Predicting the Monthly Volatility of the EuroStoxx 50 using Data Sampled at Different Frequencies

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Abstract

This paper analyses the forecastability of the EuroStoxx 50 monthly returns volatility. We consider different proxies for the unobserved volatility variable by using data sampled at different frequencies, and GARCH and AGARCH models with Normal and Student's t errors for the dynamics of returns conditional variance. We find that a method based on aggregation of multi step (daily) ahead GARCH-type forecasts provide quite accurate predictions of monthly volatility.

Key words: Asymmetry; Frequency; Model ranking; Volatility forecasting.

JEL classification: C22; C52; C53; G32

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1 Introduction

The forecast of monthly financial volatility is relevant for many economic decision making processes, from those involving macroeconomic analysis to those on risk management. In relation to the former, economic and monetary policy decisions on GDP growth and inflation targets must take into account the financial markets volatility forecast since it is closely related to interest rates expectations (see Schwert (1989) for a comprehensive analysis on the relation between monthly stock returns volatily and macroeconomic variables). On the other hand, long-horizon financial volatilities forecast is the cornerstone for many risk management decisions, such as portfolio and hedge funds composition (equity and bonds), strategic firm position, regulatory and internal capital allocation, and risk-adjusted performance measurement schemes.

The conditional volatility literature has undergone extensive development since the seminal Engle's (1982) ARCH and Bollerslev's (1986) GARCH models. Those models and their extensions (for instance, Glosten et al.'s (1993) asymmetric GARCH (AGARCH), Ding et al.'s (1993) asymmetric power ARCH (APARCH), Baillie et al.'s (1996) fractionally integrated GARCH (FIGARCH), Davidson's (2004) hyperbolic GARCH (HYGARCH)) have been successful at explaining and forecasting the dynamics of return variance. More recent methodologies for forecasting volatility focus on data-driven models of realized volatility computed from high-frequency (intra-daily) returns (see Andersen and Bollerslev (1998), Andersen et al. (2001, 2003), Meddahi (2002) and Barndorff-Nielsen and Shephard (2004)), and on mixed data sampling (MIDAS) regressions (Ghysels et al., 2006).

Specifically, when it comes to forecasting monthly volatility, a simple method consists on using returns sampled at monthly frequency and perform 1 step ahead forecast using a GARCH-type model (see e.g. Schwert, 1989). This method requires of a very long sample to find the volatility clustering observed in higher frequency returns. Alternatively, one can use daily returns and perform multi step ahead volatility forecasts using a GARCH-type model (see Baillie and Bollerslev, 1992). Operationally, the 1 day ahead GARCH forecast can be converted to longer horizons by scaling by the square root of horizon (for instance, as in J.P. Morgan's (1996) RiskMetrics). Christoffersen et al. (1998) assessed those both methods and found that volatility forecastability declines quickly with the horizon, and it seems to vanish beyond horizons of ten or fifteen days.

In this paper, I follow the methodology based on data-driven models, to predict EuroStoxx 50 monthly volatility by aggregating multi step (daily) ahead volatility forecasts.[1] Similarly, measures of the unobservable target volatility, are calculated by aggregating (future) squared or absolute returns (see Ghysels et al. (2006) and Barndorff-Nielsen and Shephard (2004)).

The EuroStoxx 50 returns conditional variance is modeled by assuming either a GARCH model or an AGARCH to account for "leverage effects", together with two distributions: the Student's t that is flexible to account for the excess kurtosis not explained by GARCH processes (Bollerslev, 1987), and the Normal distribution. Models are also estimated for filtered returns by outliers to eliminate the known bias in GARCH parameter estimates caused by those extreme observations (Carnero, Peña and Ruiz, 2007), forecasts based on those estimations are also analysed.

The models performance is evaluated by using the mean squared error (MSE hereafter) loss function, and the Minzer-Zarnowitz (M-Z hereafter) (1969) regression method.

The remainder of the paper is organised as follows. Section 2 presents the data and the models for conditional heteroscedasticity, and discusses the estimation results. Section 3 presents the method to measure the unobservable volatility variable. Section 4, presents the methodology and results of the monthly volatility forecasting. Finally, Section 5 summarises the conclusions.

2 Stock returns volatility modeling

Let daily returns be denoted by $r_t = \log(P_t) - \log(P_{t-1})$. Throughout the paper the time index t will refer to daily sampling. We also use data sampled at a lower frequency (monthly), with each month having m days, then we will denote the monthly return as $r_{mt} = \log(P_{mt}) - \log(P_{m(t-1)})$. To make our analysis more realistic we consider months with their actual number of working days, so m_{ij} (number of observations of month i of year j) is not constant and ranges from 20 to 23 days. For the sake of simplifying notation we drop

[1]Models based on monthly returns and on square-root scaling are not presented given its known inferior performance for monthly volatility forecast. Results are available from the author. Only GARCH 1 step ahead monthly forecasts are presented as a benchmark to illustrated the performance of the rest of considered models.

the ij subindices of m.

Our data set consists of daily and monthly returns of the EuroStoxx 50 index over a 19 years period, from February 2, 1988 to December 2, 2007, for a total of $5,326$ daily and 236 monthly observations. The data were downloaded from Datastream International. Figure I presents plots and descriptive statistics of the data, which show the known stylized features of financial returns: volatility clustering (Engle, 1982), heavy-tailed distribution (Mandelbrot, 1963), and asymmetric response of returns to positive and negative shocks (Black, 1976).

Both daily and monthly returns are filtered by their conditional mean to remove small linear dependences attributed to non-synchronous trading in the stocks that form the index (see Sentana and Wadhwani, 1992). The Akaike Information Criterion (AIC hereafter) selects the following process for the conditional mean: $r_t = \mu + \varepsilon_t$; all AR and MA parameters, but for the intercept, were not statistically significant at least at 10 per cent level. For the sake of simplicity, in the rest of the paper we use r_t, instead of $\widehat{\varepsilon}_t = r_t - \widehat{\mu}$, to denote filtered returns.

