

Venue

FILTECH 2016 will be held again at the Koelnmesse in **Cologne**, Germany. Welcome to Cologne where the cathedral spires tower over Germany's oldest city. Take a trip through 2000 years of history and visit cultural monuments from the Roman Empire to modern times.

FILTECH Travel Discounts

Save up to 20% on Travel with Star Alliance™ Members Airlines.

Travel to FILTECH 2016 by Deutsche Bahn from 99 \in with 100% green Power.







Scientific Committee Chairmen

Dr.-Ing. Harald Anlauf

Karlsruhe Institute of Technology – Institute for Mechanical Process Engineering and Applied Mechanics, Germany

Prof. Eberhard Schmidt University of Wuppertal – Institute of Particle Technology, Germany Join the largest Filtration Event world-wide

FILTECH

October 11 – 13, 2016 Cologne – Germany

The Filtration Event

www.Filtech.de

Organiser: Filtech Exhibitions Germany, PO Box 1225, 40637 Meerbusch - Germany

phone: +49 (0)2132 93 57 60, fax: +49 (0)2132 93 57 62, e-mail: info@filtech.de

FILTECH

October 11 – 13, 2016 Cologne – Germany

The Filtration Event

NEW VENUE

The Filtration E

www.Filtech.de

5

5

FILTECH 2016 · Conference Programme

Tuesday, October 11, 2016

08:30-10:15 Registration

10:15 - 10:45 Opening Session



Gas cleaning with pulse-jet filters:

Emission sources and dust abatement strategies

Prof. Gerhard Kasper, Karlsruhe Institute of Technology, Germany

13:00	room
14:15	1A
	13:00 14:15

Long-term stability and energy efficiency of filtration solutions Dr. Christof Asbach, IUTA Germany

Cake Filtration I – Characterization of Filterability Session Chair: Hans Théliander

Cake forming filtration - A theory-based approach for correct characterization of the suspension filtrability, I. Nicolaou*, NIKIFOS Ltd., Cyprus

Properties of microbial cell cake under high pressure conditions, N. Katagiri*, Y. Kuwajima, H. Kawahara, E. Iritani, Nagoya University, Japan

Improvement of filtration properties of mine tailings by grinding with a stirred media mill, T. Kinnarinen*, R. Tuunila, A. Häkkinen, Lappeenranta University of Technology, Finland



Magnetophoretic velocity determined by space and time-resolved extinction profiles, D. Lerche*, T. Detloff, D. Krause, M. Wolff, LUM GmbH, O. Mykhaylyk, D. Vlaskou, V. Schoemig, T. Jonas, S. Berensmeier, C. Plank, Technical University Munich, Germany

Characterization of solid-liquid separation of flocculated mineral slurries by NIR-transmission and x-ray concentration profiling, T. Sobisch^{*}, D. Kavianpour, D. Lerche, LUM GmbH, Germany

Experimental evaluation of effect of continous phase rheology on sedimentation behaviour, T. Sobisch*, D. Lerche, LUM GmbH, Germany

C1	Surface Filtration	13:00 room
	Session Chair: Gernot Krammer	14:15 4A

Baghouse filtration: Correlation between pulse intensity, filter movement and regeneration by highly time resolved measurements, S. Sobich*, J. Meyer, G. Kasper, Karlsruhe Institute of Technology (KIT), Germany

Influence of an added fraction of hygroscopic salt particles on the operating behavior of surface filters for dust separation, Q. Zhang, University of Wuppertal, Germany

Economic use of cleanable filter, S. Holfeld*, R. Heidenreich, Institute of Air Handling and Refrigeration (ILK), Germany

<u>C</u> 2	Flue Gas Cleaning	13:00 room
UZ	Session Chair: Jörg Meyer	14:15 4B

Regenerative filtering separators for small wood combustions, T. Laminger*, L. Kromer, S. Kolmanz, G. Mauschitz, Vienna University of Technology, Austria

Development of a filtration system for the combined separation of ultrafine dust particles and gaseous pollutants from biomass combustion processes, F. Prill*, S. Schiller, H.J. Schmid, University of Paderborn, Germany

TopFrax[™] Catalytic ceramic filters: Simultaneous fine dust separation and NOx, CO and VOCs emission abatement, A.Z. Friedberg, F. Castellino, J. Skotte*, Haldor Topsoe A/S, Denmark

