

# Legal vs Ownership Unbundling in Network Industries

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## Abstract

This paper studies the impact of legal unbundling vs ownership unbundling on the incentives of a network operator to invest and maintain its assets. We consider an industry where the upstream firm first chooses the size of a network, while several downstream firms then compete in selling goods and services that use this network as a necessary input. We contrast the (socially) optimal allocation with several equilibrium situations, depending on whether the upstream firm owns zero, one or two downstream firms. The first situation corresponds to ownership unbundling between upstream and downstream parts of the market. As for the other two cases, we equate legal unbundling with the following two assumptions. First, each downstream firm maximizes its own profit, without taking into account any impact on the upstream firm's profit. Second, the upstream firm is not allowed to discriminate between downstream firms by charging different access charges for the use of its network. On the other hand, we assume that the upstream firm chooses its network size in order to maximize its total profit, including the profit of its downstream subsidiaries.

Our main results are as follows. Because the investment in the network is not protected, at the time at which it is made, by a contract, the upstream firm will not take into account the interests of its clients when choosing its size. This effect can be mitigated by allowing it to own part of the downstream industry. In other words, ownership separation is more detrimental to welfare than legal unbundling. We also obtain that these results are robust to the introduction of asymmetry in network needs across downstream firms, imperfect downstream competition and downstream investments.

# 1 Introduction

This paper studies the impact of legal unbundling vs ownership unbundling on the incentives of a network operator to invest and maintain its assets. We consider an industry where the upstream firm invests in and maintains a network, while several downstream firms compete in selling goods and services that use this network as a *necessary* input (i.e., no bypass technology is available, at least at an economically relevant price). Many network industries fit this description (telecommunications, railways, electricity, etc.) but we have in mind particularly the natural gas industry.

There are many papers in the regulation, industrial economics and economics of organizations literature that study the impact of various ownership structures in network industries. The simplest such structure is one in which an upstream firm (firm  $U$ ) provides an input to a downstream firm (firm  $D$ ). These papers often compare the behavior of a vertically integrated firm with the equilibrium situation where the upstream and downstream activities are undertaken by separate firms (i.e., firms whose ownership differ from one another).

There are two types of considerations that might induce firm  $D$  and  $U$  to merge. First, they might want to use the combined weight of the two firms for strategic purposes. For instance, when the upstream firm has market power in the supply of the input, but the downstream firm faces competition, the merger can be a way to prevent a form of “trickling up” effect of competition. We will call this view the “antitrust perspective”, as it is the fear of this type of consequences that prompts competition authorities to disallow some mergers. Second, there might be some efficiency gains to running the two firms as a single unit, and the aim of the merger is to take advantage of these efficiency gains. This type of merger can arise in a competitive market, whereas the first type could not. To stress the fact that authors who write on this topic are interested in the internal functioning of the firm, we will label this branch of the literature the “managerial perspective”

In this paper, we will be considering a situation where firm  $U$  is regulated, and

where several downstream firms compete with each other. Regulatory practice has typically analyzed the ownership of a downstream firm by the upstream firm in the antitrust perspective and with suspicion: this ownership is seen as an open door to anticompetitive discrimination; we think that it is fair to say that regulators have often accepted vertical integration as a political compromise. The aim of this paper is to begin exploring what the managerial perspective can bring to the debate.

>From the managerial perspective, this paper mainly draws on the insights linked to the notions of incomplete contracts and specific capital. In many circumstances, upstream and downstream firms must make investments in order to improve the benefits they derive from their relationships. For instance, they need to conduct some research and development. This investment is *specific* if it is productive exclusively within the context of this relationship.<sup>1</sup> Because of the incompleteness of contracts, the two firms, if they are not integrated, will choose suboptimal levels of investment. Vertical integration will incite them to take into account the interests of their partner, and will therefore mitigate the resulting inefficiency.<sup>2</sup>

What the managerial perspective calls vertical disintegration corresponds to the *ownership unbundling* scenario that we study in the current paper. On the other hand, the intermediate situation of *legal unbundling* has, to the best of our knowledge, not been studied previously in the literature. By legal unbundling, we mean the situation where the upstream and one or many downstream firms belong to the same owners and where these owners, although they are the residual claimants over the financial returns generated by the firms' assets (i.e., they keep the firms's profits), do not have the full control rights over the firms' decisions.

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<sup>1</sup>There are degrees of specificity depending on the usefulness of the investment outside of the relationship with the other firm. For simplicity, we assume that the investment is only useful in the framework of the relationship that we are considering.

<sup>2</sup>The notion of incomplete contract is introduced in the economic literature by Simon (1951). Among the classical early references on vertical integration one can cite Klein, Crawford & Alchian (1978), Williamson (1985) and Grossman & Hart (1986) (which is criticized by Riordan (1990)). For accessible surveys see Crémer (1995) and Tirole (1995).

More precisely, in our context legal unbundling between upstream and downstream would mean that the upstream firm does not control its downstream subsidiaries' actions, such as their pricing or investment decisions. That this intermediate (between integration and full divestiture) situation has not been studied before is all the more surprising that it is at the heart of most European directives on network industries. For instance, the 2003/55 European Directive on natural gas states "In order to ensure efficient and non-discriminatory network access it is appropriate that the transmission and distribution systems are operated through legally separate entities where vertically integrated undertakings exist. It is important however to distinguish between such legal separation and ownership unbundling. Legal separation implies neither a change of ownership of assets [...]. However, a non-discriminatory decision-making process should be ensured through organizational measures regarding the independence of the decision-makers responsible."

The way we model legal unbundling is as follows. We consider a sequential game where the upstream firm first chooses the size of its network, and where two downstream firms then compete by selling goods that use this network as an essential input. We contrast the (socially) optimal allocation with several equilibrium situations, depending on the ownership structure in the industry. More precisely, we consider the market equilibria when the upstream firm owns zero, one or two downstream firms. The first situation corresponds to ownership unbundling between upstream and downstream parts of the market. As for the other two cases, we equate legal unbundling with the following two assumptions. First, each downstream firm maximizes its own profit, without taking into account any impact on the upstream firm's profit. Second, the upstream firm is not allowed to discriminate between downstream firms by charging different access charges for the use of its network. On the other hand, we assume that the upstream firm chooses its network size in order to maximize its total profit, including the profit of its downstream subsidiaries. In other words, the regulator is unable to prevent the network operator from choosing the dimension of its network that maximizes the total profit of its owner.

We show that the same concerns as those raised by the managerial perspective on vertical integration are at play here. Because the investment in the network is not protected by a contract at the time it is made, the upstream firm will not take into account the interests of its clients when choosing its size. This effect can be mitigated by allowing it to own part of the downstream industry. In order to show this, we present four different models. After introducing our general framework in section 2, in section 3 we explore the strategies of the firms when the two downstream firms face the same cost functions, use the network with the same intensity and are price takers on the market for the final output (on which they sell their production). Section 4 revisits the same model assuming that the firms have the same “non-network” cost function, but have different network utilization requirements. In section 5, we relax the assumption that the downstream market is competitive. Section 6 assumes that the downstream firms can make some investments that reduce their use of the network at given output. In all these cases, we obtain the same results: disallowing joint ownership of network and downstream facilities reduces the investment in the network. The conclusion, section 7, discusses the limits of our work and the extensions that would be necessary for a more complete comparison of legal and ownership unbundling.