We assume two distributions for the returns, the Normal and the Student's t (Bollerslev, 1987). To account for volatility clustering the conditional variance of the returns distribution is specified to follow the GARCH process of Engle (1982) and Bollerslev (1986), and "leverage effects" are modeled by using the asymmetric GARCH (AGARCH) model of Glosten et al. (1993). To fix notation the return process is defined as,

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \\ \varepsilon_t &= h_t^{\frac{1}{2}} \eta_t, \quad \varepsilon_t|\Omega_{t-1} \sim N(0, h_t), \quad \varepsilon_t|\Omega_{t-1} \sim t_\nu(0, h_t), \end{aligned} \tag{1}$$

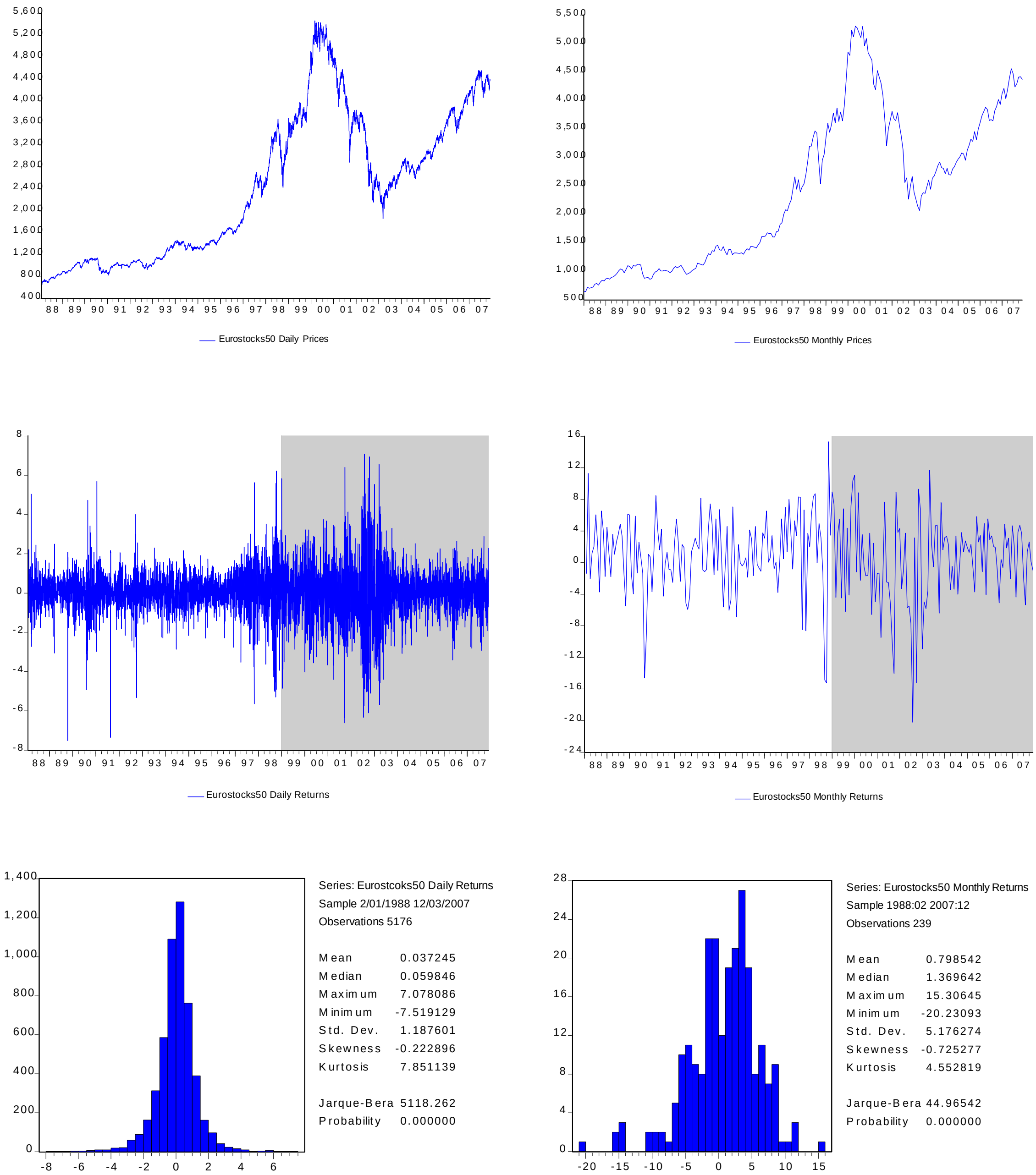
where Ω_{t-1} denotes the econometrician information set up to time $t-1$, ν is the degrees of freedom parameter of the Student's t distribution, and h_t is the variance of the conditional distribution of r_t, which follows a GARCH(1,1) model (eq. 2) or an AGARCH(1,1) (eq. 3).[2]

$$h_t = \omega + \alpha u_{t-1}^2 + \beta h_{t-1}, \tag{2}$$

$$h_t = \omega + \alpha (u_{t-1} - \gamma)^2 + \beta h_{t-1}. \tag{3}$$

[2] Other possibilities include FIGARCH and APARCH models. Note that GARCH-in-Mean models (Engle et al., 1986) are not likely to provide significantly different performance than GARCH models for monthly volatility forecasts, since the effect of the mean factor vanishes after 2 steps ahead volatility forecast.

Figure I. EuroStoxx 50 index daily & monthly prices and returns plots and descriptive statistical information. Data downloaded from Datastream International. Monthly returns sample is from February 1988 to December 2007 (observations 239), the out-of-sample period is from February 1999 to December 2007 (107 months). Daily returns sample spans from February 02, 1988 to December 1, 2007 (Observations 5,176), the out-of-sample period is from February 2, 1999 to December 1, 2007 (2,326 daily observations corresponding to 107 months).



The models are estimated by maximum likelihood (ML) techniques and robust standard errors are calculated by using Bollerslev and Wooldridge (1992) formula. When the Normal distribution is assumed, ML estimation of GARCH and AGARCH models provides quasy-ML estimates (QMLE) which are consistent and asymptotically normal although not efficient (see Straumann and Mikosch (2006) for the analysis of the AGARCH case). Much less is known about the theoretical properties of the MLE when the Student's t distribution is assumed (see Newey and Steigerwald (1997) for further details).