VJ	Keynote Lecture 2	14:45 room
NZ	Session Chair: Harald Anlauf	16:00 IA

Role of solid/liquid separation technology in circular economy Prof. Anti Häkkinen, Lappeenranta University, Finland



Cake Filtration II – Washing and Multicomponent Separation Chair: Urs Peuker

Displacement washing of porous particles, S. Heinisch*, U.A. Peuker, Technical University Bergakademie Freiberg, Germany

Washing with immiscible liquids - Equilibrium, limitations and opportunities, M. Burisch, U.A. Peuker*, Technical University Bergakademie Freiberg, Germany

Cake filtration of multi-component-suspensions, S. Kühne*, U.A. Peuker, Technical University Bergakademie Freiberg, Germany



Improved decanter operations by reduced particle fragmentation, M. Kuhn*, M. Reichenbach, S. Weis, A. Siebelitz, Andritz Separation, Germany; J. Grossalber, Andritz AG, Austria



Development of a dynamic process model for the mechanical fluid separation in decanter centrifuges, M. Glei β^* , H. Nirschl, Karlsruhe Institute of Technology (KIT), Germany

An efficient numerical approach for transient simulation of multiphase flow behavior in centrifuges, S. Hammerich*, H Nirschl, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany

FILTECH 2016 · October 11–13 · Cologne · Germany

FILTECH 2016 October 11-13, 2016



Nanofiltration and Reverse Osmosis 13:00 Session Chair: Siegfried Ripperger 13:00

Decentralized production of potable water based on membrane technology, T. Peters, Membrane Consulting, Germany

Inhibitors for CaCO₃-scaling in reverse osmosis plants - Influence of suspended matter on membrane clogging, W. Hater, K. Zimmer, J. Schallenberg, Kurita Europe GmbH; J. Jaworski, N. Kruse, G. Braun*, Cologne University of Applied Science, Germany

Production integrated filtration for resource recovery from rinsing waters - A progress towards environmental sustainability, L. Q. T. Nguyen*, M. Engelhart, M. Wagner, P. Cornel, Technical University Darmstadt; Germany; Linclau, P&G, Belgium

Session Chair: Ioannis Nicolaou	1 <u>4:45</u> 16:45	IA
---------------------------------	------------------------	-----------

Depth Filtration

Industrial waste water treatment with a Fuzzy Filter: A flexible costand space-saving solution, K. Gantner, Bosman Watermanagement GmbH, Germany

Microstructured iron hydroxide containing agglomerates for water cleaning processes, A. Gerbeth*, B. Gemende, T. Riedel, N. Pausch, University of Zwickau; M. Leiker, R. Heiduschke, B. Bäde, P.U.S. Produktions- und Umweltservice GmbH; P. Ay, C. Glaser, F. Logsch, University of Cottbus-Senftenberg, Germany

Robust filtration improves helicopters reliability & safety, K. Suri*, M. Gao, M. Sandford, B. Mouhebaty, PTI Technologies, Inc, USA

Cake Filtration

Gas consumption, solids throughput and residual cake moisture -Re-lation between operating expenses and process results of rotary filters, H. Anlauf*, Karlsruhe Institute of Technology (KIT), Germany

Evaluation of compression and relaxation properties of compressible filter cake, N. Katagiri^{*}, G. Inagaki, E. Iritani, Nagoya University, Japan

Constant pressure expression of power law non-newtonian fluid/ solid mixture, M. Iwata*, T. Shimo, T. Tanaka, Osaka Prefecture University, Japan; M.S. Jami, Islamic University Malaysia, Malaysia

High pressure squeezing in the membrane filtration in pilot scale, A. Grosso^{*}, F. Kaswalder, D. Collini, Aqseptence Srl / Diemme[®] Filtration, Italy

Tuning and understanding mining cake filtration by means of X-rays techniques, F. Kaswalder*, D. Collini, Aqseptence Group S.r.l.; N. Masciocchi, A. Guagliardi, Università dell'Insubria, To.Sca.Lab, Italy

Crystallization and mechanical dewatering, H. Meldau, Germany

Centrifugal Separation

Study of a Novel Centrifugal Filtration Device, C.-C. Lin, R.-M. Wu*, Tamkang University, Taiwan