## 2 The model

Consider an industry where one firm (referred to as “upstream”, indexed by  $U$ ) is in charge of building and maintaining a network, while two firms (“downstream”, indexed by  $i = 1, 2$ ) sell goods or services that use the network. One prominent example of such an industry is the natural gas sector, where the upstream firm builds the pipeline network while the downstream firms sell natural gas to households and industrial customers. In order to bring gas to their customers, downstream firms need to transport this gas from the place where it is injected into the upstream firm’s network to the consumption node.

The upstream firm chooses the size  $l$  of the network it builds and maintains.

The (constant) per-unit cost of the network is denoted by  $k$ , so that its total cost is  $K = kl$ . Downstream firm  $i$  sells  $x_i$  units of its product at price  $p_i$ . Production technology is such that each unit of good  $i$  uses one unit of network: there is no bypass technology available at an economically relevant cost, so that the network is an essential facility. In addition to network costs, firm  $i$  incurs downstream costs of  $C_i(x_i)$ . In the natural gas sector, these downstream costs are the costs of buying the gas and all other costs not related to transport, such as the distribution or marketing costs. We assume that the downstream technology shows decreasing returns to scale, so that  $C_i'(x_i) > 0$  and  $C_i''(x_i) < 0$ .<sup>3</sup> To ensure concavity of the profit functions, we will also often assume that  $C_i'''(x)$  is positive. As for the network costs, downstream firms pay to the upstream firm a constant access charge  $a$  (that is endogenous in our model) for each unit of the network that they use.

The products sold by both downstream firms are perfect substitutes.<sup>4</sup> This appears to be a sensible assumption in the natural gas market, since natural gas is a homogenous product.<sup>5</sup> Let  $X$  denote the total quantity in the downstream market, so that  $X = x_1 + x_2$ . We denote by  $X(p)$  the aggregate demand for the downstream product, and by  $p(X)$  the aggregate inverse demand. We assume that the revenue functions  $pX(p)$  and  $Xp(X)$  are concave.

We model the following sequential game: first, the upstream firm chooses the size of the network and then the downstream firms choose their price. This timing is natural given the nature of the decisions involved. We solve this game for various scenarios concerning the downstream competitive conditions and the symmetry between downstream firms. In Sections 3 and 4, we assume that the downstream

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<sup>3</sup>This assumption is crucial in the first part of the paper, since it guarantees that competitive downstream firms earn a positive profit. However, it is not important for our argument *per se*. To show this, the assumption is relaxed in section 5 where we introduce imperfect downstream competition.

<sup>4</sup>This assumption is not crucial: our results would carry through if the downstream goods were sold on totally separate, unrelated markets or if they were imperfect substitutes.

<sup>5</sup>However, note that the services that are sold together with the gas molecules can be differentiated, for instance by adding interruptibility clauses. We abstract from these considerations for the time being.

firms are perfectly competitive (price takers). Consequently, they choose their output level to equate marginal cost and market price. Section 3 is concerned with the case where both downstream firms are symmetrical: they share the same downstream cost function and have the same needs in terms of network usage. Section 4 considers the case where the network is more adapted to one of the downstream firms than to the other, while the non network related cost functions of the two downstream firms are the same. This allows us to have a first go at understanding the impact of the ability for the upstream firm to discriminate between downstream firms. Section 5 then studies the situation where downstream firms have market power and play a Cournot game. Finally, section 6 analyzes the impact of allowing downstream firms to make investments that would allow them to decrease their need of network usage for any given output level.

We proceed similarly in Sections 3 to 6. We first study the surplus-maximizing allocation. We then solve for the downstream equilibrium, to obtain prices and quantities as a function of the network size. We then study successively the equilibrium allocation when the upstream firm owns both downstream firms, when it owns none of them and when it owns one but not the other. As mentioned above, we impose legal unbundling for the two cases where the upstream firm owns at least one downstream firm. Our objective is to assess how legal and ownership unbundling affect the equilibrium network size.

### 3 Symmetric Equilibria

>From this point on, we assume that both downstream firms have the same (non network related) cost function  $C_i$  and drop the subscript  $i$ . We begin by studying the outputs that would be chosen by a welfare maximizing planner before turning to the analysis of the game between the firms.



### 3.1 Social Optimum

The social optimum is the allocation that maximizes total surplus  $S$  in the economy. Assuming quasi-linear preferences for consumers of the downstream products, total surplus is equal to consumers gross surplus minus upstream and downstream costs. The social planner chooses a network size  $l$  that solves

$$\max_l S = \int_0^l p(s)ds - 2C\left(\frac{l}{2}\right) - kl.$$

Denote the optimal level of variables by a \*. The solution is given by  $x_1^* = x_2^* = X^*/2$  where  $X^* = l^*$  is defined by

$$p(X^*) = C'\left(\frac{X^*}{2}\right) + k = C'(x_i^*) + k.$$

This condition is easy to interpret; it requires marginal cost to equal marginal willingness to pay for the final good. The marginal cost is equal to the sum of marginal upstream and downstream costs. Further, observe that, at the optimum, the marginal cost is the same for both firms.<sup>6</sup> The optimal network size equals the volume of goods sold at this optimal price.

### 3.2 Equilibrium in the downstream market

In the remainder of this section, we shall study different ownership structures. For all of them, once the size of the network has been chosen, the downstream firms act as price-takers; in this subsection, we study the prices which will prevail given a choice of a network size  $l$ .

Because the downstream firms are price-takers, they consider both the market price  $p$  of their output and the network access  $a$  to as given. Consequently, they choose their output in order to equalize their marginal cost with the market price  $p$ :

$$p = C'(x_i) + a. \tag{1}$$

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<sup>6</sup>This will hold true also when we introduce an asymmetry between the downstream firms. In other words, productive efficiency is necessary for social optimality.

Their total production will be  $X(p)$ , which is equal to their total demand for the services of the network since equilibrium on the network input market requires

$$X(p) = l. \quad (2)$$

Given a size  $l$  chosen for the network in the first stage of the game, equations (1) and (2) simultaneously determine the access charge  $a$  and the downstream price  $p$  (and therefore also the quantity sold  $X$ ) as functions of  $l$ ; we denote these functions by  $\tilde{a}(l)$  and<sup>7</sup>  $\tilde{p}(l) = p(l)$ : they denote the prices that will prevail as a function of the choice of  $l$ .

We now turn to the equilibrium when the upstream firm owns both downstream firms.

### 3.3 Equilibrium when $U$ owns both downstream firms

If  $U$  owns both downstream firms, it chooses  $l$  so as to maximize the sum of its profits,

$$\pi_U = \tilde{a}(l)l - kl,$$

and those of the two downstream firms,  $\pi_1$  and  $\pi_2$ , where

$$\pi_i = x_i(\tilde{p}(l)) [\tilde{p}(l) - \tilde{a}(l)] - C\left(\frac{x_i(\tilde{p}(l))}{2}\right), \quad i = 1, 2.$$

This sum is equal to

$$\pi_U + \pi_1 + \pi_2 = \tilde{a}(l)l - kl + X(\tilde{p}(l)) [\tilde{p}(l) - \tilde{a}(l)] - 2C\left(\frac{X(\tilde{p}(l))}{2}\right),$$

where  $\tilde{a}(l)$  and  $\tilde{p}(l)$  are the solutions to equations (1) and (2).

