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EXPLORING ICA FOR TIME SERIES DECOMPOSITION

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Keywords: ICA, Signal Extraction, Multivariate Time Series, Forecasting

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Exploring ICA for time series decomposition

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In this paper, we apply independent component analysis (ICA) for prediction and signal extraction in multivariate time series data. We compare the performance of three different ICA procedures, JADE, SOBI, and FOTBI that estimate the components exploiting either the non-Gaussianity, or the temporal structure of the data, or combining both, non-Gaussianity as well as temporal dependence. Some Monte Carlo simulation experiments are carried out to investigate the performance of these algorithms in order to extract components such as trend, cycle, and seasonal components. Moreover, we empirically test the performance of those three ICA procedures on capturing the dynamic relationships among the industrial production index (IPI) time series of four European countries. We also compare the accuracy of the IPI time series forecasts using a few JADE, SOBI, and FOTBI components, at different time horizons. According to the results, FOTBI seems to be a good starting point for automatic time series signal extraction procedures, and it also provides quite accurate forecasts for the IPIs.

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1 Introduction

In many applications of empirical sciences such as Medicine, Engineering, and Economics, when the data are observed with a high level of noise, extracting the relevant patterns from the observations becomes an important task. The problem of estimating those underlying components (components of interest) from the observations is known as signal extraction or feature extraction problem. Thus, considering the additive decomposition,

$$x_t = \chi_t + \nu_t, \tag{1}$$

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where x_t is the observed data, χ_t is the set of interesting components (signal), and ν_t is the noise process (not necessarily white) which is assumed to be independent of χ_t, the aim of signal extraction is to isolate the signal from the noise. The estimates of the signal will be obtained by filtering the observations, $\widehat{\chi}_t = \mathbf{F}x_t$, in such a way that the signal estimates satisfy the minimum mean square error (MMSE) criterion.

If x_t is a univariate time series process, model (1) might represent the decomposition of x_t as the sum of some underlying components of interest, which are usually interpreted in terms of trend, seasonality, and cycle, among others. Then, some economic applications such as seasonal adjustment, detrending, and analysis of the business cycles, can be seen as particular cases of signal extraction problems, where the interesting signals (χ_t) are, respectively, seasonally adjusted components, trends, and cycles.

Several approaches have been developed for solving the signal extraction problem in the univariate framework. The first one, called 'ad-hoc' filter design approach, includes methods that use moving-average smoothing filters to estimate the signal. These methods are supported by the main central statistical agencies for trend extraction and seasonal adjustment in time series. The X-11 filter (Shiskin et al. (1967)) for seasonal adjustment and the Beveridge-Nelson (Beveridge and Nelson (1981)), the Baxter and King (Baxter and King (1995)), and the Hodrick-Prescott (Hodrick and Prescott (1997)) filters, which were used to estimate the trend-cycle components, are some well-known examples of the 'ad-hoc' filter design approach. The main disadvantage of these filters is that they do not take into account the structure of the time series process and they could produce spurious results and over/under-estimated components. Trying to solve this important limitation, it has been developed the so-called model-based procedures, where the filter is derived from statistical models and it is adapted to the particular structure of the time series processes. Two directions emerge within the model-based procedures: the ARIMA-model-based approach and the structural modelling approach.

On the one hand, the ARIMA-model-based procedures (Box et al. (1978), Burman (1980), Bell and Hillmer (1984), Hillmer and Tiao (1982), among others) directly identify a parsimonious ARIMA model for the observations. Then, univariate models for the components are derived with the restriction that the aggregation of those models yields the ARIMA model identified for the data. Because there is not a unique admissible decomposition, these methods apply the 'canonical decomposition' (see Box et al. (1978)) to solve identifiability problems. Within this approach, the most popular algorithm is the SEATS/TRAMO software (Gómez and Maravall (1996), Maravall (1993)) that is based on the filter developed by Burman (1980).

On the other hand, the structural modelling approach (Harvey (1989), Young et al. (1999), Bujosa et al. (2007), among others), instead of using a-priori information to specify a model for the observations, directly assumes different stochastic linear models for the unobserved components. These models are formulated within an stochastic state space setting, and the Kalman filter is used to estimate the parameters. STAMP (Koopman et al. (1995)) is a well known software that directly specifies structural models for the components of interest in the time domain framework. Another implementations of this approach, such as the CAPTAIN MatLab Toolbox program (Young and Pedregal (1999), Taylor et al. (2007)) and the linear dynamic harmonic regression algorithm (Bujosa et al. (2007)), are developed in the spectral framework assuming that the data are periodic time series.

When we move to the multivariate framework, where the issue of information redundancy in the observed data set is usually arising, capturing the most 'interesting' features of the data might be as important as (or even more than) it was in the univariate case. In particular, when we observe multiple time series data where dynamic relationships are involved, the components of interest might be common to different time series. Thus, extracting those underlying common components, which probably may have a useful interpretation in terms of common trends or common seasonality, becomes an important task in multivariate time series analysis. Dynamic factor models (see Forni et al. (2000) and Peña and Poncela (2006), among others) and multivariate structural time series models (Harvey (1989)) have traditionally dealt with this topic. However, it is hard to develop 'automatic' (or quasi-automatic) procedures for signal extraction in the multivariate framework, and STAMP (Koopman et al. (1995)) is the only model-based procedure that can handle this problem.

As an alternative to model-based procedures, principal component analysis (PCA) is usually applied to multivariate data sets with the aim of noise and/or dimension reduction, and signal extraction. PCA can be seen as an 'automatic' procedure for signal extraction, where the relevant information is given by those components that explain the largest amount of variance in the data. PCA is quite successful in multivariate linear data but, when the data are non-Gaussian (non-linear), PCA has difficulty in separating the underlying components. Empirical applications show that, under non-Gaussianity assumption, the components extracted by PCA are quite far away from the real ones (see for example, Oja (1982) and Särelä and Valpola (2005), among others). Moreover, these empirical results reveal that independent component analysis (ICA) estimates the underlying components better than PCA does.

In this chapter, we explore the performance of ICA in multivariate time series signal extraction, and analyze how the ICA components could be useful to predict the observations. ICA seems to be appropriate when we observed several economic time series data, where some components of interest, such as trend or seasonal variations, can be assumed to be fairly independent.

This paper is organized as follows. Section 2 reviews the main approaches that have been presented in the literature for signal extraction. Then, we introduce the procedure to forecast the data using a set of ICA components. In Section 4, we carry out some simulation experiments to support the idea that ICA could be seen as the first step for automatic signal extraction procedures. Next, we apply ICA to extract the components of interest in the industrial production indexes of several European countries. In addition, we analyze how these data are forecasted using a few ICA components. Finally, Section 6 gives some concluding remarks.

2 Model-based methods for signal extraction

Most of the latest signal extraction algorithms are model-based procedures where the observations are decomposed as the sum of some components of interest, such as trend, cycle, and seasonal components. For example, for time series data, estimating the trend and the seasonality is important to analyze the main movements of the time series, and to obtain seasonal adjusted data, respectively. In general, since an infinite number of decompositions is possible, the identification of the components is not unique, and additional assumptions should be made.

An attractive feature of model based-approaches is that, since they are based on specific statistical models for the observations and/or the components, model-based approaches could facilitate analysis and inference. Next, we review the ARIMA-model based and the structural modelling approaches, paying attention to some of their well-known implementations.

2.1 ARIMA-model based methods

The ARIMA-model based methodology (Box et al. (1978), Hillmer and Tiao (1982), Burman (1980), Maravall and Pierce (1987), amongst others) came up as an alternative procedure for seasonal adjustment of time series data. The ARIMA-model based approach starts by applying the Box and Jenkins methodology to specify an ARIMA model that describes the behavior of the time series data. Then, univariate models for the components are derived so that their aggregation should be consistent with the original ARIMA model. Two assumption are made to guarantee the unique identification of the components: first, it is assumed that the components of interest are mutually uncorrelated; second, it is applied the canonical principle (Box et al. (1978)) which maximizes the variance of the noise component and leads the 'interesting' components to be as stable as possible (Hillmer and Tiao (1982)). The underlying components are computed by the Wiener-Kolmogorov filter (Box et al. (1978)) that provides the MMSE estimators of the components, even for non-stationary time series (Bell (1984)).

Popular procedures that take the ARIMA-model based approach are the X-11-ARIMA (Dagum (1980)), the X-12-ARIMA (Findley et al. (1998)) and the SEATS/TRAMO software (Gómez and Maravall (1996), Maravall (1993)). These methods are commonly used by official statistical agencies to get seasonally adjusted data (for example, Statistics Canada, US Bureau of the Census, and Bank of Spain are well-known examples of official agencies that apply, respectively, X-11-ARIMA, X-12-ARIMA, and SEATS/TRAMO programs, to seasonal adjustment).

The first two procedures, the X-11-ARIMA and X-12-ARIMA, are based on moving averages filters and then, they are not ARIMA-model based procedures themselves. However, since at the first stage the two procedures identify an ARIMA model for the observations and the definitions of the signals are 'implicit', the X-11- and the X-12-ARIMA are considered as ARIMA-model based procedures. Both X-11- and X-12-ARIMA uses the X-11 filter (Cleveland and Tiao (1976)), that applies a set of centered moving averages to estimate the seasonal components. The problem is that when moving averages filters are used, many observations of the beginning and the end of the series are lost and the seasonal effect could be underestimated. The X-11-ARIMA, trying to avoid the loss of observations, uses the ARIMA model fitted to the original series for extending the length of the data set (forecasting and backcasting). The X-12-ARIMA follows the same idea that the X-11-ARIMA but introduces a pre-adjustment program, REGARIMA, that is applied to the original time series data (before the identification of the ARIMA model) to detect outliers and to estimate some deterministic effects (for example, the calendar effect).

The SEATS/TRAMO programs (Gómez and Maravall (1996), Maravall (1993)) are efficient and automatic procedures which are mainly applied for seasonal adjustment and trend-cycle estimation. First, TRAMO (Time series Regression with ARIMA noise, Missing values and Outliers) is a pre-adjustment program that is applied to the univariate time series data to pre-test for the log-level specification, to detect and correct outliers (additive outliers, transitory changes, and level shifts), to interpolate missing values, and to correct other deterministic effects such as Trading Day, Leap Year, and Easter effects. Then, TRAMO specifies a set of possible models for the pre-adjusted data, estimates them by maximum likelihood, and selects the 'optimal' one based on AIC and BIC criteria. Finally, according to the selected model, TRAMO forecasts the data to extend the time series and thus, it reduces the bias when a new observation enters to the model. Next, SEATS (Signal Extraction in ARIMA Time Series) derives univariate ARIMA models for the stochastic components so that they reflect the usual structures associated to trend, cyclical (or trend-cycle), and seasonal components. SEATS uses the canonical principle (Box et al. (1978)) to avoid identifiability problems and applies the Burman-Wilson algorithm (Burman (1980)) to estimate the components (MMSE estimators). The final estimates for the unobserved components are obtained by the aggregation of the deterministic effects (computed by TRAMO) of each individual component to the stochastic components given by SEATS.

ARIMA-model based procedures have two important drawbacks: first, since the models for the components are not directly specified (they are derived from the original ARIMA model for the observations and should be consistent with it) those components could not be easily interpretable, a-posteriori, in terms of trend or seasonality; second, since the ARIMA-model based procedures consider a common noise for all the components, the components' estimates could be correlated, and therefore, the assumption of uncorrelated components would not be satisfied. In structural modelling procedures, this problem is solved considering independent noises for each component.

2.2 Structural modelling approach

The structural modelling approach is an alternative model-based methodology for signal extraction that is based on unobserved components models. Contrary to the ARIMA-model based methodology the structural modelling procedures directly specify univariate stochastic models for the underlying components and then, their interpretability in terms of trends, seasonalities and cycles is guaranteed.

We distinguish two structural modelling specifications: the structural time series approach (Harvey (1989)) that is implemented in the STAMP software (Koopman et al. (1995)), and the dynamic harmonic regression approach (Young et al. (1999)), that is implemented in the CAPTAIN Toolbox for Matlab (Young and Pedregal (1999), Taylor et al. (2007)) as well as in the new linear dynamic harmonic regression algorithm (Bujosa et al. (2007)). The main differences between the dynamic harmonic regression model (Young et al. (1999)) and Harvey's structural model (Harvey (1989)) rely on the model specification for the periodic components and the optimization method used to estimate the parameters. In the following, we discuss these two approaches.