L11/F1 Short Oral + Poster Presentation 14:45 Session Chair: Wilhelm Höflinger

Cake Filtration

Insights into dewatering and characterization of waste activated sludge from Indian perspective, S. Jadhav *, S. Haramkar, A. Kamble, B. Thorat, Institute of Chemical Technology Mumbai - India

Slurry Pretreatment - Fractionation and Sorting

Continuous phosphorus recovery by heterogeneous nucleation: Challenges in solid-liquid separation, E. Förster, S. Illies*, H. Anlauf, H. Nirschl, Karlsruhe Institute of Technology (KIT), Germany

Separation of chromite particles from olivine by flotation, L. Turri^{*}, F. Lapicque, H. Muhr, University of Lorraine, France; C. Rech, University of Caxias do Sul, Brazil

Depth Filtration - Adsorption

Methylene blue adsorption study in chitin and shrimp shell pre-treated, M.S. Zamprognio, R.F. Vaccari, L. Rodrigues, M.C.R. Tosta, A. P. Meneguelo, S.M.S. Rocha, Federal University of Espirito Santo; E.R. Nucci*, Federal University of São João Del Rei, Brazil

The use of coffee husks as bioadsorbent of oily wastewater, A.S. Almagro, A.N.F. Mendes, S.M.S. Rocha*, Federal University of Espirito Santo, Brazil

Water Treatment by Electrocoagulation and Aeration

Oily wastewater: Separation by electroflotation, I.L. Nascimento, E.V.O. Mattedi, E.R. Nucci, A.P. Meneguelo, S.M.S: Rocha*, Federal University of Espirito Santo, Brazil

Filter Media - Functionalization

Novel Short Chain/C6 Fluoroproducts with improved properties, F.F. Alemdaroglu^{*}, M. Wagner, O. Friedrichs, Daikin Chemical Europe GmbH, Germany

Filter Media - Pore Size Analysis

Ultrafiltration membranes: Pore structure characterization by liquidliquid porometry, V. Davey*, K. Gupta, PMI Porous Materials, Inc., USA

The "true" pore size of textile filter media and its relevance for the filtration process with respect to the interaction with apparatus and suspension, H. Anlauf, Karlsruhe Institute of Technology (KIT), Germany

Measurement of filtration efficacy by simultaneous qualitative laser diffraction and quantitative image analysis, T. D. Benen*, Microtrac GmbH, Germany; P.E. Plantz, Microtrac Inc, USA



10 Sh

Short Oral + Poster Presentation Session Chair: Hans-Joachim Schmid 14:45 16:45 **2**

Investigation into the operating performance of periodic regenerated surface filters as a function of the filter media structure, J. Schelp*, Q. Zhang, E. Schmidt, University of Wuppertal, Germany

Online pulse jet cleaning of filter bags with venturi and no cleaning nozzle, M. Staben, L. Gamborg*, FLSmidth A/S, Denmark

New flexible test system for cleanable filter media in extreme process conditions- Temperature up to 250°C; rel. Humidity up to 80% (90°C), M. Schmidt, Palas® GmbH, Germany

The new ISO 16890 test Norm for air filters, T. Stoffel, Denco Happel GmbH, Germany

Standardized testing of filter systems regarding their separation efficiency of allergenic particles and airborne germs, J. Mertl*, OFI – Austrian Research Institute for Chemistry and Technology, Austria; A. Wiegers, S.Denninger, Daimler AG; K.C. Bergmann, European Centre For Allergy Research Foundation (ECARF); U. Häfner, Freudenberg Filtration Technologies SE & Co. KG, Germany

Generation of a cooling lubricant-aerosol for separation tests, M. Kaul*, E. Schmidt, University of Wuppertal, Germany

CONSTANT PRESSURE EXPRESSION OF POWER LAW NON-NEWTONIAN FLUID/SOLID MIXTURE

Masashi Iwata^{a*}, Takayuki Shimo^a, Takanori Tanaka^a, Mohammed Saedi Jami^b

 ^a Department of Chemical Engineering, Faculty of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Nakaku, Sakai-shi, Osaka 599-8531, Japan
 ^b Department of Biotechnology Engineering, Faculty of Engineering, International Islamic University Malaysia, P.O. Box 10, 50728 Kuala Lumpur, Malaysia