Observe that firm  $U$  has some market power, since it anticipates the equilibrium downstream prices (access charge  $a$  and final price  $p$ ) induced by its choice of  $l$ . Further, the assumption that  $C''(x) > 0$  means that downstream firms make a positive profit even when they act as price takers.

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<sup>7</sup>Notice the difference:  $p(l)$  represents the price at which consumers will choose to consume  $l$  units of the final good whereas  $\tilde{p}(l)$  represents the price which will prevail if  $l$  units of network services are provided. In the model of this section, they are equal; with other technology of productions they need not be.

This scenario of legal unbundling differs from the classical vertical integration case because the upstream firm  $U$  does not control the pricing policy of the two downstream firms. In other words, the managers of the downstream firms maximize their profit given the market price.

We reorganize this optimization problem to obtain

$$\begin{aligned} \max_{l,a,p} \quad & \pi_U + \pi_1 + \pi_2 = al - kl + X(p)(p - a) - 2C\left(\frac{X(p)}{2}\right), \\ \text{s. t.} \quad & p = C'\left(\frac{X(p)}{2}\right) + a, \\ & X(p) = l. \end{aligned}$$

Simplifying and using the inverse demand function yield

$$\pi_U + \pi_1 + \pi_2 = lp(l) - 2C\left(\frac{l}{2}\right) - kl. \quad (3)$$

Maximizing this expression with respect to  $l$  gives the following first-order condition:

$$p + lp' = C'\left(\frac{l}{2}\right) + k. \quad (4)$$

Equation (4) is the same condition that we would obtain if we assumed that the three firms acted as an integrated profit maximizing monopolist and maximized  $p(l)l - 2C(l/2) - kl$ . Total marginal cost is the sum of the downstream marginal cost  $C'$  and of the upstream marginal cost  $k$ , rather than the access charge paid by the downstream firm: when setting its network size, the upstream firm understands that the access charge is a pure transfer between its subsidiary and itself.

Using the superscript  $e2$  to index the equilibrium levels of the different variables, we obtain

$$l^{e2} < l^* \quad \text{and} \quad p^{e2} > p^*.$$

In words, the equilibrium network size is lower than optimal while the equilibrium retail price is larger than optimal. Intuitively, the upstream firm chooses a lower-than-optimal network size in order to reduce the downstream output level and to

increase downstream profits. This result holds even with legal unbundling between downstream and upstream firms i.e., even when managers of the downstream firms do not take into account the profits of the upstream firm when they set their profit-maximizing prices.

We now turn to the situation where downstream and upstream ownerships are separated.

### 3.4 Equilibrium with ownership unbundling

When the upstream firm owns neither of the downstream firms, it sets the network size in order to maximize its *own* profits,

$$\pi_U = \tilde{a}(l)l - kl.$$

Using equations (1) and (2) together with the symmetry between the downstream firms, this optimization program can be rewritten as

$$\begin{aligned} \max_{l,a,p} \pi_U &= al - kl \\ \text{s.t. } p &= C' \left( \frac{X(p)}{2} \right) + a, \\ X(p) &= l. \end{aligned}$$

The two constraints imply

$$a = p(l) - C' \left( \frac{l}{2} \right)$$

which we substitute in  $\pi_U$  to obtain

$$\pi_U = \left[ p(l) - C' \left( \frac{l}{2} \right) \right] l - kl, \tag{5}$$

$$= \left[ lp(l) - 2C \left( \frac{l}{2} \right) - kl \right] - 2 \left[ C' \left( \frac{l}{2} \right) \frac{l}{2} - C \left( \frac{l}{2} \right) \right]. \tag{6}$$

Observe that the first term in the right hand side of (6) corresponds to  $\pi_U + \pi_1 + \pi_2$  as defined in (3). Because the two downstream firms are price takers, their downstream prices reflect their marginal costs: per unit of output, they each

charge  $C'(l/2)$  to their customers to reflect their costs. The second bracketed term represents the difference between the resulting revenue and their true cost. These are profits that the network must abandon to the downstream firms.

From (6), we obtain

$$\frac{d\pi_U}{dl} \Big|_{l=l^{e2}} = -\frac{l^{e2}}{2} C'' \left( \frac{l^{e2}}{2} \right) < 0.$$

Denote by  $l^{e0}$  the equilibrium levels of variables in the ownership unbundling scenario. If the function  $\pi_U$  is concave, which it will be if the revenue function is concave and if  $C''' \geq 0$ ,<sup>8</sup> this implies

$$l^{e0} < l^{e2} < l^*.$$

In words, the fact that the upstream firm does not share in the downstream profits induces it to further decrease  $l$  and  $X$ , compared to the legal unbundling situation. Ownership unbundling is thus more detrimental to welfare than legal unbundling in our setting. The intuition for this result is as follows. With ownership unbundling, the upstream firm's only source of profit is the selling of access to its network. Total revenue of the upstream firm is given by  $2ax_i = 2(px_i - x_i C')$ , with  $x_i = l/2$ . This is lower than downstream profit, which equals  $2(px_i - C)$ , because decreasing returns to scale imply that  $x_i C' > C$ . The gap between upstream revenue and downstream profit increases with the difference between  $x_i C'$  and  $C$ , which is itself increasing<sup>9</sup> with  $x_i$  and  $l$ . This explains why the upstream firm has an incentive to further decrease its network's size when it does not own any downstream firm.

We now look at the intermediate situation where the upstream firm owns only one of the two downstream firms. We continue to assume legal unbundling

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<sup>8</sup>Let  $R(x) = p(x)x$  be the revenue function. From (5), the second derivative of  $\pi_U$  with respect to  $l$  is

$$R''(l) - C'' \left( \frac{l}{2} \right) - \frac{1}{2} C''' \left( \frac{l}{2} \right).$$

It is negative if  $R$  is concave and both  $C''$  and  $C'''$  are positive.

<sup>9</sup>The derivative of  $x_i C'(x_i) - C(x_i)$  with respect to  $x_i$  is  $x_i C''(x_i)$ , which is positive by convexity of  $C$ .

between the upstream and the downstream firm it owns.

### 3.5 Equilibrium when the upstream firm owns one of the downstream firms

To study the situation where the upstream firm owns only one downstream firm, one can proceed as in the previous section to obtain that

$$\pi_U + \pi_i = \left[ lp(l) - 2C\left(\frac{l}{2}\right) - kl \right] - \left[ C'\left(\frac{l}{2}\right) \frac{l}{2} - C\left(\frac{l}{2}\right) \right], \quad i = 1, 2. \quad (7)$$

Differentiating this equation, and denoting equilibrium levels by the superscript  $e1$ , one shows<sup>10</sup>

$$l^{e0} < l^{e1} < l^{e2} < l^*.$$

Another way to proceed will prove easier and more general. Note that the objective function of  $U$  in this section, given by (7) is a convex combination of the objectives in the previous two sections, which are given by (3) and (6):

$$\pi_U + \pi_i = \frac{1}{2}(\pi_U + \pi_1 + \pi_2) + \frac{1}{2}\pi_U, \quad i = 1, 2.$$

This in turn gives the same ranking of equilibrium and optimal network sizes, provided that the objective functions are concave.