Returns are also filtered to eliminate outlier observations that are known to bias the (Q)ML parameter estimates of GARCH processes (see Carnero, Peña and Ruiz, 2007). In our empirical application, outliers are identified by the simple rule of 3 times the sample standard deviation ($3^*\widehat{h}^{\frac{1}{2}}$) and substituted by the returns sample mean, 117 outliers were substituted for the daily returns and 5 for the monthly data. This filtering procedure will not affect the models ranking regarding their out-of-sample forecasting performance, since it is same for all models. More formal procedures, applicable in a forecasting volatility context, to correct for outliers in returns series with GARCH effects are proposed in Franses and Ghysels (1999).

2.1 Estimation results

Table I reports the estimation results for the models using the daily returns. Hereafter 'n' and 't' preceded by the conditional variance process denote that the assumed distribution for the returns is either the Normal or the Student's t, respectively. Panel 1 presents the results for the unfiltered returns. Consistent with the prior literature, the sum of $\widehat{\alpha}$ and $\widehat{\beta}$ is near 1, which indicates high persistence in the EuroStoxx 50 returns daily volatility. The degrees of freedom coefficient, $\widehat{\nu}$, is around 6, confirming the existence of leptokurtosis in the returns conditional distribution. The asymmetry parameter estimate, $\hat{\gamma}$, is statistically different from zero or one in both asymmetric models, confirming the existence of "leverage effect" in the index return daily volatility.

The estimation results in Panel 2 for the filtered returns show that the returns distribution is nearer to normality, since $\widehat{\nu}$ is higher than when unfiltered returns are considered. Furthermore, we note that the bias of $\widehat{\alpha}$ and $\widehat{\beta}$ is not corrected for the first sample window, although careful monitoring of estimation reveals that it does correct for other windows over the out-of-sample period, for which $\widehat{\nu}$ take values above 25. According to the AIC, asymmetry an Student's t distribution both provide the GARCH with more flexibility to fit to the data.

Table I. Models estimation results for daily returns across the in-sample period (2/2/1999 1/12/2007, observations 2,326). (Q)MLE estimates, and t-statistics (in parenthesis) obtained from robust standard errors. Panel 1 presents the model estimation for the original sample, and Panel 2 for the sample corrected by substituting outliers (returns larger than 3 sample standard deviations) by the returns sample mean.

	GARCH-n	AGARCH-n	GARCH-t	AGARCH-t
Panel 1: No Filter				
ω	.0475 (2.48)	.0279 (2.11)	.0226 (3.65)	.0189 (3.25)
α	.1113 (4.47)	.0889 (3.36)	.0226 (5.68)	.0934 (5.95)
β	.8370 (26.7)	.8616 (24.2)	.8797 (44.5)	.8789 (45.5)
γ		.4445 (3.08)		.2661 (4.10)
ν			6.037 (7.47)	6.207 (7.33)
LogL	-3641.9	-3623.9	-3483.5	-3476.1
AIC	2.5578	2.5459	2.4474	2.4428
Panel 2: Filter $3^*\widehat{h}^{\frac{1}{2}}$				
ω	.0110 (2.78)	.0073 (1.93)	.0085 (2.48)	.0062 (1.91)
α	.0607 (5.43)	.0567 (5.51)	.0617 (5.17)	.0612 (5.43)
β	.9238 (63.1)	.9284 (67.8)	.9275 (62.9)	.9272 (65.1)
γ		.2625 (5.40)		.2375 (3.56)
ν			9.333 (6.23)	9.7201 (5.93)
LogL	-3383.4	-3374.8	-3356.8	-3350.6
AIC	2.3764	2.3711	2.3584	2.3548