ABSTRACT

Expression is the separation of liquid from a two-phase solid/liquid system by compression due to movement of the retaining wall. Expression of non-Newtonian fluid/solid mixture is often encountered in the fields of polymer and food industries. However, a reliable method for the design of the expression process has not been established. In this study, we conducted expression experiments of non-Newtonian fluid/solid mixture under a constant pressure condition. As a model suspended particle, cellulose or kieselguhr powder was used. The powder was mixed with an aqueous solution of sodium carboxymethyl cellulose or sodium polyacrylate. The mixture was first preconsolidated in the compression-permeability cell that consisted of a cell cylinder and a piston of 6 cm diameter and then expressed under constant pressure. As the expression advanced, the time course of the thickness of the sample was measured by a dial gauge fitted on the cylinder. The basic consolidation equation was derived by combining the fundamental equation for power law non-Newtonian flow in the cake with the equation of continuity, and was solved numerically using the Runge-Kutta method. The progress of the expression is represented by an average consolidation ratio $U_{\rm c}$. The agreement between calculated and experimental $U_{\rm c}$ was satisfactory when the creep deformation of the material was taken into consideration. It was elucidated that the consolidation time required for attaining a certain degree of primary consolidation depends on the (N+1)/N-th power of the total volume of the solid material ω_0 , where N is the flow behavior index of the squeezed liquid. On the other hand, the creep constants B, the ratio of creep deformation to the total deformation, and η , a measure of the rate of creep deformation, both were found to be ω_0 -independent parameters, supporting the validity of the assumption made in the theoretical derivation.

KEYWORDS

Non-Newtonian Fluids, Expression, Creep Effect, Rheology, Consolidation

1. INTRODUCTION

Expression is the separation of liquid from a two-phase solid/liquid system by compression due to movement of a retaining wall rather than pumping the solid/liquid system into a fixed chamber as in filtration. Although the mathematical modelling of expression of Newtonian fluid/solid mixtures has been established several decades ago [1, 2, 3], the mechanism of expression of a non-Newtonian fluid/solid mixture which is often encountered in the fields of polymer and food industries (e.g. polysaccharide is produced by expression of seaweed) is not yet explained fully.

The principal objective of this study is to present an analytical method of a constant pressure expression of a homogeneous semi-solid material which consists of power law non-Newtonian fluid/solid mixture.

2. EXPERIMENTAL

The experimental apparatus used in this study is shown in **Fig. 1**. It consists essentially of a cell cylinder and a piston of 6 cm diameter. As a model solid/liquid mixture, cellulose powder (KC Flock, W-300G, Nippon Paper Industries Co.) or kieselguhr powder (Hyflo Super Cel, Wako) mixed with an aqueous solution of sodium carboxymethyl cellulose (Nacalai Tesque Co.) or sodium polyacrylate (Nacalai Tesque Co.) was used. The mixture was preconsolidated under a constant pressure p_{s1} , resulting in a homogeneous cake of thickness L_1 . Then it was expressed under a constant pressure p. The time course of the thickness L of the sample was measured by a dial gauge fitted on the cylinder. The flow behavior index N and the fluid consistency index K of squeezed liquid were measured by using the cone-plate viscometer (DV2T, Brookfield).



Fig. 1 Compression permeability cell.

3. THEORY

In the design of expression operation, it is essential to understand the time course of the thickness *L* of the cake. The decrease in the cake thickness L_1 -*L* is equal to the total liquid volume removed per unit medium area, and it is given by Eq. (1) using the total decrease of the local void ratio e_1 -*e* integrated over the whole thickness

$$L_1 - L = \int_0^{\omega_0} (e_1 - e) \mathrm{d}\omega \tag{1}$$

where L_1 and e_1 are the thickness and void ratio at the beginning of the expression, respectively, while *L* and *e* are the thickness and void ratio at time θ_c , respectively. ω is a moving coordinate which represents a net solid volume per unit cross-sectional area extending from the drainage surface up to an arbitrary position in the solid/liquid mixture, and ω_0 is the total volume of the solid per unit cross-sectional area. The variation of *e* is caused by both the change in local solid compressive pressure p_s and the simultaneous effect of creep of the material; the former and the latter are called the primary and the secondary consolidations, respectively. The time rate of change in *e* at ω can be written as

