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TESIS DOCTORAL

Market Share Discounts, Separation, and Equilibrium Structure in Successive Oligopolies

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SUMMARY

This thesis comprises three essays on Industrial Organization. The first chapter contributes to the literature on efficiency of a market share discounts use. The second chapter is a joint research papers with Emmanuel Petrakis and Chrysovalantou Milliou, where we study an equilibrium structure in multiproduct successive oligopolies. The third chapter investigates firms' incentives for vertical separation and integration. Each chapter can be considered independently of the rest.

The first chapter investigates the pro- and anticompetitive effects of market share discounts (MSD's). While MSD's can be used for exploiting a dominant position and lead to a welfare reduction, MSD's can also serve as an efficient device for the creation of an investment incentive. Particularly, if the final demand for an upstream manufacturer's good depends on retailer's promotional effort, the manufacturer can effectively use MSD's to induce the optimal level of the retailer's effort. Moreover, it is possible that MSD's have a positive impact both on the consumers' surplus and total industry profits. Thus, the main result of the chapter is that MSD's should not be treated anticompetitive apriori, but rather they should be judged on a case-by-case basis.

In the second chapter, we develop a successive oligopoly model in which multi-product upstream manufacturers sell their products to consumers through downstream retailers. The product variety offered by each manufacturer and the entry in the upstream market are both endogenous. We show that the equilibrium configuration of the upstream market depends crucially on the economies of scope in the process of new product creation. When the economies of scope are weak the number of manufacturers increases and each manufacturer produces a single product. Manufacturers produce multiple products only if the economies of scope are sufficiently strong. Furthermore, we examine how a number of other market characteristics, such as the market size, the product substitutability and the number of retailers affect product variety, entry, firms' profits and welfare.

The third chapter explores firms' incentives for strategic vertical separation in duopoly settings. Each firm chooses either to be a retailer of its own good (vertical integration) or to sell its good through an independent exclusive retailer (vertical separation). In the latter case, a two-part tariff contract is applied. Retailers compete in quantities, goods are perfect substitutes and firms' cost functions are quadratic. It is shown that the equilibrium crucially depends on the degree of the (dis)economies of scale and the asymmetry in firms' costs. Two asymmetric equilibria arise, in which one firm separates while the other integrates, when both firms' cost functions exhibit sufficiently high diseconomies of scale, or extreme asymmetry of their costs. When the cost asymmetry is moderate, a unique equilibrium exists in which the firm with the lower degree of diseconomies of scale separates, while its rival integrates. When instead the diseconomies of scale are low for both firms, in the unique equilibrium both firms separate. Robustness analysis demonstrates that the results hold also under mild assumptions of a demand and cost functions.

La tesis se compone de tres ensayos en el marco de Organizacion Industrial. El primer capítulo, contribuye a la literatura existente sobre descuendo por cuota de mercado. El segundo capítulo representa un trabajo hecho junto con Emmanuel Petrakis y Chrysovalantou Milliou, donde estudiamos el equilibrio para el mercado con oligopolios multiproductos sucesivos. El tercer capítulo investiga los incentivos de las empresas para los casos de la separación vertical y de la integración. Cada capítulo puede ser considerado independientemente del resto.

El primer capítulo estudia los efectos favorables y contracompetitivos de descuendo por cuota de mercado (market share discounts, MSD). Aunque MSD se puede utilizar para la explotación de una posición dominante y resulta en una disminución del bienestar, MSD también puede servir como un instrumento eficaz para crear los incentivos a la inversión. En particular, si la demanda final de un fabricante upstream depende del esfuerzo de promoción del minorista, el fabricante puede utilizar MSD con eficacia para inducir el nivel óptimo de esfuerzo del minorista. A parte, es posible que MSD tenga un impacto positivo tanto sobre el excedente del consumidor y los beneficios de la industria. Por lo tanto, el resultado principal del capitulo es que MSD no debe ser tratado únicamente contracompetitivo a priori, sino que debe ser juzgado caso por caso.

En el segundo capítulo, se desarrolla un modelo de oligopolio sucesivo en el que los fabricantes multiproductos upstream venden sus productos a los consumidores a través de minoristas downstream. La variedad de productos que ofrece cada fabricante y la entrada en el mercado upstream son endogenas. Se demuestra que el equilibrio del mercado upstream depende fundamentalmente del marco de las economías en el proceso de la creación de nuevos productos. Cuando las economías son débiles aumenta el número de fabricantes y cada fabricante produce un solo producto. Los fabricantes producen varios productos sólo si las economías son lo suficientemente fuertes. Además, se examina cómo una serie de otras caracteristicas del mercado, tales como el tamaño del mercado, la posibilidad de sustitución del producto y el número de minoristas, afectan variedad de productos, la entrada, los beneficios de las empresas y el bienestar.

El tercer capítulo analiza los incentivos de las empresas para la separación vertical estratégica en el marco de duopolio. Cada empresa elige uno de los dos casos: ser un minorista de su propio bien (integración vertical) o vender su bien a través de un distribuidor exclusivo independiente (separación vertical). En este último caso se aplica la tarifa por partes. Los minoristas compiten en cantidades, los bienes son sustitutos perfectos y funciones de costes de las empresas son cuadráticas. Se muestra que el equilibrio depende del grado de las (des)economias de escala y la asimetria de los costes de las empresas. Dos equilibrios asimétricos surgen, en los que una empresa se separa mientras que la otra se integra, cuando las funciones de coste de ambas firmas exhiben las des economias de escala lo suficientemente alta, o asimetría extrema de sus costes. Cuando la asimetría de costes es moderada, existe un único equilibrio en el que la empresa con el menor grado de las des economías de escala separa, mientras que su rival se integra. Sin embargo cuando las des economías de escala son bajas para las ambas empresas, en el único equilibrio ambas empresas separan. Análisis de robustez demuestra que estos resultados se mantienen bajo supuestos leves de la demanda y las funciones de coste.

CHAPTER I

MARKET SHARE DISCOUNTS AND INVESTMENT INCENTIVES

1.1 Introduction

Vertical restraints, such as loyalty rebates, resale price maintenance, exclusive dealing and exclusive territories are often used in dealing between manufacturers and retailers. In some cases, vertical restraints serve an anti-competitive purpose by excluding competitors or creating entry barriers. In other cases, vertical restraints are used to increase efficiency by eliminating double price marginalization, reaching an optimal level of production or by creating the "right" incentive for vertically related firms. In all these cases, vertical restraints are of considerable interest to antitrust practitioners.

In this paper, I analyze a special type of vertical restraint, i.e. market share discounts (MSD's) (also known as loyalty rebates). MSD's are discounts that a manufacturer offers to its distributors, or retailers, if their sales of the manufacturer's brand comprise a sufficiently high percentage of their total sales for a given class of goods. Thus, MSD's are a special type of discount which is based on the quantity of goods that the retailer buys from both the manufacturer and its competitors.

The increasing number of antitrust cases related to such restraints confirms that manufacturers are using such arrangements increasingly in recent years[1]. The case of the Concord Boat Corporation versus the Brunswick Corporation is one of the well-known examples of the use of MSD's[2]. Brunswick manufactured and sold stern drive engines for recreational boats; it had a large share of the market (i.e., 75% in 1983). Beginning in the early 1980s, Brunswick offered market share discounts. Boat builder customers who agreed to purchase

[1] See "Roundtable on loyalty or fidelity discounts and rebates", DAFFE meeting, May, 2002, Tom et al. (2000) and Kobayashi (2005) for a review.

[2] See Concord BoatCorp. v. Brunswick Corp., 207 F.3d 1039 (8th Cir. 2000)

a certain percentage of their engine requirements from Brunswick for a period of time (often a year, sometimes longer) received a discount off the list price for all engines purchased[3]. Some of the boat builders sued Brunswick, alleging among other claims, that these discount programs excluded competing stern drive engine manufacturers from the market and amounted to monopolization. A court ruled that Brunswick's pricing amounted to de facto exclusive dealing, and foreclosed rival suppliers of marine engines from the market. On appeal, that ruling was reversed on grounds that market conditions were not conducive to foreclosure.

An additional example is the case of Virgin Atlantic Airways Ltd. versus British Airways[4]. British Airways (BA) used incentive programs that provided travel agencies with commissions, and corporate customers with discounts, for meeting specified thresholds for sales of BA tickets (sometimes expressed in terms of market share). Virgin Atlantic claimed that the result was below cost pricing on certain transatlantic routes where Virgin and BA competed, with BA's attendant losses being subsidized by monopoly pricing on other BA routes. Virgin alleged that the below cost pricing slowed its expansion on the competitive routes. Both a district court and a court of appeals concluded that Virgin had failed to demonstrate that pricing was below cost.

In this paper, I consider a vertically related two-level industry. At the upstream level, a manufacturer and a competitive fringe produce imperfect substitutes. At the downstream level, there is only one retailer which trades both goods to final consumers[5]. The manufacturer may offer to the retailer either a wholesale price contract or a market share discount contract. Further, the retailer can make a costly effort investment which results in an increase in the demand of the manufacturer's good. By assumption this effort has no effect on the demand for the competitive sector's firms good. The effort level is noncontractible; hence, neither the wholesale price nor MSD's may be contingent on the retailer's effort level. This allows us to analyze the role of MSD's as a tool for the creation of incentives,

[3] Particularly, an agreement to buy 70% of engine requirements from Brunswick might result in a 3% discount, agreement for 65% in a 2% discount, and an agreement for 60% in a 1% discount.

[4] Virgin Atlantic Airways Ltd. v. British Airways F.3d 256 (2d Cir. 2001).

[5] The similar setup is adopted in papers of Mills (2009) and Chioveanu and Akgun (2006).

as well as to consider the welfare effects of MSD's. To highlight this role I begin with a consideration of a benchmark case in which the retailer's effort has no impact on demand. Then I analyze the case in which the retailer's effort results in an increase in the demand of the manufacturer's good.

In the benchmark case, I find that if MSD's are applied, both the quantity of the manufacturer's sales and the manufacturer's profit increase; the quantity of the good sold by the competitive sector's firm decreases. The total industry profit decreases as does consumers' surplus. Thus, only the manufacturer gains from the use of MSD's. This allows us to conclude that MSD's have an anticompetitive character in this setting. For the case when the retailer's effort is productive I obtain the following results. First, if the wholesale price contract is applied, the manufacturer may not be able to motivate the retailer to undertake the desired level of effort. In this case, the market outcome is the same as in the benchmark case with wholesale price contracts. If MSD's are applied, then the manufacturer can design the menu of prices in such a way that the retailer undertakes the desired level of effort, i.e. efficient from the social point of view. Moreover, in this case both the industry profit and consumer surplus are higher with MSD's then only with wholesale price contract. Another important result is that while the use of MSD's increases the manufacturer's market share, it does not completely drive competing firms out of the market. Thus a use of the MSD contract is socially more preferable than a use of wholesale price contract.

Combining the above results, one can see that the judgments on whether MSD's have an anti- or procompetitive effect depends crucially on the features of the market environment. While, in some cases, MSD's may serve for a redistribution of profit between the manufacturer and the retailer and may lead to a decrease in social welfare, in other cases, they may also serve as an efficient instrument for the creation investment incentives and may result in an increase in total social welfare. Thus, the treatment of MSD's should be deduced on a case-by-case basis.

Recently some papers have examined different aspects of MSD's. For example, Marx and Shaffer (2004) and Greenlee and Reitman (2004) analyze the rent-shifting effects of MSD's. Marx and Shaffer (2004) examine the use of MSD's, slotting allowances and predatory pricing in a three-party sequential contracting environment. In their model two sellers negotiate sequentially with one buyer. MSD's and slotting allowances are used to shift rents between the contracting parties, with no short run consequences for social welfare. They find that this type of rent shifting equilibrium generally results in both sellers remaining in the market. In the long run, the authors suggest that preventing the use of such devices will result in the adoption of strategies that are more likely to result in one of the sellers being excluded. However, the model does not explicitly analyze the welfare effect of such long term effects.

Greenlee and Reitman (2004) analyze the case of two competing firms selling their goods to final consumers by using loyalty rebates or wholesale price contracts. They find that in equilibrium only one firm applies market share discount. Moreover, as me show, that welfare effects of MSD use depends on the demand structure.

Majumdar and Shaffer (2007) analyze a case where one manufacturer and a competitive fringe supply goods to a retailer who has private information about the state of demand. They examine the conditions under which market-share contracts are profitable, and show that the full-information outcome can be obtained. They show as well that MSD's contracts are more profitable than all-units discounts contract.

Chioveanu and Akgun (2006) compare a manufacturer's incentives to apply MSD's, all-unit discounts and incremental-unit discounts. They show that in a situation where there is full information, all discounts are equivalent from both manufacturer's and social viewpoints. However, under uncertainty, the attitude toward risk of the retailer can play a crucial role in the form of the loyalty discount applied by the manufacturer.

Greenlee et al. (2004) analyze the use of bundled market share discounts by a multi-product monopolist. They show that it may exclude an equally efficient competitor that produces a single-product, and that the welfare effects are ambiguous.

Ordover and Shaffer (2007) show that when MSD's are implemented by a dominant firm, who may have easier access to financing compared to a rival, they can sometimes exclude an equally-efficient rival and lower overall welfare.

MSD's could also be used for efficiency reasons. Mills (2009) examines the competitive effects of a vertically differentiated product manufacturer implementing MSD's in its sales to its distributors. His central idea is that MSD's are not mainly an exclusionary device, but rather a device for inducing merchandising services that help consumers make well-informed decisions. Mills (2009) assumes that each consumer has unit demand and the retailer's effort increases a share of consumers who prefer the manufacturer's good. Mills shows that MSD's induce increased selling effort and improve efficiency comparing to the use of the wholesale price contract. In some cases MSD's may lead to redistribution of gross industry benefit but in Mills' setup they never reduce social welfare comparing to a wholesale price use. This result is crucially depends on assumption that consumers have a unit demand.

Like Mills (2009), I examine efficiency of MSD. In contrast to Mills (2009), I consider linear demand functions and demonstrate that the impact of MSD's on social welfare is in general ambiguous and crucially depends on model specification. In my model, the retailer may reach any required market threshold (by decreasing the quantity of other goods it sells) without undertaking any effort. In fact, this possibility is a base for arguments on anticompetitive nature of MSD's and must be taken into account.

The rest of the paper proceeds as follows. Section 1.2 describes the model. Section 1.3 considers the benchmark case. Section 1.4 analyses the case with a productive effort. Section 1.5 includes the welfare analysis and a numerical example and section 1.6 concludes.

1.2 The Model

There is one retailer, R, which sells two substitutable goods to final consumers. The first good is produced by a brand-name upstream manufacturer, M. The brand manufacturer produces with a constant marginal cost, $c \geq 0$. The second good is produced by a competitive fringe. The marginal cost of production of the second good is zero.

The retailer can undertake a costly investment effort which will increase the demand for the manufacturer's good. For example, consumers may not be perfectly informed about the quality of the manufacturer's good and the retailer can provide consumers with that information by offering promotion, better visibility etc. It is reasonable to assume that this effort is made by the retailer and not the manufacturer. The level of the effort is discrete, $e = \{0, 1\}$ and not contractible. The cost of effort is $E > 0$.

A representative consumer has utility function of the form:

$$U(q_1, q_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2), \tag{1}$$

where q_1, q_2 are the quantities purchased by the consumer, $b \in (0, 1)$,is the degree of goods differentiation. I assume that the parameter A depends on the retailer's effort level with $A(1) = A_1 \geq A(0) = 1$. I assume that $1 - b - c > 0$.

The utility function (1) generates the inverse demand for the manufacturer's good, $p_1 = A(e) - q_1 - bq_2$, which depends on the retailer's effort level, and the demand for the competitive sector's firms good $p_2 = 1 - q_2 - bq_1$. The latter does not depend on the effort level.

The manufacturer may use two types of contracts in dealing with the retailer, i.e. a wholesale price contract, which specifies a constant per-unit price, ω, and market-share discounts. Denote $s = q_1/(q_1+q_2)$ then the the manufacturer's contract specifies parameters $\{t_L, t_H, \overline{s}\}$ that form a menu of prices:

$$t_{MSD} = \begin{cases} t_L & \text{if } s \geq \overline{s} \\ t_H & \text{if } s < \overline{s} \end{cases} \quad t_H > t_L, \tag{2}$$

where s denote the share of the manufacturer's good as a proportion of the total sales of the retailer, $\overline{s}$ is the market share threshold that the retailer must meet in order to buy at the two prices t_L, and t_H. Meanwhile, t_L (t_H) state the manufacturer's price when the retailer does (does not) meet the market share requirement.

Let t denote either the single price ω or the menu of prices t_{MSD}. All producers in the fringe compete in prices. As a result the competitive sector's firms set prices equal to marginal cost and obtain zero profit.

The retailer's profit is:

$$\pi^R = (A(e) - q_1 - bq_2 - t)q_1 + (1 - q_2 - bq_1)q_2 - eE.$$

The profit of the brand manufacturer is:

$$\pi^M = q_1(t - c),$$

where t is either the wholesale price or the menu of prices.

The timing in the model is the following. In the first stage, the manufacturer and competitive sector's firms simultaneously set their prices. The manufacturer sets the menu of prices, t_{MSD} (as in (2)) or the wholesale price ω. In the second stage, the retailer chooses its effort level $e = \{0, 1\}$ and the levels of quantities, q_1 and q_2.

In what follows, I first look at the special case of the model where $A_1 = A_0 = 1$.[6] The condition $A_1 = A_0$ implies that the retailer's effort has no effect on the consumer demand and that the manufacturer has no reason to motivate the retailer to undertake the costly effort. Then, I examine the general case where $A_1 > A_0 = 1$.In this case consumer demand depends on the retailer's effort level. Profit functions are subscribed by indexes MSD and WP for case when MSD's and the wholesale price contract is applied respectively.

1.3 Benchmark case: No investment effort

1.3.1 Wholesale price contract

First, I consider the retailer's problem:

$$\max_{q_1, q_2} \pi^R_{WP}(q_1, q_2; \omega) = (1 - q_1 - bq_2 - \omega)q_1 + (1 - q_2 - bq_1)q_2.$$

The solution for the first order conditions is: $q_1(\omega) = \frac{1-b-\omega}{2(1-b^2)}, q_2(\omega) = \frac{1-b+b\omega}{2(1-b^2)}$.

The profit of the retailer as a function of the price ω can be rewritten as:

$$\pi^R_{WP}(\omega) = \frac{2 - 2b(1-\omega) - 2\omega + \omega^2}{4(1-b^2)}.$$

Now the problem of the manufacturer can be written as:

$$\max_{\omega} \pi^R_{WP}(\omega) = q_1(\omega)(\omega - c) = \frac{1 - b - \omega}{2(1-b^2)}(\omega - c)$$

and it has the solution: $\omega = \frac{1}{2}(1 - b + c)$.

The market outcome is characterized by quantities produced by the manufacturer and competitive sector's firms,

$$\left\{ q_1^{WP} = \frac{1 - b - c}{4(1-b^2)}, q_2^{WP} = \frac{2 - b - b^2 + bc}{4(1-b^2)} \right\},$$

[6] This case is considered as a benchmark for a comparison with the general case where $A_1 > A_0 = 1$.

the price of the manufacturer,

$$\omega = \frac{1}{2}(1 - b + c),$$

the final markets prices

$$\{p_1^{WP} = \frac{1}{4}(3 - b + c), p_2^{WP} = \frac{1}{2}\}$$

and the profits of the retailer and the manufacturer,

$$\{\pi_{WP}^R = \frac{5 - 3b^2 - 2b(1 - c) - (2 - c)c}{16(1 - b^2)}, \pi_{WP}^M = \frac{(1 - b - c)^2}{8(1 - b^2)}\}.$$

1.3.2 MSD contract

The retailer's profit maximization problem is:

$$\max_{q_1, q_2} \pi_{MSD}^R(q_1, q_2; t_{MSD}) = (1 - q_1 - bq_2 - t_{MSD})q_1 + (1 - q_2 - bq_1)q_2$$

$$\text{s.t. } t_{MSD} = \begin{cases} t_L & \text{if } s \geq \overline{s} \\ t_H & \text{if } s < \overline{s} \end{cases}$$

Note, if the retailer trades the good from the competitive sector then its profit is a solution of the problem:

$$\max_{q_2} \pi^R = (1 - q_2)q_2.$$

This is equal to 1/4. It is the retailer's "reservation profit" in the sense that the retailer is guaranteed at least this profit level in equilibrium.

Lemma 1 *The equilibrium values of $\{t^L, t^H, \overline{s}\}$ are such that the retailer meets the market share threshold, $s \geq \overline{s}$.*

Lemma 1 says that in equilibrium the manufacturer's price t_{MSD} is such that the retailer always meets the market share threshold and buys at the price t_L. The intuition is the following. If the retailer does not meet the threshold, that is $s < \overline{s}$, and it buys at the price t_H then the outcome does not change if the manufacturer sets prices $\{t_L', t_H', \overline{s}'\}$ such that $t_L' = t_H$, $\overline{s}' = s$ and t_H' is the prohibitively high. Now, if the manufacturer increases the market share threshold slightly $\overline{s}' > s$ then the retailer buys more for the same price and the manufacturer's profit is higher. Thus, $s < \overline{s}$ cannot stay in equilibrium. Hence, the exact value of the manufacturer's price t_H does not play a role provided it is high enough. Without loss of generality we can set $t_H = +\infty$.

Corollary 2 *In equilibrium it must be that* $s = \overline{s}$.

Note that if under equilibrium we obtain $s \neq \overline{s}$, this would imply that the market outcome is the same as in the case of wholesale price. In this case the manufacturer has no possibility of increasing its profit by setting appropriate levels of $\overline{s}$ and t_L, which is contra-intuitive.

As a result of Corollary 2, the profit of the retailer can be written as:

$$\max_{q_1,q_2} \pi^R_{MSD}(q_1, q_2; t_L) = (1 - q_1 - bq_2 - t_L)q_1 + (1 - q_2 - bq_1)q_2$$
$$\text{s.t. } \frac{q_1}{(q_1 + q_2)} = \overline{s}.$$

The first order condition gives the solution:

$$q_1(t_L, \overline{s}) = \frac{\overline{s}(1 - \overline{s}t_L)}{2(1 - 2\overline{s}(1 - b)(1 - \overline{s}))}.$$

Thus, the retailer's profit as a function of t_L and $\overline{s}$ is

$$\pi^R_{MSD}(t_L, \overline{s}) = \frac{(1 - \overline{s}t_L)^2}{4(1 - 2\overline{s}(1 - b)(1 - \overline{s}))}.$$

The manufacturer's profit maximization problem now can be written as:

$$\max_{t_L,\overline{s}} \pi^M_{MSD} = q_1(t_L, \overline{s})(t_L - c),$$
$$\text{s.t. } q_1(t_L, \overline{s}) = \begin{cases} \frac{\overline{s}(1-\overline{s}t_L)}{2(1-2\overline{s}(1-b)(1-\overline{s}))} & \text{if } \pi^R_{MSD}(t_L, \overline{s}) \geq \frac{1}{4} \\ 0 & \text{if } \pi^R_{MSD}(t_L, \overline{s}) < \frac{1}{4} \end{cases}.$$

Lemma 3 *In equilibrium, the manufacturer extracts the entire retailer's profit above the reservation profit level.*

The Lemma 3 says that in equilibrium, the equality $\pi^R_{MSD}(t_L, \overline{s}) = \frac{1}{4}$ holds. This gives a correspondence between a price t_L and market share threshold $\overline{s}$ which must hold in the equilibrium:

$$\overline{s}(t_L) = \frac{2(1-b-t_L)}{2(1-b)-t_L^2}. \tag{3}$$

Now, the profit maximization problem of the manufacturer becomes:

$$\max_{t_L} \pi^M_{MSD}(t_L) = q_1(\overline{s}(t_L), t_L)(t_L - c) = \frac{1-b-t_L}{2(1-t_L)(1-b)+t_L^2}(t_L - c)$$

and it has the following unique solution:

$$t_L^* = \frac{(2-c)(1-b) - D_1}{1-b-c}, \tag{4}$$

where $D_1 = const = \sqrt{(1-b^2)\left(2(1-c)(1-b)+c^2\right)}$.

