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# THE POLITICAL ECONOMY OF INTERNATIONAL PRIVATE INSURANCE AND FISCAL POLICY

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Abstract

We consider a two-country model in which international risk-sharing is beneficial. Even though complete contingent markets exist to trade private wealth, the fact that fiscal policy voting decisions have an impact on contingent wealth prices implies that government spending will be inflated in good states and deflated in bad ones, with the following general implications: (i) Prices of contingent wealth are distorted; (ii) Volatility of public spending increases; (iii) Incomplete insurance arises. An example shows that apart from the increase in the volatility of public spending, it is also possible that average spending increases in both countries. These distortions are shown to be stronger the more similar the two countries are in ex ante terms. We compare the decentralized system with a fiscal union contrasting equilibrium properties in terms of government spending and allocation of risk.

Key Words

Private International Insurance; Market; Fiscal Union; Decentralized Fiscal Policy; Risksharing.

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# 1 Introduction

Fiscal policy is often advocated as a means to provide households with insurance in situations in which a private insurance market is not viable. Fiscal policy is argued to provide insurance both in the form of production of goods and services and in the form of cash transfers, for instance through unemployment benefits. While this kind of fiscal policy has undeniable positive effects in that it increases risk-sharing, it is also well understood that it may have negative efficiency implications due to the fact that it normally relies on distortionary taxation and it leads to (possibly undesirable) redistribution of income through possibly inefficient instruments.

Existing research on insurance and fiscal policy has concentrated on the case in which optimal risk-sharing cannot be achieved because markets are incomplete. Eaton and Rosen (1980) and Easley, Kiefer, and Possen (1985, 1993) study redistribution of income through taxation or through unemployment insurance; Wright (1986) studies public unemployment insurance in a dynamic model; Sachs and Sala-i-Martin (1992) provide estimates of federal insurance against shocks to state GDP in the US and show that about 40% of GDP reduction in absorbed via reduction in federal taxes and increase in federal transfers.

Similar arguments are also used to show how fiscal unions can provide insurance to the residents of the member states and how they can solve the *fiscal externalities problem* first introduced by Stigler (1957). This approach has been followed among others by Oates (1972), Mintz and Tulkens (1986), Gordon (1993), Alesina and Perotti (1994), Bolton and Roland (1995), Persson and Tabellini (1996a, b) and Inman and Rubinfeld (1996).

Making the assumption that no form of insurance is available at all is of course only a stylized description of a situation in which markets are incomplete. This paper tries to highlight the fact that making this extreme assumption in fact hides two important elements of the problem: (i) The way in which available private insurance affects fiscal policy decisions; (ii) The way in which fiscal policy decisions have an impact on private insurance markets. To analyze these effects we consider a situation in which the existence of contingent wealth markets enables individuals to insure their wealth and we study the equilibrium determination of fiscal policy in a direct democracy model in which taxing (or equivalently, spending) decisions are made by the median voter.

The general idea that will be pursued in this paper is that a median voter is in the position of affecting the distribution of wealth not only across individuals but also across states of nature with the implication that he has the ability to affect demand for insurance and as a consequence its equilibrium price. This general intuition is pursued in a specific setting in which two countries have perfect negative correlation in their wealth levels, so that perfect insurance is a feasible outcome, and in which fiscal policy is decided by the median voter in each of the two countries. We consider a number of situations differing in terms of tax bases, and in terms of the distribution of wealth across states of nature when voting takes place. We characterize situations in which voting decisions are motivated by the desire to manipulate contingent wealth prices and analyze equilibrium outcomes in terms of fiscal policy and contingent wealth prices and trades.

Our results show that if voting takes place when individual households are not already perfectly insured, the median voters distort contingent wealth prices by setting tax rates in such a way as to manipulate contingent wealth excess demand functions of households in their same country. As compared to a situation in which individuals have perfectly insured wealth levels, distortions are shown to exist in equilibrium, where both countries choose higher government spending in good states of nature and lower in bad ones and the result of this competitive manipulation is a minor effect on contingent wealth prices, but higher average public spending. Median voters are shown to be worse off than in a situation in which tax rates (or government expenditures) are set once households have insured their wealth levels in competitive markets.

The general result of the paper is that whenever voting takes place at a time in which individuals do not hold perfectly insured portfolios, voting decisions may be affected by the desire to manipulate contingent wealth prices and this has two main implications: (i) Even though complete contingent markets exist and perfect insurance is feasible and desirable, equilibrium prices and fiscal policies imply that incomplete insurance arises; (ii) Fiscal policy is different from what it would be if individuals could insure their wealth levels before voting (in the specific examples we analyze, average public spending turns out to be higher).

Our results show how competitive tax setting in a decentralized system leads to a fiscal externality problem in the sense that it creates distortions in insurance markets and ultimately leads to higher public spending. This idea leads us to study the potential benefits of fiscal integration. Centralized fiscal policy is known to lead to higher spending whenever integration leads to higher income inequality which is likely to happen when the different countries have very different income distributions.[1]

While impossible in a centralized system, manipulation of contingent wealth prices and the increase in average public spending associated to it in a decentralized system will be shown to be highest when the two countries are very similar to each other in the sense that they have prior probabilities of experiencing a positive shock which are approximately equal and tend to disappear as the probability of one country experiencing the good shock goes to 1, i.e., as they become very different in ex ante terms.

The combination of these two effects has the following implications: (a) While a fiscal union always leads to higher risk sharing, this efficiency gain is higher the more similar the two countries are in ex ante

[1] Perotti (1994) highlights how "redistribution takes up the largest share of the government budget virtually everywhere in the industrialized world."

terms and tends to disappear when they are very different; (b) When countries are very similar in ex ante terms government spending is higher under decentralized fiscal policy than under centralized fiscal policy as the manipulation motive dominates the redistribution motive; When countries are very different in ex ante terms, the opposite holds.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the case in which taxes are levied on wealth gross of contingent wealth trades. Section 4 presents numerical computations for a specific example and discusses the results of the previous section. Section 5 considers the case of a fiscal union in which tax rates and spending are constrained to be equal across countries and compares to the case of decentralized fiscal policy. Section 6 summarizes and concludes. The Appendix analyzes the equilibrium when taxes are levied on wealth net of contingent wealth trades.

# 2 The Model

Consider two countries which we will refer to as the home country and the foreign country. Each country has a unit mass of individuals which are heterogenous in terms of their endowed wealth. Each individual has "reference wealth" $y$; the distribution of reference wealth is equal across the two countries and is described by the cumulative distribution function $G(y)$ with mean $\overline{y}$ and median $y^m$.

An individual's realized wealth is equal to his reference wealth multiplied by a factor which can be equal either to $1-\varepsilon$ or to $1+\varepsilon$, where $\varepsilon$ is a constant between 0 and 1. In what follows we will assume that shocks are regional in the sense that there is perfect positive correlation between shocks within the same country (all individuals in the same country will experience the same shock) and that there is perfect negative correlation across countries (if the home country experiences the positive shock, the foreign country experiences the negative shock with probability 1). This implies that there are only two relevant states of nature: One in which the home country experiences the positive shock and the foreign country the negative one, and one in which the opposite is true. Given our focus on the home country, we will call the first state of nature the good state ($s=\gamma$) and the second the bad state ($s=\beta$).

To avoid confusion in what follows we will denote variables referring to the foreign country with $^*$. Letting $Q$ denote the prior probability of the good state, an individual with reference wealth $y$ living in the home country will have realized wealth $y(1+\varepsilon)$ with probability $Q$ and realized wealth $y(1-\varepsilon)$ with probability $1-Q$; Similarly, an individual with reference wealth $y^*$ living in the foreign country will have realized wealth $y^*(1-\varepsilon)$ with probability $Q$ and realized wealth $y^*(1+\varepsilon)$ with probability $1-Q$. These somewhat extreme assumptions on the stochastic structure of the model simplify calculations significantly and let us focus on a situation in which, since aggregate wealth is constant across states of nature, there are clear gains from international insurance which is one of the main concerns of this paper. Assuming less than perfect positive correlation within each country or less than perfect positive correlation across countries would significantly complicate our algebra without leading to qualitatively different results.

Fiscal policy is constrained to equal treatment in the sense that consumption of publicly provided goods and services is equal for all the individuals living in the same country. We will assume that all individuals of both countries have the same preferences over private consumption $c$ and public consumption $g$ represented by a separable utility function

$$U(c, g) = H(c) + V(g)$$

with $H'(.) > 0$, $H''(.) < 0$, $V'(.) > 0$ and $V''(.) < 0$, where the strict inequality on $H''(.)$ guarantees that, because of individuals' strict aversion to risk on private consumption, international risk-sharing is desirable. Government spending is assumed to be financed through proportional taxation either at the national level (in the decentralized case) or at the aggregate level (in the centralized case).

Individuals can trade in contingent wealth markets exchanging units of wealth in one state of nature for units of wealth in the other state at an (endogenously determined) equilibrium price. Given our setup, individual preferences over fiscal policy are single-peaked and we will assume that fiscal policy will be the median voter's most preferred outcome. Since tax rates are decided by the median voter, it is easy to see that incentives to set tax rates are different depending on the time at which such decisions are made. This opens different modeling options depending on whether the tax rate is set

1. Before trading in contingent wealth markets (and therefore before the realization of uncertainty)

2. Before the realization of uncertainty but after trading in contingent wealth markets

3. After the realization of uncertainty (and therefore after trading in contingent wealth markets).

In all these cases it is important to decide what the tax base is, i.e., whether it should be wealth *gross* or *net* of contingent wealth trades. Moreover, in cases 1 and 2 it is important to determine whether fiscal policy is set in terms of tax rates or government spending and whether these variables are independent of the state of nature or can be made dependent on it. For reasons that will become clear in the following, we consider that the most interesting case is the one in which tax rates are set before trading in contingent wealth markets and wealth gross of contingent wealth trade is the relevant tax base. In the case in which tax rates are set independently in the two countries we will moreover allow tax rates to be contingent on the realization of uncertainty, whereas when considering the centralized system in which a single tax rate (and spending level) is voted for both countries we will not allow tax rates to depend on which country experiences the positive shock. We do not try to explain these choices now, but we will motivate them as the different models will be introduced and will alert the reader of the different results that the alternative models imply.

# 3 Fiscal policy and insurance

The goal of this section is to analyze the equilibrium determination of fiscal policy in the two countries when individuals can trade contingent wealth in competitive markets and in particular to study the equilibrium relationship among fiscal policy, contingent wealth prices and insurance of both private and public consumption. Since we have two states of nature, $s \in \{\gamma, \beta\}$, we will index wealth, consumption, public spending, and tax rates in the two states of nature with $\gamma$ and $\beta$.

Constraining the tax rate to be constant across states of nature in the decentralized model would introduce an inefficiency as it can make it impossible for individuals to insure their consumption levels. Given this, from now on we will concentrate on the case in which (country) tax rates are allowed to be contingent on the realization of uncertainty in the decentralized system. We will consider in turn the following cases

1. Tax rates are set after trading in contingent wealth markets (but before the realization of uncertainty)

2. Tax rates are set before trading in contingent wealth markets (and therefore before the realization of uncertainty)

While our main interest is in the second case we start with the first case to establish a reference framework. The reason we think the second case is the most interesting is that it deals with a situation in which when votes are cast, individuals do not hold perfectly insured positions and are therefore in a position to benefit from subsequent trading in contingent wealth before uncertainty is resolved. This is a most natural assumption as it is reasonable to think, in the spirit of the literature on frequent trading of long-lived securities (e.g. Hart (1975), Harrison and Kreps (1979) and Kreps (1982)), that even though markets are open at all dates the number of assets available for trade at each moment may be sufficient to complete markets through sequential trading strategies but is not sufficient to allow individuals to hold perfectly insured positions at all dates. Making the opposite assumption would imply that before any voting decision is made, all voters have been able to insure their wealth in such a way that they will not want to trade again after voting. This assumption is used as a reference framework in what follows but it is easy to see that it is much less reasonable than the one we will pursue.

Throughout this section we will assume that the tax base is realized wealth gross of contingent wealth trades. This assumption is made only for tractability convenience and equivalent results hold also in the case in which the tax base is realized wealth net of contingent wealth trades although the existence of an additional effect makes it difficult to give general predictions on contingent wealth prices and trades. The Appendix deals with this case.