In words, incentives for the proper determination of the network size increase with the number of downstream firms that the upstream firm owns. The intuition for this result is as explained at the end of the previous subsection: because of decreasing returns to scale, the difference between the access revenues of the upstream firm and the downstream profit increases with output. If the upstream firm does not share in this downstream profit, it is induced to under-invest in its network. As the upstream firm acquires more downstream firms, its incentives to invest in the network increase, and the equilibrium network size increases toward the optimal level. Observe that we have assumed throughout the analysis

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<sup>10</sup>This requires to prove that the relevant objective functions,  $(\pi_U, \pi_U + \pi_i$  and  $\pi_U + \pi_i + \pi_j)$  are concave. This is a straightforward consequence of the concavity of the revenue function, and of the convexity of  $C$  and  $x C'$  (whose second derivative is  $2C'' + C'''$ ).

that legal unbundling prevails in the absence of ownership unbundling. We have also obtained that, with legal unbundling, the equilibrium network size when the upstream firm owns both downstream firms falls short of the optimal network size.

## 4 Downstream firms' asymmetry and discrimination

Let us now introduce some asymmetry between downstream firms, in the form of different needs in terms of network access. We assume that the investments made in the network by the upstream firm benefit more one firm than the other. In the natural gas sector, this situation could arise because of the localization of the investments (new pipelines built in a region where one downstream firm has a larger share of its customers' portfolio than the other firm) or their type (investing in LNG rather than pipelines for instance). The objective in this section is to understand how the existence of discrimination affects the optimal and equilibrium size of the network, and how it relates with legal and ownership unbundling.

We model asymmetry in network needs as follows. We assume that downstream firm 1 benefits more than downstream firm 2 from investments in the network: firm 1 needs only  $(1 - \alpha)$  unit of network for each unit of final good that it sells. On the other hand, downstream firm 2 needs one unit of network use for each unit of final good sold, as previously. The parameter  $\alpha \in [0, 1[$  measures the intensity of the additional benefit that firm 1 gets from the network. We assume that this parameter  $\alpha$  is set exogenously (given by the technology, for instance). An extension to our analysis would be to endogenize the setting of this parameter by letting the upstream firm choose its profit-maximizing level.

Except for the introduction of the parameter  $\alpha$ , we maintain all the assumptions made in the previous section. We proceed as in the previous section by looking first at the optimal allocation before turning to the equilibrium allocations in the various unbundling scenarios.

## 4.1 Social Optimum

The social planner's optimization program is

$$\max_{x_1, x_2} S = \int_0^{x_1+x_2} p(s)ds - C(x_1) - C(x_2) - k[(1-\alpha)x_1 + x_2],$$

yielding the following first-order conditions

$$\begin{aligned} p(x_1 + x_2) &= C'(x_1) + (1 - \alpha)k, \\ p(x_1 + x_2) &= C'(x_2) + k. \end{aligned}$$

These are the usual conditions that price should equal marginal costs. Together, they imply that

$$C'(x_1^*) + (1 - \alpha)k = C'(x_2^*) + k,$$

i.e., that we have productive efficiency at the optimum.

The first order conditions allow us to obtain the optimal downstream quantities and network size, which we denote as previously with a \*:  $x_1^*, x_2^*$  and  $l^* = (1 - \alpha)x_1^* + x_2^*$ .

## 4.2 Equilibrium in the downstream market

As in section 3, we need to compute the equilibrium of the game played by the downstream firms as a function of  $l$ .

Competition in the downstream market generates the following equilibrium conditions, which replace (1) and (2):

$$p = C'(x_1) + a(1 - \alpha), \tag{8}$$

$$p = C'(x_2) + a, \tag{9}$$

$$l = (1 - \alpha)x_1 + x_2, \tag{10}$$

$$X(p) = x_1 + x_2. \tag{11}$$

The solution to these four simultaneously equations, yields the equilibrium levels of the access charge, retail price and downstream quantities as functions of the



network size and the parameter  $\alpha$ . Given that  $\alpha$  is treated as exogenous in this section, we denote these relationships by  $\tilde{a}(l)$ ,  $\tilde{x}_1(l)$ ,  $\tilde{x}_2(l)$  and

$$\tilde{p}(l) = p(\tilde{x}_1(l) + \tilde{x}_2(l)). \quad (12)$$

Observe that unlike in the earlier section we now have that  $\tilde{p}(l) \neq p(l)$  since  $l \neq x_1 + x_2$ .

We now look at the equilibrium where the upstream firm owns the two downstream firms, with legal unbundling between the upstream and downstream segments.

### 4.3 Equilibrium when $U$ owns the two downstream firms

We start by using the equilibrium quantities and price in the downstream markets in order to obtain the profit levels of the three operators as a function of the network size:

$$\begin{aligned} \pi_U &= \tilde{a}(l)l - kl, \\ \pi_1 &= \tilde{x}_1(l) [\tilde{p}(l) - (1 - \alpha)\tilde{a}(l)] - C(\tilde{x}_1(l)), \\ \pi_2 &= \tilde{x}_2(l) [\tilde{p}(l) - \tilde{a}(l)] - C(\tilde{x}_2(l)). \end{aligned}$$

The objective of the upstream firm is to find the network size  $l$  that maximizes the sum of the three operators' profits:

$$\max_l \pi_U + \pi_1 + \pi_2 = [\tilde{x}_1(l) + \tilde{x}_2(l)]\tilde{p}(l) - C(\tilde{x}_1(l)) - C(\tilde{x}_2(l)) - kl. \quad (13)$$

The first order solution of this program is given by

$$\begin{aligned} \tilde{p}(l)[\tilde{x}'_1(l) + \tilde{x}'_2(l)] + [\tilde{x}_1(l) + \tilde{x}_2(l)]\tilde{p}'(l) \\ - C'(\tilde{x}_1(l))\tilde{x}'_1(l) - C'(\tilde{x}_2(l))\tilde{x}'_2(l) - k = 0. \end{aligned} \quad (14)$$

To simplify this expression, we use (12) and

$$\tilde{p}'(l) = [\tilde{x}'_1(l) + \tilde{x}'_2(l)]p'(\tilde{x}_1(l) + \tilde{x}_2(l)), \quad (15)$$

$$(1 - \alpha)\tilde{x}'_1(l) + \tilde{x}'_2(l) = 1, \quad (16)$$

where (15) and (16) are obtained by differentiating, respectively, (12) and (10).