Plugging (4) into (3), we obtain the equilibrium values of $\overline{s}^*$ which together with t_L^* determine the rest of the equilibrium values. The market outcome is characterized by quantities produced by the manufacturer and competitive sector's firms,

$$\left\{q_1^* = \frac{1-b-c}{2D_1},\, q_2^* = \frac{1-b+bc}{2D_1}\right\},$$

the market share threshold,

$$\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)},$$

the manufacturer's price,

$$t_L^* = \frac{(2-c)(1-b) - D_1}{1-b-c},$$

the final market prices

$$\{p_1^* = \frac{2D_1 - (1-b^2)(1-c)}{2D_1}, p_2^* = \frac{2D_1 + b^2 - 1}{2D_1}\}$$

and the profits of the retailer and the manufacturer,

$$\{\pi^R_{MSD} = 1/4, \pi^M_{MSD} = \frac{D_1 + b^2 - 1}{2(1-b^2)}\}.$$

Let's note that $b(1-c) < 1$ implies $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)} < 1$. Hence we can formulate the following Proposition.

Proposition 4 *Although the share of the manufacturer is higher in the case of MSD's than in the case of wholesale price contract, the manufacturer never sets the market threshold equals to 1.*

Thus, the competitive sector is never foreclosed from the market completely and MSD's do not result in an exclusive relation[7]. The intuition here is the following. According to (3), the higher is the market share threshold $\overline{s}$ the lower the price t_L must be in order to provide the retailer's with its reservation profit level. Thus, to implement $\overline{s} = 1$, the manufacturer has to set $t_L = 0$, which does not maximize its profit.

1.3.3 MSD's vs. wholesale price contracts

In what follows, I compare the outcomes in the case of MSD's with those in the case of the wholesale price contract.

Proposition 5 *Relative to a wholesale price contract, MSD's leads to:*

i) an increase in the manufacturer's market share, s,

ii) the retailer buys at higher price, that is $t_L > \omega$,

iii) an increase in the manufacturer's output, q_1,

iv) an increase in the manufacturer's profit,

v) a decrease in the final market price for the manufacturer's good p_1,

vi) an increase in the final market price for the good p_2,

vii) a decrease in the output of competitive sector's firms q_2,

viii) a decrease in the retailer's profit,

ix) a decrease in the consumer surplus.

According to Proposition 5 the manufacturer, which has some degree of market power, uses MSD's to increase both its output and price in order to extract the entire profit of the retailer above the reservation level. In the case of MSD's all agents, with the exception of the manufacturer, lose. Hence MSD's can be treated as an anticompetitive tool.

[7] This result contributes to a discussion in the antitrust law literature (see for example Tom at el [2000]) on a relation between MSD's and exclusive dealing. See Bernheim and Whinston [1998], Katz's [1989] survey, Marvel [1982], Mathewson and Winter [1987] on exclusive dealing.

1.4 *Investment Effort*

1.4.1 Wholesale price contract

The profit maximization problem for the retailer is:

$$\max_{q_1,q_2,e} \pi^R_{WP} = (A(e) - q_1 - bq_2 - \omega)q_1 + (1 - q_2 - bq_1)q_2 - eE$$

where $e \in \{0,1\}$, $A(0) = 1$, $A(1) = A_1$.

The first order conditions with respect to q_2 and q_1 are:

$$\begin{cases} A(e) - 2q_1 - 2bq_2 - \omega = 0 \\ 1 - 2bq_1 - 2q_2 = 0 \end{cases}.$$

The solution of the system is: $q_1(\omega, e) = \frac{A(e)-b-\omega}{2(1-b^2)}$, $q_2(\omega, e) = \frac{1-A(e)b+b\omega}{2(1-b^2)}$.

The retailer's profit as a function of the effort level e and the price ω is:

$$\pi^R_{WE}(\omega, e) = \frac{(A(e) - \omega)(A(e) - b - \omega)}{4(1-b^2)} + \frac{1 - A(e)b + b\omega}{4(1-b^2)} - eE.$$

Retailer's profit functions for different levels of the investment effort, $\pi^R_{WP}(\omega, e)\big|_{e=0}$ and $\pi^R_{WP}(\omega, e)\big|_{e=1}$, are decreasing in ω functions with:

$$\left.\frac{\partial \pi^R_{WP}(\omega, e)}{\partial \omega}\right|_{e=1} = \frac{-2A_1 + 2b + 2t}{4(1-b^2)} < \frac{-2 + 2b + 2t}{4(1-b^2)} = \left.\frac{\partial \pi^R_{WP}(\omega, e)}{\partial \omega}\right|_{e=0}. \tag{5}$$

Let $\widehat{\omega}$ denote the price such that the retailer is indifferent either to make the investment effort $e = 1$ or $e = 0$. The solution of $\pi^R_{WP}(\widehat{\omega}, e)\big|_{e=1} = \pi^R_{WP}(\widehat{\omega}, e)\big|_{e=0}$ is

$$\widehat{\omega} = \frac{1}{2}(A_1 + 1 - 2b) - \frac{2E(1-b^2)}{A_1 - 1},$$

and the following conditions hold: if $\omega < \widehat{\omega}$ then the retailer's profit is higher if it makes the effort $e = 1$ and if $\omega > \widehat{\omega}$ then the retailer's profit is higher if its level of the effort is $e = 0$. Together with (5) it implies that the lower is the price ω, the higher is the retailer's gain from the investment effort, $\pi^R_{WP}(\omega, 1) - \pi^R_{WP}(\omega, 0)$.

Now let's restrict the parameters of the model to rule out trivial cases.

Assumption 1.1. $\widehat{\omega} > 0$.

Assumption 1.2. $\widehat{\omega} < \frac{1}{2}(1 - b + c)$.

Assumption 1.3. $\frac{A_1-b-\widehat{\omega}}{2(1-b^2)}(\widehat{\omega} - c) < \frac{(1-b-c)^2}{8(1-b^2)}$.

Assumption 1.4. $b(A_1 + c) < 1$.

Assumption 1.1 implies that if the manufacturer's price is low enough, $\omega \in (0, \widehat{\omega}]$, then the retailer makes the investment effort. It may be rewritten in the form: $\frac{(A_1-1)(A_1+1+2b)}{4(1-b^2)} > E$ and it rules out cases when the cost of effort is "too high" ($E \to +\infty$) or the result of the effort investment is "too small" ($A_1 \approx 1$). If Assumption 1.1 does not hold there is no possibility of implementing the level of effort $e = 1$.

Assumption 1.2 may be rewritten in the form $\frac{(A_1-1)(A_1-b-c)}{4(1-b^2)} < E$ and it implies that the effort cost is not "too small" or that the effect of the effort investment is not "too high". It is outside of our interest because in this case the retailer makes the effort investment regardless of the type of contract with the manufacturer.

Assumption 1.3 may be written in the form:

$$E > \frac{(A_1 - 1)[(1 - b - c) + \sqrt{(A_1 - 1)(A_1 + 1 - 2b - 2c)}}{4(1 - b^2)}.$$

This implies that neither the effect of the effort should be too high nor the cost of effort too low. In addition, it implies that the rate of substitution between goods should not be close to 1. While Assumption 1.1 implies the possibility of implementation of the effort level $e = 1$, and Assumptions 1.2 implies that the effort $e = 1$ is not implemented with necessity under equilibrium, Assumption 1.3 allows us to concentrate on a case that reveals the role of MSD's as a tool for the creation of investment incentives. Assumption 1.4 states that the degree of goods substitution should not be close to 1.

The profit maximization problem of the manufacturer may be written in the form:

$$\max_{\omega} \pi^M_{WP} = q_1(\omega, e)(\omega - c) = \begin{cases} \frac{A_1-b-\omega}{2(1-b^2)}(\omega - c) & \text{if } \omega \leq \widehat{\omega} \\ \frac{1-b-\omega}{2(1-b^2)}(\omega - c) & \text{if } \omega > \widehat{\omega} \end{cases}.$$

Let's note that in equilibrium the optimal manufacturer's price does not exceed the level $(A_1 - b + c)$. Thus, the manufacturer's profit function has the following properties: it is kinked at point $\widehat{\omega}$ and it increases at both intervals $\omega \in [0, \widehat{\omega}]$ and $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$. If $\omega \in [0, \widehat{\omega}]$ then the equilibrium effort level is $e = 1$, while if $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$ then $e = 0$.

The immediate result of Assumption 1.2 is that in order to implement the level of effort $e = 1$ the manufacturer sets the price $\omega = \widehat{\omega}$ and gets the profit:

$$\max_{\omega\in[0,\widehat{\omega}]} \pi^M_{WP} = \pi^M_{WP}\big|_{\omega=\widehat{\omega}} = \frac{A_1 - b - \widehat{\omega}}{2(1-b^2)}(\widehat{\omega} - c). \tag{6}$$

Now, let's consider the manufacturer's profit for the price $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$. While, $\omega > \widehat{\omega}$, the retailer does not undertake the investment effort and the manufacturer's profit, $\pi^M_{WP} = \frac{1-b-\omega}{2(1-b^2)}(\omega - c)$, reaches the maximum at the point $\omega = \frac{1}{2}(1 - b + c)$ with,

$$\max_{\omega>\widehat{\omega}} \pi^M_{WP}(\omega, e) = \frac{(1-b-c)^2}{8(1-b^2)}. \tag{7}$$

According to Assumption 1.3 the manufacturer's profit is higher if it sets the price $\omega = \frac{1}{2}(1 - b + c)$ and a investment level of zero is implemented in equilibrium. Thus, given Assumptions 1.1-1.3, the wholesale price contract which maximizes manufacturer's profit implies zero level of the retailer's effort. This immediately implies that the equilibrium outcome coincides with the benchmark case with wholesale price contract.

1.4.2 MSD contract

The profit maximization problem of the retailer is:

$$\max_{q_1,q_2,e} \pi^R_{MSD} = (A(e) - q_1 - bq_2 - t_{MSD})q_1 + (1 - q_2 - bq_1)q_2 - eE. \tag{8}$$

Lemma 6 *In equilibrium, the condition $s = \overline{s}$ holds and the manufacturer's price t_H is prohibitively high.*

Hence, in equilibrium the retailer chooses q_1, q_2 such that $q_1/(q_1 + q_2) = \overline{s}$ and buys at the price t_L. Plugging $q_2 = q_1\frac{1-\overline{s}}{\overline{s}}$ into (8) and solving the first order conditions we get the optimal level of q_1:

$$q_1(\overline{s}, t_L, e) = \frac{\overline{s}(1 + \overline{s}(A(e) - 1 - t_L))}{2(1 - 2\overline{s}(1-b)(1-\overline{s}))}.$$

Given $\{t_L, \overline{s}\}$ the retailer makes the effort if and only if

$$\begin{cases} \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=1} \geq \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} \\ \pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=1} \geq \frac{1}{4} \end{cases}, \tag{9}$$

where the retailer's profit is:

$$\pi^R_{MSD}(e;t_L,\overline{s}) = \frac{(1-\overline{s}+\overline{s}(A(e)-t_L)^2}{4(1-2\overline{s}(1-b)(1-\overline{s}))} - eE.$$

The first inequality in (9) is an incentive constraint and it implies that for the retailer it is profitable to make the effort $e = 1$. The second inequality in (9) is a participation constraint and it implies that the profit of the retailer is greater or equal to its reservation profit.

If the values of $\{t_L, \overline{s}\}$ are such that

$$\begin{cases} \pi^R_{MSD}(e;t_L,\overline{s})\big|_{e=1} \geq \pi^R_{MSD}(e;t_L,\overline{s})\big|_{e=0} \\ \pi^R_{MSD}(e;t_L,\overline{s})\big|_{e=0} \geq \frac{1}{4} \end{cases}, \tag{10}$$

then the retailer chooses the effort $e = 0$.

Now the manufacturer's profit maximization problem is:

$$\max_{t_L,\overline{s}} \pi^M_{MSD} = q_1(t_L,\overline{s})(t_L - c), \tag{11}$$

$$\text{s.t. } q_1(t_L,\overline{s}) = \begin{cases} \frac{\overline{s}(1+\overline{s}(A(e)-1-t_L))}{2(1-2\overline{s}(1-b)(1-\overline{s}))} & \text{if the condition (9) holds} \\ \frac{\overline{s}(1-\overline{s}-t_L)}{2(1-2\overline{s}(1-b)(1-\overline{s}))} & \text{if the condition (10) holds} \\ 0 & \text{otherwise} \end{cases}$$

In what follows I consider MSD contract parameters that implement the retailers effort $e = 1$ and $e = 0$. Then I compare profits under these two levels of effort to determine which one provides manufacturer with higher profit. Let's first consider the manufacturers profit in the case where the price t_{MSD} is such that condition (9) holds which implies that the retailer makes the effort $e = 1$.

Lemma 7 *In equilibrium, the manufacturer extracts the entire retailer's profit above the reservation level.*

The condition $\pi^R_{MSD}(e;t_L,\overline{s})\big|_{e=1} \geq \frac{1}{4}$ binds and this determines the equilibrium correspondence on t_L and $\overline{s}$ of the form:

$$t_L(\overline{s}) = A_1 - 1 + \frac{1-\sqrt{1+4E}\sqrt{1-2\overline{s}(1-b)(1-\overline{s})}}{\overline{s}} \tag{12}$$

Plugging (12) into the manufacturer's profit function (11) and solving the first order conditions we get the optimal value of $\overline{s}$ and t_L:

$$\overline{s}^* = \frac{A_1 - b - c}{(1-b)(1+A_1-c)},$$
$$t_L^* = A_1 - 1 + \frac{(1-b)(A_1+1-c) - D_2}{A_1 - b - c}$$

where $D_2 = const = \sqrt{(1-b^2)(1+(A_1-c)(A_1-c-2b)}\sqrt{(1+4E)}$.

The profit of the manufacturer is

$$\pi^R_{MSD}(t_L^*, \overline{s}^*, e)\big|_{e=1} = \frac{(1+4E)(1+(A-c)(A-c-2b)-D_2)}{2D_2}.$$

By setting

$$\{t_L^* = A_1 - 1 + \frac{(1-b)(A_1+1-c)-D_2}{A_1-b-c}, t_H^* = \infty, \overline{s}^* = \frac{A_1-b-c}{(1-b)(1+A_1-c)}\}$$

the manufacturer motivates the retailer to make the effort investment $e = 1$. Then equilibrium profits, prices, and outputs are the following.

$$\{\pi^M_{MSD}\big|_{e=1} = \frac{(1+4E)(1+(A-c)(A-c-2b)-D_2)}{2D_2}, \pi^R_{ME}\big|_{e=1} = 1/4\},$$
$$\{q_1 = \frac{(A_1-b-c)(1+4E)}{2D_2}, q_2 = \frac{(1-Ab+bc)(1+4E)}{2D_2}\},$$
$$\{p_1 = A_1 - \frac{(4E+1)(A_1-c)(1-b^2)}{2D_2}, p_2 = 1 - \frac{(4E+1)(1-b^2)}{2D_2}\}.$$

Now, let's consider the profit of the manufacturer in the case when it motivates the retailer to choose a zero-level effort. Given the price t_L and the threshold $\overline{s}$ the retailer's profit is $\pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} = \frac{(1-\overline{s}t_L)^2}{4(1-2\overline{s}(1-b)(1-\overline{s}))}$. The manufacturer's maximization problem is:

$$\max_{t_L, \overline{s}} \pi^M_{MSD} = \frac{\overline{s}(1-\overline{s}t_L)}{2(1-2\overline{s}(1-b)(1-\overline{s}))}(t_L - c),$$

s.t. (10) holds

The participation constraint $\pi^R_{MSD}(e; t_L, \overline{s})\big|_{e=0} = \frac{1}{4}$ gives the correspondence $t_L(\overline{s})$ that guarantees the reservation level of the profit to the retailer:

$$t_L(\overline{s}) = \frac{1 - \sqrt{1-2\overline{s}(1-b)(1-\overline{s})}}{\overline{s}}. \quad (13)$$

Plugging (13) into (11) and solving the first order conditions we get the optimal value of $\overline{s} = \frac{1-b-c}{(1-b)(2-c)}$.

The optimal value of t_L is:

$$t_L = \frac{(1-b)(2-c) - D_3}{1-b-c},$$

where $D_3 = const = \sqrt{(1-b^2)(1+(1-c)(1-c-2b)}$.

The profit of the manufacturer in this case is

$$\pi^M_{MSD}(t_L, \overline{s}, e)\big|_{e=0} = \frac{1+(1-c)(1-c-2b) - D_3}{2D_3}.$$

Thus, if

$$\pi^M_{MSD}(t^*_L(e), \overline{s}^*(e), e)\big|_{e=1} \geq \pi^M_{MSD}(t^*_L(e), \overline{s}^*(e), e)\big|_{e=0} \tag{14}$$

the manufacturer sets the price $t^*_L = A_1 - 1 + \frac{(1-b)(A_1+1-c)-D_2}{A_1-b-c}$, market share threshold $\overline{s}^* = \frac{A_1-b-c}{(1-b)(1+A_1-c)}$ and the equilibrium retailer's level of the effort is $e = 1$, otherwise the manufacturer sets the price $t^*_L = \frac{(1-b)(2-c-D_3)}{1-b-c}$, the market share threshold $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)}$ and the equilibrium level for the retailer's effort is $e = 0$. For the purpose of the paper I am interested in the former case.

Let Ω denote the set of parameters (A_1, b, c, E) for which conditions (14), Assumptions 1.1-1.3 and the incentives compatibility constraint hold. The following technical lemma states that the set Ω is the non-degenerated set.

Lemma 8 *There is a compact set of the parameters of the model* $(A_1, b, c, E) \in \Omega$ *where inequalities (14), (9) and Assumptions 1.1, 1.2 and 1,3 are compatible.*

Proof. The numerical example in part 1.5.1 proves that it contains at least one point. Moreover because all functions used in (14) and Assumptions 1.1-1.3 are continuous the required conditions hold in the neighborhood of the provided point. ■

The set Ω is characterized by the following properties. For given levels of the marginal cost c and the degree of substitution b the set specifies the cost of effort as a function of the efficiency of the effort $A_1 : 0 < \underline{E}(A_1; b, c) < E \leq \overline{E}(A_1; b, c)$, where bounds $\underline{E}, \overline{E}$ increase in A_1.For a given level of A_1 the higher level of b corresponds to a smaller interval $[\underline{E}, \overline{E}]$. For instance:

if $c = 0, A_1 = 1.5, b = 0.5$ then $\Omega = \{E : E \in [0.166,\ 0.253]\}$;

if $c = 0, A_1 = 1.5, b = 0.6$ then $\Omega = \{E : E \in [0.175, 0.255]\}$ and

if $c = 0, A_1 = 1.4, b = 0.5$ then $\Omega = \{E : E \in [0.12, 0.1892]\}$.

Thus, MSD's allow the manufacturer to design the menu of prices such that the retailer makes the level of the effort $e = 1$.While, if only the wholesale price contract is applied, the manufacturer implements the level of the effort $e = 0$. Hence, we can conclude that MSD's can be used by the manufacturer as an efficient device for the creation of investment incentives. Certainly, the manufacturer gains from the use of MSD's. In order to analyze MSD's impacts from the social point of view I conduct a welfare analysis.

1.5 Welfare analysis

The consumer surplus is given by:

$$U(q_1, q_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - q_1p_1 - q_2p_2.$$

Proposition 9 *For the set of parameters Ω the following statements hold:*

1. When MSD's are applied, the manufacturer designs the menu of prices such that the retailer's level of the effort is $e = 1$ When the wholesale price is applied the level of effort $e = 0$ is implemented under equilibrium.

2. The total industry profit is higher when MSD's are applied.

3. The total output is higher when MSD's are applied.

4. Both the consumers' surplus and the total welfare are higher when MSD's are applied.

Proof. The proposition immediately follows the numerical example in part 1.5.1 and Lemma 8 ■

The intuition for the result is the following. The retailer is motivated to make the costly effort only if the quantity of the manufacturer's good that it resells is high enough. That means that the manufacturer's wholesale price should be small enough to achieve this. Thus, the manufacturer faces a trade-off: either to set a lower wholesale price to shift the demand upward or to set a higher price and to remain on the same demand curve. The gain by the manufacturer from an increase in the demand can be smaller than its losses from the price reduction. Thus, the wholesale price contract may not be enough to implement the desired level of effort from the retailer. If MSD's are applied then the manufacturer may use the market threshold to enforce the retailer to buy more of the manufacturer's good, up to the level where the costly effort becomes profitable for the retailer. The investment effort shifts the demand for the manufacturer's good and increases both the manufacturer's profit and the consumer's surplus.

1.5.1 A numerical example

Let's consider a numerical example with the following values for the parameters: $A_1 = 1.5, b = 0.7, c = 0.14, E = 0.2$.

First I consider the case of the wholesale price ω. Given these parameters for the model the retailer is indifferent to making the effort investment or not, if and only if

$$\pi^R_{WP}(\omega, e)\big|_{e=0} = \pi^R_{WP}(\omega, e)\big|_{e=1},$$

that is

$$\frac{(1-\omega)(1-b-\omega)}{4(1-b^2)} + \frac{1-b+b\omega}{4(1-b^2)} = \frac{(A_1-\omega)(A_1-b-\omega)}{4(1-b^2)} + \frac{1-A_1 b+b\omega}{4(1-b^2)} - E,$$

with the solution

$$\widehat{\omega} = \frac{(A_1-1)[A_1+1-2b)] - 4E(1-b^2)}{2(A_1-1)} = 0.142.$$

The manufacturer's profit in this case is:

$$\pi^M_{WP}(\widehat{\omega}) = q_1(\widehat{\omega})(\widehat{\omega}-c) = \frac{A_1-b-\widehat{\omega}}{2(1-b^2)}(\widehat{\omega}-c) = 0.0009.$$

For any price above the $\widehat{\omega} = 0.142$ the retailer chooses a level of effort of zero.

The wholesale price ω that maximizes the manufacturer's profit is $\omega = \frac{1}{2}(1-b+c) = 0.22$ and the profit is

$$\pi^M_{WP} = \frac{1-b-\omega}{2(1-b^2)}(\omega-c) = 0.61 \cdot 0.078 = 0.006.$$

Thus, the investment effort $e = 0$ is implemented.

