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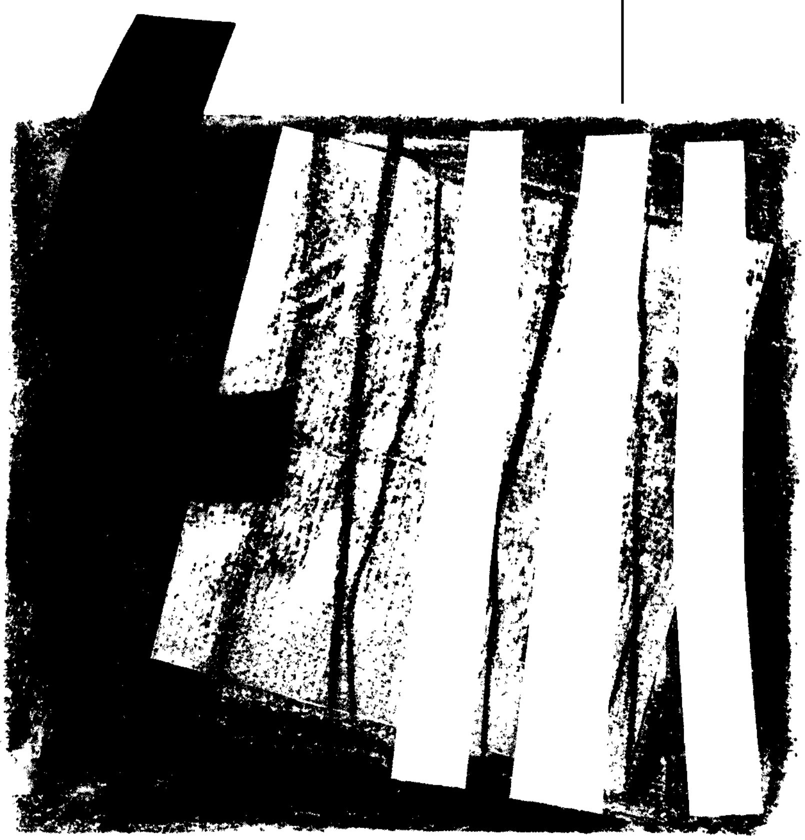

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Kaiser, R.*

Abstract

In the framework of decomposing a time series into the sum of signal components plus noise as in detrending or seasonal adjustment, we analyze the situation in which the unobserved components may be subject to the influence of sudden shifts. The kind of perturbation that such shifts cause on the observed series can be classified as an *outlier*, when the shift affects the noise component, or as a *structural change*, when the shift affects one of the signal components. The consequences of ignoring these perturbations are important for model specification, parameter estimation and forecasting. We extend and modify the iterative procedure of Chen and Liu (1993) to allow the location, classification and estimation of outliers and structural changes affecting the unobserved components of a time series.

Key Words
Signal extraction; state space models; Kalman filter; structural changes; outliers; generalized least squares; iterative procedure.

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Detection and Estimation of Structural Changes and Outliers in Unobserved Components

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February 1998

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Abstract

In the framework of decomposing a time series into the sum of signal components plus noise as in detrending or seasonal adjustment, we analyze the situation in which the unobserved components may be subject to the influence of sudden shifts. The kind of perturbation that such shifts cause on the observed series can be classiffied as an *outlier*, when the shift affects the noise component, or as a *structural change*, when the shift affects one of the signal components. The consequences of ignoring these perturbations are important for model specification, parameter estimation and forecasting. We extend and modify the iterative procedure of Chen and Liu (1993) to allow the location, clasiffication and estimation of outliers and structural changes affecting the unobserved components of a time series.

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1 Introduction

Two basic kinds of phenomena that occur frequently in seasonal time series and must be dealt with adequately are outlying observations and structural changes. The former are the result of non-repetitive events whose effects on the series rest only for one period; for example, a recording error or occurrence of a disaster. The latter suppose a change in the dynamic pattern of the series; the most well known case is the level shift which can arise, for example, as the consequence of a change in the definition for the magnitude being described by the data. Other structural changes, such as change in slope or change is seasonality may also be needed to be dealt with. Proper treatment of outlying observations and structural changes is important for model specification, parameter estimation and crucial for forecasting purposes. The main goal of this article is to devise automatic tools for handling outlying observations and structural changes in the hope that the proposed procedure will prove useful in the seasonal adjustment of economic time series.

The state-space representation provides a flexible general framework within which to model and analyze seasonal time series subject to the presence of outlying observations and abrupt changes in pattern, see, for instance, Smith and West (1983), Kitagawa and Gersch (1984); Harvey and Durbin (1986), Harvey and Koopman (1992) or Durbin and Cordero (1993).

Our basic tool is the iterative procedure for detecting outliers in a time series initially proposed by Chang (1982) and further modified by Chang et al. (1988), Tsay (1986), Chen and Liu (1993) and Gómez (1997). The more recent versions of the iterative procedure allow the identification and estimation of four types of perturbations, *Additive Outlier*, *Innovative Outlier*, *Level Shift* and *Transitory Change*. Although these four types are usually encountered in real economic time series, we believe that this classification is too restrictive when a seasonal time series is considered, since does not allow the identification of perturbations associated with the trend or the seasonal structure of the series. In this paper, the definition of the different types of perturbations that may affect the observed series is based on the particular structure of the series under study. Under this approach, structural changes are seeing as unusual large values for the noise of the signal unobserved components, and outliers are seeing as unusual large values for the irregular or noise component of the series. Further, this approach allows the "complete " decomposition of the observed series, in the sense that the observed series can be decomposed into several unobserved components that incorporate the deterministic effects of the structural changes, for the case of signal components, and the outliers, for the case of the irregular component, that are affecting them.

Section 2 describes the basic model; also, in this section, the state-state representation and the Kalman filter needed to estimate the model and obtain the residuals are introduced. In section 3, outliers and structural changes are defined and their effects on the unobserved components and the observed series are described. Section 4 presents the iterative procedure for detecting and estimating the effect of outliers and structural changes. Section 5 presents some simulation results and, in section 6, two real examples are investigated. Finally, in section 7 some conclusions are given.

2 State-space representation

In a very general unobserved components model the observed series can be thought as the result of summing up the trend, seasonal and irregular components as follows,

$$\begin{aligned} y_t &= \mu_t + s_t + \epsilon_t \qquad t = 1, \ldots, N & (1) \\ \phi_\mu(B)\delta_\mu(B)\mu_t &= \theta_\mu(B)b_t & (2) \\ \phi_s(B)\delta_s(B)s_t &= \theta_s(B)c_t & (3) \end{aligned}$$

where y_t is the tth observation and μ_t, s_t and ϵ_t are, respectively, the trend, seasonal and irregular components of y_t. The irregular component ϵ_t is white noise with mean zero and variance σ_ϵ^2 and it is assumed to be uncorrelated with b_t and c_t. The model for the trend component is described in equation (2), where $\delta_\mu(B)$ is a polynomial in the backshift operator B of degree d_μ whose zeros are on the unit circle, $\phi_\mu(B)$ is a polynomial of degree p_μ whose zeros are outside the unit circle, $\theta_\mu(B)$ is a polynomial of degree q_μ whose zeros are on or outside the unit circle and $b_t \sim N(0, \sigma_\epsilon^2 * b)$, where

b is the trend-noise ratio. Similarly, let (3) be the model for the seasonal component. The polynomials $\delta_s(B)$, $\phi_s(B)$ and $\theta_s(B)$ are of degree d_s, p_s and q_s respectively, and are defined in an analogous way to the correspondent trend component polynomials; finally, $c_t \sim N(0, \sigma_\epsilon^2 * c)$ where c is the seasonal-noise ratio. We assume $\delta_\mu(B)$ and $\delta_s(B)$ have no common zeros and b_t and c_t are uncorrelated with each other. Note that the unobserved components representation in (1)-(3) is general and admits as particular cases most frequent characterizations of the trend and seasonal components, see Maravall (1993) for a complete description of possible specifications for models in (2) and (3).