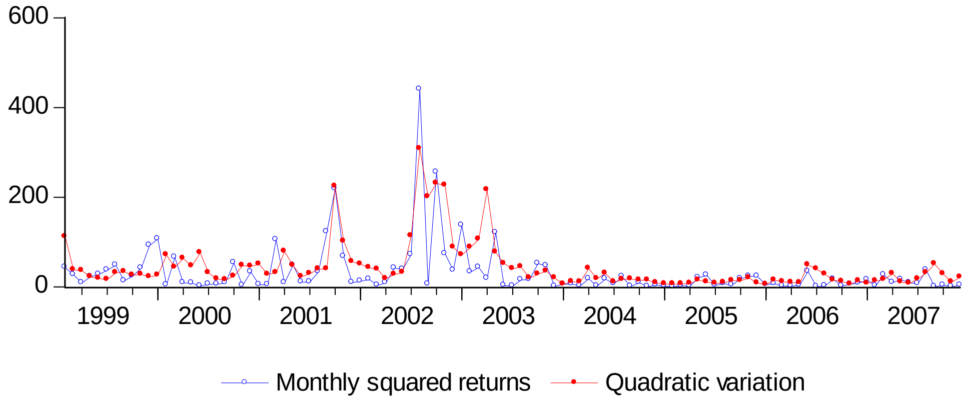
3 The volatility proxy

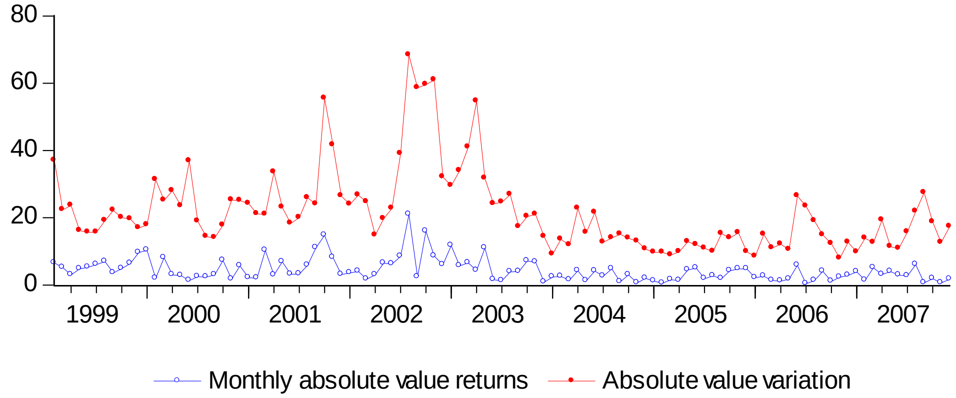
Our goal is to predict a measure of volatility over a monthly horizon, which corresponds to 1 step ahead forecast for monthly frequency data, $\widetilde{h}_{m(t+1)}$, and to m days ahead forecast for daily frequency, $\widetilde{h}_{t+m}$. As volatility is an unobservable variable and in order to make our analysis comparable to a large body existing literature, we consider several measures of volatility for the period t to $t+m$ (daily frequency), or mt to $m(t+1)$ (monthly frequency). As primary measures we consider the squared and absolute value monthly returns, denoted as $r^2_{m(t+1)}$ and $\left|r_{m(t+1)}\right|$, respectively. Following the volatility literature on high-frequency (intra-daily) data, we also consider the increments in the quadratic variation, H_{t+m}, (see Ghysels et al., 2006) and the "realized power" variation, P_{t+m}, of the return process (see Barndorff-Nielsen and Shephard, 2004). Those variables are not observed directly but can be measured with some discretization error. Both such measures would be the sum of (future) daily squared or absolute value returns, denoted as $\widetilde{H}_{t+m} = \sum_{j=1}^{m} \left(r_{t+j}\right)^2$ and $\widetilde{P}_{t+m} = \sum_{j=1}^{m} \left|r_{t+j}\right|$, respectively. Note that we aggregate daily returns to obtain measures of "realized" monthly volatility since our goal is monthly volatility forecast. We could even use higher frequency data (intra-daily), but we discard this possibility since for monthly-horizon forecasts there are not significant differences in performance, as shown in Ghysels et al. (2006). Figure II presents the four different EuroStoxx 50 returns monthly volatility measures over the out-of-sample period for unfiltered (Panel 1) and filtered returns (Panel 2). It is observed that monthly volatility tend to be underestimated when using punctual monthly squared returns in relation to the aggregated measure of quadratic variation. This underestimation is systematic when considering punctual monthly absolute value returns in relation to the "realized power" volatility.

It is worth noting that, in the case of discrete time processes, $r^2_{m(t+1),mt}$ and $\widetilde{H}_{t+m,t} = \sum_{j=1}^{m} \left(r_{t+j/t}\right)^2$ are unbiased measures for the implied (true) underlying monthly volatility, and although the former is noisier, both ensure a correct ranking of models, in a quadratic loss function case (see e.g. Andersen and Bollerslev (1998), Awartani and Corradi (2005) and Hansen and Lunde (2005)).

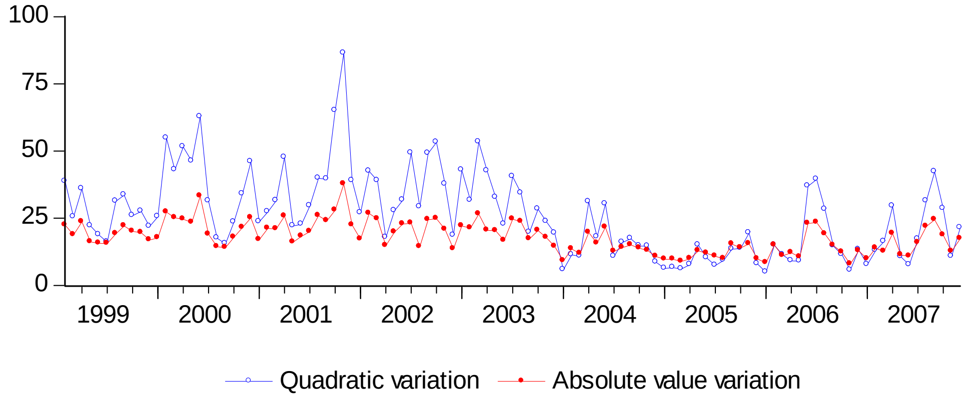
Figure II. Volatility proxies: Quadratic/Absolute value variation proxies from filtered and unfiltered for outliers daily returns, and monthly squared monthly returns proxy.

Panel 1: Outliers non-filtered returns





Panel 2: Outliers filtered returns



4 Volatility forecast methodology

For the monthly forecasts using the monthly frequency, we use the first 129 observations to estimate the parameters of the GARCH models, and compute $N = 107$ out-of-sample 1 step ahead forecasts of the conditional variance, $\mathbf{E}_{mt}(r^2_{m(t+1)}) = \widehat{h}_{m(t+1)}$, by using a rolling window of constant size $238 - N - 1$ that discards old observations.

For the daily frequency, we use the first $2,850$ observations to estimate the parameters of the GARCH models, and compute $N = 107$ out-of-sample m step ahead forecasts of the conditional variance by using a rolling window of non-constant size $5,326 - (N * m) - 1$ that discards the m oldest observations and incorporates the newest m observations of the month that just went.[3] Then we use a recursive multi step ahead forecasting procedure (Baillie and Bollerslev, 1992) where the optimal predictor for the 1 step ahead GARCH(1,1) conditional variance is given by

$$\widehat{h}_{t+1} \equiv \mathbf{E}_t(h^2_{t+1}) = \omega + \alpha u^2_t + \beta h_t \tag{4}$$

and the m step ahead optimal predictor is

$$\widehat{h}_{t+m} \equiv \mathbf{E}_t(h^2_{t+m}) = \omega + (\alpha + \beta)h_{t+m-1}. \tag{5}$$

For the AGARCH(1,1) the optimal m step ahead predictor of the conditional variance is given by,

$$\mathbf{E}_t(h^2_{t+m}) = \begin{cases} \omega + \alpha(u_t - \gamma)^2 + \beta h_t, & \text{for } m = 1 \\ \omega + (\alpha + \beta)h_{t+m-1} + \alpha\gamma^2, & \text{for } m > 1 \end{cases} \tag{6}$$

Then, a monthly conditional variance forecast is obtained by adding the previous m steps ahead conditional variance forecasts, $\widehat{H}_{t+m} = \sum^m_{t+j} \widehat{h}_{t+j}$. On the other hand, when the target volatility is $\widetilde{P}_{t+m}$ or $\left|r_{m(t+1)}\right|$, the monthly forecast predictor is, $\widehat{H}^{\frac{1}{2}}_{t+m} = \sum^m_{t+j} \widehat{h}^{\frac{1}{2}}_{t+j}$. In summary, this procedure simulates monthly forecasts of EuroStoxx 50 volatility from

[3]Note that in this case windows sizes may differ at most in 3 daily observations, a small number given the window size. But all windows have the same number of months.