The equilibrium prices and quantities are $(p_1^{WP}, p_2^{WP}) = (0.61, 0.5)$ and $(q_1^{WP}, q_2^{WP}) = (0.0784, 0, 445)$ respectively; the profit of the retailer is $\pi_{WP}^R = 0.2531$; the consumer surplus is $CS^{WP} = 0.1266$. Thus, the total surplus is $TS^{WP} = 0.3857$.

If MSD's applied then the manufacturer sets the price $t_L = 0.161$ and the market share threshold $\overline{s} = 0.9322$ in order to implement the effort investment level $e = 1$.The retailer may choose either scenario indifferently. The first being to make the effort ($e = 1$) and to set the optimal prices $(p_1^{MSD},\ p_2^{MSD}) = (0.83, 0.507)$. The quantities in this case are $(q_1^{MSD}, q_2^{MSD}) = (0.6375, 0, 046)$. The second scenario is not to trade the manufacturers good at all and to set $p_2 = 1/2$ and $q_2 = 1/2$. The retailer's profit in both cases is $\pi_{MSD}^R = 1/4$. It is assumed that in this case the retailer makes the investment effort . Then the manufacturer's profit is $\pi_{WSD}^M = 0.0134$, the consumer surplus is $CS^{MSD} = 0.225$. Thus, the total surplus is $TS^{MSD} = 0.488$. If the retailer chooses the effort level $e = 0$ its profit is $0.1878 < 0.25$. Thus, given $\{t_L = 0.161, \overline{s} = 0.9322\}$ the equilibrium level of the effort is $e = 1$. To implement the effort level $e = 0$ the manufacturer may set the price $t_L = 0.221$ and $\overline{s} = 0.287$. The manufacturer's profit in this case is $0.0123 < 0.0134$. Thus, if MSD's are applied then the equilibrium effort level is $e = 1$.

The results confirmed in the example are the following: relative to the wholesale price contracts, MSD's result in:

1) an increase in the manufacturer's output, q_1, and a decrease in the competitive sector's firms output q_2,

2) an increase in the manufacturer's profit and a decrease in the retailer's profit,

3) the retailer buys at the lower price, that is $t_L < \omega$,

4) an increase in both the final market prices p_1 and p_2,

5) an increase in the total industry's profit, an increase in the consumer surplus and, as a result, an increase in the total welfare.

1.6 Conclusion

The paper investigates the effects of MSD's on market competition and welfare. First, we consider the case without the possibility of the productive effort investment. It is shown that the manufacturer, who has some degree of market power, can use MSD's to extract an additional profit through an increase in its market share and a decrease in the market share of its competitors. Interpreted in this way, MSD's can be treated as anticompetitive as they lead to a decrease in both the total industry profit and the consumer surplus.

However, if we consider the case where retailer can make effort investment that increases the demand for the manufacturer's good, we find that the MSD's can be used to motivate the retailer to make an efficient level of investment effort. This happens because the MSD's use guarantees that the quantity of the manufacturer's good sold by the retailer is high enough and this provides the incentives for the retailer to make the effort investment. It is shown that this outcome can not always be reached through the use of a wholesale price contract. The main result is that MSD's can lead to an increase in both the total industry profit and the social surplus. Hence the total welfare in the case of MSD's may be higher relative to the case of the wholesale price.

A possible extension of the model is to consider the case of many heterogeneous retailers. Probably, in this case the optimal menu of prices may include as many non-degenerated price as well as market thresholds, as many retailers are at the downstream level in order to provide incentives compatibility constraints for each retailer.

Another possible extension of the model is the comparison of the result of MSD's with the results of other non-linear price schemes. There is particular interest in comparison of MSD's and quantity discounts. Quantity discounts usually are not considered as anticompetitive discounts and their use is not restricted by law. If it is shown that MSD's are more preferable from the social point of view, than quantity discounts, it will provide more reasons to treat MSD's as an efficiency increasing, procompetitive tool. One possibility of getting this result may be the consideration of a case of stochastic demand when the use of quantity discounts can involve difficulties related to the absolute value of a discount threshold. MSD's may not suffer from this drawback in the case where both demands, for the manufacturer's good and for the competitive sector's firms good, have the same shock. I leave these extensions for future investigation.

1.7 Appendix

Proof of Lemma 1. I proof the statement by contradiction. Suppose, in the equilibrium the manufacturer sets $\{t^e_L, t^e_H, s^e\}$ and the retailer does not meet the market share threshold. That is, $s = \frac{q^e_1}{q^e_1+q^e_2} < s^e$, where $\{q^e_1, q^e_2\}$ and s are equilibrium quantities and the equilibrium market share of the manufacturer respectively.

Because in the equilibrium the market threshold restriction is not met, the level of the market threshold s^e has no effect on market outcome. In this case the equilibrium price t^e_H coincides with one in the case of wholesale price $t^e_H = \frac{1}{2}(1-b+c)$. As a result, the equilibrium retailer's profit equals one in the case of the wholesale price, $\pi^R_{MSD} = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)}$.

Let's note that the retailer's profit is higher than its reservation profit. It is because $\frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4} \Leftrightarrow 5-3b^2-2b(1-c)-(2-c)c > 4(1-b^2) \Leftrightarrow (b+c)^2-2(b+c)+1 > 0$ $\Leftrightarrow (1-b-c)^2 > 0$, were the last inequality is obviously true.

Let's construct new menu of prices $t' = \{t'_L, t'_H, s'\}$ in the form:

$$\begin{cases} t'_L = t^e_H \\ t'_H = +\infty \\ s' = s^e + \delta \end{cases},$$

where $\delta > 0$.

Now let's show that the new price t' gives the higher profit to the manufacturer.

Because $t'_H = +\infty$, the retailer has either to meet the market share threshold or to trade the competitive sector's firms good only. In the latter case its profit equals to the reservation profit. In the former case, the retailer faces the same manufacturer's price $t'_L = t^e_H$ but it has to adjust quantities q^e_1, q^e_2 to meet the market share threshold. The optimal adjustment implies a decrease in the quantity q_2 and an increase in the manufacturer's quantity q_1. Because of continuity of the retailer's profit function in $q_1$and q_2, for δ small enough we have that the new retailer's profit is still higher than the reservation profit. Thus, if the new price t' is offered then the retailer chooses new quantity $q_1' > q_1^e$. Given the manufacturer's price remains the same, $t_L' = t_H^e$, the profit of the manufacturer is higher. Thus, $\{t_L^e, t_H^e, s^e\}$ were not the equilibrium values which contradicts to the assumption. ■

Proof of Corollary 2. I proof the statement by contradiction. Let's $\{t_L, t_H, \overline{s}\}$ and s be the equilibrium manufacturer's menu of prices and the equilibrium manufacturer's market share respectively. By Lemma 1 $s \geq \overline{s}$ and the retailer buys at the price t_L.

Suppose $s > \overline{s}$. Note that small changes in t_L result in small changes in the equilibrium quantities of q_1, q_2 and the condition $s > \overline{s}$ still holds.

If t_L is higher (lower) than the equilibrium manufacturer's wholesale price (which is $\frac{1}{2}(1-b+c)$) then a small decrease (increase) in t_L leads to an increase in the manufacturer's profit π_{MSD}^M with $s > \overline{s}$ still holding. Thus, in equilibrium $t_L = \frac{1}{2}(1-b+c)$ and the condition $\pi_{MSD}^R > \frac{1}{4}$ holds. Now, if the manufacturer sets $\overline{s}' = s + \delta$ then the retailer has either to trade the good 2 only or to adjust quantities q_1, q_2 to meet new threshold requirement. In the former case the retailer obtains its reservation profit only while in the latter case its profit decreases only slightly and it still remains higher than the reservation profit. Thus, the retailer chooses to by more the manufacturer's good at the same price. The profit of the manufacturer is higher that contradict to the assumption that $\{t_L, t_H, \overline{s}\}$ was the equilibrium menu of prices. ■

Proof of Lemma 3. By Lemma 1 and Corollary 2 we have $s = \overline{s}^e$ and hence $\pi_{MSD}^M = q_1(t_L^e, \overline{s}^e)(t_L^e - c) = \frac{\overline{s}^e(1-\overline{s}^e t_L^e)}{2(1-2\overline{s}^e(1-b)(1-\overline{s}^e))}(t_L^e - c)$.

Let's show that $\frac{\partial \pi_{MSD}^M}{\partial t_L} = \frac{s^2(c-2t_L)+s}{2(1-2s(1-b)(1-s))} \geq 0$.

First, because $2s(1-b)(1-s) \leq \max_s 2s(1-s)(1-b) = \frac{1-b}{2} < 1$, we have that

$$2(1-2s(1-b)(1-s)) > 0.$$

Hence, the denominator is positive. Second, the nominator is positive because

$$s^2(c-2t_L) + s > \min_s s^2(c-2t_L) + s = [s^2(c-2t_L)+s]\Big|_{s=\frac{1}{2(2t_L-c)}} = 0$$

for any $t_L \geq c$.

Thus, for any given level of $\overline{s}^e$, π_{MSD}^M is a non-decreasing in t_L^e function. Therefore, the manufacturer sets t_L^e to be as high as possible until $\pi_{MSD}^R \geq \frac{1}{4}$. As the retailer's profit

π^R_{MSD} deceases in t^e_L for any $0 < t_L < 1$, the manufacturer sets price such that $\pi^R_{MSD} = \frac{1}{4}$.

■

Proof of Proposition 5. *i*) The equilibrium manufacturer's market share is $\overline{s}^* = \frac{1-b-c}{(1-b)(2-c)}$ in the case of MSD's and $s^{WP} = \frac{q_1}{q_1+q_2} = \frac{1-b-c}{(1-b)(3-c+b)}$ in the case of the WP contract. As $(3-c+b) > 2 > (2-c)$, we have that $\overline{s}^* > s^{WP}$.

ii) Now, I show that $t^*_L = \frac{(2-c)(1-b)-D_1}{1-b-c} > \frac{1}{2}(1-b+c) = \omega$, where $D_1 = \sqrt{(1-b^2)(2(1-c)(1-b)+c^2)}$.

$\frac{(2-c)(1-b)-D_1}{1-b-c} > \frac{1}{2}(1-b+c) \Leftrightarrow 2(2-c)(1-b) - (1-b)^2 + c^2 > 2D_1 \Leftrightarrow$

$(1-b^2) + (2(1-b)(1-c)+c^2) > 2D_1 \Leftrightarrow \sqrt{(1-b^2)}^2 + \sqrt{(2(1-c)(1-b)+c^2)}^2 > 2D_1 \Leftrightarrow$

$(\sqrt{(1-b^2)} - \sqrt{(2(1-c)(1-b)+c^2)})^2 > 0.$

Moreover, $1-b^2 = 2(1-c)(1-b)+c^2 \Leftrightarrow 1-b-c = 0$, which contradicts to the assumption. Thus, $t^*_L > \omega$.

iii) $q_1^{MSD} = \frac{1-b-c}{2D_1} > \frac{1-b-c}{4(1-b^2)} = q_1^{WP} \Leftrightarrow D_1 < 2(1-b^2) \Leftrightarrow 2(1-c)(1-b)+c^2 < 4(1-b^2)$.

By assumptions, $c < 1-b \Rightarrow$

$2(1-c)(1-b)+c^2 < 2(1-c)(1-b)+(1-b)^2 =$

$= (1-b)[2(1-c)+(1-b)] = (1-b)[3-2c-b] <$

$< 3(1-b) < 4(1-b^2)$.

iv) $q_1^{MSD} > q_1^{WP}$ and $t^*_L > \omega$ give that $\pi^M_{MSD} > \pi^M_{WP}$.

v) Competitive sector's firms outputs in cases of WP and MSD's contracts are $q_2^{WP} = \frac{2-b-b^2+bc}{4(1-b^2)}$ and $q_2^{MSD} = \frac{1-b+bc}{2D_1}$, respectively.

First, let's note that $D_1 > 1-b^2$ because of

$\sqrt{(1-b^2)[2(1-c)(1-b)+c^2]} \geq \min_c \sqrt{(1-b^2)[2(1-c)(1-b)+c^2]} =$

$= \sqrt{(1-b^2)[2(1-c)(1-b)+c^2]}\,|_{c=1-b} = (1-b^2)$

Thus, $q_2^{MSD} = \frac{1-b+bc}{2D_1} < \frac{1-b+bc}{2(1-b^2)} = \frac{2-2b+2bc}{4(1-b^2)}$. Let's note that

$$2-2b+2bc < 2-b-b^2+bc$$

$$\Leftrightarrow b(b+c-1) < 0,$$

which holds by assumptions and therefore $q_2^{MSD} < q_2^{WP}$.

vi) and (*vii*) The changes in prices are the immediate result of changes in quantities. Thus, both the increase in q_1 and the decrease in q_2 result in the decrease in p_1 and the increase in p_2.

viii). $\pi^R_{WP} = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > 1/4 = \pi^R_{MSD}$ because of $\frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4} \Leftrightarrow$ $5-3b^2-2b(1-c)-(2-c)c > 4(1-b^2) \Leftrightarrow$ $(b+c)^2-2(b+c)+1>0 \Leftrightarrow (1-b-c)^2>0$.

ix). Substituting equilibriums values of prices and quantities for both cases of the wholesale price and MSD's we get that the consumers' surpluses are:

$CS^{WP} = \frac{5-4b^2+(b+c)^2+(b+c)}{32(1-b^2)}$ and $CS^{MSD} = \frac{1}{8}$. Hence, $CS^{WP} > CS^{MSD} \Leftrightarrow \frac{5-4b^2+(b+c)^2+(b+c)}{32(1-b^2)} > \frac{1}{8} \Leftrightarrow 1+(b+c)^2-2(b+c)>0 \Leftrightarrow (1-b-c)^2>0$, and the last inequality is obviously true.

■

Proof of Lemma 6. Suppose, in the equilibrium the manufacturer sets $\{t^e_L, t^e_H, \overline{s}^e\}$ and the retailer does not meet the market share threshold, $s \neq \overline{s}^e$. Because in the equilibrium $s \neq \overline{s}^e$, the level of the market threshold $\overline{s}^e$ has no effects on quantities q^e_1, q^e_2.

Suppose that $e^* = 1$. In this case the equilibrium price (either t^e_H if $s < \overline{s}^e$ or t^e_L if $s > \overline{s}^e$) coincides with the price $\widehat{\omega}$. But this contradicts to the assumption 1.3 which says that the manufacturer's profit is higher if its price is $\omega^* = \frac{1}{2}(1-b+c)$ and $e=0$. Thus, if $e^*=1$ it must be that $s = \overline{s}^e$.

Suppose that $e^* = 0$. Then because of $s \neq \overline{s}^e$ the manufacturer's price (either t^e_H if $s < \overline{s}^e$ or t^e_L if $s > \overline{s}^e$) equals to its wholesale price $\omega^* = \frac{1}{2}(1-b+c)$. As a result, the equilibrium retailer's profit is $\pi^R_{MSD} = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4}$.

Let's consider new menu of prices $t' = \{t'_L, t'_H, \overline{s}'\}$ in the form:

$$\begin{cases} t'_L = t^e_H \\ t'_H = +\infty \\ \overline{s}' = s^e + \delta \end{cases},$$

where $\delta > 0$.

Now let's show that the new price t' gives the higher profit to the manufacturer.

Because $t'_H = +\infty$, the retailer have either to meet the market share threshold $\overline{s}'$ or to trade the competitive sector's firm good only. To exclude the manufacturer's good from the trade is not profitable because in this case the retailer obtains its reservation profit only. In the former case, the retailer has to adjust quantities q^e_1, q^e_2 to meet the market share threshold. The retailer may change its effort level to $e = 1$ also. Regardless changes in the effort level, the optimal adjustment implies an increase in the manufacturer's quantity q_1. Because of the continuity of the retailer's profit function in $q_1$and q_2, for δ small enough the new retailer's profit is still higher than its reservation profit. Thus, if the new menu of prices t' is offered then the retailer chooses new quantity $q_1' > q_1^e$. Given the manufacturer's price remains the same, $t_L' = t_H^e$, the profit of the manufacturer is higher. Thus, $\{t_L^e, t_H^e, s^e\}$ were not the equilibrium values which contradicts to assumption. ■

Proof of Lemma 7. Suppose, in the equilibrium the manufacturer sets $\{t_L^e, t_H^e, \overline{s}^e\}$. By Lemma 6 $s = \overline{s}^e$.

Suppose the equilibrium level of the retailer's effort is $e^* = 0$. Then all arguments of the Lemma 3 applied with small difference in the following way.

First, $\pi_{MSD}^M(t_L^e, \overline{s}^e) = q_1(t_L^e, \overline{s}^e)(t_L^e - c) = \frac{\overline{s}^e(1-\overline{s}^e t_L^e)}{2(1-2\overline{s}^e(1-b)(1-\overline{s}^e))}(t_L^e - c)$,
with $\frac{\partial \pi_{MSD}^M}{\partial t_L} = \frac{s^2(c-2t_L)+s}{2(1-2s(1-b)(1-s))} \geq 0$ for any $t_L < \frac{(1+cs)}{2s}$.

Thus, for any given level of $\overline{s}^e$, π_{MSD}^M is non-decreasing in t_L^e function for $t_L < \frac{(1+cs)}{2s}$ If $\pi_{MSD}^R(t_L^e, \overline{s}^e)\big|_{e^*=0} > 1/4$, then the manufacturer may increase its profit by raising t_L^e. The retailer's response on an increase in t_L^e may imply changes in the effort level and in quantities q_1, q_2. Regardless changes in the retailer's effort level, the profit of the manufacturers increases for any $t_L < \frac{(1+cs)}{2s}$. The profit of the retailer decreases in t_L and it is less then $\frac{1}{4}$ at $t_L = \frac{(1+cs)}{2s}$.Thus, if $e^* = 0$, the optimal manufacturer's price is such that $\pi_{MSD}^R\big|_{e=0} = \frac{1}{4}$.

Now, suppose the equilibrium level of the retailer's effort is $e^* = 1$, and the retailer's profit is $\pi_{MSD}^R(t_L^e, \overline{s}^e)\big|_{e=1} > 1/4$. If the manufacturer increases its price to $t_L^* = t_L^e + \delta$, then the retailer's response may imply changes in the effort level and quantities q_1, q_2. If the retailer changes the effort level, then $\pi_{MSD}^R\big|_{e=0} = \frac{1}{4}$ as it was shown above. Given that $\delta > 0$ is small enough, an adjustment in quantities still provide the retailer with $\pi_{MSD}^R(t_L^*, \overline{s}^e)\big|_{e=1} > 1/4$. The profit of the manufacturer $\pi_{MSD}^M(t_L, \overline{s})\big|_{e=1}$ increases, while the retailer's profit $\pi_{MSD}^R\big|_{e=1}$ decreases for $t < \frac{1+(A_1-1)s+cs}{2s}$, and $\pi_{MSD}^R\big|_{e=1} < 1/4$ if $t = \frac{1+(A_1-1)s+cs}{2s}$. Therefore, if $e^* = 1$, the manufacturer's optimal price is such that $\pi_{MSD}^R(t_L, \overline{s})\big|_{e=1} = 1/4$. ■

CHAPTER II

EQUILIBRIUM PRODUCT VARIETY AND MARKET STRUCTURE IN SUCCESSIVE OLIGOPOLIES

2.1 Introduction

In real world markets most of the firms produce multiple products. Yet, the economic literature has traditionally assumed that firms are single-good producers (see e.g., Hotelling, 1929, Salop, 1979). In real world markets also most of the product manufacturers do not sell their products directly to consumers. They sell them instead through retailers. In other words, in most real world markets manufacturers produce multiple products, they sell their products to retailers and the latter sell them to consumers. A typical example is the food industry, where food processing firms produce a line of food products which they sell through food retailers and supermarkets. Another example is car producers which sell their products through independent retailers. The literature that has studied product variety in markets characterized by successive oligopoly is scarce and has assumed that manufacturers are single-product firms (see e.g., Reisinger and Schnitzer, 2008).

This paper aims to fill the gap between real world markets and economic theory by analyzing successive oligopolies in which the manufacturers that operate in the market's upstream tier have the option to produce a line of goods. All the manufacturers' goods are imperfect substitutes and entry in the upstream market is endogenous. The number of retailers that operate in the market's downstream tier is exogenously fixed.[1] Each retailer can buy and resell to final consumers all the manufacturers' goods. Each manufacturer incurs a cost that is increasing in the number of varieties that it produces.[2] There are

[1] As we demonstrate, imperfect competition in a retailers' level has a non-trivial impact on a market outcome. One-tier multiproduct oligopoly can be obtained as a partial case by assuming perfect competition among retailers.

[2] A similar approach is used in Alanson and Montagna (2005), Ottaviano et al. (2002), Chemla (2003), Feenstra and Ma (2007).

thus economies of scope in the creation of new products. Reselling costs are assumed to be null. Consumers' preferences are described by a quadratic Dixit (1979)-type utility function defined over all the varieties of the substitute goods offered in the market. A multi-stage game with observable actions is analyzed. In the beginning of the game, manufacturers decide whether or not they will enter in the upstream market. In the following stage, each manufacturer chooses the number of its products, i.e., product variety, and the wholesale price of each of its products. In the final stage, retailers buy the manufacturers goods and resell them in the final market by setting their quantities.[3]

Using the above described framework, we study the equilibrium market structure in the upstream tier, that is, the number of manufacturers and the product variety offered by each manufacturer. Moreover, we explore the role of a number of market characteristics such as the economies of scope, the degree of product substitutability and the number of downstream retailers for market outcomes (i.e., number of upstream manufacturers, product variety and wholesale prices). Their impact on consumers' surplus and total welfare is also investigated. A comparison with the benchmark case of single-product manufacturers is conducted.

We demonstrate that the equilibrium number of manufacturers as well as the number of goods produced by each manufacturer depends crucially on the economies of scope. When the economies of scope are weak, the number of manufacturers increases and each manufacturer is single-product. When instead the economies of scope are too strong, a single manufacturer produces all the goods. Intuitively, the strong economies of scope translate into a lower cost of introducing an additional product in the market. Clearly, a manufacturer has higher incentives to introduce more products in the market when the cost of introducing them is lower. However, the higher product variety offered by a manufacturer increases the competition in the upstream tier and decreases in turn the entry incentives.

We also demonstrate that the product variety offered by each manufacturer is higher when there are more retailers in the downstream tier, as well as when the market is large.

[3] We assume that manufacturers make take-it-or-leave-it offer and retailers have no bargaining power. This assumption is reasonable if a number of retailers is significantly greater than a number of manufacturers.

Both of these results are quite intuitive since when the market size is large and there are more downstream customers the manufacturers enjoy higher demand for their products. More surprising result is that both the market size and the downstream concentration have no direct impact on the equilibrium number of manufacturers but they affect it through the equilibrium number of goods only. This allows showing that in fact the impact of the market size and the downstream concentration on the equilibrium number of manufacturer is negligible.

Comparing the case of multi-product manufacturers with the benchmark case in which all the manufacturers are single-product, we find that the wholesale prices are lower in latter case then in the former one. This occurs simply because a multi-product manufacturer internalizes the positive effect of an increase in the wholesale price of one of its products on the demand of the rest of its products. It turns outs that the total number of products, the total industry's output and the retailers' profits are higher, in the case of multi-product manufacturers than the respective ones in the case of single-product manufacturers. Regarding welfare, numerical simulations indicate that both the consumers' surplus and the total welfare increase with the intensity of the economies of scope, the product differentiation and the number of downstream firms.