### 3.1 Contingent wealth markets equilibrium

Equilibrium in contingent wealth markets is determined by individual optimality and market clearing conditions given a pair of state contingent tax rates for each country. Depending on whether voting takes place before or after trading, these pairs will be interpreted as givens or expectations of tax rates to be voted after trade, but the definition of equilibrium in contingent wealth markets will be the same. It should be made clear that even though voting decisions which are cast after market trading are actually affected by wealth trades, given we use the notion of competitive equilibrium in contingent wealth markets, individuals understand that they cannot influence equilibrium prices and they therefore maximize their expected utility given their expectations of equilibrium prices and equilibrium tax rates.

Let $P_\gamma$ denote the price of wealth in state $\gamma$ and $P_\beta$ the price of wealth in state $\beta$, and let $P = P_\beta / P_\gamma$. Let $c_\gamma$ and $c_\beta$ denote private consumption in state $\gamma$, $\beta$ and $x_\gamma$ and $x_\beta$ excess demand of contingent wealth in state $\gamma$, $\beta$. The maximization for individual $i$ with reference wealth $y$ living in the home country is therefore:

$$\begin{aligned} \max_{c_\gamma, c_\beta, x_\gamma, x_\beta} \quad & Q\left[H(c_\gamma) + V(g_\gamma)\right] + (1-Q)\left[H(c_\beta) + V(g_\beta)\right] \\ \text{s.t.} \quad & c_\gamma \leq y(1+\varepsilon)(1-t_\gamma) + x_\gamma \\ & c_\beta \leq y(1-\varepsilon)(1-t_\beta) + x_\beta \\ & x_\gamma + P x_\beta \leq 0. \end{aligned} \tag{1}$$

In the rest of the paper we will always assume that the solutions to all maximization problems are interior. Given this, the solution $(c_\gamma (t_\gamma, t_\beta, y, P), c_\beta (t_\gamma, t_\beta, y, P), x_\gamma (t_\gamma, t_\beta, y, P), x_\beta (t_\gamma, t_\beta, y, P))$ will be characterized by the following conditions:

$$\begin{aligned} \frac{H'(c_\beta)}{H'(c_\gamma)} &= \frac{PQ}{(1-Q)} \\ c_\gamma &\leq y^m(1+\varepsilon)(1-t_\gamma) + x_\gamma \\ c_\beta &\leq y^m(1-\varepsilon)(1-t_\beta) + x_\beta \\ 0 &\geq x_\gamma + P x_\beta. \end{aligned}$$

Similarly, the conditions for $\left(c^*_\gamma \left(t^*_\gamma, t^*_\beta, y, P\right), c^*_\beta \left(t^*_\gamma, t^*_\beta, y, P\right), x^*_\gamma \left(t^*_\gamma, t^*_\beta, y, P\right), x^*_\beta \left(t^*_\gamma, t^*_\beta, y, P\right)\right)$ to be an optimum for an individual with reference wealth $y$ living in the foreign country are:

$$\begin{aligned}
\frac{H'(c^*_\beta)}{H'(c^*_\gamma)} &= \frac{PQ}{(1-Q)} \\
c^*_\gamma &\leq y^m(1-\varepsilon)(1-t^*_\gamma)+x^*_\gamma \\
c^*_\beta &\leq y^m(1+\varepsilon)(1-t^*_\beta)+x^*_\beta \\
0 &\geq x^*_\gamma + Px^*_\beta .
\end{aligned}$$

Combining individual optimality with market clearing conditions we get the following equilibrium conditions.

$$\begin{aligned}
z_\gamma(P) &= \int x_\gamma(t_\gamma,t_\beta,y,P)\,dG(y) + \int x^*_\gamma(t^*_\gamma,t^*_\beta,y,P)\,dG(y) \leq 0 \\
z_\beta(P) &= \int x_\beta(t_\gamma,t_\beta,y,P)\,dG(y) + \int x^*_\beta(t^*_\gamma,t^*_\beta,y,P)\,dG(y) \leq 0
\end{aligned}$$

In the following we will denote by $x_s$, $x^*_s$ the excess demand of wealth in state $s \in \{\gamma,\beta\}$ of an individual living respectively in the home and the foreign country, whereas we will let $\widetilde{x}_s$, $\widetilde{x}^*_s$ denote the corresponding equilibrium excess demands.

## 3.2 Fiscal policy

Consider an individual living in the home country with reference wealth $y$ and with excess demands $(x_\gamma,x_\beta)$; Recalling that we are considering the case in which the tax base is his realized wealth gross of contingent wealth trades, it is easy to see that his preferred tax rates $(t_\gamma,t_\beta)$ will be given by the solution of the following problem:

$$\begin{aligned}
&\max_{t_\gamma,t_\beta} Q\left[H\left(y^i(1+\varepsilon)(1-t_\gamma)+x_\gamma\right)+V(g_\gamma)\right]+(1-Q)\left[H\left(y^i(1-\varepsilon)(1-t_\beta)+x_\beta\right)+V(g_\beta)\right] \\
&\text{s.t. } g_\gamma = \overline{y}(1+\varepsilon)t_\gamma \text{ and } g_\beta = \overline{y}(1+\varepsilon)t_\gamma
\end{aligned}$$

As before the vector of excess demands $(x_\gamma,x_\beta)$ will be interpreted either as the vector of observed excess demands in the case in which voting takes place after trading, or as the individual's expectations over the trades that he will carry out in equilibrium given the observed tax rates in the case in which trading takes place after voting. We will present each of these two cases in the following subsections.

Since we want to be able to characterize equilibrium in terms of prices and fiscal policy we will now concentrate on a case in which it is guaranteed that the median voter will be the individual with median reference income in all voting settings we will be considering, i.e., irrespective of whether voting takes place before or after trading and of what the definition of tax base is.

**Assumption 1** $H'(\cdot)$ *is homogeneous.*

The proof of the fact that this assumption is sufficient to be able to say that the median voter is the individual with median reference income will only be provided for the case in which tax base is wealth gross of contingent wealth trades and voting takes place before trading (Lemma 1). The proof of the same result for the other cases is a straightforward adaptation of the same argument. In the following it will also be understood that preferences satisfy Assumption 1 and explicit reference to it in the statements of Propositions will be omitted.

### 3.2.1 Voting after trading

If voting takes place after trading and given a vector of excess demands $(x_\gamma, x_\beta)$, the equilibrium tax rates will be determined by the first order condition of the above maximization problem for the median voter who, given our setting, is the individual with the median reference income in each of the two countries. In the home country we have therefore:

$$\begin{aligned} H'\left(y^m(1+\varepsilon)(1-\widehat{t}_\gamma)+x_\gamma\right)y^m(1+\varepsilon)-V'\left(\overline{y}(1+\varepsilon)\widehat{t}_\gamma\right)\overline{y}(1+\varepsilon) &= 0 \\ H'\left(y^m(1-\varepsilon)(1-\widehat{t}_\beta)+x_\beta\right)y^m(1-\varepsilon)-V'\left(\overline{y}(1-\varepsilon)\widehat{t}_\beta\right)\overline{y}(1-\varepsilon) &= 0 \end{aligned}$$

Symmetric conditions characterize the tax rates for the foreign country.

**Proposition 1** *When voters choose state contingent tax rates, the tax base is wealth gross of contingent wealth trades and voting takes place after trading, in equilibrium*

1. $\widetilde{P}=(1-Q)/Q$;
2. $\widetilde{c}_\gamma(y)=c_\gamma\left(y,\widetilde{P}\right)=\widetilde{c}_\beta(y)=c_\beta\left(y,\widetilde{P}\right)$ *for all* $y$,
   $\widetilde{c}^*_\gamma(y)=c^*_\gamma\left(y,\widetilde{P}\right)=\widetilde{c}^*_\beta(y)=c^*_\beta\left(y,\widetilde{P}\right)$ *for all* $y$;
3. $\widehat{t}_\gamma=\widehat{t}_\beta\frac{(1-\varepsilon)}{(1+\varepsilon)}$ *and* $\widehat{t}^*_\gamma=\widehat{t}^*_\beta\frac{(1+\varepsilon)}{(1-\varepsilon)}$;
4. $\widehat{g}_\gamma=\widehat{g}_\beta$ *and* $\widehat{g}^*_\gamma=\widehat{g}^*_\beta$.

**Proof:** Suppose that $P=\frac{(1-Q)}{Q}$. This implies that $c_\gamma(y,P)=c_\beta(y,P)$ for all $y$ and $c^*_\gamma(y,P)=c^*_\beta(y,P)$ for all $y$. Under this assumption, from the first order condition of the median voter of the home country, we get $V'\left(\overline{y}(1+\varepsilon)\widehat{t}_\gamma\right)=V'\left(\overline{y}(1-\varepsilon)\widehat{t}_\beta\right)$, which implies that $\widehat{t}_\gamma=\widehat{t}_\beta\frac{(1-\varepsilon)}{(1+\varepsilon)}$. Similarly from the first order condition of the median voter of the foreign country we get $V'\left(\overline{y}(1-\varepsilon)\widehat{t}^*_\gamma\right)=V'\left(\overline{y}(1+\varepsilon)\widehat{t}^*_\beta\right)$ which implies that $\widehat{t}^*_\gamma=\widehat{t}^*_\beta\frac{(1+\varepsilon)}{(1-\varepsilon)}$. On the other hand, given $\widehat{t}_\gamma=\widehat{t}_\beta\frac{(1-\varepsilon)}{(1+\varepsilon)}$ and $\widehat{t}^*_\gamma=\widehat{t}^*_\beta\frac{(1+\varepsilon)}{(1-\varepsilon)}$ it is easy to see that aggregate wealth net of tax revenues is constant across states which implies that $P=\frac{(1-Q)}{Q}$ and $c_\gamma(y,P)=c_\beta(y,P)$ for all $y$, and $c^*_\gamma(y,P)=c^*_\beta(y,P)$ for all $y$. ■

Proposition 1 says that when tax rates are set after trading the median voter has no ability to manipulate net wealth in his country thereby affecting his country excess demand functions for contingent wealth and he therefore ultimately lacks any ability to manipulate contingent wealth prices. Since equilibrium in the contingent wealth market is determined given expectations of the tax rates that will be set by the median voters in the two countries, it is easy to see that in equilibrium individuals in the two countries insure their wealth perfectly given the contingent wealth equilibrium price ratio is equal to the ratio of probabilities of the two states of nature, $\widetilde{P} = \widetilde{P}_\beta / \widetilde{P}_\gamma = (1 - Q)/Q$. Contingent tax rates are not equal across states but it is easy to recognize that they are adjusted to take care of the fact that tax base is not constant across states with the ultimate goal to insure both private consumption and public spending across states.

**Corollary 1** *When voters choose state contingent tax rates, the tax base is wealth gross of contingent wealth trades and voting takes place after the realization of uncertainty, the same results as in Proposition 1 hold.*

This Corollary depends trivially on the fact that under the conditions of Proposition 1, by voting state contingent tax rates before the realization of uncertainty, individuals anticipate perfectly their preferences over tax rates once each of the two states of nature is realized.

**Corollary 2** *When voters choose state contingent government expenditure levels, the tax base is wealth gross of contingent wealth trades the same results as in Proposition 1 hold independently of whether voting takes place after trading but before the realization of uncertainty or after the realization of uncertainty.*

The fact that voting over government expenditure levels is equivalent to voting over tax rates is a straightforward consequence of the fact that in the voting maximization problem each individual is subject to an independent budget constraint, so that government expenditure can be substituted for tax rate and vice versa. The fact that the results are equivalent regardless of whether voting takes place after trading but before the realization of uncertainty or after the realization of uncertainty depends on the same argument behind Corollary 1.[2]

Summarizing, the previous results show that if no trade in contingent wealth takes place after voting, the median voter cannot manipulate equilibrium contingent wealth prices and individuals are in the position to use contingent wealth markets to efficiently reallocate risk.

[2] In the setting that we are considering the same results would also hold even if we constrained government spending to be equal across states of nature. This equivalence, however, depends crucially on the hypothesis that aggregate wealth is constant across states of nature and does not generalize to a setting in which aggregate wealth is not constant.