Let $\tilde{\phi}_\mu(B) = \phi_\mu(B)\delta_\mu(B)$ be a polynomial of degree $p_\mu + d_\mu$. Following Bell and Hillmer (1990), the state-space representation of minimum degree r_μ, where $r_\mu = max(p_\mu + d_\mu, q_\mu + 1)$, for the trend component μ_t could be written as,

$$\mathbf{x}_{\mu,t} = \mathbf{F}_\mu \mathbf{x}_{\mu,t-1} + \mathbf{G}_\mu b_t \quad (4)$$

$$\mu_t = \mathbf{H}'_\mu \mathbf{x}_{\mu,t} \quad (5)$$

where the $r_\mu \times 1$ state-vector $\mathbf{x}_{\mu,t}$ has the following components, $x_{\mu,t}(1) = \mu_t$ and,

$$x_{\mu,t}(i) = \sum_{j=i}^{r_\mu} \tilde{\phi}_{\mu,j}\mu_{(t-1+i-j)} - \sum_{j=i-1}^{q_\mu} \theta_{\mu,j} b_{(t-1+i-j)} \qquad i = 2, \ldots, r_\mu$$

and $\mathbf{H}'_\mu = (1, 0, \ldots, 0)$' and $\mathbf{G}_\mu = (1, \theta_{\mu,1}, \ldots, \theta_{\mu,r_\mu})$ are $r_\mu \times 1$ vectors with $\theta_{\mu,j} = 0$ for $j > q_\mu$. Finally, $\mathbf{F}_\mu$ is a $r_\mu \times r_\mu$ matrix as follows,

$$\mathbf{F}_\mu = \begin{bmatrix} \tilde{\phi}_{\mu,1} & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\phi}_{\mu,r_\mu-1} & 0 & 0 & 1 \\ \tilde{\phi}_{\mu,r_\mu} & 0 & 0 & 0 \end{bmatrix}$$

where $\tilde{\phi}_{\mu,j} = 0$ for $j > p_\mu + d_\mu$. Letting $\tilde{\phi}_s(B) = \phi_s(B)\delta_s(B)$ be a polynomial of degree $p_s + d_s$, a state-space representation of minimum degree $r_s = max(p_s + d_s, q_s + 1)$ for the seasonal component s_t is possible with the $r_s \times 1$ vectors $\mathbf{x}_{s,t}$, $\mathbf{H}'_s$ and $\mathbf{G}_s$, and the $r_s \times r_s$ matrix $\mathbf{F}_s$ defined as previously for the trend component.

Considering model (1)-(3) and the state-representation for the individual components μ_t and s_t, the state-space representation of minimum degree $r = r_\mu + r_s$ for the observed series y_t can be written as,

$$\begin{aligned} \mathbf{x}_t &= \mathbf{F}\mathbf{x}_{t-1} + \mathbf{G}\mathbf{W}_t \\ y_t &= \mathbf{H}'\mathbf{x}_t \end{aligned}$$

where the $r \times 1$ state-vector is $\mathbf{x}_t = [\mathbf{x}'_\mu, \mathbf{x}'_s]$; $\mathbf{H}' = [\mathbf{H}'_\mu, \mathbf{H}'_s]$ is a $r \times 1$ vector ; $\mathbf{F} = diag(\mathbf{F}_\mu, \mathbf{F}_s)$ is a $r \times r$ matrix ; $\mathbf{G} = diag(\mathbf{G}_\mu, \mathbf{G}_s)$ is a $r \times 2$ matrix; and $\mathbf{W}'_t = [b_t, c_t]$.

For given values of the unknown parameters in model (1)-(3), the likelihood function can be evaluated using the Kalman filter recursions, see Jones (1980), Gómez and Maravall (1994), etc. Since the variances of the trend and seasonal components have been specified in terms of σ^2_ϵ, the latter can be concentrated out of the likelihood function and therefore, we can set, without loss of generality, $\sigma^2_\epsilon = 1$ in the Kalman filter recursions and obtain it once the unknown parameters have been estimated. The Kalman filter recursions consist on the initial conditions $\mathbf{x}_{1|0} = E(\mathbf{x}_0 = 0)$ and $\mathbf{P}_{1|0} = var(\mathbf{x}_0) = \mathbf{P}_{0|0}$, and then,

$$\mathbf{x}_{t|t-1} = \mathbf{F}\mathbf{x}_{t-1|t-1}, \qquad \mathbf{P}_{t|t-1} = \mathbf{F}\mathbf{P}_{t-1|t-1}\mathbf{F}' + \mathbf{G}\mathbf{G}'$$

$$\mathbf{x}_{t|t} = \mathbf{x}_{t|t-1} + \mathbf{K}_t(y_t - \mathbf{H}'\mathbf{x}_{t|t-1}), \qquad \mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}_t\mathbf{H}'\mathbf{P}_{t|t-1}$$

where, $\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}(\mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + 1)^{-1}$. To evaluate the likelihood function at each iteration, it is necessary to obtain the innovation $\nu_t = y_t - y_{t|t-1}$ where $y_{t|t-1} = \mathbf{H}'\mathbf{x}_{t|t-1}$ and its variance $\Sigma_t = \mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + 1$. Setting the standardized residuals $e_t = \nu_t/\Sigma_t^{1/2}$ and the vector $\mathbf{e} = (e_1, \ldots, e_N)'$, the prediction error decomposition of the concentrated likelihood is,

$$l = -\frac{1}{2}\left\{ln\left(\prod_{t=1}^{N}\Sigma_t\right) + Nln(\mathbf{e}'\mathbf{e})\right\}$$

The variance of the irregular component can then be estimated as $\sigma^2_\epsilon = (1/N)\mathbf{e}'\mathbf{e}$. For non-stationary unobserved components, the likelihood can not be properly defined since the initial conditions are unknown. There are several ways to handle this situation as transforming the data to eliminate the influence of initial observations or placing a diffuse prior on the variance of the initial state; see, for instance, Bell (1984), Harvey and Pierse (1984), Kohn and Ansley (1986), Harvey (1989) pg.127-128, or Gómez and Maravall (1994).

3 Outliers and structural changes

Assume the time series y_t is as described in (1)-(3); y_t may be subject to the influence of outliers and structural changes that should be needed to deal with. The unobserved components model suggests a natural definition of outliers and structural changes as being unusual large values for the white noise series ϵ_t, b_t and c_t.

(i) Outliers. Can be defined as a perturbation affecting the irregular or noise component and can be represented by an unusual large value for the white noise series ϵ_t. For the case of one outlier at time $t = T$, a convenient way of modelling the observed or "contaminated" series, y_t^*, is as follows

$$y_t^* = \mu_t + s_t + \epsilon_t + \omega I_t^{(T)} = \mu_t + s_t + \epsilon_t^* \tag{6}$$

where ω is the magnitude of the outlier, $I_t^{(T)}$ is an indicator variable with $I_t^{(T)} = 1$ if $t = T$ and $I_t^{(T)} = 0$ otherwise, and $\epsilon_t^* = \epsilon_t + \omega I_t^{(T)}$ can be considered as the contaminated irregular component which includes the original irregular component plus the effect of the outlier at time $t = T$. Note that the trend and seasonal components result unaffected by the perturbation and, therefore, (2) and (3) still hold. This type of perturbation is equivalent to the *Additive Outlier*, see, for instance, Fox (1972), Tsay (1986), Chen and Liu (1993).

(ii) Structural changes in the trend. Can be defined as a perturbation affecting the trend component model (2) and imply an abrupt change or a break in the dynamic pattern of the trend and therefore, a change in the dynamic pattern of the observed series. Note that if $\delta_\mu(B) \neq 1$, that is, if the trend follows a nonstationary process, the change will be permanent. For a trend structural change at time $t = T$, the observed time series can be described as the outcome of the following model,

$$\begin{aligned} y_t^* &= \mu_t + \frac{\theta_\mu(B)}{\tilde{\phi}_\mu(B)} \omega I_t^{(T)} + s_t + \epsilon_t \\ &= \mu_t^* + s_t + \epsilon_t \end{aligned} \tag{7}$$

with (3) still applying and the contaminated trend component as follows,

$$\mu_t^* = \frac{\theta_\mu(B)}{\tilde{\phi}_\mu(B)} \left\{ b_t + \omega I_t^{(T)} \right\}. \tag{8}$$

Expression (8) makes clear that a structural change in the trend can be naturally modelled as an unusual large value for the trend disturbance b_t.

