Study of Localized Solutions of the Nonlinear Discrete Model for Dipolar Bose-Einstein Condensate in an Optical Lattice by the Homoclinic Orbit Method

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Introduction. It is well known that the dynamics of the Bose Einstein condensate (BEC) trapped in an optical lattice can be described in tight-binding approximation by the discrete Nonlinear Schrödinger Equation (DNLSE) [1,2]. This model opens the way to study different aspects of the BEC dynamics in an optical lattice, such as discrete solitons and nonlinear localized modes and their stability and dynamics, modulational instability, superfluid-insulator transition, etc.[3]. Also recently the nonlinearity caused by long range dipole-dipole interaction attracts much interest. In this case the corresponding discrete model for dipolar BEC in an optical lattice was introduced in the work [4], and further investigated in [5]. The equation for one dimensional optical lattice case written in the dimensionless variables has the following form [6]

$$i\frac{d}{dx}\Phi_n + k(\Phi_{n+1} + \Phi_{n-1}) + q|\Phi_n|^2\Phi_n + g(|\Phi_{n+1}|^2 + |\Phi_{n-1}|^2)\Phi_n = 0. \quad (1)$$

In the paper [6] the nonlinear localized modes of Eq.(1) has been investigated in details by application of numerical Newton-Raphson method.

In this work we use homoclinic orbits to find the localized solutions of Eq.(1). The equation of motion is transformed to a two-dimensional map and the homoclinic orbits for this map are computed numerically. Each homoclinic orbit leads to a different solution. Also the linear stability of the found solutions is investigated.

References

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