### 3.2.2 Voting before trading

We now consider the case in which national fiscal policy is voted before trading in contingent wealth markets, i.e., a situation in which after voting, individuals can gain from reallocating risk. In this situation voters can affect excess demand functions in their own country and are therefore in the position to manipulate contingent wealth equilibrium prices. The following Lemma shows that under Assumption 1 the median voter in this setting is the individual with median reference income.

**Lemma 1** *Under Assumption 1 the pivotal voter is the voter with the median income.*

**Proof.** From individual maximization $x_\gamma$ is such that Then individual contingent wealth excess demand functions are such that:

$$H'\left(y(1-\varepsilon)(1-t_\beta)-\frac{x_\gamma}{P}\right)=\frac{PQ}{(1-Q)}H'\left(y(1+\varepsilon)(1-t_\gamma)+x_\gamma\right) \tag{2}$$

If $H'(\cdot)$ is homogeneous of degree $r$, it is easy to see that (2) implies

$$x_\gamma=y\left[\frac{(1-\varepsilon)(1-t_\beta)-\left(\frac{PQ}{(1-Q)}\right)^{\frac{1}{r}}(1+\varepsilon)(1-t_\gamma)}{\left(\frac{PQ}{(1-Q)}\right)^{\frac{1}{r}}+\frac{1}{P}}\right]$$

so that $x_\gamma$ can be rewritten as

$$x_\gamma=yf(t) \tag{3}$$

where $t=\left(t_\gamma,t_\beta,t_\gamma^*,t_\beta^*\right)$ and $f(t)$ does not depend on $y$. Since from the budget constraint $x_\beta=-\frac{x_\gamma}{P}$ we also have that

$$x_\beta=-y\frac{f(t)}{P} \tag{4}$$

Recall that if $H'(\cdot)$ is homogeneous of degree $r$, $H(\cdot)$ can be expressed as the sum of a function $\widetilde{H}(\cdot)$ which is homogeneous of degree $r+1$ and a constant. Using this and given (3) and (4), indirect expected utility for an individual with reference income equal to $y$ for a given $t=\left(t_\gamma,t_\beta,t_\gamma^*,t_\beta^*\right)$ can be shown to be equal to

$$v(t)=y^{r+1}w(t)+z(t)$$

where

$$\begin{aligned}
w(t) &= \left[Q\widetilde{H}\left(((1+\varepsilon)(1-t_\gamma)+f(t))\right)+(1-Q)\widetilde{H}\left(((1-\varepsilon)(1-t_\beta)+f(t))\right)\right]\\
z(t) &= QV(g_\gamma)+(1-Q)V(g_\beta)+K
\end{aligned}$$

where $K$ is the constant mentioned above.

Consider a $t' = \left(t'_\gamma, t'_\beta, t^*_\gamma, t^*_\beta\right)$ with $\left(t'_\gamma, t'_\beta\right) \neq (t_\gamma, t_\beta)$; Then we have that

$$v(t) - v(t') = (y)^{r+1}\left[w(t) - w(t')\right] + \left[z(t) - z(t')\right].$$

Now suppose that $t$ is such that $(t_\gamma, t_\beta)$ is the preferred vector of the individual with median income given $\left(t^*_\gamma, t^*_\beta\right)$; Then

$$v(t) - v(t') = (y^m)^{r+1}\left[w(t) - w(t')\right] + \left[z(t) - z(t')\right] \geq 0 \tag{5}$$

Consider the following two cases $r+1 \geq 0$ y $r+1 \leq 0$.

1. $r+1 \geq 0$,

   (a) Let $t' = \left(t'_\gamma, t'_\beta, t^*_\gamma, t^*_\beta\right)$ be such that $\left(t'_\gamma, t'_\beta\right)$is the preferred fiscal policy of an individual with income $\widetilde{y} > y^m$ given $\left(t^*_\gamma, t^*_\beta\right)$; Then

   $$v(t) - v(t') = (\widetilde{y})^{r+1}\left[w(t) - w(t')\right] + \left[z(t) - z(t')\right] \leq 0$$

   which together with (5) implies that $\left[w(t) - w(t')\right] \leq 0$ and $\left[z(t) - z(t')\right] \geq 0$ and this implies that

   $$v^i(t) - v^i(t') > 0 \text{ iff } y^i \leq y(t,t') = \left(\frac{z(t') - z(t)}{w(t) - w(t')}\right)^{\frac{1}{r+1}}$$

   and since $y^m < y(t,t')$, all individuals with income less than or equal to $y^m$ also prefer $t$ to $t'$.

   (b) Let $t' = \left(t'_\gamma, t'_\beta, t^*_\gamma, t^*_\beta\right)$ be such that $\left(t'_\gamma, t'_\beta\right)$is the preferred fiscal policy of an individual with income $\widetilde{y} < y^m$ given $\left(t^*_\gamma, t^*_\beta\right)$; Then

   $$v(t) - v(t') = (\widetilde{y})^{r+1}\left[w(t) - w(t')\right] + \left[z(t) - z(t')\right] \leq 0$$

   which together with (5) implies that $\left[w(t) - w(t')\right] \geq 0$ and $\left[z(t) - z(t')\right] \leq 0$ and this implies that

   $$v^i(t) - v^i(t') > 0 \text{ iff } y^i \geq y(t,t') = \left(\frac{z(t') - z(t)}{w(t) - w(t')}\right)^{\frac{1}{r+1}}$$

   and since $y^m > y(t,t')$, all individuals with income greater than or equal to $y^m$ also prefer $t$ to $t'$.

2. A similar argument shows that the same is true when $r+1 \leq 0$ ■

In subsection 3.1 we derived equilibrium excess demands given the pairs of state contingent tax rates in the home and the foreign country, $(t_\gamma, t_\beta)$, $\left(t^*_\gamma, t^*_\beta\right)$. Letting $\widetilde{x}_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, y\right)$, $\widetilde{x}_\beta\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, y\right)$, $\widetilde{x}^*_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, y\right)$, and $\widetilde{x}^*_\beta\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, y\right)$ denote such equilibrium excess demands as functions of the state contingent tax rates in the home and the foreign country. and applying the median voter theorem, we have that the state contingent tax rates in the home country will be given by the median voter's first order conditions of expected utility maximization:

$$Q\left[H'\left(y^m(1+\varepsilon)(1-t_\gamma)+\widetilde{x}_\gamma\right)y^m(1+\varepsilon)-V'\left(\overline{y}(1+\varepsilon)t_\gamma\right)\overline{y}(1+\varepsilon)\right]-QH'(c_\gamma)\frac{\partial\widetilde{x}_\gamma}{\partial t_\gamma}-(1-Q)H'(c_\beta)\frac{\partial\widetilde{x}_\beta}{\partial t_\gamma}=0$$

$$(1-Q)\left[H'\left(y^m(1-\varepsilon)(1-t_\beta)+\widetilde{x}_\beta\right)y^m(1-\varepsilon)-V'\left(\overline{y}(1-\varepsilon)t_\beta\right)\overline{y}(1-\varepsilon)\right]-QH'(c_\gamma)\frac{\partial\widetilde{x}_\gamma}{\partial t_\beta}-(1-Q)H'(c_\beta)\frac{\partial\widetilde{x}_\beta}{\partial t_\beta}=0$$

Let $\widetilde{P}(t)=\widetilde{P}\left(t_\gamma,t_\beta,t_\gamma^*,t_\beta^*\right)$ denote the equilibrium price ratio as a function of the state contingent tax rates. Notice that since $\widetilde{x}_\beta=-\frac{\widetilde{x}_\gamma}{P}$ we have $\frac{\partial\widetilde{x}_\beta}{\partial t_\gamma}=-\frac{\frac{\partial\widetilde{x}_\gamma}{\partial t_\gamma}\widetilde{P}(t)-\widetilde{x}_\gamma\frac{\partial\widetilde{P}(t)}{\partial t_\gamma}}{\left[\widetilde{P}(t)\right]^2}$. Simple algebra then shows that the above conditions are equivalent to

$$Q\left[H'\left(\widetilde{c}_\gamma\right)y^m(1+\varepsilon)-V'\left(g_\gamma\right)\overline{y}(1+\varepsilon)\right]-\frac{(1-Q)H'\left(\widetilde{c}_\beta\right)\widetilde{x}_\gamma\frac{\partial\widetilde{P}(t)}{\partial t_\gamma}}{\left[\widetilde{P}(t)\right]^2}=0$$

$$(1-Q)\left[H'\left(\widetilde{c}_\beta\right)y^m(1-\varepsilon)-V'\left(g_\beta\right)\overline{y}(1-\varepsilon)\right]-\frac{(1-Q)H'\left(\widetilde{c}_\beta\right)\widetilde{x}_\gamma\frac{\partial\widetilde{P}(t)}{\partial t_\beta}}{\left[\widetilde{P}(t)\right]^2}=0$$

Using the above conditions we can now state the following result:

**Proposition 2** *When voters choose state contingent tax rates, the tax base is wealth gross of contingent wealth trades, and voting takes place before trading, in equilibrium*

1. $g_\gamma \geq g_\beta$

2. $g_\gamma^* \leq g_\beta^*$

**Proof.** Let $\widetilde{c}_s\left(t_\gamma,t_\beta,t_\gamma^*,t_\beta^*,y\right)$ and $\widetilde{x}_s\left(t_\gamma,t_\beta,t_\gamma^*,t_\beta^*,y\right)$ denote respectively equilibrium contingent consumption and equilibrium excess demand function of contingent wealth in state $s$ of an individual with reference wealth $y$ living in the home country for given $t_\gamma$, $t_\beta$, $t_\gamma^*$, $t_\beta^*$.

As shown before, the four equations which characterize the political equilibrium are,

$$Q\left[H'\left(\widetilde{c}_\gamma\right)y^m(1+\varepsilon)-V'\left(g_\gamma\right)\overline{y}(1+\varepsilon)\right]-\frac{(1-Q)H'\left(\widetilde{c}_\beta\right)\widetilde{x}_\gamma\frac{\partial\widetilde{P}(t)}{\partial t_\gamma}}{\left[\widetilde{P}(t)\right]^2}=0 \tag{6}$$

$$(1-Q)\left[H'\left(\widetilde{c}_\beta\right)y^m(1-\varepsilon)-V'\left(g_\beta\right)\overline{y}(1-\varepsilon)\right]-\frac{(1-Q)H'\left(\widetilde{c}_\beta\right)\widetilde{x}_\gamma\frac{\partial\widetilde{P}(t)}{\partial t_\beta}}{\left[\widetilde{P}(t)\right]^2}=0 \tag{7}$$

$$Q\left[H'\left(\widetilde{c}_\gamma^*\right)y^m(1-\varepsilon)-V'\left(g_\gamma^*\right)\overline{y}(1-\varepsilon)\right]-\frac{(1-Q)H'\left(\widetilde{c}_\beta^*\right)\widetilde{x}_\gamma^*\frac{\partial\widetilde{P}(t)}{\partial t_\gamma^*}}{\left[\widetilde{P}(t)\right]^2}=0 \tag{8}$$

$$(1-Q)\left[H'\left(\widetilde{c}_\beta^*\right)y^m(1+\varepsilon)-V'\left(g_\beta^*\right)\overline{y}(1+\varepsilon)\right]-\frac{(1-Q)H'\left(\widetilde{c}_\beta^*\right)\widetilde{x}_\gamma^*\frac{\partial\widetilde{P}(t)}{\partial t_\beta^*}}{\left[\widetilde{P}(t)\right]^2}=0 \tag{9}$$

To compute the sign of $\frac{\partial \widetilde{P}(t)}{\partial t_\beta}$, $\frac{\partial \widetilde{P}(t)}{\partial t_\gamma}$, $\frac{\partial \widetilde{P}(t)}{\partial t^*_\gamma}$ and $\frac{\partial \widetilde{P}(t)}{\partial t^*_\beta}$, let $z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)$ denote the aggregate excess demand function for wealth in state $\gamma$ and totally differentiate the equilibrium condition

$$z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right) = 0$$

with respect to $P$ and $t_\gamma$, $t_\beta$, $t^*_\gamma$ and $t^*_\beta$ respectively. For the home country we get

$$\frac{\partial \widetilde{P}(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta)}{\partial t_\gamma} = -\frac{\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\gamma}}{\frac{z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial P}}$$

$$\frac{\partial \widetilde{P}(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta)}{\partial t_\beta} = -\frac{\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\beta}}{\frac{z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial P}}$$