We substitute equations (12) to (16) in (14). Using the superscript  $e2$  to denote the equilibrium levels of the variables in this scenario, we obtain after simplifications that

$$\begin{aligned} p(x_1^{e2} + x_2^{e2}) + (x_1^{e2} + x_2^{e2})p'(x_1^{e2} + x_2^{e2}) &= C'(x_1^{e2}) + (1 - \alpha)k \\ &= C'(x_2^{e2}) + k, \end{aligned}$$

i.e., the same conditions as if  $x_1$  and  $x_2$  were directly controlled by the upstream operator. Marginal revenue is equal to marginal cost for both downstream operators. Observe that productive efficiency is maintained by the combined firm, since marginal costs are the same at equilibrium for the two downstream operators.

Comparing these profit-maximizing downstream quantities with their optimal levels, we obtain that  $x_1^{e2} < x_1^*$  and  $x_2^{e2} < x_2^*$ , which implies that  $l^{e2} = (1 - \alpha)x_1^{e2} + x_2^{e2} < l^* = (1 - \alpha)x_1^* + x_2^*$ . This is the same ranking of downstream quantities and network sizes as in the symmetric case. Although the mechanism is made more complex by the existence of asymmetric network needs between downstream operators, the intuition for the result is not affected by this asymmetry: the upstream firm under-invests in the network, anticipating that lower downstream quantities will generate larger profits for the two downstream firms that it owns.

#### 4.4 Equilibrium with ownership unbundling

One could proceed as in the symmetric situation in order to solve for equilibrium quantities when ownership is unbundled between upstream and downstream segments. It will prove easier to use the indirect method introduced in 3.5.

The upstream firm maximizes its own profit, which can be expressed as

$$\pi_U = (\pi_U + \pi_1 + \pi_2) - (\pi_1 + \pi_2). \tag{17}$$

We can also rewrite the profit functions of the two downstream firms as

$$\begin{aligned}
\pi_1 &= \tilde{x}_1(l) [\tilde{p}(l) - (1 - \alpha)\tilde{a}(l)] - C(\tilde{x}_1(l)), \\
&= \tilde{x}_1(l)C'(\tilde{x}_1(l)) - C(\tilde{x}_1(l)), \\
\pi_2 &= \tilde{x}_2(l) [\tilde{p}(l) - \tilde{a}(l)] - C(\tilde{x}_2(l)), \\
&= \tilde{x}_2(l)C'(\tilde{x}_2(l)) - C(\tilde{x}_2(l)).
\end{aligned}$$

Differentiating profits with respect to network size, we then obtain

$$\begin{aligned}
\pi'_1 &= \tilde{x}_1(l)C''(\tilde{x}_1(l))\tilde{x}'_1(l) > 0, \\
\pi'_2 &= \tilde{x}_2(l)C''(\tilde{x}_2(l))\tilde{x}'_2(l) > 0.
\end{aligned}$$

Using equation (17), we show

$$\begin{aligned}
\pi'_U(l^{e2}) &= [\pi'_U(l^{e2}) + \pi'_1(l^{e2}) + \pi'_2(l^{e2})] - [\pi'_1(l^{e2}) + \pi'_2(l^{e2})], \\
&= 0 - [\pi'_1(l^{e2}) + \pi'_2(l^{e2})] < 0,
\end{aligned}$$

which by concavity of the function  $\pi_U$  implies

$$l^{e0} < l^{e2}.$$

This shows that the intuition obtained in section 3.4 carries over to the case of asymmetric downstream cost functions: with ownership unbundling, the upstream firm fails to take into account downstream profits, with the difference between upstream and downstream profit levels increasing with downstream volumes. The upstream firm has then an incentive to invest less in its network than in the case where it owns the two downstream firms.

## 4.5 Equilibrium when the upstream firm owns one of the downstream firms

We distinguish between the case where the upstream firm owns the downstream firm 1 (denoted by  $e11$ ) and the case where  $U$  owns firm 2 (denoted by  $e12$ ). In the  $e11$  scenario, the objective of the upstream operator is to maximize

$$\pi_U = (\pi_U + \pi_1 + \pi_2) - \pi_2, \tag{18}$$

which allows us to use the same argument as in the  $e0$  scenario where ownership is totally unbundled between the upstream and downstream segments. More precisely, we evaluate  $\pi'_U(l^{e0})$  and  $\pi'_U(l^{e2})$ , with  $\pi_U$  given by equation (18), to show that (provided that  $\pi_U$  is concave in  $l$ )

$$l^{e0} < l^{e11} < l^{e2}.$$

Similarly, one can show that

$$l^{e0} < l^{e12} < l^{e2}.$$

The general conclusion that we draw from this section is that the relative ranking of the equilibrium network sizes is robust to the introduction of asymmetry between downstream firms. With such an asymmetry, ownership unbundling leads to more under-investment than legal unbundling: the more integrated the industry is, the closer the equilibrium network size comes to its optimal level.

We now study the robustness of our results to the introduction of imperfect competition in the downstream market.

## 5 Imperfect competition in the downstream market

In this section, we assume that both downstream firms compete *à la Cournot* on the final market. We maintain the assumption that the products they offer are perfect substitutes. We retain the assumption that they are totally symmetric: they have the same cost function and require the same use of the network. We further assume that the downstream cost function is linear, with  $C_i(x_i) = cx_i$ . Finally, we assume that the downstream firms act as price takers in their purchase of network services.

### 5.1 Social Optimum

The social planner's objective is to

$$\max_l S = \int_0^l p(s)ds - cl - kl.$$

The solution  $X^* = l^*$  to this problem is defined by

$$p(X^*) = c + k.$$

This is the usual condition that marginal willingness to pay should equal marginal cost. Since the (constant) marginal cost is the same for both downstream firms, the socially optimal allocation is concerned with the total downstream quantities and not with the individual quantities sold by each firm.

## 5.2 Equilibrium in the downstream market

With Cournot competition, each downstream firm chooses its output level  $x_i$  in order to maximize

$$\begin{aligned}\pi_i &= x_i p(x_i + x_j) - ax_i - cx_i, \\ &= x_i [p(x_i + x_j) - a - c],\end{aligned}$$

given the output level  $x_j$  supplied by its competitor  $j$ . The fact that firm  $i$  acts as a price taker in the market for network services implies that it takes the access charge  $a$  as given, and independent of its own demand for these services.

The first order condition for downstream profit maximization is

$$x_i = \frac{p(x_i + x_j) - a - c}{-p'(x_i + x_j)}. \quad (19)$$

Equation (19) together with the condition  $X(p) = l$  determine as previously the access charge and retail price as a function of network size. These relationships are denoted by  $\tilde{a}(l)$  et  $\tilde{p}(l) = p(l)$ .

The symmetry between the two firms, together with the equilibrium condition on the market for input  $l$  imply that both firms choose the same output at equilibrium

$$x_1 = x_2 = l/2.$$

This relationship allows us to simplify equation (19) to obtain

$$\tilde{p}(l) = a + c - \frac{l}{2}\tilde{p}'(l),$$

with  $\tilde{p}'(l) < 0$ .

The intuition for this result is that each firm sells its product at a price larger than its marginal cost  $a + c$ , with the mark-up being inversely proportional to (half) the demand-price elasticity of output.

We now proceed to study equilibrium network size under various integration scenarios.