$$\left(\frac{\partial e}{\partial \theta_{\rm c}}\right)_{\omega} = \left(\frac{\partial e}{\partial p_{\rm s}}\right)_{\theta_{\rm c}} \left(\frac{\partial p_{\rm s}}{\partial \theta_{\rm c}}\right)_{\omega} + \left(\frac{\partial e}{\partial \theta_{\rm c}}\right)_{p_{\rm s}}$$
(2)

where the term $\left(\frac{\partial e}{\partial p_s}\right)_{\theta_c} \left(\frac{\partial p_s}{\partial \theta_c}\right)_{\omega}$ represents the time rate of change in e due to the

primary consolidation and the term $\left(\frac{\partial e}{\partial \theta_{\rm c}}\right)_{p_{\rm s}}$ represents the time rate of change in e

due to the creep effect. This variation of *e* can be represented by the Terzaghi-Voigt combined model; the rheological constitution of the primary consolidation is represented by the Terzaghi element, while that of the creep deformation is represented by the Voigt element as shown in **Fig. 2**.



Fig. 2 Terzaghi-Voigt model

Eq. (2) is rewritten as

$$\left(\frac{\partial e}{\partial \theta_{\rm c}}\right)_{\omega} = \left(\frac{\partial e}{\partial p_{\rm s}}\right)_{\theta_{\rm c}} \left(\frac{\partial p_{\rm s}}{\partial \theta_{\rm c}}\right)_{\omega} + \left(\frac{\partial e}{\partial \theta_{\rm c}}\right)_{p_{\rm s}} = -a_{\rm E}\frac{\partial p_{\rm s}}{\partial \theta_{\rm c}} - \frac{\partial}{\partial \theta_{\rm c}} \left[a_{\rm c}\eta\int_{0}^{\theta_{\rm c}} \left\{p_{\rm s}\left(\omega,\tau\right) - p_{\rm s1}\right\}\exp\left\{-\eta\left(\theta_{\rm c}-\tau\right)\right\}d\tau\right]$$
(3)

where $a_{\rm E} = -\left(\frac{\partial e}{\partial p_{\rm s}}\right)_{\theta_{\rm c}}$, $a_{\rm c} = \frac{1+e}{E_2}$, and $\eta = \frac{E_2}{G}$. Detailed explanations of Eq. (3) have

been methodically reported elsewhere [4, 5, 6]. For simplicity of calculation, we assume that $a_{\rm E}$, $a_{\rm c}$ and η are all constant. From Eq. (3), (e_1 -e) in Eq. (1) can be written as

$$e_{1} - e = a_{\rm E} \int_{0}^{\theta_{\rm c}} \frac{\partial p_{\rm s}}{\partial \theta_{\rm c}} \mathrm{d}\theta_{\rm c} + a_{\rm c} \eta \int_{0}^{\theta_{\rm c}} \left\{ p_{\rm s}(\omega,\tau) - p_{\rm s1} \right\} \exp\left\{-\eta \left(\theta_{\rm c} - \tau\right)\right\} \mathrm{d}\tau \tag{4}$$

[4, 5, 6]. Substituting Eq. (4) into Eq. (1) gives

$$L_{1} - L = a_{\rm E} i \int_{0}^{\omega_{0}/i} \left\{ p_{\rm s}\left(\omega, \theta_{\rm c}\right) - p_{\rm s1} \right\} \mathrm{d}\omega + a_{\rm c} \eta i \int_{0}^{\omega_{0}/i} \int_{0}^{\theta_{\rm c}} \left\{ p_{\rm s}\left(\omega, \tau\right) - p_{\rm s1} \right\} \exp\left\{ -\eta\left(\theta_{\rm c} - \tau\right) \right\} \mathrm{d}\tau \mathrm{d}\omega$$
(5)

where *i* is number of drainage surfaces.