February 1999 to December 2007, performed the first day of each month over that period, with an updating window that discards the oldest month observations and incorporates the daily data from the latest month.

The model performance is measured by using the MSE with respect to the volatlity proxies described in Section 2, namely $\widetilde{H}_{t+m}$, $\widetilde{P}_{t+m}$, $r^2_{m(t+1)}$ and $\left|r_{m(t+1)}\right|$. MSE are calculated for returns series free of outliers (filtered returns hereafter) and unfiltered returns. The performance of models forecasts obtained by using the filtered series is measured with respect either to filtered or to unfiltered volatility proxies. We also consider the M-Z regression as a measure for models forecasting performance. It consists on estimating the following equation,

$$\widetilde{H}_{t+m} = \vartheta_0 + \vartheta_1 \widehat{H}_{t+m} + u_{t+m}. \tag{7}$$

Thus, the forecast from a model is optimal with respect to the available information set (Ω_{T+i-1}) if the null $H_0 : (\vartheta_0, \vartheta_1) = (0, 1)$ is accepted.[4]

4.1 Forecast results

Figure III presents the plots of the monthly volatility forecasts, $\widehat{H}_{t+m}$, obtained from a GARCH-n, and punctual 1 step ahead monthly forecasts obtained from a GARCH-n model fitted to monthly data, with respect to proxies $\widetilde{H}_{t+m}$ and $r^2_{m(t+1)}$. We clearly observed that $\widehat{H}_{t+m}$ is much more flexible to capture periods of high volatility in relation to both proxies and, both methods (aggregation and monthly punctual forecasts) provide similar results for periods of low volatility.

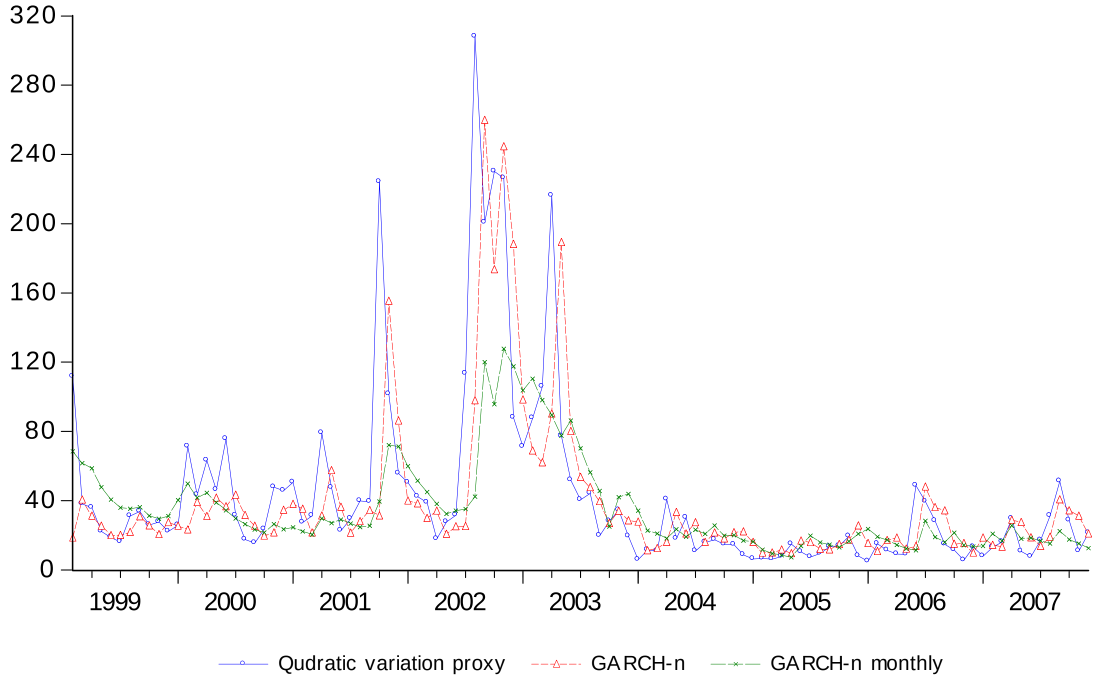
Figure IV presents plots of $\widehat{H}_{t+m}$ obtained from AGARCH-n and AGARCH-t models against the proxy $\widetilde{H}_{t+m}$. From the plots we observe that both models provide reasonably good forecasts being difficult to discriminate between them.

[4] Note that without loss of geneality the M-Z equation is specified for the proxy $\widetilde{H}_{t+m}$ and the forecasts $\widehat{H}_{t+m}$.

Figure V presents monthly forecasts $\widehat{H}_{t+m}$ and $\widehat{H}^{\frac{1}{2}}_{t+m}$, from a GARCH-$n$ model and filtered returns against (filtered and unfiltered) proxies $\widetilde{H}_{t+m}$ and $\widetilde{P}_{t+m}$, respectively. As we expected, we observe a better fit of forecasts $\widehat{H}_{t+m}$ to the proxy $\widetilde{H}_{t+m}$ than when unfiltered data are considered. But, $\widehat{H}_{t+m}$ obtained from filtered data are not able to capture "real" periods of high volatility, as shown by the large discrepancy between $\widehat{H}_{t+m}$ and $\widehat{H}^{\frac{1}{2}}_{t+m}$ from filtered data and proxies $\widetilde{H}_{t+m}$ and $\widetilde{P}_{t+m}$ from non-filtered returns. Note that $\widetilde{H}_{t+m}$ and $\widetilde{P}_{t+m}$ for non-filtered returns are the actual (observed) proxies of volatility and so the target variables.

