Introduction to

LATTICE STATISTICAL MECHANICS

This book is an introduction to statistical mechanics suitable for readers knowing basic measure theory. The mathematical prerequisites are summarized in Chapter 1. It is hoped the reader will be ready to tackle research papers after reading the book. The first part of the text (Chapters 2-4) is concerned with 1-D Ising model. Some examples are described. The second part (Chapter 5) of the text focuses on Ising model on the Cayley tree and 2-D Ising model. Limit Gibbs measure and problem of phase transition are discussed and the proofs of existing of phase transition for these models are also given. The third part (Appendix) outlines some applications of Ising model and Islamic aspects of the considered subject.

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