It is recognized that $p_s(\omega,\tau)$ in Eq. (5) can be replaced by the expression pressure p when the time rate of change in e due to the creep effect is much smaller than the time rate of change in e due to the primary consolidation for expression of a Newtonian fluid/solid mixture under constant pressure condition. This simplification

may hold true for the expression of non-Newtonian fluid/solid mixture, since the experimental results shown later can be explained well under the simplification. Thus, Eq. (5) is rewritten as

$$L_{1} - L = a_{\rm E} i \int_{0}^{\omega_{0}/i} \left\{ p_{\rm s}\left(\omega, \theta_{\rm c}\right) - p_{\rm s1} \right\} \mathrm{d}\omega + a_{\rm c} \eta i \int_{0}^{\omega_{0}/i} \int_{0}^{\theta_{\rm c}} \left(p - p_{\rm s1}\right) \exp\left\{-\eta\left(\theta_{\rm c} - \tau\right)\right\} \mathrm{d}\tau \mathrm{d}\omega \tag{6}$$

At $\theta_{\rm c} = \infty$, Eq. (6) becomes

$$L_{1} - L_{\infty} = (a_{\rm E} + a_{\rm c})i \int_{0}^{\omega_{0}/i} (p - p_{\rm s1}) d\omega = (a_{\rm E} + a_{\rm c})(p - p_{\rm s1})\omega_{0}$$
(7)

since $p_s(\omega,\infty) = p$. Here L_{∞} is the final thickness of compressed cake at $\theta_c = \infty$.

Combining Eqs. (6) and (7), we obtain the following equation of an average consolidation ratio U_c , which is a measure of the progress of expression

$$U_{c} = \frac{L_{1} - L}{L_{1} - L_{\infty}} = (1 - B) \frac{i \int_{0}^{\omega_{0}/i} \{ p_{s}(\omega, \theta_{c}) - p_{s1} \} d\omega}{(p - p_{s1})\omega_{0}} + B \{ 1 - \exp(-\eta \theta_{c}) \}$$
(8)

Here, *B* is a creep constant defined by $B \equiv \frac{a_c}{a_c + a_E}$, which is the ratio of creep deformation to the total deformation. (1-*B*) is the ratio of primary consolidation to the total deformation. The first term of the right-hand side of Eq. (8) is the contribution of the primary consolidation, while the second term is the contribution of the creep deformation. Eq. (8) simplifies approximately to Eq. (9) when $\theta_c >> 0$.

$$U_{c} = \frac{L_{1} - L}{L_{1} - L_{\infty}} = 1 - B \exp(-\eta \theta_{c}) \quad \text{at} \quad \theta_{c} >>0$$
(9)

Eq. (8) can be modified as

$$U_{\text{c-corr}} = \frac{U_{\text{c}} - B\left\{1 - \exp\left(-\eta\theta_{\text{c}}\right)\right\}}{\left(1 - B\right)} = \frac{i\int_{0}^{\omega_{0}/i} \left\{p_{\text{s}}\left(\omega, \theta_{\text{c}}\right) - p_{\text{s}1}\right\} d\omega}{\left(p - p_{\text{s}1}\right)\omega_{0}}$$
(10)

where $U_{c \cdot corr}$ indicates the progress of the primary consolidation.

To calculate Eq. (8) or Eq. (10), it is essential to obtain $p_s(\omega, \theta_c)$ value. Eqs. (6) and (8) imply that the flow resistance in the cake affects only the progress of the primary consolidation, provided that the creep deformation is much slower than the primary consolidation. Under such condition, combining the fundamental equation for power law non-Newtonian flow in the cake

$$u^{N} = \frac{1}{K\gamma\rho_{\rm s}} \left(-\frac{\partial p_{\rm s}}{\partial \omega} \right) \tag{11}$$

[7, 8, 9,10,11] with the equation of continuity

$$\left(\frac{\partial e}{\partial \theta_c}\right)_{\text{primary}} = -a_E \left(\frac{\partial p_s}{\partial \theta_c}\right)_{\omega} \approx \frac{\partial u}{\partial \omega}$$
(12)

leads to the basic Eq. (13) for primary consolidation, where *u* is the apparent velocity of fluid in the cake, γ is the local flow resistance in the cake, and ρ_s is the solid density.