A sharp result that emerges from Table II is that for both filtered and unfiltered returns Normal models provide a lower MSE than their Student's t counterparts, being the AGARCH-n generally preferred to the GARCH-n model. This result is consistent with those in the existing literature that show that the heavy-tail assumption in GARCH models helps better to forecasts measures as value-at-risk rather than conditional variance (see, e.g., Brooks and Persand (2003), Awartani and Corradi (2005), and Ñíguez (2008)). In relation to the M-Z regression criteria is worth noting that the null of optimal forecasts is accepted only for Normal models when the target variable is $\widetilde{H}_{t+m}$, in the rest of the cases it is rejected at any reasonable significance level. It also stands out the high R^2 found from models when either $\widetilde{H}_{t+m}$ and $\widetilde{P}_{t+m}$ are used as proxies, these values are higher than those generally found in the literature. On the other hand, when proxies $r^2_{m(t+1)}$ and $\left|r_{m(t+1)}\right|$ are used (see Table III) our values are in line with those in the literature (see, for instance, Andersen and Bollerslev, 1998). Furthermore, R^2 from models using filtered data are higher, as expected. It is also worth mentioning that for unfiltered returns the same model ranking is found with respect the MSE when using either $\widetilde{H}_{t+m}$ or $r^2_{m(t+1)}$ as proxies for the implied (underlying) volatility.

Figure III. Daily/monthly fitted GARCH-n for m days ahead/1 month ahead predictions of quadratic variation and monthly square returns.



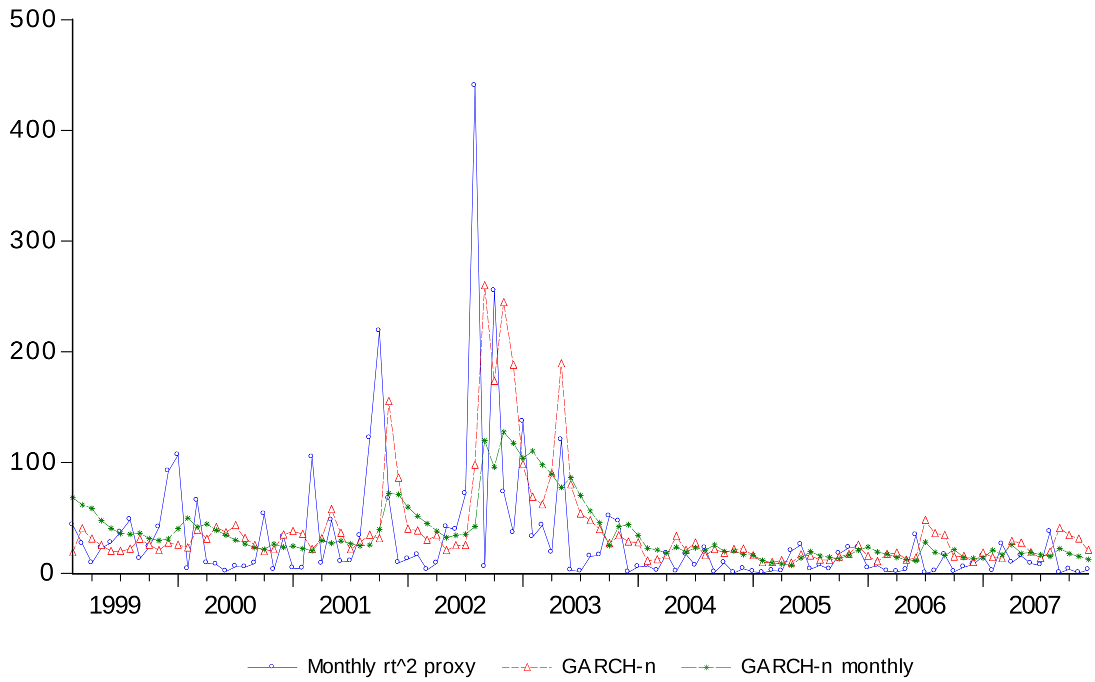
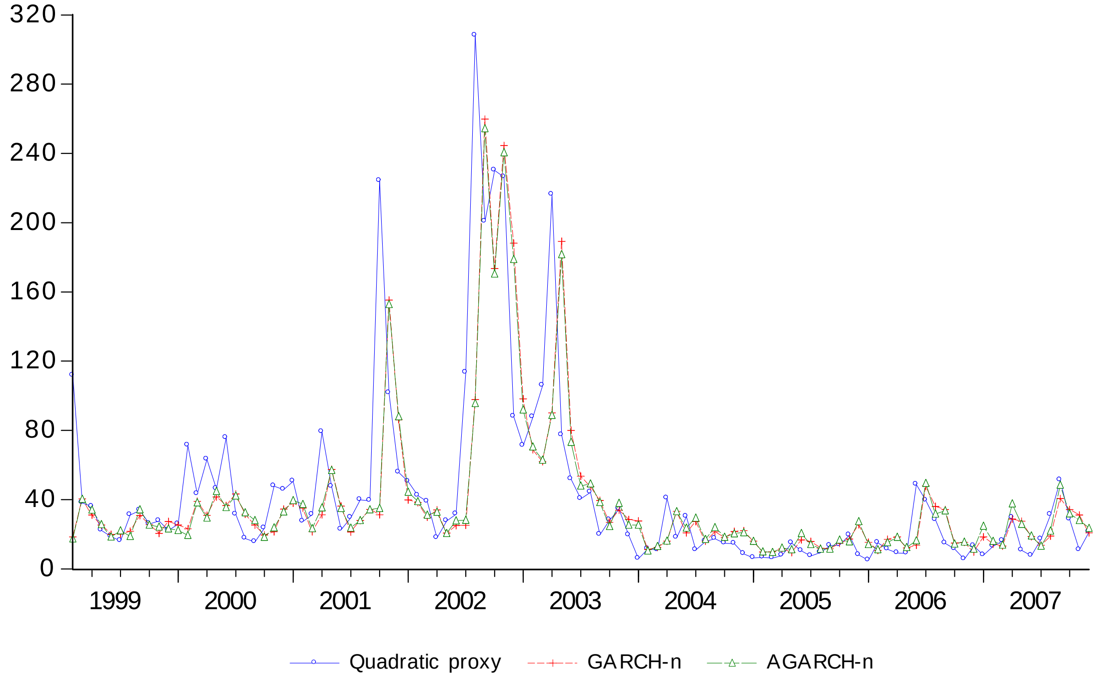


Figure IV. GARCH models m days ahead prediction of quadratic variation.