### 5.3 Equilibrium when $U$ owns both downstream firms

In its choice of network size, the upstream firm internalizes the downstream profit and solves

$$\begin{aligned} \max_{l,a,p} \pi_U + \pi_1 + \pi_2 &= al - kl + X(p) [p - a - c] \\ \text{s.t. } p &= a + c - \frac{l}{2}\tilde{p}'(l), \\ X(p) &= l. \end{aligned}$$

After simplification and using the inverse demand function, we obtain

$$\pi_U + \pi_1 + \pi_2 = l(\tilde{p}(l) - c - k), \quad (20)$$

whose maximization with respect to  $l$  gives the condition

$$\tilde{p} + l\tilde{p}' = c + k. \quad (21)$$

This condition is the usual profit-maximization solution of a monopoly, equalizing marginal revenue and marginal cost.

Observe that the second order condition for total (downstream plus upstream) profit maximization is given by

$$2\tilde{p}' + l\tilde{p}'' < 0. \quad (22)$$

We will use this result later.

As in the previous two sections, we obtain  $l^{e2} < l^*$  and  $p^{e2} > p^*$ . The intuition for this result is also the same as previously: the upstream firm under-invests in

the network in order to decrease downstream quantities and increase downstream prices. The main difference with the previous section lies in the fact the downstream firms make a profit because of imperfect competition, not because returns to scale are decreasing.

## 5.4 Equilibrium with ownership unbundling

If ownership is separated between upstream and downstream segments of the markets, the upstream firm chooses the network size that maximizes

$$\pi_U = \tilde{a}(l)l - kl.$$

We can rewrite the optimization problem as

$$\begin{aligned} \max_{l,a,p} \pi_U &= al - kl \\ \text{s.t. } p &= a + c - \frac{l}{2}p'(l), \\ X(p) &= l. \end{aligned}$$

We obtain after substitution that

$$\pi_U = [p(l) - c - k]l + \frac{l^2}{2}p'(l). \quad (23)$$

The first term in the right hand side of (23) is equal to the total profit  $\pi_U + \pi_1 + \pi_2$  as specified by equation (20) when the upstream firm owns both downstream firms.

This implies

$$\left. \frac{d\pi_U}{dl} \right|_{l=l^{e2}} = l^{e2} \left[ p'(l^{e2}) + \frac{l^{e2}}{2} p''(l^{e2}) \right] < 0,$$

where the inequality is a consequence of condition (22). Therefore, by concavity of  $\pi_U$ ,

$$l^{e0} < l^{e2} < l^*.$$

Although this ranking of network sizes is the same than under downstream perfect competition coupled with decreasing returns to scale, the intuition differs. Note first that the upstream firm revenue is given by  $ax_i = (px_i - cx_i + l^2 p'(l)/4)$

(with  $x_i = l/2$ ), which is lower than the downstream profit (equal to  $px_i - cx_i$ ) because of the mark-up posted downstream. Moreover, the second order condition for (total) profit maximization guarantees that the difference between the two increases with  $x_i$  and  $l$ . In other words, the reason why the difference between downstream profit and upstream revenue increases with the network size varies with the downstream cost structure and competitive situation: under perfect competition, it is due to the (assumed) convexity of costs while under imperfect competition, it is due to the increase in the downstream mark-up.

## 5.5 Equilibrium when the upstream firm owns one of the downstream firms

If the upstream firm owns one of the downstream firms, one can replicate the argument mentioned in section 3.5: the constraints faced are the same in the cases where the upstream firm owns zero, one and two downstream firms, while the objective in the case  $e1$  is a convex combination of the objectives in the scenarios  $e0$  and  $e2$ :

$$\pi_U + \pi_i = \frac{1}{2}(\pi_U + \pi_1 + \pi_2) + \frac{1}{2}\pi_U \quad , i = 1, 2.$$

We then obtain that, provided that the profit functions are concave in  $l$ ,

$$l^{e0} < l^{e1} < l^{e2} < l^*.$$

We then conclude from this section that the ranking of network sizes according to the number of downstream firms owned by the upstream firm is robust to the introduction of imperfect competition in the downstream market.

## 6 Investments by the downstream firms

We now study the robustness of our results to the introduction of a second decision by the downstream firms, beyond the setting of their prices. This decision is how much to invest in an activity that, although costly by itself, allows the downstream



firm to economize on its network usage for any given level of output. The kind of downstream investment we have in mind for the natural gas market consists in offering to final clients interruptible contracts or alternatively buying insurance to cover risks such as transport congestion due to a peak demand. These two types of contracts are obviously costly for the downstream firm (in the first case because they decrease its output price, in the second because of the direct outlays they represent) but allow it to decrease its needs in terms of network usage for any level of output sold to clients.

We maintain the assumption of legal unbundling throughout the analysis, so that the upstream firm cannot control the investment decisions of its downstream subsidiaries. We model this extension to downstream investments as follows. The profit of downstream firm  $i$  is given by

$$\pi_i = px_i - C(x_i) - \beta(y_i)ax_i - y_ix_i,$$

where, as above, the non network cost function  $C$  is convex, with  $C''' > 0$ .

## 6.1 Social Optimum

The social planner chooses the network size  $l$  and the downstream investment that solve the problem

$$\max_{l,y} S = \int_0^X p(s)ds - 2C\left(\frac{l}{2\beta(y)}\right) - \frac{yl}{\beta(y)} - kl,$$

with  $X = l/\beta(y)$ .

Denoting the optimal level of variables by a  $*$  as previously, the first order condition with respect to network size is

$$p^* = p(X^*) = C'\left(\frac{X^*}{2}\right) + y + k\beta(y) \quad (24)$$

i.e., marginal willingness to pay should equal social marginal cost. With constant marginal costs, the social optimum determines total downstream output but not

how much is produced by firm 1 or firm 2. For later use, we express condition (24) in terms of mark-up over the marginal non network cost:

$$p^* - C' \left( \frac{X^*}{2} \right) = y + k\beta(y) \quad (25)$$

The first-order condition with respect to downstream investment is

$$\beta'(y^*)k = -1. \quad (26)$$

Both firms should invest the same per-unit of output amount, which equalizes marginal benefit and marginal cost per unit of output.

## 6.2 Equilibrium in the downstream market

The two downstream firms, which are price takers both on the downstream market and on the market for the network input, simultaneously choose their profit-maximizing levels of investment,  $y_i$ . Using the symmetry between downstream firms, the first-order condition for  $y$  is

$$\beta'(y)a = -1, \quad (27)$$

which is very intuitive, since it calls for equalization of the monetary marginal benefit from the investment with its marginal cost.

The price taking behavior of downstream firms implies

$$p = C' \left( \frac{X(p)}{2} \right) + a\beta(y) + y, \quad (28)$$

i.e., that the equilibrium price equals total marginal cost for the downstream firms.

Equilibrium on the input  $l$  market implies

$$\beta(y)X(p) = l. \quad (29)$$

Equations (27) to (29) simultaneously determine the access charge  $a$ , the downstream price  $p$  and the amount of downstream investment  $y$  (and thus also  $X$ ) as function of  $l$ . We denote these functions by  $\tilde{a}(l)$ ,  $\tilde{y}(l)$  and  $\tilde{p}(l)$ . Observe that, as in section 4,  $\tilde{p}(l) \neq p(l)$  because  $l \neq X = x_1 + x_2$ .