Since by definition

$$z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right) = \int \left[x_\gamma\left(t_\gamma, t_\beta, P, y\right) + x^*_\gamma\left(t^*_\gamma, t^*_\beta, P, y\right)\right] dG(y)$$

we have

$$\begin{aligned}
\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\gamma} &= \int \frac{\partial x_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\gamma} dG(y) \\
\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\beta} &= \int \frac{\partial x_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\beta} dG(y)
\end{aligned}$$

and from the first order conditions

$$H'\left(y(1-\varepsilon)(1-t_\beta) - \frac{x_\gamma}{P}\right) - \frac{PQ}{(1-Q)} H'\left(y(1+\varepsilon)(1-t_\gamma) + x_\gamma\right) = 0$$

$$H'\left(y^m(1+\varepsilon)(1-t^*_\beta) - \frac{x^*_\gamma}{P}\right) - \frac{PQ}{(1-Q)} H'\left(y^m(1-\varepsilon)(1-t^*_\gamma) + x^*_\gamma\right) = 0$$

we get

$$\begin{aligned}
\frac{\partial x_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\gamma} &= \frac{PQ}{(1-Q)}(1+\varepsilon)\frac{H''\left(\widetilde{c}_\gamma, y\right) y}{\frac{H''\left(\widetilde{c}_\beta, y\right)}{P} + H''\left(\widetilde{c}_\gamma, y\right)\frac{PQ}{(1-Q)}} \geq 0 \\
\frac{\partial x_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\beta} &= -(1-\varepsilon)\frac{H''\left(\widetilde{c}_\beta, y\right) y}{\frac{H''\left(\widetilde{c}_\beta, y\right)}{P} + H''\left(\widetilde{c}_\gamma, y\right)\frac{PQ}{(1-Q)}} \leq 0
\end{aligned}$$

which implies that,

$$\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\gamma} = \frac{PQ}{(1-Q)}(1+\varepsilon)\int \frac{H''\left(\widetilde{c}_\gamma, y\right) y}{\frac{H''\left(\widetilde{c}_\beta, y\right)}{P} + H''\left(\widetilde{c}_\gamma, y\right)\frac{PQ}{(1-Q)}} dG(y) \geq 0 \tag{10}$$

$$\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\beta} = -(1-\varepsilon)\int \frac{H''\left(\widetilde{c}_\beta, y\right) y}{\frac{H''\left(\widetilde{c}_\beta, y\right)}{P} + H''\left(\widetilde{c}_\gamma, y\right)\frac{PQ}{(1-Q)}} dG(y) \leq 0 \tag{11}$$

Provided that contingent consumption is a normal good, $\frac{\partial z_\gamma(t_\gamma,t_\beta,t_\gamma^*,t_\beta^*,P)}{\partial P} \geq 0$, for both the home and the foreign country, we get that,

$$\frac{\partial \widetilde{P}(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*)}{\partial t_\gamma} \leq 0 \text{ and } \frac{\partial \widetilde{P}(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*)}{\partial t_\beta} \geq 0$$

In a similar way it can be shown that

$$\frac{\partial \widetilde{P}(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*)}{\partial t_\gamma^*} \leq 0 \text{ and } \frac{\partial \widetilde{P}(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*)}{\partial t_\beta^*} \geq 0$$

Now suppose that $\widetilde{x}_\gamma(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*, y^m) \leq 0$; then from (6)-(9), necessary conditions for equilibrium are

$$H'(\widetilde{c}_\gamma)\, y^m - V'(g_\gamma)\, \overline{y} \;>\; 0 \tag{12}$$

$$H'(\widetilde{c}_\beta)\, y^m - V'(g_\beta)\, \overline{y} \;<\; 0 \tag{13}$$

$$H'(\widetilde{c}_\gamma^*)\, y^m - V'(g_\gamma^*)\, \overline{y} \;<\; 0 \tag{14}$$

$$H'(\widetilde{c}_\beta^*)\, y^m - V'(g_\beta^*)\, \overline{y} \;>\; 0 \tag{15}$$

Even though we cannot determine the equilibrium price $\widetilde{P}(t)$, we will show that for all values that it could get we would have $g_\gamma \geq g_\beta$ and $g_\gamma^* \leq g_\beta^*$.

i) If $\widetilde{P}(t) = \frac{(1-Q)}{Q}$, this implies that $\widetilde{c}_\gamma = \widetilde{c}_\beta$ and $\widetilde{c}_\gamma^* = \widetilde{c}_\beta^*$, then from (12)-(15) we have $g_\gamma \geq g_\beta$ and $g_\gamma^* \leq g_\beta^*$.

ii) If $\widetilde{P}(t) > \frac{(1-Q)}{Q}$, this implies that $\widetilde{c}_\gamma > \widetilde{c}_\beta$ and $\widetilde{c}_\gamma^* > \widetilde{c}_\beta^*$. Inequalities (12) and (13) then imply that $g_\gamma \geq g_\beta$, and for $\widetilde{P}(t) > \frac{(1-Q)}{Q}$ to be an equilibrium, the aggregate net wealth in state $\beta$ must be less than aggregate net wealth in state $\gamma$, that is, $g_\beta^* + g_\beta > g_\gamma^* + g_\gamma$ which implies that $g_\gamma^* \leq g_\beta^*$.

iii) If $\widetilde{P}(t) < \frac{(1-Q)}{Q}$, this implies that $\widetilde{c}_\gamma < \widetilde{c}_\beta$ and $\widetilde{c}_\gamma^* < \widetilde{c}_\beta^*$. Inequalities (14) and (15) then imply that $g_\gamma^* \leq g_\beta^*$, and for $\widetilde{P}(t) < \frac{(1-Q)}{Q}$ to be an equilibrium, the aggregate net wealth in state $\beta$ must be greater than aggregate net wealth in state $\gamma$, that is, $g_\beta^* + g_\beta < g_\gamma^* + g_\gamma$ which implies that $g_\gamma \geq g_\beta$.

A similar argument shows that assuming that $\widetilde{x}_\gamma(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*, y^m) \geq 0$ leads to a contradiction. ■

Proposition 2 shows that when voting takes place before trading, state contingent fiscal policies are distorted away from their equilibrium levels in the case in which voting takes place after trading. The proof highlights that the median voter inflates government expenditure in the good state of nature and deflates it in the bad state of nature with the goal to increase the relative price of the wealth he holds in larger amount. The following Proposition provides an additional characterization in terms of aggregate government expenditure when the two countries are ex ante identical (i.e., when $Q = 1/2$) and shows that aggregate government expenditure is higher when voting takes place before trading than when it takes place after trading.

**Proposition 3** *Under the conditions of Proposition 2, if countries are identical ex-ante and $V(.)$ exhibits D.A.R.A., in equilibrium*

1. $g^*_\beta + g_\beta > \widehat{g}^*_\beta + \widehat{g}_\beta$

2. $g^*_\gamma + g_\gamma > \widehat{g}^*_\gamma + \widehat{g}_\gamma$

**Proof.** If countries are ex-ante identical, $Q = \frac{1}{2}$, by symmetry $g^*_\beta = g_\gamma$ and $g^*_\gamma = g_\beta$, which implies that $\widetilde{P}(t) = 1$ and therefore $\widetilde{c}_\gamma = \widetilde{c}_\beta = \widetilde{c}^*_\gamma = \widetilde{c}^*_\beta = c$. When voting takes place after trading, in equilibrium $\widehat{c}_\gamma = \widehat{c}_\beta = \widehat{c}^*_\gamma = \widehat{c}^*_\beta = \widehat{c}$ and $\widehat{g}^*_\beta = \widehat{g}_\beta = \widehat{g}_\gamma = \widehat{g}_\gamma = \widehat{g}$. Then the statement of the proposition is equivalent to $\widehat{c} > c$.

Since $\widetilde{c}_\gamma = \widetilde{c}_\beta = \widetilde{c}^*_\gamma = \widetilde{c}^*_\beta = c$ then from (10) and (11) we get

$$\frac{\partial \widetilde{P}(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta)}{\partial t_\gamma} = -\frac{\int \frac{y(1+\varepsilon)}{\frac{1}{P}+1} dG(y)}{\frac{\partial z_\gamma(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P)}{\partial P}}$$

$$\frac{\partial \widetilde{P}(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta)}{\partial t_\beta} = \frac{\int \frac{y(1-\varepsilon)}{\frac{1}{P}+1} dG(y)}{\frac{\partial z_\gamma(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P)}{\partial P}}$$

and substituting these into (6) and (7) we get

$$V'(g_\gamma)\,\overline{y} = H'(c)\left[y^m + \frac{Q\widetilde{x}_\gamma \overline{y}}{\frac{\partial z_\gamma(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P)}{\partial P}}\right]$$

$$V'(g_\beta)\,\overline{y} = H'(c)\left[y^m - \frac{Q\widetilde{x}_\gamma \overline{y}}{\frac{\partial z_\gamma(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P)}{\partial P}}\right]$$

which imply that

$$V'(g_\gamma)\,\overline{y} - H'(c)\,y^m = H'(c)\,y^m - V'(g_\beta)\,\overline{y} \tag{16}$$

We know that $H'(\widehat{c}) = V'(\widehat{g})\,\frac{\overline{y}}{y^m}$. Now, suppose contrary to the claim that $\widehat{c} \leq c$. By concavity of $H(.)$, this implies that $H'(\widehat{c}) \geq H'(c)$ and this together with (16) implies that

$$\overline{y}\left[V'(g_\gamma) + V'(g_\beta)\right] = 2y^m H'(c) \leq 2y^m H'(\widehat{c}) = 2\overline{y}V'(\widehat{g})$$

which in turn implies that

$$V'(g_\gamma) + V'(g_\beta) \leq 2V'(\widehat{g}) \tag{17}$$

Since $V(.)$ exhibits D.A.R.A., $V'''(.) > 0$ so that $V'(.)$ is convex; this implies that if $g_\gamma \neq \widehat{g}$ , $g_\beta \neq \widehat{g}$, we have

$$V'(g_\gamma) \quad > \quad V'(\widehat{g}) + V''(\widehat{g})(g_\gamma - \widehat{g}) \tag{18}$$

$$V'(g_\beta) \quad > \quad V'(\widehat{g}) + V''(\widehat{g})(g_\beta - \widehat{g}) \tag{19}$$

Adding (18) and (19) we get

$$V'(g_\gamma) + V'(g_\beta) - 2V'(\widehat{g}) > V''(\widehat{g})(g_\gamma + g_\beta - 2\widehat{g})$$

From (17) we know that $V'(g_\gamma) + V'(g_\beta) - 2V'(\widehat{g}) \leq 0$ which implies that

$$V''(\widehat{g})(g_\gamma - \widehat{g} + g_\beta - \widehat{g}) < 0$$

Since $V''(\widehat{g}) \leq 0$ then $g_\gamma + g_\beta > 2\widehat{g}$, and this implies that $\widehat{c} > c$ leading to a contradiction. ■

The following corollary is a straightforward implication of Proposition 3:

**Corollary 3** *Under the conditions of Proposition 2, if countries are identical ex-ante and $V(.)$ exhibits D.A.R.A., in equilibrium*

1. $t_\gamma \geq \widehat{t}_\gamma$ , $t_\beta \leq \widehat{t}_\beta$;
2. $t^*_\beta \geq \widehat{t}^*_\beta$, $t^*_\gamma \leq \widehat{t}^*_\gamma$;
3. $g_\gamma \geq \widehat{g}_\gamma = \widehat{g}_\beta \geq g_\beta$;
4. $g^*_\beta \geq \widehat{g}^*_\beta = \widehat{g}^*_\gamma \geq g^*_\gamma$.

While it is impossible to give a precise characterization of equilibrium prices, trades, private and public consumption without specifying functional forms for the utility functions, the equilibrium values of these variables are very important to have a perception of the inefficiencies that this form of manipulation can induce. Section 4 considers a specific example to show the equilibrium outcome of this competitive manipulation process and shows that, although it is possible that these manipulation attempts almost cancel each other out (in the sense that the impact on equilibrium prices is only minor), significant distortions in fiscal policy and significant inefficiencies in terms of risk sharing of both private and public consumption may arise in equilibrium.