The trend component in (2) can be further decomposed as being the result of summing up the level and the slope. Among others, Harvey and Todd (1983) or Smith and West (1983) use the *local linear trend*,

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + b_t &(9)\\ \beta_t &= \beta_{t-1} + d_t &(10) \end{aligned}$$

where d_t is white noise with variance σ_d^2 and it is assumed to be uncorrelated with b_t, ϵ_t and c_t. Note this model for the trend is equivalent to model (2) with $\tilde{\phi}_\mu(B) = \nabla^2$ and $\theta_\mu(B) = (1 - \theta_{\mu,1}B)$, see Harvey (1989). Two different kind of structural changes affecting the trend can be considered in this framework: a structural change in the level, caused by an unusual large value for the disturbance level b_t; and a structural change in the slope, caused by an unusual large value for the slope disturbance d_t. Smith and West (1983) present an example where a trend component as the one described by (9)-(10) is subject to the influence of both types of structural changes.

(iii) Structural changes in the seasonal component. Can be defined as a perturbation affecting the noise in the seasonal equation (3) and imply an abrupt change or break in the dynamic pattern of the seasonal component. For the case of a seasonal structural change at time t=T, the observed series y_t^* can be described as follows,

$$\begin{aligned} y_t^* &= \mu_t + s_t + \frac{\theta_s(B)}{\tilde{\phi}_s(B)}\omega I_t^{(T)} + \epsilon_t \\ &= \mu_t + s_t^* + \epsilon_t &(11) \end{aligned}$$

with (2) still applying and,

$$s_t^* = \frac{\theta_s(B)}{\tilde{\phi}_s(B)}\left\{c_t + \omega I_t^{(T)}\right\} \qquad (12)$$

If $\delta_s(B) \neq 1$ the effect of the abrupt change will be permanent from $t = T$ onwards, otherwise, if $\delta_s(B) = 1$ and $\phi_s(B) \neq 1$, the effect of the perturbation on the seasonal component will decay proportionally to the coefficient in the autoregressive polynomial $\phi_s(B)$.

Apart from those types, depending on the nature of the series under study, would be possible to define structural changes in a cyclical component. Also the decomposition of the seasonal component in several components, each of them associated to one seasonal root, would allow the definition of structural changes affecting each of these subcomponents. But the more obvious and interesting extension would be a case where the structural change simultaneously affects two or more components. To illustrate the case of a simultaneous change, suppose a perturbation affecting at time $t = T$ the seasonal and the trend disturbances. The model for the observed series is then,

$$\begin{aligned} y_t^* &= \mu_t + s_t + \epsilon_t + \left\{ \frac{\theta_\mu(B)}{\tilde{\phi}_\mu(B)}\omega_\mu + \frac{\theta_s(B)}{\tilde{\phi}_s(B)}\omega_s \right\} I_t^{(T)} \\ &= \mu_t^* + s_t^* + \epsilon_t \end{aligned}$$

where ω_μ and ω_s are the magnitudes for the initial impact of the structural change on the trend and on the seasonal component, respectively; and μ_t^* and s_t^* are as in (8) and (12) respectively. Note that a perturbation causing structural changes in each of the signal components in a time series will have an effect on the observed series equivalent to the one produced by the *innovative outlier* -an important difference is that the effect of a simultaneous structural change may be decomposed into all the affected components-, see Kaiser (1995).

4 Detection and estimation

In this section we discuss how standard tools in the time series literature may be used to detect and estimate the kind of outliers and structural changes described in the previous section. In particular the iterative procedure described in, among others, Chang (1982), Tsay (1986), Chen and Liu (1993) or Gómez (1994) can be extended and modified to allow both the detection and estimation of these perturbations. The key point in our analysis is to consider the observed series as being the sum of several unobserved components which can be subject to deterministic effects as outliers or structural changes. Suppose, for instance, we base our analysis on the seasonally adjusted series, then we should provide ourselves with tools to extract the seasonal component and all the deterministic effects related to it from the observed series.

4.1 One perturbation parameters known

Let $\mathbf{\Psi} = (\tilde{\phi}_{\mu,1}, \ldots, \tilde{\phi}_{\mu,d_\mu+p_\mu}, \theta_{\mu,1}, \ldots, \theta_{\mu,q_\mu}, \tilde{\phi}_{s,1}, \ldots, \tilde{\phi}_{s,d_s+p_s}, \theta_{s,1}, \ldots, \theta_{s,q_s}, b, c, \sigma_\epsilon^2)$ be the vector of parameters in model (1)-(3) and let us suppose, by the moment, that it is known. Further, suppose that the *i-th* unobserved component, $i = \mu, s, \epsilon$, is subject to the influence of a structural change at time $t = T$. Then the state-space representation of the observed series y_t^* consists on equation (4) and,

$$\begin{aligned} y_t^* &= \psi_i(B)\omega I_t^{(T)} + \mathbf{H}'\mathbf{X}_t + \epsilon_t \\ &= Z_{i,t}^*(T)\omega + \mathbf{H}'\mathbf{X}_t + \epsilon_t \end{aligned}$$

where $\psi_i(B) = \theta_i(B)/\tilde{\phi}_i(B)$ is the ARIMA polynomial for the contaminated component model; and $Z_{i,t}^*(T) = \psi_i(B)I_t^{(T)}$ is an $N \times 1$ vector describing the change in the dynamic pattern due to the perturbation. The state-space model above can be rewritten as a linear regression model (see Khon and Ansley 1985, Harvey 1989 or Gómez and Maravall 1994) as follows,

$$y_t^* = Z_{i,t}^*(T)\omega + y_t \tag{13}$$

$$y_t = \mathbf{H}'\mathbf{X}_t + \epsilon_t \tag{14}$$

where y_t is as described in expressions (1)-(3). Let $\mathbf{y}^* = (y_1^*, \ldots, y_N^*)'$; $\mathbf{y} = (y_1, \ldots, y_N)'$ and $\mathbf{Z}^* = (Z_{i,1}^*(T), \ldots, Z_{i,N}^*(T))'$. Writing (13) in matrix terms yields,

$$\mathbf{y}^* = \mathbf{Z}^*\omega + \mathbf{y} \tag{15}$$

The model in (15) is a regression model with autocorrelated residuals and, therefore, the problem of estimating ω can be solved by Generalized Least Squares (GLS), see Kohn and Ansley (1985) or Gómez (1994). Let $var(\mathbf{y}) = \sigma_\epsilon^2\mathbf{\Omega}$ with $\mathbf{\Omega}$ a $N \times N$ matrix which depends on $\mathbf{\Psi}$ and which is assumed to be positive defined; and let $\mathbf{\Omega} = \mathbf{L}'\mathbf{L}$ be the Cholesky decomposition of $\mathbf{\Omega}$ with $\mathbf{L}$ lower triangular. Premultiplying (15) by $\mathbf{L}^{-1}$, and setting $\mathbf{e} = \mathbf{L}^{-1}\mathbf{y}^*$, $\mathbf{Z} = \mathbf{L}^{-1}\mathbf{Z}^*$ and $\mathbf{e}^* = \mathbf{L}^{-1}\mathbf{y}$, it is obtained the Ordinary Least Squares (OLS) model,

$$\mathbf{e} = \mathbf{Z}\omega + \mathbf{e}^* \tag{16}$$

where $var(\mathbf{e}^*) = \sigma_\epsilon^2 \mathbf{I}_N$. The OLS estimator of ω and its variance are obtained from (16) as

$$\hat{\omega} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{e} \qquad\qquad var(\hat{\omega}) = (\mathbf{Z}'\mathbf{Z})^{-1}\sigma_\epsilon^2 \tag{17}$$

As argued in Gómez and Maravall (1994), to move from the GLS model in (15) to the OLS model in (16) there is no need to evaluate the matrix $\mathbf{\Omega}$ since the application of the Kalman filter recursions on the observed series $\mathbf{y}^*$ yields the vector of standardized residuals $\mathbf{e} = \mathbf{L}^{-1}\mathbf{y}^*$ and, similarly, the application of the same filter on vector $\mathbf{Z}^*$ provides the vector $\mathbf{Z} = \mathbf{L}^{-1}\mathbf{Z}^*$ from which (17) can be obtained.