$$\frac{\partial p_{\rm s}}{\partial \theta_{\rm c}} = \frac{1}{-a_{Eav}} \left\{ \frac{1}{K\gamma_{\rm av}\rho_{\rm s}} \right\}^{\frac{1}{N}} \frac{\partial}{\partial\omega} \left(-\frac{\partial p_{\rm s}}{\partial\omega} \right)^{\frac{1}{N}} = C_{\rm e} \left\{ -\frac{\partial}{\partial\omega} \left(-\frac{\partial p_{\rm s}}{\partial\omega} \right)^{\frac{1}{N}} \right\}$$
(13)

where $C_{\rm e}$ is the modified consolidation coefficient defined by $C_{\rm e} = \frac{1}{a_{Eav}} \left\{ \frac{1}{K \gamma_{\rm av} \rho_s} \right\}^{\frac{1}{N}}$,

and γ_{av} is the average flow resistance in the cake. Eq. (13) suggests that the time for attaining a specific degree of primary consolidation is proportional to $(\omega_0 / i)^{\frac{N+1}{N}}$ if other parameters are held constant. The time course of p_s can be determined by using the Runge-Kutta method for solving Eq. (13) numerically. Substituting the value of p_s into Eq. (8) yields the time course of U_c .

4. RESULTS AND DISCUSSION

The creep constants *B* and η and the modified consolidation coefficient C_e can be determined from experimental data. **Fig. 3** illustrates the experimental result of consolidation of a homogeneous cake of KC Flock mixed with a sodium polyacrylate aqueous solution. This result supports the validity of Eqs. (8) and (9), since the slope of $\ln(1-U_c)$ vs. θ_c is a straight line when $\theta_c >> 0$. In consideration of Eq. (9), the values

of both *B* and η can be graphically determined from the later stage of the experimental result of $\ln(1-U_c)$ vs. θ_c as illustrated in Fig. 3. The effect of total solid volume per unit area ω_0 on the creep constants *B* and η is shown in **Figs. 4** and **5**. Both *B* and η seem not to depend on ω_0 . η is the ratio of Young's modulus E_2 of the spring and the viscosity *G* of the dash pot of the Voigt element shown in Fig. 2, and a measure of the rate of the creep deformation. The fact that η does not depend on ω_0 implies that the liquid flow resistance through the material is not a rate-determining factor in the later part of expression process and the expression rate depends only on the creep deformation of the solid network [12].



Fig. 3 Determination of creep constants, *B* and η .



Fig. 4 Effect of the amount of total solid on creep constants for Hyflo Super Cel-SPA aq. sol. mixture.



Fig. 5 Effect of the amount of total solid on creep constants for Hyflo Super Cel-CMC aq. sol. mixture.



Fig. 6 Determination of $C_{\rm e}$ value by fitting method.

In determining the value of C_{e} , a fitting method was used. The time course of calculated $U_{c\text{-corr}}$ was obtained by assuming the value of C_{e} and evaluating Eqs. (10) and (13) simultaneously. In **Fig. 6**, the calculated value of $U_{c\text{-corr}}$ is shown. C_{e} value was determined as the calculated value of $U_{c\text{-corr}}$ coincides with the empirical one at $U_{c\text{-corr}} = 0.5$. The theoretical values of U_{c} calculated from Eq. (8) are compared with experimental results as shown in **Figs. 7** and **8**. In the figures, the solid line represents when the creep effect of the materials is taken into consideration, while the broken line represents when it is neglected (B = 0). The agreement between calculated and experimental U_{c} is satisfactory when the creep effect is considered.



Fig. 7 Average consolidation ratio U_c of KC Flock-SPA (0.2 wt%) aq. sol. mixture.



Fig. 8 Average consolidation ratio U_c of KC Flock-CMC (1.9 wt%) aq. sol. mixture.

Figs. 9 and **10** illustrate the relationship between the consolidation time θ_c required for attaining a certain amount of primary consolidation and ω_0 . In the figures, θ_{c40} , θ_{c50} , θ_{c60} are the time required for attaining 40%, 50%, 60% of $U_{c\cdot corr}$, respectively. Since the agreement between slopes of these figures and calculated (N+1)/N was satisfactory, Figs. 9 and 10 support the validity of Eq. (13) which implies that the time for attaining a specified degree of primary consolidation is proportional to $(\omega_0/i)^{\frac{N+1}{N}}$ if other parameters are held constant.



Fig. 9 Relationship among θ_{c40} , θ_{c50} , θ_{c60} and ω_0 .