Before concluding this section it is interesting to mention the following

**Corollary 4** *When voters choose state contingent government expenditure, the tax base is wealth gross of contingent wealth trades and voting takes place before trading, equilibrium outcomes are the same as in the case in which voters choose state contingent tax rates.*

The intuition behind this result is the same that was discussed in relation to Corollary 2.

# 4 Discussion and numerical examples

The goal of this section is to discuss the results of the previous section computing equilibrium values of the relevant variables. In what follows we will consider the case in which the Bernoulli utility function of individuals living in both countries is $U(c, g) = c^\alpha + g^\alpha$ with $\alpha = .5$ and $\varepsilon = .2$. We will moreover assume that reference income is distributed according to a generalized uniform on $(0, y_{\max}]$, i.e., with a distribution function $F(y) = \left(\frac{y}{y_{\max}}\right)^m$ with $y_{\max} = 1$ and $m = .7$; these assumptions imply that mean reference income is .4118 and median reference income is .3715

As we hinted in the previous section it is interesting to find out the equilibrium impact on contingent wealth prices of the manipulation attempts of the two median voters when voting takes place before trading. We suggested that it could well be that in equilibrium the two effects tended to cancel each other out; In the example we consider in this section this is exactly the case as the equilibrium price ratios when voting takes place before trading and when voting takes place after trading differ only at the third decimal figure, that is to say less than 1%. Equilibrium prices are on the other hand always strictly lower than $(1 - Q)/Q$ when $Q > 1/2$, meaning that the price of wealth in the good state ($s = \gamma$, the state in which the home country has the positive shock) is slightly inflated.

Figures 1 and 2 plot the equilibrium values of the tax rates when voting takes place before trading $\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta\right)$ and when voting takes place after trading $\left(\widehat{t}_\gamma, \widehat{t}_\beta, \widehat{t}^*_\gamma, \widehat{t}^*_\beta\right)$against $Q$; Since the problem is symmetric we let $Q$ only vary between $Q = 1/2$ (perfect ex-ante symmetry) to $Q = 1$ (most asymmetric case, in which there is in fact no uncertainty). In this case we have that $t_\gamma \geq \widehat{t}_\gamma$ , $t_\beta \leq \widehat{t}_\beta$, $t^*_\beta \geq \widehat{t}^*_\beta$, $t^*_\gamma \leq \widehat{t}^*_\gamma$. The tax rates of the home country are all increasing in $Q$ as a consequence of the fact that, as $Q$ increases, the home country becomes richer; the opposite holds true for the foreign country. It is also interesting to realize that, as $Q$ increases, $t_\gamma$ tends to $\widehat{t}_\gamma$; this is a consequence of the fact that as $Q$ increases the relevance of uncertainty decreases while the probability of state $\gamma$ increases thus making $t_\gamma$ the tax rate that will be implemented with higher probability. As $Q$ increases, on the other hand, the difference between $t_\beta$ and $\widehat{t}_\beta$ increases but it should also be emphasized that as $Q$ increases $t_\beta$ is the tax rate that will be implemented with lower and lower probability.

Figures 3 and 4 plot equilibrium government spending levels when voting takes place before trading $\left(g_\gamma, g_\beta, g^*_\gamma, g^*_\beta\right)$ and when voting takes place after trading $\left(\widehat{g}_\gamma, \widehat{g}_\beta, \widehat{g}^*_\gamma, \widehat{g}^*_\beta\right)$against $Q$. As was stated in Proposition 2 we have $g_\gamma \geq g_\beta$ and $g^*_\beta \geq g^*_\gamma$; In this case, moreover we have $g_\gamma \geq \widehat{g}_\gamma = \widehat{g}_\beta \geq g_\beta; g^*_\beta \geq \widehat{g}^*_\beta = \widehat{g}^*_\gamma \geq g^*_\gamma$. It is also important to notice that when voting takes place before trading, government spending is more volatile across states and this implies a higher volatility of private consumption with the obvious efficiency implications for risk averse agents.

As was argued in the previous section, the median voter manipulates contingent wealth equilibrium prices by inflating government expenditure in the good state and deflating it in the bad state. Figures 5 and 6 below plot the differences between expected government spending for the home country when voting takes place before trading and when voting takes place after trading , i.e., $g - \widehat{g}$, where $g = Qg_\gamma + (1-Q)\, g_\beta$ and $\widehat{g} = Q\widehat{g}_\gamma + (1-Q)\, \widehat{g}_\beta$, and the same difference for the foreign country, i.e., $g^* - \widehat{g}^*$, where $g^* = Qg^*_\gamma + (1-Q)\, g^*_\beta$ and $\widehat{g}^* = Q\widehat{g}^*_\gamma + (1-Q)\, \widehat{g}^*_\beta$, against $Q$. As can be seen, such differences are always positive showing that average government spending is higher when voting takes place before trading than when voting takes place after trading *in both countries* which in turn highlights the possibility that voting before trading may lead to a general tendency to increase public spending.

# 5 Insurance and centralized vs. decentralized fiscal policy

In this section we study the equilibrium outcomes when the two countries join to form a fiscal union. In this case a natural restriction is that both taxing and spending have to be equal across countries. For this reason, it no longer makes sense to assume that fiscal policy can be made contingent on the aggregate state of nature, as in this case aggregate wealth is constant across states of nature.

As will be seen in the following subsection, the main implication of the fiscal union is that since tax rates are constrained to be equal across countries and across states of nature there is no possibility to manipulate equilibrium prices of contingent wealth. On the other hand the higher dispersion of endowment that can arise in the fiscal union when the countries are not ex-ante identical may have the implication that higher spending levels will be chosen by the median voter and will therefore arise in equilibrium.

The next subsection characterizes the equilibrium for the fiscal union and the following subsection uses an example to compare the equilibrium in the fiscal union with the equilibrium in the decentralized case when voting takes place before trading.

## 5.1 Equilibrium in the fiscal union

Consider the case in which the two countries form a fiscal union whose constitution prescribes that fiscal policy has to be chosen by majority, that tax rates cannot be made contingent on the aggregate state of nature, and that both tax rates and spending levels have to be equal across countries. We will consider the case in which the tax base is realized wealth gross of contingent wealth trades; this assumption has the implication that the tax base is exogenously given and is equal to $\overline{y}$ in both states of nature. The tax rate preferred by an individual with reference wealth $y$ living in the home country is given by the solution of the following problem:

$$\max_{\tau} Q\left[H\left(y(1+\varepsilon)(1-\tau)+\widetilde{x}_{\gamma}(y,\tau)\right)+V(\tau\overline{y})\right]+(1-Q)\left[H\left(y(1-\varepsilon)(1-\tau^{C})+\widetilde{x}_{\beta}(y,\tau)\right)+V(\tau\overline{y})\right]$$

Since in equilibrium $\widetilde{x}_{\gamma}(y,\tau)=-\widetilde{x}_{\gamma}^{*}(y,\tau)=-2\varepsilon(1-Q)y(1-\tau)$ we have $\widetilde{c}_{\gamma}(y,\tau)=\widetilde{c}_{\beta}(y,\tau)=\widetilde{c}(y,\tau)=y(1+\varepsilon(2Q-1))(1-\tau)$ and this implies that the above maximization problem can be rewritten as

$$\max_{\tau} H(\widetilde{c}(y,\tau))+V(\tau\overline{y})$$

so that the preferred tax rate of an individual with reference wealth $y$ living in the home country is given by the following first order condition

$$\frac{(1+\varepsilon(2Q-1))\,H'(\widetilde{c}(y,\tau))}{V'(\tau\overline{y})}=\frac{\overline{y}}{y}$$

A similar argument shows that the preferred tax rate of an individual with reference wealth $y$ living in the foreign country is given by the following first order condition

$$\frac{(1-\varepsilon(2Q-1))\,H'(\widetilde{c}^{*}(y,\tau))}{V'(\tau\overline{y})}=\frac{\overline{y}}{y}$$

Now let $\Gamma(\tau,Q)$ be the fraction of individuals in the home country whose preferred tax rate is less than or equal to $\tau$ and let $\Gamma^{*}(\tau,Q)$ be the corresponding fraction of individuals in the foreign country. Given this the equilibrium tax rate, $\tau$, will be given by

$$\Gamma(\tau,Q)+\Gamma^{*}(\tau,Q)=1.$$

## 5.2 Centralized vs. decentralized fiscal policy

Given the results of the previous subsection we are now in a position to compare spending levels in a centralized system and in a decentralized system when voting takes place before trading. We will consider the same functional forms for utility functions and for the distribution of reference income as in section 4.

When $Q=1/2$ the equilibrium in a fiscal union coincides with the decentralized case in which voting takes place after trading, i.e., with the case in which no manipulation of equilibrium prices is possible. Expenditure can be expected to increase with ex-ante inequality, i.e., with $Q$, as the increasing difference between the median voter's expected income and expected income implies a higher level of redistribution through public spending.

From above we know that with decentralized fiscal policy, when $Q=1/2$ the additional expected spending is highest and that it converges to 0 as $Q$ tends to 1 as uncertainty matters less and less and contingent wealth equilibrium prices manipulation becomes less and less important.

These effects are summarized in Figure 7 that plots expected government spending in the decentralized system and in the centralized one against $Q$.

As we claimed in the introduction while higher spending arises in the fiscal union when the two countries are ex-ante very different (high values of $Q$) lower government spending can be expected in the fiscal union when the two countries are ex-ante similar (low values of $Q$). This result is due to the following two effects

- Government spending in the fiscal union is increasing with ex-ante inequality;
- Government spending is increasing with ex-ante equality in the decentralized system as when the two countries are very similar ex ante (values of $Q$ around $1/2$), median voters have the strongest incentives to manipulate contingent prices through fiscal policy manipulation.

### 5.3 The case of ex-ante identical countries

To further evaluate the results of our comparison we now want to contrast the equilibrium outcomes in the centralized and the decentralized system when countries are ex-ante identical, i.e., when $Q = 1/2$. In this case we can actually show, even without making any specific assumptions on utility functions, that the median voters strictly prefer the centralized system to the decentralized one.

**Proposition 4** *The median voters of the two countries strictly prefer the equilibrium outcome of the fiscal union to the equilibrium outcome of the decentralized system in which voters choose state contingent tax rates, the tax base is wealth gross of contingent wealth trades and voting takes place before trading.*

**Proof:** We use a revealed preference argument. Recall that in the decentralized system, when $Q = 1/2$, the contingent wealth equilibrium price is equal to $(1 - Q)/Q = 1$ regardless of whether voting takes place before or after trading. In the latter case, however, after trading at price ratio $(1 - Q)/Q = 1$, the median voter could set the same tax rates as in the former case but chooses not to do so which implies that he prefers the equilibrium outcome when trading takes place after trading to the equilibrium outcome when voting takes place before trading. Noticing that the equilibrium outcome in the centralized system perfectly coincides with the outcome in the decentralized system when voting takes place after trading completes the argument. ■

## 6 Conclusions

We consider a two-country model in which the residents of each country can benefit from reallocating risk with residents of the other country. Even though complete contingent markets exist to trade private wealth the fact that fiscal policy voting decisions have an impact on contingent wealth prices implies that government spending will be inflated in good states and deflated in bad ones, with the following general implications:

- Prices of contingent wealth are distorted;
- Volatility of public spending increases;
- Incomplete insurance arises.

An example shows that apart from the increase in the volatility of public spending, it is also possible that average spending increases in both countries. These distortions have been shown to be stronger the more similar the two countries are in ex ante terms. This result coupled with a parallel result on spending and insurance in a fiscal union, highlighting that spending is higher the more different the two countries are in ex ante terms and that perfect insurance always results, implies that a fiscal union may be preferable to a decentralized system if the joining members are sufficiently similar, whereas a decentralized system may be preferred if they are sufficiently different.