We now turn to the equilibrium when the upstream firm owns the two downstream firms.

### 6.3 Equilibrium when $U$ owns both downstream firms

When the upstream firm owns the two downstream firms, it maximizes the sum of its profit,  $\pi_U$  and of profits of the two downstream firms,  $\pi_1$  and  $\pi_2$ :

$$\begin{aligned} \pi_U + \pi_1 + \pi_2 = & \tilde{a}(l)l - kl + X(\tilde{p}(l)) [\tilde{p}(l) - \beta(\tilde{y}(l))\tilde{a}(l)] \\ & - 2C\left(\frac{X(\tilde{p}(l))}{2}\right) - \tilde{y}(l)X(\tilde{p}(l)), \end{aligned}$$

where  $\tilde{a}(l)$ ,  $\tilde{y}(l)$  and  $\tilde{p}(l)$  are the solutions to equations(27) to (29).

We reorganize this optimization problem to obtain

$$\begin{aligned} \max_{l,a,p,y} \pi_U + \pi_1 + \pi_2 = & al - kl + X(p)(p - \beta(y)a) - 2C\left(\frac{X(p)}{2}\right) - yX(p) \\ \text{s. t. } p = & C'\left(\frac{X(p)}{2}\right) + a\beta(y) + y, \\ l = & \beta(y)X(p), \\ 1 = & -\beta'(y)a. \end{aligned}$$

After simplification, and using the inverse demand function, we obtain

$$\pi_U + \pi_1 + \pi_2 = \frac{l}{\beta(y)}\tilde{p}(l) - 2C\left(\frac{l}{2\beta(y)}\right) - kl - \frac{yl}{\beta(y)}$$

whose maximization with respect to  $l$  gives the following first order condition

$$\tilde{p}(l) + l\tilde{p}'(l) = C'\left(\frac{l}{2\beta(y)}\right) + y + k\beta(y), \quad (30)$$

where  $y$  is determined by

$$\beta'(y)a = -1.$$

This corresponds to the profit-maximizing condition of a monopoly, where marginal revenue equals total marginal cost. In order to compare with the socially optimal price, we denote as usual the equilibrium levels with the  $e2$  superscript and reformulate (30) into

$$p^{e2} - C'\left(\frac{l^{e2}}{2\beta(y^{e2})}\right) = y^{e2} + k\beta(y^{e2}) - l^{e2}\tilde{p}'(l^{e2}). \quad (31)$$

We now compare the right hand sides of (25) and (31) term by term. The sum of the first two terms is the (per unit of output) network cost, including the investment cost. Note that  $y^* + k\beta(y^*) < y^{e2} + k\beta(y^{e2})$  if  $a \neq k$ , since  $y^*$  precisely minimizes  $y + k\beta(y)$ . This calls for a profit maximizing price  $p^{e2}$  larger than its optimal level, because in the  $e2$  scenario the downstream firms base their investment decision on the access charge rather than the marginal social cost  $k$ , and end up (when  $a \neq k$ ) with a social marginal cost that is larger than its socially optimal level. The third term in (31) pushes  $p^{e2}$  in the same direction since it represents the classical impact of a profit-maximizing firm concentrating on marginal revenue rather than considering that its final price is exogenously set. We then conclude that the mark-up over the non network cost is larger when the upstream firm owns the downstream firms than its socially optimal level.

In section 3, the observation that the mark-up over non network marginal cost  $C'$  was larger in the  $e2$  scenario than its optimal level was enough to deduct that  $p^{e2} > p^*$  and  $l^* > l^{e2}$ . This is not sufficient in the framework of this section, since such comparisons also depend on the comparison between  $y^{e2}$  and the socially optimal downstream investment level  $y^*$ . This comparison in turn hinges on whether the access charge  $a$  is larger or smaller than the network marginal cost  $k$ . Observe that, with legal unbundling, the upstream firm cannot control the pricing decisions of its downstream subsidiaries. In the absence of downstream investment, the upstream firm induces a positive mark-up on the downstream market by decreasing the size of its network and at the same time increasing the (market clearing) access charge  $a$ , so that  $a > k$ . Introducing downstream investment, we obtain that a further effect of increasing  $a$  above  $k$  is to induce the downstream firm to invest more than would be socially optimal:  $y^* < y^{e2}$ . This in turn implies that the downstream firm is able to sell more output for a given network size  $l$  than with the optimal downstream investment level, which counteracts the effect of a higher access charge  $a$  on  $p$ .

We have not been able to obtain analytically unambiguous results with respect to the comparison between optimal and  $e2$  levels of  $a$ ,  $y$ ,  $l$  and  $X$ . We surmise that

the new effect mentioned above mitigates only partially the direct effects described in section 3, so that the most likely situation is the one where  $a^{e2} > k$ ,  $y^* < y^{e2}$ ,  $l^* > l^{e2}$ ,  $p^{e2} > p^*$  and  $X^* > X^{e2}$  — i.e., where the relationships between prices and quantities obtained in section 3 carry through to the case where downstream firms make an investment. We show in section 6.6 that it is the case for the numerical example we develop there.

## 6.4 Equilibrium with ownership unbundling

We proceed as in section 4.4, noting that the objective of the upstream firm is to maximize its own profit, which can be expressed as

$$\pi_U = (\pi_U + \pi_1 + \pi_2) - (\pi_1 + \pi_2).$$

We can also rewrite the profit functions of the two downstream firms as

$$\begin{aligned} \pi_i &= \tilde{x}_i(l) [\tilde{p}(l) - \beta (\tilde{y}(l)) \tilde{a}(l) - \tilde{y}(l)] - C(\tilde{x}_i(l)), \\ &= \tilde{x}_i(l) C'(\tilde{x}_i(l)) - C(\tilde{x}_i(l)), \end{aligned}$$

where

$$\tilde{x}_i(l) = \frac{l}{2\beta (\tilde{y}(l))}.$$

Differentiating profits with respect to network size, we obtain

$$\pi'_i = \tilde{x}_i(l) C''(\tilde{x}_i(l)) \tilde{x}'_i(l),$$

where

$$\tilde{x}'_i(l) = \frac{2\beta (\tilde{y}(l)) - 2l\beta'(\tilde{y}(l)) \tilde{y}'(l)}{4\beta^2 (\tilde{y}(l))}$$

is of an ambiguous sign since  $\beta'(\tilde{y}(l)) < 0$  and  $\tilde{y}'(l) < 0$ .

Observe that, if  $\tilde{x}'_i(l) > 0$ , then we can use the same reasoning as in section 4.4 to obtain, provided that the objective function  $\pi_U$  is concave,

$$l^{e0} < l^{e2}.$$

In that case, we would also have

$$l^{e0} < l^{e1} < l^{e2}.$$

Finally, it is easy to see that  $y^{e0} > y^*$  because, with ownership unbundling, the only way for the upstream firm to make a profit is to charge an access price larger than its marginal cost,  $a > k$ .