The existing theoretical literature on product diversity suggests that product diversity may be excessive or insufficient depending on the relative strength of various effects. Studies of product diversity have been traditionally conducted using two alternative families of models. On the one hand, spatial models of localized competition, similar to those proposed by Hotelling (1929) and Salop (1979) have been extensively used. On the other hand, a large literature has followed Spence (1976) and Dixit and Stiglitz (1977) and assumed the existence of a representative consumer with well defined preferences over all possible varieties. In this setup neighboring effects are absent and each firm competes against "the market". Both types of studies were made based typically on the assumption that an individual firm produces one good only. However, as mentioned above the single-good producing firm assumption is in stark contrast to reality where multi-product lines are a commonplace.

A number of more recent papers have started to investigate theoretically the behavior of multi-product firms in the industrial organization literature, as well as in the literature on international trade. Helpman (1985) has analyzed how a multinational firm will expand over multiple product lines. He has constant-elasticity demands (CES preferences) in his analysis and for this reason his model has not taken into account the implied effect on the markups of the firms, i.e., it has ignored the interaction of multiple products in demand. Instead, Helpman has relied on diseconomies of scope to limit firms' expansion into new product lines. Different versions of Helpman (1985) there exist in more recent literature dealing with CES preferences (see e.g., Allanson and Montagna, 2005, Bernard et al., 2006, Brambilla, 2006).[4] Departing from CES preferences, Nocke and Yeaple (2006) have used a partial equilibrium inverse demand curve for every product produced by a firm. They likewise have not taken into account the effect of increases in a firm's varieties on the demand for its existing products but have assumed decreasing returns to the range of products. Endogenous markups have been introduced using alternative preferences. More specifically, Anderson and de Palma (1992 and 2006) have considered a nested logit demand function. Ottaviano at al. (2002) and Melitz and Ottaviano (2005) have assumed linear-quadratic preferences. Eaton and Schmitt (1994), Norman and Thisse (1999), Eckel (2006), and Eckel and Neary (2006) have analyzed multi-product firms in models of spatial product differentiation in a Salop-type circular market.[5] In their settings, marginal costs increase with the distance from a firms' core competence, such that diseconomies of scope limit firms' expansion over the product space in addition to the cannibalization effect. Doraszelski and Draganska (2006) have analyzed product differentiation strategies in a duopoly by assuming that firms can either produce general purpose products or products that are targeted to a certain market segment. Finally, Hansen and Jurgensen (2001) and Hansen and Nielsen (2007) have considered a linear demand function. In their model production strategies of multi-product firms are determined by the influence of the number of goods or the number of plants on fixed and variable costs. All of these papers have considered one-tier industries.

[4] Erkel-Rousse (1997) has considered vertical product differentiation with multi-product firms and CES-preferences.

[5] Blanchard at al. (2007) have analyzed product differentiation in a Hotelling's model with a linear market.

A recent paper by Reisinger and Schnitzer (2008) has analyzed product variety in vertically related industries. They have developed a model of successive oligopolies with endogenous market entry, allowing for varying degrees of product differentiation and entry costs in both the upstream and the downstream market. They have analyzed how different forms of vertical restraints influence the endogenous market structure and show when they are welfare enhancing. Although this paper has dealt with product variety in successive oligopolies, in contrast to ours, it has assumed that upstream manufacturers are single-product.

Summing up, the existing literature has not provided a general model with multi-product firms in vertically related oligopolies. One of the reasons for this lack is that such models quickly become very complicated. A contribution of our paper is to provide such a model that can be used for addressing a variety of issues that arise in vertically related industries.

The rest of the paper is organized as follows. In Section 2.2, we describe our model. In Section 2.3, we characterize the equilibrium when the number of manufacturers is given. In Section 2.4, we endogenize the upstream market structure, that is, the number of manufacturers. We conclude in Section 2.5.

2.2 The Model

We consider a two-tier industry. The industry's upstream tier consists of $M \geq 1$ product manufacturers, each denoted by m, with $m = 1, ..., M$. The manufacturers sell their products to consumers through $R \geq 1$ retailers that operate in the industry's downstream tier. Each retailer is denoted by r, with $r = 1, ..., R$.

Each manufacturer m produces $n_m \geq 1$ different products. We assume that a marginal cost of production of each unit of each product is zero[6]. The total number of products produced by all manufacturers is $N = \sum_{m=1}^{M} n_m$. The total cost faced by each manufacturer depends on the number of its products. More specifically, the total cost of manufacturer m is given by $TC_m = c(n_m) = bn_m^\alpha$, where $\alpha > 0$ determines the rate of economies of scope in the products creation process and $b \in (0, 1]$ defines the scale of the cost function. This cost

[6] As all goods in the model are symmetric we assume that marginal cost of production is the same for each good and we normalize it to zero.

can be thought as the cost of investing in R&D for the creation of new products. Obviously, $c'(n_m) > 0$ and $c(1) > 0$. The last condition implies that entry in the upstream tier is costly.

Each manufacturer m sells its products to the retailers through linear wholesale price contracts. That is, it sets a wholesale price, w_i^m, per unit of product i, $i = 1, 2, ..., n_m$, that it sells. We denote by $\{w_1^m, ..., w_{n_m}^m\}$ the vector of wholesale prices of manufacturer m. The respective total vector of wholesale prices of all the products produced by all the manufacturers is denoted by $\{w_1, ..., w_{n_1}, w_{n_1+1}, ..., w_{n_1+n_2}, ..., w_N\} \equiv \{w_1^1, ..., w_{n_1}^1, w_1^2, ..., w_{n_2}^2, ..., w_1^M, ...w_{n_N}^M\}$, where the first n_1 numbers are the wholesale prices of the products of the manufacturer 1, the next n_2 numbers are the wholesale prices of the products of the manufacturer 2 etc.

We assume that each retailer r faces no other cost than the cost of obtaining the products from the manufacturers, i.e., the wholesale price w_i per unit of product i[7]. Each retailer may buy and resell any number of products that are produced by the manufacturers and may choose any quantity of each product[8]. We assume that each retailer will trade all products in equilibrium. We denote by $\{q_1^r, ..., q_{n_1}^r, q_{n_1+1}^r, ..., q_{n_1+n_2}^r, ..., q_N^r\}$ the vector of quantities that the retailer r trades. The retailer r buys quantities $\{q_1^r, ..., q_{n_1}^r\}$ from the manufacturer 1, quantities $\{q_{n_1+1}^r, ..., q_{n_1+n_2}^r\}$ from the manufacturer 2 etc. Q_i stays for the total quantity of product i sold in the market by all retailers, i.e., $Q_i = \sum_{r=1}^R q_i^r$, where $i = 1, 2, ..., N$. Let's to determine Q_j^m such that

$$\{Q_1^1, ..., Q_{n_1}^1, Q_1^2, ..., Q_{n_2}^2, ..., Q_1^M ..., Q_{n_N}^M\} \equiv$$
$$\equiv \{Q_1, ..., Q_{n_1}, Q_{n_1+1}, ..., Q_{n_1+n_2}, ..., Q_{N-n_m}, ..., Q_N\},$$

i.e. if the manufacturer m produces n_m goods then $\{Q_1^m, ..., Q_{n_m}^m\}$ is a vector of its quantities sold by the market.

The representative consumer has the following quadratic utility function:

$$U = A\sum_{i=1}^{N} Q_i - \frac{1}{2}\left(\sum_{i=1}^{N} Q_i^2 + \gamma \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} Q_i Q_j\right) + L, \tag{15}$$

[7]We use this assumption as a standard simplification to increase tractability of the model.

[8]This setting is referred to as "multilateral transaction" in Fauli-Oller and Mesa-Sanchez (2007) because there is no restriction in the products that retailers can buy.

where L is the income spend on outside goods, $A > 0$, is the size of the market, and $\gamma \in (0, 1)$, is the degree of product substitutability. Namely, the higher is γ the closer substitutes the products are. Note that for simplification reasons, γ denotes the degree of product substitutability both among the products of different manufacturers and among the products of the same manufacturer.

From (15), we obtain the demand function for each product variety i sold by any retailer:

$$p_i = A - Q_i - \gamma \sum_{\substack{j=1 \\ j \neq i}}^{N} Q_j.$$

It follows from the above that the profit function of each retailer r is given by:

$$\pi_r^D = \sum_{i=1}^{N} (p_i - w_i) q_i^r.$$

Respectively, the profit function of each manufacturer m is given by:

$$\pi_m^U = \sum_{i=1}^{n_m} w_i^m Q_i^m - c(n_m).$$

Competitive interactions are modeled as a two-stage game with observable actions. In stage one, each manufacturer m chooses how many goods it will produce, n_m, and sets the wholesale prices of its products, $\{w_1^m, ..., w_{n_m}^m\}$. In the following stage, stage two, each retailer r buys the manufacturers' products and chooses the quantities of each product $\{q_1^r, ..., q_N^r\}$ that it sells to the final consumers.

The solution concept that we use is the subgame perfect Nash Equilibrium in pure strategies which we obtain using backward induction.

Note that in what follows we consider two different scenarios regarding entry in the upstream tier. In the first scenario, we assume that the number of manufacturers is exogenous, i.e., it is fixed and equal to M. In the second scenario, we endogenize the number of manufacturers using the free-entry condition. More specifically, we add one stage on the above described game, stage zero, where manufacturers decide whether or not they will enter in the upstream market.

Throughout, we use as a benchmark for comparisons the case in which each manufacturer produces a single product. Manufacturer m's cost function in this case is $TC_m = c(1)$. If N^s

is the equilibrium number of manufacturers under the free-entry condition in the benchmark case then clearly N^s is also the equilibrium number of products.

2.3 Equilibrium with M Manufacturers

In the last stage of the game, each retailer r chooses the quantity of each product in order to maximize its profits, taking as given wholesale prices:

$$\max_{q_1^r,\dots,q_N^r} \pi_r^D = \sum_{i=1}^{N}(A - Q_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} Q_j - w_i)q_i^r, \quad r = 1, ..., R; \quad i = 1, ..., N, \tag{16}$$

where q_i^r is the quantity of product i sold by retailer r and w_i is the wholesale price of good i. The first order conditions are:

$$\frac{\partial \pi_r^D}{\partial q_i^r} = -q_i^r + A - Q_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} Q_j - w_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} q_j^r. \tag{17}$$

Looking for the symmetric equilibrium, we assume that each retailer sells the same amount of each product, that is, $q_i^1 = q_i^2 = ... = q_i^R = q_i$. Therefore, $Q_i = Rq_i$. Given this, the first order conditions (17) can be rewritten in the following way:[9]

$$\frac{\partial \pi_r^D}{\partial q_i^r} = -q_i + A - Rq_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} Rq_j - w_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} q_j.$$

Rearranging terms we have:

$$(R+1)(q_i + \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} q_j) = A - w_i. \tag{18}$$

The above system of first order conditions (18) determines the equilibrium quantities as functions of the wholesale prices and of the total number of goods N:

$$q_i(w_1, ..., w_N) = \frac{(1-\gamma+\gamma(N-1))(A-w_i) - \gamma \sum_{\substack{k=1 \\ k\neq i}}^{N}(A-w_k)}{(1+R)(1-\gamma)(1-\gamma+\gamma N)}. \tag{19}$$

The respective total demand for manufacturer's product i is $Q_n(w_1, ..., w_N) = Rq_i(w_1, ..., w_N)$.

[9] We discuss the conditions that ensure the existence of interior solutions in the model later on.

In the previous stage, stage two, each manufacturer m chooses the wholesale prices of its products, as well as the number of its products in order to maximize its profits:

$$\max_{w_1^m,\dots,w_{n_m}^m,\, n_m} \pi_m^U = \sum_{i=1}^{n_m} w_i^m Q_i^m - c(n_m), \tag{20}$$

where $Q_i^m = Rq_i^m$ is the total amount of the good i of manufacturer m sold by all retailers.

To derive the first order conditions for the manufacturers' problems for a symmetric equilibrium it is convenient to assume that all manufacturers but the manufacturer 1 choose the same product variety, $n_2 = n_3 = ... = n_M = n$ and set the same prices for all their goods $(w_{n_1+1} = w_{n_1+2} = ... = w_N = w)$ while manufacturer 1 chooses its variety n_1 and sets the price w_1^1 for its first good and the price w^1 for the rest of its goods, $w_2^1 = ... = w_{n_1}^1 = w^1$. Given this, the total number of goods may be written as $N = (M-1)n + n_1$. Now by (19) we have that:

$$q_1^1(w_1^1, w^1, n_1, w, n) = \frac{(1-\gamma)(A - w_1^1) + \gamma(M-1)n(w - w_1^1) + \gamma(n_1 - 1)(w^1 - w_1^1)}{(1+R)(1-\gamma)(1-\gamma+\gamma((M-1)n+n_1))}$$
$$q^1(w_1^1, w^1, n_1, w, n) = \frac{(1-\gamma)(A - w^1) + \gamma(M-1)n(w - w^1) + \gamma(w_1^1 - w^1)}{(1+R)(1-\gamma)(1-\gamma+\gamma((M-1)n+n_1))},$$

where q_1^1 is the demand for the good 1 of the manufacturer 1 and q^1 is the demand for the rest of its goods in terms of w_1^1, w^1, n_1, w and n.

Here and thereafter we abstract from the fact that n_1 is integer-valued. Then we rewrite manufacturer 1's profit as:

$$\pi_1^U(w_1^1, w^1, n_1, n) = w_1^1 R q_1^1 + (n_1 - 1) w^1 R q^1 - c(n_1). \tag{21}$$

Differentiating (21) with respect to w_1 and n_1 we obtain:

$$\begin{cases} \frac{\partial \pi_1^U}{\partial w_1^1} = \frac{R}{1+R} \frac{A(1-\gamma)+\gamma(M-1)nw-2\gamma(n_1-1)w_1-2(1-\gamma+\gamma((M-1)n+n_1-1))w_1^1}{(1-\gamma)(1-\gamma+\gamma((M-1)n+n1)} \\ \frac{\partial \pi_1^U}{\partial n_1} = \frac{R}{1+R} \frac{(A(1-\gamma)+\gamma(M-1)n(w-w^1)-w^1+\gamma w_1^1))((1+\gamma(M-1)n)w^1-\gamma w_1^1)}{(1-\gamma)(1-\gamma+\gamma((M-1)n+n1)^2} - abn_1^{-1+a} \end{cases}. \tag{22}$$

The system of the first order conditions for the symmetric equilibrium is:

$$\begin{cases} \frac{\partial \pi_1^U}{\partial w} = \frac{R}{1+R} \frac{A(1-\gamma)-(2(1-\gamma)+\gamma(M-1)n)w}{(1-\gamma)(1-\gamma+\gamma Mn)} \leq 0 \\ \frac{\partial \pi_1^U}{\partial n} = \frac{R(A-w)w}{1+R} \frac{1-\gamma+\gamma(M-1)n}{(1-\gamma+\gamma Mn)^2} - abn^{-1+a} \leq 0 \end{cases}. \tag{23}$$

Whenever there exists an *internal* solution to (23), it is determined implicitly by the following system of equations:

$$\begin{cases} w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma} \\ \frac{R(A-w^*)w^*}{1+R}\frac{1-\gamma+\gamma n^*(M-1)}{(1-\gamma+\gamma M n^*)^2} = abn^{*-1+a} \end{cases} \tag{24}$$

Note that the derivative of the manufacturer's profits with respect to w_1^1 is positive at point $w_1^1 = 0$:

$$\left.\frac{\partial \pi_1^U}{\partial w_1^1}\right|_{n_1=n; w_1=w; w_1^1=0} = \frac{R}{1+R}\frac{A(1-\gamma)+\gamma w(Mn+n-2)}{(1-\gamma)(1-\gamma+\gamma Mn)} > 0.$$

Therefore, the optimal w^* is always positive. It follows that if there exists a corner solution of (23) then it is given by $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)\gamma}$ and $n^* = 1$.

Next, we discuss the second order conditions and the existence of a symmetric equilibrium. It is easy to see that the second derivative of the manufacturer 1's profit function with respect to any of its wholesale prices is negative: $\frac{\partial^2 \pi_1^U}{\partial w_i^2} = -\frac{2(1-\gamma+\gamma((M-1)n+n_1-1))}{(1-\gamma)(1-\gamma+\gamma((M-1)n+n1)} < 0$, for any $M \geq 1, n \geq 1, n_1 \geq 1$ and $i = 1, ..., n_1$. Thus, the manufacturer's profit function is strictly concave in every of its wholesale prices for any n_1 and therefore there exists a unique point of maximum with respect to any its wholesale price. From this we conclude that, whenever a symmetric equilibrium exists, the wholesale price $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ is indeed the *maximizer* of the manufacturer's profits function for any n_1.

Remark 1. The internal symmetric equilibrium exists for any fixed n^* with equilibrium price $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$. In particular this implies that in the case of single-good producers we have that $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)\gamma}$.

Plugging $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ into (21) we get the profits of manufacturer 1 as a function of n_1 and n^* :

$$\begin{aligned} &\pi_1^U(n_1, n^*)\big|_{w_1^1=w^1=w^*(n^*)} = \\ &= \frac{A^2 R}{1+R}\frac{(1-\gamma)(1-\gamma+\gamma n^*(M-1))n_1}{(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma(M-1)n^*+n_1)} - bn_1^a. \end{aligned} \tag{25}$$

Note that the manufacturer 1's profit function (25) in general is neither concave nor quasi-concave in n_1. Therefore, it is not necessary that the solution of the first order conditions

(w^*, n^*) provides a point of maximum. Indeed, the solution of (23), (w^*, n^*), is the symmetric solution of the manufacturer's problem (20) if and only if $n_1 = n^*$ is the *maximizer* of (25).

Example 10 *Let's consider the set of parameters:* $M = 2, \gamma = 0.6, A = 10, b = 0.1, R = 4$ *and* $a = \{0.43; 0.55; 0.7\}$*.*

- $a = 0.43$ *(Figure 1):* $\pi_1^U(n_1, n)$ *is neither concave nor quasiconcave in* n_1*; the solution of the FOCs (24) is* $\{n^* = 181.7, w^* = 0.08\}$ *and it is a point of local maximum of* $\pi_1^U(n_1, n)$*. The point of global maximum of* $\pi_1^U(n_1, n^*)\big|_{n^*=181.7}$ *is* $\{n_1^* = 1, w^* = 0.08\}$*.*

- $a = 0.55$ *(Figure 2):* $\pi_1^U(n_1, n)$ *is neither concave nor quasiconcave in* n_1*; the solution of the FOC (24) is* $\{n^* = 102.3, w^* = 0.14\}$ *and it is a point of global maximum of* $\pi_1^U(n_1, n^*)\big|_{n^*=102.3}$*.*

- $a = 0.7$ *(Figure 3):.* $\pi_1^U(n_1, n)$ *is quasiconcave in* n_1*; the solution of the FOC (24) is* $\{n^* = 57.9, w^* = 0.25\}$ *and it is a point of global maximum of* $\pi_1^U(n_1, n^*)\big|_{n^*=57.9}$*.*

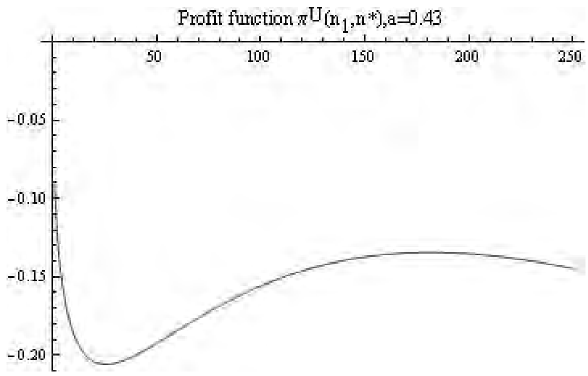

Figure 1: Case 1.

The system (24) is equivalent to the following whenever there exists an internal solution to the manufacturer's maximization problem (20):

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M-1)n^*))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2(1+\gamma(Mn^*-1))^2} = abn^{*a-1}, \tag{26}$$

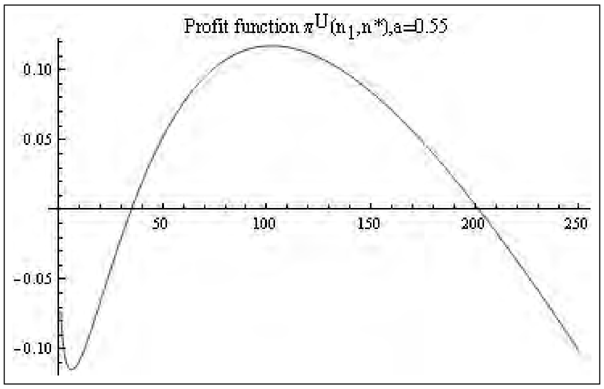

Figure 2: Case 2.

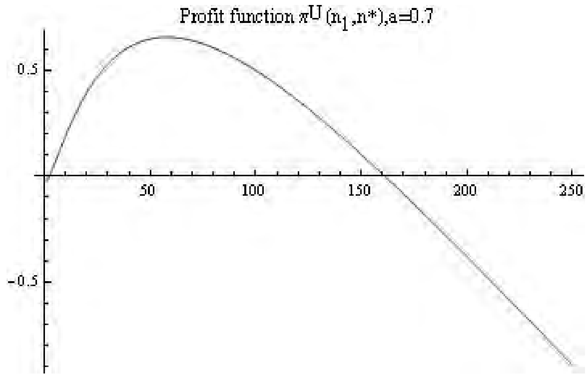

Figure 3: Case 3.

where the left and the right sides are the marginal revenue and the marginal cost (in terms of the product variety n^*) of each manufacturer in the symmetric equilibrium.

Assumption 2.1. *Suppose that a set of parameters* $\{A, R, M, a, b, \gamma\}$ *is such that*

$$\frac{A^2R(1-\gamma)(1-\gamma+\gamma(M-1))^2}{(1+R)(2(1-\gamma)+\gamma(M-1))^2(1+\gamma(M-1))^2} > ab.$$

The condition implies that the marginal revenue of each manufacturer is greater than its marginal cost at the point $n = 1, w^*|_{n=1} = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)\gamma}$ and thus the profit of each manufacturer increases in n.

Lemma 11 *The system of the first order conditions (23) has a unique internal solution on* $n^* \in (1, +\infty)$ *if and only if Assumption 2.1 holds.*

Assumption 2.2. *Suppose that* $\{A, R, M, a, b, \gamma\}$ *and* $n^* > 1$ *determined by (26) are* *such that*

$$\left.\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\right|_{\substack{w_1^1=w^*(n^*)\\ w^1=w^*(n^*)\\ n_1=1}} =$$

$$= \frac{A^2R(1-\gamma)(1-\gamma+\gamma(M-1)n^*))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2(1+\gamma(M-1)n^*)^2} - ab > 0.$$

The condition $\left.\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\right|_{\substack{w_1^1=w^1=w^*(n^*)\\ n_1=1}} > 0$ means that the profit of the manufacturer 1 increases in n_1 at the point $n_1 = 1$ given that $n^* > 1$, where n^* is determined by (26).