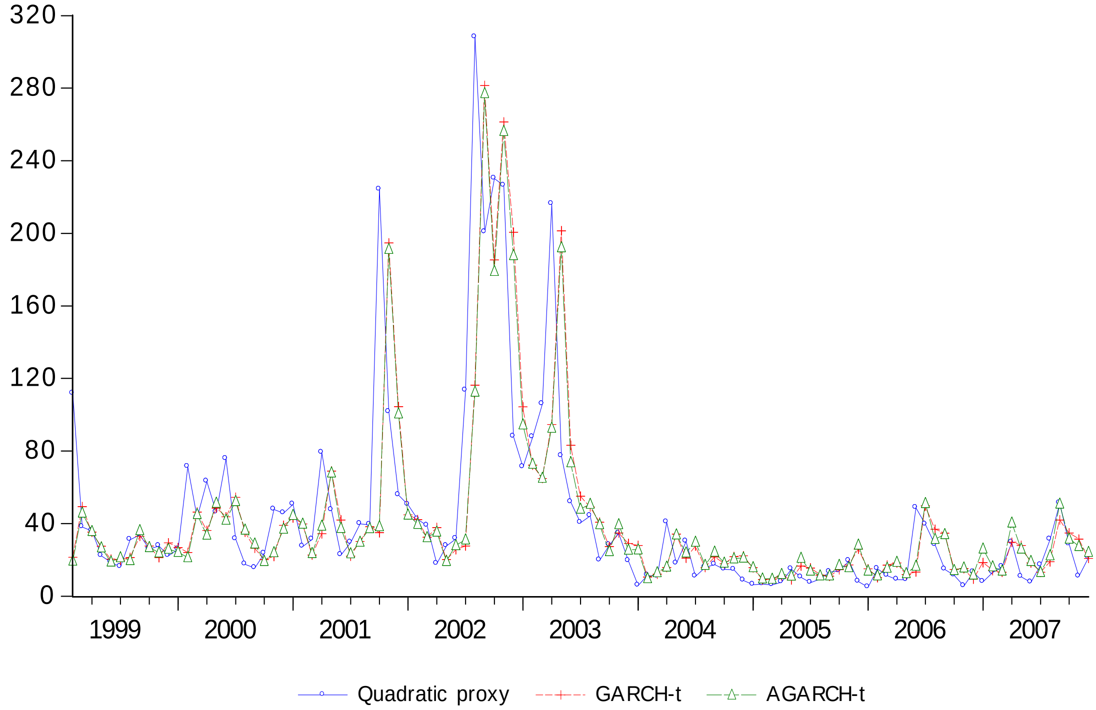
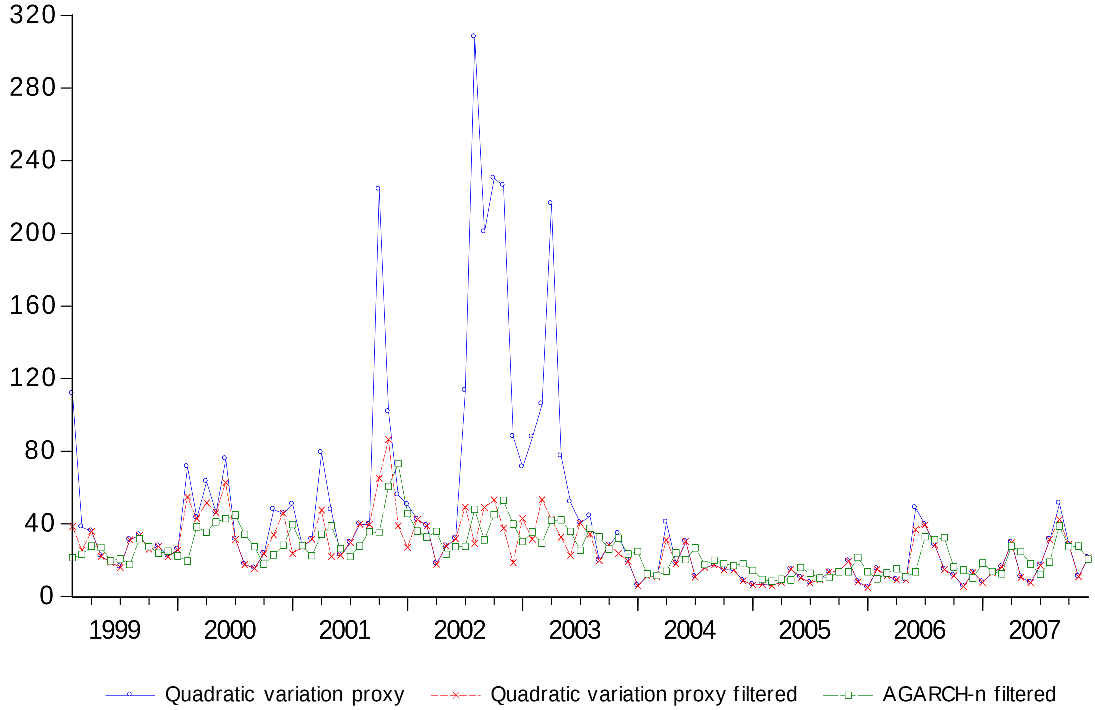


Figure V. Filtered daily returns fitted AGARCH–n monthly predictions of variance/standard deviation against (filtered and unfiltered) quadratic/absolute value variation