2.2.1 Structural time series approach

Structural time series models (Harvey (1989)) are formulated in terms of unobserved components which have a direct interpretation. According to Harvey (1989), the structural time series models 'are not more than regression models in which explanatory variables are a function of time and the parameters change with time'. These explanatory variables represent dynamic features of the data (such as stochastic trends, cycles, and/or seasonalities). The starting point in structural time series models is to identify those features and model them in such a way that we can obtain useful predictions for the time series data. Structural time series models are usually formulated as state space models and the parameters of the unobserved components models are estimated using the Kalman filter and related algorithms (see Harvey (1989) for a detailed description of the state space and the Kalman filter methodologies).

STAMP (Structural Time Series Analyzer, Modeler and Predictor) (Koopman et al. (1995)) is a standard signal extraction procedure that is implemented according to structural time series models (as they are defined in Harvey (1989)). STAMP, contrary to alternative signal extraction procedures that are only developed in the univariate framework (e.g. SEATS/TRAMO), can be applied to extract the components of interest in both univariate as well as multivariate time series data.

The basic structural time series model assumes that univariate time series can be decomposed into additive stochastic components as

$$y_t = \mu_t + \psi_t + \gamma_t + \epsilon_t. \tag{2}$$

where μ_t represents the trend, ψ_t the cycle, γ_t the seasonality, and ϵ_t the irregular component (a structural time series model should not be necessarily defined in terms of these four UCs; it may be defined only by some of them). There are different specifications to formulate the stochastic process for each component. By default, for univariate time series data, STAMP considers a basic structural time series model which chooses the local linear trend (LLT) model for the trend, a stochastic cyclical component, a stochastic trigonometric model for the seasonality, and a white noise process for the irregular term, $\epsilon_t \sim NID(0, \sigma_\epsilon^2)$.

According to the LLT model, the stochastic trend is given by

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t,\ \eta_t \sim NID(0, \sigma_\eta^2), \\ \beta_t &= \beta_{t-1} + \xi_t,\ \xi_t \sim NID(0, \sigma_\xi^2), \end{aligned} \tag{3}$$

where β_t is the stochastic slope of the trend. Here, the two noises, η_t and ξ_t, and the irregular component in (2), ϵ_t, are assumed to be mutually uncorrelated. Different specifications for the trend are possible: either the level (μ_t) or the slope (β_t) could be deterministic instead of stochastic, and the slope might not be included in the model (see Harvey (1989) for a complete revision of different specifications).

The stochastic cyclical component is given by

$$\begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} = \rho^\psi \begin{pmatrix} cos\lambda^c & sin\lambda^c \\ -sin\lambda^c & cos\lambda^c \end{pmatrix} \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa_t^* \end{pmatrix} \tag{4}$$

where ρ^{ψ} and λ^c represent, respectively, the damping factor and the cyclical frequency (measured in radians) which take values $0 < \rho^{\psi} \leq 1$ and $0 \leq \lambda^c \leq \pi$, respectively. The period of the cycle is given by $2\pi/\lambda^c$. The cyclical disturbances, $\kappa_t \sim NID(0, \sigma_\kappa^2)$ and $\kappa_t^* \sim NID(0, \sigma_\kappa^2)$, are assumed to have the same variance and to be mutually uncorrelated.

The trigonometric formulation for the seasonal component is

$$\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{j,t}, \tag{5}$$

where $[s/2] = \begin{cases} s/2, & \text{if } s \text{ is even} \\ (s-1)/2, & \text{if } s \text{ is odd} \end{cases}$ (s is the number of seasonal frequencies in a period), and $\gamma_{j,t}$ is defined as a non-stationary stochastic cycle, for each $j = 1, 2, ..., [s/2]$. That is, it is given by (4) where $\rho^{\psi} = 1$, and the frequency for $\gamma_{j,t}$, in radians, is $\lambda^c \equiv \lambda_j = 2j\pi/s$. As an alternative to the trigonometric form, the seasonality may be formulated using the dummy variable form (see Harvey (1989) for more details).

When we have more than one time series, dynamic interactions usually appear among most (or all) of them and capturing those relationships requires the joint estimation of the multiple time series within a multivariate framework. Multivariate structural time series models are straightforward generalized from the univariate ones as follows: the data, that is now a vector of time series, $\mathbf{y}_t$, decompose as in (2), but considering vector components instead of scalars. The models that are specified for each vectorial component generalize the ones formulated in the univariate case (for instance, models (3), (4), and (5) for the trend, the cycle, and the seasonal components, respectively), replacing the scalar components with vectors. In the particular, for multivariate cycles, the damping factor, ρ^{ψ}, and the cyclical frequency, λ^c, are assumed to take the same value for all the series. This kind of models, called SUTSE (Seemingly Unrelated Time Series Equations), assumes that the disturbances of different components are multivariate normally distributed and mutually uncorrelated in all time periods.

In SUTSE models, the disturbance covariance matrices, in particular their ranks, play an important role to determine the presence of common factors. On the one hand, if the disturbance covariance matrices are of full-rank, then each individual time series of $\mathbf{y}_t$ will have its own components (trend, and/or cycle, and/or seasonality, and/or irregular components), and the interactions among the different time series are reflected as non-zero off-diagonal elements in the covariances matrices of the disturbances. On the other hand, if there is any disturbance covariance matrix with reduced rank, then the component associated to this disturbance term will be common to more than one series. Thus, multivariate structural time series models consider the possibility of dealing with cointegrated time series. The cointegration restrictions, that are interpreted as a lower rank of the disturbance covariance matrix, can be imposed a-priori, but it may also be given by the result of the model estimation. The general multivariate unobserved components model nests more specific models with a restricted number of common components. For instance, the non-stationary dynamic factor models (Peña and Poncela (2006)), where the common factors can be formulated in terms of UC with a useful interpretation.

STAMP solves the signal extraction problem in both cases: general multivariate structural time series models (SUTSE) and multivariate structural time series models with common factors and cointegration. STAMP deals with common factor models writing them in terms of SUTSE models with reduced rank disturbance covariance matrices.

The problem of structural time series models (either univariate or multivariate) the a-priori structure imposed to the components (which makes easier their interpretation) may not be appropriate for the particular series at hand, and wrong specifications could produce serious misleading errors.

2.2.2 Dynamic harmonic regression approach

As in Harvey's structural time series approach, the dynamic harmonic regression approach (Young et al. (1999)) directly specifies unobserved components models for the components within an stochastic state space setting. However, whereas structural time series models formulate the unobserved components models in the time domain (see previous section for more details), the whole process of identification and estimation for the dynamic harmonic regression model is formulated in the frequency domain.

The dynamic harmonic regression model assumes that the univariate time series, y_t, can be decomposed as in (2). According the dynamic harmonic regression approach, these additive unobserved components (trend, cycle, seasonal and irregular components) have a so-called dynamic harmonic representation. That is, each component is defined by a linear combination of sines and cosines with time varying coefficients, which are modelled as generalized random walk (GRW) stochastic processes (Young et al. (1999)). More formally, the general definition of the dynamic harmonic regression components is given by

$$s_t^{p_j} = a_{jt} cos(w_j t) + b_{jt} sin(w_j t) \tag{6}$$

where p_j and $w_j = 1/p_j$ are, respectively, the period and the frequency associated with the jth dynamic harmonic regression component, and $\{a_{jt}, b_{jt}\}$ follow generalized random walk (GRW) processes, that include the random walk (RW), integrated random walk (IRW), and smoothed random walk (SRW) processes as special examples. The trend component corresponds to the zero frequency component, s_t^{∞}, that is described by a GRW process of the form:

$$\begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ 0 & \gamma \end{pmatrix} \begin{pmatrix} \mu_{t-1} \\ \beta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_t \\ \xi_t \end{pmatrix}, \text{ where} \begin{pmatrix} \eta_t \\ \xi_t \end{pmatrix} \sim \text{WN} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\xi^2 \end{pmatrix} \right) \tag{7}$$

where μ_t and β_t are, respectively, the changing level and the slope of the trend component.

The periodic components (cycle, ψ_t, and seasonality, γ_t) are given by

$$\psi_t \equiv \gamma_t = \sum_{j=1}^{R} s_t^{p_j}, \tag{8}$$

where $j = 1, 2, ..., R$ are the associated periodic frequencies and $s_t^{p_j}$ are defined as in (6). The time varying coefficients, $\{a_{jt}, b_{jt}\}$, that define the seasonal component, are usually assumed to be random walk (RW) processes,

$$\begin{aligned} a_{jt} &= a_{jt-1} + \eta_{jt}^a, \text{ where } \eta_{jt}^a \sim N(0, \sigma_{\eta_a}^2), \\ b_{jt} &= b_{jt-1} + \eta_{jt}^b, \text{ where } \eta_{jt}^b \sim N(0, \sigma_{\eta_b}^2). \end{aligned} \tag{9}$$

From the state space formulation of the dynamic harmonic regression model, Young et al. (1999) derive an algorithm that combines the Kalman filter and the fixed interval smoothing to estimate the structural parameters (usually called hyper-parameters) of the unobserved components models. The dynamic harmonic regression algorithm estimates the autoregressive spectrum of the observed time series, and computes the hyper-parameters as the minimum non-linear least squares estimates of the difference between the logarithmic pseudo-spectrum of the dynamic harmonic regression model and the logarithmic autoregressive spectrum of the data (see Young et al. (1999) for more details). The dynamic harmonic regression algorithm is implemented in the CAPTAIN Toolbox for Matlab (see Young and Pedregal (1999), Taylor et al. (2007), among others). An alternative algorithm for the identification and estimation of dynamic harmonic regression models is the linear dynamic harmonic regression (Bujosa et al. (2007)) that simplifies and reduces the computational complexity of the basic dynamic harmonic regression algorithm by using an alternative cost function. The advantages of the linear dynamic harmonic regression algorithm are twofold: first, it eliminates the poles in the objective function of the dynamic harmonic regression algorithm by considering a quadratic cost function (that it is obtained by a linear algebraic transformation, using the ARIMA reduced-form representation of the components). Second, it requires less input information than other existing alternatives. In fact, the linear dynamic harmonic regression only needs the time series data (in a row) and the nature of its periodicity to extract the dynamic harmonic regression components (for a detailed description of the linear dynamic harmonic regression algorithm see Bujosa et al. (2007)).

3 ICA for prediction and signal extraction

In the literature, we can find many applications which use ICA to separate the components of interest in multivariate data sets (see, for example, Bingham (2001), Funaro et al. (2001) Hyvärinen (1999), and Vigàrio et al. (1998), among others). However, ICA has never been applied to extract the basic components in time series data. In this chapter, we explore the performance of ICA for decomposing multivariate time series data in terms of trend, cycle, and seasonal components. Moreover, we present an alternative procedure to forecast multivariate time series data using a small number of independent components (ICs).

3.1 Definition and estimation procedures

Let $\mathbf{x}_t = (x_{1t}, \ldots, x_{mt})'$ be an m-dimensional vector of time series processes. It is assumed that there are some underlying components, $\mathbf{s}_t = (s_{1t}, \ldots, s_{rt})'$, with $r \leq m$, which are statistically independent, that affect approximately linearly to the m observed time series, but with different impact from one series to another. That is:

$$\mathbf{x}_t = \mathbf{A}\mathbf{s}_t, \text{ for } t = 1, \ldots, T \tag{10}$$

where $\mathbf{A}$ is a full rank $m \times r$ matrix whose elements, $\{a_{ij}\}_{i=1,\ldots,m}^{j=1,\ldots,r}$, represent the effect of each independent component (IC), s_{jt}, on the observations, x_{it}. If $\mathbf{A}$ is known, the unobserved components, $\mathbf{s}_t$, can be easily obtained just by (pseudo)inverting the matrix $\mathbf{A}$. If the loading matrix is unknown, the basic idea of ICA is that both, $\mathbf{A}$ and $\mathbf{s}_t$, can be estimated only from the observations, assuming statistical independence of the underlying components. Thus, the solution to the ICA problem computes the estimates of the components as those linear combinations of the data that are maximally independent. That is,

$$\widehat{\mathbf{s}}_t = \mathbf{W}\mathbf{x}_t \tag{11}$$

where $\mathbf{W}$ is an $r \times m$ matrix that maximizes the statistical independence of $\widehat{\mathbf{s}}_t$. To avoid identifiability problems, additional assumptions such as that the ICs have unit variance and that no more than one IC can be Gaussian distributed, should be made (Comon (1994)).