To test the null hypothesis that there is no a structural change in component i at time $t = T$ one can use the standardized statistic,

$$\lambda = \frac{\hat{\omega}}{\sqrt{var(\hat{\omega})}}$$

which for known $\mathbf{\Psi}$, follows a standard normal distribution. In practice, the true parameters in $\mathbf{\Psi}$ are usually unknown in the modelling stage, but they can be estimated by any consistent estimator, see Chang et al (1988); in that case the λ-statistic above is still asymptotically normal.

If the objective of the analysis is to determine the component affected by the structural change at time $t = T$, one possibility is, as suggested by Chang et al (1988) for the case of outliers affecting the observed series, to calculate the estimates $\hat{\omega}_i(T)$ and its respective statistics λ_i, where the subscript i makes reference to the unobserved component which results contaminated by the perturbation and then use the statistic,

$$\eta(T) = \max_i \{ | \lambda_i | \}$$

if $\eta(T) > C$, where C is a predetermined critical value, then there is a possibility that component i is subject to the influence of a structural change at time $t = T$. Since the vector $\mathbf{Z} = \mathbf{L}^{-1}\mathbf{Z}^*$ in (16) depends on the ARIMA polynomial for the contaminated component, it is clear that in order to compute the statistic $\eta(T)$, the Kalman filter needs to be run four times, once on the vector of observations $\mathbf{y}^*$ and once on each of the three possible vectors $\mathbf{Z}_i^* = \psi_i(B)I_t^{(T)}$, for $i = \mu, s, \epsilon$.

Finally, the timing T is seldom known a priori, but as suggested by Chang et al (1988), the likelihood ratio criteria leads to the criteria,

$$\eta = \max_t \eta_t(T) \qquad t = 1, \ldots, N$$

Then, if $\eta > C$, there is a possibility that component i is subject to the influence of one perturbation at time $t = T$. In order to compute the η statistic above, the Kalman filter should be applied on the vector of observations to obtain the vector $\mathbf{e}$ and on $N \times 3$ different $\mathbf{Z}_{i,t}$ vectors, for $i = \mu, s, \epsilon$ and $t = 1, \ldots, N$. The procedure is therefore inefficient in terms of computing time. Nevertheless, this difficulty can be avoided if the reduced ARIMA form of model in (1)-(3) is known. Let $\Pi(B)y_t = a_t$ where a_t is a white noise process with zero mean and variance σ_a^2; and $\Pi(B) = \tilde{\phi}(B)/\theta(B)$ is an ARIMA polynomial such that $\tilde{\phi}(B) = \tilde{\phi}_\mu(B)\tilde{\phi}_s(B)$ and $\theta(B)a_t = \theta_\mu(B)\tilde{\phi}_s(B)b_t + \theta_s(B)\tilde{\phi}_\mu(B)c_t + \tilde{\phi}_\mu\tilde{\phi}_s(B)\epsilon_t$. The GLS estimator of ω can then be computed by using the Kalman filter to obtain the exact residuals $\mathbf{e}$, and then applying the truncated filter $\Pi(B)$ to vector $\mathbf{Z}^*$ to obtain the vector $\mathbf{Z}$ in (16).

Once the location and the component affected by the perturbation are known its effect can be adjusted from the residuals using (16). The adjusted series and its unobserved components can also be obtained from (13) and the expressions in section 3.

4.2 Multiple perturbations

In a more general framework, one can consider the observed series as being the result of summing up the trend, seasonal and irregular components plus k deterministic effects at times $t = t_1, \ldots, t_k$. In such a situation the state-space representation of y_t^* consists on equation (4) and,

$$y_t^* = \sum_{j=1}^{k} Z_{i,t}^*(j)\omega_i + \mathbf{H}'\mathbf{X}_t + \epsilon_t \tag{18}$$

where $Z_{i,t}(j) = \psi_i(B)I_t^{(t_j)}$ represents the effect of the structural change in component i at time $t = t_j$. The practical procedure we propose in this paper does not detect the k perturbations at the same time but proceeds in several iterations detecting one by one the possible outliers and structural changes. In the detection stage, the procedure starts by applying the Kalman filter to the vector of observations to obtain the residuals and the truncated filter $\Pi(B)$ to vectors $Z^*_{i,t}(j)$ to determine the location and type of the k perturbations in (19). Once the detection stage is completed, in order to avoid possible masking effects see, Chen an Liu (1993), the final ω_i's are obtained within the following multiple regression model,

$$\mathbf{y}^* = \mathbf{Z}^*\boldsymbol{\omega} + \mathbf{y}$$

where $\mathbf{Z}^*$ is a $N \times k$ matrix with columns $\mathbf{Z^*}_i = (Z^*_{i,1}(j), \ldots, Z^*_{i,N}(j))$ and $\boldsymbol{\omega}$ is a $k \times 1$ vector with elements ω_j for $i = 1, \ldots, k$. The application of the Kalman filter recursions on the vector of observations $\mathbf{y}^*$ and on the k columns of matrix $\mathbf{Z}^*$ allows the specification of an OLS model from which the vector $\boldsymbol{\omega}$ can be estimated as in (17). An efficient way to estimate the vector $\boldsymbol{\omega}$, see Kohn and Ansley (1985), makes use of the QR algorithm in which an orthogonal $N \times k$ matrix $\mathbf{Q}$ is obtained, such that $\mathbf{Q}'\mathbf{L}^{-1}\mathbf{Z}^* = (\mathbf{R}', 0)'$, where $\mathbf{R}$ is a non-singular $k \times k$ upper triangular matrix. Then $\hat{\omega} = \mathbf{R}^{-1}\mathbf{v}_1$, where $\mathbf{v}_1$ consists of the first k elements of the vector $\mathbf{v} = \mathbf{Q}\mathbf{L}^{-1}\mathbf{y}^*$.

4.3 A procedure for joint detection and estimation

The proposed procedure starts with the specification of the models for the unobserved components as if were no shocks and the following stages:

I.1 Obtain the maximum likelihood estimators for the unknown parameters in vector $\boldsymbol{\Psi}$ based on the original or the adjusted vector of standardized residuals $\mathbf{e}$. In the first iteration, the residuals obtained from the application of the Kalman filter on the observed series are used; after the first iteration the adjusted residuals are used to evaluate the likelihood function.

Detection inner loop

I.2 For $t = 1, \ldots, N$, compute $\lambda_i(T)$ for $i = \mu, s, \epsilon$ and the statistic $\eta_t = max_i \{|\lambda_i|\}$. If $\eta = max_t \eta_t = |\eta_i(T)| > C$ where C is a predetermined critical value, then there is a possibility of one perturbation affecting component i at time $t = T$.

In the presence of outlying observations or structural changes, the prediction errors obtained via the Kalman filter are contaminated and hence $\hat{\sigma}_\epsilon^2 = \{1/N\}\mathbf{e}'\mathbf{e}$ may be overestimated. One method to overcoming this problem which it is not time consuming is the omit-one method in which $\hat{\sigma}_\epsilon^2$ is calculated with the $e(t_i)$ omitted. Other alternatives include the MAD method or the α % trimmed method, see Chen and Liu (1993).

I.3 If a possible structural change or outlier is found, remove its effect from the residuals and obtain the adjusted residuals $\mathbf{e}^*$ using,

$$\mathbf{e}^* = \mathbf{e} - \mathbf{Z}\hat{\omega},$$

and go back to I.2 to iterate. Otherwise, proceed to I.4.