Lemma 12 *The profit function (25) is quasiconcave and has a unique internal maximum on* $n_1 \in [1, +\infty)$ *if and only if Assumption 2.2 holds.*

Combining Assumptions 2.1 and 2.2 we obtain the following result.

Proposition 13 *Suppose that Assumption 2.1 and 2.2 hold together. Then the system of the first order conditions (23) determines the unique internal symmetric equilibrium.*

Proof of Proposition 13. Follows immediately from Lemma 11 and Lemma 12. ■

Let's note that a unique internal solution of the manufacturers' maximization problem (20) may exist even if Assumptions 2.1-2.2 do not hold and manufacturer's profit functions are not quasiconcave. Thus, Assumption 2.1 and 2.2 together are sufficient for quasiconcavity of the manufacturers' profit functions and for uniqueness of the *internal* solution of the problem (20).

Plugging $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ into (19), (20) and (16), we obtain the equilibrium quantities, as well as the equilibrium profits and the total industry's output as functions of n^*:

$$q^* = \overline{q}_n = \frac{A}{1+R}\frac{1-\gamma+\gamma n^*(M-1)}{(2(1-\gamma)+\gamma(M-1)n^*)(1-\gamma+\gamma M n^*)}; \tag{27}$$

$$\pi^{U*} = \pi_m^U = \frac{A^2R}{1+R}n^*\frac{(1-\gamma)(1-\gamma+\gamma n^*(M-1))}{(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma M n^*)} - c(n^*); \tag{28}$$

$$\pi^{D*} = \pi_r^D = \frac{A^2Mn^*}{(1+R)^2}\frac{(1-\gamma+\gamma n^*(M-1))^2}{(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma M n)}; \tag{29}$$

$$TQ^* = Mn^*R\overline{q} = Mn^*\frac{AR}{1+R}\frac{1-\gamma+\gamma n^*(M-1)}{(2(1-\gamma)+\gamma(M-1)n^*)(1-\gamma+\gamma M n^*)}. \tag{30}$$

The respective final price is:

$$p^* = \frac{A((2+R)(1-\gamma)+\gamma(M-1)n^*)}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)}. \tag{31}$$

Finally, one could obtain the equilibrium consumers' surplus using (15), as well as the total welfare defined as the sum of the consumers surplus and the firms' profits:

$$CS^* = \frac{A^2MR^2n^*}{2(1+R)^2}\frac{(1-\gamma+\gamma n^*(M-1))^2}{(2(1-\gamma)+\gamma n^*(M-1))^2(1-\gamma+\gamma Mn^*)}, \tag{32}$$

$$W^* = \frac{A^2MR(2+R)n^*}{2(1+R)^2}\frac{(1-\gamma+\gamma n^*(M-1))^2}{(2(1-\gamma)+\gamma n^*(M-1))^2(1-\gamma+\gamma Mn^*)} + M\pi^{U^*}. \tag{33}$$

Setting $n^* = 1$ in the equilibrium expressions (27)-(33), we obtain the respective equilibrium expressions for the benchmark case with single-product manufacturers. In particular, the equilibrium wholesale price and the manufacturer's profits in the benchmark are:

$$w^{*s} = \frac{A(1-\gamma)}{2(1-\gamma)+\gamma(N^s-1)}; \tag{34}$$

$$\pi^{U*s} = \frac{RA^2(1-\gamma)(1+\gamma(N^s-2))}{(1+R)((2+\gamma(N^s-3))^2(1+\gamma(N^s-1))} - c(1) \tag{35}$$

.

The next Proposition compares the multi-product manufacturer's wholesale prices with the ones of a single-product manufacturer for the same level of total product variety.

Proposition 14 *Suppose that Assumptions 2.1 and 2.2 hold. When the number of manufacturers in the benchmark case N^s is such that $N^s = Mn^*$ then the equilibrium wholesale price in the benchmark case is lower than the equilibrium wholesale price in the case of multi-product manufacturers, $w^{*s} < w^*$.*

Proof. $N^s = Mn^* \Rightarrow N^s - 1 = Mn^* - 1 > Mn^* - n^* = (M-1)n^* \Rightarrow w^s < w^*$. ■

According to Proposition 14, the equilibrium wholesale prices are higher when the manufacturers are multi-product than when they are single-product. This finding is driven by the impact of a change in the wholesale price of a product on the demand for the rest of the products. More specifically, if a multi-product manufacturer increases the wholesale price of one of its products then it will increase the demand for the products of its rival manufacturers, as well as in the demand for the rest of its own products. The latter is a positive effect. The multi-product manufacturer internalizes this effect and keeps its wholesale prices higher than a single good producer.

Next, we analyze the role of a number of market characteristics for the market equilibrium. We start by examining the impact of concentration in the upstream market sector.

Proposition 15 *Suppose Assumptions 2.1 and 2.2 hold for some* $M_1, M_2 \in \mathbb{N}$ *with* $1 < M_1 < M_2$*. Then the product variety offered by each manufacturer* n^**, as well as the equilibrium profits of each manufacturer decrease in* M *in the sense that* $n^*(M_1) > n^*(M_2) > 1$ *and* $\pi_m^{U*}(M_1) > \pi_m^{U*}(M_2)$*.*

Remark 2. Let's note that the result of the Proposition holds for $M \in \mathbb{Z}$ and does not imply that $LS(n, M)$ decreases on $[1, 2]$. Actually for a fixed n the left side of (12) is not monotone in $M \in [1; +\infty)$, it has one point of maximum at $\widetilde{M} : 1 < \widetilde{M} < 2$ and thus $LS(n, M)$ increases on the interval $[1; \widetilde{M}]$ and decreases on $[\widetilde{M}; +\infty]$.

Proposition 15 asserts that the more manufacturers are in the upstream market, and thus the less concentrated the upstream market is, the less is the product variety offered by each of them, as well as the lower are each manufacturer's profits. Intuitively, a higher number of manufacturers clearly means stronger competition among them. When competition is strong, the manufacturer's incentives to insert a new product in the market are reduced. This occurs because due to the intensity of competition the new product will not be so profitable.

Proposition 16 *Suppose that Assumptions 2.1 and 2.2 hold on some set of* $\{A, R, M, a, b, \gamma\}$*. Then the equilibrium variety of each firm* n^* *increases in* R, A *and decreases in* α, b*; the equilibrium wholesale price* w^* *increases in* α, b *and decreases in* R *on this set.*

In order to examine the role of the number of retailers, as well as of the economies of scope and of product substitutability we resort to numerical simulations. We do so because the system (26) that determines endogenously the equilibrium values of the wholesale prices and of the number of products (w^*, n^*) does not have a solution in closed form. Thus, it is not possible to perform a comparative statics analysis for all of the market characteristics analytically. We start by setting values for of parameters round the point: $R = 4, M = 2, \gamma = 0.6$, $A = 10$, $b = 0.1$, and $\alpha = 0.7$. In order to examine the impact of the number of retailers R we allow for different values for R.

R	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
2	25.43	50.87	.37	.25	10.69	17.38	34.75	70.26
3	27.35	54.69	.39	.23	12.06	9.81	44.18	74.41
4	28.45	56.90	.40	.22	12.88	6.29	50.38	76.37
5	29.16	58.34	.41	.22	13.43	4.37	54.74	77.12
6	29.67	59.35	.41	.21	13.82	3.22	57.97	78.11
50	32.21	64.42	.44	.20	15.85	.061	76.15	80.08
∞	32.6	65.20	.44	.19	16.17	0	79.27	80.16

Table 1: The impact of R for given M.

It follows from Table 1 that the more retailers (higher R) are in the market, the higher is the product variety of each manufacturer and the total variety, the manufacturer's profits, the total output, the consumers surplus and the total welfare. These results are quite intuitive: the stronger is competition among retailers the lower are their prices and therefore the higher are quantities sold. Thus manufacturers' profits and consumer's surplus are higher.

Next, we examine the impact of the intensity of the economies of scope by considering different values of a.

α	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
0.7	28.45	56.89	.400	.22	12.88	6.30	50.38	76.37
0.8	21.68	43.36	.675	.28	12.75	6.19	49.52	75.64
0.9	17.12	34.25	0.99	.36	12.61	6.08	48.60	74.88
1	13.92	27.84	1.32	.44	12.45	5.95	47.63	74.11
1.1	11.59	23.18	1.68	.51	12.29	5.83	46.63	73.32

Table 2: The impact of alpha for given M.

As Table 2 indicates the smaller is α, and thus, the stronger are the economies of scope, the higher is product variety and the lower are the manufacturer's profits. The intuition is that the strong economies of scope translate into a lower cost of introducing an additional product in the market. Clearly, a manufacturer has higher incentives to introduce more products in the market when the cost of introducing them is lower. However, the higher product variety offered by each manufacturer increases the competition in the upstream level leading in turn to lower manufacturer's profits. As Table 2 also shows, due to the higher product variety, the total output, the consumers' surplus and the total welfare are also higher when the economies of scope are stronger.

γ	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
0.5	40.10	80.20	0.50	.23	15.42	7.53	60.24	91.38
0.6	28.44	56.89	0.40	.22	12.88	6.29	50.38	76.37
0.7	20.11	40.21	0.31	.20	11.08	5.42	43.40	65.73
0.8	13.60	27.21	0.24	.17	9.73	4.78	38.24	57.84
0.9	7.93	15.87	0.16	.13	8.71	4.29	34.34	51.63

Table 3: The impact of gamma for given M.

In the above table, Table 3, we examine the role of product substitutability γ. As one can see, the smaller is γ and thus the more differentiated are the products, the higher is product variety, the higher are the manufacturer's profits as well as the consumers surplus and the total welfare. Intuitively, the manufacturers have stronger incentives to offer more products when these products do not compete fiercely among them. However, not only the manufacturers but also the consumers and the retailers benefit from the higher product variety. Thus, welfare is also higher when γ is low.

Considering the impact of the cost function parameter b we find the following:

Table 4 indicates that the higher is the cost of production b the lower is the total variety produced by each manufacturer and the total number of goods. Moreover the higher b

b	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
0.2	18.53	37.07	0.58	.33	12.65	6.12	48.93	74.56
0.3	14.35	28.71	.71	.42	12.47	5.97	47.79	73.11
0.4	11.94	23.88	.82	.5	12.32	5.85	46.80	71.85
0.5	10.33	20.66	.91	.57	12.18	5.74	45.93	70.73
0.6	9.16	18.33	.99	.63	12.04	5.64	45.13	69.69
0.7	8.27	16.54	1.06	.69	11.93	5.55	44.40	68.73
0.8	7.56	15.12	1.13	.75	11.81	5.46	43.71	67.83
0.9	6.98	13.95	1.19	.80	11.70	5.38	43.06	66.97

Table 4: The impact of b for given M.

corresponds to the higher manufacturers' profits while both the consumers' surplus and the total welfare decrease in b.

Finally, we analyze the impact of changes in M for the set of parameters $R = 4, \gamma = 0.6$, $A = 10$, $b = 0.1$, and $\alpha = 0.9$.

M	n^*	Mn^*	π^{U*}	w^*	TQ^*	π^{D*}	CS^*	W^*
2	17.12	34.25	.99	.36	12.60	6.08	48.6	74.88
3	11.27	33.82	.30	.28	12.71	6.18	49.42	75.03
4	8.35	33.40	.13	.25	12.74	6.21	49.68	75.04
5	6.61	33.04	.07	.24	12.75	6.22	49.80	75.02
6	5.46	32.75	.03	.23	12.76	6.23	49.86	75.00
7	4.64	32.49	.02	.22	12.77	6.24	49.90	74.98
8	4.03	32.27	.01	.23	12.77	6.24	49.92	74.96
9	3.56	32.07	.003	.22	12.77	6.24	49.94	74.94
10	3.19	31.90	-.0006	.22	12.77	6.24	49.95	74.92
11	2.98	31.74	-.003	.22	12.77	6.24	49.95	74.90
12	2.63	31.59	-.005	.22	12.77	6.24	49.96	74.89

Table 5: The impact of M.

Table 5 demonstrates that while an increase in M has a strong negative effect on the variety of each firm n^* it has only week negative effects on the total variety, the total quantity and the final price and therefore the week (positive) effect on the consumers' surplus. The manufacturer's profit decreases in M while the total welfare in non-monotone in M. Also Table 5shows that the highest variety is offered by a monopoly and that the socially optimal number of firms is such that all manufacturers get positive profit.

2.4 Endogenous Number of Manufacturers

In this Section, we do no longer treat the number of manufacturers M as exogenous. Instead, we endogenize it by imposing the free entry condition, that is, $\pi_m^U(M^*) = 0$.

Whenever there exists the internal solution of FOCs (22), that is $n^* > 1$, it is determined by the system of (26) which may be written as

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma M^* n^* - \gamma n^*)n^*}{(1+R)(2-2\gamma+\gamma M^* n^* - \gamma n^*)^2(1-\gamma+\gamma M^* n^*)} = \alpha b n^{*\alpha} \frac{1-\gamma+\gamma M^* n^*}{1-\gamma+\gamma(M^*-1)n^*}.$$

Then the manufacturer's profit function (**??**) is $\pi^U = bn^{*\alpha}(\alpha \frac{1-\gamma+\gamma M n^*}{1-\gamma+\gamma(M-1)n^*} - 1)$ and clearly it is positive for any $\alpha \geq 1$.

Therefore, if $\alpha \geq 1$ then if the free-entry equilibrium exists it must be that $n^* = 1$. The following proposition asserts that such an equilibrium indeed exists for any $\alpha \geq 1$.

Proposition 17 *Suppose that $A^2 \geq 8b$. Under the free-entry condition for any $\alpha \geq 1$ the system (22) has a corner solution only, that is for any $\alpha \geq 1$, $n^* = 1$. Moreover, the equilibrium number of manufacturers is the same as in the benchmark case, that is $M^* = N^{s*}$.*

The Proposition 17 says that if the production function exhibits diseconomy of scope then each manufacturer produces one good only and the equilibrium is the same as one in the case of singe-good producers. The intuition for this result is the following. A cost of entering market (b) is lower than the cost of creation of one additional product (bn^α) by existing producers. Thus each manufacturer produces one good only and the number of firms coincides with equilibrium number of firms in the benchmark case.

Now let's consider the case $\alpha < 1$. Whenever an internal solution of FOCs which satisfies the free-entry condition (26) exists it is determined by the system:

$$\begin{cases} \pi^U = \frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M^*-1)n^*)n^*}{(1+R)(2(1-\gamma)+\gamma(M^*-1)n^*)^2(1-\gamma+\gamma M^* n^*)} - bn^{*\alpha} = 0 \\ \frac{\partial \pi^U}{\partial n^*} = \frac{A^2 R(1-\gamma)(1-\gamma+\gamma M^* n^* - \gamma n^*)^2}{(1+R)(2-2\gamma+\gamma M^* n^* - \gamma n^*)^2(1-\gamma+\gamma M^* n^*)^2} - \alpha b n^{*\alpha-1} = 0 \end{cases}, \quad (36)$$

where the first equation is the free entry condition and the second is the first order condition in terms of n^*. Equivalently the system (36) may be written in the form of:

$$\begin{cases} M^* = \frac{1}{1-\alpha} - \frac{1-\gamma}{\gamma n^*} \\ \frac{(1-\alpha)^2 \alpha A^2 R(1-\gamma)n}{(1+R)((1-\gamma)(1-\alpha)+\alpha\gamma n^*)^2} = bn^{*\alpha} \end{cases}, \tag{37}$$

and the equilibrium wholesale price as a function of n^* is given by:

$$w^* = \frac{(1-\alpha)(1-\gamma)A}{(1-\alpha)(1-\gamma)+\alpha\gamma n^*}. \tag{38}$$

Using $M^* = \frac{1}{1-\alpha} - \frac{1-\gamma}{\gamma n^*}$ we may rewrite Assumptions 2.1 and 2.2 as the following.

Assumption 2.1'. *Suppose that a set of parameters* $\{A, R, a, b, \gamma\}$ *is such that*

$$\frac{(1-\alpha)^2 \alpha A^2 (1-\gamma)R}{(1+R)((1-\gamma)(1-\alpha)+\alpha\gamma)} > b.$$

Assumption 2.2'. *Suppose that* $\{A, R, a, b, \gamma\}$ *and* $n^* > 1$ *determined by the second equation in (37) are such that*

$$\frac{(1-\alpha)^2 \alpha A^2 (1-\gamma)Rn^{*2}}{(1+R)(1-\alpha+\alpha n^*)^2((1-\alpha)(1-\gamma)+\alpha\gamma n^*)^2} \geq b.$$

It is straightforward corollary of Proposition 4 that Assumptions 2.1' and 2.2' ensure that manufacturers' profit functions are quasiconcave and there exists the unique internal solution. Let's note that in the benchmark case each manufacturer produces one good only and thus the equilibrium number of producers is determined by

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma(N^{s*}-1))}{(1+R)(2(1-\gamma)+\gamma(N^{s*}-1))^2(1-\gamma+\gamma N^{s*})} = b$$

while the equilibrium wholesale price is $w^{s*} = \frac{A(1-\gamma)}{2(1-\gamma)+\gamma(N^s-1)}$.

In order to obtain additional results we need an assumption that guarantees that α and γ are not close to zero together.

Assumption 2.3. *Suppose* $\gamma + \alpha > 1$.

Lemma 18 *Under Assumption 2.3 the free entry condition* $\pi^U(M, n) = 0$ *determined implicitly the function* $M(n)$ *such that* $\frac{dM}{dn} < 0$ *and* $\frac{d(Mn)}{dn} > 0$.

Lemma 18 asserts that the higher the variety produced by each manufacturer the lower the equilibrium number of manufacturers and the higher the total variety. It follows immediately that $\frac{d(Mn)}{dM} < 0$ and thus the higher the equilibrium number of manufacturers in the equilibrium the lower the total variety.

Proposition 19 *Suppose Assumptions 2.1',2.2' and 2.3 hold together. Then the number of manufacturers in the benchmark case is higher than the one in the case of multiproduct firms while the total variety in the benchmark case is lower than the one in the case of multiproduct firms, that is* $M^* < N^{s*}; M^*n^* > N^{s*}$.

Proof of Proposition. Under Assumption 2.1' and 2.2' there exists an internal equilibrium with $n^* > 1$. According to Lemma 18 under Assumption 3.3 the equilibrium number of manufacturers decreases in n^* while the total variety increases in n^* and thus $M^* < N^{s*}$ and $M^*n^* > N^{s*}$. ■

Proposition 19 says that, while the number of manufacturers in the benchmark case is higher, the total variety produced is lower comparing to the case of multiproduct firms.

In order to describe the effect of the economy of scale on the equilibrium outcome we provide the following Lemma.

Lemma 20 *Suppose Assumptions 2.1',2.2' and 2.3 hold together for any* $\alpha : \underline{\alpha} < \alpha < \overline{\alpha}$. *Then the equilibrium values* M^* *and* n^* *are such that* $\frac{dM^*}{d\alpha} > 0, \frac{d(M^*-1)n^*}{d\alpha} < 0, \frac{d(M^*n^*)}{d\alpha} < 0, \frac{dn^*}{d\alpha} < 0$ *for* $\underline{\alpha} < \alpha < \overline{\alpha}$.

Plugging (37) into (27)-(33) we obtain the following characterization of the symmetric equilibrium when M is endogenous:

$$q^* = \overline{q} = \frac{aA\,(1-a)}{(1+R)(a\gamma n^* + (1-\gamma)(1-a))}; \tag{39}$$

$$TQ^* = \frac{RaA(\gamma n^* - (1-\gamma)(1-a))}{(1+R)(a\gamma n^* + (1-\gamma)(1+a))}; \tag{40}$$

$$\pi^{D*} = \frac{a^2A^2n^*(\gamma n^* - (1-\gamma)(1-a))}{(1+R)^2(a\gamma n^* + (1-\gamma)(1-a))^2}; \tag{41}$$

$$CS^* = \frac{R^2a^2A^2n^*(\gamma n^* - (1-\gamma)(1-a))}{2(1+R)^2(a\gamma n^* + (1-\gamma)(1-a))^2}; \tag{42}$$

$$W^* = \frac{R(2+R)a^2A^2n^*(\gamma n^* - (1-\gamma)(1-a))}{2(1+R)^2(a\gamma n^* + (1-\gamma)(1-a))^2}. \tag{43}$$

One might wonder how the competition in the downstream market (measured in terms of both product substitutability and number of retailers), the economies of scope and the market size affect the manufacturers' entry incentives, the retailer's profits, the consumers' surplus and the total welfare. We are in the position to obtain analytical results for the case in which each manufacturer produces a high number of products. More specifically, when the parameters $\{\alpha, \gamma, b, A, R\}$ are such that n^* is high enough (or in other words $1/n^*$ is negligible) then in the internal equilibrium we have: $TQ^* = \frac{RA}{\gamma(1+R)}$, $\pi^{D^*} = \frac{A^2}{\gamma(1+R)^2}$, $CS^* = \frac{A^2R^2}{2\gamma(1+R)^2}$, $TW^* = \frac{A^2R(2+R)}{2\gamma(1+R)^2}$, and $M^* = \frac{1}{1-\alpha}$. From these equilibrium expressions, it follows that the stronger are the economies of scope (lower a), the fewer manufacturers enter into the market (lower M^*). Moreover, it follows that the market size A, the scale of production cost b, as well the number of retailers R do not affect the equilibrium number of manufacturers M^*. Instead, the total industry output, the consumers' surplus and the total welfare increase in both A and R and decrease in γ.

In order to draw conclusions for lower values of n^*, we have to resort again to numerical simulations. This is so because the system (37 - 38) that describes implicitly the equilibrium values of the wholesale prices and of the number of goods (w^*, n^*) when the number of manufacturers is endogenous does not have a closed form solution. We provide simulation for parameters values around point $\gamma = 0.8$, $A = 10$, $b = 0.1$, and $\alpha = 0.8$. Table 6 includes some results regarding the role of the number of retailers R. The number of retailers has a significant impact on total output, consumers' surplus and welfare in both multi- and single-product manufacturers cases.

It is not surprising that in the benchmark case an increase in downstream competition (i.e. higher R) leads to an increase in both the number of manufacturers N^s and their total output TQ^s. In the case of multiproduct manufacturers it influences each manufacturer's product variety n^* and the total output TS^* but in contrast to the benchmark case it almost has no effect on the number of manufacturers M^*. These findings are in line with Proposition 19 and with our discussion of the case in which $n^* \longrightarrow \infty$. Thus one prediction of our model is that while a stronger competition in the downstream tier results in a higher manufacturer's product variety its effect on the number of manufacturers is negligible.

The following Table provides some results regarding the impact of the economies of

R	n^*	M^*	Mn^*	N^s	TQ^*	TQ^s	π^{D*}	π^{Ds}	CS^*	CS^s	TW^*	TW^s
1	3.06	4.92	15.07	12.50	6.02	6.00	29.52	29.37	14.76	14.69	44.28	44.06
2	3.61	4.93	17.78	14.43	8.08	8.05	13.23	13.17	26.47	26.33	52.93	52.66
3	3.85	4.94	19.03	15.31	9.11	9.07	7.47	7.43	33.60	33.43	56.00	55.72
4	4.00	4.94	19.74	15.81	9.72	9.68	4.79	4.76	38.29	38.10	57.44	57.15
5	4.09	4.94	20.21	16.14	10.13	10.01	3.33	3.31	41.59	41.38	58.23	57.94
6	4.16	4.94	20.54	16.37	10.43	10.38	2.45	2.43	44.03	43.81	58.71	58.42
51	4.48	4.94	22.18	17.51	11.96	11.91	0.05	0.04	57.83	57.54	60.10	59.80
∞	4.54	4.94	22.43	17.68	12.19	12.15	0	0	60.14	59.85	60.14	59.85

Table 6: The impact of R with endogenous M.

scope.