Different approaches such as using higher-order statistics, temporal structure, or mutual information criteria, among others, have been used in the literature to develop algorithms for solving the ICA problem. Although those ICA algorithms are based on different optimization criteria, all of them often begin by a pre-processing step that standardizes the data and transforms them into a set of uncorrelated components. This transformation, that is always possible, is usually performed by PCA. After applying PCA, the dimension of the data may be reduced and the new loading matrix will be orthogonal.

Here, we will use three different ICA algorithms to estimate the underlying components: JADE (Cardoso and Souloumiac (1993)), SOBI (Belouchrani et al. (1997)), and FOTBI (García-Ferrer et al. (2011)). JADE and SOBI are well-known algorithms which obtain the ICs estimates by utilizing information of higher-order statistics and time structure of the data, respectively. On the one hand, JADE looks for the independence of the components by maximizing their non-Gaussianity. Since second-order information is not enough to achieve independence under non-Gaussianity assumption, JADE introduces higher-order statistics in terms of fourth-order cumulants. Cardoso and Souloumiac (1993) define the fourth-order cumulant matrices, whose off-diagonal elements are given by linear combinations of several fourth-order cross-cumulants, and they propose to estimate the ICs by the simultaneous diagonalization of several cumulant matrices. Then, since the independence of a set of variables is achieved when their cross-cumulants, of order higher than two, are equal to zero, it is clear that the JADE ICs will be as independent as possible (see Cardoso and Souloumiac (1993) for more details).

On the other hand, SOBI seeks the solution to the ICA problem exploiting the temporal structure of the data. It can be seen as a de-correlation method (then, it is based on second-order moments) that obtain the ICs as in (11), where the separation matrix, $\mathbf{W}$, is the joint diagonalizer of a set of time-delayed covariance matrices. Thus, the underlying components estimated by SOBI will be instantaneous and temporally uncorrelated, but under non-Gaussianity, they will be not statistically independent (see Belouchrani et al. (1997) a complete explanation of the SOBI algorithm).

The third algorithm, FOTBI (García-Ferrer et al. (2011)), combines both, higher-order information as well as temporal structure, to obtain the ICs estimates. FOTBI can be seen as an extension of JADE that incorporates temporal dependence. FOTBI introduces the time-delayed fourth-order cumulant matrices and proposes to estimate the ICs by the joint diagonalization of some of them. Then, according to that estimation principle, if the data are non-linear and have significant autocorrelation structure (like multivariate time series data are), FOTBI may provide good estimates for the underlying components in the sense that they will be maximally temporally independent under non-Gaussianity assumption.

3.2 Signal extraction with ICA

The ICA model given by (10) is quite realistic for being applied in many practical situations. In particular, the classical problem of time series signal extraction, where the time series data are given by the sum of some basic unobserved components, such as trend, cycle, and seasonality, fits to the ICA model formulation. The motivation for applying ICA to multivariate time series signal extraction is twofold: we are looking for the trend, the cycle, and the seasonal components that should be non-Gaussian and statistically independent

The main advantage of ICA with respect to existing signal extraction procedures is that it is 'automatic' in the sense that it is able to extract the components without assuming any a-priori structure either in the components nor in the loading matrix. ICA identifies the signal components as those linear combinations of the data that are maximally independent. In addition, it requires that each of the components explains the largest amount of variance in the data. Thus, if we apply ICA to extract the basic components in multivariate time series data, the estimates for the trend, cycle and seasonal components will be mutually independent. Then, ICA can be seen as an 'automatic' procedure for time series decomposition where the ICA components do not share common information and each of them represent different features of the data. Throughout this chapter, we will explore the idea of presenting ICA as an automatic method for multivariate time signal extraction.

Previous empirical applications proposed in the ICA literature assume that the ICs are stationary stochastic processes. However, our proposal applies ICA to extract the trend, cycle, and seasonal component in multivariate economic time series and some of the components could be non-stationary. Therefore, we propose applying ICA to perform the separation of possible non-stationary components but, does it make sense to think about non-stationary ICA? This is an open question that we will try to explore next.

One of the first approaches to deal with non-stationary unobserved components was proposed by Peña and Poncela (2006). They present the non-stationary dynamic factor model (DFM) that extends the stationary factor model introduced by Peña and Box (1987) to the non-stationary case. The non-stationary DFM assumes that the dynamic structure of a vector of time series can be explained by a small number of stationary and/or non-stationary latent factors. Peña and Poncela (2006) define the generalized covariance matrices, $\mathbf{C}_{\mathbf{x}}(k)$, that converges to a random matrix which can be diagonalized. Moreover, since ICA can be seen as dynamic factor model (DFM) with non-linear latent factors (see Section 2.2.3), it may have sense to think about non-stationary ICA. That is, ICA could be seen as a dynamic factor model with non-linear ICs that may be non-stationary. In the simulation experiments of the previous chapter (in particular, in the third experiment) we explore how ICA could deal with non-stationary components, and it seems that it performs quite well. However, from a theoretical point of view, non-stationary ICA is an open question that should be studied deeply.

3.3 Forecasting with ICA

In this section, we present the procedure that we will use to forecast multivariate time series data with some components of interest, that are estimated by ICA. This approach was firstly applied by Malaroiu et al. (2000) to forecast financial time series data. The idea is to make the forecasts in the space of the unobserved components, and then transforming back to the observed dataset. The main advantage of this methodology, in comparison to other procedures that also used a small number of factors to forecast large dataset, is that here the components are statistically independent. Then, they can be forecasted separately, fitting different univariate models for each one of them. In the following, we summarize this three-steps procedure:

1. We apply any ICA algorithm to the observations (it is convenient to choose the algorithm which, a-priori, fits better to the features of the data), and we obtain estimates for both the ICs, $\widehat{\mathbf{s}}_t$, and the loading matrix, $\widehat{\mathbf{A}}$.

2. In this step, we make the ICs forecasts. Since the ICs are statistically independent, they can be modelled separately. Then, we fit a univariate ARIMA$(p,d,q) \times (P,D,Q)_s$ model for each $\widehat{s}_{jt}$, for $j = 1, ..., r$,

$$(1 - \phi_1^{(j)} B - \ldots - \phi_p^{(j)} B^p)\Delta^d \Delta_s^D \widehat{s}_{jt} = (1 - \theta_1^{(j)} B - \ldots - \theta_q^{(j)} B^q) a_{jt}, \; t = 1, ..., T. \quad (12)$$

   For each ARIMA model, we estimate the parameters and, according to (12), the h-step-ahead forecasts for each IC are given by,

$$\widehat{s}_{jT}(h) = E[\widehat{s}_{j(T+h)} | \mathbf{I}_T].$$

3. The forecasts of the observed data set, $\widehat{\mathbf{x}}_T(h)$, are obtained by weighting the ICs forecasts, $\widehat{\mathbf{s}}_T(h)$, with the loading matrix. That is, according to model (10),

$$\widehat{\mathbf{x}}_T(h) = \widehat{\mathbf{A}} \widehat{\mathbf{s}}_T(h), \quad (13)$$

   or equivalently,

$$\widehat{x}_{it}(h) = \sum_{j=1}^{r} a_{ij}^2 \widehat{s}_{jt}(h). \quad (14)$$

4 Simulation Study

In this section we present some simulation experiments to illustrate the performance of ICA as an automatic procedure in multivariate time series signal extraction. Since PCA is commonly used to estimate the components of interest in large data set, we will also apply PCA to the simulations in order to compare the performance of the two methodologies. We design four simulation experiments where the components are generated by the two different unobserved components formulations: whereas in two experiments the components are defined according to Harvey's structural model (Harvey (1989)), in the other two, they follow the dynamic harmonic regression specifications (Young et al. (1999)). For each experiment, we generate $R = 1000$ realizations, and the components are generated with three different sample sizes, $T = 150, 300, 500$.

The procedure to design the Monte Carlo experiments is summarized as follows: once the m components are generated, they are mixed according to basic ICA model, given by (10), to obtain the observations, $\mathbf{x}_t$. Then, the unobserved components are estimated using PCA and the three ICA procedures considered in this paper: JADE (Cardoso and Souloumiac (1993)), SOBI (Belouchrani et al. (1997)), and FOTBI(García-Ferrer et al. (2011)). The performance of each procedure is analyzed by computing the correlation coefficient and the MSE between the original and the estimated components.

First, we consider the two Monte Carlo experiments where the components follow the Harvey's structural time series approach. Experiments 1 and 2 are defined in Table 1. After mixing the m component of each experiment (see the loading matrices in Table 10 in the appendix), PCA, JADE, FOTBI, and SOBI, are applied to extract the unobserved components from the observations. Table 2 presents the average results (measured over the m components) for the correlation coefficients and the MSE between the original and the corresponding estimated component by each procedure (Table 11 in the appendix shows the results for each individual component). We see that the results for the two experiments are quite similar: independently of the sample size, PCA has the worst signal extraction performance overall the procedures. It is specially significant the value of the MSE of PCA (around 0.52 and 0.73 in Experiments 1 and 2, respectively) that doubles, and sometimes triples, the values of the MSEs of the ICA procedures. Moving on the ICA procedures, independently of the sample size, FOTBI provides better unobserved components estimates than JADE and SOBI do (see Table 2). The performance of JADE and SOBI depends on T. Whereas SOBI performs better (or quite similar) than JADE for small sample sizes ($T = 150$), when the sample size increases ($T = 300, 500$) JADE estimates the components more accurately than SOBI. This is because higher-order methods (as JADE and FOTBI), in order to reduce the variance associated to their estimates, requires longer data sets than the second-order methods (as SOBI and PCA). Supporting this argument, whereas the values of correlation coefficients and the MSE for SOBI and PCA are quite similar for all T, the performance of JADE and FOTBI improves when the sample size increases (see Table 2).

Next, we focus on the two Monte Carlo experiments where the components of interest are generated as dynamic harmonic regression components. These experiments are defined in Table 3. For the first dynamic harmonic regression experiment (Experiment 3), we generate the four basic components In Experiment 4, we would like to investigate how PCA and ICA procedures separate two periodic components with weekly and monthly periodicity. The loading matrices of each experiment are in the appendix (see Table 10). Table 4 presents the average results for both measures, the correlation coefficients and the MSEs. The conclusions from the dynamic harmonic regression experiments are similar to those obtained from Experiments 1 and 2: PCA and FOTBI have, respectively, the worst and the best performance to extract the components of interest. Comparing Tables 2 and 4, we see that the three ICA procedures provide more accurate estimates for the dynamic harmonic regression components than for the structural time series Harvey's components.

According to the results, any of the three ICA procedures which have been considered here, provides better estimates of the trend, cycle and seasonal components than PCA does. Moreover, within the ICA procedures, we conclude that FOTBI outperforms JADE and SOBI algorithms.

Table 1: Definition of the unobserved components-structural time series components (Harvey (1989)) in the Monte Carlo simulation experiments. The components are defined according to models 3, 5, and 4 for the trend, seasonal, and cyclical components, respectively.

Experiment 1: m=6 monthly time series	
$s_{1t} \sim$ LLT trend	$\sigma^2_\eta = 7.49 \times 10^{-4}$, $\sigma^2_\xi = 2.75 \times 10^{-6}$
$s_{2t} \sim$ seasonal component	s=12 (monthly seasonality), $\rho^\psi = 1$, $\sigma^2_\kappa = 0.0109$
$s_{3t} \sim$ cyclical component	$\lambda^c = \frac{2\pi}{72}$ (7-years cycle), $\rho^\psi = 0.9$, $\sigma^2_\kappa = 0.0278$
$s_{4t} \sim$ AR(1)	$\phi_1 = 0.7$, $n_{4t} \sim t_9$
$s_{5t} \sim$ AR(2)	$\phi_1 = 0.6$, $\phi_2 = -0.2$, $n_{5t} \sim U(0,1)$
$s_{6t} \sim$ irregular component	$s_{6t} \sim t_5$

Experiment 2: m=7 quarterly time series	
$s_{1t} \sim$ RW trend	$\sigma^2_\eta = 0.0515$, $\sigma^2_\xi = 0$
$s_{2t} \sim$ seasonal component	s=4 (quarterly seasonality), $\rho^\psi = 1$, $\sigma^2_\kappa = 0.8$
$s_{3t} \sim I(1)_4$	$s_{3t} = s_{3t-4} + n_{3t}$, $n_{3t} \sim N(0,1)$
$s_{4t} \sim$ cyclical component	$\lambda^c = \frac{2\pi}{16}$ (4-years cycle), $\rho^\psi = 0.75$, $\sigma^2_\kappa = 0.25$
$s_{5t} \sim$ AR(2)	$\phi_1 = 0.5$, $\phi_2 = 0.35$, $n_{5t} \sim t_9$
$s_{6t} \sim$ irregular component	$s_{6t} \sim U(0,1)$
$s_{7t} \sim$ irregular component	$s_{7t} \sim N(0,1)$

(*) s is the number of seasonal frequencies in a period.