I.4 If no structural change or outlier is found in the first iteration, then stop. If the presence of one or more structural changes or outliers have been detected in the previous iterations from steps I.2-I.3, then go back to I.1 to revise the parameters estimates. Continue to repeat I.1-I.3 until no new perturbations are found, then go to II.1.

Joint estimation stage

II The effects of the identified structural changes are jointly estimated in the multiple regression model in (19) by Generalized Least Squares applying the Kalman filter, both on the vector of observations and on each of the columns of matrix Z, and the QR algorithm. Then compute the t-statistic for the estimated effects and check if there is any perturbation for which the t-statistic is less than C, where C is the same critical value used in I.2. If there is not, then obtain the adjusted series, check whether the initial specification in the models for the unobserved components is still valid, apply the Kalman filter on the adjusted series, obtain the new residuals and go back to stage I to repeat the complete process. Otherwise, delete one by one the insignificant effects and reestimate the multiple regression model until all the ω_i's are significant, then obtain the vector of adjusted observations, apply the Kalman filter on it, obtain the new residuals and go back to I to iterate.

5 Simulations

In this section we design simulations to study the performance of the proposed iterative procedure. We focus our analysis in two aspects of the procedure: a) the relative frequency of detection of one outlier or structural change while no one is effectively present, which is a measure of the error type I, and b) the relative frequency of correct detection, which is a measure of the power.

The simulations were performed using the quarterly "Basic Structural Model" (BSM), see Harvey (1989) in which the observed series is $y_t = \mu_t + s_t + \epsilon_t$ and the models for the unobserved components are as follows,

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$$

$$\beta_t = \beta_{t-1} + \xi_t$$

$$s_t = -\sum_{j=1}^{s-1} s_{t-j} + \omega_t$$

where $\epsilon_t, \eta_t, \xi_t$ and ω_t are white noise series uncorrelated with each other and with zero mean and variance $\sigma^2_\epsilon, \sigma^2_\eta, \sigma^2_\xi$ and σ^2_ω respectively. The BSM was chosen because it is simple, very general -in the sense that many economic series can be described with this model- and it has, at least in the cases we have considered, a well known reduced form, the so called airline model of Box and Jenkins (1976). In all the cases we have considered, the simulated series also admit a canonical decomposition. The results using these canonical models are almost identical to the ones that are presented and therefore they are not reported here.

The performance of the iterative procedure was found to be associated with i) the number of observations ii) the magnitude of the variances $\sigma^2_\epsilon, \sigma^2_\eta, \sigma^2_\xi$ and σ^2_ω, and iii) the critical value C. In the first simulation exercise we consider different values of the variances for the unobserved components. The six models are described in table 1. For each model we generated 1000 series with N observations ($N = 50, 100, 200, 300$) and considered critical values $C = 2.5, 3.0, 3.5, 4.0, 4.5$ and 5.0. Table 2 gives the relative frequency of error type I for every possible combination of models I to VI, number of observations and critical values.

The worst result is obtained in model I, where $\sigma_\epsilon^2 = 20$, for which a frequency of error type I smaller than 10% is obtained for critical values bigger than 4.0 and 4.5 for 50 and 100 observations, respectively; even using a critical value of 5.0, the frequency of error type I is bigger than 10% for 200 and 300 observations. In models IV, V and VI, when $\sigma_\epsilon^2 < 1$, we obtain the best results. In these models, using a critical value of $C \geq 3.5$, the frequency of error type I is smaller than 5% and 10% for samples sizes of 50 and 100 observations, respectively; for a sample size of 200 observations and critical value $C = 4.0$, the frequency of error type I is smaller than 5%, and it varies from 4.2% in model IV to 6.2% V for a sample size of 300 observations. Models II and III can be considered intermediate cases, for which a critical value of $C = 4.0$ yields a frequency of error type I less than 1% in the case of 50 observations, less than 5% in the case of 100 observations, 6-7% in 200 the case of 200 observations, and 10-11% in the case of 300 observations.

In the models we have considered the influence of the variances $\sigma_\eta^2, \sigma_\xi^2$ and σ_ω^2, in the size of error type I is less evident than the influence of σ_ϵ^2 but, in general, one should expect that higher values of the four variances cause a higher relative frequency of error type I, e.g. although models IV and V yield similar results, model IV is slightly better than model V while the only difference between them is the value of σ_ξ^2 ($\sigma_\xi^2 = 0$ in model IV and $\sigma_\xi^2 = 0,25$ in model V).

To investigate the power of the iterative procedure in terms of perturbation detection, we studied first the relative frequency of correct detection (type an location are correctly identified) of one outlier or structural change. The BSM with $\sigma_\epsilon^2 = \sigma_\eta^2 = \sigma_\omega^2 = 1$ and $\sigma_\xi^2 = 0$ was used to generate simulated series of length 100 observations. We considered outliers and structural changes in the trend and seasonal components, affecting one observation at the middle of the sample $t = 50$ or one observation near the end of the series $t = 90$. The size of the initial impact ω was considered to vary from 3.0 to 5.0. For each combination of type and location we generated 1000 simulated series.

Table 3 reports the mean relative frequency of correct detection for critical values $C = 2.5, 3.0, 3.5, 4.0, 4.5$ and 5.0. In general, the power of the iterative procedure increases with the size of the initial impact ω and decreases with the critical value C, whereas the location of the perturbation, at the middle or near the end of the series, does not seem to have any significant effect. The power of the iterative procedure is greater in the cases of structural changes in the seasonal or trend component than in the case of one outlier. Going back to table 2, for a sample size of 100 observations a critical value of $C = 4.0$ yields a frequency of error type I smaller than 4% in all the considered models except in model I. Using the same critical value, the power of the iterative procedure in the case of one outlier is greater than 50% for $\omega = 3.0$, greater than 75% for $\omega = 3.5$ and, for $\omega \geq 4.0$, the power is greater than 90%. In the cases of structural changes in the seasonal or the trend component, the power is around 70% for $\omega = 3.0$; around 90% for $\omega = 3.5$ and greater than 99% for $\omega \geq 4.0$.

The power of the procedure when two perturbations are present was also investigated. The BSM with $\sigma_\epsilon^2 = \sigma_\eta^2 = \sigma_\omega^2 = 1$ and $\sigma_\xi^2 = 0$ was used to generated simulated series of 100 observations. We considered six cases with different combinations of outliers and structural changes in trend and seasonal component that are listed bellow,

1. Two outliers at t=50 and t=80.
2. Two structural changes in the seasonal component at t=50 and t=80.
3. Two structural changes in the trend component at t=50 and t=80.
4. One outlier at t=50 and one structural change in the seasonal component at t=80.
5. One structural change in the trend component at t=50 and one structural change in the seasonal component at t=80.
6. One structural change in the trend component and one structural change in the seasonal component both at t=50.

As before, the size of the initial impact ω was considered to vary from 3.0 to 5.0 and for each combination we generated 1000 simulated series. Table 4 reports, for critical values $C = 3.0, 4.0$ and 5.0, in the columns marked with $\mathbf{F_2}$, the mean relative frequency of correct detection of the two perturbations that affect the simulated series and, in the columns marked with $\mathbf{F_1}$, the mean relative frequency of correct detection of one of the two perturbations. The performance of the iterative procedure in correctly detecting the two perturbations is an increasing function of ω and a decreasing function of C. In cases 1-3, where the two perturbations are of the same type, it is again clear that it is more difficult to detect outliers than structural changes, specially when ω is small. In case 1, for a critical value of $C = 4.0$ and size $\omega = 4.0$, $\mathbf{F_2} = 75\%$ and $\mathbf{F_1} = 24\%$ while, for the same values of C and ω, $\mathbf{F_2} = 90\%$ and $\mathbf{F_1} = 9.7\%$ in case 2 and, $\mathbf{F_2} = 94\%$ and $\mathbf{F_1} = 5\%$ in case 3. Cases 4 and 5 are very similar; for small values of ω are worse than cases 2 and 3 but, as ω increases, the differences among the four cases tend to decrease. Finally, for case 6, the power is greater than it was expected. For a critical value of $C = 4.0$ and size $\omega = 4.0$, the procedure detects in the first two iterations the two perturbations with a relative frequency of 76%, and for size $\omega = 5.0$ and the same critical value, the relative frequency of correct detection of the two perturbations increases to 95%. Note that the power in case 6 is greater than the power in case 1 for small values of ω and it is only slightly worse when $\omega \geq 4.0$. This good performance of the iterative procedure in case 6 may imply that there is no need to define a new type of structural change to capture the effect of a perturbation affecting two or more unobserved components at the same time (see the end of section 3) since the power of the iterative procedure in terms of perturbation detection is still high when a simultaneous perturbation occurs.

