_L+\alpha_R}{2}\right).$$

We have already shown in the proof for party L in the two party case that the sum of the second and third terms is increasing in α_L provided that $f(x)/F(x)$ is decreasing with x. Using the same technique (i.e. dividing by $F((\alpha_{EL}+\alpha_L)/2)$ and then differentiating with respect to α_L), it is easy to show that the first term is also increasing in α_L under the same conditions. We thus obtain that the second-order conditions are satisfied in this case. Using similar arguments, one can verify that the second-order conditions for parties R and ER are satisfied if $f(x)/(1-F(x))$ is increasing with x, which is equivalent to the requirement that $f(x)/F(x)$ is decreasing with x for a symmetrical $f(x)$.

Proof of Proposition 6 The first-order conditions are still given by Eqs. (26) and (27) in the Proof of Proposition 4. Together, they imply that $F\left(\frac{\alpha_L+\alpha_R}{2}\right) = \frac{1}{2}$ so that $\alpha_L = 2med - \alpha_R$. Put together with (26), one then obtains (18) and (19). These two values are admissible if they both belong to the interval $[-1,1]$, i.e. if $f(med) > \max\left\{\frac{1}{2(1+med)}, \frac{1}{2(1-med)}\right\}$.

If $med < 0$, we can have a full corner equilibrium ($\alpha_L = -1$, $\alpha_R = 1$) or a partial corner equilibrium. In this last case, $\frac{1}{2(1-med)} < f(med) < \frac{1}{2(1+med)}$ and $\alpha_L = -1$. First order conditions are then given by

$$\frac{d\alpha}{d\alpha_L} = F\left(\frac{\alpha_R-1}{2}\right) - \frac{1}{2}f\left(\frac{\alpha_R-1}{2}\right)(1+\alpha_R) \geq 0, \tag{30}$$

$$\frac{d\alpha}{d\alpha_R} = 1 - F\left(\frac{\alpha_R-1}{2}\right) - \frac{1}{2}f\left(\frac{\alpha_R-1}{2}\right)(1+\alpha_R) = 0 \tag{31}$$

which together imply that

$$F\left(\frac{\alpha_R-1}{2}\right) \geq \frac{1}{2},$$

i.e. that the left party gets more than one half of the votes. It then follows that $\alpha < 0$. If $med < 0$ and $f(med) < \frac{1}{2(1-med)} < \frac{1}{2(1+med)}$, we can have a full corner equilibrium ($\alpha_L = -1$, $\alpha_R = 1$) with $\alpha = 1 - 2F(0) < 0$.

The proof for the second-order condition is identical to that of Proposition 4 since $\alpha_L < med < \alpha_R$.

Proof of Proposition 7 First-order conditions for a symmetric interior equilibrium are still given by Eqs. (28) and (29) in Proposition 5's proof. Together, they imply that

$$-2F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right) + (\alpha_L - med)f(med) + \frac{1}{2} = 0$$

which means that

$$\alpha_L = med - \frac{1 - 4F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right)}{2f(med)}.$$

Comparing this with the result for two party competition, one immediately sees that moderate platforms are closer to 0 with four parties than with 2. Furthermore, we obtain that $F\left(\frac{\alpha_{EL}+\alpha_L}{2}\right) < \frac{1}{4}$ at an interior equilibrium.

We next verify that the above result (moderate platforms closer to the center with four parties) holds true when some parties propose corner platforms. We concentrate on the positively skewed case where two party competition leads to an interior equilibrium $\left(f(med) > \frac{1}{2(1+med)}\right)$ and compute $\frac{d\alpha}{d\alpha_L}$ in the four party case where $\alpha_{EL} = -1$, $\alpha_{ER} = 1$ and α_L and α_R are located at their interior equilibrium location with two parties:

$$\left.\frac{d\alpha}{d\alpha_L}\right|_{\alpha_{EL}=-1,\alpha_L=med-1/2f(med)} = \frac{1}{2}f\left(\frac{med - 1 - \frac{1}{2f(med)}}{2}\right)\left(-1 - med + \frac{1}{2f(med)}\right)$$
$$-F\left(\frac{med - 1 - \frac{1}{2f(med)}}{2}\right).$$

As soon as $f(med) > \frac{1}{2(1+med)}$, this expression is negative. Hence the conclusion that L's platform is closer to med than in the two party case. The proof for party R runs similarly.

The analysis of the second-order condition is identical to Proposition 5's proof since $\alpha_{EL} < \alpha_L <$ med $< \alpha_R < \alpha_{ER}$.

Proof of Proposition 8 Condition (21) implies that the two moderate parties have the same number of votes. We can replicate the part of the proof of Proposition 2 (ii) that excludes deviations from EL. The sufficient condition for this is given by (22). Similarly, Condition (23) guarantees that ER does not win by deviating.

We can replicate the part of the proof of Proposition 2 (ii) that excludes deviations from L. The sufficient condition for this is condition (24). Similarly, Condition (25) guarantees that R does not win by deviating.

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