Fig. 10 Relationship among θ_{c40} , θ_{c50} , θ_{c60} and ω_0 .

5. CONCLUSIONS

The basic equation for the expression of non-Newtonian fluid/solid mixture has been derived and solved numerically by using the Runge-Kutta method. The agreement between calculated and experimental U_c was satisifactory when the creep effect was considered. It has been elucidated that the consolidation time required for attaining a certain degree of primary consolidation depends on the (N+1)/N-th power of

the total volume of the solid material. The model equation developed in this study could be used in the design of the expression process for a non-Newtonian fluid/solid mixture.

NOMENCLATURE

В	creep constant
$C_{ m e}$	modified consolidation coefficient, m ^{(N+1)/N} /(Pa ^{1-1/N} ·s)
e	void ratio at consolidation time θ_{c}
e_1	void ratio at the beginning of consolidation period
i	number of drainage surfaces
Κ	fluid consistency index, Pa·s ^N
L	thickness of cake at consolidation time θ_{c} , m
L_1	thickness of cake at the beginning of consolidation period, m
L_{∞}	final thickness of compressive cake, m
Ν	flow behavior index
$p_{ m s}$	local solid compressive pressure, Pa
p_{s1}	preconsolidation pressure, Pa
и	apparent liquid velocity, m/s
$U_{ m c}$	average consolidation ratio
Greek Letters	
γ	local flow resistance, m ^{2-N} /kg
$\gamma_{ m av}$	average flow resistance, m ^{2-N} /kg
η	creep constant, s ⁻¹
$ heta_{ m c}$	consolidation time, s
$ ho_{ m s}$	solid density, kg/m ³
ω	net solid volume per unit cross-sectional area extending from the
	drainage surface up to an arbitrary position in the solid-liquid
	mixture, m ³ /m ²
ω_0	total volume of the solid material per unit cross-sectional area, $\mbox{m}^3\slash\mbox{m}^2$

REFERENCES

- [1] M. Shirato, T. Murase, H. Kato and S. Fukaya, Studies on expression of slurries under constant pressure, *Kagaku Kogaku*, **31**, 1125-1131 (1967).
- [2] M. Shirato, T. Murase, H. Kato and S. Fukaya, Fundamental analysis for expression under constant pressure, *Filtration & Separation*, 7, 277-282 (1970).
- [3] M. Shirato, T. Murase, A. Tokunaga and O. Yamada, Calculations of consolidation period in expression operations, *J. Chem. Eng. Japan*, **7**

229-231(1974).

- [4] S. Murayama and T. Shibata, A solution on the consolidation settlement, Transactions of the Japan Society of Civil Engineers, No.14,24-28(1952).
- [5] Y. Ishii, S. Kurata and T. Fujishita, Researches on the engineering properties of alluvial clays, *Transactions of the Japan Society of Civil Engineers*, No.30,1-92(1955).
- [6] R. E. Gibson and K. Y. Lo, A theory of consolidation for soils exhibiting secondary consolidation, Norwegian Geotechnical Institute, Publication No.41, pp. 1-16 (1961).
- [7] W. Kozicki, C. J. Hsu and C. Tiu, Non-Newtonian flow through packed beds and porous media, *Chem. Eng. Sci.*, **22**, 487-502 (1967).
- [8] W. Kozicki, C. Tiu and A. R. K. Rao, Filtration of non-Newtonian fluids, *Can. J. Chem. Eng.*, **46**, 313-321 (1968).
- [9] W. Kozicki, A. R. K. Rao and C. Tiu, Filtration of polymer solutions, *Chem. Eng. Sci.*, 27, 615-625 (1972).
- [10] M. Shirato, T. Aragaki, E. Iritani, Constant pressure filtration of power-law non-Newtonian fluids, *J. Chem. Eng. Japan*, **10**, 54-60 (1977).
- [11] M. Shirato, T. Aragaki and E. Iritani, Analysis of constant pressure filtration of power-law non-Newtonian fluids, *J. Chem. Eng. Japan*, **13**, 61 (1980).
- [12] M. Shirato, T. Murase, M. Iwata and S. Nakatsua, The Terzaghi-Voigt combined model for constant-pressure consolidation of filter cakes and homogeneous semi-solid materials, *Chem. Eng. Sci.*, **41**, 3213-3218 (1986).