Table 2: Unobserved components-Harvey's simulation experiments: comparison of the mean average of the correlation coefficients and the MSE between the original and the estimated components by PCA, JADE, FOTBI, and SOBI, measured over the m components. $\mathrm{Corr}(\cdot) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{R}\sum_{r=1}^{R}\mathrm{Corr}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$
$\mathrm{MSE}(\cdot) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{R}\sum_{r=1}^{R}\mathrm{MSE}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$

Experiment 1

	T=150		T=300		T=500	
	Corr	MSE	Corr	MSE	Corr	MSE
PCA	0.7264	0.5436	0.7407	0.5169	0.7492	0.5006
JADE	0.7798	0.4390	0.8433	0.3128	0.8681	0.2634
FOTBI	**0.8761**	**0.2471**	**0.9231**	**0.1535**	**0.9375**	**0.1248**
SOBI	0.8204	0.3579	0.8241	0.3513	0.8266	0.3465

Experiment 2

	T=150		T=300		T=500	
	Corr	MSE	Corr	MSE	Corr	MSE
PCA	0.6290	0.7371	0.6304	0.7367	0.6308	0.7370
JADE	0.7918	0.4151	0.8458	0.3080	0.8692	0.2612
FOTBI	**0.8537**	**0.2917**	**0.8903**	**0.2190**	**0.9105**	**0.1787**
SOBI	0.7818	0.4349	0.7895	0.4204	0.7970	0.4056

Table 3: Definition of the dynamic harmonic regression components in the Monte Carlo simulation experiments. The components are defined according to models 7 for the trend, and 8 for the periodic components.

Experiment 3: m=4 monthly time series	
$s_{1t} \sim$ SRW trend	$0 < \alpha < 1$ randomly generated, $\beta = \gamma = 1$, $\delta = 0$, $\sigma^2_\xi = 0.0015$
$s_{2t} \sim$ periodic component	p=12 $a_t, b_t \sim$ RW, $\sigma^2_{\eta_a} = 0.01$, $\sigma^2_{\eta_b} = 0.0005$
$s_{3t} \sim$ periodic component	p=60 $a_t, b_t \sim$ RW, $\sigma^2_{\eta_a} = 3$, $\sigma^2_{\eta_b} = 12$
$s_{4t} \sim$ irregular component	$s_{4t} \sim U(0, 1)$

Experiment 4: m=5 daily time series	
$s_{1t} \sim$ IRW trend	$\alpha = \beta = \gamma = 1$, $\delta = 0$, $\sigma^2_\xi = 0.00035$
$s_{2t} \sim$ periodic component	p=7 $a_{jt}, b_{jt} \sim$ RW, $\sigma^2_{\eta_a} = 0.1$, $\sigma^2_{\eta_b} = 0.05$
$s_{3t} \sim$ periodic component	p=30 $a_t, b_t \sim$ RW, $\sigma^2_{\eta_a} = 3$, $\sigma^2_{\eta_b} = 12$
$s_{4t} \sim$ AR(5)	$\phi_1 = 0.2$, $\phi_2 = 0.5$, $\phi_3 = -0.11$, $\phi_4 = 0.01$, $\phi_5 = 0.005$
$s_{5t} \sim$ irregular component	$s_{5t} \sim U(0, 1)$

(*) p denotes the periodicity

These results are as we expected. On the one hand, since PCA estimates the components by maximizing the total variance of the observations, the first PC will increase its percentage of explained variability by mixing the trend and the peaks of seasonality. Then, PCA cannot separate the trend, seasonal, and cyclical components from a vector of time series. On the other hand, since the signals in previous experiments are clearly non-linear and have a significant autocorrelation structure, FOTBI will provide more reliable component estimates than the other two ICA procedures do. In addition, as in previous experiments, the performance of PCA and SOBI does not depend on the sample size, whereas the performance of JADE and FOTBI improves when T increases.

5 Empirical application

In this section we apply the ICA methodology to extract the signal in a set of economic time series. First, we introduce the data and describe the estimates of the components obtained by different ICA algorithms. Then, we evaluate the forecasting performance of those different estimation procedures to predict the industrial production index of each country.

5.1 Data and components estimates

We consider the industrial production indexes (IPI) in four European countries: France, Germany, Spain, and Italy. They represent the four main economies of the Euro Area, and in all of them, the IPI is a highly quality indicator of their industrial activity. The data are monthly

Table 4: Unobserved components-dynamic harmonic regression simulation experiments: comparison of the mean average of the correlation coefficients and the MSE between the original and the estimated components by PCA, JADE, FOTBI, and SOBI, measured over the m components. $\mathrm{Corr}(\cdot) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{R}\sum_{r=1}^{R}\mathrm{Corr}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$ $\mathrm{MSE}(\cdot) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{R}\sum_{r=1}^{R}\mathrm{MSE}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$

Experiment 3

	T=150		T=300		T=500	
	Corr	MSE	Corr	MSE	Corr	MSE
PCA	0.6477	0.7000	0.6591	0.6795	0.6598	0.6791
JADE	0.9299	0.1397	0.9555	0.0889	0.9609	0.0782
FOTBI	**0.9721**	**0.0555**	**0.9859**	**0.0281**	**0.9879**	**0.0242**
SOBI	0.9083	0.1828	0.9139	0.1718	0.9167	0.1663

Experiment 4

	T=150		T=300		T=500	
	Corr	MSE	Corr	MSE	Corr	MSE
PCA	0.7355	0.5254	0.7286	0.5410	0.7231	0.5527
JADE	0.9470	0.1058	0.9661	0.0678	0.9716	0.0568
FOTBI	**0.9573**	**0.0851**	**0.9789**	**0.0422**	**0.9831**	**0.0338**
SOBI	0.8521	0.2948	0.8590	0.2816	0.8612	0.2773

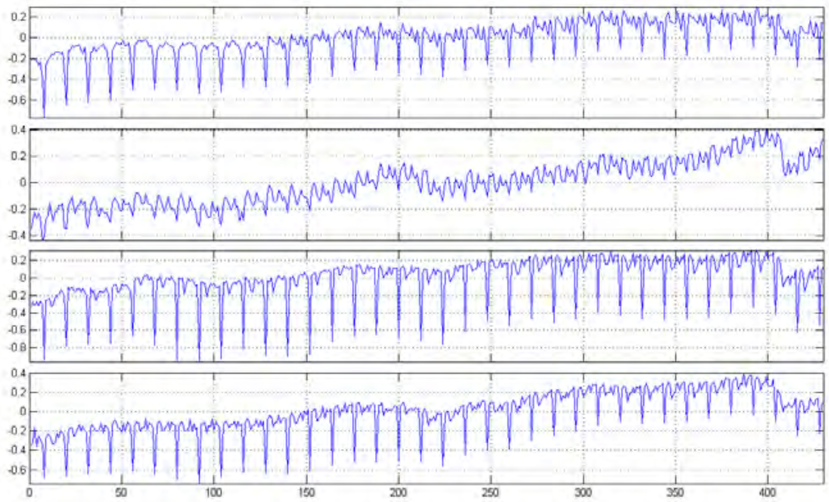


Figure 1: Series of 4 monthly IPI time series from 1975:01 to 2010:10 (France, Germany, Spain, and Italy)

Table 5: Jarque-Bera skewness-kurtosis statistic of the IPI (in logs)

	France	**Germany**	**Italy**	**Spain**
Jarque-Bera	191.3013	7.9463	416.5863	53.9601
p-value	(0.0001)	(0.0233)	(0.0001)	(0.0001)

time series from the period January 1975 to October 2010 (430 monthly observations). Then, we have a 4×430 vector of time series, which is denoted by $\mathbf{y}_t$. We transform the dataset taking logs and subtracting the mean from the observations:

$$\mathbf{x}_t = \log(\mathbf{y}_t) - \overline{\log(\mathbf{y}_t)}.$$

The IPI time series (in logs) are shown in Figure 1. They are clearly non-stationary time series which are characterized by strong trend and seasonality patterns. Our aim is to extract those relevant features and isolate the less interesting ones. For this purpose, we will apply PCA and ICA, which extract the underlying signals directly from the observations, without assuming any a-priori model for the components of interest. Thus, we could compare the PCs and the ICs components estimates.

To motivate the use of ICA in our data, we compute the Jarque-Bera skewness-kurtosis statistics of $\mathbf{x}_t$ to test for normality on each individual series. The results, which are displayed in the Table 5, show that the null hypothesis of normality is rejected at the 1% significance level for each time series. Therefore, since the dataset is non-Gaussian distributed, it is reasonable applying ICA to extract the interesting features from the data.

We apply JADE, SOBI and FOTBI to extract the unobserved signal from the observations. These three ICA procedures decompose the multivariate time series data into a set of approxi-

Table 6: Individual and accumulate percentage of variability explained by the PCs

	$\widehat{s}_{1t}^{PCA}$	$\widehat{s}_{2t}^{PCA}$	$\widehat{s}_{3t}^{PCA}$	$\widehat{s}_{4t}^{PCA}$
% Variability	89.21	8.57	1.26	0.96
% Accumulate Variability	89.21	97.78	99.04	100

mately independent components, but none of them provide a formal criterion to sort the ICs and to identify the more relevant ones. In this empirical application, the interest is to separate the trend (or trend-cycle) and the seasonal component of the IPIs time series. Since these patterns explain most of the variance of the observations and PCA sorts the components in terms of the total explained variability, we will use PCA as an intermediate step in the ICA signal extraction procedures. Thus, our proposal can be summarized by the following steps:

1. Applying PCA to the data and choose the optimal number of PCs, r, that depends on the percentage of the total variance that we would like to be explained. In time series signal extraction, usually no more that two or three components are selected.

2. Applying any ICA algorithm to the data to extract the m ICs.

3. Computing the correlation between the PCs and the ICs, and sorting the ICs according to the maximum correlation criterion. That is, for each $i = 1, ..., m$, the i-th IC satisfies:

$$\max_{1 \leq j \leq m} corr(\widehat{s}_{it}^{PCA}, \widehat{s}_{jt}^{ICA}). \tag{15}$$

   Thus, the first IC will be the component that is maximally correlated to the first PC, the second IC will have maximum correlation to the second PC, and so on. Once the ICs are sorted, we could select the r ICs that provides the estimates for the underlying signals.

Applying previous procedure for our data, $\mathbf{x}_t$, we firstly estimate the four PCs that are sorted in terms of the total explained variability. From Table 6 we have that the two first PCs explain almost the 98% of total variability, so we can fix r equal to two. Second, we estimate the four ICs using JADE, FOTBI, and SOBI. Then, we compute the correlation between the PCs and the different ICA components, and sort them according to the criterion (15). In Table 7 we report the value of the correlation coefficients between the two first PCs and the corresponding ICs. In this particular example, the two SOBI and FOTBI ICs that have been selected, correspond to the two first ICs which were given automatically by those ICA algorithms (for the JADE ICs the order is not preserved). However, this fact cannot be generalized to any empirical application.

The PCs and the ICs that represent the relevant patterns of the IPI time series data are shown on Figure 2. The desirable results would provide estimates for the trend in the first component of interest, and estimates for the seasonal component in the second one. However, as we can see in Figure 2, the results are not very convincing, specially those corresponding to the first component estimates. Just by graphical inspection of the estimated components, it is clear that PCA is not able to separate the trend and the seasonal component. The first PC is

Table 7: Correlation coefficients between the two first PCs and the corresponding ICs which are maximally correlated to each of them.

	$\widehat{s}_{jt}^{JADE}$	$\widehat{s}_{jt}^{FOTBI}$	$\widehat{s}_{jt}^{SOBI}$
$\widehat{s}_{1t}^{PCA}$	0.4690	0.7346	0.7034
$\widehat{s}_{2t}^{PCA}$	0.6774	0.7398	0.5937

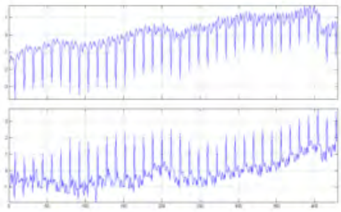
a mixture of the trend and seasonality patterns; the second one is dominated by accentuated seasonality but some evidences of the trend component still remain. According to the PCA optimization criterion, that looks for the components that maximize the total variability, those are the expected results.