α	n^*	M^*	n^*M^*	N^s	TQ^*	TQ^s	π^{D*}	π^{Ds}	CS^*	CS^s	TW^*	TW^s
0.8	4.00	4.93	19.74	15.81	9.73	9.68	4.79	4.76	38.29	38.10	57.44	57.15
0.85	2.73	6.58	17.95	15.81	9.71	9.68	4.78	4.76	38.21	38.10	57.31	57.15
0.90	1.68	9.85	16.58	15.81	9.69	9.68	4.77	4.76	38.14	38.10	57.20	57.15
0.93	1.13	14.07	15.94	15.81	9.68	9.68	4.76	4.76	38.11	38.10	57.16	57.15

Table 7: The impact of alpha with endogenous M.

From Table 7 it follows that α influences both the number of manufacturers and the manufacturer's product variety and its impact on both is significant. In line with the case in which $n^* \longrightarrow \infty$ we see again that the stronger are the economies of scope (lower a), the higher is both the total product variety and the product variety offered by each manufacturer. Moreover, we see that the stronger are the economies of scope, the fewer manufacturers enter into the market (lower M^*). At the same time, a has a "moderate" impact on total output, consumers' surplus and welfare.

γ	n^*	M^*	M^*n^*	N^s	TQ^*	TQ^s	π^{D^*}	π^{Ds}	CS^*	CS^s	W^*	W^s
0.8	4.00	4.94	19.74	15.81	9.72	9.68	4.79	4.76	38.29	38.10	57.44	57.15
0.85	3.19	4.94	15.79	13.04	9.18	9.15	4.53	4.51	36.22	36.09	54.33	54.14
0.90	2.40	4.95	11.89	10.22	8.71	8.69	4.30	4.29	34.42	34.35	51.63	51.53
0.95	1.54	4.97	7.65	7.07	8.29	8.29	4.11	4.11	32.89	32.87	49.34	49.30

Table 8: The impact of gamma with endogenous M.

Table 8 shows that product substitutability almost has no effect on the equilibrium number of manufacturers M. However, it has a big impact on each manufacturer's equilibrium product variety n^*. As expected, the closer substitutes the products are (higher γ) and thus the fiercer is the competition the lower are the retailer's profits.

2.5 Conclusion

In this paper we have developed and analyzed a successive oligopoly model with multi-product manufacturers. Both the case of exogenous number of manufacturers and endogenous entry in the market's upstream tier are considered. For each case we provide sufficient conditions for quasiconcavity of manufacturers' profit functions and for existence and uniqueness of the symmetric equilibrium in pure strategies. Also we have analyzed the impact of the downstream market structure, the market size, parameters of both consumer's preferences and production on the product variety, the number of manufacturers as well as the firms' profits and welfare.

For the case of exogenous number of manufacturers the main results are the following. For any degree of the economy of scope there may exists equilibrium where each manufacturer produce more then one good when the number of manufacturers is not big. The product variety produced by each manufacturer, the total product variety, the total output decrease as the number of manufacturers increases while the social surplus and the total welfare increases provided that manufacturers' profits are positive. The higher degree of diseconomy of scope corresponds to lower variety produced by each manufacturer, total product variety, both social and total welfare but to higher manufacturers' profits.

We demonstrate that under free entree condition the equilibrium configuration of the upstream tier crucially depends on the economies of scope in the process of new products creation. When the production technology exhibits diseconomy of scope then in equilibrium each manufacturer produces only one product. Thus the sufficiently high degree of the economy of scope is necessary in order each manufacturer produces more than one good. The stronger is competition at the downstream level the higher is product variety produced by each manufacturer and the total product variety while the number of manufacturers increases very slightly.

Throughout we have restricted our attention to the case where the number of downstream retailers is given and trading between the manufacturers and the retailers takes place through linear wholesale price contracts. It would be interesting to attempt to endogenize the structure of the downstream tier too by including a fixed entry cost and to consider different forms of vertical contracts such as two-part tariffs.

Our model provides a useful framework for addressing a variety of questions that arise in vertically related markets as well as for empirical analysis and policy experiments. For instance, it would be interesting to introduce different tax regimes for the upstream and downstream firms and analyze the optimal tax structure.

In our model we assume that manufacturers makes take-it-or-leave-it offer to retailers. This seems to be reasonable whenever we consider industries where the number of retailers is greater then the number of manufacturers. In contrast, if the number of retailers is comparable to (or lower than) the number of manufacturers then it is a restrictive assumption. To cover such cases, we need to extend the model assuming that retailers have some bargaining power. We leave it for the further research.

2.6 Appendix

Proof of Lemma 11. First, let's show that the equation (26) has no more than two roots on $n^* \in R_+$. Taking into account that all expressions insight brackets with power two are positive, that is $(2(1-\gamma)+\gamma(M-1)) > 0$, $1-\gamma+\gamma(M-1) > 0$, $1+\gamma(M-1) > 0$, on $n^* \in R_+$, w.l.g. we may rewrite (26) in the form of

$$(\sqrt{\frac{R(1-\gamma)}{1+R}}Aab)n^{*\frac{a-1}{2}}(1-\gamma+\gamma(M-1)n^*)) = (2(1-\gamma)+\gamma(M-1)n^*)(1+\gamma(Mn^*-1))$$

where the left side is a strictly increasing function and the right side is a quadratic function. It is obvious that there can not exist more than two roots of the last equation on $n^* \in R_+$ and therefore (26) has also not more than two roots on $n^* \in R_+$.

Sufficiency. Suppose Assumption 2.1 holds. Then the left side of (26) is greater than its right side at $n^* = 1$. Obviously, the left side of (26) is smaller than its right side for big n^*

and thus there are odd number of roots of (26) on $n^* \in (1, +\infty)$. Given that there (26) has no more than two roots on $n^* \in R_+$ we conclude that if Assumption 2.1 holds then there exists a unique root on $n^* \in (1, +\infty)$.

Necessity holds trivially. ■

Proof of Lemma 12. First, let's show that the equation $\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{\substack{w_1^1 = w^*(n^*) \\ w^1 = w^*(n^*)}} = 0$ has not more than two roots on $n_1 \in R_+$.

On $n_1 \in R_+$ the number of roots of the equation

$$\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{\substack{w_1^1 = w^*(n^*) \\ w^1 = w^*(n^*)}} =$$
$$= \frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M-1)n^*)^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2(1-\gamma+\gamma(M-1)n^*+\gamma n_1)^2} - abn_1^{a-1} = 0$$

is not bigger than the maximum number of roots of

$$\frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M-1)n^*)^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n^*)^2}\frac{1}{ab}n_1^{1-a} = (1+\gamma(M-1)n^*+n_1)^2$$

where the left side is a strictly increasing function and the right side is a quadratic function. Therefore the maximum number of roots is two. Second, let's note that $\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{w_1^1 = w^1 = w^*(n^*)} < 0$ when n_1 big enough .

Sufficiency. Suppose Assumption 2.2 holds. Then $\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{w_1^1 = w^1 = w^*(n^*)} > 0$ at $n_1 = 1$ and therefore the equation $\frac{\partial \pi_1^U}{\partial n_1}(n_1, n^*)\Big|_{w_1^1 = w^1 = w^*(n^*)} = 0$ has an odd number of roots on $n_1 \in (1, +\infty)$. Because there can not exist more than two roots we conclude that there exists a unique root on $n_1 \in (1, +\infty)$. By Assumption 2.1, we have also that $\pi_1^U(n_1, n^*)$ increases at $n_1 = 1$ and therefore its unique point of extremum is the point of maximum. Therefore (25) is a quasiconcave function.

Necessity holds trivially. ■

Proof of Proposition 15. First, let's show that the left side of (26) is a decreasing function in M for any $M > 2$ and any *fixed* n. Let's $LS(n, M) = \frac{A^2 R(1-\gamma)(1-\gamma+\gamma(M-1)n))^2}{(1+R)(2(1-\gamma)+\gamma(M-1)n)^2(1+\gamma(Mn-1))^2}$. Then it is easy to see that

$$\frac{\partial LS}{\partial M} = -\frac{2A^2 R(1-\gamma)}{(1+R)} \times$$
$$\times \frac{\gamma n(1-\gamma+\gamma n(M-1))((1-\gamma)^2+\gamma((1-\gamma)(2M-3)n+(M-1)^2 n^2))}{(2(1-\gamma)+\gamma n(M-1))^3(1-\gamma+\gamma Mn)^3} < 0$$

for any $M > 2$.

Now, let's to show that $LS(n,M)|_{M=1} = \frac{A^2R(1-\gamma)}{4(1+R)(1-\gamma+\gamma n)^2} > \frac{A^2R(1-\gamma)(1-\gamma+\gamma n)^2}{(1+R)(2-2\gamma+\gamma n)^2(1-\gamma+2\gamma n)^2} = LS(n,M)|_{M=2}$.

$\frac{A^2R(1-\gamma)}{4(1+R)(1-\gamma+\gamma n)^2} > \frac{A^2R(1-\gamma)(1-\gamma+\gamma n)^2}{(1+R)(2-2\gamma+\gamma n)^2(1-\gamma+2\gamma n)^2} \Leftrightarrow (2-2\gamma+\gamma n)(1-\gamma+2\gamma n) > 2(1-\gamma+\gamma n)(1-\gamma+\gamma n) \Leftrightarrow (1-\gamma)\gamma n > 0$ which is obvious.

This implies that the graph of $LS(n,M)$ shifts downward as M increases for all $M \in \mathbb{N}$. Therefore a point of intersection of $LS(n,M)$ and abn^{*a-1} shifts left as M increases. Thus the equilibrium variety $n^*(M)$ decreases in M.

Using the FOC (26) the manufacturer's profits (**??**) can be rewritten as:

$$\pi^U = nc'(n)\frac{1-\gamma+\gamma Mn}{1-\gamma+\gamma Mn-\gamma n} - c(n).$$

By the envelope theorem we have

$$\frac{d}{dM}\pi^U(M,n^*(M)) = \frac{\partial}{\partial M}\pi^U(M,n^*(M)) = -\frac{nc'(n)(\gamma n)^2}{(1-\gamma+\gamma Mn-\gamma n)^2} < 0$$

and thus $\pi_m^{U*}(M_1) > \pi_m^{U*}(M_2)$ if $1 < M_1 < M_2$. ■

Proof of Proposition 16. On the picture bellow the dotted and the bold line represent the graphs of the left and the right side of (26) respectively. The intersection point is the point of the symmetric equilibrium. An increase in R or A shifts the graph of the left side upward and thus the equilibrium point shifts right while an increase in α or b shifts the graph of the right side upward and thus the equilibrium point moves left. Therefore the equilibrium variety of each firm n^* increases in R, A and decreases in α, b. As $w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M-1)n^*\gamma}$ inversely depends on n^* and does not depends on α, b, R directly we have that w^* increases in α, b and decreases in R.

■

Proof of Proposition 17. As we show before the internal equilibrium (with $n^* > 1$) cannot satisfy the free-entry condition. Thus we need to show that for any $\alpha \geq 1$ there exist $M^c > 1$ such that (i) (23) satisfied at point $\{M^c, n^* = 1\}$, (ii) it satisfies the free entry condition, that is $\pi^U\big|_{M^c,n^*=1} = 0$ and (iii) $\{M^c, n^* = 1\}$ indeed solves the manufacturer 1's maximization problem or, in other words, that $n_1 = 1$ maximizes (25) given $\{M^c, n^* = 1\}$.

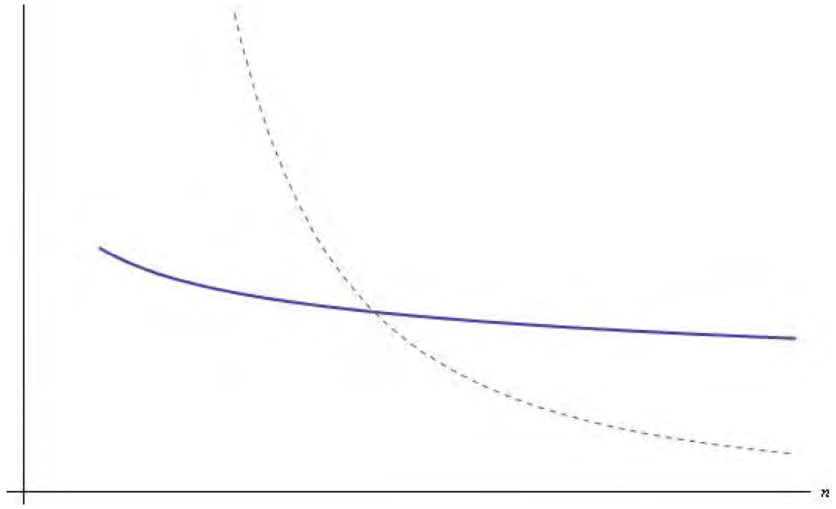

Figure 4: The dotted (bold) line is the graph of the left (right) side of (12).

First let's note that if $n^* = 1$ the manufacturers profit function (**??**),

$$\pi^U\big|_{M,n^*=1} = \frac{A^2R}{1+R}\frac{(1-\gamma)(1-\gamma+\gamma(M-1))}{(2(1-\gamma)+\gamma(M-1))^2(1-\gamma+\gamma M)} - b,$$

has the following properties: $\pi^U\big|_{M=1,n^*=1} \geq 0$, it monotonically decreases in M and it goes to $-b < 0$ as $M \to \infty$. Therefore there exists the unique $M = M^c$ such that $\pi^U\big|_{M=M^c,n^*=1} = 0$.

Second, it is easy to see that if $M = M^c, n^* = 1, w^* = \frac{A(1-\gamma)}{2(1-\gamma)+(M^c-1)\gamma}$ then the system (23) is satisfied with $\frac{\partial \pi^U}{\partial w} = 0$ and $\frac{\partial \pi^U}{\partial n} < 0$.

Next, let's note that the manufacturer's profit $\pi_1^U(n_1, n^*)\big|_{n^*=1}$ described by (25) is concave in n_1 for any $\alpha \geq 1$. Therefore it has unique extremum point which is the point of maximum. Combining these facts we conclude that for any $\alpha \geq 1$ under the free entry condition there exists unique solution and it is the corner solution with $n^* = 1$.

Finally as $\pi^U(M^*, 1) = 0$ we have that $M^* = N^{s*}$ by definition of N^{s*}. ■

Proof of Lemma 18. By the implicit function theorem $\frac{dM}{dn} = -\frac{d\pi^U/dn}{d\pi^U/dM}$. Calculating derivatives, using $\pi^U(M, n) = 0$ and rearranging terms we obtain that $\frac{dM}{dn} = \frac{\theta(M,n;\gamma,\alpha)}{\varphi(M,n;\gamma)}$, where

$$\theta(M,n;\gamma,\alpha)=2(1-\alpha)(1-\gamma)^3+(1-\gamma)^2\gamma(3(M-1)-\alpha(5M-3))n-$$
$$-\alpha\gamma^2(1-\gamma)(M-1)(4M-1)n^2-(1+\alpha)\gamma^3(M-1)^2Mn^3,$$
$$\varphi(M,n;\gamma)=2\gamma n^2+\gamma^2n^2(4(M-1)n-4)+\gamma^3n(2+(M-1)n(-4+(2M-1)n).$$

As $\varphi(M,n;\gamma)>0$ we have that $\frac{dM}{dn}<0\Leftrightarrow\theta(M,n;\gamma,\alpha)<0$. As $(1+\alpha)\gamma^3(M-1)^2Mn^3>0$ the last holds for any $M>1$ and any $n>1$ whenever

$$2(1-\alpha)(1-\gamma)^3+(1-\gamma)^2\gamma(3(M-1)-\alpha(5M-3))n-\alpha\gamma^2(1-\gamma)(M-1)(4M-1)n^2<0\Leftrightarrow$$
$$2(1-\alpha)(1-\gamma)^2-(1-\gamma)\gamma\alpha(5M-3)n-(M-1)n\left(\alpha\gamma^2(4M-1)n+3(1-\gamma)\gamma\right)<0\Leftrightarrow$$
$$2(1-\alpha)(1-\gamma)-\gamma\alpha(5M-3)n<0\Leftrightarrow$$
$$2(1-\alpha)(1-\gamma)-2\gamma\alpha<0\Leftrightarrow$$
$$(1-\alpha)(1-\gamma)-\gamma\alpha<0\Leftrightarrow$$
$$1-\alpha-\gamma<0,$$

which holds by Assumption.

Now, $\frac{d(Mn)}{dn}=\frac{dM}{dn}n+M=-\frac{d\pi^U/dn}{d\pi^U/dM}n+M>0\Leftrightarrow\theta(M,n;\gamma,\alpha)n+\varphi(M,n;\gamma)M>0$.

Rearranging terms we get

$$\theta n+\varphi M=$$
$$=\gamma^3\left(M-1\right)M\left(\alpha+M(1-\alpha)\right)n^4+\gamma^2\left(1-\gamma\right)\left(M-1\right)\left(4\left(1-\alpha\right)M+\alpha\right)n^3+$$
$$+\left(1-\alpha\right)\gamma\left(1-\gamma\right)^2\left(5M-3\right)n^2+2\left(1-\alpha\right)\left(1-\gamma\right)^3>0$$

for all $M>1,n>1,\gamma\in(0,1),\alpha\in(0,1)$. ■

Proof of Lemma 20. $n^*(\alpha)$ is determined by the second equation of (37). Taking logarithms of both the left and the right sides and applying the implicit function theorem we obtain

$$\frac{dn^*}{da}=-\frac{n^*}{\alpha(1-\alpha)}+\frac{n^*((1-\gamma)(1-\alpha)+\alpha\gamma n^*)}{(1-\alpha)^2(1-\gamma)-\alpha(1+\alpha)\gamma n^*}\log(n^*)<0,$$

as $(1-\alpha)^2(1-\gamma)-\alpha(1+\alpha)\gamma<0$ and $(1-\gamma)(1-\alpha)+\alpha\gamma>0$ for any $\alpha,\gamma:\alpha\in(0,1),\gamma\in(0,1),\alpha+\gamma>1$.

The first equation in the (37) implies that $Mn^* = \frac{n^*}{1-\alpha} - \frac{1-\gamma}{\gamma}$ and thus

$$\begin{aligned}
&\frac{d(M^* n^*)}{da} = \frac{dn^*}{da}\frac{1}{1-\alpha} + \frac{n^*}{(1-\alpha)^2} = \\
&= \frac{n^*}{(1-\alpha)^2}\left(1 - 1/\alpha + (1-\alpha)\frac{(1-\gamma)(1-\alpha) + \alpha\gamma n^*}{(1-\alpha)^2(1-\gamma) - \alpha(1+\alpha)\gamma n^*}\log(n^*)\right) < 0
\end{aligned}$$

as $(1-\alpha)^2(1-\gamma) - \alpha(1+\alpha)\gamma < 0, (1-\gamma)(1-\alpha) + \alpha\gamma > 0$ and $1 - 1/\alpha < 0$ for any $\alpha, \gamma : \alpha \in (0,1), \gamma \in (0,1), \alpha + \gamma > 1$.

Also from the first equation of (37) we obtain that $(M^* - 1)n^* = \frac{n^*}{1-\alpha} - \frac{1-\gamma}{\gamma} - n^*$ and therefore

$$\begin{aligned}
&\frac{d((M^* - 1)n^*)}{da} = \frac{dn^*}{da}(\frac{1}{1-\alpha} - 1) + \frac{n^*}{(1-\alpha)^2} = \\
&= \frac{\alpha n^*(1-\gamma)(1-\alpha) + \alpha\gamma n^*}{(1-\alpha)^3(1-\gamma) - \alpha(1-\alpha^2)\gamma n^*}\log(n^*) < 0,
\end{aligned}$$

as $(1-\alpha)^3(1-\gamma) - \alpha(1-\alpha^2)\gamma < 0$ for any $\alpha, \gamma : \alpha \in (0,1), \gamma \in (0,1), \alpha + \gamma > 1$.

Finally $M = \frac{1}{1-\alpha} - \frac{1-\gamma}{\gamma n^*}$ implies that $\frac{dM^*}{d\alpha} = \frac{1}{(1-\alpha)^2} + \frac{1-\gamma}{\gamma n^{*2}}\frac{dn^*}{d\alpha}$. Plugging expression for $\frac{dn^*}{d\alpha}$ and rearranging terms we get that

$$\begin{aligned}
&\frac{dM^*}{d\alpha} > 0 \Leftrightarrow \\
&\frac{(\alpha-1)(1-\gamma) + \alpha\gamma n^*}{-(\alpha-1)(1-\gamma) + \alpha\gamma n^*} > \frac{(1-\alpha)^2\alpha(1-\gamma)\log(n^*)}{(1-\alpha)^2(1-\gamma) - \alpha(1+\alpha)n^*}.
\end{aligned}$$

The last inequality holds as for any $\alpha, \gamma : \alpha \in (0,1), \gamma \in (0,1), \alpha + \gamma > 1,\ n^* > 1$ we have that $\frac{(\alpha-1)(1-\gamma)+\alpha\gamma n^*}{-(\alpha-1)(1-\gamma)+\alpha\gamma n^*} > 0 > \frac{(1-\alpha)^2\alpha(1-\gamma)\log(n^*)}{(1-\alpha)^2(1-\gamma)-\alpha(1+\alpha)n^*}$. Therefore $\frac{dM^*}{d\alpha} > 0$. ■

CHAPTER III

STRATEGIC VERTICAL SEPARATION

3.1 Introduction

This paper examines firms' choice between vertical separation and vertical integration in duopoly setting. Vertical separation is defined as selling through an independent exclusive retailer and vertical integration as selling directly to final consumers. The subject of the possible advantages of vertical separation in strategic duopoly games has been receiving growing attention in the recent economic literature on firms behavior. Bonanno and Vickers (1988) consider a duopoly model with linear costs in which each manufacturer makes a decision of whether to separate or integrate. Assuming price competition among retailers, they show that vertical separation is both in the collective and in the individual interest of firms. Thus, in the unique equilibrium both firms sell their products through independent retailers.

Fershtman and Judd (1987) consider vertical separation under Cournot competition with linear demand and constant marginal costs. They show that both manufacturers have an incentive to separate and that the resulting equilibrium generates greater output, lower prices and more efficient allocation of production than the Cournot equilibrium. If firms' cost functions are symmetric, both firms receive lower profits compared to the ones in the Cournot equilibrium. Under asymmetry of costs, the more efficient firm's profit may be higher than in the Cournot equilibrium. The authors also show that in the case of Bertrand competition with differentiated products both manufacturers separate also and in a unique equilibrium profits are higher and outputs are lower compared to Bertrand equilibrium.

Sklivas (1987) studies a delegation problem in which owners set objective functions for their managers at the first stage and then managers make a decision. His findings are close to those of Fershtman and Judd (1987): owners always take advantage of separation; in the case of competition in quantity (prices) both firms earn lower (higher) profits.