Although throughout the paper we have concentrated on the case in which individuals' utility functions are separable in private and public consumption, similar results can be obtained for the case in which they are not separable. In this case, however, while it is possible to show that distortions occur in equilibrium it is not possible to characterize equilibrium outcomes in terms of prices and public spending unless functional forms for utility functions and for the distribution of reference income are specified. The paper concentrates on the case in which fiscal policy consists in thee public provision of goods or services. An alternative formulation, considering the case in which fiscal programs consist in cash transfers financed through general (proportional) taxation confirms the general results provided in this paper.

# 7 Figures

**Figure 1:** Contingent tax rates in local country. t_gamma = $\widehat{t}_\gamma$; t_beta = $\widehat{t}_\beta$; T_gama = $t_\gamma$ and T_beta = $t_\beta$.

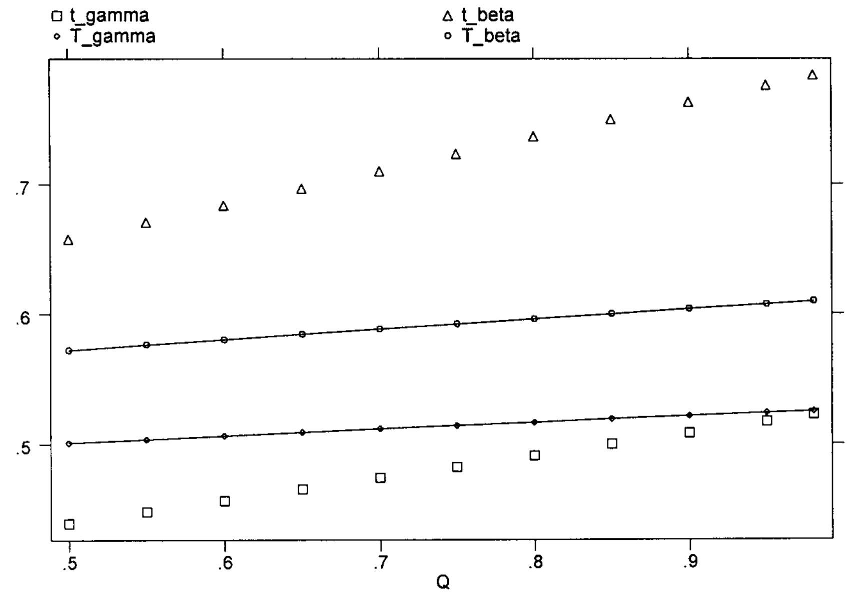



**Figure 2:** Contingent tax rates in foreign country. t_gammax = $\widehat{t}^*_\gamma$; t_betax = $\widehat{t}^*_\beta$; T_gamax = $t^*_\gamma$ and T_betax = $t^*_\beta$.

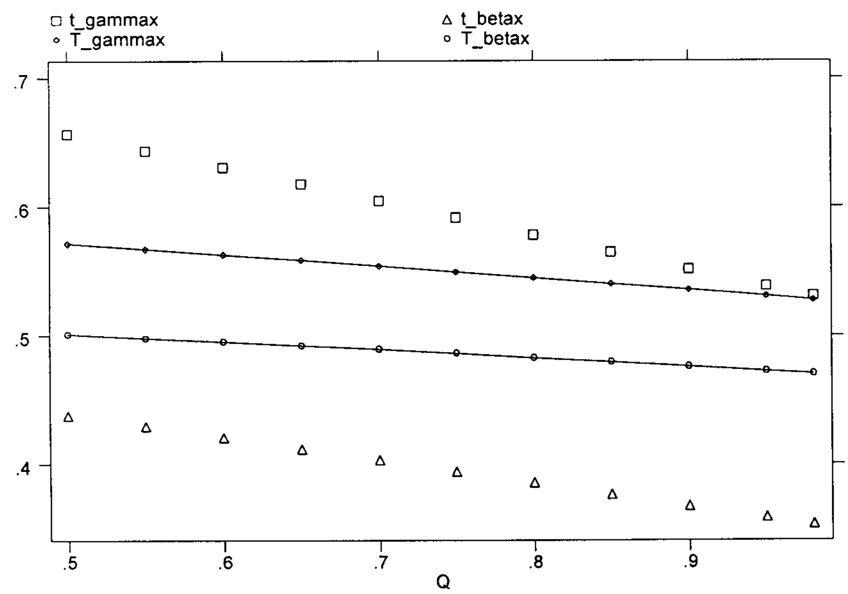


**Figure 3:** Contingent government spending in home country. g $= \widehat{g}_\gamma = \widehat{g}_\beta$; G_gama $= g_\gamma$ and G_beta $= g_\beta$.

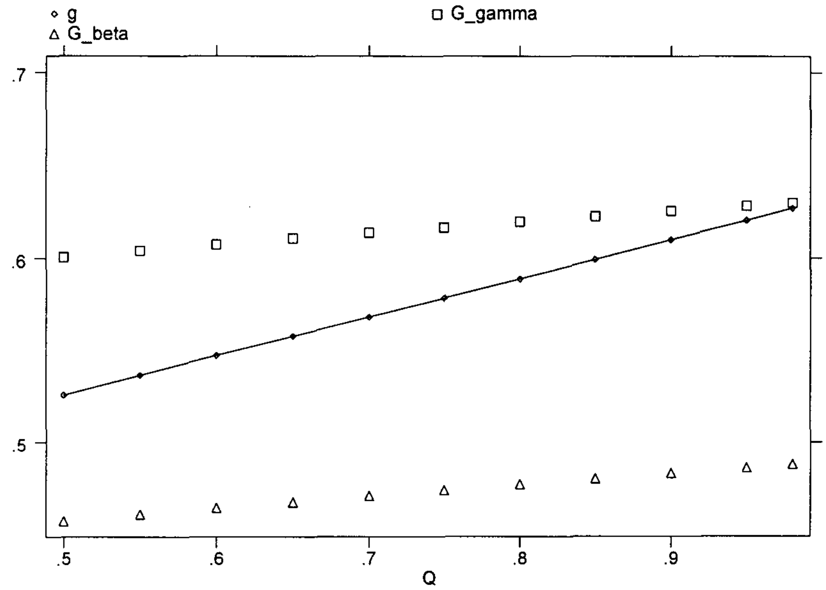



**Figure 4:** Contingent government spending in foreign country. gx $= \widehat{g}^*_\gamma = \widehat{g}^*_\beta$; G_gamax $= g^*_\gamma$ and G_betax $= g^*_\beta$.

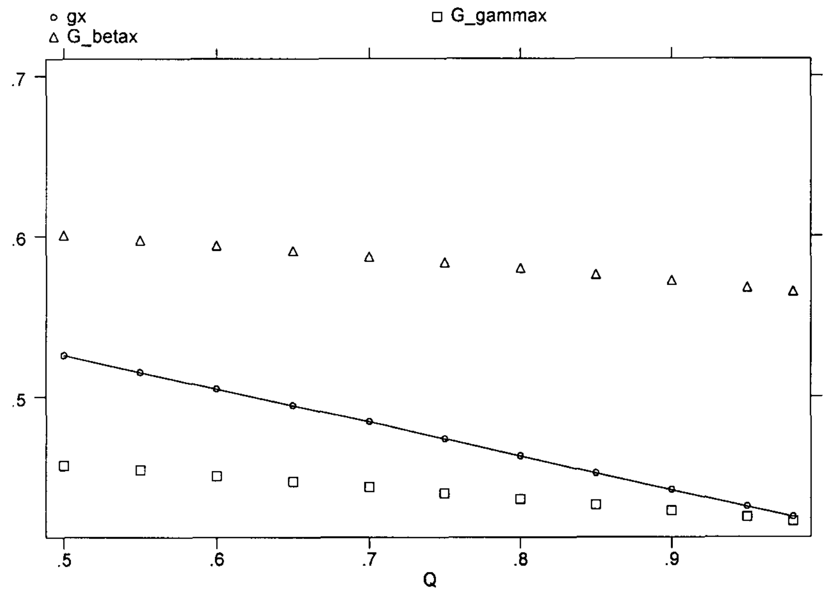


**Figure 5:** Increase in average public spending in the home country. g_G $= g - \widehat{g}$.

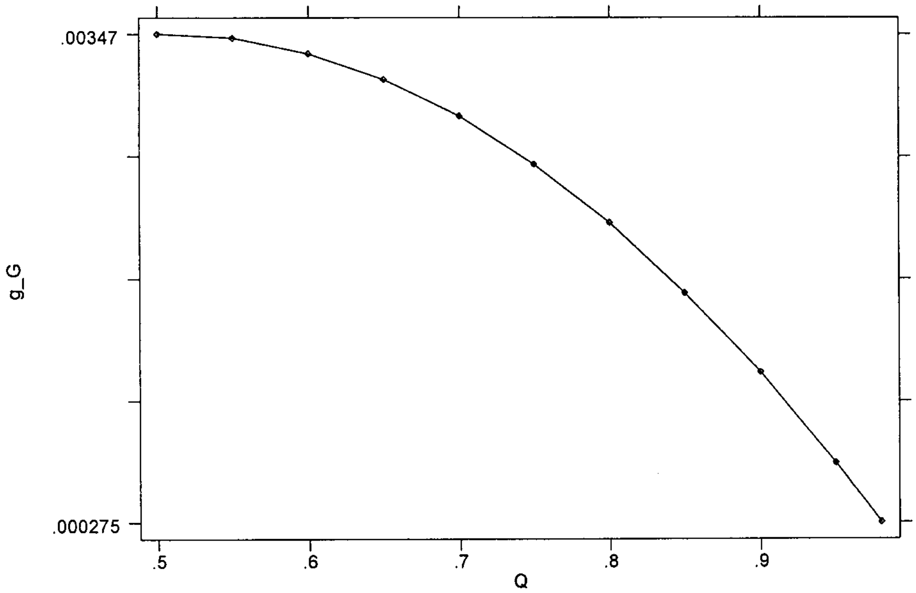



**Figure 6:** Increase in average public spending in the foreign country. gx_Gx $= g^* - \widehat{g^*}$.

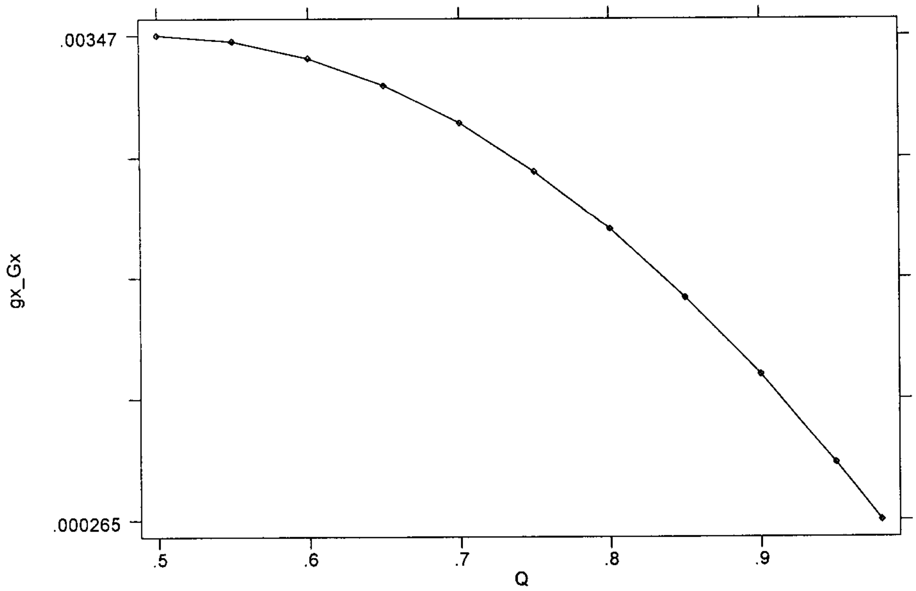
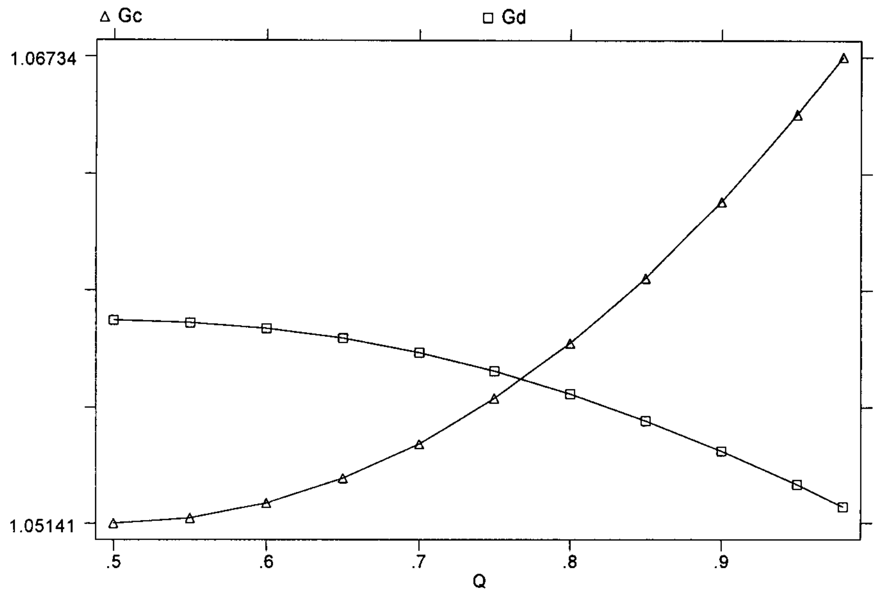


**Figure 7:** Expected government expenditure: Centralized vs. decentralized.



# 8 Appendix: Taxing wealth net of contingent trades

As we remarked in the text, the reason we concentrated on the case in which the tax base is realized wealth gross of contingent trades is its tractability. The main complication in dealing with the case in which the tax base is realized wealth net of contingent trades is that the tax base is no longer exogenous but is determined in equilibrium and therefore ultimately depends on the (state contingent) tax rates. This additional effect complicates the analytics of the problem and has the implication that while it is possible to show that in equilibrium state contingent tax rates differ from the ones that would be set in the case in which voting takes place after trading, it is not possible to rank government spending in the two states of nature.