Although the results of the three ICA algorithms for our data are quite different, it seems that ICA provide more encouraging results than PCA for signal extraction purposes. The differences among the ICs extracted by JADE, FOTBI, and SOBI are due to the different estimation principle used by each procedure. On the one hand, JADE does not take into account the time structure of the data and it have the worse performance for the IPIs signal extraction: JADE cannot separate the trend and the seasonal patterns, and the two components are mixed in the first JADE IC (see Figure 2(b)). On the other hand, FOTBI and SOBI exploit the autocorrelation structure of the observations, and they would provide, a-priori, better estimates for the trend and the seasonal components than JADE and PCA do. In addition, since economic time series are usually non-Gaussian and the trend and seasonality are non-linear components, FOTBI seems to be more appropriate than SOBI for the IPI time series signal extraction. Figures 2(c)-2(d) confirm this fact: the first FOTBI IC seems to provide the most reliable estimate for the trend component overall the estimated ICs. The second best performance is given by SOBI, where the first SOBI IC still exhibits some evidence of seasonality, although it is less accentuated than in the first component given by JADE and PC. Then, FOTBI can be seen as a first step for an automatic multivariate signal extraction procedure.

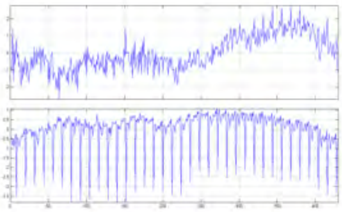
In the following subsection, we will analyze how the IPIs of four European countries can be forecasted using the underlying signals extracted by the previous procedures.

5.2 Forecasting results

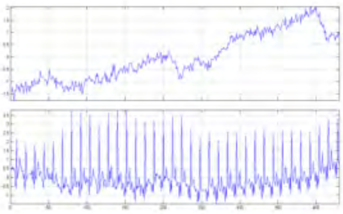
The IPI is usually published with some significant delay, and this fact motivates the interest in providing accurate forecasts. Here, we analyze the forecasting performance of PCA, JADE, FOTBI, and SOBI, to predict the IPI of the four main European countries, using the first two components estimated by each procedure. We use simple univariate ARIMA models for the IPI of each country as benchmark models. We compute the forecasts at different time horizons, $h = 1, 3, 6, 12$ steps ahead. We apply a three-step iterative forecasting procedure: estimating the components of interest using the whole sample (as it is explained in previous section), making forecasts in the space of the components, and transforming back to the original data set.



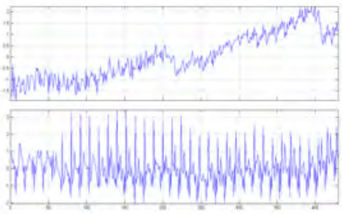
(a) PCs



(b) JADE



(c) FOTBI



(d) SOBI

Figure 2: The two estimated components that have been selected for each procedure. We have the PCs in Figure 2(a), the JADE componentes in Figure 2(b), the FOTBI components in Figure 2(c), and the SOBI ones in Figure 2(d).

Since the ICs are statistically independent, they can be forecasted separately fitting a different model for each IC. Then, to compute the forecasts for the components, we first apply the automatic procedure of TRAMO/SEATS program to fit univariate ARIMA$(p, d, q) \times (P, D, Q)_s$ models to the components (since ICA and PCA are automatic procedures, we decide to use the automatic specification given by the TRAMO/SEATS program). For each component, we estimate the univariate ARIMA model using observations from 1975:01 to 2007:10 (see Table 13 in the appendix for a detailed description of the models), and compute the $h = 1, 3, 6, 12$ steps ahead forecasts. This procedure is repeated following a rolling window approach. That is, after getting the first set of h-steps ahead forecasts ($h = 1, 3, 6, 12$) for each component, the estimation sample is extended by one further observation, the parameters of the corresponding ARIMA model are re-estimated each time (keeping constant the automatic specification for the ARIMA models thought the procedure), and the new $1-, 3-, 6-$, and 12-monthly steps ahead forecasts are built recursively until the end of the sample.

Then, we have computed the forecasts for the components of interest and we will use them to predict the IPIs time series. By (13), the h-steps ahead forecasts for the IPI of each country can be obtained just weighting the univariate forecasts of the components by the corresponding loading matrix coefficients. That is, for the IPIs, a sequence of h-steps ahead forecasts for $h = 1, 3, 6, 12$ is performed by:

$$\widehat{\mathbf{x}}_{t_0}(h) = \widehat{\mathbf{A}}\widehat{\mathbf{s}}_{t_0}(h), \quad h = 1, 3, 6, 12, \; t_0 = 2007:10, \ldots, 2010:10-h,$$

or equivalently, $\widehat{x}_{it_0}(h) = \sum_{j=1}^{2} a_{ij}^2 \widehat{s}_{jt_0}(h)$, for $i = 1, \ldots, 4$.

In order to evaluate the accuracy of each procedure to forecast the IPIs time series, we compare the $h - step - ahead$ prediction error associated to each method, given by:

$$e_{it_0} = x_{it_0+h} - \widehat{x}_{it_0}(h), \quad i = 1, \ldots, 4, \; h = 1, 3, 6, 12, \; t_0 = 2007 : 10, \ldots, 2010 : 10 - h,$$

to the one associated to some benchmark model. Here, the benchmark models will be the univariate ARIMA models fitted to each IPI time series using the automatic TRAMO/SEATS identification procedure. To compute the h-steps ahead forecasts and prediction errors associated to the benchmark models, we apply the same recursive procedure that we used to obtain the forecasts of the components.

To analyze the forecasting performance of PCA and ICA procedures with respect to the benchmark model, we propose to measure the forecasting accuracy of each procedure by the following criteria (see Hyndman and Koehler (2006) for a complete revision of measures of forecast accuracy). For each $i = 1, \ldots, 4$, and $h = 1, 3, 6, 12$,

1. Root Mean Squared Error: $RMSE_{ih} = \sqrt{\sum_{t=1}^{36-h+1} e_{it}^2}$.
2. Mean Absolute Percentage Error: $MAPE_{ih} = \sqrt{\sum_{t=1}^{36-h+1} |p_{it}|}$, where $p_{it} = \frac{e_{it}}{x_{it}}$.
3. Mean Absolute Scale Error: $MASE_{ih} = \sqrt{\sum_{t=1}^{36-h+1} |q_{it}|}$, where $q_{it} = \frac{e_{it}}{\frac{1}{t-1}\sum_{l=2}^{t-1} |x_{il} - x_{il-1}|}$.
4. Geometric Mean Absolute Error: $GMAE_{ih} = geomean(|e_{it}|)$.

We consider the relative values of the four criteria: RelRMSE, RelMAPE, RelMASE, and RelGMAE. That is, we use the ratios of the corresponding criterion for PCA, JADE, FOTBI, and SOBI, with respect to the corresponding one for the benchmark model (the value of each criterion for the univariate ARIMA models):

$$RelRMSE_{(\cdot)} = \frac{RMSE_{(\cdot)}}{RMSE_{benchmark}}; \quad RelMAPE_{(\cdot)} = \frac{MAPE_{(\cdot)}}{MAPE_{benchmark}}$$

$$RelMASE_{(\cdot)} = \frac{MASE_{(\cdot)}}{MASE_{benchmark}}; \quad RelGMAE_{(\cdot)} = \frac{GMAE_{(\cdot)}}{GMAE_{benchmark}}$$

Table 8 shows the average results for the relative criteria, measured overall the IPIs of the four European countries, at different time horizons, $h = 1, 3, 6, 12$. We obtain similar results, independently of the criterion used to evaluate the forecasting performance of the different procedures. The forecasting performance of the PCA and ICA procedures, with respect to the univariate one, depends on the time horizon, h. It is known that the univariate models produce quite accurate short-term forecasts ($h = 1, 3$), but not in the medium- and long-term. This fact is pointed out in our results, where the forecasting performance of the PCA and ICA procedures, relative to the univariate models, improves when h increases (Table 8).

Within the ICA procedures, FOTBI performs better than JADE and SOBI at any time horizon, $h = 1, 3, 6, 12$. However, the forecasting performance of FOTBI in comparison to the univariate ARIMA models (benchmark models) depends on h. In the short-term ($h = 1, 3$), both procedures, FOTBI and univariate models, have similar forecasting performance. They provide more accurate short-term forecasts than PCA, JADE, and SOBI do. In addition, note that, for

Table 8: Relative values of the different criteria for each of the procedures (Univariate=1). The results represent the average values measured over the IPIs of the four main European countries: France, Germany, Italy, and Spain.

		RelRMSE	RelMAPE	RelMASE	RelGMAE
h=1	PCA	1.7982	1.1469	2.0040	1.7410
	JADE	3.7799	1.8392	4.6462	4.5830
	FOTBI	1.0439	**0.9561**	1.0462	**0.9934**
	SOBI	2.6876	2.0061	2.4041	2.5748
	UNIV	**1.0000**	1.0000	**1.0000**	1.0000
h=3	PCA	1.1509	0.9665	1.2660	1.1772
	JADE	2.1590	0.8465	2.5701	2.5803
	FOTBI	**0.9827**	**0.8261**	1.0176	**0.9834**
	SOBI	1.7867	1.3246	1.6353	1.8116
	UNIV	1.0000	1.0000	**1.0000**	1.0000
h=6	PCA	0.9992	0.9034	1.0033	0.9456
	JADE	1.2020	0.7009	1.3301	1.1830
	FOTBI	**0.8817**	**0.5039**	**0.8895**	**0.7982**
	SOBI	1.3271	1.0703	1.1608	1.0937
	UNIV	1.0000	1.0000	1.0000	1.0000
h=12	PCA	1.0924	1.0362	1.0628	1.0191
	JADE	0.7922	0.7209	0.7657	0.6155
	FOTBI	**0.7897**	**0.4502**	**0.7335**	**0.6145**
	SOBI	1.0757	0.9474	0.8759	0.6713
	UNIV	1.0000	1.0000	1.0000	1.0000

$h = 1, 3$, PCA performs better than SOBI, and SOBI performs better than JADE (Table 8). In the medium-, and long- term, the results are slightly different. On the one hand, for $h = 6$, FOTBI has the best forecasting performance followed by PCA and the univariate models, which have similar performance and outperform JADE and SOBI. On the other hand, any of the ICA procedures (although the smallest values of the different criteria correspond to FOTBI) provide more accurate long-term forecasts ($h = 12$) than PCA and the benchmark models do.

The results for each individual IPI time series are provided in the appendix (see Table 14). The conclusions are analogous to the ones explained above for the average results.

According to previous results, our main interest is to compare the forecasting performance of FOTBI and the univariate models. It seems that both procedures have similar forecasting performance in the short-term ($h = 1, 3$), but FOTBI outperforms the univariate models in medium- and long-term forecasting ($h = 6, 12$) (Table 8). However, we would like to investigate whether or not these differences are statistically significant applying the Diebold-Mariano test (Diebold and Mariano (1995)), that is used to compare the forecast accuracy of two competing models. Under the 'equal accuracy' null hypothesis of the Diebold-Mariano test, there are no-differences in the predictive accuracy of the two models. In this paper, we carry out the Diebold-Mariano test taking into account two different, squared and absolute error, loss functions. The outputs of the Diebold-Mariano test applied to the average results given in Table 8 are presented in Table 9. We applied the Diebold-Mariano test to all procedures, two by two, and we report the value of the Diebold-Mariano test statistic, the p-value (between brackets), and the procedure that produces better forecasts in each comparison (= means that the two procedures have equal predictive accuracy).

The results of the Diebold-Mariano test to compare the forecast accuracy of the different procedures for each individual IPI time series are in the appendix (Tables 15 and 16 present the results of the Diebold-Mariano test considering the squared error and the absolute error loss functions, respectively). These results are consistent to the previous ones.

Summarizing the results given by Tables 8 and 9, we cannot conclude that there is a procedure which outperforms the others for any time horizons. However, the FOTBI procedure seems to have quite promising performance: it provides similar forecasts than the univariate models do in the short-term ($h = 1, 3$), the best medium-term forecasts ($h = 6$) overall the procedures, and more accurate 12-steps ahead forecasts than PCA and the univariate models (the other ICA procedures, JADE and SOBI, perform equal than FOTBI in the long-run).

