

Brain Tumor Quantification Equation: Modeled on Complete Step Response Algorithm

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Abstract— In Image Guided neuro-Surgery (IGnS) protocol relating to tumor, the planning stage is the bottleneck where most times are spent reconstructing the slices in order to; quantify the tumor, get the tumor shape and location relative to adjacent cells, and determine best incursion route among others. This time consuming assignment is handled by a surgeon using any of the standardized IGnS software. It has been observed that the approach taken to quantify tumor in those software are simply replicating the surgeons' experience-based brain tumor quantification technique fashionable in the pre-imaging era. The result is a quantification method that is time consuming, and at bests an approximation. What is presented here is a novel brain tumor quantification method based on step response algorithm utilizing a model which itself was based on step response model resulting in smart and rapid quantification of brain tumor volume.

Keywords- Image Guided neuro-Surgery; Brain Tumor Quantification; Step Response Model; Smart quantification.

I. INTRODUCTION

One of the core principles in maintenance engineering is that the time to detect fault and restore the system back to good working condition should be as small as possible to reduce down-time. When the whole IGnS protocol is observed, the time spent at the planning stage is phenomenal [1] especially when the surgeon is new or inexperienced in IGS protocol. Many approaches have been suggested for reducing the time spent on Image Guided Surgery (IGS) as a whole [2][3][4][5]. Moreover several works has equally been done that were aimed at getting a growth model or tumor treatment [6][7][8]. However, we observed that since one of the work at that stage is to know the size and shape of tumor, for tumor related operation, we thus observed that, having a good model of brain tumor growth pattern as in [9], and a mathematical equation based on the model will help in quickly determining the size of tumor in the patients' brain without having to spent time at the planning stage. The rest of the paper is arranged thus: Brain tumor growth model was discussed in section two, section three is on the derivation of mathematical equation for tumor quantification, application of the equation as a proof of concept and design of validation technique was the subject of section

four while section five discussed the result and gave a summary of the work. Conclusion was draw in section six together with our future work.

II. BRAIN TUMOR GROWTH MODEL

The work in [9] proposed that the general pattern of growth of brain tumor on slice by slice basis is governed by equation (1):

$$\varphi = -\frac{h}{\ln\left(\frac{\alpha}{\beta}\right)} \dots \dots \dots (1)$$

Where $T_u(0) - T_u(\infty)$ and $T_u(h) - T_u(\infty)$ are β and α respectively, h is slice thickness, and φ is the tumor growth and decay model called behavioural function. It is expanded as:

$$\varphi = -\frac{h}{\ln\left(\frac{T_u(h) - T_u(\infty)}{T_u(0) - T_u(\infty)}\right)} \dots \dots \dots (2)$$

Where $T_u(h)$, $T_u(0)$, and $T_u(\infty)$ stand for tumor area at any point within slice thickness (h), tumor area at the lower surface of slice, and tumor area at the upper surface of slice assuming that the tumor area at the lower surface is less than that of the upper surface.

The work went further to decompose φ into a function of ∇ (nabla) and ρ (rho) as in:

$$\varphi = \nabla f(\rho) \dots \dots \dots (3)$$

in which nabla is a constant called behavioural factor and static part of φ , it is a kind of generic curve for tumor growth and decay whose numerical value was experimentally fixed at 1.1 as in figure 1, and rho is the dynamic part of φ which moves nabla within the quadrant. This resulted into an exponential model for finding the instantaneous area of tumor in a slice as in equation (4) [last work]

$$T_{um}(h) = T_u(\infty) + \beta e^{(-h/\varphi/\rho)} \dots \dots \dots (4)$$

Where β is $Tu(0) - Tu(\infty)$, and ρ is $[Tu(\infty) - Tu(0)]$

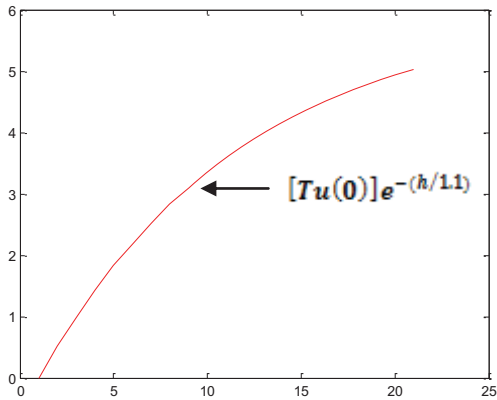


Fig.1: The Brain Tumor Model Curve

III. MATHEMATICAL EQUATION FOR TUMOR QUANTIFICATION

The brain tumor model equation in equation (4) is an equation for determining the instantaneous tumor area at any given point within the slice thickness (h). Conversely, the intended equation for tumor quantification is meant to find the volume of tumor within and between slice(s). This makes equation (4) unsuitable for the job at hand, hence we find its integral function to make it conform to the present need.