## 8.1 Competitive equilibrium in contingent wealth markets

Equilibrium in contingent wealth markets is determined by individual optimality and market clearing conditions given a pair of state contingent tax rates for each country. The maximization for individual $i$ with reference wealth $y$ living in the home country is therefore:

$$\begin{aligned}
&\max_{c_\gamma, c_\beta, x_\gamma, x_\beta} \quad Q\left[H(c_\gamma)+V(g_\gamma)\right]+(1-Q)\left[H(c_\beta)+V(g_\beta)\right] \\
\text{s.t.} \quad & c_\gamma \le \left(y(1+\varepsilon)+x_\gamma\right)(1-t_\gamma) \\
& c_\beta \le \left(y(1-\varepsilon)+x_\beta\right)(1-t_\beta) \\
& x_\gamma + P x_\beta \le 0
\end{aligned}$$

and the solution $(c_\gamma(t_\gamma,t_\beta,y,P), c_\beta(t_\gamma,t_\beta,y,P), x_\gamma(t_\gamma,t_\beta,y,P), x_\beta(t_\gamma,t_\beta,y,P))$ will be characterized by the following conditions:

$$\begin{aligned}
\frac{H'(c_\beta)}{H'(c_\gamma)} &= \frac{PQ(1-t_\gamma)}{(1-Q)(1-t_\beta)} \\
c_\gamma &\le \left(y(1+\varepsilon)+x_\gamma\right)(1-t_\gamma) \\
c_\beta &\le \left(y(1-\varepsilon)+x_\beta\right)(1-t_\beta) \\
0 &\ge x_\gamma + P x_\beta .
\end{aligned}$$

Similarly, the conditions for $\left(c_\gamma^*\left(t_\gamma^*,t_\beta^*,y,P\right), c_\beta^*\left(t_\gamma^*,t_\beta^*,y,P\right), x_\gamma^*\left(t_\gamma^*,t_\beta^*,y,P\right), x_\beta^*\left(t_\gamma^*,t_\beta^*,y,P\right)\right)$ to be an optimum for an individual with reference wealth $y$ living in the foreign country are:

$$\begin{aligned}
\frac{H'(c_\beta^*)}{H'(c_\gamma^*)} &= \frac{PQ(1-t_\gamma^*)}{(1-Q)(1-t_\beta^*)} \\
c_\gamma^* &\le \left(y(1-\varepsilon)+x_\beta\right)(1-t_\beta) \\
c_\beta^* &\le \left(y(1+\varepsilon)+x_\gamma\right)(1-t_\gamma) \\
0 &\ge x_\gamma^* + P x_\beta^* .
\end{aligned}$$

Combining individual optimality with market clearing conditions we get the following equilibrium conditions.

$$z_\gamma(P) = \int x_\gamma(t_\gamma, t_\beta, y, P)\, dG(y) + \int x_\gamma^*(t_\gamma^*, t_\beta^*, y, P)\, dG(y) \le 0$$
$$z_\beta(P) = \int x_\beta(t_\gamma, t_\beta, y, P)\, dG(y) + \int x_\beta^*(t_\gamma^*, t_\beta^*, y, P)\, dG(y) \le 0$$

As before we will denote by $x_s$, $x_s^*$ the excess demand of wealth in state $s \in \{\gamma, \beta\}$ of an individual living respectively in the home and the foreign country, whereas we will let $\widetilde{x}_s$, $\widetilde{x}_s^*$ denote the corresponding equilibrium excess demands.

## 8.2 Fiscal policy and insurance

Let $\widetilde{x}_s\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*, y\right)$ be the equilibrium excess demand for wealth in state $s$ of an individual with reference income $y$ living in the home country, given a vector of state contingent tax rates $\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right)$. Let $T_s\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right)$ denote the equilibrium tax base in state $s$ in the home country, given the vector of state contingent tax rates $\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right)$ :

$$T_\gamma\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right) = \int \left[y(1+\varepsilon) + \widetilde{x}_\gamma\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*, y\right)\right] dG(y)$$
$$T_\beta\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right) = \int \left[y(1-\varepsilon) + \widetilde{x}_\beta\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*, y\right)\right] dG(y)$$

As was remarked above $T_\gamma$ and $T_\beta$ are a function of the tax rates. In the setting we are describing the preferred state contingent tax rates of an individual with reference income $y$ living in the home country are given by the following problem:

$$\max_{t_\gamma^C, t_\beta^C} \quad Q\left\{H\left((y(1+\varepsilon)+x_\gamma)(1-t_\gamma)\right) + V\left(T_\gamma\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right) t_\gamma\right)\right\}$$
$$+(1-Q)\left\{H\left((y(1-\varepsilon)+x_\beta)(1-t_\beta)\right) + V\left(T_\beta\left(t_\gamma, t_\beta, t_\gamma^*, t_\beta^*\right) t_\beta\right)\right\}$$

### 8.2.1 Voting after trading

If voting takes place once individuals have already traded contingent wealth, neither equilibrium trades nor tax bases can be affected by the voting outcome. Let $\widetilde{x}_s(y)$ and $T_s$ denote respectively the equilibrium excess demand for wealth in state $s$ of an individual with reference income $y$ living in the home country and the tax base in state $s$ in the home country. Since the median voter is the individual with the median reference wealth, the state contingent tax rates will be determined by the following problem

$$H'\left((y^m(1+\varepsilon)+\widetilde{x}_\gamma)(1-t_\beta)\right)(y^m(1+\varepsilon)+\widetilde{x}_\gamma) - V'(T_\gamma t_\gamma)T_\gamma = 0$$

$$H'\left((y^m(1-\varepsilon)+\widetilde{x}_\beta)(1-t_\beta)\right)(y^m(1-\varepsilon)+\widetilde{x}_\beta) - V'(T_\beta t_\beta)T_\beta = 0$$

Similar conditions characterize the problem for the median voter of the foreign country.

**Proposition 5** *When voters choose state contingent tax rates, the tax base is wealth net of contingent wealth trades and voting takes place after trading in equilibrium*

1. $\widetilde{P} = (1-Q)/Q$

2. $\widetilde{c}_\gamma(y) = c_\gamma\left(y, \widetilde{P}\right) = \widetilde{c}_\beta(y) = c_\beta\left(y, \widetilde{P}\right)$ *for all* $y$ *and*
   $\widetilde{c}^*_\gamma(y) = c^*_\gamma\left(y, \widetilde{P}\right) = \widetilde{c}^*_\beta(y) = c^*_\beta\left(y, \widetilde{P}\right)$ *for all* $y$

3. $\widehat{t}_\gamma = \widehat{t}_\beta$ *and* $\widehat{t}^*_\gamma = \widehat{t}^*_\beta$

4. $\widehat{g}_\gamma = \widehat{g}_\beta$ *and* $\widehat{g}^*_\gamma = \widehat{g}^*_\beta$

**Proof:** Suppose in equilibrium $P = \frac{(1-Q)(1-t_\beta)}{Q(1-t_\gamma)} = \frac{(1-Q)(1-t^*_\beta)}{Q(1-t^*_\gamma)}$. Given $P = \frac{(1-Q)(1-t_\beta)}{Q(1-t_\gamma)}$ we have $c_\gamma(y, P) = c_\beta(y, P)$ for all $y$ and this implies that from the first order condition of the median voter maximization problem we have

$$\frac{V'\left(T_\gamma \widehat{t}_\gamma\right) T_\gamma}{(y^m(1+\varepsilon) + x_\gamma)} = \frac{V'\left(T_\beta \widehat{t}_\beta\right) T_\beta}{(y^m(1-\varepsilon) + x_\beta)}$$

and since $T_\gamma = T_\beta$ this implies $\widehat{t}_\gamma = \widehat{t}_\beta$. A similar argument shows that if $P = \frac{(1-Q)(1-t^*_\beta)}{Q(1-t^*_\gamma)}$ then $\widehat{t}^*_\gamma = \widehat{t}^*_\beta$ and the assumption that $P = \frac{(1-Q)(1-t_\beta)}{Q(1-t_\gamma)} = \frac{(1-Q)(1-t^*_\beta)}{Q(1-t^*_\gamma)} = \frac{1-Q}{Q}$ is satisfied. Part 4 is a straightforward consequence of the previous parts. ■

Comparing Propositions 1 and 5 it is easy to see that, when voting takes place after trading, the only difference between the case in which tax base is gross wealth and the case in which it is net wealth is that given in equilibrium in the latter the tax base is constant across states so are tax rates, while in the former since tax base is not constant across states, tax rates vary across states to raise tax revenue which is constant across states.

#### 8.2.2 Voting before trading

When voting takes place before trading, the median voter's maximization problem also takes into account two additional effects

1. The state contingent tax rates have an impact on contingent wealth equilibrium prices

2. The state contingent tax rates have an impact on equilibrium tax base.

As a consequence differentiating the median voter's objective function, one also has to keep into account the fact that both $\widetilde{x}_s\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, y\right)$ and $T_s\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta\right)$ depend on $t_\gamma$ and $t_\beta$. In what follows we will nevertheless let this dependence be implicit in the sense that we will write $\widetilde{x}_s(y)$ and $T_s$ for notational convenience. With this notational convention the first order condition of the home country median voter's maximization problem turn out to be:

$$
\begin{aligned}
&Q\left\{-H'\left(\left(y^m(1+\varepsilon)+\widetilde{x}_\gamma\left(y^m\right)\right)\left(1-t_\gamma\right)\right)\left(y^m(1+\varepsilon)+\widetilde{x}_\gamma\left(y^m\right)\right)+V'\left(\widetilde{T}_\gamma t_\gamma\right)\widetilde{T}_\gamma\right\}+\\
&+QH'\left(\widetilde{c}_\gamma\right)\frac{\partial\widetilde{x}_\gamma\left(y^m\right)}{\partial t_\gamma}(1-t_\gamma)+(1-Q)H'\left(\widetilde{c}_\beta\right)\frac{\partial\widetilde{x}_\beta\left(y^m\right)}{\partial t_\gamma}(1-t_\beta)+\\
&QV'\left(\widetilde{T}_\gamma t_\gamma^C\right)t_\gamma\int\frac{\partial\widetilde{x}_\gamma(y)}{\partial t_\gamma}dG(y)+(1-Q)V'\left(\widetilde{T}_\beta t_\beta\right)t_\beta\int\frac{\partial\widetilde{x}_\beta(y)}{\partial t_\gamma}dG(y)=0\\
&(1-Q)\left\{-H'\left(\left(y^m(1-\varepsilon)+\widetilde{x}_\beta\left(y^m\right)\right)\left(1-t_\beta\right)\right)\left(y^m(1-\varepsilon)+\widetilde{x}_\beta\left(y^m\right)\right)+V'\left(\widetilde{T}_\beta t_\beta\right)\widetilde{T}_\beta\right\}+\\
&+QH'\left(\widetilde{c}_\gamma\right)\frac{\partial\widetilde{x}_\gamma\left(y^m\right)}{\partial t_\beta}(1-t_\gamma)+(1-Q)H'\left(\widetilde{c}_\beta\right)\frac{\partial\widetilde{x}_\beta\left(y^m\right)}{\partial t_\beta}(1-t_\beta)\\
&QV'\left(\widetilde{T}_\gamma t_\gamma\right)t_\gamma\int\frac{\partial\widetilde{x}_\gamma(y)}{\partial t_\beta}dG(y)+(1-Q)V'\left(\widetilde{T}_\beta t_\beta\right)t_\beta\int\frac{\partial\widetilde{x}_\beta(y)}{\partial t_\beta}dG(y)=0
\end{aligned}
$$