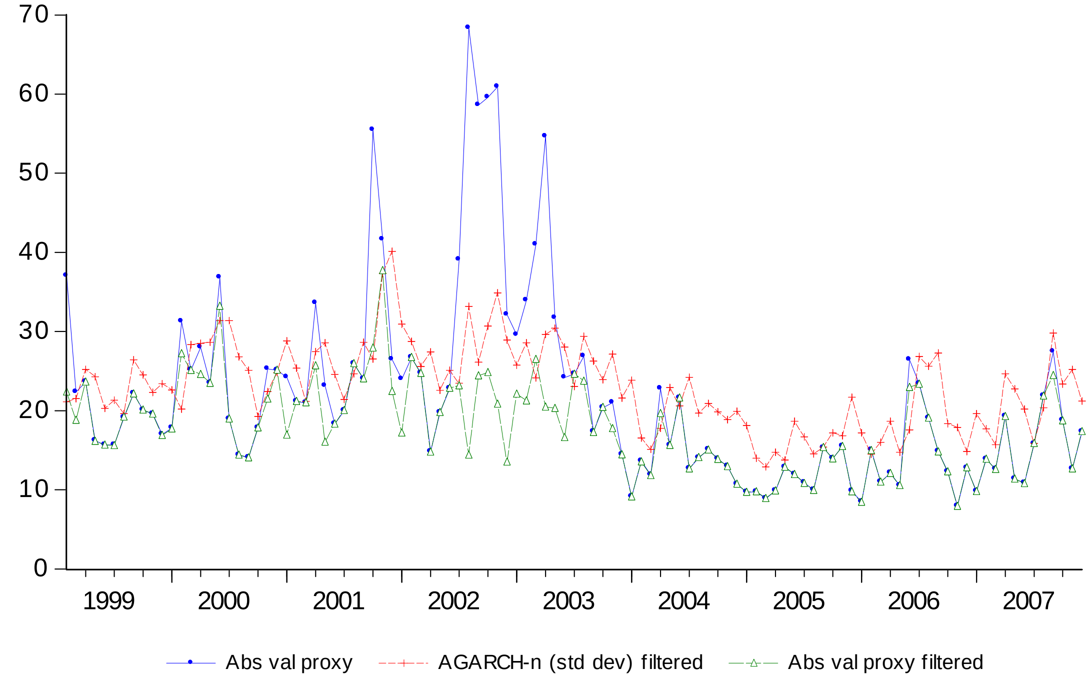


Table II. Out-of-sample monthly forecasting performance with respect to quadratic variation (Panel 1) and absolute value variation(Panel 2) proxies, for filter and unfiltered returns. Predictions are produced by using GARCH models fitted to daily filtered and unfiltered returns. P-value and R^2 from the Minzer-Zarnowitz regression, and MSE stands for Mean Square Error.

	GARCH-n	AGARCH-n	GARCH-t	AGARCH-t
Panel 1: Proxy $\widetilde{H}_t$				
No Filter				
P-value	0.0726	0.1601	0.0071	0.0306
R^2	0.4726	0.4832	0.4809	0.4904
MSE	1566.9	1512.4	1612.4	1539.3
Filter $3{*}\widehat{h}^{\frac{1}{2}}$				
P-value	0.2287	0.1816	0.0369	0.0142
R^2	0.3816	0.4022	0.3927	0.4118
MSE	147.3	143.0	149.7	147.7
Panel 2: Proxy $\widetilde{P}_t$				
No Filter				
P-value	0.0000	0.0000	0.0000	0.0000
R^2	0.5448	0.5622	0.5621	0.5754
MSE	99.3	96.4	111.3	106.2
Filter $3{*}\widehat{h}^{\frac{1}{2}}$				
P-value	0.0000	0.0000	0.0000	0.0000
R^2	0.3602	0.3793	0.3724	0.3791
MSE	49.3	51.3	54.7	56.9

Table III. Out-of-sample monthly forecasting performance with respect to squared (Panel 1) absolute value (Panel 2) monthly returns. Predictions are produced by using GARCH models fitted to daily filtered and unfiltered returns. P-value and R^2 from Minzer-Zarnowitz regression, and MSE stands for Mean Square Error.

	GARCH-n	AGARCH-n	GARCH-t	AGARCH-t
Panel 1: Proxy monthly r^2_{mt}				
No Filter				
P-value	0.0000	0.0000	0.0000	0.0000
R^2	0.1239	0.1232	0.1322	0.1304
MSE	3495.8	3435.6	3730.5	3636.9
Filter $3{*}\widehat{h}^{\frac{1}{2}}$				
P-value	0.0000	0.0000	0.0000	0.0002
R^2	0.3892	0.3670	0.3818	0.3577
MSE	550.6	547.1	522.6	524.1
Panel 2: Proxy monthly $\|r_{mt}\|$				
No Filter				
P-value	0.0000	0.0000	0.0000	0.0000
R^2	0.1548	0.1558	0.1632	0.1646
MSE	617.6	617.8	672.7	665.0
Filter $3{*}\widehat{h}^{\frac{1}{2}}$				
P-value	0.0000	0.0000	0.0000	0.0000
R^2	0.4588	0.4295	0.4593	0.4281
MSE	315.0	323.6	332.8	341.6

5 Conclusions

This paper provides an analysis of the predictability of the EuroStoxx 50 stock index monthly volatility. We consider different measures for the unobservable target monthly volatility including, monthly squared returns and an estimate of the increments of the returns quadratic (absolute value) variation calculated using daily future squared (absolute value) returns.

We analyse the forecasting performance of GARCH and AGARCH models with Normal and Student's t errors together with a procedure that aggregate Baillie and Bollerslev's (1992) multi step ahead volatility optimal forecasts to predict monthly volatility. We find that this method provides quite accurate results of monthly volatility in relation to other methods based on either 1 step ahead GARCH-type forecasts using monthly frequency, or multi step ahead (without aggregation) GARCH-type forecasts using daily returns (see Christoffersen et al. (1998)). Normal AGARCH models provide more accurate forecasts according to MSE loss functions and M-Z regression criteria.

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