Concluding this section, we would recommend to base the election of the critical value C in both the length of the series and the variances of the noises in the models for the unobserved components. The critical value $C = 4.0$ may be adequate for a series of moderate length, i.e. 100 to 200 observations, and moderate values of the variances. In practice, it is recommended to use more than one critical value in the analysis, to allow the investigation of the results sensitivity.

6 Examples

In this section we consider two examples discussed in Harvey (1989), the log-transformed quarterly UK gas consumption by other final users and the log-transformed monthly number of car drivers killed or seriously injured.

The gas consumption series starts in the first quarter of 1960 and ends in the fourth quarter of 1986. The plot of the observed series is represented in figure 1.a with a solid line. Note that from the beginning of the seventies, the pattern of the seasonal fluctuations changes coinciding with the introduction of natural gas. The programs TRAMO and SEATS were used to decompose the observed series into unobserved components of trend, seasonal and irregular following the ARIMA model based approach (see Gómez and Maravall, 1996). The models for the trend and seasonal components are, respectively,

$$\nabla^2 \mu_t = (1 + 0.0786B - 0.9214B^2) b_t$$

$$\sum_{j=0}^{3} s_{t-j} = (1 - 0.1792B - 0.4755B^2 - 0.3453B^3) c_t$$

where the variances are $\sigma_b^2 = 0.00942, \sigma_c^2 = 0.11937$ and $\sigma_\epsilon^2 = 0.27473$,and the reduced form is,

$$\nabla \nabla_4 log z_t = (1 - 0.9196B)(1 - 0.2446B^4) a_t$$

where z_t is the gas consumption series. The variances for the unobserved components are obtained using that $\theta(B)a_t = \theta_\mu(B)\tilde{\phi}_s(B)b_t + \theta_s(B)\tilde{\phi}_\mu(B)c_t + \tilde{\phi}_\mu \tilde{\phi}_s(B)\epsilon_t$, and are expressed in units of σ_a^2, the variance of the noise in the reduced form. The variance for the noise associated with the irregular component represents, for the gas consumption, the 27% of the noise variance in the reduced form model, whilst the variances associated with the trend and seasonal component represent the 3.8% and the 0.8%, respectively. Applying the iterative procedure with a critical value of $C = 4.0$ two perturbations were detected,

First iteration.	Seasonal Structural change at t=1971.1 with $\omega = -0.3510$
Second iteration.	Seasonal Structural change at t=1970.2 with $\omega = -0.1951$

Figure 1.a shows that the change in the seasonal pattern is less pronounced for the adjusted series plotted with a dashed line. Figure 1.b shows, for a reduced sample beginning in 1968.1 and ending in 1979.4, the original and adjusted seasonal components, and makes clearer the effects of the two interventions that reduce the magnitude

of the seasonal fluctuations. Finally figure 1.c shows the sum of the effects of the two structural changes. The models for the adjusted trend and seasonal components are, respectively,

$$\nabla^2 \mu_t = (1 + 0.1349B - 0.8641B^2)b_t$$

$$\sum_{j=0}^{3} s_{t-j} = (1 - 0.1748B - 0.4763B^2 - 0.3488B^3)\left\{c_t - 0.351I_t^{(71.1)} - 0.195I_t^{(70.2)}\right\}$$

where the variances are now $\sigma_b^2 = 0.00941$, almost equal to the original, $\sigma_c^2 = 0.0854$, reduced by the effect of the two seasonal structural changes that have been corrected, and $\sigma_\epsilon^2 = 0.3214$.

The second series, the log-transformed number of car drivers killed or seriously injured, starts in January 1960 and ends in December 1984. This series has been used by several authors, see Harvey and Durbin (1986), Harvey and Koopman (1992) or Balke (1993). The observed series is plotted with a solid line in figure 2.a. The trend pattern of the series seems to be composed of three different periods; during the first five years the series exhibits an upwards trend, the second period is characterized for a very stable level without a clear upwards or downwards trend and finishes with the introduction of the seat belt law on January 1983, the third period lasts for about two years and exhibits a soft upwards trend. The models for the trend and seasonal components are respectively,

$$\nabla p_t = (1 + 0.0094B - 0.9906B^2)b_t$$

$$\begin{aligned}\sum_{j=0}^{11} s_t &= (1 + 0.96B + 0.74B^2 + 0.46B^3 + 0.16B^4 - 0.10B^5 \\ &- 0.31B^6 - 0.46B^7 - 0.56B^8 - 0.60B^9 - 0.61B^{10} - 0.66B^{11})c_t\end{aligned} \tag{19}$$

where the variances are $\sigma_b^2 = 0.03806, \sigma_c^2 = 0.0026$ and $\sigma_\epsilon^2 = 0.56528$,and the reduced ARIMA form is,

$$\nabla\nabla_{12} log z_t = (1 - 0.5892B)(1 - 0.8924B^{12})a_t$$

where z_t is the number of car drivers killed or seriously injured.

Applying the iterative procedure with a critical value of $C = 4.0$ two perturbations were detected,

First iteration.	Trend Structural change at t=1983.01 with $\omega = -0.1385$
Second iteration.	Trend Structural change at t=1973.10 with $\omega = -0.1187$

In figure 2.a the adjusted series, plotted with a dashed line, exhibits an upwards trend for the entire sample. Figure 2.b shows the effect of the two structural changes in the trend component. The models for the adjusted trend and seasonal components are, respectively,

$$\nabla p_t = (1 + 0.0134B - 0.9866B^2)\left\{b_t - 0.138I_t^{(83.01)} - .118I_t^{(73.10)}\right\}$$

$$\begin{aligned}\sum_{j=0}^{11} s_t &= (1 + 0.69B + 0.41B^2 + 0.16B^3 - 0.04B^4 - 0.20B^5 \\ &- 0.30B^6 - 0.36B^7 - 0.38B^8 - 0.37B^9 - 0.33B^{10} - 0.27B^{11})c_t\end{aligned} \qquad (20)$$

where the variances are $\sigma_b^2 = 0.01609, \sigma_c^2 = 0.00846$ and $\sigma_\epsilon^2 = 0.63701$. The variance of the trend component represents now the 1.6% of σ_a^2, less than half of the proportion for the original trend component, while the proportion for the irregular and seasonal components increases.

7 Conclusion

In this paper a procedure for detecting outliers and structural changes in the unobserved components of a seasonal time series was proposed. The iterative procedure was illustrated with real and simulated series and, consequently, may be of interest for data analyst focused on the seasonal adjustment of economic time series.

A final remark must be made on the nature of the iterative procedure. An observed time series may be subject to the influence of many different types of perturbations. The method proposed here permits the treatment of some of them, not all, that have a clear and direct interpretation.