and they can be shown to be equivalent to:

$$
\begin{aligned}
&Q\left\{-H'\left(\widetilde{c}_\gamma\left(y^m\right)\right)\left(y^m(1-\varepsilon)+\widetilde{x}_\beta\left(y^m\right)\right)+V'\left(T_\gamma t_\gamma\right)T_\gamma\right\}+\frac{(1-Q)H'\left(\widetilde{c}_\beta\left(y^m\right)\right)(1-t_\beta)\widetilde{x}_\gamma\left(y^m\right)}{P^2}\frac{\partial\widetilde{P}(t)}{\partial t_\gamma}+\\
&\left[QV'\left(T_\gamma t_\gamma\right)t_\gamma-\frac{(1-Q)}{P}V'\left(T_\beta t_\beta\right)t_\beta\right]\int\frac{\partial\widetilde{x}_\gamma(y)}{\partial t_\gamma}dG(y)+\frac{(1-Q)V'\left(T_\beta t_\beta\right)t_\beta}{P^2}\int\frac{\partial P(t)}{\partial t_\gamma}\widetilde{x}_\gamma(y)\,dG(y)=0
\end{aligned}
$$

$$
\begin{aligned}
&(1-Q)\left\{-H'\left(\widetilde{c}_\beta\left(y^m\right)\right)\left(y^m(1-\varepsilon)+\widetilde{x}_\beta\left(y^m\right)\right)+V'\left(T_\beta t_\gamma\right)T_\beta\right\}+\frac{(1-Q)H'\left(\widetilde{c}_\beta\left(y^m\right)\right)(1-t_\beta^C)\widetilde{x}_\gamma\left(y^m\right)}{P^2}\frac{\partial\widetilde{P}(t)}{\partial t_\beta}+\\
&\left[QV'\left(T_\gamma t_\gamma\right)t_\gamma-\frac{(1-Q)}{P}V'\left(T_\beta t_\beta\right)t_\beta\right]\int\frac{\partial\widetilde{x}_\gamma(y)}{\partial t_\beta}dG(y)+\frac{(1-Q)V'\left(T_\beta t_\beta\right)t_\beta}{P^2}\int\frac{\partial P(t)}{\partial t_\beta}\widetilde{x}_\gamma(y)\,dG(y)=0
\end{aligned}
$$

The following Lemma establishes results that will be used in the following Propositions:

**Lemma 2** *When voters choose state contingent tax rates, the tax base is wealth net of contingent wealth trades and voting takes place before trading, then*

1. $sign\frac{\partial\widetilde{P}(t)}{\partial t_\gamma}\begin{cases}=0 & \textit{if } r_R(x,H(.))=1 \textit{ for all } x\\ >0 & \textit{if } r_R(x,H(.))<1 \textit{ for all } x\\ <0 & \textit{if } r_R(x,H(.))>1 \textit{ for all } x\end{cases}$

2. $sign\frac{\partial\widetilde{P}(t)}{\partial t_\beta}\begin{cases}=0 & \textit{if } r_R(x,H(.))=1 \textit{ for all } x\\ <0 & \textit{if } r_R(x,H(.))<1 \textit{ for all } x\\ >0 & \textit{if } r_R(x,H(.))>1 \textit{ for all } x\end{cases}$

3. $sign\frac{\partial\widetilde{P}(t)}{\partial t_\gamma^*}\begin{cases}=0 & \textit{if } r_R(x,H(.))=1 \textit{ for all } x\\ <0 & \textit{if } r_R(x,H(.))<1 \textit{ for all } x\\ >0 & \textit{if } r_R(x,H(.))>1 \textit{ for all } x\end{cases}$

4. $sign\frac{\partial\widetilde{P}(t)}{\partial t_\beta^*}\begin{cases}=0 & \textit{if } r_R(x,H(.))=1 \textit{ for all } x\\ >0 & \textit{if } r_R(x,H(.))<1 \textit{ for all } x\\ <0 & \textit{if } r_R(x,H(.))>1 \textit{ for all } x\end{cases}$

**Proof:** Part 1: Let $z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)$ denote the aggregate excess demand function for wealth in state $\gamma$ and totally differentiate the equilibrium condition with respect to $P$ and $t_\gamma$

$$z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right) = \int \left[x_\gamma\left(t_\gamma, t_\beta, P, y\right) + x^*_\gamma\left(t^*_\gamma, t^*_\beta, P, y\right)\right] dG(y) = 0$$

which implies that

$$\frac{\partial P\left(t_\beta, t_\gamma, t^*_\beta, t^*_\gamma\right)}{\partial t_\gamma} = -\frac{\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\gamma}}{\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial P}}$$

$$\frac{\partial z_\gamma\left(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P\right)}{\partial t_\gamma} = \int \left[\frac{\partial x_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\gamma} + \frac{\partial x^*_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\gamma}\right] dG(y)$$

However $\frac{\partial x^*_\gamma(t_\gamma, t_\beta, P, y)}{\partial t_\gamma} = 0$ for all $y$ and

$$\frac{\partial x_\gamma\left(t_\gamma, t_\beta, P, y\right)}{\partial t_\gamma} = \frac{QP\left(H'(\widetilde{c}_\gamma(y)) + \widetilde{c}_\gamma(y) H''(\widetilde{c}_\gamma(y))\right)}{H''(\widetilde{c}_\beta(y))\left(\frac{(1-t_\beta)^2(1-Q)}{P}\right) + H''(\widetilde{c}_\gamma(y))(1-t_\gamma)Q}$$

Since the denominator is always negative, this derivative will be positive if and only if

$$H'(\widetilde{c}_\gamma(y)) + \widetilde{c}_\gamma(y) H''(\widetilde{c}_\gamma(y)) < 0$$

and this condition is equivalent to

$$r_R(x, H(.)) = -\frac{\widetilde{c}_\gamma(y) H''(\widetilde{c}_\gamma(y))}{H'(\widetilde{c}_\gamma(y))} > 1$$

where $r_R(x, H(.))$ is the coefficient of relative risk aversion.

If contingent consumption is a normal good $\frac{\partial z_\gamma(t_\gamma, t_\beta, t^*_\gamma, t^*_\beta, P)}{\partial P} \geq 0$ , and the sign of $\frac{\partial \widetilde{P}(t)}{\partial t_\gamma}$ will be the opposite of the sign of $\frac{\partial x_\gamma(t_\gamma, t_\beta, P, y)}{\partial t_\gamma}$ for all $y$ and therefore we have

$$sign\frac{\partial \widetilde{P}(t)}{\partial t_\gamma}\begin{cases} = 0 & \text{if } r_R(x, H(.)) = 1 \text{ for all } x \\ > 0 & \text{if } r_R(x, H(.)) < 1 \text{ for all } x \\ < 0 & \text{if } r_R(x, H(.)) > 1 \text{ for all } x \end{cases}$$

and Part 1 follows. Since $sign\left[\frac{\partial \widetilde{x}_\gamma(t_\gamma, t_\beta, P, y)}{\partial t_\beta}\right] = -sign\left[\frac{\partial \widetilde{x}_\gamma(t_\gamma, t_\beta, P, y)}{\partial t_\gamma}\right]$ we have $sign\left[\frac{\partial \widetilde{P}(t)}{\partial t_\beta}\right] = -sign\left[\frac{\partial \widetilde{P}(t)}{\partial t_\gamma}\right]$ and Part 2 follows.

The same argument can be applied to the foreign country to obtain the symmetric results of Parts 3 and 4. ■

We are now in the position to state the following result

**Proposition 6** *When voters choose state contingent tax rates, the tax base is wealth net of contingent wealth trades, voting takes place before trading, and $r_R(x, H(.)) = 1$ for all $x$, in equilibrium*

1. $\widetilde{P} = (1-Q)/Q$

2. $\widetilde{c}_\gamma(y) = c_\gamma\left(y, \widetilde{P}\right) = \widetilde{c}_\beta(y) = c_\beta\left(y, \widetilde{P}\right)$ *for all* $y$ *and*

$\widetilde{c}^*_\gamma(y) = c^*_\gamma\left(y, \widetilde{P}\right) = \widetilde{c}^*_\beta(y) = c^*_\beta\left(y, \widetilde{P}\right)$ *for all* $y$

3. $\widehat{t}_\gamma = \widehat{t}_\beta$ *and* $\widehat{t}^*_\gamma = \widehat{t}^*_\beta$

4. $\widehat{g}_\gamma = \widehat{g}_\beta$ *and* $\widehat{g}^*_\gamma = \widehat{g}^*_\beta$

**Proof:** From Lemma 2 when $r_R(x, H(.)) = 1$ for all $x$ then $\frac{\partial \widetilde{P}(t)}{\partial t_s} = \frac{\partial \widetilde{P}(t)}{\partial t^*_s} = 0$ for $s \in \{\gamma, \beta\}$, and the first order condition of median voters' maximization problems when voting takes place before trading coincides with the first order condition of median voters' maximization problems when voting takes place after trading. ■

The following proposition shows that when the tax base is wealth net of contingent wealth trades, voting takes place before trading, and $r_R(x, H(.))$ is either greater than or smaller than 1 for all $x$ , the equilibrium cannot coincide with the equilibrium when voting takes place after trading.

**Proposition 7** *When voters choose state contingent tax rates, the tax base is wealth net of contingent wealth trades, voting takes place before trading, if either* $r_R(x, H(.)) < 1$ *for all* $x$*, or* $r_R(x, H(.)) > 1$ *for all* $x$*, then* $t_\gamma = \widehat{t}_\gamma$, $t^*_\beta = \widehat{t}^*_\beta$, $t_\beta = \widehat{t}_\beta$, $t^*_\gamma = \widehat{t}^*_\gamma$ *cannot all hold in equilibrium.*

**Proof:** Contrary to the statement, suppose that $t_\gamma = \widehat{t}_\gamma$, $t^*_\beta = \widehat{t}^*_\beta$, $t_\beta = \widehat{t}_\beta$, $t^*_\gamma = \widehat{t}^*_\gamma$ and evaluate the first order condition of the home country median voter's maximization problem when voting takes place before trading at $\widehat{t}_\gamma$. The left hand side of the home country median voter's first order condition with respect to $t_\gamma$ is equal to

$$\frac{(1-Q)H'\left[\widetilde{c}_\beta\right](1-t_\beta)\widetilde{x}_\gamma}{P^2}\frac{\partial \widetilde{P}(t)}{\partial t_\gamma} + (1-Q)V'\left[T_\beta t_\beta\right]t_\beta \int \frac{\partial P(t)}{\partial t_\gamma}\widetilde{x}_\gamma(y)\, dG(y_i) > 0$$

and is therefore strictly positive if $r_R(x, H(.)) < 1$ for all $x$ and strictly negative if $r_R(x, H(.)) > 1$ for all $x$, thus leading to a contradiction. ■

As is clear the last proposition can only give an idea of the inefficiencies in fiscal policy and both private and public consumption smoothing that arise when voting takes place before trading and the tax base is wealth net of contingent wealth trades. The additional effect of tax rates on equilibrium tax bases makes it impossible to give a more precise characterization of the result unless some additional assumptions on preferences are introduced.

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