6 Concluding remarks

In this study we have explored how ICA performs for prediction and signal extraction in multiple non-stationary time series data.

ICA assumes that the observations are linearly generated by a set of underlying components which are statistically independent. It has been traditionally used in different areas of research, such as medical, biological, and engineering applications, where the data are observed with high level of noise. ICA is a powerful technique that is able to extract the underlying components only from the observations, and just by making the assumption of statistically independence on the components.

Here we have applied ICA to multivariate time series data in which the underlying components can be interpreted in terms of trends and seasonality patterns. Most of the procedures (e.g. TRAMO/SEATS, STAMP, and linear dynamic harmonic regression) found in the signal extraction literature, are model-based procedures, developed in the univariate case, that specify directly stochastic linear models either on the observations or on the underlying components. Despite that those procedures are, in general, quite successful, modelling the components a-priori could produce specification problems that culminate in crucial estimation errors.

We present ICA as an alternative methodology for multivariate time series signal extraction. The advantage of ICA with respect to the so called model-based signal extraction procedures relies on the fact that ICA is an automatic procedure that does not specify any a-priori struc-

Table 9: Results of the Diebold-Mariano test carried out to evaluate the forecast accuracy (measured as an average over the four IPIs time series) of the different procedures

	Squared Error Loss Function				Absolute Error Loss Function			
MET A vs MET B	h=1	h=3	h=6	h=12	h=1	h=3	h=6	h=12
PCA vs UNIV	2.4316	0.3024	-0.0021	0.0440	2.1510	0.5897	-0.0013	0.1115
	(0.0075)	(0.3812)	(0.4992)	(0.4824)	(0.0157)	(0.2777)	(0.4995)	(0.4556)
	UNIV	=	=	=	**UNIV**	=	=	=
JADE vs UNIV	5.8395	4.0193	1.1808	-2.0155	7.5484	4.2871	1.0567	-1.9712
	(0.0000)	(0.0000)	(0.1188)	(0.0347)	(0.0000)	(0.0000)	(0.1453)	(0.0244)
	UNIV	**UNIV**	=	**JADE**	**UNIV**	**UNIV**	=	**JADE**
FOTBI vs UNIV	-0.0023	-0.0032	-2.1340	-4.4494	-0.0284	-0.0214	-1.9594	-3.9709
	(0.4991)	(0.4987)	(0.0164)	(0.0000)	(0.4887)	(0.4915)	(0.0485)	(0.0000)
	=	=	**FOTBI**	**FOTBI**	=	=	**FOTBI**	**FOTBI**
SOBI vs UNIV	3.1194	2.7339	1.7258	0.4367	4.0058	2.7723	1.1120	-0.9176
	(0.0009)	(0.0031)	(0.0522)	(0.3312)	(0.0000)	(0.0028)	(0.1331)	(0.1794)
	UNIV	**UNIV**	=	=	**UNIV**	**UNIV**	=	=
PCA vs SOBI	-2.2202	-2.4832	-1.8237	0.3216	-2.3833	-2.2104	-1.1275	1.2683
	(0.0132)	(0.0065)	(0.0541)	(0.3739)	(0.0086)	(0.0135)	(0.1298)	(0.1023)
	PCA	**PCA**	=	=	**PCA**	**PCA**	=	=
JADE vs SOBI	2.6451	1.5609	-0.2912	-1.3380	3.1134	1.9802	0.1126	-0.8315
	(0.0041)	(0.0593)	(0.3854)	(0.0904)	(0.0009)	(0.0375)	(0.4552)	(0.2029)
	SOBI	=	=	=	**SOBI**	**SOBI**	=	=
FOTBI vs SOBI	-3.2780	-3.1143	-2.5614	-1.9808	-4.4839	-3.3646	-2.0629	-1.2128
	(0.0005)	(0.0009)	(0.0052)	(0.0238)	(0.0000)	(0.0004)	(0.0196)	(0.1126)
	FOTBI	**FOTBI**	**FOTBI**	**FOTBI**	**FOTBI**	**FOTBI**	**FOTBI**	=
PCA vs FOTBI	2.9578	1.2520	2.2474	4.1526	2.8958	1.3144	1.9822	4.5209
	(0.0015)	(0.1053)	(0.0123)	(0.0000)	(0.0019)	(0.0944)	(0.0299)	(0.0000)
	FOTBI	=	**FOTBI**	**FOTBI**	**FOTBI**	=	**FOTBI**	**FOTBI**
JADE vs FOTBI	5.9747	4.3287	2.7698	0.0540	8.0409	4.6709	2.4767	-0.1718
	(0.0000)	(0.0000)	(0.0028)	(0.4785)	(0.0000)	(0.0000)	(0.0066)	(0.4318)
	FOTBI	**FOTBI**	**FOTBI**	=	**FOTBI**	**FOTBI**	**FOTBI**	=
PCA vs JADE	-6.0001	-4.7388	-1.4072	2.4965	-7.9074	-5.3897	-1.1535	2.4902
	(0.0000)	(0.0000)	(0.0797)	(0.0063)	(0.0000)	(0.0000)	(0.1243)	(0.0064)
	PCA	**PCA**	=	**JADE**	**PCA**	**PCA**	=	**JADE**

ture either in the data nor in the components. ICA looks for the trend, cycle, and seasonal components by assuming only their statistical independence.

As different ICA algorithms provide different components estimates, we have implemented three different ICA algorithms, JADE, FOTBI, and SOBI, to analyze their performance as automatic signal extraction procedures. We have tested the three ICA procedures on four Monte Carlo simulation experiments, and the results show that FOTBI performs quite well. Then, it seems that the FOTBI procedure could be considered as a first-step for an automatic procedure in multivariate time series signal extraction.

We have empirically assess the ability of PCA and the three different ICA procedures to extract the dynamic relationships among the IPIs of the four main European countries. In this analysis, the contribution of the paper are two fold. On the one hand, as it was expected, since these data were non-Gaussian and they had a pronounced autocorrelation structure, FOTBI provided the best estimates for the trend and the seasonal components. On the other hand, we have analyzed the forecasting performance of PCA and ICA, using the univariate ARIMA models for the IPIs as benchmark models. We have computed $h = 1, 3, 6, 12$ steps-ahead forecasts for the IPIs and the results are very promising. When we forecast the IPIs using the FOTBI ICs, we have: (i) short-term forecasts ($h = 1, 3$) given by the FOTBI components are similar to the ones obtained by the univariate models (we know that univariate models perform well in short-term forecasting); (ii) in medium-forecasting ($h = 6$), FOTBI outperforms overall the procedures; and (iii) any of the ICA procures (JADE, FOTBI, and SOBI have equally predictive power according to the Diebold-Mariano test) provide more accurate long-term forecasts of the IPIs ($h = 12$) than the benchmark models does.

Then, FOTBI seems to perform quite well for prediction and signal extraction in multivariate time series data, which may be non-stationary.

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A Appendix

Table 10: Mixing matrices using in the simulation experiments.

	Experiment 1	Experiment 2
A	$\begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}$	$\begin{pmatrix} -4 & 3 & -1 & 1 & -1 \\ 2 & -1 & 1 & 0 & 1 \\ 3 & 1 & -2 & 1 & 0 \\ -1 & -1 & 1 & 1 & 0 \\ -2 & -4 & 3 & 0 & -1 \end{pmatrix}$
	Experiment 3	**Experiment 4**
A	$\begin{pmatrix} 2 & 1 & -1 & 1 & 0 & 0 \\ 3 & 2 & 2 & 1 & 0 & 1 \\ -2 & 1 & -1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 & 0 & 1 \\ 2 & -1 & -1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 & 3 & -2 & 1 & 1 & 0 & -1 \\ -2 & 1 & 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 1 & -1 & 0 & 1 \\ -3 & -2 & 4 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & -2 & 1 & -1 \\ 0 & 1 & 2 & 3 & -1 & 0 & 2 \end{pmatrix}$

Table 11: Unobserved components-Harvey's simulation experiments: comparison of the correlation coefficients and the MSE between the original and the estimated components by PCA, JADE, FOTBI, and SOBI. For each component, these values corresponds to the mean average values measured over the R realizations.
$\mathrm{Corr}(\cdot) = \frac{1}{R}\sum_{r=1}^{R}\mathrm{Corr}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$; $\mathrm{MSE}(\cdot) = \frac{1}{R}\sum_{r=1}^{R}\mathrm{MSE}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$

Experiment 1

		Correlation Coefficient				MSE			
T	$\mathbf{s}_t$	PCA	JADE	FOTBI	SOBI	PCA	JADE	FOTBI	SOBI
T=150	s_{1t}	0.8839	0.8614	0.9504	0.8374	0.2307	0.2762	0.0989	0.3240
	s_{2t}	0.7689	0.8366	0.9161	0.8419	0.4592	0.3257	0.1672	0.3151
	s_{3t}	0.7068	0.7212	0.8295	0.8150	0.5825	0.5558	0.3398	0.3687
	s_{4t}	0.6822	0.7110	0.8245	0.7940	0.6313	0.5760	0.3499	0.4106
	s_{5t}	0.6343	0.7026	0.8301	0.8009	0.7266	0.5928	0.3388	0.3969
	s_{6t}	0.6823	0.8457	0.9058	0.8334	0.6312	0.3077	0.1879	0.3321
T=300	s_{1t}	0.8813	0.9165	0.9627	0.8337	0.2366	0.1667	0.0745	0.3320
	s_{2t}	0.7785	0.8992	0.9769	0.8457	0.4416	0.2013	0.0462	0.3082
	s_{3t}	0.7391	0.7862	0.8743	0.8240	0.5201	0.4270	0.2509	0.3514
	s_{4t}	0.6937	0.7649	0.8708	0.7928	0.6106	0.4695	0.2580	0.4138
	s_{5t}	0.6361	0.7671	0.8988	0.8076	0.7253	0.4651	0.2021	0.3842
	s_{6t}	0.7154	0.9262	0.9554	0.8406	0.5673	0.1473	0.0891	0.3183
T=500	s_{1t}	0.8817	0.9407	0.9651	0.8405	0.2362	0.1185	0.0698	0.3187
	s_{2t}	0.7870	0.9217	0.9887	0.8498	0.4252	0.1565	0.0226	0.3002
	s_{3t}	0.7534	0.8075	0.8895	0.8221	0.4923	0.3847	0.2208	0.3554
	s_{4t}	0.6975	0.7878	0.8907	0.7985	0.6038	0.4241	0.2185	0.4027
	s_{5t}	0.6377	0.7932	0.9215	0.8087	0.7231	0.4131	0.1568	0.3822
	s_{6t}	0.7380	0.9581	0.9698	0.8398	0.5229	0.0837	0.0604	0.3200

Experiment 2

		Correlation Coefficient				MSE			
T	$\mathbf{s}_t$	PCA	JADE	FOTBI	SOBI	PCA	JADE	FOTBI	SOBI
T=150	s_{1t}	0.6670	0.8043	0.9112	0.8517	0.6615	0.3900	0.1770	0.2956
	s_{2t}	0.6651	0.8591	0.9026	0.7820	0.6653	0.2808	0.1941	0.4345
	s_{3t}	0.5047	0.6967	0.7735	0.7301	0.9841	0.6045	0.4514	0.5380
	s_{4t}	0.5431	0.7732	0.8462	0.8329	0.9077	0.4521	0.3066	0.3332
	s_{5t}	0.5718	0.7166	0.8153	0.7786	0.8508	0.5650	0.3682	0.4412
	s_{6t}	0.8290	0.8832	0.8411	0.7317	0.3398	0.2329	0.3168	0.5349
	s_{7t}	0.6222	0.8092	0.8857	0.7658	0.7506	0.3803	0.2279	0.4669
T=300	s_{1t}	0.6545	0.8432	0.9298	0.8502	0.6888	0.3130	0.1401	0.2991
	s_{2t}	0.6607	0.8786	0.9097	0.7889	0.6764	0.2423	0.1802	0.4215
	s_{3t}	0.4950	0.7192	0.7802	0.7410	1.0066	0.5607	0.4390	0.5172
	s_{4t}	0.5381	0.8373	0.8982	0.8320	0.9206	0.3248	0.2032	0.3355
	s_{5t}	0.5802	0.7772	0.8708	0.7852	0.8369	0.4448	0.2579	0.4289
	s_{6t}	0.8419	0.9700	0.9067	0.7498	0.3151	0.0599	0.1862	0.4995
	s_{7t}	0.6424	0.8947	0.9368	0.7792	0.7127	0.2103	0.1262	0.4408
T=500	s_{1t}	0.6448	0.8619	0.9443	0.8570	0.7089	0.2759	0.1113	0.2858
	s_{2t}	0.6513	0.8804	0.9115	0.7994	0.6959	0.2390	0.1768	0.4008
	s_{3t}	0.4896	0.7257	0.7886	0.7368	1.0188	0.5481	0.4224	0.5258
	s_{4t}	0.5353	0.8674	0.9292	0.8418	0.9274	0.2649	0.1414	0.3162
	s_{5t}	0.5878	0.8262	0.9015	0.7955	0.8227	0.3472	0.1968	0.4087
	s_{6t}	0.8572	0.9869	0.9395	0.7602	0.2850	0.0262	0.1210	0.4791
	s_{7t}	0.6493	0.9363	0.9593	0.7884	0.7000	0.1273	0.0814	0.4227