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Model I.	$\sigma_\epsilon^2 = 20, \sigma_\eta^2 = 1, \sigma_\xi^2 = 0$ and $\sigma_\omega^2 = 0.25$
Model II.	$\sigma_\epsilon^2 = 3, \sigma_\eta^2 = 1, \sigma_\xi^2 = 0$ and $\sigma_\omega^2 = 0.75$
Model III.	$\sigma_\epsilon^2 = 1.13, \sigma_\eta^2 = 1, \sigma_\xi^2 = 0$ and $\sigma_\omega^2 = 0.01$
Model IV.	$\sigma_\epsilon^2 = 0.70, \sigma_\eta^2 = 1, \sigma_\xi^2 = 0$ and $\sigma_\omega^2 = 0.25$
Model V.	$\sigma_\epsilon^2 = 0.70, \sigma_\eta^2 = 1, \sigma_\xi^2 = 0.25$ and $\sigma_\omega^2 = 0.25$
Model VI.	$\sigma_\epsilon^2 = 0.5, \sigma_\eta^2 = 0.25, \sigma_\xi^2 = 0.25$ and $\sigma_\omega^2 = 0.25$

Table 1. List of models in the first simulation exercise. * Models I to IV were taken from Maravall(1985)

	Model I						Model II					
$N\backslash C$	2.5	3.0	3.5	4.0	4.5	5.0	2.5	3.0	3.5	4.0	4.5	5.0
50	1.000	0.862	0.494	0.166	0.032	0.006	0.906	0.348	0.060	0.004	0.002	0.000
100	1.000	0.980	0.838	0.436	0.144	0.034	0.992	0.684	0.168	0.020	0.004	0.000
200	1.000	1.000	0.980	0.780	0.378	0.114	1.000	0.926	0.390	0.064	0.008	0.002
300	1.000	1.000	0.998	0.916	0.574	0.252	1.000	0.964	0.488	0.100	0.016	0.000
	Model III						Model IV					
$N\backslash C$	2.5	3.0	3.5	4.0	4.5	5.0	2.5	3.0	3.5	4.0	4.5	5.0
50	0.880	0.346	0.060	0.004	0.000	0.000	0.870	0.260	0.036	0.004	0.000	0.000
100	0.990	0.644	0.180	0.042	0.002	0.000	0.980	0.504	0.092	0.010	0.000	0.000
200	1.000	0.868	0.340	0.070	0.012	0.000	1.000	0.826	0.226	0.040	0.006	0.000
300	1.000	0.974	0.516	0.110	0.020	0.006	1.000	0.912	0.344	0.042	0.002	0.000
	Model V						Model VI					
$N\backslash C$	2.5	3.0	3.5	4.0	4.5	5.0	2.5	3.0	3.5	4.0	4.5	5.0
50	0.856	0.294	0.042	0.000	0.000	0.000	0.858	0.288	0.036	0.002	0.000	0.000
100	0.976	0.522	0.098	0.002	0.000	0.000	0.986	0.550	0.114	0.008	0.000	0.000
200	0.998	0.850	0.244	0.024	0.002	0.000	1.000	0.840	0.240	0.040	0.000	0.000
300	1.000	0.918	0.320	0.062	0.008	0.000	1.000	0.924	0.358	0.054	0.002	0.000

Table 2. Mean relative frequency of error type I using the iterative procedure.

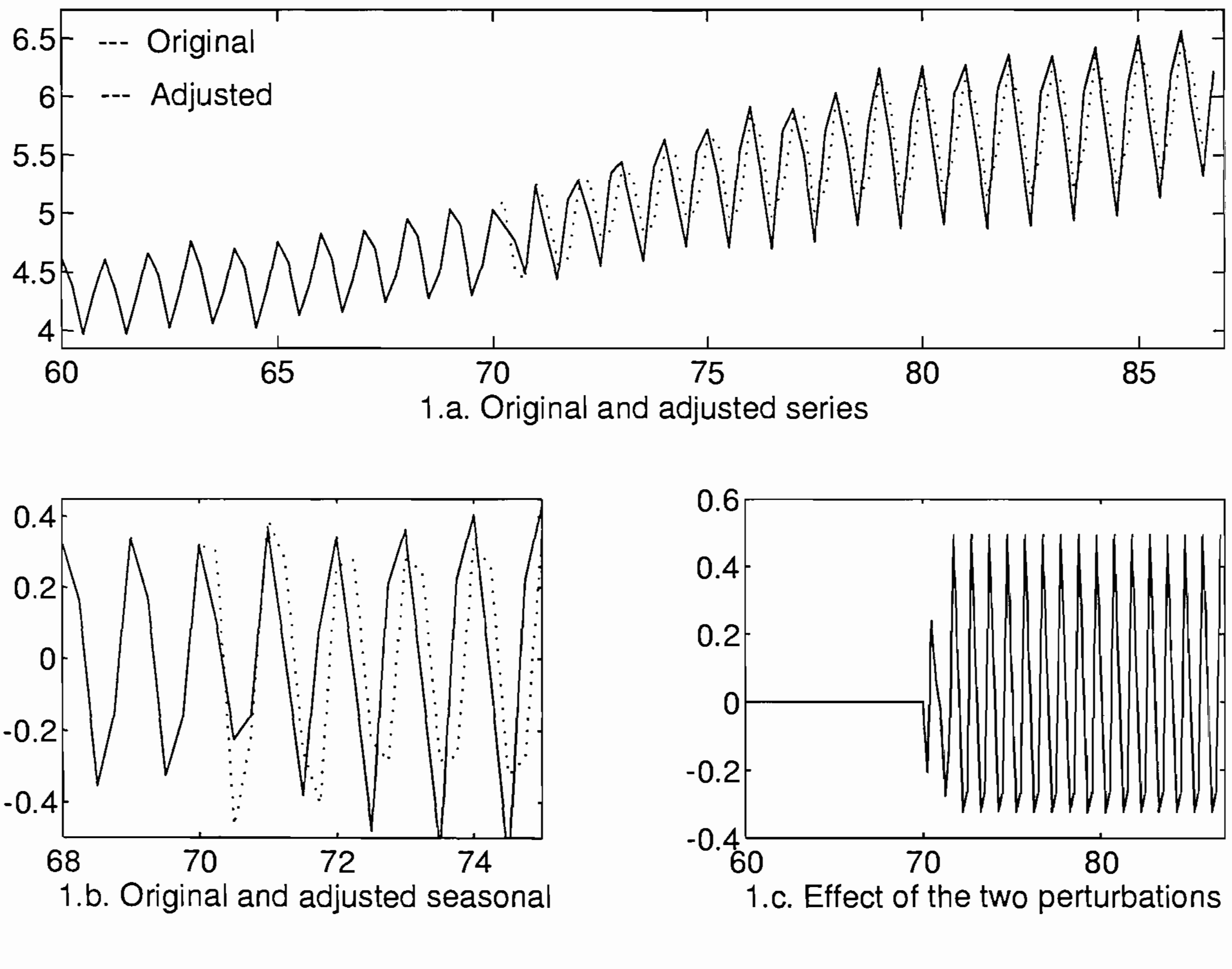
	Outlier $T = 50$						Outlier $T = 90$					
$\omega \backslash C$	2.5	3.0	3.5	4.0	4.5	5.0	2.5	3.0	3.5	4.0	4.5	5.0
3.0	0.951	0.873	0.738	0.552	0.351	0.199	0.963	0.894	0.755	0.556	0.322	0.154
3.5	0.994	0.969	0.910	0.785	0.583	0.389	0.993	0.969	0.906	0.781	0.583	0.358
4.0	1.000	0.995	0.977	0.930	0.827	0.656	1.000	0.996	0.979	0.926	0.803	0.616
4.5	1.000	0.998	0.996	0.981	0.938	0.855	1.000	1.000	0.999	0.982	0.941	0.833
5.0	1.000	0.999	0.998	0.996	0.983	0.952	1.000	0.999	0.999	0.999	0.986	0.943
	ST in Seasonal $T = 50$						**ST in Seasonal** $T = 90$					
$\omega \backslash C$	2.5	3.0	3.5	4.0	4.5	5.0	2.5	3.0	3.5	4.0	4.5	5.0
3.0	0.961	0.926	0.839	0.671	0.445	0.236	0.984	0.959	0.883	0.686	0.430	0.225
3.5	0.990	0.988	0.972	0.885	0.733	0.500	0.995	0.991	0.972	0.910	0.756	0.525
4.0	0.999	0.999	0.997	0.973	0.919	0.773	0.998	0.997	0.995	0.980	0.915	0.780
4.5	1.000	1.000	1.000	0.997	0.975	0.916	1.000	1.000	1.000	0.999	0.991	0.944
5.0	1.000	1.000	1.000	1.000	0.997	0.982	1.000	1.000	1.000	1.000	1.000	0.991
	ST in Trend $T = 50$						**ST in Trend** $T = 90$					
$\omega \backslash C$	2.5	3.0	3.5	4.0	4.5	5.0	2.5	3.0	3.5	4.0	4.5	5.0
3.0	0.984	0.949	0.885	0.751	0.546	0.307	0.986	0.946	0.869	0.720	0.481	0.258
3.5	1.000	0.992	0.967	0.912	0.764	0.528	0.997	0.988	0.963	0.894	0.761	0.529
4.0	1.000	0.999	0.996	0.989	0.941	0.829	1.000	1.000	0.992	0.977	0.916	0.777
4.5	1.000	1.000	0.999	0.992	0.976	0.937	1.000	1.000	1.000	0.997	0.983	0.934
5.0	1.000	1.000	1.000	1.000	0.998	0.989	1.000	1.000	1.000	0.999	0.998	0.985