## 6.5 Equilibrium when the upstream firm owns one of the downstream firms

We can proceed as in sections 3.5 and 5.5, to show that the objective in the case  $e1$  is a convex combination of the objectives in the scenarios  $e0$  and  $e2$ , with the same constraints in all three cases. Provided that the objective is concave, we then obtain that the  $e1$  levels of the variables  $p$ ,  $y$ , and  $l$  should be in between their equilibrium levels in scenarios  $e0$  and  $e2$ .

## 6.6 A numerical example

The new effects generated by the introduction of downstream investments have prevented use from reaching unambiguous analytical conclusions when comparing equilibrium and optimal levels of prices, network size and output. We therefore present a numerical example where the comparison of the equilibrium levels in the various scenarios is the same as in the previous sections.

We use the following functional forms

$$\begin{aligned} C(x) &= x^2, \\ \beta(y) &= 1 - \frac{\sqrt{y}}{10}, \\ X(p) &= 100 - 5p. \end{aligned}$$

We first study the case where  $y$  is set exogenously equal to zero i.e., the case developed in section 3. This allows us to show graphically the equilibrium and optimal levels of the network size  $l$  and of output price  $p$  as a function of the

marginal network cost  $k$ . Figure 1 shows that  $l^{e0} < l^{e1} < l^{e2} < l^*$  while figure 2 illustrates that  $p^{e0} > p^{e1} > p^{e2} > p^*$ .

[Insert Figures 1 and 2 around here]

We now turn to the case where  $y$  is chosen by the downstream firms. In Table 1, we compare the optimum and equilibrium values of  $y$ ,  $X$ ,  $p$ ,  $l$  and  $a$  when  $k$  is set equal to 5.<sup>11</sup>

Table 1: Equilibrium levels when  $k = 5$ .

	Scenarios			
	*	e2	e1	e0
$y$	0.062	0.126	0.283	0.409
$X$	12.552	10.853	8.037	6.345
$p$	17.490	17.830	18.393	18.731
$l$	12.238	10.467	7.609	5.939
$a$		7.103	10.639	12.796

Table 1 shows that we obtain the following relationship:  $k < a^{e2} < a^{e1} < a^{e0}$ . Intuitively, as the number of downstream firms owned by the upstream firm decreases, the upstream firm relies more and more on the access charge to increase its profit. At the limit, with ownership unbundling (case  $e0$ ), the access charge is the only way for the upstream firm to obtain revenues. In all scenarios, the equilibrium access charge is larger than its optimal level. It follows directly from this that we obtain  $y^* < y^{e2} < y^{e1} < y^{e0}$  i.e., the equilibrium level of downstream investment is too large and increases with ownership separation. The intuition is that downstream firms react to large access charges by over-investing in activities whose objective is to limit their network usage.

Table 1 also shows that  $l^* > l^{e2} > l^{e1} > l^{e0}$  i.e., the main result of the paper carries through to the case of downstream investments: the more ownership is unbundled, the larger is the incentive for the upstream firm to decrease its network size in order to raise its profit. We also obtain  $p^* < p^{e2} < p^{e1} < p^{e0}$ :

<sup>11</sup>We obtain the same qualitative results for any value of  $k$  between 0 and 20.

prices increase with ownership unbundling. Finally, observe that, even though downstream investment increases with ownership unbundling, total downstream quantity decreases with ownership unbundling:  $X^* > X^{e2} > X^{e1} > X^{e0}$ . In words, the main effect at work when ownership is unbundled is the incentive for the upstream firm to decrease its network size. The impact on the downstream investment mitigates only partially the consequences of a smaller network size, so that total quantity sold decreases with ownership unbundling.

## 7 Conclusion

In all the models that we have developed in this paper, we find that full control of the downstream industry by the upstream firm would be socially efficient. This is of course too strong a conclusion, but we still believe that our analysis highlights important considerations for economic analysis. In this conclusion, we would like both to discuss these lessons and explain how we believe our model should be expanded and/or modified.

In all our models, we assume that the regulator has a strong control over the behavior in the downstream market. In particular, it can completely prevent the network from favoring one of the downstream firms and, in the models of sections 3 and 4 it can impose on the downstream firms that they behave competitively. On the other hand, it has less control over the long term decisions of the network, in our case new investment. We believe that this is a fair, if caricatural, characterization of the powers of most regulators. Our model stresses the fact that under these circumstances making the upstream firm internalize the profits of its client can be a powerful method for inducing it to invest more. Even if the upstream firm owns only one of the two downstream firms, both firms benefit from this vertical integration.

To analyze in more details the tradeoffs involved, we would need to modify the model so that there is positive reasons why competition in the downstream market is beneficial. This would involve introducing explicitly some degree of asymmetric



information, while preserving our emphasis on incomplete contracts and specific investment, and will be the topic for future research.

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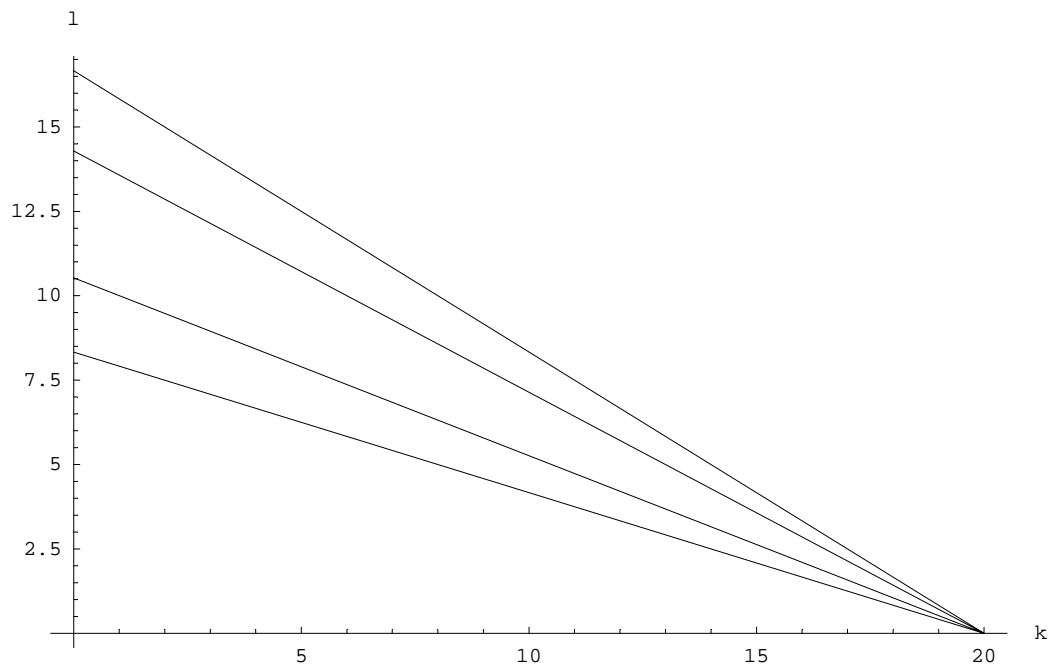
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**Figure 1 : Network size  $l$  as a function of network cost  $k$**



**Figure 2 : Output price  $p$  as a function of network cost  $k$**