Table 12: Unobserved components-dynamic harmonic regression simulation experiments: comparison of the correlation coefficients and the MSE between the original and the estimated components by PCA, JADE, FOTBI, and SOBI. For each component, these values corresponds to the mean average values measured over the R realizations. $\text{Corr}(\cdot) = \frac{1}{R}\sum_{r=1}^{R} \text{Corr}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$; $\text{MSE}(\cdot) = \frac{1}{R}\sum_{r=1}^{R} \text{MSE}(s_{it}^{(\cdot)}, \widehat{s_{it}^{(\cdot)}})$

Experiment 3

T	$\mathbf{s}_t$	Correlation Coefficient PCA	JADE	FOTBI	SOBI	MSE PCA	JADE	FOTBI	SOBI
T=150	s_{1t}	0.7129	0.9250	0.9602	0.9061	0.5703	0.1496	0.0794	0.1871
	s_{2t}	0.7235	0.9362	0.9807	0.9285	0.5493	0.1271	0.0385	0.1425
	s_{3t}	0.5923	0.9586	0.9766	0.8842	0.8100	0.0825	0.0467	0.2309
	s_{4t}	0.5620	0.8998	0.9711	0.9144	0.8702	0.1997	0.0576	0.1705
T=300	s_{1t}	0.6998	0.9517	0.9777	0.9112	0.5983	0.0965	0.0446	0.1774
	s_{2t}	0.7044	0.9558	0.9938	0.9289	0.5893	0.0882	0.0125	0.1420
	s_{3t}	0.6252	0.9768	0.9847	0.8970	0.7470	0.0464	0.0306	0.2056
	s_{4t}	0.6070	0.9375	0.9876	0.9187	0.7834	0.1247	0.0248	0.1623
T=500	s_{1t}	0.7065	0.9583	0.9807	0.9104	0.5859	0.0833	0.0385	0.1789
	s_{2t}	0.6711	0.9552	0.9937	0.9282	0.6565	0.0894	0.0127	0.1435
	s_{3t}	0.6547	0.9815	0.9854	0.9067	0.6891	0.0369	0.0292	0.1864
	s_{4t}	0.6067	0.9484	0.9917	0.9216	0.7850	0.1030	0.0165	0.1566

Experiment 4

T	$\mathbf{s}_t$	Correlation Coefficient PCA	JADE	FOTBI	SOBI	MSE PCA	JADE	FOTBI	SOBI
T=150	s_{1t}	0.7848	0.9568	0.9814	0.8814	0.4276	0.0861	0.0370	0.2365
	s_{2t}	0.6322	0.9423	0.9704	0.8691	0.7307	0.1151	0.0590	0.2609
	s_{3t}	0.5904	0.9786	0.9854	0.8457	0.8138	0.0426	0.0292	0.3075
	s_{4t}	0.9696	0.8818	0.9176	0.8317	0.0603	0.2356	0.1643	0.3356
	s_{5t}	0.7007	0.9753	0.9318	0.8327	0.5945	0.0493	0.1359	0.3335
T=300	s_{1t}	0.7656	0.9666	0.9862	0.8814	0.4672	0.0667	0.0276	0.2369
	s_{2t}	0.6173	0.9576	0.9873	0.8714	0.7629	0.0847	0.0254	0.2567
	s_{3t}	0.5919	0.9816	0.9899	0.8557	0.8134	0.0367	0.0202	0.2881
	s_{4t}	0.9810	0.9308	0.9554	0.8390	0.0378	0.1382	0.0890	0.3214
	s_{5t}	0.6871	0.9937	0.9756	0.8473	0.6236	0.0126	0.0486	0.3050
T=500	s_{1t}	0.7634	0.9680	0.9857	0.8746	0.4723	0.0639	0.0285	0.2505
	s_{2t}	0.6048	0.9605	0.9843	0.8701	0.7889	0.0790	0.0314	0.2596
	s_{3t}	0.5936	0.9820	0.9881	0.8710	0.8111	0.0359	0.0237	0.2578
	s_{4t}	0.9865	0.9504	0.9705	0.8391	0.0270	0.0991	0.0590	0.3214
	s_{5t}	0.6671	0.9969	0.9867	0.8513	0.6645	0.0062	0.0265	0.2971

Table 13: Univariate models for the different components and for the IPI time series data. The optimal specification for the $ARIMA(p,d,q) \times (P,D,Q)_s$ models as well as the estimation of the parameters are carried out by the TRAMO/SEATS automatic identification procedure.

	1^{st} component	2^{nd} component
PCA	ARIMA(3,1,1)$\times(0,1,1)_{12}$	ARIMA(0,1,1)$\times(1,1,1)_{12}$
	$(1-\underset{(0.07)}{0.20}\,B^2-\underset{(0.05)}{0.23}\,B^3)\Delta\Delta_{12}s_{1t}^{PCA}=(1+\underset{(0.11)}{0.77}\,B)(1+\underset{(0.05)}{0.41}\,B^{12})a_{1t}^{PCA}$	$(1-\underset{(0.04)}{0.41}\,B^{12})\Delta\Delta_{12}s_{2t}^{PCA}=(1+\underset{(0.05)}{0.67}\,B)(1+\underset{(0.00)}{0.89}\,B^{12})a_{2t}^{PCA}$
JADE	ARIMA(0,1,1)$\times(0,1,1)_{12}$	ARIMA(0,1,2)$\times(0,1,1)_{12}$
	$\Delta\Delta_{12}s_{1t}^{JADE}=(1+\underset{(0.04)}{0.69}\,B)(1+\underset{(0.04)}{0.50}\,B^{12})a_{1t}^{JADE}$	$\Delta\Delta_{12}s_{2t}^{JADE}=(1+\underset{(0.05)}{0.84}\,B-\underset{(0.05)}{0.18}\,B^2)(1+\underset{(0.03)}{0.73}\,B^{12})a_{2t}^{JADE}$
FOTBI	ARIMA(0,1,1)$\times(1,0,0)_{12}$	ARIMA(0,1,1)$\times(1,1,1)_{12}$
	$(1+\underset{(0.04)}{0.58}\,B^{12})\Delta s_{1t}^{FOTBI}=(1-\underset{(0.05)}{0.16}\,B)a_{1t}^{FOTBI}$	$(1+\underset{(0.03)}{0.81}\,B^{12})\Delta\Delta_{12}s_{2t}^{FOTBI}=(1+\underset{(0.03)}{0.75}\,B)(1-\underset{(0.03)}{0.85}\,B^{12})a_{2t}^{FOTBI}$
SOBI	ARIMA(0,1,1)$\times(0,1,1)_{12}$	ARIMA(3,0,1)$\times(0,1,1)_{12}$
	$\Delta\Delta_{12}s_{1t}^{SOBI}=(1+\underset{(0.04)}{0.66}\,B)(1+\underset{(0.03)}{0.70}\,B^{12})a_{1t}^{SOBI}$	$(1-\underset{(0.16)}{0.36}\,B-\underset{(0.06)}{0.24}\,B^3)\Delta_{12}s_{2t}^{SOBI}=(1+\underset{(0.15)}{0.31}\,B)(1+\underset{(0.05)}{0.27}\,B^{12})a_{2t}^{SOBI}$
	Observed time series data (IPI)	
FRA	ARIMA(3,1,1)$\times(0,1,1)_{12}$	
	$(1-\underset{(0.06)}{0.16}\,B^2-\underset{(0.05)}{0.44}\,B^3)\Delta\Delta_{12}x_{1t}=(1+\underset{(0.04)}{0.89}\,B)(1+\underset{(0.05)}{0.49}\,B^{12})a_{1t}$	
GER	ARIMA(2,1,0)$\times(0,1,1)_{12}$	
	$(1+\underset{(0.05)}{0.54}\,B+\underset{(0.05)}{0.21}\,B^2)\Delta\Delta_{12}x_{2t}=(1+\underset{(0.04)}{0.59}\,B^{12})a_{2t}$	
ITA	ARIMA(0,1,1)$\times(0,1,1)_{12}$	
	$\Delta\Delta_{12}x_{3t}=(1+\underset{(0.04)}{0.60}\,B)(1+\underset{(0.05)}{0.41}\,B^{12})a_{3t}$	
SPA	ARIMA(0,1,1)$\times(0,1,1)_{12}$	
	$\Delta\Delta_{12}x_{4t}=(1+\underset{(0.04)}{0.58}\,B)(1+\underset{(0.04)}{0.51}\,B^{12})a_{4t}$	

The standard deviations for the estimates of the parameters are given between brackets

Table 14: Ratios of the four different evaluation criteria with respect to the benchmark (univariate ARIMA) model, *RelRMSE*, *RelMAPE*, *RelMASE*, and *RelGMAE*, measured over the IPI of each country

		RelRMSE				RelMAPE				RelMASE				RelGMAE			
		FRA	GER	ITA	SPA	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA
h=1	PCA	1.5296	2.6222	1.6941	1.4963	1.4479	1.7489	1.3800	1.2992	1.4345	3.0073	1.6438	1.4564	1.3506	3.4082	1.6978	1.0025
	JADE	2.7092	6.1680	3.2358	3.5241	1.6819	3.7333	2.1143	1.5590	2.7691	7.3742	3.3784	3.8814	2.7822	8.6476	3.5968	5.2167
	FOTBI	1.0786	1.1645	0.8055	1.1573	1.1830	1.1200	0.6876	0.8337	0.9438	1.2282	0.7864	1.1041	0.6978	1.3043	0.8182	1.4128
	SOBI	2.4338	1.8017	3.3811	3.0215	2.5811	1.6473	2.5078	1.5994	2.0687	2.0049	3.1210	2.9926	1.5812	2.3487	3.2827	3.3993
	UNIV	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
h=3	PCA	1.1682	1.3892	1.0792	0.9580	0.8442	1.0211	0.7301	1.1179	1.1112	1.5797	1.1045	0.9760	1.0292	1.8151	1.0473	0.9922
	JADE	1.7996	3.0036	1.8179	2.0294	0.8827	0.9970	0.8673	0.8054	1.7386	3.5878	1.9270	2.1996	1.6782	4.6292	2.3344	2.3937
	FOTBI	0.9550	0.9844	0.7654	1.2676	0.9064	1.8008	1.1037	0.5474	0.8903	1.0831	0.7485	1.2965	0.8074	1.2739	0.6918	1.3127
	SOBI	1.8172	1.1094	2.1030	2.1164	1.5383	1.0767	1.4430	1.2197	1.5605	1.2532	2.0290	2.1898	1.3391	1.4202	2.3602	2.1913
	UNIV	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
h=6	PCA	0.9198	0.9790	0.8908	1.2431	0.6400	0.9605	0.7078	1.1772	0.8954	0.9930	0.8582	1.3006	0.7870	0.8968	0.7138	1.4837
	JADE	1.0099	1.6256	1.0595	1.0989	0.5990	0.7166	0.6779	0.7695	0.9672	1.7407	1.0692	1.0867	0.8688	1.8461	0.9966	1.0125
	FOTBI	0.8589	0.9085	0.8467	0.9183	0.4419	0.9693	0.5747	0.4074	0.8291	0.9208	0.8640	0.9204	0.5952	0.8258	0.8814	0.9111
	SOBI	1.2670	0.8618	1.5423	1.6752	0.9781	0.8641	1.0054	1.2015	1.1397	0.8749	1.4205	1.6343	0.9689	0.8446	1.1621	1.4570
	UNIV	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
h=12	PCA	0.9914	1.0430	1.0619	1.2963	0.8345	0.9808	1.1730	1.0675	0.9700	1.0204	1.0326	1.3220	0.9059	0.8889	1.0366	1.3087
	JADE	0.7476	0.7625	0.7792	0.8913	0.4868	0.6830	0.6939	0.8814	0.6537	0.8633	0.6682	0.7441	0.5314	0.6877	0.5253	0.7312
	FOTBI	0.7125	0.9517	0.7178	0.7631	0.3713	0.5652	0.4875	0.4489	0.6580	0.7164	0.7446	0.8675	0.4348	0.5847	0.7028	0.7587
	SOBI	0.9887	0.8038	1.1965	1.3538	0.7165	0.7960	0.9448	1.1097	0.8402	0.7234	0.9990	1.2127	0.6338	0.5398	0.7030	0.8543
	UNIV	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 15: Results of the Diebold-Mariano test considering the squared error loss function (for each IPI time series)