Table 3. Mean relative frequency for correct detection of one perturbation in a simulated series of length N=100.

	Case 1						Case 2					
C	3.5		4.0		4.5		3.5		4.0		4.5	
ω	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$
3.0	0.486	0.435	0.221	0.543	0.077	0.426	0.794	0.206	0.571	0.425	0.284	0.677
3.5	0.744	0.248	0.488	0.441	0.242	0.538	0.934	0.066	0.777	0.223	0.518	0.479
4.0	0.905	0.301	0.753	0.242	0.497	0.448	0.975	0.025	0.903	0.097	0.697	0.303
4.5	0.972	0.028	0.894	0.105	0.732	0.262	0.994	0.006	0.957	0.043	0.837	0.163
5.0	0.992	0.008	0.966	0.034	0.890	0.110	0.999	0.001	0.988	0.012	0.933	0.067
	TC in $T = 50$ and in $T = 80$						**IC in $T = 50$ and SC in $T = 80$**					
C	3.5		4.0		4.5		3.5		4.0		4.5	
ω	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$
3.0	0.793	0.084	0.565	0.157	0.339	0.160	0.724	0.265	0.476	0.463	0.212	0.573
3.5	0.937	0.042	0.807	0.118	0.570	0.212	0.905	0.094	0.753	0.236	0.456	0.050
4.0	0.986	0.011	0.940	0.049	0.785	0.164	0.973	0.027	0.895	0.105	0.731	0.266
4.5	0.997	0.002	0.973	0.025	0.894	0.088	0.995	0.005	0.970	0.030	0.872	0.128
5.0	1.000	0.000	0.994	0.006	0.970	0.028	0.997	0.003	0.989	0.011	0.936	0.064
	TC in $T = 50$ and SC in $T = 80$						**TC and SC in $T = 50$**					
C	3.5		4.0		4.5		3.5		4.0		4.5	
ω	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_1}$
3.0	0.705	0.284	0.431	0.502	0.180	0.591	0.582	0.392	0.339	0.550	0.145	0.570
3.5	0.882	0.118	0.699	0.030	0.426	0.535	0.763	0.235	0.560	0.427	0.319	0.604
4.0	0.947	0.053	0.859	0.141	0.651	0.347	0.903	0.097	0.760	0.240	0.550	0.439
4.5	0.989	0.011	0.958	0.042	0.833	0.167	0.960	0.040	0.873	0.127	0.732	0.268
5.0	0.997	0.003	0.984	0.016	0.920	0.080	0.985	0.015	0.950	0.050	0.855	0.145

Table 4 Mean relative frequency for correct detection of two perturbations in a simulated series of length N=100. $\mathbf{F_2}$ Correct detection of the two structural changes; $\mathbf{F_1}$ Correct detection of only one of the two changes.

Figure 1. UK gas consumption by other final users

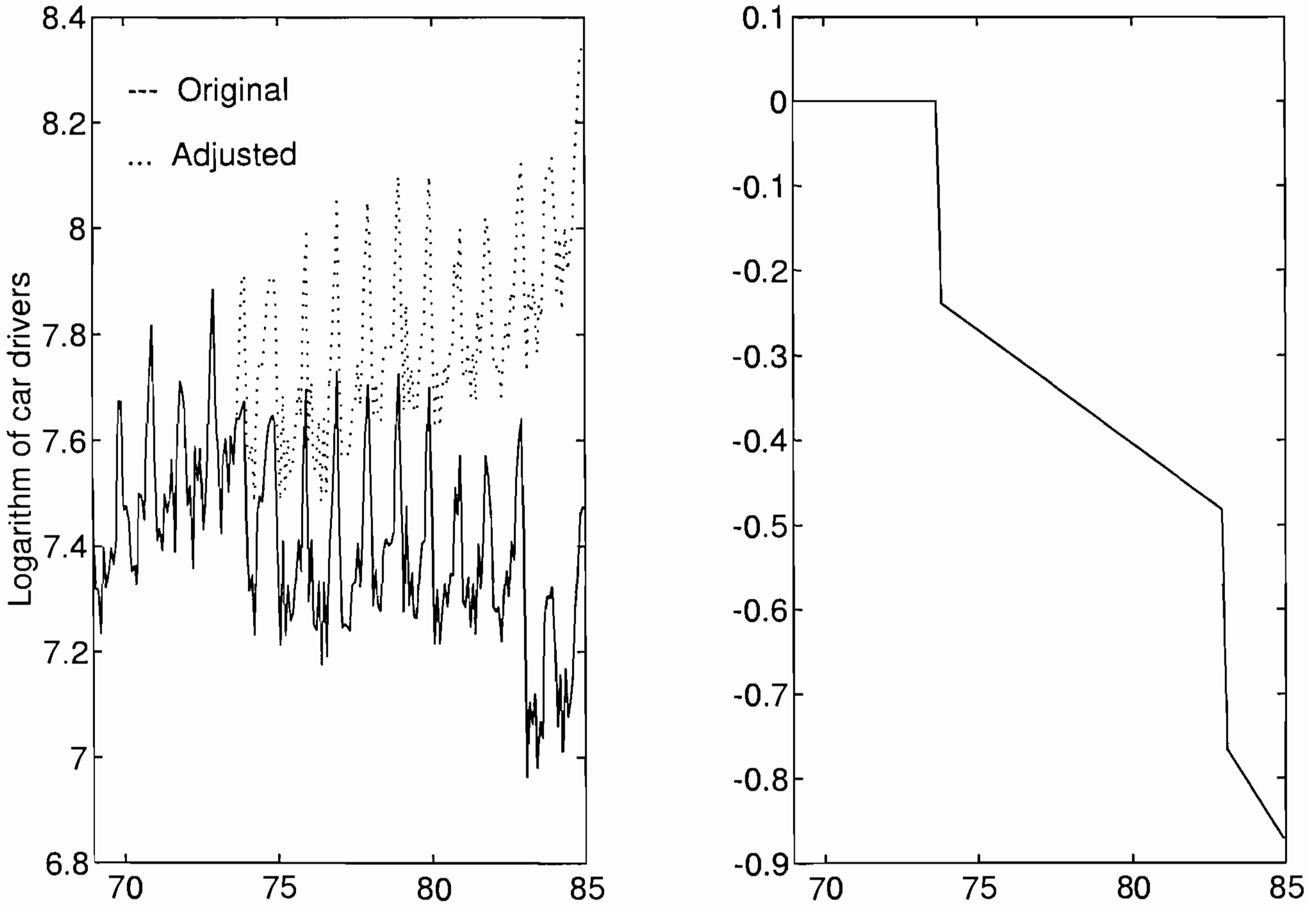


1.a. Original and adjusted series

1.b. Original and adjusted seasonal

1.c. Effect of the two perturbations

Figure 2. Car drivers killed and seriously injured



2.a. Original and adjusted series

2.b. Effect of the two trend changes