Lin (1988) considers a model in which the consumers have the discrete choice of buying either one unit of good or not at all and retailers compete in prices. He shows that in equilibria either both firms separate or both integrate.

Basu (1995) considers a model of managerial delegation in a duopoly with a linear demand, constant marginal costs and competition in quantities. Assuming that separation is associated with a fixed cost, the author shows that asymmetric equilibria arise, even in the case of symmetric firms' costs. In this model, in the absence of fixed costs, if only one firm separates, the profit of the separated (integrated) firm is higher (lower) compared to the Cournot equilibrium. If both firms separate, each firm's profit is strictly lower than in the Cournot equilibrium. Basu, further, shows that there exists a level of the fixed cost such that, with only one firm separating, the final profit of the separated firm is still higher than its Cournot profit. Moreover, the profit of the integrated firm is higher than in the case of both firms separating. Thus, if separation is associated with additional costs, asymmetric equilibria arise and outputs levels are as in Stackelberg equilibrium. Similarly to this, Jansen (2003) considers a Cournot oligopoly game with a linear demand and constant marginal costs and assumes that separation implies a fixed contracting cost. He shows that asymmetric equilibria emerge, when the Cournot oligopolists supply close substitutes.

The following summarizes existing results. When goods are imperfect substitutes, under Bertrand competition with constant marginal costs, it is both in the private and the collective interest of firms to separate. In the case of Cournot competition with a linear demand and constant marginal costs, it is in the private, but not in the collective, interest of firms to separate. In the symmetric case firms' profits are strictly lower than in the Cournot equilibrium. In the presence of a fixed cost associated with separation, asymmetric equilibria may arise.

In this paper we extend the existing literature by allowing for increasing marginal costs and keeping all other assumptions of a standard separation game unchanged. We follow a traditional approach in assuming that, in the case of vertical separation, a firm chooses both a wholesale price, at which it will supply to its retailer, and a franchise fee. The retailer chooses quantity to trade in order to maximize its own profit. In fact, separation implies that the firm delegates the decision on the quantity to its retailer and controls retailer's objective: using the franchise fee the firm extracts the entire retailer's profit, with the wholesale price being used to set the optimal incentive scheme offered to the retailer. In the case of integration the firm is a retailer of its own good and, as such, the firm's objective is maximization of own profit.

The main results of the paper are formulated for the quadratic cost functions. This form of cost functions allows, first, to capture effects of decreasing economy of scale on a structure of equilibrium and, second, still to have a solution in closed form.

I show that the optimal strategy, either to separate or to integrate, for each firm depends on both its own and its rival's cost structure. More specifically, the equilibrium market structure critically depends on slopes of marginal cost functions, i.e., on the degree of diseconomies of scale, and on asymmetry of firms' costs. If slopes of both marginal cost functions are sufficiently low in unique equilibrium both firms separate. Under moderate asymmetry of costs, in unique equilibrium, the more efficient firm separates, whereas the less efficient one integrates. Asymmetric equilibria (with one firm separating and the other integrating) arise in two cases: either if the slope of each manufacturer's marginal cost is sufficiently high or if asymmetry of costs is sufficiently high. When firms are symmetric, equilibrium is determined by the degree of diseconomy of scale: if this degree is low, then both firms separate, whereas if it is high, two asymmetric equilibria exist.

The intuition for these results is as follows. If only one firm separates, firms get the same profits as in a Stackelberg game, with the separated firm being a Stackelberg leader.[1] If two symmetric firms separate, each one has an incentive to set a low enough wholesale price in order to increase its retailer's output and profit. This results in a higher total output and, actually, lower profits comparing to the Cournot outcome. A central question is whether competition in wholesale prices between firms is such strong that firms' profits fall bellow a Stackelberg follower profit. An answer depends on degree of diseconomy of scale for the following reason. A decrease in, say, the firm 2's wholesale price results in

[1]Separation serves as a commitment mechanism in this case: the separated firm commits its retailer's high output by setting a low wholesale price.

a decrease in the retailer 1's output, and hence in the firm 1's marginal cost.[2] Both the retailer 2's output and the total output increase, and thus a marginal revenue declines. The firm 1's best reply is to restore a balance between its marginal cost and revenue. If the slope of the firm 1's marginal cost function is high enough, the decline in its marginal cost level is higher than that in marginal revenue, and the firm 1 should increase its output, that is, decrease its wholesale price. Thus, firms' wholesale prices are strategic complements. In this case, competition in wholesale prices between firms may be so tough that it results very low firms' profits, namely, lower than a Stackelberg follower profit. In such a situation, each firm prefers to integrate (and to obtain the Stackelberg follower profit), given that its rival separates. Therefore, asymmetric equilibria arise in a completely symmetric game.

Now, suppose that there is a cost asymmetry. Suppose also the firm 1 separates and the firm 2 integrates and let's consider an incentive of the firm 2 to deviate. Separation of the firm 2 has a twofold effect: firstly, in the absence of the firm 1's reaction, the firm 2 could increase its profit by setting its wholesale price at an appropriate level. The possible firm 2's gain depends on its own cost structure: the lesser efficient the firm 2 is, the lesser the gain obtained. Secondly, if the firm 1's wholesale price is a complement to the firm 2's wholesale price, the firm 1's reaction may imply a significant decrease in its wholesale price. This may lead to an increase in the total output, which decreases the firm 2's profit. The latter effect may dominate the former if the marginal curve of at least one firm is steep enough. In this case each firm prefers to integrate, given its rival separates, implying the existence of two asymmetric equilibria.

If a firm's marginal curve is flat, without its rival's marginal cost curve being very steep, then it may be that the positive effect of separation is always dominates the negative effect for the more efficient firm but not for the less efficient one. In this case, separation is a dominant strategy for the more efficient firm, while the less efficient one prefers to integrate. Then, there is a unique asymmetric equilibrium. Finally, if both marginal curves are sufficiently flat, the first effect dominates the second for both firms and in the unique equilibrium both firms separate.

[2] This obviously cannot occur if the firms' marginal costs are constant.

Summing up, this paper provides a possible explanation for the widely observed asymmetry in the sales strategies among firms[3] based on the decreasing economy of scale. Moreover, it predicts multiplicity of equilibrium when cost functions exhibits certain properties. In particular, under some conditions any firm, either the less efficient or the more efficient, may sell through its retailer while its rival sells directly to consumers.

The organization of the paper is as follows. Section 3.2 describes the model and provides the characterization of equilibrium under general assumptions on the demand and cost functions. Section 3.3 analyzes the case of quadratic cost functions and liner demand functions as well as robustness of results. Finally, Section 3.4 concludes.

3.2 The general model and characterization of equilibrium

We consider an industry consisting of two firms. Each firm is denoted by i, with $i = 1, 2$. Firms produce a homogeneous good with costs $C_i(q_i), i = 1, 2$ and face the inverse demand function given by $P(Q)$, where $Q = q_1 + q_2$ with q_1 and q_2 denoting the quantity of the firm 1 and the firm 2, respectively. The demand and cost functions satisfy the following assumptions:

Assumption 3.1. $\exists \overline{Q} > 0$: $P(Q) > 0$ for $Q \in [0, \overline{Q})$ and $P(Q) = 0$ for $Q \geq \overline{Q}$; $P''(Q)$ is continuous; $P(0) = \overline{P} > 0$, $-P'(Q) > \delta > 0$, $P'(Q) + P''(Q)q_i < 0$ for $Q \in [0, \overline{Q})$.

Assumption 3.2. $C_i(q)$ is a twice continuously differentiable increasing convex function, $C_i(0) = 0$, $C_i'(0) = 0$, $0 < C_i''(Q) < b$ for all $q_i \in (0, \overline{Q}]$ and some $b > 0$.

Assumption 3.3. $P^{(3)}(Q) \geq 0$ for all $Q \in [0, \overline{Q})$.

Assumptions 3.1 and 3.2 are sufficient conditions for existence of a unique equilibrium in a Cournot game[4] and together with Assumption 3.3 ensure existence of a pure strategy subgame perfect Nash equilibrium in a whole game.

Firms play the following three-stage game. At the first stage, each firm decides whether it will sell its good through an independent exclusive retailer or it will sell it directly to the final consumers. Following Bonanno and Vickers (1988), we refer to the former case as

[3] See Buehler and Schmutzler (2005) and Jansen (2003) for a detailed discussion of the empirical observations over the asymmetry in vertical structures.
[4] See Van Long and Soubeyranb (2000) for details.

vertical separation and to the latter case as *vertical integration*. In other words, at the first stage each firm chooses the action $m_i \in \{S, I\}, i = 1, 2$, where S and I denoting respectively vertical separation and vertical integration. If at the first stage the firm i chooses $m_i = I$, it becomes the retailer of its own good.

At the second stage, the results of the first stage are observed and each separated firm sets the terms of a two-part tariff contact to trade with its retailer. More specifically, it sets a wholesale price w_i and a franchise fee A_i.

At the third stage, all previous decisions are observed[5] and retailers compete choosing their quantities simultaneously and independently.

The profit of the integrated firm i is $P(q_1 + q_2)q_i - C_i(q_i)$. If the firm i separates, its own and its retailer's profits are $w_i q_i + A_i - C_i(q_i)$ and $P(q_1 + q_2)q_i - w_i q_i - A_i$, respectively.

3.2.1 Subgame outcomes

There are four subgames depending on the choice of $m_i \in \{I, S\}$ $i = 1, 2$ at the first stage: $[I, I], [S, S], [S, I], [I, S]$. Next, I will analyze each of them.

If both firms vertically integrate, i.e., in the $[I, I]$-subgame, the firm i's maximization problem is:

$$\max_{q_i} \pi_i = P(q_1 + q_2)q_i - C_i(q_i), i = 1, 2. \tag{44}$$

Given Assumptions 3.1 and 3.2, the game (44) has a unique Nash-Cournot equilibrium. This equilibrium is characterized by the following first order conditions:

$$P' q_i + P - C_i' = 0, i = 1, 2. \tag{45}$$

Let's denote by $\{q_1^{*C}, q_2^{*C}\}$ and $\{\pi_1^{*C}, \pi_1^{*C}\}$ equilibrium quantities and profits in this subgame, respectively.

If both firms separate, i.e., in the $[S, S]$-subgame, retailers maximization problems are

$$\max_{qi} \pi_i^R = Pq_i - w_i q_i - A_i, i = 1, 2,$$

[5] It is assumed that all decisions are irreversible and therefore there is no commitment problem.

where w_1, w_2 are set by firms at the previous stage. Without loss of generality we assume that $w_i \leq \overline{P}$. Then a solution of the retailers' problem is determined by the system of first order conditions:

$$\begin{cases} P'q_1 + P - w_1 = 0 \\ P'q_2 + P - w_2 = 0 \end{cases}. \tag{46}$$

The Jacobian matrix of (46) is

$$\mathbf{J} = \begin{pmatrix} P''q_1 + 2P' & P''q_1 + P' \\ P''q_2 + P' & P''q_2 + 2P' \end{pmatrix}$$

with $\det(\mathbf{J}) > 0$ for any (q_1, q_2) and by the implicit function theorem we have that

$$\frac{\mathbf{dq}}{\mathbf{dw}} = \frac{1}{\det(\mathbf{J})} \begin{pmatrix} P''q_2 + 2P' & -(P''q_1 + P') \\ -(P''q_2 + P') & P''q_1 + 2P' \end{pmatrix}, \tag{47}$$

where $\mathbf{q} \equiv (q_1, q_2)$ and $\mathbf{w} \equiv (w_1, w_2)$.

Let $\{q_1^S(w_1, w_2), q_2^S(w_1, w_2)\}$ denote the solution of (46). Then, optimal values of w_1, w_2 satisfy:

$$\frac{\partial \pi_i^S}{\partial w_i} = P' \left(\frac{\partial q_i^S}{\partial w_i} + \frac{\partial q_j^S}{\partial w_i} \right) q_i^S + P \frac{\partial q_i^S}{\partial w_i} - C_i' \frac{\partial q_i^S}{\partial w_i} = 0, i \neq j. \tag{48}$$

Directly differentiating (48) in respect to w_i and using (47) one can obtain that $\partial^2 \pi_i / \partial w_i^2 < 0$ under Assumptions 1 and 2 and provided $P^{(3)}(Q) \geq 0$. Therefore Assumptions 3.1-3.3 ensure existence of a pure strategy equilibrium in the $[S, S]$-subgame. It is convenient to rewrite (48) in the form

$$P'q_i^S + P - C_i' + P'q_i^S \left(\frac{\partial q_j^S}{\partial w_i} / \frac{\partial q_i^S}{\partial w_i} \right) = 0 \tag{49}$$

where

$$\frac{\partial q_j^S}{\partial w_i} / \frac{\partial q_i^S}{\partial w_i} = -\frac{P''q_j^S + P'}{P''q_j^S + 2P'}. \tag{50}$$

Let $\{q_1^{*S}, q_2^{*S}\}$ and $\{\pi_1^{*S}, \pi_1^{*S}\}$ be the equilibrium values in the $[S, S]$-subgame.

Plugging $P'q_i^S + P = w_i$ into (49) and rearranging terms we obtain that

$$w_i = C_i' - P'q_i^S \frac{\partial q_j^S}{\partial w_i} / \frac{\partial q_i^S}{\partial w_i}. \tag{51}$$

Under Assumptions 3.1-3.3, $q_i^S(w_1, w_2)$ and $q_j^S(w_1, w_2)$ satisfy $\frac{\partial q_i^S}{\partial w_i} < 0 < \frac{\partial q_j^S}{\partial w_i} < \left|\frac{\partial q_i^S}{\partial w_i}\right|$ and therefore $w_i < C_i'$. In other words, if both firms separate, in equilibrium each firm sets its wholesale price lower than its marginal cost.

If the firm 1 separates and the firm 2 integrates then the retailers' game

$$\begin{cases} \max\limits_{q_1} \pi_1^R = Pq_1 - w_1 q_1 \\ \max\limits_{q_2} \pi_2^R = Pq_2 - C_2 \end{cases}$$

has first order conditions

$$\begin{cases} P'q_1 + P - w_1 = 0 \\ P'q_2 + P - C_2' = 0 \end{cases}.$$

Let's note, that the retailer 2 has the Cournot reaction curve while a position of the retailer 1's reaction curve depends on the firm 1's choice of w_1. Thus, by choosing w_1, the firm 1 determines a point of intersection of reaction curves. Clearly, the optimal w_1 is such that an equilibrium outcome replicates the Stackelberg outcome of the $[I, I]$-subgame. Therefore, the solution of the $[S, I]$-subgame may be also characterized as the following: $q_2^F(q_1)$ solves

$$P'q_2 + P - C_2' = 0$$

and q_1^L is such that

$$P'q_1 + P - C_1' + P'q_1 \frac{dq_2^F(q_1)}{dq_1} = 0. \tag{52}$$

By the implicit function theorem we have that

$$\frac{dq_2^F}{dq_1} = -\frac{P''q_2^F + P'}{P''q_2^F + 2P' - C_2''}.$$

Let $\{q_1^{*L}, q_2^{*F}\}$ and $\{\pi_1^{*L}, \pi_2^{*F}\}$ denote equilibrium quantities and profits, respectively, with the upper index F (L) referring to the separated (integrated) firm.[6] The same arguments apply to $[I, S]$-subgame, so let $\{q_1^{*F}, q_2^{*L}\}$ and $\{\pi_1^{*F}, \pi_2^{*L}\}$ be equilibrium values in the $[I, S]$-subgame.

[6] The upper index F (L) indicates that the integrated (separated) firm obtains the Stackelberg follower's (leader's) profit.

3.2.2 Subgame Perfect Nash Equilibrium

The previous results may be summarized in the following table:

		Firm 2	
		Separate	*Integrate*
Firm 1	*Separate*	π_1^{*S}, π_2^{*S}	π_1^{*L}, π_2^{*F}
	Integrate	π_1^{*F}, π_2^{*L}	π_1^{*C}, π_2^{*C}

As it was argued above, if one firm separates and another integrates, then the equilibrium outcome coincide with the Stackelberg equilibrium with the separated firm being a leader. As it always hold that $\pi_i^{*L} > \pi_i^{*C}$, $i = 1, 2$, we have the following result.

Proposition 21 *Under Assumptions 1-3 the* $[I, I]$*-subgame is never played in equilibrium.*

Equilibrium is determined by the relation of firm' profits π_i^{*L}, π_i^{*F} and π_i^{*S}. More specifically, $[S, I]$ is equilibrium if $\pi_2^{*F} \geq \pi_2^{*S}$; $[I, S]$ if $\pi_1^{*F} \geq \pi_1^{*S}$; $[S, S]$ if $\pi_1^{*F} \leq \pi_1^{*S}, \pi_2^{*F} \leq \pi_2^{*S}$. Let's note that if $\pi_i^{*F} > \pi_i^{*S}, i = 1, 2$, there are two asymmetric strict equilibria $[S,\ I]$ and $[I,\ S]$, whereas if $\pi_i^{*F} > \pi_i^{*S}$ and $\pi_j^{*F} < \pi_j^{*S}$ there is a unique asymmetric strict equilibrium, in which the firm i integrates and the firm j separates. In order to compare firm's profits, and thus, fully determine equilibrium structure, we resort to the use of specific functions.

3.3 Linear demand and quadratic costs

In this Section, we consider linear demand function, $P(Q) = 1 - Q$, and quadratic cost functions, $C_i(q_i) = \frac{1}{2} d_i q_i^2$, with $d_i \geq 0$, in order to characterize fully the equilibrium.

If both firms vertically integrate, they play a standard Cournot game:

$$\max_{q_i} \pi_i = (1 - q_i - q_j) q_i - \frac{1}{2} d_i q_i^2, i, j = 1, 2.$$

Substituting the specific functions into (45) we obtain equilibrium quantities and profits

$$q_i^C = \frac{1 + d_j}{(3 + 2d_i + d_j d_i + 2d_j)} \text{ and } \pi_i^C = \frac{(2 + d_i)(1 + d_j)^2}{2(3 + 2d_j + d_j d_i + 2d_i)^2}, i, j = 1, 2.$$

If both firms vertically separate, then first order conditions of the retailers' problem (46) is:

$$q_i = \frac{1 - 2w_i^s + w_j^s}{3}, i, j = 1, 2,$$

with the total output and the final price, respectively:

$$Q = \frac{2 - w_1^s - w_2^s}{3} \text{ and } P = \frac{1 + w_1^s + w_2^s}{3}.$$

Then, the firm i's maximization problem is given by:

$$\max_{w_i} \pi_i = \frac{1 + w_i^s + w_j^s}{3} \frac{1 - 2w_i^s + w_j^s}{3} - \frac{d_i}{2} \left(\frac{1 - 2w_i^s + w_j^s}{3} \right)^2,$$

and first order conditions (48) determine firms' reaction curves in the space $\{w_1^s, w_2^s\}$:

$$w_i^s = \frac{(-1 + 2d_i)(1 + w_j^s)}{4(1 + d_i)}, i, j = 1, 2. \tag{53}$$

Let's note that $\frac{dw_i^s}{dw_j^s} = \frac{(-1+2d_i)}{4(1+d_i)}$ is strictly increasing in d_i with $\left.\frac{dw_i^s}{dw_j^s}\right|_{d_i=0} = -\frac{1}{4}$, $\left.\frac{dw_i^s}{dw_j^s}\right|_{d_i=1/2} = 0$, $\frac{dw_i^s}{dw_j^s} \underset{d_i \to +\infty}{\longrightarrow} \frac{1}{2}$. Hence, the degree of substitution between w_i^s and w_j^s decreases in d_i. Moreover, if $d_i > \frac{1}{2}$ and $d_i < \frac{1}{2}$, then $\frac{dw_i^s}{dw_j^s} > 0$ and $\frac{dw_j^s}{dw_i^s} < 0$. That is, w_i^s is a strategic complement to w_j^s whereas w_j^s is a strategic substitute for w_i^s.

The system (53) has a solution:

$$w_i^{s*} = \frac{2d_i - 2d_j + 4d_i d_j - 1}{5 + 6d_i + 6d_j + 4d_i d_j}.$$

Thus, the equilibrium quantities and profits are given by:

$$q_i^S = \frac{2 + 4d_j}{(5 + 6d_i + 4d_i d_j + 6d_j)},$$
$$\pi_i^S = \frac{2(1 + d_i)(1 + 2d_j)^2}{(5 + 6d_i + 4d_i d_j + 6d_j)^2},$$
$$i, j = 1, 2, i \neq j.$$

If the firm i separates and the firm j integrates, the retailers' profit maximization problems:

$$\begin{cases} \max_{q_i} \pi_i^L = (1 - q_i^L - q_j^F) q_i^L - \ w_i^L q_i^L \\ \max_{q_j} \pi_j^F = (1 - q_j^F - q_i^L) q_j^F - \frac{1}{2} d_j \left(q_j^F \right)^2 \end{cases},$$

have a solution:

$$\begin{cases} q_i^L = \frac{1+d_j - w_i^L(2+d_j)}{3+2d_j} \\ q_j^F = \frac{1+w_i^L}{3+2d_j} \end{cases}. \tag{54}$$

Plugging (54) into $\pi_i^L = (1 - q_i^L - q_j^F)q_i - \frac{1}{2}d_i \left(q_i^L\right)^2$ and optimizing with respect to w_i^L, we obtain:

$$w_i^{L*} = \frac{(1+d_j)(2d_i + d_i d_j - 1)}{(2+d_j)(2+2d_j+d_i d_j+2d_i)}.$$

The respective equilibrium quantities and profits are given by:

$$\begin{cases} q_i^{L*} = \frac{1+d_j}{(2+2d_j+d_i d_j+2d_i)} \\ q_j^{F*} = \frac{1+d_j+2d_i+d_i d_j}{(2+d_j)(2+2d_j+d_i d_j+2d_i)} \end{cases},$$

$$\begin{cases} \pi_i^L = \frac{(1+d_j)^2}{2(2+d_j)(2+2d_j+d_i d_j+2d_i)} \\ \pi_j^F = \frac{(1+d_j+2d_i+d_i d_j)^2}{2(2+d_j)(2+2d_j+d_i d_j+2d_i)^2} \end{cases}.$$

It is easy to see that $\pi_i^L > \pi_i^C > \pi_i^F$ and, as it is prescribed by Proposition 1, the outcome of the subgame $[I, I]$ is never played in SPNE. To determine equilibrium we compare π_i^F and π_i^S. Let's consider the set (d_1, d_2) such that the firm i is indifferent between separating and integrating given that the firm j separates:

$$\pi_i^F = \frac{(1+d_i+2d_j+d_i d_j)^2}{2(2+d_i)(2+2d_j+d_i d_j+2d_i)^2} = \frac{2(1+d_i)(1+2d_j)^2}{(5+6d_i+4d_i d_j+6d_j)^2} = \pi_i^S. \tag{55}$$

It can be shown that $d_i = \theta_i(d_j)$, $i, j = 1, 2$ determined by (55) are such that: (i) $d_i = \theta_i(d_j), i, j = 1, 2$ are strictly concave and have a unique maximum; (ii) $\theta_i(0) > 0$ and (iii) $\exists \overline{d_j} < +\infty : \theta_i(\overline{d_j}) = 0$. Figure 5 gives a graphical representation of $\theta_i(d_j)$ and $\theta_j(d_i)$.