A. Finding the Integral of Equation (4)

It is mathematically possible to use the equation for instantaneous value to determine the value of a range by simple repetitive method and changing the time-step bit-by-bit. However, this is unprofessional, tedious, and probably inaccurate as it may be impossible to pick all the points in a range one after the other. A more professional and mathematically acceptable method is to integrate the equation and fix the range into it. In this way a whole range or length would be evaluated at ones. Since we are confronted with knowing the volume of tumor within a slice having known the tumor area at both the upper and lower surfaces of the slice together with the slice thickness, integration of equation (4) with respect to slice thickness (h) and use of the of limits of (h) solves the problem.

$$V_t(h) = \int_{h=0}^{\infty} Tu(\infty) + \beta e^{-0.91\rho h} dh \dots \dots (5)$$

Where β is $Tu(0) - Tu(\infty)$, and $V_t(h)$ denotes tumor volume with respect to h. Therefore equation (5) becomes

$$V_t(h) = \int_{h=0}^{\infty} Tu(\infty) + [Tu(0) - Tu(\infty)] e^{-0.91\rho h} dh \dots \dots (6)$$

Hence, equation (6) becomes:

$$V_t(h) = \int_{h=0}^{\infty} Tu(\infty) + [Tu(0) - Tu(\infty)] e^{-0.91((Tu(\infty)-Tu(0))h} dh \dots \dots (7)$$

IV. APPLICATION OF EQUATION, PROVE OF CONCEPT / VALIDATION

The application of the equation and proof of this concept shall be done using known mathematical concept to solve tumor volume problem and compare the result with the one using our equation ($V_t(h)$), that is equation (7). The validation design is presented stage-wise in the proceeding sub section.

A. Validation Design

The images used are non-IGS protocol images because of the fairly large intervals (6mm) compare with IGS protocol images. However, since the radiological images are not naturally fit into a form that will allow direct use of any known mathematical concept, the first step was is to render radiological images into known shapes and apply mensuration to solve for the parameters. In connection to this, we carefully selected patients whose two consecutive tumor images are fairly circular (see figures 2) with slice thickness of 6mm. The five steps taken in designing the validation technique are:

- 1) Extract the circular shapes of tumor from the two surfaces of the slice – figure 2
- 2) Link the edges of the shapes together to form a pipe or vestige cone of slant heights 6mm (slice thickness).
- 3) Project the slant heights to meet on a line linking the radii of the two circles and perpendicular to the diameters of the circles to form a full cone.
- 4) Measure the height of the full cone, and the radii of the two circles.
- 5) Then, apply an appropriate mensuration formula to find the volume of the full cone, the smaller cone, and the vestige cone (the tumor volume).

The rendition of the image property into known shapes is shown in figure 3, and figure 4 shows how mensuration formula is used in determining tumor volume. However, we made use of engineering drawing software called Solidworks which made the whole process of this validation design easier than listed above.

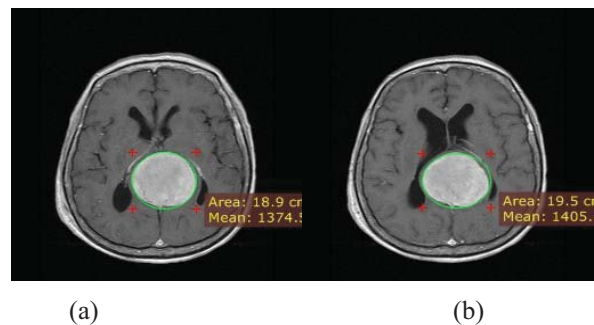


Fig.2. Images depict tumor growth from 18.9cm² (a) to 19.5cm² (b) within 6mm interspaces or slice thickness [(a) is lower surface, and (b) is upper surface].

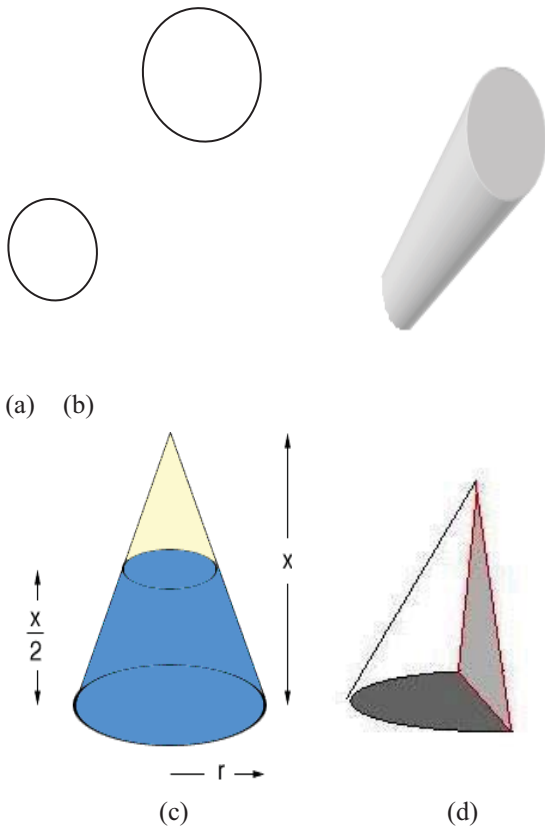
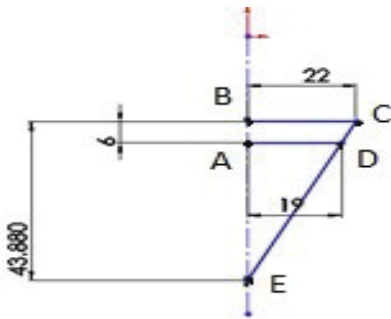