	h=1				h=3				h=6				h=12			
Met A vs Met B	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA
PCA vs UNIV	2.3497	4.0582	2.7108	3.0295	1.1791	1.9856	0.5329	-0.5160	-0.9598	-0.3329	-1.2318	3.9932	-0.1251	0.6337	0.7034	3.4339
	0.0094	0.0000	0.0034	0.0012	0.1192	0.0297	0.2971	0.3029	0.1686	0.3696	0.1090	0.0000	0.4502	0.2631	0.2409	0.0003
	UNIV	UNIV	UNIV	UNIV	=	UNIV	=	=	=	=	=	UNIV	=	=	=	UNIV
JADE vs UNIV	4.9888	6.0409	5.5669	5.8904	3.3830	4.6911	3.4075	3.6509	0.0703	2.6840	0.3728	0.5615	-2.5792	-0.3302	-2.6525	-2.0162
	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000	0.0003	0.0001	0.4720	0.0036	0.3547	0.2872	0.0050	0.3706	0.0040	0.0431
	UNIV	UNIV	UNIV	UNIV	UNIV	UNIV	UNIV	UNIV	=	UNIV	=	=	JADE	=	JADE	JADE
FOTBI vs UNIV	0.5283	1.9763	-2.0656	1.1201	-0.4561	-0.2305	-2.1041	2.7656	-1.3319	-0.6652	-1.6072	-1.5766	-2.5218	-2.8283	-3.7047	-3.4430
	0.2986	0.0478	0.0428	0.1313	0.3242	0.4088	0.0177	0.0028	0.0914	0.2529	0.0540	0.0574	0.0058	0.0023	0.0001	0.0003
	=	UNIV	FOTBI	=	=	=	FOTBI	UNIV	=	=	=	=	FOTBI	FOTBI	FOTBI	FOTBI
SOBI vs UNIV	2.7632	2.9532	3.1009	4.1265	2.4961	0.8862	2.7992	4.1411	1.3025	-1.9860	2.2890	3.6752	-0.0684	-4.7883	1.1814	2.4961
	0.0029	0.0016	0.0010	0.0000	0.0063	0.1878	0.0026	0.0000	0.0964	0.0459	0.0110	0.0001	0.4727	0.0000	0.1187	0.0063
	UNIV	UNIV	UNIV	UNIV	UNIV	=	UNIV	UNIV	=	SOBI	UNIV	UNIV	=	SOBI	=	UNIV
PCA vs SOBI	-2.2734	4.0572	-2.4816	-3.3368	-2.4146	2.4288	-2.6180	-4.2146	-1.9606	1.6346	-2.5658	-2.4014	0.0199	3.0383	-0.6505	-0.3403
	0.0115	0.0000	0.0065	0.0004	0.0079	0.0076	0.0044	0.0000	0.0250	0.0511	0.0051	0.0082	0.4921	0.0012	0.2577	0.3668
	PCA	PCA	PCA	PCA	PCA	SOBI	PCA	PCA	PCA	=	PCA	PCA	=	SOBI	=	=
JADE vs SOBI	0.5795	6.1554	-0.2352	1.0548	-0.0509	5.0861	-0.7549	-0.2679	-1.0950	3.7811	-1.9718	-2.2744	-1.4222	1.0027	-1.9706	-2.6061
	0.2811	0.0000	0.4070	0.1458	0.4797	0.0000	0.2251	0.3944	0.1368	0.0001	0.0382	0.0115	0.0775	0.1580	0.0244	0.0046
	=	SOBI	=	=	=	SOBI	=	=	=	SOBI	JADE	JADE	=	=	JADE	JADE
FOTBI vs SOBI	-2.8171	-2.8627	-3.2383	-3.9357	-2.7606	-1.2698	-3.2179	-3.2692	-2.4519	0.5558	-2.7485	-3.9552	-2.0161	-0.5534	-2.1555	-3.0296
	0.0024	0.0021	0.0006	0.0000	0.0029	0.1021	0.0006	0.0005	0.0071	0.2892	0.0030	0.0000	0.0277	0.2900	0.0156	0.0012
	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	=	FOTBI	FOTBI	FOTBI	=	FOTBI	FOTBI	FOTBI	=	FOTBI	FOTBI
PCA vs FOTBI	2.5796	4.1188	3.4147	1.9996	2.0113	2.3335	2.2074	-2.1988	1.1261	0.7519	0.4969	3.7811	3.7934	4.6473	2.8276	4.5043
	0.0049	0.0000	0.0003	0.0469	0.0221	0.0098	0.0136	0.0139	0.1301	0.2261	0.3096	0.0001	0.0001	0.0000	0.0023	0.0000
	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	PCA	=	=	=	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI
JADE vs FOTBI	4.6019	6.0225	6.0089	5.6832	3.3801	4.7980	4.4571	3.0783	1.1519	4.3926	2.0532	1.1216	-0.3513	2.0363	-0.7183	-1.0066
	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000	0.0000	0.0010	0.1247	0.0000	0.0254	0.1310	0.3627	0.0413	0.2363	0.1571
	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	=	FOTBI	FOTBI	=	=	FOTBI	=	=
PCA vs JADE	-4.0967	-6.3429	-5.3449	-5.7935	-3.0023	-5.3856	-4.3005	-3.9148	-0.6330	-2.9818	-1.2330	0.9099	2.4825	0.6789	2.8513	3.7156
	0.0000	0.0000	0.0000	0.0000	0.0013	0.0000	0.0000	0.0000	0.2634	0.0014	0.1088	0.1815	0.0065	0.2486	0.0022	0.0001
	PCA	PCA	PCA	PCA	PCA	PCA	PCA	PCA	=	PCA	=	=	JADE	=	JADE	JADE

Table 16: Results of the Diebold-Mariano test considering the absolute error loss function (for each IPI time series)

	h=1				h=3				h=6				h=12			
Met A vs Met B	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA	FRA	GER	ITA	SPA
PCA vs UNIV	2.1373	5.3360	2.6293	2.7013	0.7404	2.3572	0.6419	-0.2934	-1.0582	-0.0942	-1.4252	3.3397	-0.4278	0.2729	0.3815	3.5522
	0.0163	0.0000	0.0043	0.0035	0.2295	0.0092	0.2605	0.3846	0.1450	0.4625	0.0771	0.0004	0.3344	0.3925	0.3514	0.0002
	UNIV	UNIV	UNIV	UNIV	=	UNIV	=	=	=	=	=	UNIV	=	=	=	UNIV
JADE vs UNIV	5.5998	8.2868	5.9620	7.6683	3.0917	5.7584	3.2256	3.8976	-0.1905	2.8435	0.3358	0.4073	-2.8070	-0.8230	-2.6861	-1.6274
	0.0000	0.0000	0.0000	0.0000	0.0010	0.0000	0.0006	0.0000	0.4244	0.0022	0.3685	0.3419	0.0025	0.2053	0.0036	0.0518
	UNIV	UNIV	UNIV	UNIV	UNIV	UNIV	UNIV	UNIV	=	UNIV	=	=	JADE	=	JADE	=
FOTBI vs UNIV	-0.3956	1.8663	-1.6333	0.8101	-1.0205	0.9736	-2.0890	2.2913	-1.4041	-0.5308	-1.2025	-1.4893	-3.1589	-3.2807	-3.6754	-3.0610
	0.3462	0.0310	0.0512	0.2089	0.1537	0.1651	0.0184	0.0110	0.0801	0.2978	0.1146	0.0682	0.0008	0.0005	0.0001	0.0011
	=	UNIV	=	=	=	=	FOTBI	UNIV	=	=	=	=	FOTBI	FOTBI	FOTBI	FOTBI
SOBI vs UNIV	3.0770	3.9510	4.5951	5.6577	2.2336	1.5892	3.4326	4.8873	0.6714	-1.2569	1.8914	3.5057	-1.0163	-5.4505	-0.0072	1.5122
	0.0010	0.0000	0.0000	0.0000	0.0128	0.0560	0.0003	0.0000	0.2510	0.1044	0.0293	0.0002	0.1547	0.0000	0.4971	0.0652
	UNIV	UNIV	UNIV	UNIV	UNIV	=	UNIV	UNIV	=	=	UNIV	UNIV	=	SOBI	=	=
PCA vs SOBI	-2.2328	3.8989	-2.9377	-3.8455	-2.2240	2.1032	-2.9232	-4.6030	-1.4307	1.1933	-2.4611	-1.5995	0.9528	3.5269	0.1780	0.5575
	0.0128	0.0000	0.0017	0.0001	0.0131	0.0177	0.0017	0.0000	0.0763	0.1164	0.0069	0.0549	0.1703	0.0002	0.4294	0.2886
	PCA	SOBI	PCA	PCA	PCA	SOBI	FOTBI	FOTBI	=	=	PCA	=	=	SOBI	=	=
JADE vs SOBI	1.3624	7.9452	0.4053	1.5365	0.4894	6.2437	-0.2497	0.0233	-0.6621	3.9422	-1.2044	-1.8415	-0.9909	0.8027	-1.4930	-1.9846
	0.0865	0.0000	0.3426	0.0622	0.3123	0.0000	0.4014	0.4907	0.2539	0.0000	0.1142	0.0328	0.1609	0.2111	0.0677	0.0236
	=	SOBI	=	=	=	SOBI	=	=	=	SOBI	=	JADE	=	=	=	JADE
FOTBI vs SOBI	-3.6466	-3.7743	-5.2761	-4.9129	-3.0846	-1.3158	-4.5742	-3.2987	-2.0922	0.4087	-2.4859	-3.6825	-1.6137	-0.0875	-1.6304	-2.2050
	0.0001	0.0001	0.0000	0.0000	0.0010	0.0941	0.0000	0.0005	0.0182	0.3414	0.0065	0.0001	0.0533	0.4651	0.0515	0.0137
	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	=	FOTBI	FOTBI	FOTBI	=	FOTBI	FOTBI	=	=	=	FOTBI
PCA vs FOTBI	2.9152	5.5287	3.6604	1.9812	1.9701	2.4154	2.3030	-1.9289	0.9025	0.6548	-0.0550	3.7394	4.6161	4.6611	3.1174	4.5286
	0.0018	0.0000	0.0001	0.0493	0.0244	0.0079	0.0106	0.0269	0.1834	0.2563	0.4781	0.0001	0.0000	0.0000	0.0009	0.0000
	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	PCA	=	=	=	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI
JADE vs FOTBI	5.3447	8.1754	7.1663	7.2710	3.3415	5.7275	4.5488	3.0097	0.8434	5.0193	1.3695	0.8491	-0.0351	1.1287	-0.7052	-0.8387
	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000	0.0000	0.0013	0.1995	0.0000	0.0854	0.1979	0.4860	0.1295	0.2403	0.2008
	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	FOTBI	=	FOTBI	=	=	=	=	=	=
PCA vs JADE	-4.4548	-9.0368	-5.2800	-6.8152	-2.8985	-7.3121	-3.9471	-4.1657	-0.4043	-3.0683	-1.1390	1.1032	2.4334	0.9492	2.8769	3.3237
	0.0000	0.0000	0.0000	0.0000	0.0019	0.0000	0.0000	0.0000	0.3430	0.0011	0.1273	0.1350	0.0075	0.1712	0.0020	0.0004
	PCA	PCA	PCA	PCA	PCA	PCA	PCA	PCA	=	PCA	=	=	JADE	=	JADE	JADE