In zone A (low d_1 and low d_2) both firms have relatively flat marginal cost curves. In this case each firm prefers to separate even if its rival separates and hence the unique equilibrium is $[S, S]$. The equilibrium profit of each firm is lower than in the Cournot equilibrium, yet higher than the Stackelberg follower's profit: $\pi_i^F < \pi_i^S < \pi_i^C$. Although within zone A firms may differ in efficiency, this difference is sufficiently small. In zone C (low d_1 and moderate d_2), the firm 1 is more efficient than the firm 2, with the difference in efficiency being not too high. Then, strategy $m_1 = S$ is dominant for the the firm 1, while the firm 2 chooses to

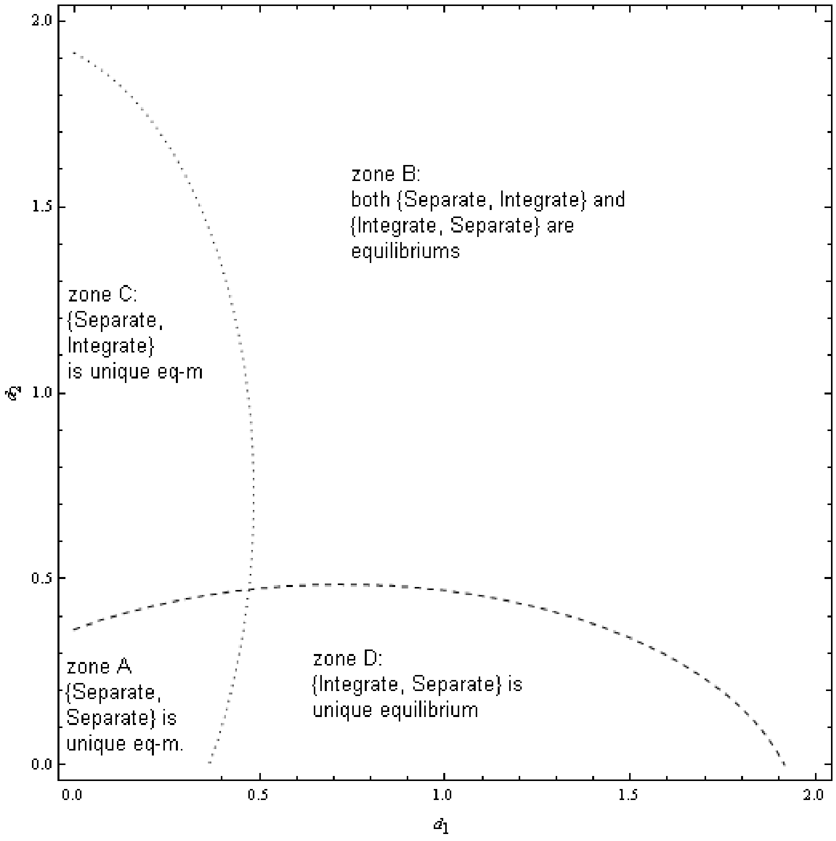

Figure 5: $\pi_2^F > \pi_2^S$ above the dashed line, $\pi_1^F > \pi_1^S$ right at the dotted line.

integrate if the firm 1 separates, $\pi_1^F < \pi_1^S$ and $\pi_2^F > \pi_2^S$. Thus, the $[S, I]$-subgame is played in SPNE. In zone D (low d_1 and moderate d_2), the situation is the opposite of that in zone C and the $[I, S]$-subgame is played in SPNE. Finally, zone B is such that (either d_1 or d_2 or both are sufficiently high), each firm chooses to integrate provided its rival separates. Therefore, two asymmetric equilibria, $[I, S]$ and $[S, I]$, exist.

Next Proposition summarizes the above results:

Proposition 22 *(i)* $[S, S]$ *is an equilibrium if* $(d_1, d_2) \in A = \{(d_1, d_2) | \, d_1 \leq \theta_1(d_2), d_2 \leq \theta_2(d_1)\}$*;*

(ii) both $[S, I]$ *and* $[I, S]$ *are equilibria if* $(d_1, d_2) \in B = \{(d_1, d_2) | \, d_1 \geq \theta_1(d_2), d_2 \geq \theta_2(d_1)\}$*;*

(iii) $[S, I]$ *is an equilibrium if* $(d_1, d_2) \in C = \{(d_1, d_2) | \, d_1 \leq \theta_1(d_2), d_2 \geq \theta_2(d_1)\}$*;*

(iv) $[I, S]$ *is an equilibrium, if* $(d_1, d_2) \in D = \{(d_1, d_2) | \, d_1 \geq \theta_1(d_2), d_2 \geq \theta_2(d_1)\}$.

Intuition for this result is the following. If the firm 1 separates and the firm 2 integrates, then they get Stackelberg leader and follower profits, respectively. Let's consider $\widetilde{w}_1$ and $\widetilde{w}_2$ which replicates this Stackelberg outcome in the $[S,S]$-subgame, that is such that $q_1^S(\widetilde{w}_1,\widetilde{w}_2) = q_1^{L*}$ and $q_2^S(\widetilde{w}_1,\widetilde{w}_2) = q_2^{F*}$. Now, a deviation of the firm 2 from strategy I to strategy S is equivalent to switching from the outcome determined by $\{\widetilde{w}_1,\widetilde{w}_2\}$ to the outcome determined by $\{w_1^{s*}, w_2^{s*}\}$ in the $[S,S]$-subgame. It may be shown that $\widetilde{w}_2 > w_2^{s*}$ for any d_1, d_2 while either $\widetilde{w}_1 < w_1^{s*}$ or $\widetilde{w}_1 > w_1^{s*}$ depends on values of d_1 and d_2.

Let's consider effects of changes in w_1 and w_2 on the firm 2 profit. Clearly, the decrease in w_2 raises the output of the firm 2 and lowers one of the firm 1. If $\widetilde{w}_1 < w_1^{s*}$ then an increase in w_1 has the same qualitative effect on outputs: q_1 decreases and q_2 increases. As both changes raise the firm 2's profit, the firm 2 strictly prefers to deviate to separation. This holds on the subset of zone A with d_1 and d_2 close enough to zero.

In contrast, if $\widetilde{w}_1 > w_1^{s*}$ then a decrease in w_1 rises the firm 1's output and decreases the firm 2's output. This has a negative impact on the firm 2's profit. In this case, the total effect of changes in both w_1 and w_2 depends on values of d_1 and d_2. If both d_1 and d_2 are sufficiently low (the rest of zone A and zone D) then the positive effect of the change in w_2 dominates the negative effect of the change in w_1 and the deviation is profitable. Otherwise, the firm 2 prefers to integrate given the firm 1 separates (zones C and B).

The symmetric arguments apply for the case when the firm 1 chooses between separation and integration given the firm 2 separates. Therefore, the zone A is the set $\{d_1, d_2\}$ such that each firm strictly prefers to separate and thus $[S,S]$ is unique equilibrium. In the zone C separation is a dominant strategy for the firm 1 while the firm 2's profit is higher if it integrates given the firm 1 separates. Thus, there exists a unique asymmetric equilibrium. Similarly, in the zone D, there exists a unique asymmetric equilibrium where the firm 2 separates and the firm 1 integrates. Finally, each firm prefers to integrate given that its rival separates in the zone B and there exist two asymmetric equilibria.

Corollary 23 *In the symmetric game with $d_1 = d_2 = d$, there exists a unique $\widehat{d}$ such that, if $d < \widehat{d}$, then $[S,S]$ is the unique equilibrium while if $d > \widehat{d}$, then there are two asymmetric equilibria $[I,S]$ and $[S,I]$.*

In particular, $\widehat{d} \approx 0.47 < \frac{1}{2}$ and $\left.\frac{dw_i}{dw_j}\right|_{d_1=d_2=\widehat{d}} = \left.\frac{dw_i}{dw_j}\right|_{d_1=d_2=\widehat{d}} \approx -0.01 < 0$. Thus, asymmetric equilibria in the symmetric game arise even if firms' wholesale prices are not strategic complements; it is enough that the degree of substitution between them is sufficiently low.

3.3.1 Welfare analysis

Using explicit solutions for outputs in every subgame we have that for any fixed d_1, d_2 the following relation holds

$$q_1^C + q_2^C < q_1^L + q_2^F < q_1^S + q_2^S,$$

that is, consumer surplus is maximized when both firms separate. Thus, if $(d_1, d_2) \in A$ then firms actions$\{Separate, Separate\}$ is optimal from the point of view of the consumer surplus maximization. In respect to profits, we have reverse relationship

$$\pi_1^C + \pi_2^C > \pi_1^L + \pi_2^F > \pi_1^S + \pi_2^S.$$

This implies that the total industry profit maximized in the Cournot setting. Calculating the total welfare as the sum of consumers surplus and firms' profits for each subgame we obtain that the total welfare in the $[S, S]$-subgame (TW^{SS}) is always greater than the ones in $[S, I]$-subgame (TW^{SI}) and $[C, C]$-subgame (TW^{CC}):

$$TW^{SS}(d_1, d_2) > TW^{SI}(d_1, d_2); TW^{SS}(d_1, d_2) > TW^{CC}(d_1, d_2) \text{ for any } d_1, d_2 \geq 0.$$

Thus, if $(d_1, d_2) \in A$ then firm's choice of separation at the first stage maximizes also the total welfare.

3.3.2 Robustness

In this section, we come back to a general formulation of the model to provide arguments on a robustness of main results[7]. Suppose $P(Q), C_1(q_1)$ satisfy Assumptions 3.1-3.3. Let's consider the the firm 2's cost function in the following form: $C_2(q_2) = \alpha c_2(q_2)$, where $\alpha > 0$ and $c_2(q_2)$ satisfies Assumptions 3.1 and 3.2. Clearly, for any fixed α, $C_2(q_2, \alpha)$ satisfies these Assumptions also.

[7] This section is based on a recent paper of Sloev (2010), to which we refer for more technical details.

Let's consider the $[S, I]$-subgame. By (52) equilibrium of the subgame coincides with a Stackelberg solution of the $[I, I]$-subgame and it is determined by the system

$$\begin{cases} P'(Q)q_1 + P(Q) - C_1' + P'(Q)q_1 \frac{dq_2^F(q_1)}{dq_1} = 0 \\ P'(Q)q_2^F + P(Q) - \alpha c_2'(q_2^F) = 0 \end{cases}, \tag{56}$$

with

$$\frac{dq_2^F}{dq_1} = -\frac{P''(Q)q_2^F + P'(Q)}{P''(Q)q_2^F + 2P'(Q) - \alpha c_2''(q_2^F)}.$$

Suppose that $\alpha \to \infty$. Then, both the firm 2's output and a change in q_2^F in respect to change in q_1 go to zero, that is $q_2^F(\alpha) \to 0$, $\frac{dq_2^F}{dq_1} \to 0$. Therefore, the second equation in (56) uniformly converges to $P'(q_1)q_1 + P(q_1) - C_1'(q_1) = 0$, which implies that the firm 1's output converges to a monopolistic one.

Now, let's consider the $[S, S]$-subgame. According to (49) the solution is determined by the system

$$\begin{cases} P'(Q)q_1^S + P(Q) - C_1' + P'(Q)q_1^S \left(\frac{\partial q_2^S}{\partial w_1} / \frac{\partial q_1^S}{\partial w_1}\right) = 0 \\ P'(Q)q_2^S + P(Q) - \alpha c_2' + P'(Q)q_2^S \left(\frac{\partial q_1^S}{\partial w_2} / \frac{\partial q_2^S}{\partial w_2}\right) = 0 \end{cases},$$

with $\frac{\partial q_j^S}{\partial w_i} / \frac{\partial q_i^S}{\partial w_i} = -\frac{P''(Q)q_j^S + P'(Q)}{P''(Q)q_j^S + 2P'(Q)}$. As $\alpha \to \infty$, the output of the firm 2 again goes to zero, and $\frac{\partial q_j^S}{\partial w_i} / \frac{\partial q_i^S}{\partial w_i} \to -\frac{1}{2}$. Thus, the first order condition for the firm 1 uniformly converges to $P'(q_1)q_1/2 + P(q_1) - C_i'(q_1) = 0$, which implies that the firm 1's output is greater than the monopolistic output.

Therefore, for α big enough, the output of the firm 1 in the $[S, S]$-subgame is greater than it is in the $[S, I]$-subgame. As the Stackelberg follower profit decreases in the Stackelberg leader output, we have that the firm 2's profit in the $[S, S]$-subgame is lower than it is in the $[S, I]$-subgame. Thus, if the firm 1 separates, then the firm 2 prefers to integrate provided its marginal cost curve is steep enough.

Now, let's consider the $[I, S]$-subgame. Again, as $\alpha \to \infty$, the firm 2's output goes to zero and the first order condition for the firm 1 uniformly converges to

$$P'(q_1^F)q_1^F + P(q_1^F) - C_1'(q_2^F) = 0,$$

which implies that the firm 1' output converges to the monopolistic output. As it was shown above, the firm 1's output in the $[S, S]$- subgame exceeds the monopolistic output and therefore the profit of the firm 1 in the $[S, S]$-subgame is lower than it is in the $[I, S]$-subgame. Thus, the firm 1 prefers to integrate given that the firm 2 having very steep marginal curves separates.

This analysis demonstrates that vertical separation and vertical integration coexist under very mild assumptions on a demand and cost functions provided that diseconomy of scale is high enough at least for one firm.

3.4 Conclusion

In this paper, we have analyzed the firms' incentives to vertically separate (i.e., sell their products through independent exclusive retailers), or vertically integrate (be retailers of their own products) in a framework of Cournot model with a linear-demand and quadratic cost functions.

We have demonstrated that the equilibrium market structure critically depends on firms' cost structures. Considering quadratic cost functions, we have shown that if firms' cost asymmetry is small and degrees of diseconomy of scale are sufficiently low for both firms, in the unique equilibrium of the game both firms vertically separate. Under moderate asymmetry in costs, when diseconomy of scale is low for the first firm and it is high for the second firm, there is a unique equilibrium in which the first firm separates, whereas the second integrates. If instead either the asymmetry in costs is extremely high or diseconomy of scale is high for both firms then each firm prefers to integrate, given that its rival separates and two asymmetric equilibria arise.

As a result, in the symmetric case in the unique equilibrium both firms separate if the degree of diseconomy of scale is relatively low, and there are two asymmetric equilibria if the degree of diseconomy is high. The intuition for these results is as follows.

If one firm separates and another integrates then a separated firm gets the Stackelberg leader profit and the integrated firm gets the Stackelberg follower profit. Obviously, each firm prefers to separate given that its rival integrates and therefore the case when both firms integrate is never equilibrium.

If two symmetric firms separate, their profits depend on strength of competition in the wholesale prices among producers. The strength of this competition is in turn determined by the degree of diseconomy of scale; higher diseconomy implies stronger competition. Strong competition results in low wholesale prices, high output levels and profits lower than the Stackelberg follower's profit. Hence, if the degree of diseconomy is high, there are two asymmetric equilibria in which one firm separates and the other integrates. If the competition in wholesale prices is weak (which is the case if diseconomy of scale is low), each firm prefers to separate given that its rival separates. Hence, there exists the unique symmetric equilibrium where both firms separate.

Our analysis provides a possible explanation for the widely observed asymmetry in firms' sales strategies based on decreasing economies of scales and cost asymmetry. It is worth to note that in the model, separation neither implies a change in the production function nor is associated with additional costs. In this sense, we have shown the existence of asymmetric equilibria in a "pure" separation game.

REFERENCES

[1] Allanson, P. and C. Montagna (2005), Multiproduct Firms and Market Structure: An Explorative Application to The Product Life Cycle, International Journal of Industrial Organization 23, 587-597.

[2] Anderson, S. P. and A. de Palma (1992), Multiproduct Firms: A Nested Logit Approach, Journal of Industrial Economics 40, 261–271.

[3] Anderson, S. and A. de Palma (2006), Market Performance with Multiproduct Firms, Journal of Industrial Economics 54, 95-124.

[4] Bernard, A.B., R. Stephen, and S. Peter (2006), Multi-Product Firms and Trade Liberalization, NBER Working Paper 12782.

[5] Basu, K. (1995), Stackelberg equilibrium in oligopoly: An explanation based on managerial incentives, Economic Letters, Vol. 49, pp. 459-464.

[6] Bernheim, B. D., M. D. Whinston, (1998), Exclusive Dealing, Journal of Political Economy, vol. 106, 64-103.

[7] Blanchard, P., C. Gaigne, and C. Mathieu (2007), Endogenous Product Differentiation, Multiproduct Multinationals and FDI, unpublished manuscript.

[8] Bonanno, G., and J. Vickers (1988), Vertical Separation, The Journal of Industrial Economics, Vol. 36, No. 3, pp. 257-265.

[9] Brambilla, I. (2006), Multinationals, Technology, and the Introduction of Varieties of Goods, NBER Working Paper 12217.

[10] Buehler, S., and A. Schmutzler (2005), Asymmetric Vertical Integration, Advances in Theoretical Economics, Vol. 5, pp. 1164-1164.

[11] Chioveanu, I., and U. Akgun (2006), Loyalty Discounts, working paper, Alicante University.

[12] Dixit, A. (1979), A model of duopoly suggesting a theory of entry barriers, The Bell Journal of Economics, Vol. 10, No. 1 , pp. 20-32.

[13] Dixit, A. and J. Stiglitz (1977), Monopolistic Competition and Optimum Product Diversity, American Economic Review 67, 297-308.

[14] Doraszelski, U. and M. Draganska (2006), Market Segmentation Strategies of Multiproduct Firms, Journal of Industrial Economics 54, 125–149.

[15] Eaton, J., S. Kortum, and F. Kramarz (2004), Dissecting Trade: Firms, Industries, and Export Destination, American Economic Review (Papers and Proceedings) 94, 150-154.

[16] Eckel, C. (2006), Endogenous Sunk Costs, Flexible Manufacturing and the Productivity Effects of International Trade, unpublished manuscript.

[17] Eckel, C. and P. Neary (2006), Multi-Product Firms and Flexible Manufacturing in the Global Economy, CEPR Discussion Paper 5941.

[18] Erkel-Rousse, H. (1997), Endogenous Differentiation Strategies, Comparative Advantage and the Volume of Trade, Annales D' Economie et de Statistique 47, 121–149.

[19] Fauli-Oller, R. and B. Mesa-Sanchez (2007), Downstream Mergers in Markets with Multilateral Transactions, unpublished manuscript, University of Alicante.

[20] Fershtman, C. and K. L. Judd (1987), Equilibrium Incentives in Oligopoly, The American Economic Review, Vol. 77, No. 5 , pp. 927-940.

[21] Greenlee, P., D. Reitman and D. Sibley (2004), An Antitrust Analysis of Bundled Loyalty Discounts, working paper, U.S. Department of Justice.

[22] Greenlee, P., and D. Reitman, 2004, Competing with Loyalty Discounts, working paper, U.S. Department of Justice.

[23] Hansen J.D. and J.G. Jorgensen (2001), Market Integration and Industrial Specialization on a Monopolistic Competitive Market, Journal of Economic Integration 163, 279-298.

[24] Hansen, J. D. and J. U.-M. Nielsen (2007), Economies of Scale and Scope, Firm Heterogeneity and Exports, unpublished manuscript, University of Aarhus.

[25] Helpman, E. (1985), Multinational Corporations and Trade Structure, Review of Economic Studies 52, 443-457.

[26] Hendricks, K. and R. P. McAfee (2007), A Theory of Bilateral Oligopoly, Working Paper, California Institute of Technology.

[27] Hotelling, H. (1929), Stability in Competition, The Economic Journal 39, 41-57.

[28] Jansen, J. (2003), Coexistence of Strategic Vertical Separation and Integration, International Journal of Industrial Organization, Vol. 21, pp. 699-716.

[29] Katz, M. L. (1989), Vertical Contractual Relations, In the Handbook of Industrial Organization, vol.1, edited by Richard Shmalensee and Robert D. Willig. Amsterdam: North-Holland.

[30] Kobayashi, B. H., 2005, The Economics Of Loyalty Discounts And Antitrust Law In The United States, George Mason Univ., School of Law, working paper.

[31] Lin, Y. J. (1988), Oligopoly and Vertical Integration: Note, American Economic Review, Vol. 78, pp. 251-254.

[32] Majumdar, A., and G. Shaffer (2007), Market-Share Contracts with Asymmetric Information, CCP working paper 07-17.

[33] Marx, L. M. and G. Shaffer (2004), Rent-Shifting, Exclusion, and Market-Share Discounts, working paper, Duke University.

[34] Mathewson, G. F, and R. A. Winter (1987), The Competitive effect of Vertical Agreements: Comment, American Economic Review, 77.

[35] Melitz, M. J. and G. I. P. Ottaviano (2005), Market Size, Trade, and Productivity, NBER Working Paper Series, Working Paper 11393.

[36] Mills, D. E. (2009), Inducing Downstream Selling Effort with Market Share Discounts, Mimeo, University of Virginia.

[37] Nocke, V. and S. Yeaple (2006), Globalization and Endogenous Firm Scope, NBER Working Paper 12322.

[38] Norman, G. and J.-F. Thisse (1999), Technology Choice and Market Structure: Strategic Aspects of Flexible Manufacturing, Journal of Industrial Economics 47, 345–372.

[39] Ordover, J., and G. Shaffer, 2007, Exclusionary Discounts, CCP Working Paper No. 07-13.

[40] Ottaviano G. P., T. Tabucchi, and J.F. Thisse (2002), Agglomeration and Trade Revisited, International Economic Review 43, 409-435.

[41] Ottaviano G. P. and J.-F. Thisse (1999), Monopolistic Competition, Multiproduct Firms And Optimum Product Diversity, CEPR Discussion Paper 2151.

[42] Reisinger, M. and M. Schnitzer (2008), A Model of Vertical Oligopolistic Competition, University of Munich Discussion Paper 3189.

[43] Salop, S. (1979), Monopolistic Competition with Outside Goods, The Bell Journal of Economics 10, 141-156.

[44] Schmalensee, R. (1978), Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry, The Bell Journal of Economics 9, 305-327.

[45] Sklivas, S. D. (1987), The strategic Choice of Managerial Incentives, The RAND Journal of Economics, Vol. 18, No. 3, pp.452-458.

[46] Sloev Igor (2010), On Coexistence of Vertical Separation and Vertical Integration, SFB-15 working paper..

[47] Spence, M. (1976), Product Selection, Fixed Costs and Monopolistic Competition, Review of Economic Studies 43, 217-235.

[48] Tom, W.K., D.A. Balto, and N.W. Averitt, 2000, Anticompetitive Aspects of Market-Share Discounts and Other Incentives to Exclusive Dealing, Antitrust Law Journal, 67, 615-640.

[49] Roundtable on loyalty or fidelity discounts and rebates, DAFFE meeting, May, 2002.

[50] Van Long, N. and A. Soubeyranb (2000), Existence and uniqueness of Cournot equilibrium: a contraction mapping approach, Economics Letters, 67, pp. 345–348.

[51] Vickers, J. (1985), Delegation and the Theory of the Firm, The Economic Journal, Vol. 95, Supplement: Conference Papers, pp. 138-147.