Fig. 3a-c: (a) Circles extracted from image surfaces; (b) Circles rendered into vestigial cone; (c) Vestigial cone projected into full cone; (d) Full cone made into half cone.



$$V_c(H) = \frac{\pi r^2_{BC} BE}{3} \dots\dots\dots (8)$$

$$V_c(h_1) = V_c(H) - V_c(h_2) \dots\dots\dots (9)$$

$$V_c(h_2) = \frac{\pi r^2_{AD} AE}{3} \dots\dots\dots (10)$$

Fig. 4: Arrangement for Mathematically Determining Tumor Area. Equation 8 is for the whole cylinder BCDEAB, equation 9 is for the small volume bounded by BCDAB, and equation 10 for ADEA.

Figures 5 and 6 were also subjected to the same procedure and the volumes obtained for the three, that is, figures 2, 5, and 6 are compared with our equation in table 1.

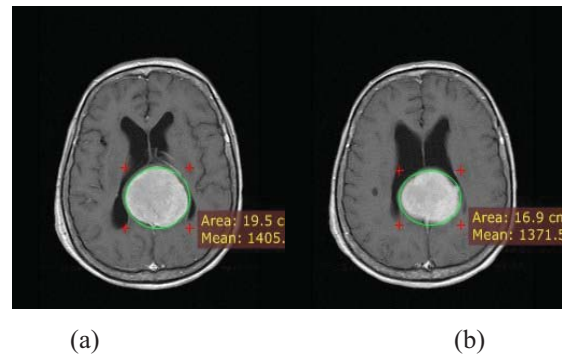


Fig. 5: Images show tumor decay from 19.5cm² (a) to 16.9cm² (b) within 6mm interspaces or slice thickness [(a) is upper surface, and (b) is lower surface].

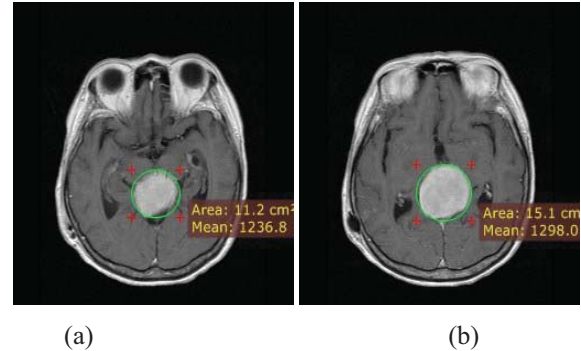


Fig. 6: Images depict tumor growth from 11.2cm² (a) to 15.1cm² (b) within 6mm interspaces or slice thickness [(a) is lower surface, and (b) is upper surface].

B. Use of Instantaneous Tumor Area (T_{um}(h)) Formula

In addition to integrating equation (4), a logical approach is required in solving the dynamic part of equation. The logical approach is in the use of j operator to arrive at a desired solution to equation (7) when h = 0. The normal solution to equation (7) under this condition does not tally with both experimental and mathematical solutions hence, the need to adjust the solution. The adjustment is made by making [-e⁰] = j². Therefore, the solution to equation (7) includes a special kind of complex number whose the imaginary part is j√-1β. Hence, the tumor volume is the real part of the solution.

V. RESULTS DISCUSSION

At this preliminary stage, the results using standard formula were compared with the result obtained using our new formula for tumor quantification. The results showed a high level of correlation between our new formula and the standard formula. The significance of this result is that our goal of devising a quicker way of finding the volume of tumor in both intra and inter slice is close to being achieved. This would be a novel and landmark achievement in that the need for slice reconstruction is eliminated in IGS planning, once this idea is incorporated into the IGS planning software.

VI. CONCLUSION AND FUTURE WORK

This paper started by studying the equation for deriving the instantaneous area of tumor within an image slice, and later advanced it for determining the volume of tumor for both intra

slice and inter slice by evolving its integral form. Moreover, in order to make comparison between our equation and method of using it to obtain accurate tumor volume with the mathematical concept, some smooth edge circular shape tumor slices were used (figure 2, 5, and 6). Our formula and methods were applied on one hand, and for the mathematical concept on the other hand, a full cone is drawn (by projection) from the parameters of $Tu(0)$ and $Tu(\infty)$. The cone is then considered cut into two part namely; the tumor part and the projected part. Mensuration formula was then applied to get the tumor part out of the cone. As seen in figure 4, a half cone (symmetrical by the y-axis (BAE)), and the equations used in getting out the tumor volume. We intend to concretize our validation by further comparing our result with many other software that are used for medical image analysis

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